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A FAST, MORE MEANINGFUL TWENTY-YEAR NET COST FORMULA

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ABSTRACT

The main features of the proposed formula follow:

1. Cost obtained in one operation of a desk calculator as a linear function of

a) the premium,
b) the sum of the first ten years' dividends,
c) the sum of the first twenty years' dividends,
d) the twentieth-year cash value, and
e) the twentieth-year terminal dividend (if any).

2. Interest and survivorship taken account of in the coefficients of *b*, *c*, *d*, and *e*. (Coefficients for quinquennial issue ages are given in the Appendix.)
3. Data needed readily available in the trade publications.
4. Increases in death benefit due to apportionable premiums, post-mortem dividends, and terminal dividends taken into account by varying the coefficients in the cost formula. (Required coefficients are included in the Appendix.)

The effect of using the proposed costs instead of either twenty-year ledger costs or one-thirtieth-method costs to compare twenty-four companies' whole life policies is analyzed with reference to (1) the relative ranking of costs and (2) causes of distortion in ranking.

To facilitate calculation of the proposed costs by clerical staff, the Appendix gives working instructions with illustrative calculations.

This paper develops a technique for calculating interest-survivorship, specific, twenty-year policyholder costs for participating insurance. The time required to apply the method is only fractionally longer than that required under the traditional ledger cost method, and the data needed are readily available in the trade publications.

The basic form of the proposed formula is

$$\pi - D_f^{10}\left(\sum_1^{10} \text{Div}\right) - D_f^{20}\left(\sum_1^{20} \text{Div}\right) - P_{x:\overline{20}|}^{\;\;1}(\text{CSV}_{20}) \qquad (1)$$

where x is the issue age and π,

$$\sum_{1}^{10} \text{Div}, \quad \sum_{1}^{20} \text{Div},$$

and CSV_{20} are, respectively, the premium charged, the ten-year and twenty-year dividend totals, and the cash-surrender value at the end of twenty years, all per $1,000 face amount. The coefficients D_f^{10} and D_f^{20} are designed to produce a close approximation to the "equivalent level dividend." They are a function of interest rate, mortality rates, issue age, and the policy year f in which the first dividend is payable.

Section I of the paper shows how an equivalent level dividend may be approximated from ten-year and twenty-year dividend totals and tests the accuracy of the approximation. Section II deals with sundry matters concerning the interpretation of policyholder costs. In Section III, considerations that affect the choice of mortality and interest rates for use in applying the proposed formula are discussed, and, on the basis of these considerations, specific rates are recommended for current use. Section IV shows how, with virtually no extra work, adjustments in the proposed method of calculating policyholder costs can be made to take account of increases in the death benefit due to apportionable premiums, post-mortem dividends, and terminal dividends. In Section V the effect of using the proposed costs instead of either twenty-year ledger costs or one-thirtieth-method costs is shown for twenty-four companies' whole life policies at various issue ages. The relative ranking of costs is tabulated, and the causes of distortion in ranking inherent in the ledger-cost and one-thirtieth methods are analyzed.

I. EQUIVALENT LEVEL DIVIDEND

The equivalent level dividend may be defined for the present purpose as a level amount payable at the beginning of each of the first twenty policy years such that the present value of the twenty level payments equals the present value of the dividends payable in the first twenty policy years. It is equal to

$$\sum_{t=f}^{20} (\text{Dividend payable in } t\text{th policy year})\, D_{x+t}/(N_x - N_{x+20}) \qquad (2)$$

where the first dividend is payable in policy year f.

Approximation of the Equivalent Level Dividend

To approximate the equivalent level dividend as

$$D_f^{10}\left(\sum_{1}^{10} \text{Div}\right) + D_f^{20}\left(\sum_{1}^{20} \text{Div}\right),$$

it is assumed that the present value of the given dividend scale equals the present value of a dummy scale having the same ten-year and twenty-year dividend totals, with the dividends falling at the ends of policy years f through 20 but with the individual dividends forming an arithmetical progression. If the first dividend (for policy year f) under this dummy scale is a and the common difference between consecutive dividends is d, then

$$\sum_1^{10} \text{Div} = (11-f)a + \sum_{t=1}^{10-f} td$$

$$= (11-f)a + \frac{(10-f)(11-f)}{2}d$$

and

$$\sum_1^{20} \text{Div} = (21-f)a + \frac{(20-f)(21-f)}{2}d\,.$$

It follows that

$$a = 0.1\left[\frac{20-f}{11-f}\left(\sum_1^{10} \text{Div}\right) - \frac{10-f}{21-f}\left(\sum_1^{20} \text{Div}\right)\right]$$

and

$$d = 0.2\left(\frac{\sum_1^{20} \text{Div}}{21-f} - \frac{\sum_1^{10} \text{Div}}{11-f}\right).$$

The approximate equivalent level dividend is

$$\frac{a(N_{x+f} - N_{x+21}) + d(S_{x+f+1} - S_{x+21} - \overline{20-f}\cdot N_{x+21})}{N_x - N_{x+20}}$$

or, on substituting the values of a and d just deduced,

$$D_f^{10}\left(\sum_1^{10} \text{Div}\right) + D_f^{20}\left(\sum_1^{20} \text{Div}\right), \tag{3}$$

where

$$D_f^{10} = \frac{(20-f)(N_{x+f} - N_{x+21}) - 2(S_{x+f+1} - S_{x+21} - \overline{20-f}\cdot N_{x+21})}{10(11-f)(N_x - N_{x+20})}$$

and

$$D_f^{20} = \frac{2(S_{x+f+1} - S_{x+21} - \overline{20-f}\cdot N_{x+21}) - (10-f)(N_{x+f} - N_{x+21})}{10(21-f)(N_x - N_{x+20})}.$$

Values of D_1^{10}, D_1^{20}, D_2^{10}, and D_2^{20} (all multiplied by 10^5 for conciseness) are given in Table A of the Appendix for quinquennial issue ages at interest rates of 4 and 5 per cent.

Accuracy of the Approximate Equivalent Level Dividend

To appraise the accuracy in practice of the approximate equivalent level dividend as given by formula (3), equivalent level dividends have been calculated for a number of dividend histories, both precisely by formula (2) and approximately by formula (3), using the 1958 C.S.O. Table and 4 per cent interest. Dividend histories have been employed partly because they are readily available and partly because they tend inherently to be less regular in their patterns than illustrative dividend scales. The histories used are those of twenty-three companies which, in 1967, each wrote more than $300 million of insurance on participating life and endowment plans. Twenty-one of these companies are mutual, and two are stock companies issuing only participating insurance. Two are Canadian companies. (All histories are for business issued in the United States.) The histories are for issues on ordinary life, twenty-payment life, and twenty-year endowment plans, with issue ages 25, 35, 45, and 55.

TABLE 1

FREQUENCY OF EQUIVALENT LEVEL DIVIDEND ERRORS

Error (per $1,000 Face Amount)	Whole Life				Life 20 and Endowment 20			
	Issue Age							
	25	35	45	55	25	35	45	55
$0.00–$0.03	22	20	17	10	35	33	32	11
.04	1	1	2	1	6	4	1	3
.05		2	1	3	1	4	4	4
.06			2		2	2	2	3
.07				3		1	1	2
.08			1					1
.09				1			1	
.10				2			1	
.11				1			2	2
.12				1				1
.13								1
.14								1
.15								1
0.21								2
All	23	23	23	22	44	44	44	32

NOTE.—Mortality: 1958 C.S.O. Table; interest rate: 4 per cent; data: 1948–68 dividend histories of twenty-three large companies.

Table 1 shows the extent to which errors are present in the approximate equivalent level dividends derived from the histories of the twenty-three companies, the data being subdivided by type of plan and issue age.

It may be noted that histories were not available for all company-plan combinations falling within the scope of the test.

From the error frequencies given in Table 1 it may be inferred that approximate equivalent level dividends for the whole life plan are most unlikely to be in error, per $1,000 face amount, by more than $0.05 for issue ages under 40 or by more than $0.07 and $0.10 for issue ages in the forties and fifties, respectively. Table 1 also shows that the errors under higher-premium plans may be about 50 per cent larger.

Thus at the ages at which most business is issued the error is small. The somewhat larger error at the higher issue ages is unimportant by comparison with the greater lack of precision at these ages in whatever mortality basis is employed.

II. INTERPRETATION OF POLICYHOLDER COSTS

The proposed cost formula (formula [1]) may, for greater intelligibility, be cast in the form

$$0.05\left(\sum_1^{20}\pi\right)-(D_f^{10}+D_f^{20})\left(\sum_1^{10}\text{Div}\right)-D_f^{20}\left(\sum_{11}^{20}\text{Div}\right)-P_{x:\overline{20}|}^{\;1}(\text{CV}_{20})\,,$$

where

$$\sum_1^{20}\pi$$

is the total of the premiums charged in the first twenty policy years and

$$\sum_{11}^{20}\text{Div}$$

is the total of the dividends payable in policy years eleven to twenty, each per $1,000 face amount. The layman will not appreciate precisely how the values of all the coefficients in this formula are arrived at, but an explanation of their order of magnitude, such as that given in the next paragraph, will make the underlying rationale of the method evident to him.

If the dividends payable in the first ten policy years were, like the premiums, distributed evenly through the full twenty policy years, the factor applied to their total would also be 0.05. As their average time of payment is earlier, however, they are worth more to the policyholder than they would be if they were evenly distributed through the twenty years, as a result of both the interest he can earn on money paid earlier and his greater chance of living to receive such earlier payments. Thus the factor applied to the total of the first ten years' dividends is somewhat larger than 0.05. On the other hand, similar reasoning shows that, since the average time of payment of the dividends payable in the second ten policy years is later than that of the twenty years' premiums, the factor applicable to their total must be less than 0.05. Since the time of payment of the twentieth-year cash-surrender value is later than the average time of payment of the second ten years' dividends, the factor applied to it is smaller still.

Guide 10 of the Society's "Guides to Professional Conduct" says, in part, "The member will include in any report quoting actuarial costs a statement describing or clearly identifying the assumptions employed." Quotations of policyholder costs calculated by the proposed method should specify the mortality table and interest rate assumed. If these assumptions had to be explicitly stated whenever traditional twenty-year surrendered net costs are quoted, such policyholder costs would lose much of their appeal.

Surrendered-net-cost formulas are sometimes criticized on the grounds that they assume surrender of the policy at the end of twenty years, whereas it is not desirable for insurance to be sold on the assumption that it will be surrendered at a particular time. However, since "the best laid schemes o' mice an' men Gang aft a-gley," a prospective policyholder with the firmest of intentions cannot be certain that his policy will remain in force for the full twenty years or until his earlier death. Thus, realistically, a prospect who plans to surrender his policy at the end of twenty years should compare costs for a somewhat shorter period. Twenty-year surrendered-net-cost comparisons should be regarded as appropriate for prospects who plan to keep their policies in force much longer than twenty years.

The above argument assumes that the cash-surrender values give a fair measure of the policyholder's equity. Where this is not the case and, for example, the twentieth-year cash-surrender value exceeds the then asset share, the proposed cost would generally not be as equitable a basis of comparison for a policy not surrendered at the end of twenty years. The distortion in such a cost resulting from inconsistency between twentieth-year cash value and asset share would, however, be less than that in the case of the cost calculated by the traditional formula, since the factor $P_{x:\overline{20}|}^{\;\,1}$ applicable to the twentieth-year cash value under the proposed formula is less than the factor of 0.05 applicable under the traditional formula.

III. CHOICE OF MORTALITY AND INTEREST RATES

Any choice of mortality and interest rates for use in calculating policyholder costs is, to some extent, arbitrary. Some choices, however, such as that implicit in ledger costs, are much more arbitrary than others. In what follows, some general considerations affecting the choice of mortality and interest rates are stated, and, then, on the basis of these considerations, specific rates are suggested for current use.

The governing consideration in the selection of mortality and interest rates for the calculation of *policyholder* costs is that the rates should be such as most prospective policyholders would regard as reasonable. As the typical prospect is not in a position to appraise the suitability of mortality rates, these should be chosen to meet requirements that an actuarially informed prospect could be expected to regard as reasonable.

Choice of Mortality Table

The mortality table used to compare policyholder costs should reflect what an increase in the amount at risk is worth to a prospective policyholder. If we assume, as will generally be the case, that the prospect feels that he has an insurable interest in the largest amount of net future protection provided by any of the contracts under comparison, we may also assume that he would consider it fair to pay for the extra insurance element in the contracts with the lower cash values (or flatter dividend scales) at appropriate market rates.

The extra insurance element in question is in the nature of a supplementary benefit comparable to a renewable term rider. Such a rider, however, is worth somewhat more to the policyholder, since he can terminate the rider benefit without terminating the basic policy. The opportunity which renewable term insurance gives the policyholder to select against the company is especially valuable at the higher ages. Thus the mortality table used to compare policyholder costs should have rates slightly below renewable term rider rates at the younger ages, with the difference increasing with increasing age.

In considering the choice of a mortality table, it is important to bear in mind that, as the pure endowment values of which the coefficients of the cost formula are composed can be factorized into terms of the form $(1 - q_x)(1 + i)^{-1}$, a constant increase in the mortality rates assumed changes values of policyholder cost by approximately the same amounts as the same increase in the interest rate assumed. Thus, accepting the fact that the mortality rates used through the twenty years of age relevant to a policyholder cost calculation may be, on the average, in error by \$2.00 per \$1,000 is equivalent to tolerating an error from the interest rate assumed of only about 0.2 per cent. (The use of a zero interest rate implicit in the traditional net cost formula is in error by about 20 times this amount.)

The fact that it is impossible precisely to specify mortality rates appropriate for use in making policyholder cost comparisons is unimportant for the younger ages at which the bulk of the business is written. A larger absolute error in the mortality rates has to be tolerated at the higher ages. This is no reason, however, to ignore mortality at these ages. Indeed, it is at the higher ages that it is most important to recognize the mortality factor. The error in a policyholder cost calculated for a high issue age from mortality rates 15 per cent in error is less than one-fifth the error had the cost been calculated ignoring mortality.

Use of the 1958 C.S.O. Table is suggested, partly because the mortality rates satisfy the criteria mentioned above (e.g., for ages below 45, 1958 C.S.O. net term premiums fall short of the renewable term rider rates of one large company by from $0.30 to $1.00) and partly because (as the standard valuation table) it is the one best known to agents and the public.

Choice of Interest Rates

The method by which the interest rate assumed in calculating policyholder costs is selected should not be inconsistent with the method by which the dividends themselves have been obtained. Thus the interest rate assumed in calculating policyholder costs based on illustrative dividends should not be based on any estimate of conditions over the next twenty years.

Just as the choice of mortality rates may be related to rates for supplementary term insurance benefits, so also the choice of interest rates may be related to the rate for dividend accumulations. In this regard it may be noted that in recent years United States policyholders have placed 28 per cent of their dividends under the deposit option and have applied a further 25 per cent to purchase paid-up insurance.[1] (For Canada the corresponding proportions are 50 and 20 per cent.[2]) A substantial proportion of the remaining dividends is spent rather than invested elsewhere. Of the dividends applied to purchase paid-up insurance, only a small fraction is used to pay for the cost of protection, the balance being credited with interest at a rate close to the dividend-accumulation rate.

The above facts show that most policyholders consider the dividend-accumulation rate an acceptable one. Furthermore, this rate is likely to be mentioned during the agent's presentation, and the prospect may be expected to compare it with the policyholder cost rate. He is unlikely to

[1] See *Life Insurance Fact Book, 1968* (New York: Institute of Life Insurance), p. 47.

[2] See *Canadian Life Insurance Facts, 1968* (Toronto: The Canadian Life Insurance Association), p. 20.

expect these rates to be different, except for considerations of liquidity and personal income tax.

While the liquidity of dividend accumulations is doubtless of importance to some policyholders, the fact that the total of policy dividend accumulations at the end of 1967[3] is greater than the accumulation of all the dividends deposited in the six preceding years[4] shows that for many policyholders the deposit option is not just a short-term investment. Of more importance to most policyholders than the difference in liquidity is the fact that the policyholder cost rate can in large part be regarded as an after-tax rate. Consequently, the rate used for cost comparisons should be slightly below the bulk of companies' dividend-accumulation rates.

It is desirable that the interest rate used to calculate policyholder costs remain constant over fairly long intervals. For this reason and because a high degree of refinement is inconsistent with the rather tentative nature of policyholder cost comparisons, it is suggested that the rate be a multiple of 0.5 per cent.

From the above considerations it follows that the current interest rate for policyholder cost comparisons should be 4 per cent in the United States and 5 per cent in Canada. It is desirable that each of these rates be reviewed annually and be increased by 0.5 per cent when the dividend-accumulation rates of, say, 90 per cent of the respective country's twenty largest companies, increase to a level 0.5 per cent or more above these rates. On the other hand, either rate would be decreased by 0.5 per cent should the dividend-accumulation rates of, say, 20 per cent of the respective companies, drop below the prevailing policyholder cost interest rate.

In calculating policyholder costs based upon dividend histories, the interest rate used should in theory be based upon interest levels through the past twenty years. Much more weight would, however, have to be given to the level of interest rates in the more recent years, when the fund developed under the policy would be larger. On this account, and for simplicity, it is suggested that the choice of interest rates for costs based upon dividend histories be the same as that for costs based upon dividend illustrations.

IV. ANCILLARY DEATH BENEFITS

At the higher issue ages the value of the death benefit due to apportionable premiums, post-mortem dividends, and terminal dividends is by no means negligible. Adjustments to take account of these benefits may be

[3] See *Life Insurance Fact Book, 1968*, p. 60.

[4] See the section "Living Benefit Payments to Policyholders" in the *Life Insurance Fact Book* for each of the years 1963–68.

incorporated into costs calculated by the present method with hardly any increase in calculation time.

Apportionable Premiums

Fourteen of the companies that wrote more than $300 million of insurance on participating life and endowment plans in 1967 give the apportionable premium benefit. Three of these refund premiums paid beyond the date of death, and eleven refund premiums paid beyond the end of the policy month of death. The approximate average refund is, for the former, 50 per cent of the annual premium and, for the latter, just under 46 per cent of the annual premium. Assuming that, on average, 47 per cent of the annual premium is refunded, credit for this benefit may be given by reducing the factor in the cost formula (formula [1]) applicable to the premium from 1.00 to

$$1.00 - 0.47(1 + i)^{2/3} \cdot P^1_{x:\overline{20|}} .$$

Values of this function[5] (multiplied by 10^5) are included in Table A of the Appendix under the heading "π^{AP}."

Recognition (on the 1958 C.S.O. 4 per cent basis) in the cost formula of the apportionable premium benefit results for the above-mentioned fourteen companies in an average reduction in costs per $1,000 face amount on the whole life plan (policy size $10,000) at issue ages 25, 35, 45, and 55 of $0.02, $0.06, $0.19, and $0.65, respectively.

Post-Mortem Dividends

Allowance for payment of a post-mortem dividend may be made by modifying the dividend redistribution factors, D^s_f. When no post-mortem dividend is payable, these factors (see formula [3]) may be expressed as

$$D^s_f = \frac{F_s(x, f)}{N_x - N_{x+20}} .$$

When a post-mortem dividend of, on average, half the year's dividend is paid, the factors become (with $F_s[x, f]$ defined as above)

$$D^s_f = \frac{F_s(x, f) + v \cdot F_s(x - 1, f)}{2(N_x - N_{x+20})} .$$

Values of D^s_f applicable when a post-mortem dividend is payable are included in Table A of the Appendix.

Recognition (on the 1958 C.S.O. 4 per cent basis) in the cost formula

[5] The factor $(1 + i)^{2/3}$ takes account of the decreasing death benefit during each policy year.

of post-mortem dividends results, for the sixteen large companies offering this benefit, in an average reduction in cost per $1,000 face amount on the whole life plan (policy size $10,000) at issue ages 25, 35, 45, and 55, of $0.01, $0.02, $0.06, and $0.20.

Terminal Dividends

The cash-surrender value used in the proposed cost formula (formula [1]) at the end of twenty years includes, of course, any terminal dividend then payable. In the trade publications the twentieth-year cash value (CV_{20}) and terminal dividend (TD_{20}) are generally quoted separately, so that, in the calculation of costs on a desk calculator, they are handled most conveniently in the operating sequence:

$$-P_{x:\overset{1}{\overline{20|}}}(CV_{20}) - P_{x:\overset{1}{\overline{20|}}}(TD_{20}) .$$

Allowance for payment of a terminal dividend on death may be made by changing this to

$$-P_{x:\overset{1}{\overline{20|}}}(CV_{20}) - P_{x-r:\overset{1}{\overline{20|}}}(TD_{20}) ,$$

where r equals 5, 10, 15, 20, or 25 years, according to the slope of the terminal dividend scale.

The feasibility of the above age-rating technique was tested (on the 1958 C.S.O. 4 per cent basis) for issue ages 25, 35, 45, and 55 on the whole life plan, using the widely varying terminal dividend scales of eight large companies. For each company-age combination the value of r (a multiple of 5) was found for which the adjustment in cost

$$(P_{x-r:\overset{1}{\overline{20|}}} - P_{x:\overset{1}{\overline{20|}}})TD_{20}$$

most nearly equaled the value of

$$\frac{\sum_{t=1}^{20} TD_t^d \cdot C_{x+t-1}}{N_x - N_{x+20}} ,$$

where TD_t^d is the terminal dividend payable on death in the tth policy year.

It was generally found that for each company r took the same value at issue ages 45 and 55. (At the younger issue ages the value of the benefit is so small that the age rating is inconsequential.) Thus for each company a value of r may be specified appropriate for all issue ages. Furthermore, since it is customary for each company to apply the same or a similar formula to obtain terminal dividends for all its plans, the age ratings appropriate for whole life may be used in calculating costs under other plans.

The values of r obtained range from five years when terminal dividends are first payable after fifteen policy years on a steeply sloping scale to twenty-five years when terminal dividends are payable after ten policy years as a constant proportion of the cash value.

Recognition (on the 1958 C.S.O. 4 per cent basis) in the cost formula of the terminal dividend death benefit results for the above-mentioned eight companies in an average reduction in cost per $1,000 face amount on the whole life plan at issue ages 25, 35, 45, and 55 of $0.01, $0.02, $0.07, and $0.17.

V. COMPARISON OF POLICYHOLDER COST METHODS

To conclude, we show the effect on policyholder cost comparisons of the use of the proposed method (with recognition of the ancillary death benefits discussed above) rather than either the traditional ledger cost method or the one-thirtieth method. This effect is analyzed for 1968 issues at ages 25, 35, 45, and 55 on the participating whole life plan (except as noted below) for policies of $10,000 of the following twenty-four companies:

a) The twenty United States mutual companies that issued more than $330 million of insurance on life and endowment plans in 1967.

b) The two United States stock companies that issued more than $330 million of insurance on life and endowment plans in 1967 and whose business is solely on a participating basis.

c) The two Canadian mutual companies operating in the United States that issued more than $330 million of insurance on life and endowment plans (in all territories) in 1967.

The whole life plan is not issued by two of the above companies in the amount $10,000. The nearest equivalents issued by these companies are, in one case, life at 85 and, in the other, life at 90. However, at each issue age considered these plans have lower twentieth-year cash values than at least one of the other companies' whole life plans. The twentieth-year cash-surrender value (cash value plus terminal dividend) for each of these two plans at each issue age is also less than the corresponding value of at least one of the other companies' whole life plans, except at issue age 55 on the life at 85 plan. On account of this case, footnotes have been added to Tables 2 and 3.

Effect on Company Ranking

When moving from one cost formula to another, the effect on a company's ranking may be expressed either as the number of places that it moves up or down in the over-all ranking or as the number of companies whose order of precedence with respect to it changes. Table 2 utilizes each

TABLE 2

Effect of the Proposed Method on Company Ranking

n	Moving from 20-Year Ledger Costs to Proposed Costs: No. Companies Changing Rank by *n* Places	Moving from 20-Year Ledger Costs to Proposed Costs: No. Companies Changing Rank with Respect to *n* Companies	Moving from One-thirtieth-Method Costs to Proposed Costs: No. Companies Changing Rank by *n* Places	Moving from One-thirtieth-Method Costs to Proposed Costs: No. Companies Changing Rank with Respect to *n* Companies
	Issue Age 25			
0	2	...	13	13
1	5	3	10	10
2	5	3	1	1
3	4	3	...	...
4	3	5	...	...
5	...	3	...	...
6	2	4	...	...
7	3	3	...	...
	Issue Age 35			
0	2	1	12	11
1	9	6	10	11
2	3	5	2	2
3	2	1	...	...
4	5	4	...	...
5	1	3	...	...
6	...	2	...	...
7	1	1	...	...
10	1	1	...	...
	Issue Age 45			
0	3	2	11	10
1	5	3	4	3
2	3	3	4	2
3	3	3	2	4
4	5	2	2	4
5	1	2	1	1
6	...	4	...	...
7	...	1	...	...
8	3	3	...	...
9	1	1	...	...
	Issue Age 55			
0	2	2	5	3
1	8	2	10	8
2	6	4	3	4
3	...	4	3	3
4	4	5	...	3
5	1	3	1	1
6	1	2	1	1
7	...	...	...	...
8	...	...	1*	1*
9	1	1	...	...
13	1*	1*	...	...

Note.—Data: Values for 1968 issues on the participating whole life plan (except for one life at 85 plan and one life at 90 plan) of twenty-four large companies (policy size $10,000). Proposed costs on 1958 C.S.O. 4 per cent basis.

* Life at 85.

of these measures to show the effect of moving to the proposed costs from either ledger costs or one-thirtieth-method costs.

Considering the change in ranking associated with a move from ledger costs to the proposed costs, we may note that, of the 276 pairs of companies that may be compared at each issue age, the numbers misranked (with proposed costs as control) by the traditional formula at issue ages 25, 35, 45, and 55 are 49, 39½, 50, and 45½, respectively. (The halves derive from tied costs.) Thus over all the probability of incorrect ranking on the whole life plan is one in six. Since the probability of incorrect ranking with costs allocated at random would be one in two, the traditional formula may be regarded as only two-thirds successful (the Kendall rank coefficient τ being, on average, two-thirds).

Table 2 shows that the ranking given by the one-thirtieth method accords closely with that given by the proposed method at issue ages 25 and 35, with the Kendall rank coefficients being 0.957 and 0.946 at these respective ages. However, the one-thirtieth method gives a less satisfactory ranking at issue ages 45 and 55, with Kendall rank coefficients of 0.855 and 0.797, respectively. (The probability of the relative ranking of two companies not being the same under both methods equals half the difference between this coefficient and 1.0.)

Causes of Distortion in Ranking

Changes in ranking when moving from one cost method to another may be due to differences in the twentieth-year cash value or the annual dividend scale, or they may be due to the allocation of terminal dividends. Table 3 is designed to show the relative importance of these causes with regard to both their average and maximum impact on policyholder cost comparisons on the whole life plan.

Under the heading "Total Change," Table 3 shows, for the 276 pairs of companies that can be drawn from the group of twenty-four companies, average and maximum values of the change in the difference between the policyholder costs of two companies when moving to the proposed costs from either ledger costs (cols. headed "L.C.") or one-thirtieth-method costs (cols. headed "One-thirtieth"). It may be noted that, when moving from ledger costs to the proposed costs, the change in the difference between policyholder costs increases markedly with increasing issue age, while the change in ranking is about the same at each issue age. This accords with the wider spread of policyholder costs at the higher issue ages.

The last six columns of Table 3 give the average and maximum changes in the difference between the policyholder costs of two companies when moving from ledger costs and one-thirtieth-method costs to the proposed costs, with account taken first of differences in the twentieth-year cash value only, then of differences in the annual dividend scale only, and finally (for the twelve companies of the twenty-four offering terminal dividends) of the allocation of terminal dividends only. The changes given in the last two columns of Table 3 apply to comparisons between one company with and another without the terminal dividend benefit. The remainder of Table 3 applies to comparisons between any two of the twenty-four companies. The total changes given are not equal to the sums of the changes from the separate causes for this reason and because the changes due to different causes do not always reinforce one another.

Table 3 shows that, when ledger costs are used at issue ages 25, 35, and 45, the maximum distortion in relative costs due to differences in twentieth-year cash value is more than five times the average distortion on this account. This reflects the fact that, while seventeen of the companies base

TABLE 3

EFFECT OF THE PROPOSED METHOD ON THE DIFFERENCE BETWEEN THE POLICYHOLDER COSTS OF TWO COMPANIES

Issue Age	Category	Change in Difference between Costs of Two Companies							
		Total Change		Changes due to Differences in: 20th C.V.		Changes due to Differences in: Annual Divs.		Change due to Terminal Divs.	
		L.C.	One-thirtieth	L.C.	One-thirtieth	L.C.	One-thirtieth	L.C.	One-thirtieth
25	Average	$0.38	$0.09	$0.26	$0.03	$0.13	$0.11	$0.27	$0.03
	Maximum	2.04	0.33	1.50	.18	0.53	0.32	.44	.04
35	Average	0.42	0.13	0.29	.05	0.18	0.14	.34	.04
	Maximum	2.17	0.44	1.60	.29	0.69	0.41	.51	.06
45	Average	0.57	0.21	0.34	.10	0.33	0.23	.46	.08
	Maximum	2.38	0.70	1.76	.53	1.12	0.69	.71	.13
55	Average	1.09	0.61	0.51	.23	0.80	0.57	.60	.17
	Maximum*	5.07	2.61	2.08	0.95	2.47	1.67	0.96	0.29

NOTE.—This table shows the average and maximum changes in the difference between whole life policyholder costs (per $1,000 face amount) of pairs of companies (*a*) when moving from the traditional twenty-year ledger cost method to the proposed method (cols. headed "L.C.") and (*b*) when moving from the one-thirtieth method to the proposed method (cols. headed "One-thirtieth").

Data: Values for 1968 issues on the participating whole life plan (except for one life at 85 plan and one life at 90 plan) of twenty-four large companies (policy size $10,000). Proposed costs on 1958 C.S.O. 4 per cent basis.

* With the life at 85 plan omitted, this line reads: 3.77, 1.93; 2.08, 0.95; 2.27, 1.60; 0.96, 0.29.

their cash values on a level $2\frac{1}{2}$ per cent interest rate, one company uses a level $3\frac{1}{2}$ per cent interest rate (with curtate functions) and two use 3 per cent for the first twenty policy years and 2 per cent thereafter (both with continuous functions). It may be noted that, while the former company does not allot terminal dividends, one of the latter companies does. For issue ages 22 to 37 the differences between the twentieth-year cash-surrender values (on the whole life plan) of these two large companies are more than $95 per $1,000 face amount.

Differences in annual dividend scales are shown by Table 3 to be the most important cause of distortion in relative costs when the one-thirtieth method is used. This distortion arises partly as a result of the fact that, while dividends are currently (1968) accumulated at rates of from 4.0 to 4.6 per cent, the one-thirtieth method in effect discounts the accumulated dividends at 3.7 per cent. If, instead of one-thirtieth, a factor of 0.03 (corresponding to a rate of 4.6 per cent) were used, distortion from this source would be reduced, but the neglect of survivorship would still lead to significant misranking of companies at the higher issue ages, as may be seen by comparing the factor 0.03 with the values of $P_{x:\overset{1}{\overline{20}|}}$ given in Table A of the Appendix. It is indeed anomalous that an industry should compare prices for its product in ways that ignore the value to the buyer of differing amounts of the very ingredient which gives the industry its *raison d'être.*

APPENDIX

WORKING INSTRUCTIONS FOR THE CALCULATION OF POLICYHOLDER COSTS

1. Where applicable, adjust the data to allow for the inclusion of the waiver of premium disability benefit and ages on a "last birthday" basis.
2. *a*) Calculate the policyholder cost as

$$(1.0 \; or \; \pi^{AP})(\text{Premium}) - D_f^{10}\left(\sum_1^{10} \text{Div}\right) - D_f^{20}\left(\sum_1^{20} \text{Div}\right) - P_{x:\overset{1}{\overline{20}|}}(CV_{20}) - P_{x-r:\overset{1}{\overline{20}|}}(TD_{20}) \; .$$

b) In this formula the second factor in each product is a policy datum, the symbolized data being

$\sum_1^s \text{Div}$ = sum of the first *s* policy years' dividends,

CV_{20} = cash value at the end of twenty years, and

TD_{20} = terminal dividend at the end of twenty years.

c) The interpretation of the coefficients applicable to the data is given by

π^{AP} = factor applicable to the premium when a premium refund is payable on death;

D_f^s = factor applicable to the sum of the first s policy years' dividends, where the first dividend is payable for policy year f; and

r = rating (5, 10, 15, 20, or 25 years) in age to allow for terminal dividends payable on death ($x - r$ taken to be 0 when $r > x$). Values of the coefficients for quinquennial issue ages are given in Table A below.

Illustrative Calculations (1958 C.S.O. 4 Per Cent Basis)

a) Data: Issue age... 45

Premium charged per $1,000.......................... $ 32.94

Total dividends first ten years........................ $ 55.24

Total dividends first twenty years..................... $180.70

First dividend payable at end of first year.

Twentieth-year cash value........................... $463.00

No terminal dividends and no ancillary death benefits payable.

Calculation:

$$\text{Cost} = 32.94 - 0.03259(55.24) - 0.03120(180.70) - 0.02619(463.00)$$
$$= \$13.38.$$

b) Data: Issue age, premium charged per $1,000, total dividends first ten and first twenty years, and year of payment of first dividend as for *a* above.

Twentieth-year cash value........................... $440.00

Twentieth-year terminal dividend..................... $ 23.00

Age rating to allow for terminal dividend payable on death 10 years

Premiums are apportionable and a post-mortem dividend is payable.

Calculation:

$$\text{Cost} = 0.99435(32.94) - 0.03244(55.24) - 0.03156(180.70)$$
$$- 0.02619(440.00) - 0.02966(23.00)$$
$$= \$13.05.$$

It may be noted that the difference of $0.33 in cost between illustrations *a* and *b* arises solely from the inclusion of ancillary death benefits in *b*.

TABLE A

Policyholder Cost Factors

1958 C.S.O. 4 PER CENT (FOR UNITED STATES POLICIES)

Issue Age x	π^{AP*}	No Post-Mortem Dividend				With Post-Mortem Dividend				$10^5 \times P^{\,1}_{x:\overline{20\rceil}}$
		A		B		C		D		
		D_1^{10*}	D_1^{20*}	D_2^{10*}	D_2^{20*}	D_1^{10*}	D_1^{20*}	D_2^{10*}	D_2^{20*}	
0–10....	99,933	2,570	3,516	2,396	3,557	2,570	3,519	2,396	3,560	3,168
15.......	99,913	2,597	3,500	2,421	3,542	2,598	3,504	2,422	3,546	3,152
20.......	99,900	2,612	3,491	2,434	3,534	2,613	3,496	2,435	3,538	3,138
25.......	99,878	2,641	3,474	2,463	3,517	2,641	3,481	2,462	3,523	3,109
30.......	99,835	2,701	3,440	2,522	3,482	2,698	3,450	2,519	3,493	3,055
35.......	99,759	2,810	3,378	2,627	3,421	2,803	3,394	2,620	3,437	2,966
40.......	99,632	2,986	3,277	2,794	3,322	2,976	3,301	2,785	3,346	2,827
45.......	99,435	3,259	3,120	3,053	3,169	3,244	3,156	3,040	3,205	2,619
50.......	99,135	3,673	2,882	3,444	2,936	3,656	2,935	3,427	2,989	2,317
55.......	98,697	4,284	2,531	4,014	2,595	4,269	2,605	4,001	2,669	1,914
60.......	98,089	5,120	2,050	4,777	2,131	5,125	2,146	4,785	2,227	1,440

* These values are 10^5 times the values as defined above in Section 2, *c*, of the Appendix.

TABLE A—*Continued*

1958 C.S.O. 5 PER CENT (FOR CANADIAN POLICIES)

Issue Age x	π^{AP*}	No Post-Mortem Dividend				With Post-Mortem Dividend				$10^5 \times P^{\,1}_{x:\overline{20\rceil}}$
		A		B		C		D		
		D_1^{10*}	D_1^{20*}	D_2^{10*}	D_2^{20*}	D_1^{10*}	D_1^{20*}	D_2^{10*}	D_2^{20*}	
0–10....	99,933	3,126	3,192	2,897	3,246	3,127	3,195	2,898	3,249	2,825
15.......	99,914	3,152	3,177	2,921	3,232	3,154	3,180	2,923	3,235	2,810
20.......	99,901	3,166	3,169	2,934	3,224	3,168	3,173	2,935	3,228	2,797
25.......	99,880	3,194	3,153	2,961	3,208	3,194	3,159	2,961	3,214	2,771
30.......	99,839	3,251	3,120	3,016	3,175	3,249	3,129	3,014	3,185	2,722
35.......	99,766	3,354	3,061	3,115	3,117	3,350	3,075	3,111	3,132	2,641
40.......	99,643	3,522	2,964	3,273	3,023	3,514	2,986	3,266	3,045	2,516
45.......	99,452	3,781	2,815	3,517	2,877	3,771	2,848	3,508	2,911	2,327
50.......	99,161	4,175	2,588	3,884	2,657	4,163	2,637	3,874	2,706	2,055
55.......	98,735	4,754	2,255	4,418	2,334	4,747	2,323	4,413	2,402	1,693
60.......	98,140	5,543	1,799	5,131	1,896	5,560	1,886	5,149	1,984	1,268