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AN EMPIRICAL APPROACH TO THE DETERMINATION OF CREDIBILITY FACTORS

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ABSTRACT

This paper is concerned with the determination of credibility factors applicable to the estimation of the true claim level of a particular risk, based on both the actual observed claim experience of that risk and the expected claim level that would be predicted for the risk according to its exposure characteristics.

The theoretical nature of credibility is explored, with particular emphasis on the roles of chance fluctuation in actual claim levels and of imperfection in expected claim level determination. The inherent importance of this second element leads the author to conclude that the determination of credibility factors must be empirical in approach.

A formula for estimating credibility factors is derived. This formula requires actual claim experience data in two separate periods on each risk in various subsamples. Each subsample would consist of risks of approximately uniform size and other relevant characteristics, such that approximately the same credibility factor would be applicable to each risk within the subsample.

A simple example is offered that is intended to illustrate the concepts underlying this empirical approach to credibility determination. In addition, the results of applying the formula to a sample of 413 group life insurance risks are presented and discussed. It is pointed out that the general principles and formulas are equally applicable to other forms of insurance, particularly group accident and health and casualty-property.

CREDIBILITY theory is concerned with the weight, or credence, that should be attached to a particular body of statistical data in relation to the weight that should be assigned to prior knowledge. Of considerable importance to the actuary is the application of credibility theory to the problem of estimating the true claim level of a particular risk. This paper is addressed to the determination of the credibility factors appropriate to this application.

The standard formula for estimating the true claim level of a particular risk is as follows:

$$t \cong ka + (1 - k)e = e + k(a - e) \,, \tag{1}$$

where

t represents the true claim level of that risk,
k represents the credibility factor appropriate to the risk ($0 \leq k \leq 1$),
a represents the actual observed claim level of the risk, based on actual experience during a given period, and
e represents the expected claim level that would be predicted for the risk according to its exposure characteristics.

An examination of the alternate equality $t \cong e + k(a - e)$ reveals that the estimation of the true claim level is based on modifying the expected claim level by a portion of the past deviation of actual from expected claim levels. This modification is limited to only a part of the past deviation because random fluctuations will account on the average for at least a portion of such deviation. The reason that past deviations are considered at all is based on the well-substantiated belief that, inasmuch as the determination of expected claim levels is imperfect, the true claim level of many risks will differ substantially from the expected and on the realization that past deviations of actual from expected claim levels offer the only basis for estimating deviations of true from expected claim levels.

The foregoing paragraph merely suggests a rationale for modifying the expected claim level by some portion of the past deviation of actual from expected claim levels. However, standard practice in the application of credibility implies the following in addition:

a) The credibility factor appropriate to a particular risk depends entirely on some measure of the size of either the risk itself or the volume of its observed experience, on the degree of imperfection in expected claim level determination, and on the character of the insurance coverage. It does not depend on the deviation of the true claim level from the expected on that risk.

b) The credibility factor appropriate to a particular risk is equally applicable to all deviations of actual from expected claim levels, whether large or small, positive or negative.

The existence of a function with the above properties has not been established, to the best knowledge of the author, nor has any attempt been made in this paper to do so. However, since these properties are implied by the standard application of credibility, this paper is directed to the determination of credibility factors which, on the average, satisfy these conditions.

Consider a group of n risks with the same credibility factor applicable to each. The mean value of the true claim level for this group may be estimated from equation (1) as follows:

$$\frac{1}{n}\sum_{i=1}^{n} t_i \cong \frac{1}{n}\sum_{i=1}^{n}[e_i + k(a_i - e_i)] . \tag{2}$$

The mean deviation of the true claim level from the expected would be estimated by

$$\frac{1}{n}\sum_{i=1}^{n}(t_i - e_i) \cong \frac{1}{n} k \sum_{i=1}^{n}(a_i - e_i) . \tag{3}$$

The group of n risks may be split into two subgroups, consisting of n_1 risks with positive deviations of actual from expected claim levels and n_2 risks with negative deviations. Since we require that the same credibility factor be applicable to all deviations of actual from expected, whether positive or negative, equation (3) should hold for each subgroup separately. Accordingly, it can be shown that equation (3) may be modified to the following:

$$\frac{1}{n}\sum_{i=1}^{n}\frac{|a_i - e_i|}{a_i - e_i}(t_i - e_i) \cong \frac{1}{n} k \sum_{i=1}^{n}|a_i - e_i| . \tag{4}$$

From this equation we may develop the following formula for estimating k:

$$k \cong \frac{\sum_{i=1}^{n}[(|a_i - e_i|)/(a_i - e_i)](t_i - e_i)}{\sum_{i=1}^{n}|a_i - e_i|} . \tag{5}$$

Consideration of the following two extreme situations will demonstrate the reasonableness of equation (5) and also provide further insight into the nature of credibility.

a) If the determination of expected claim levels were completely refined, such that all characteristics affecting claim levels were perfectly evaluated, the expected claim level would be equal to the true claim level on each risk and the estimation formula would tend to produce a zero credibility factor. The use of zero credibility would be proper in this situation, since deviations of actual claim levels from the expected would be solely the result of chance fluctuation and therefore would be valueless in estimating true claim levels.

b) At the other extreme would be the situation in which there was absolutely no correlation between the expected and true claim levels. Equation (5) would tend to produce a credibility factor of 1. The use of full credibility would be proper in this situation, since the expected should be ignored entirely and full reliance placed on the actual experience.

These observations may be summarized by the statement that credibility varies according to the dispersion of true claim levels about the expected. It is interesting to note that, since the accuracy of manual rate determination in group accident and health insurance has generally been improving in recent years, with more accurate evaluation of the effects of area, wage, age, and so forth, we would expect that credibility factors for that line of insurance should have been declining. Also, any credibility factors determined from past experience would tend to be overstated for the future.

In assessing the effect of risk size on the credibility factor, the following rearrangement of equation (5) is enlightening:

$$k \cong \frac{\sum_{i=1}^{n}[(|a_i - e_i|)/(a_i - e_i)](t_i - e_i)}{\sum_{i=1}^{n}|(a_i - t_i) + (t_i - e_i)|}. \qquad (6)$$

Although the denominator is independent of t, this formula nevertheless indicates that credibility factors depend on both the dispersion of actual claim levels about the true ones and the dispersion of true claim levels about the expected. This principle must be considered in comparing credibility factors for various risk sizes. Obviously the dispersion of actual claim levels about the true ones decreases as the size increases, and this has the effect of increasing the credibility factor. However, it is reasonable to believe that the dispersion of true claim levels about the expected ones also decreases as the size increases, and this would tend to work in the opposite direction, that is, to decrease the credibility factor. Nevertheless, there should be little doubt that the net effect of these two interacting and offsetting influences is that credibility increases with increasing risk size. In any event, there is reason for skepticism regarding the assumption that credibility factors vary by risk size according to the simple relationships that have gained rather wide acceptance, such as the following formulas for k:

$$\frac{N}{N+C} \quad \text{or} \quad \frac{NC_1}{N+C_2} \quad \text{or} \quad \frac{N}{C},$$

where N is a measure of the risk size and C, C_1, and C_2 are constants.

Earlier in this paper it was noted that credibility varies according to the dispersion of true claim levels about the expected. This dispersion arises from the failure of the expected claim level determination to reflect completely and accurately all the exposure characteristics that affect claim levels. The degree of this failure, which would obviously vary by line of insurance and even by insurer, must be measured, either implicitly or explicitly, in any determination of credibility factors. It appears to this author that measurement of the imperfection in the expected claim level determination can only be accomplished empirically. The approach to credibility factor determination set forth below is based on an implicit measurement of this imperfection.

Credibility factors may be estimated from the experience of a sample of risks by applying the following formula to subsamples consisting of risks with approximately uniform size and other relevant characteristics, such that approximately the same credibility factor would be applicable to each risk within the subsamples:

$$k \cong \frac{\sum_{i=1}^{n}[(|D_i^A|)/(D_i^A)]D_i^B}{\sum_{i=1}^{n}|D_i^A|}, \tag{7}$$

where $D = a - e$ and the superscripts A and B designate experience periods that are mutually exclusive but sufficiently close to each other to warrant the assumption that the true claim level deviates from the expected by the same amount in each period.

This formula is derived from equation (5) by substituting D^A for $(a - e)$ and D^B for $(t - e)$. The second substitution is based on the fact that the mean of the observed deviations of actual from expected claim levels in Period B is an unbiased estimate of the mean of the deviations of true from expected claim levels for each subgroup of risks which were selected on the basis of observed deviations in Period A.

The estimation formula given by equation (7) may be rationalized as follows:

The credibility factor represents the proportion of the observed deviation of actual from expected claim levels that may be expected to repeat on the average in the future. This proportion may be estimated by determining the proportion that actually did repeat from one period to the next. The denominator of the estimation formula consists of the sum of the absolute values of observed deviations in Period A. The numerator consists of the sum of the observed deviations in Period B, counted as positive when the deviations on a particular risk are in the same direction in both periods and as negative when the deviations are in opposite directions.

EXAMPLE

An examination of an example (see Table 1) will help in developing an appreciation of the concepts underlying this empirical approach to credibility determination.

Assume that all ten risks in the sample shown in Table 1 are uniform in both size and other relevant characteristics and, therefore, that the same credibility factor is applicable to each. It will be noted that the above sample has been split into two groups according to whether the

TABLE 1

Risk No.	Actual Loss Ratio		Expected Loss Ratio	Deviation (Actual−Expected)	
	Period A	Period B		Period A	Period B
1	60%	65%	55%	+ 5%	+10%
2	80	55	58	+22	− 3
3	70	75	60	+10	+15
4	54	58	50	+ 4	+ 8
5	71	62	52	+19	+10
Subtotal				+60%	+40%
6	40%	53%	57%	−17%	− 4%
7	50	45	52	− 2	− 7
8	55	65	58	− 3	+ 7
9	45	50	55	−10	− 5
10	50	47	58	− 8	−11
Subtotal				−40%	−20%

deviation of the actual loss ratio in Period A from the expected was positive or negative. Period B is exclusive of Period A. The two periods are sufficiently close to each other to warrant the assumption that the true claim level deviates from the expected by the same amount in each period. In theory, the credibility factor appropriate to a particular risk represents the proportion of the observed deviation that, on average, can be expected to repeat in the future. In accordance with this concept the credibility factor applicable to the five risks with positive deviations in Period A may be estimated by the ratio of 40 per cent to 60 per cent, or $66\frac{2}{3}$ per cent. Similarly, the credibility factor applicable to the five risks with negative deviations in Period A may be estimated by the ratio of −20 per cent to −40 per cent, or 50 per cent. In practice, however, the same credibility factor is used whether the actual loss ratio is higher or lower than the expected loss ratio. The results of the entire sample may be combined to produce an estimated credibility factor of 60 per cent, the ratio of (40% + 20%) to (60% + 40%). Equation (7) accomplishes this combining effect without actually splitting the risks into those with positive and negative deviations in Period A.

APPLICATION TO A SAMPLE OF RISKS

Credibility factors have been estimated for group life insurance by applying equation (7) to a sample of 413 risks, none of which had an industry rating. Up to five years of experience on each case were included in the study; most of the cases had the full five years available. Period A consisted of many combinations of 1–4 years, and Period B consisted of the balance of the available experience.

As was true in the example given earlier, actual claim levels, expected claim levels, and deviations therefrom were measured in terms of loss ratios. The expected loss ratios used in this study were based on the 1950–59 intercompany group life experience on nonrated industries, with the loss ratios adjusted upward by 2 per cent for the PTD benefit so as to be consistent with the experience in the sample.

Two separate determinations of credibility factors were made. One was based only on the actual amounts of insurance. The second was based on adjusting the experience during Period A (but not Period B) to a flat amount of insurance for each covered employee. The theory was that such experience would not be distorted by occasional claims for large amounts and thus would be more credible. The results indicated, however, that, if there is a tendency in this direction, it is slight, since the empirically derived credibility factors under the two approaches do not appear to differ significantly from each other on the whole.

Table 2 summarizes the results of this sample study. It will be observed that, although the credibility factors computed from this sample generally increase by risk size, the pattern for the largest groups is somewhat erratic; this is undoubtedly the result of the small number of risks in the largest size categories. It would probably be necessary to combine the experience of several insurance companies in order to obtain reliable factors for the largest risks.

In view of competitive pressures it is probably not surprising that, according to rather limited knowledge of other companies' practices, credibility factors currently being used in group life insurance tend to be higher than the ones derived above.

Although this sample study was limited to group life insurance in the interest of simplicity, it is clear that the general principles and formulas are equally applicable to other forms of insurance, particularly group accident and health and casualty-property.

CONCLUSION

The purpose of this paper has been to examine the nature of credibility, to suggest that any determination of credibility factors must be based on an empirical approach, to derive an empirical formula for such determination, and to present the results of applying this formula to a rather limited

TABLE 2

No. of Life Years in Period A	Group Life Credibility Factors	
	Actual Amounts of Insurance	Uniform Amounts of Insurance
0–499	13.8%	10.1%
500–749	15.6	16.3
750–999	18.3	20.8
1,000–1,499	19.0	17.4
1,500–2,499	21.6	22.6
2,500–3,499	30.6	31.7
3,500–4,999	37.5	44.5
5,000–7,499	54.2	49.4
7,500–9,999	62.5	51.9
10,000–24,999	58.6	44.0

body of experience. It is recognized that this brief treatment will probably raise more questions than it answers, but, if it does nothing more than to stimulate further inquiry along the lines suggested herein, the paper will have served a worthwhile purpose.

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