RECORD OF SOCIETY OF ACTUARIES
1976 VOL. 2 NO. 1

FORECASTING TOOLS FOR THE WORKING ACTUARY

Teaching Session

JAMES C. HICKMAN

1. Economic forecasting and actuarial science
   (a) Interest rates and life insurance and annuity prices
   (b) Inflation and loss trends in property insurance

2. Fundamental limitations on economic forecasting and the importance of error variance estimates. These include inadequate basic data and errors in reporting.

3. The broad division between qualitative and quantitative forecasting methods.

4. Specific quantitative models
   (a) Time series models are easy to use and provide good short-term forecasts
   (b) Econometric models are more elaborate, are useful for economic planners, but sometimes are not robust.

Professor Hickman's session discussed both qualitative and quantitative models for forecasting. An outline of the session and bibliography is shown below.

I. Introduction

A. This session is not for experts. It will use ideas from the material covered in part 2 of the examinations of the Society of Actuaries.

B. The topic is the empirical part of economics [6]. Mathematics also has a role in the development of economic theory.

C. Traditional actuarial models require actuaries to make an informed assumption about the interest rate. Recent events, and the broader scope of actuarial practice, have combined to force actuaries to acquire the tools of economic analysis.

1. Life actuaries are concerned with future interest rates, expenses, and the variability of these cost determinants.
2. Pension actuaries must cope with inflation, interest, and wage rate changes.
3. Health actuaries must model claims costs.

II. Economic Data

A. In Canada and the United States we have excellent national statistics. Yet, there are fundamental problems in collecting economic statistics [8].

1. Lack of planned experiments
2. Hiding of information
3. Lack of trained observers
4. Errors from questionnaires and nonresponse

B. Even within a firm the measurement of economic variables is difficult. Financial accounting is devoted to measuring how much better off a firm is from one time to another, but what does it mean to be "better off" [10]?

C. Those who use economic statistics must strive to understand the sources of their data. In the United States, many social and economic statistics are derived from the Current Population Survey where error variances are intensely studied [4].

III. Qualitative and Quantitative Models [3]

A. Models are a necessity for rational thought.

B. Qualitative methods may involve structured techniques for bringing experts to a consensus (Delphi technique [5]). Historical analogy is a qualitative method for forecasting.

C. Quantitative methods may be theory centered or data centered.

1. Theory centered models involve specification (selection and definition of variables), data collection, estimation, and testing [6].
2. Data centered models involve tentative identification of a model, estimation of parameters, and checking the adequacy of the model [1].

D. Almost all quantitative economic models involve "regression analysis." That is, they are of the form $y = f(\underline{x}) + a$ where $\underline{x}$ is a vector of controllable or observable variables, a is a random shock term, and y is the response variable [9].

IV. Examples

A. Univariate time series, automobile property damage paid claim cost (1954-1970) [7].

1. Tentatively identify a model for the series by examining the sample autocorrelation function and compare it with the autocorrelation function of standard models. Keep in mind the dictum to economize on parameters.

2. Using the tentatively identified model, estimate the parameters using a non-linear least squares computer program.

3. Check the adequacy of the model by examining the residuals, $\hat{a}_t = Z_t - \hat{Z}_t$, where $\hat{Z}$ are the values generated by the fitted model. The residuals should be approximately independent and identically distributed.

4. Example. Insurance claims index data.
Model: $Z_t - 2Z_{t-1} + Z_{t-2} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$
Residuals: The residuals depend on the value of (θ_1, θ_2),
$a_t(\theta_1,\theta_2) = (Z_t - 2Z_{t-1} + Z_{t-2}) + \theta_1 a_{t-1}(\theta_1,\theta_2) + \theta_2\, a_{t-2}(\theta_1,\theta_2)$.
Estimation: Arbitrarily setting $a_1(\theta_1,\theta_2) = a_2(\theta_1,\theta_2) = 0$, we adopt the least squares principle and seek the value of (θ_1,θ_2) that will minimize $\sum_{t=3}^{n} a_t^2\ (\theta_1,\theta_2)$.

This yields approximate least squares estimates $\hat{\theta}_1 = .62$, $\hat{\theta}_2 = -.28$, and $\hat{\sigma} = .007$.

Testing: Assuming that the parameters of the model are those given by the estimation step, we may test the hypothesis that the autocorrelations among the residuals are zero. In this example, the hypothesis is accepted and the model building process is complete.

5. An example using a model selected to fit an insurance claims index series in Section IV.

   Model: $Z_t - 2Z_{t-1} + Z_{t-2} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$,

   that is, the model assumes that the second differences follow a MA(2) model.

   By successive substitution, we have

   $$Z_{n+\ell} - 2Z_{n+\ell-1} + Z_{n+\ell-2} = a_{n+\ell} - \theta_1 a_{n+\ell-1} - \theta_2 a_{n+\ell-2}$$

   Taking expectations, with values up to Z_n known, we have

   $$\hat{Z}_n(\ell) - 2Z_n(\ell-1) + \hat{Z}_n(\ell-2) = 0, \ \ell \geq 3.$$

   The solution of the difference equation yields

   $$\hat{Z}_n(\ell) = A_0 + A_1\ell.$$

   Using the insurance claims index data, we have the forecast

   $$\hat{Z}_n(\ell) = 2.1793 + .0429\ell.$$

   $$\mathrm{Var}(Z_{n+\ell}) = (.0049) \sum_{j=0}^{\ell-1} \Psi_j^2$$

   where $\Psi_0 = 1$, $\Psi_1 = 1.380$, $\Psi_2 = 2.043$, $\Psi_3 = 2.706$.

B. A simple linear regression [2]

1. Postulate a relationship between a response variable. In this example, the interest rate on Aa utility bonds, and the dependency ratio (number under 18 plus 65 and over / number 18 to 64 inclusive).
2. Collect data, U.S. Census, Moody's Public Utility Manual 1973.
3. Fit the model, $\hat{Y} = -8.012907 + .1792026\ (DR_{T-6})$, $r^2 = .776$.
4. Test the model.

References

1. Box, G. E. P. and G. W. Jenkins. Time Series Analysis, Forecasting and Control. San Francisco, Calif.: Holden-Day, 1970.
A full scale exposition of parametric time series modeling and forecasting plus a distinctive approach to using the data to select a model.

2. Burton, J. S. and J. R. Toth. "Forecasting Secular Trends in Long-Term Interest Rates," Financial Analysis Journal, September-October 1974.
An elementary econometric analysis of a hypothesized relationship between the dependency ratio and interest rates.

3. Chambers, J. C., S. K. Mullick, and D. D. Smith. "How to Choose the Right Forecasting Technique," Harvard Business Review, July-August 1971.
A general article with an interesting overview of a wide variety of qualitative and quantitative forecasting models.

4. Employment and Earnings, U.S. Department of Labor, Bureau of Labor Statistics.
A monthly publication that is an excellent example of statistical publications of the U.S. Government. The explanatory notes are particularly important for their review of basic concepts, historic comparability, and technical estimation methods.

5. Gordon, T. J. and R. E. LeBleu. "Employee Benefits, 1970-1985," Harvard Business Review, January-February 1970.
This article is an illustration of the Delphi technique to forecast in an area of interest to actuaries.

6. Johnston, J. Econometric Methods. 2nd Edition, New York, N.Y.: McGraw Hill, 1972
A widely used first text in econometrics.

7. Miller, R. B. and J. C. Hickman. "Time Series Analysis and Forecasting," TSA, Vol. XXV (1973).
An exposition of parametric time series modeling with the complete auto damage claims index example.

8. Morganstern, Oskar. On the Accuracy of Economic Observations. Princeton, N.J.: Princeton University Press, 1963.
A wealth of common sense about the problems in collecting economic statistics by one of the most influential economists of the century.

9. Neter, J. and W. Wasserman. Applied Linear Statistical Models. Homewood, Ill.: Richard D. Irwin, Inc., 1974.
An excellent exposition of linear statistics models with business examples.

10. Objectives of Financial Statements. New York, N.Y.: American Institute of Certified Public Accountants, 1973.
This is the report of the Trueblood Commission which was assigned by the accounting profession to ask some penetrating questions about the goals of financial accounting.