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Exploring at the Velocity of Diversification

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INTRODUCTION

In the June 2014 issue of *Product Matters!*, Doug Robbins of Pacific Life authored a fascinating article entitled "Velocity of Diversification." Therein, he proposed a pricing measure that describes the "…speed at which confidence in profitability is attained." For products with relatively low anticipated sales counts, this may be a useful measure to avoid the statistically significant risk of loss when the law of large numbers may not apply. Mr. Robbins suggests that the velocity of diversification may be expressed so as to capture the number of sales required to achieve a level of confidence determined by management (e.g., 95 percent) in the profitability of the business. He goes on to explore the connection between this approach and stochastic simulation. The implication is that the sales and marketing will be expected to achieve that minimum level of sales to satisfy the risk management goals of the company.

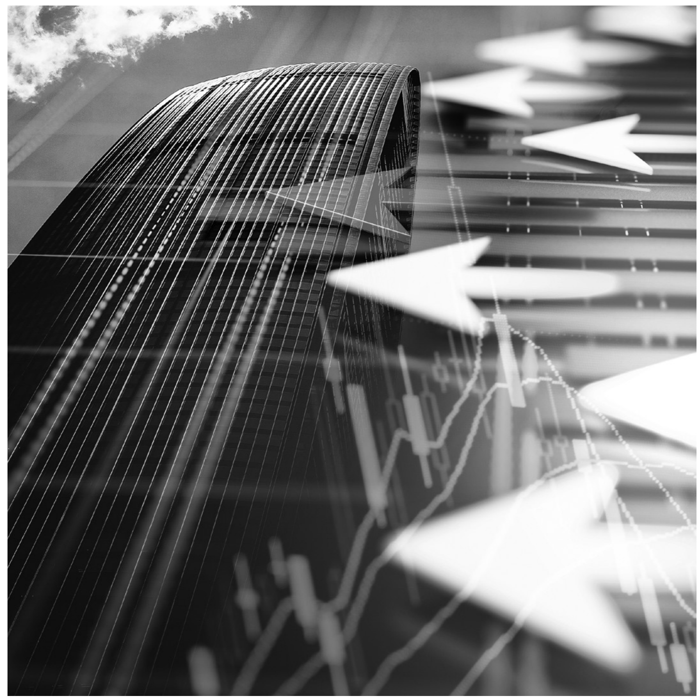
This topic was of particular interest to me. During the past several years, I, along with some of my colleagues, have been exploring stochastic simulations of future mortality, developing models that capture the effects of volatility over (a) date of death, (b) mortality table fit, (c) future mortality improvement rates, and (d) other sources (e.g., pandemics, new cures and treatments).[1] The article sent me scurrying back to my models. Specifically, how can stochastic modeling of mortality be applied to analyze the velocity of diversification, and how would that measure be affected by volatility in the underlying assumptions?

REINVENTING THE WHEEL

To start off, I reworked Mr. Robbins' analysis using a slightly more complex example for a life annuity due for a male age 65. I assumed expected mortality equal to the U.S. Annuity 2000 Basic Table with Projection Scale G.

$$q^{Best\ Estimate} = q^{Annuity2000Basic} \times (1 - Imp^{ScaleG})^{(Year-2000)}$$

The present value of profit goal was set to approximately 8.00 percent[2] of premium, at a discount rate of 3.0 percent. The expense assumptions of 3.75 percent commission plus $100 per policy per year while payments are being made are the same assumptions as used in his article.



Following this approach, for an initial premium of $100,000, annual payments and their resulting profit margins were developed as shown in Table 1:

Table 1: Life Annuity Due – Initial Pricing

	Life	Life + 10	Life + 20
Annual Benefit Payment	$ 5,335	$ 5,200	$ 4,755
Expected Profit Margin	8.07%	8.04%	8.07%

For a single annuity contract, the actual profit margin will depend on the annuitant's year of death. This is explored in Table 2, where the profit margin for a specific year of death is defined as

Profit Margin = 1 – PV@3.0%{Annuity Payments plus Expenses} / $100,000

Table 2: Life Annuity Due

Year	Attained Age	Probability of Death: in Year $_{t-1}P_x\ Q_x$	Probability of Death: by Year (Cumulative)	Profit Margin if Death in Year: Life	Profit Margin if Death in Year: Life + 10	Profit Margin if Death in Year: Life + 20
1	65	0.876%	0.876%	90.82%	49.68%	21.85%
6	70	1.366%	6.599%	65.92%	49.68%	21.85%
11	75	1.992%	15.333%	44.45%	45.74%	21.85%
16	80	2.644%	27.232%	25.93%	27.68%	21.85%
21	85	3.194%	42.201%	9.96%	12.10%	19.16%
24	88	3.419%	52.206%	1.44%	3.80%	11.56%
25	89	3.496%	55.703%	-1.23%	1.19%	9.17%
26	90	3.546%	59.249%	-3.83%	-1.34%	6.85%
29	93	3.422%	69.785%	-11.17%	-8.50%	0.30%
30	94	3.268%	73.053%	-13.47%	-10.75%	-1.76%
31	95	3.090%	76.143%	-15.71%	-12.93%	-3.77%
36	100	2.628%	89.770%	-25.97%	-22.93%	-12.93%
41	105	0.928%	98.435%	-34.81%	-31.56%	-20.83%
46	110	0.071%	99.954%	-42.44%	-39.00%	-27.64%
51	115	0.000%	100.000%	-49.03%	-45.42%	-33.52%

By this analysis, there is a positive gain for Life-Only if death occurs in the first 24 years, which has a cumulative probability of approximately 52 percent. The Life and 10 Years Certain Annuity Due is still profitable if death occurs within 25 years, which has a cumulative probability of approximately 55 percent. Finally, the Life and 20 Years Certain Annuity Due produces positive gain if death occurs within 29 years, which has an approximate cumulative probability of nearly 70 percent. The results parallel Mr. Robbins' examples. That is, although the three alternatives have the same expected profit margin of approximately 8 percent, the insurer should recognize the financial advantage of marketing the longer period certain.

Next, a stochastic projection was used to determine the number of annuity sales to provide the insurer with a 95 percent confidence of a positive total profit margin over all years of death. For each individual annuitant, the year of death is simulated to be equal to the number of years for which the cumulative survival rate is greater than a random number drawn between 0 and 1. The year of death for each sample policy corresponds to a "Profit Margin if Death in Year" for Life, Life and 10, or Life and 20, the average of which may be taken for portfolios of one or more policies. The results shown in Table 3 were produced by repeating this process for a sufficient number of trials:

Table 3: Approximate Number of Annuities Required
To Achieve 95 percent Confidence of Positive Profit Margin

Life	Life + 10	Life + 20
36	26	9

Again the result demonstrates a significant advantage to the insurer in encouraging sales of the longer periods certain, in this case by reducing the sales required to minimize risk of loss. Note that the results are shown independently for each product variation.[3] Table 4 demonstrates how the standard deviation differs as the number of annuities increases for each of the alternative periods certain.

Table 4:
Standard Deviation of Profit Margin over 2000 Trials
For Given Number of Annuities

# Annuities	Life	Life + 10	Life + 20
10	9.78%	8.20%	4.67%
25	6.25%	5.20%	2.94%
50	4.29%	3.57%	2.01%
100	3.06%	2.55%	1.44%
250	1.98%	1.65%	0.92%
500	1.38%	1.15%	0.64%
750	1.11%	0.93%	0.53%
1000	0.96%	0.81%	0.45%
1500	0.78%	0.66%	0.37%
2000	0.68%	0.58%	0.32%
2500	0.61%	0.51%	0.29%

Thus far, this article has paralleled the analysis described in the previous article by Mr. Robbins.

However, up to this point, the mortality curve has been assumed to be static. In other words, the mortality curve to be expected for these annuitants has been determined with 100 percent certainty. What happens if we acknowledge that we do not have 100 percent certainty of the mortality assumption?

VOLATILITY OF MORTALITY IMPROVEMENT

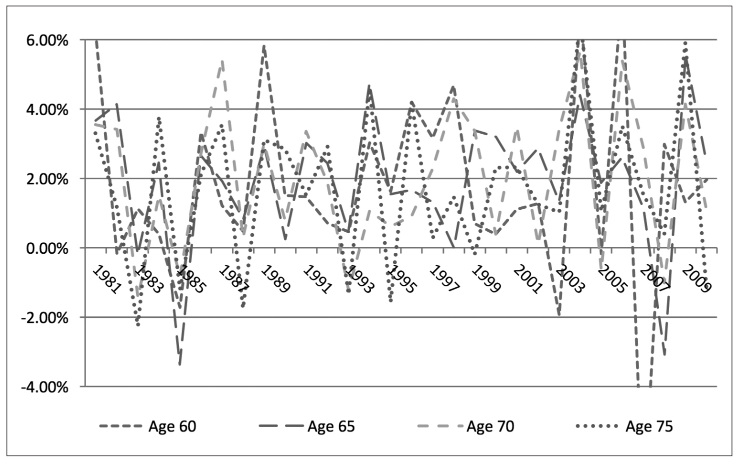
Historical mortality Improvement has been measured for many years and has demonstrated long term and short term trends. Although the long-term trends may be measured and projected, the annual change to mortality has been quite volatile from year to year and from one age to the next. Consider Graph 1 of U.S. Population annual mortality improvement rates reported 1980-2010.

Making financial projections based on "average" mortality improvement may fail to capture the variability of results based on the incidence of annual experience which could have a significant effect on the results. This is one reason to use stochastic projections reflecting volatility in mortality improvement in pricing models.

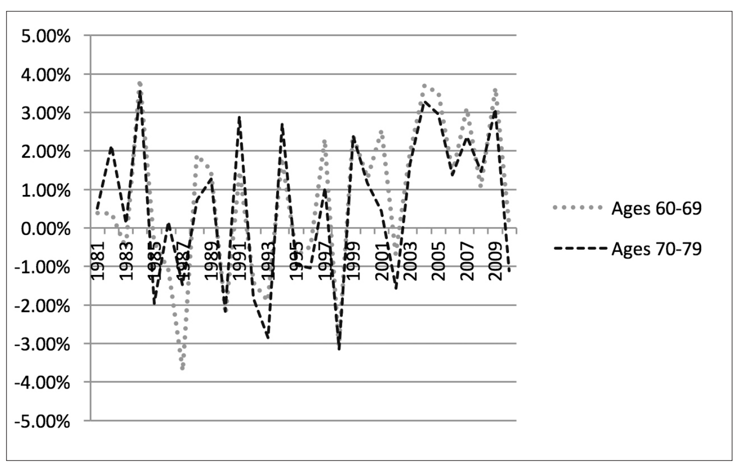
We utilized REVEAL[4], a proprietary Milliman software tool used to analyze longevity risk and the impact of volatility in future mortality rates, to stochastically generate volatile mortality curves. The methodology takes into account average long-term trends (in this case as measured over 10-year periods) and annual volatility. The average across 10-year age groups may be used to reduce statistical noise, but that does not eliminate the annual volatility. The general trend towards improving mortality is apparent, as is the high correlation between consecutive age groups in Graph 2.

Stochastic projections of annual population improvement were derived consistent with the long-term and shortterm expected values and standard deviations, also taking into account the correlation across all ages. For each scenario, the excess (shortfall) between each projected annual rate of improvement over the average historical rates are added to (subtracted from) the expected annual improvement rates used in the pricing (Projection Scale G) for all the policies being tested in that scenario.

Graph 1: Annual Mortality Improvement – U.S. Population - Male



Graph 2: Average Annual Mortality Improvement U.S. Population – Male



Assume that stochastic modeling of mortality improvement volatility is calculated using

- Expected = Best Estimate Annual Improvement (Scale G),
- Plus/Minus random fluctuation of mortality improvement rates around average historic improvement rates, reflecting standard deviations observed over long-term (10-year) and short-term (annual) intervals in U.S. Population Mortality 1970-2010.

Table 5 shows that, based on 2,000 scenarios, each of which projected the results for 2,500 policies, the stochastic analysis converged to the profit margin expected under the static mortality assumption.

Table 5: Profit Margin (Stochastic Volatility Around Expected Mortality Improvement)

	Life	Life + 10	Life + 20
No Volatility – Average Profit Margin	8.07%	8.04%	8.07%
Volatility Around Expected Mortality Improvement – Mean Profit Margin	**8.10%**	**8.07%**	**8.08%**

Note that the minimum number of policies required to achieve 95 percent confidence of a positive return increases slightly with the additional volatility, as seen in Table 6.

Table 6: Approximate Number of Annuities Required To Achieve 95 Percent Confidence of Positive Profit Margin(Stochastic Volatility Around Expected Mortality Improvement)

	Life	Life + 10	Life + 20
No Volatility	36	26	9
Volatility Around Expected Mortality Improvement	**39**	**27**	**9**

And, naturally, the additional source of volatility is reflected in the elevated standard deviations over the 2000 trials in Table 7 as compared to those shown previously in Table 4.

Table 7: Standard Deviation over 2000 Trials For Given Number of Annuities (Stochastic Volatility Around Expected Mortality Improvement)

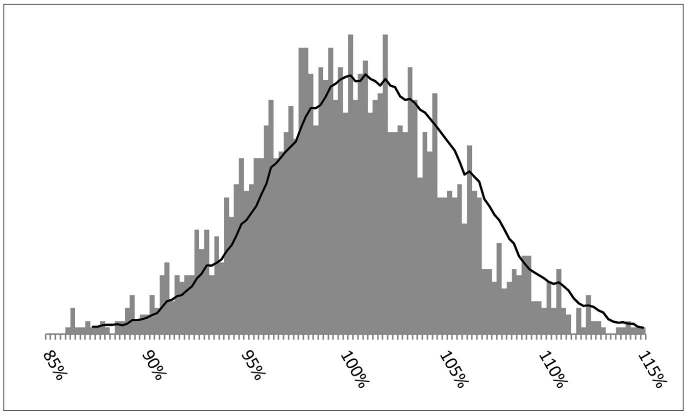
# Annuities	Life	Life + 10	Life + 20
10	9.91%	8.29%	4.71%
25	6.36%	5.35%	3.11%
50	4.59%	3.90%	2.33%
100	3.41%	2.95%	1.86%
250	2.50%	2.23%	1.50%
500	2.11%	1.93%	1.36%
750	1.95%	1.81%	1.31%
1000	1.89%	1.77%	1.30%
1500	1.82%	1.71%	1.27%
2000	1.79%	1.69%	1.27%
2500	1.76%	1.67%	1.26%

VOLATILITY OF EXPECTED RATING AND MORTALITY IMPROVEMENT

In selecting an expected mortality table (Annuity 2000 Basic in this case), an insurer is making an actuarial judgment. However, even if that selection is supported by past experience, experience may emerge that varies from that table, possibly attributable to the company characteristics and the profile of its distribution, or simply some slight skewing by region, type of employment, or other differential. Therefore, there is some risk that the "expected" table may be either higher or lower than the underlying reality of future mortality. This may be described as the level of uncertainty that the base table is 100 percent appropriate for the specific population.

This part of the analysis assumes that the starting expected mortality table is not known with full certainty. In addition to reflecting volatility in future mortality improvement patterns, the starting expected mortality table is assumed to be subject to a normal distribution around 100 percent with a standard deviation of 5.00 percent. As with the volatility of mortality improvement, a randomly generated value was used for each scenario which applied to the expected mortality in all years for all policies being tested in that scenario. The volatility of mortality improvement and the volatility of the expected mortality table are assumed to be independent.

Graph 3: Distribution of Adjustment to Expected Mortality (Over 2000 Trials with Curve of Moving Average)



While there are some discontinuities in Graph 3 that hint at the limits of using only 2,000 scenarios, the overall curve is appears normal, with nearly all of the area between 90 percent and 110 percent (i.e., 95 percent confidence interval falls within the range of the mean ± two standard deviations).

Assume that the stochastic modeling of mortality improvement is produced by a normal distribution using:

- Expected = Best Estimate Mortality (Annuity 2000 Basic), and
- Standard Deviation of Mortality Load = 5.00 percent.

Table 8: Profit Margin (Stochastic Volatility of Expected Mortality & Mortality Improvement)

	Life	Life + 10	Life + 20
No Volatility – Average Profit Margin	8.07%	8.04%	8.07%
Volatility Around Expected Mortality Improvement – Average Profit Margin	8.10%	8.07%	8.08%
Volatility Around Expected Mortality and Expected Mortality Improvement – Average Profit Margin	**8.00%**	**7.98%**	**8.02%**

As additional sources of volatility are introduced, we observe a phenomenon that I will refer to as the "Cost of Volatility" which effectively reduced the average profit margin in Table 8. As a result, the number of policies required to achieve 95 percent confidence of a positive return increases with the additional volatility, shown in Table 9.

Table 9: Approximate Number of Annuities Required To Achieve 95 Percent Confidence of Positive Profit Margin (Stochastic Volatility of Expected Mortality & Mortality Improvement)

	Life	Life + 10	Life + 20
No Volatility	36	26	9
Volatility around Expected Mortality Improvement	39	27	9
Volatility around Expected Mortality and Expected Mortality Improvement	**47**	**35**	**10**

Also, the additional volatility is expressed in the elevated standard deviations over the 2000 trials shown in Table 10 as compared to the values shown previously in Tables 4 and 7.

Table 10: Standard Deviation over 2000 Trials For Given Number of Annuities (Stochastic Volatility Around Expected Mortality & Expected Mortality Improvement)

# Annuities	Life	Life + 10	Life + 20
10	10.19%	8.52%	4.82%
25	6.58%	5.50%	3.22%
50	4.80%	4.09%	2.48%
100	3.75%	3.26%	2.05%
250	2.89%	2.58%	1.72%
500	2.53%	2.30%	1.58%
750	2.41%	2.20%	1.53%
1000	2.33%	2.14%	1.50%
1500	2.26%	2.08%	1.47%
2000	2.23%	2.07%	1.46%
2500	2.21%	2.05%	1.46%

PRICING WITH MARGINS

The SPIA provides a long-term guarantee with no mechanisms for adjustment if experience deviates from expected. Assume that the 3 percent interest assumption has adequate margin for investment risk. If U.S. Annuity 2000 Basic Table with Projection Scale G has been defined as the "best estimate" for mortality, the prudent actuary will add some margin for contingencies to the pricing assumptions. Therefore, suppose that the pricing assumption includes a margin of 10 percent on the annual mortality and 50 percent on the future annual mortality improvement rates:

$$q^{\text{Best Estimate}} = q^{\text{Annuity2000Basic}} \times (1 - Imp^{\,ScaleG})^{(Year\text{-}2000)}$$

$$q^{\text{Pricing}} = q^{\text{Annuity2000Basic}} / 110\% \times (1\text{-}Imp^{\text{ScaleG}})^{(2015\text{-}2000)} \times (1 - 150\% \times Imp^{\text{ScaleG}})^{(\text{Year-}2015)}$$

Keeping the target profit margin of 8.00 percent, in the initial premium of $100,000, the revised ("Loaded using Fixed Baseline Margin") annual payments are shown in Table 11.

Table 11: Annual Benefit Payments

	Life	Life + 10	Life + 20
Best Estimate – No Margin	$ 5,335	$ 5,200	$ 4,755
Loaded using Fixed Baseline Margin	$ 5,080	$ 4,970	$ 4,610

The resulting profit margins shown in Table 12 are comparable to those seen previously, where the Best Estimate Annual Benefit Payments are evaluated assuming Best Estimate Assumptions for mortality and improvement and the Loaded Annual Benefit Payments are evaluated using the Fixed Baseline Margins in the assumptions.

Table 12: Profit Margin (Pricing with Margins)

	Life	Life + 10	Life + 20
Best Estimate – No Margin	8.07%	8.04%	8.07%
Loaded using Fixed Baseline Margin	8.03%	8.01%	8.01%

It is worth remembering that the fixed margin serves to protect the insurer against variations in mortality and improvement experience that produce losses. The problem is that the amounts of those margins are arbitrary and are not associated with any specific range of variation. Substituting stochastic volatility projections with known parameters around the Best Estimate assumptions for mortality and improvement will give greater insight into the pricing results, as shown in Table 13.

Table 13: Approximate Number of Annuities Required To Achieve 95 percent Confidence of Positive Profit Margin

	Life	Life + 10	Life + 20
No Margin - Volatility around Expected Mortality and Expected Mortality Improvement	47	35	10
Loaded - Volatility around Expected Mortality and Expected Mortality Improvement	15	12	5

As should be expected, providing a lower annual benefit payment (developed using the Fixed Baseline Margin) reduces the number of policies required to be sold to achieve a positive profit at least 95 percent of the time when applying volatility of Expected Mortality and mortality improvement around the best estimate assumptions.

Now we can explore how we can adjust the margin to reflect expected sales results.

PRICING USING STOCHASTIC PROJECTIONS FOR MORTALITY VOLATILITY AND EXPECTED SALES

Suppose that the marketing department produces sales forecasts that exceed the minimum number of annuities to achieve the 95 percent confidence interval for positive profit margin (and is willing to commit to achieving those goals). Assume that the sales prediction is as described in Table 14.

Table 14: Assumed Sales Forecast – Estimated Number of Annuity Contracts

	Life	Life + 10	Life + 20
Forecast Sales Volume	25	15	7

The forecast sales counts exceed the respective minimum sales of 15, 12 and 5 annuities that were derived for the baseline margins of 10 percent mortality plus 50 percent margin for improvement, and less than the minimum sales of 47, 35, and 10 respectively required for the "no margins" annual payments (as shown in Table 13). Thus, while some margin may be needed, smaller margins could satisfy the 95 percent probability of positive profit margins at these higher counts, and provide for more competitive payout rates.

At this point, it is possible to develop reduced levels of "loaded" mortality and improvement around which stochastic volatility may be applied at the levels described earlier and repeated here:

- Mortality:
  - Normal Distribution with
  - Expected Mortality = Annuity 2000 Basic
  - Standard Deviation of Mortality Load = 5.00 percent;
- Mortality Improvement:
  - Expected Annual Improvement = Scale G
  - Plus/Minus random fluctuation of mortality improvement rates around average historic improvement rates, reflecting standard deviations observed over long-term (10-year) and short-term (annual) intervals in U.S. Population Mortality 1970-2010.

For example, we will consider if the earlier margin were reduced by 50 percent. That is, the alternative pricing assumption includes reduced margins of 5 percent to the annual mortality and 25 percent to the future annual mortality improvement rates:

$$q^{ReducedMargin} = \; q^{Annuity2000Basic}/105\% \times (1 - Imp^{ScaleG})^{(2015-2000)} \times (1 - 125\% \times Imp^{ScaleG})^{(Year-2015)}$$

Keeping our target profit margin of approximately 8.00 percent, we see in Table 15 that the annual payments are substantially higher than those produced using our baseline margins (but naturally still less than those paid if priced with only the unloaded "best estimate").

Table 15: Annual Benefit Payments

	Life	Life + 10	Life + 20
Best Estimate – No Margin	$ 5,335	$ 5,200	$ 4,755
Loaded Baseline Margin	$ 5,080	$ 4,970	$ 4,610
Revised Loaded	$ 5,205	$ 5,080	$ 4,680

The profit margins remain comparable as seen in Table 16.

Table 16: Profit Margin with No Volatility

	Life	Life + 10	Life + 20
Best Estimate – No Margin	8.07%	8.04%	8.07%
Loaded using Fixed Baseline Margin	8.03%	8.01%	8.01%
Revised Loaded using Fixed Reduced Margin	8.01%	8.04%	8.06%

More to the point, the number of policies to achieve the target 95 percent confidence interval is very nearly equal to the sales forecast, as seen in Tables 17.

Table 17: Approximate Number of Annuities Required To Achieve 95 percent Confidence of Positive Profit Margin

	Life	Life + 10	Life + 20
Forecast Sales Volume	*25*	*15*	*7*
Best Estimate – No Margin Volatility Around Expected Mortality and Expected Mortality Improvement	47	35	10
Baseline Margin for Contingencies Volatility Around Expected Mortality and Expected Mortality Improvement	15	12	5
Reduced Pricing Margin Volatility Around Expected Mortality and Expected Mortality Improvement	26	17	6

Therefore, the use of stochastic projections of liability cash flows may be applied to the velocity of diversification to produce more sophisticated and useful analyses of risk and profitability.

CONCLUSION

Currently most stochastic actuarial projections only utilize volatile economic assumptions. This article makes the case that it is important to reflect volatile liability assumptions when performing stochastic projections. Without taking liability assumptions into account, stochastic projections may understate the potential volatility associated with the liabilities. As such, insurers may fail to calculate a price that compensates them for the cost of volatility.

But there is an equally strong possibility that actuaries may use stochastic projections of liability cash flows to discover excesses in their explicit or implicit margins, potentially allowing them to build a more competitive and more profitable product.

Note: The views expressed herein are those of the author and do not necessarily reflect the views of Milliman, Inc.

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ENDNOTES

1 The other sources of mortality volatility are outside the scope of this article.

2 To be consistent with Mr. Robbins' analysis, the analysis was performed simply on a net cash flow basis without consideration of the effect of statutory reserves and capital.

3 We would expect that the minimum number of combined sales of the three products to achieve the 95 percent probability of positive profit margin would equal some weighted average of the independent results. That analysis is outside the scope of this article.

4 REVEAL (which stands for Risk and Economic Volatility Evaluation of Annuitant Longevity) is a system developed to analyze longevity risk. REVEAL generates stochastic projections on pension and annuity liabilities with volatile assumptions (i.e., baseline mortality, mortality improvement, extreme mortality and longevity events, and annuitant (or plan participant) behavior - such as retirement dates and benefit elections). For more information about REVEAL, *please see http://www.milliman.com/Solutions/Products/REVEAL/*.