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AN APPLICATION OF SIMULATED STOCK MARKET TRENDS TO INVESTIGATE A RUIN PROBLEM

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ABSTRACT

This paper describes a technique used to simulate stock market trends which are then applied to evaluate the adequacy of the investment risk premium for a certain type of equity-based endowment.

First a cumulative distribution function of monthly percentage changes in stock price indexes was produced from the average prices and yields of the Standard and Poor's stock price indexes (industrials). This cumulative distribution function was used to simulate by a Monte Carlo technique 1,000 stock market trends, which were in turn the basis for the calculation of possible values of the risk fund at time Ω generated by two scales of investment risk premium. The 1,000 values so calculated for each scale of investment risk premium were ranked by size, and two cumulative distribution functions of the risk fund at time Ω were obtained. By the use of these two cumulative distribution functions, the adequacy of each of the two scales of investment risk premium was assessed.

INTRODUCTION

THE purpose of this paper is to describe a Monte Carlo technique used by the author to simulate stock market trends and the manner in which the latter were then used to evaluate the adequacy of the investment risk premium for a certain type of equity-based endowment which guarantees a minimum death benefit and maturity value.

It might be argued that stock market fluctuations cannot be predicted by stochastic processes. Indeed, especially nowadays, fluctuations in stock market prices are significantly influenced by a large number of economic, political, and other forces which do not necessarily behave in a random fashion. The effect of such forces should tend to restrain extreme fluctuations and to avoid unusually prolonged periods of recession rather than to promote them. Thus it is the author's opinion that the use of simulated stock market trends to investigate the tail end of a ruin function, precisely because they allow for the random occurrence of such extremes (which in future real life may never occur), will lead to conclusions that may overstate the likelihood of ruin.

THE MONTE CARLO SIMULATION

The first step was to construct a time series of monthly indexes, reflecting the combined effect of monthly fluctuations in historical stock prices and yields. The base was set at 10.00 on December 31, 1915, and subsequent monthly indexes to December 31, 1965, were calculated reflecting the following factors:

1. The change in the monthly average of Standard and Poor's stock price indexes (industrials).
2. The monthly average yields of Standard and Poor's stock price indexes (industrials).[1] Yields were assumed to have been immediately reinvested.
3. A management fee of 0.04 per cent of the fund per month.
4. No Capital Gain or Income Taxes.

The resulting monthly indexes for each year-end are shown in Table 1.

The next step was to develop a frequency table of observed percentage changes in the six hundred and one monthly indexes of the time series. A

TABLE 1

TIME SERIES OF STOCK PRICE INDEXES (WITH DIVIDENDS REINVESTED)

(Based on Changes in Standard and Poor's Stock Price Indexes and Average Yields [Industrials])

End of Calendar Year (1)	Index* (2)	End of Calendar Year (1)	Index* (2)	End of Calendar Year (1)	Index* (2)
1915	10.00	1932	15.90	1949	103.94
1916	11.41	1933	29.13	1950	135.23
1917	8.11	1934	28.84	1951	171.21
1918	10.11	1935	41.77	1952	199.33
1919	13.75	1936	57.00	1953	198.80
1920	9.43	1937	38.74	1954	302.21
1921	10.58	1938	47.91	1955	417.79
1922	13.39	1939	47.32	1956	446.33
1923	13.62	1940	42.66	1957	402.64
1924	16.37	1941	38.86	1958	549.27
1925	21.85	1942	45.33	1959	627.77
1926	25.07	1943	55.81	1960	612.10
1927	35.25	1944	65.74	1961	789.41
1928	49.32	1945	88.42	1962	702.60
1929	43.88	1946	79.66	1963	862.02
1930	32.75	1947	85.25	1964	1,000.04
1931	18.67	1948	90.68	1965	1,128.27

* Indexes include a management fee of 0.04 per cent of the fund per month.

[1] Standard and Poor's monthly average yields are not available from 1916 to 1925; thus an arbitrary 4 per cent per annum was used.

histogram of these observed percentage changes is given in Figure 1. The observed data were then graduated by a Whittaker-Henderson formula of type A with a equal to 2, and from the graduated data the cumulative distribution function reproduced in Table 2 was obtained.

The cumulative distribution function given in Table 2 was subjected to 600,000 random numbers generated by an IBM 360 computer (employing a package program described in IBM Manual H 20-0205-0), and 1,000 simulated stock market trends, each composed of 601 monthly indexes, were constructed.

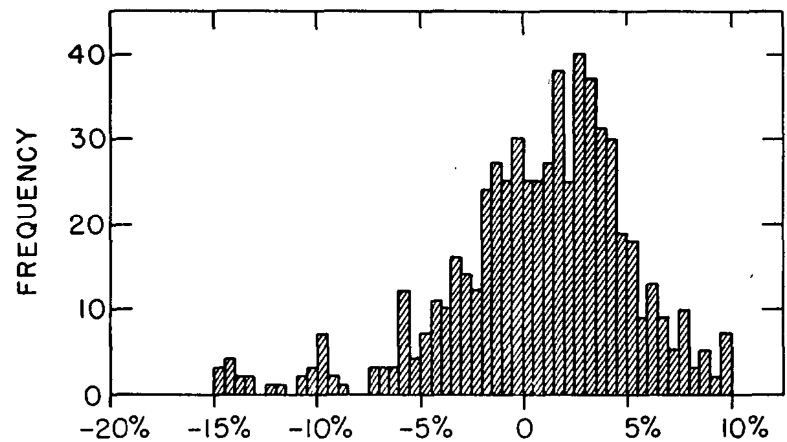

FIG. 1.—Histogram of observed monthly percentage changes in stock price indexes (with dividends reinvested).

The Monte Carlo simulation was done in the following manner:

1. The first random number N_1 was entered in Table 2, and the interval Y_s to Y_{s+1} was found, such that $Y_s \leq N_1 < Y_{s+1}$. This routine yielded the mean abscissa $\bar{X}_{s,\,s+1}$, corresponding to the randomly selected interval Y_s to Y_{s+1}.
2. $\bar{X}_{s,\,s+1}$ was taken to be the first percentage change by which the base index of 10.00 was adjusted to obtain the second monthly index.
3. The second random number N_2 was entered in Table 2 and, by following the routines described above, we produced the third monthly index and so on until the first simulated stock market trend was completed.
4. The routines described from (1) to (3) were repeated 999 times, and, as a result, 1,000 simulated stock market trends were developed.

The resulting lowest, median, mean, and highest indexes at various annual durations are shown in Table 3. The fifty-year average annual growth rate (with dividends reinvested) of the original time series given in Table 1 is 9.91 per cent, while the comparable average growth rate of the

TABLE 2

CUMULATIVE DISTRIBUTION FUNCTION OF MONTHLY PERCENTAGE CHANGES IN STOCK PRICE INDEXES (WITH DIVIDENDS REINVESTED)

$\bar{X}_{s,s+1}$ (1)	Cum. Distr. Function Y_s to Y_{s+1} (2)	$\bar{X}_{s,s+1}$ (1)	Cum. Distr. Function Y_s to Y_{s+1} (2)
−14.75%	0.0000–0.0034	−2.25%	0.1842–0.2131
−14.25	.0034– .0080	−1.75	.2131–0.2457
−13.75	.0080– .0119	−1.25	.2457–0.2820
−13.25	.0119– .0152	−0.75	.2820–0.3217
−12.75	.0152– .0179	−0.25	.3217–0.3645
−12.25	.0179– .0204	+0.25	.3645–0.4101
−11.75	.0204– .0229	+0.75	.4101–0.4583
−11.25	.0229– .0256	+1.25	.4583–0.5091
−10.75	.0256– .0287	+1.75	.5091–0.5621
−10.25	.0287– .0323	+2.25	.5621–0.6164
− 9.75	.0323– .0359	+2.75	.6164–0.6707
− 9.25	.0359– .0393	+3.25	.6707–0.7234
− 8.75	.0393– .0423	+3.75	.7234–0.7723
− 8.25	.0423– .0453	+4.25	.7723–0.8160
− 7.75	.0453– .0487	+4.75	.8160–0.8535
− 7.25	.0487– .0531	+5.25	.8535–0.8845
− 6.75	.0531– .0588	+5.75	.8845–0.9097
− 6.25	.0588– .0664	+6.25	.9097–0.9298
− 5.75	.0664– .0759	+6.75	.9298–0.9458
− 5.25	.0759– .0875	+7.25	.9458–0.9586
− 4.75	.0875– .1012	+7.75	.9586–0.9691
− 4.25	.1012– .1176	+8.25	.9691–0.9778
− 3.75	.1176– .1367	+8.75	.9778–0.9856
− 3.25	.1367– .1589	+9.25	.9856–0.9928
− 2.75	0.1589–0.1842	+9.75	0.9928–1.0000

TABLE 3

SIMULATED STOCK PRICE INDEXES (WITH DIVIDENDS REINVESTED)

(Base of 10.00; 1,000 Simulations)

At End of Year t (1)	Lowest (2)	Median (3)	Mean (4)	Highest (5)
1	5.791	10.963	10.989	16.216
2	5.596	11.818	12.044	22.828
3	4.053	12.675	13.096	27.782
4	3.926	13.783	14.340	29.352
5	3.988	14.954	15.684	34.839
10	3.310	21.566	23.984	104.265
15	4.448	30.878	36.646	308.861
20	5.305	46.484	56.444	605.207
25	5.638	71.840	90.504	1,036.908
30	8.121	103.509	140.479	1,954.278
35	12.730	158.782	221.528	3,232.158
40	18.770	231.488	355.322	4,848.439
45	22.557	336.804	560.147	7,450.170
50	29.632	491.550	868.555	10,942.400
50-year annual growth rate	2.20%	8.10%	9.34%	15.02%

mean value of the 1,000 simulated stock market trends is 9.34 per cent. The fact that the latter is smaller than the former is possibly because the number of simulations was insufficient. In the author's opinion, however, the relationship between the two growth rates suggests that the use of the 1,000 simulated stock market trends to investigate the ruin problem described in the following pages would tend to slightly overstate the probability of ruin.

THE RUIN PROBLEM

Under a certain type of equity-based endowment, the insured agrees to pay throughout the n policy years of the endowment period a level monthly premium of $SA/12n$, and the company guarantees to pay not less than the sum assured (SA) at maturity or prior death. As each contract premium is paid, certain deductions are made to cover administrative, sales, and other expenses, and the remainder is used to purchase units in a segregated fund at the then current unit value. The segregated fund, which is entirely invested in common stock, is valued at the beginning of each month, and the unit value then determined is used for all transactions taking place throughout the month.

In the event of surrender prior to maturity, the insured receives a surrender value equal to the value of the accumulated units less a small surrender charge which decreases to zero at maturity. For the purpose of this paper it is assumed that surrender charges are equal to actual surrender expenses.

In the event of death, the beneficiary receives the guaranteed sum assured plus the excess, if any, of the value of the accumulated units over the guaranteed sum assured. A level mortality premium is deducted from the contract premium and paid to the general funds of the company to purchase, as it were, decreasing term insurance. At any particular time, the amount of term insurance that this premium purchases is assumed to equal the difference between the sum assured and the value of a predetermined notional asset share, which is an estimate of the value of the accumulated units at that time. If death occurs, the general funds pay the excess of the contract's sum assured over the actual value of its accumulated units. Thus the general funds stand to realize a profit or loss, depending on whether the actual value of the accumulated units is greater or lower than the predetermined notional asset share.

The general funds also stand to realize additional mortality profits or losses, depending on the profit margin included in the decreasing term premium and the emerging experience. The ruin associated with this type of risk is, however, beyond the scope of this paper, which assumes that the decreasing term premium precisely reflects future mortality experience.

At maturity, the insured receives the value of the accumulated units. If, however, the value of these units is less than the sum assured, the general funds must make up the difference. As compensation for the investment risk assumed, with respect to both the maturity value and the death benefit, the general funds receive an investment risk premium. To provide a cushion against the inordinate occurrence of losses in the event of significant and sustained decreases in the unit value, such premiums are accumulated at interest in the general funds to build up a risk fund. In effect, maturity and mortality losses, whenever they occur, are decrements to the risk fund, while mortality profits, investment risk premiums, and interest earnings, if any, are increments. When the last contract matures, any positive balance in the risk fund represents the accumulated profit to the general funds, while a negative balance represents the accumulated loss.

To arrive at a suitable investment risk premium, the following points were taken into account:

1. An investment risk premium can be assumed to be adequate if the probability of the risk fund being in a state of ruin after the last contract matures is reasonably small, say, not greater than 0.10.

2. The investment risk premium reduces the amount available for investment in the segregated fund, and consequently it reduces the payout to contractholders in relation to the guaranteed sum assured. Thus the optimum investment risk premium is one that brings the probability of ruin within the acceptable limit and does not unnecessarily reduce the premium available for investment in the segregated fund.

3. Administrative, sales, and other expenses also reduce the portion of premium available for investment in the segregated fund. In other words, heavier expenses tend to increase the probability of maturity and mortality losses.

4. Mortality and voluntary terminations reduce the number of contractholders reaching maturity and therefore reduce the probability of maturity losses. Heavy mortality experience, however, would increase the probability of mortality losses, unless the notional asset share were calculated on the assumption of a low growth rate (in which case, heavier mortality might tend to increase mortality profits).

5. Obviously, short-term endowments involve a higher risk of maturity losses than do longer-term endowments. However, because of larger commissions and mortality premiums, the longer plans incur heavier expenses.

6. Finally, equity-based endowments are very complex to administer, and it would seem desirable that the administration of the risk fund be kept as simple as possible. Since we expected to forbid the issue of plans maturing in less than fifteen years or past age 65, it appeared reasonable that an investment

risk premium expressed as a flat percentage of the contract premium for all ages and plans would be simple to administer and would not significantly distort equities between contractholders.

The ruin function developed for the purpose of testing the effect of using as investment risk premium either of two flat percentages of the contract premium is described in the following section.

THE RUIN FUNCTION

The simulated stock market trends make it possible to estimate the probability of the risk fund being in a state of ruin at the end of the calendar year in which the last contract matures. The risk fund at Ω is given by

$$\mathrm{RF}_{\Omega} = \sum_{r=1}^{12\Omega}(\mathrm{RP}_r + \mathrm{Mor.}\,P_r - \mathrm{Mor.}\,L_r - \mathrm{Mat.}\,L_r)\left(1+\frac{i}{12}\right)^{12\Omega-r+1/2},$$

where

Ω = Number of complete calendar years in the calculation period, spanning from the beginning of the year in which the first contract is issued to the end of the year in which the last one matures.

RP_r = Investment risk premium received during the rth calendar month of the calculation period.

Mor. P_r = Mortality profits realized during the rth calendar month of the calculation period.

Mor. L_r = Mortality losses incurred during the rth calendar month of the calculation period.

Mat. L_r = Maturity losses incurred during the rth calendar month of the calculation period.

The mathematical derivation of the foregoing formula is given in the Appendix.

For each of the simulated stock market trends, and by means of a model office, a value of RF_{Ω} is calculated for each investment risk premium to be tested. Such values are then ranked by size, and the cumulative distribution function $G(X, \Omega, \alpha)$ is obtained. $G(X, \Omega, \alpha)$ yields the probability that $\mathrm{RF}_{\Omega} \leq X$, given that the investment risk premium is α per cent of the contract premium.

It is possible that the risk fund will be in a state of ruin at some point of time prior to Ω. For the purpose of the calculation described in this paper, it is assumed that the company's free surplus will always be sufficient to cover such risk-fund deficiencies. In effect, this assumption is similar to the one made in the calculation of endowment premiums where early valuation strains or other temporary losses due to mortality and/or interest fluctuations are assumed to be absorbed by the company's surplus.

The formula on page 555 was used to calculate, for each of the 1,000 simulated stock market trends, a risk fund at $\Omega = 40$, generated by the following:

1. A model block of equity-based endowments, ranging from 15 to 35 years, and not to mature past age 65, issued over a period of five years at the rate of $1,000,000 of annualized premium each year;
2. Two possible investment risk premiums, namely, a flat 1 and 2 per cent of the contract premium for all ages and plans; and
3. Testing assumptions (i.e., mortality, expenses, compensation, etc.) normally used to calculate regular endowment test premiums, as well as a notional asset share based on an interest rate of $5\frac{1}{2}$ per cent.

TABLE 4

CUMULATIVE DISTRIBUTION FUNCTION OF THE RISK FUND

Value of $G(X, 40, a)$	Risk Fund (In thousands)	
	X, when $a = 1\%$	X, when $a = 2\%$
0.001	−37,040	−33,154
0.010	−19,596	−18,416
0.020	−13,938	−13,814
0.030	−11,442	−10,108
0.040	− 8,507	− 7,390
0.050	− 6,698	− 5,283
0.060	− 4,840	− 4,442
0.070	− 3,535	− 2,739
0.080	− 1,998	− 1,364
0.090	− 1,117	− 114
0.100	− 165	998
0.150	2,679	3,945
0.200	4,179	6,061
0.250	5,081	7,353
0.300	5,494	8,207
0.350	5,735	8,699
0.400	6,016	9,068
0.450	6,216	9,476
0.500	6,343	9,691
0.550	6,469	9,824
0.600	6,623	10,003
0.650	6,730	10,120
0.700	6,849	10,224
0.750	6,995	10,374
0.800	7,139	10,532
0.850	7,307	10,719
0.900	7,484	10,899
0.950	7,769	11,190
1.000	8,623	12,043
Expected value of the risk fund at Ω	4,640	7,402

The arguments, X, of the cumulative distribution function of the risk fund at Ω for the two values of α are shown in Table 4 for successive values of $G(X, 40, \alpha)$.

It must be noted that, because of the rather low notional asset share assumed, the expected mortality profit is significant. This contributes to the relatively small difference between the two expected values of the risk fund shown in Table 4. Nevertheless, the expected value of the risk fund at Ω with $\alpha = 2$ per cent is almost 60 per cent higher than the expected value of the risk fund with $\alpha = 1$ per cent. However, $G(0, 40, 2$ per cent$)$ is approximately 0.09, which is only slightly lower than $G(0, 40, 1$ per cent$)$, which is about 0.10. Thus it seems reasonable to conclude, in the case of the equity-based endowment described in this paper, that an investment risk premium in excess of 1 per cent of the contract premium does not significantly reduce the probability of ruin, but it does affect unfavorably the eventual payout to contractholders.

ACKNOWLEDGMENT

The author wishes to express his gratitude to Mr. K. A. Wright, F.S.A., who directed the computer calculations of the data included in the tables given herein.

APPENDIX

This appendix shows the mathematical derivation of the risk fund at Ω associated with the zth simulated stock market trend.

Definitions:

$l^m_{[x]}$ = Number of lives aged x purchasing equity-based endowment contracts in the mth calendar month, measured from the beginning of the calculation period. Contracts are assumed to be issued in the middle of the calendar month.

$l^m_{[x]+(r-m)/12}$ = Survivors from $l^m_{[x]}$ who pay the premium in the rth calendar month (again measured from the beginning of the calculation period):

$$l^m_{[x]+(r-m)/12} = l^m_{[x]+(r-m-1)/12}[1 - {}_{1/12}q_{[x]+(r-m-1)/12}] \times [1 - q^{(w)}_{(r-m)/12}] .$$

${}_{1/12}q_{[x]+(r-m-1)/12}$ = Probability of dying in the contract month ending in the rth calendar month. Deaths are assumed to occur at the end of the contract month.

$q^{(w)}_{(r-m)/12}$ = Probability of voluntarily withdrawing in the contract month ending in the rth calendar month. Withdrawals are assumed to occur at the end of the contract month.

${}^n\beta^m_x$ = Proportion of $l^m_{[x]}$ lives purchasing n-year equity-based endowments.

${}^n\mathrm{SA}^m$ = Average sum assured of all n-year equity-based endowments issued in the mth calendar month.

${}^n\mathrm{SA}^m/12n$ = Average monthly premium of all n-year equity-based endowments issued in the mth calendar month.

${}^n\mathrm{NAS}^m_{(r-m)/12}$ = Value of the predetermined notional asset share for the average n-year equity-based endowment at the end of the $(r-m)$th contract month:

$$ {}^n\mathrm{NAS}^m_{(r-m)/12} = {}^n\mathrm{SA}^m \left(v^{n-(r-m)/12} - \frac{\ddot{a}^{(12)}_{\overline{n-(r-m)/12}|}}{\ddot{s}^{(12)}_{\overline{n}|}} \right) $$

${}_z\mathrm{UV}_r$ = Unit value at the beginning of the rth calendar month, based on the zth simulated stock market trend.

${}^n_zN^m_{[x]+(r-m)/12}$ = Accumulated number of units allocated to the average n-year equity-based endowment at the beginning of the $(r-m+1)$th contract month, immediately after the allocation of new units for that contract month:

$$ {}^n_zN^m_{[x]+(r-m)/12} = \sum_{s=0}^{r-m} \frac{{}^n\mathrm{SA}^m}{12n} \frac{(1 - {}^nE^m_{[x]+s/12})}{{}_z\mathrm{UV}_{s+m}} . $$

${}^nE^m_{[x]+(r-m)/12}$ = Total expenses (including the investment risk premium) incurred at the beginning of the $(r-m+1)$th contract month, per dollar of monthly contract premium.

$$ \mathrm{RP}_r = \sum_m \sum_x \sum_n {}^n\beta^m_x l^m_{[x]+(r-m)/12} \frac{{}^n\mathrm{SA}^m}{12n} \frac{\alpha}{100} . $$

$$ {}_z\mathrm{Mor.}\ L_r = \sum_m \sum_x \sum_n {}^n\beta^m_x l^m_{[x]+(r-m-1)/12} [{}^n\mathrm{NAS}^m_{(r-m)/12} $$

$$ - {}^n_zN^m_{[x]+(r-m-1)/12} \cdot {}_z\mathrm{UV}_r]_{1/12}q_{[x]+(r-m-1)/12} ; $$

whenever ${}^n\mathrm{NAS}^m_{(r-m)/12} - {}^n_zN^m_{[x]+(r-m-1)/12} \cdot {}_z\mathrm{UV}_r \leq 0$,

it is taken as 0.

$$\text{Mor. } P_r = \sum_m \sum_x \sum_n {}^n\beta_x^m l^m_{[x]+(r-m-1)/12}[{}^n_s N^m_{[x]+(r-m-1)/12} \cdot {}_s\text{UV}_r$$

$$- {}^n\text{NAS}^m_{(r-m)/12}]_{1/12}q_{[x]+(r-m-1)/12} ;$$

whenever ${}^n_s N^m_{[x]+(r-m-1)/12} \cdot {}_s\text{UV}_r - {}^n\text{NAS}^m_{(r-m)/12} \leq 0$,

it is taken as 0;

whenever ${}^n_s N^m_{[x]+(r-m-1)/12} \cdot {}_s\text{UV}_r \geq {}^n\text{SA}^m$,

it is taken as ${}^n\text{SA}^m$.

$${}_s\text{Mat. } L_r = \sum_m \sum_x \sum_n {}^n\beta_x^m l^m_{[x]+(r-m-1)/12}$$

$$\times [{}^n\text{SA}^m - {}^n_s N^m_{[x]+(r-m-1)/12} \cdot {}_s\text{UV}_r]$$

for $\frac{r-m}{12} = n$;

whenever ${}^n\text{SA}^m - {}^n_s N^m_{[x]+(r-m-1)/12} \cdot {}_s\text{UV}_r \leq 0$,

it is taken as 0.

Therefore

$${}_s\text{RF}_\Omega = \sum_{r=1}^{12\Omega} (RP_r + {}_s\text{ Mor. } P_r - {}_s\text{Mor. } L_r - {}_s\text{Mat. } L_r) \times \left(1 + \frac{i}{12}\right)^{12\Omega - r + 1/2}$$

DISCUSSION OF PRECEDING PAPER

MR. DONALD D. CODY:

Mr. DiPaolo found that the probability of ruin was virtually unchanged when the premium was doubled. Does this imply that the risk-premium determination is uncertain and undependable?

MR. DI PAOLO:

For the specific type of business that I investigated, it makes little difference whether the risk premium is 1 or 2 per cent. There are a number of factors interplaying here. As the risk premium increases, the risk fund increases, but the amount of money available for investment in the segregated fund is smaller and consequently the probability of a maturity loss is larger. Somehow the interplay of these factors is such that, for the model I used, the probability of ruin at Ω remains at about 0.10 as the risk premium is changed from 1 per cent to 2 per cent. After the paper was written, I experimented with a risk premium of $\frac{1}{2}$ per cent, and the probability of ruin jumped to about 0.15. This confirmed that, for the equity-based endowment described in the paper, the optimum risk premium is indeed 1 per cent.

MR. HARRY WALKER:

I wonder whether the question of the reserves, to be held throughout the collection of risk premiums, from which the cost of the benefits is deducted, has also been looked at? Or the question of the situation in which you are unfortunate enough to incur large losses in the early years of your contract and have to sell assets and rely on future risk premiums to meet those losses? In other words, to what extent can you expect or will you have to provide for a strain on the surplus of the company to help bail you out temporarily in anticipation of the collection of future risk premiums?

MR. DI PAOLO:

As Mr. Walker suspects, it is quite possible for the risk fund to find itself in a negative position long before Ω. The assumption is made, however, that early risk-fund deficiencies will be covered by the company's free surplus, which is assumed to be sufficiently large that it can be subjected to significant strains. This assumption is, in effect, analogous to the one made in the calculation of gross premiums for regular endowments, where early losses due to valuation strain and/or mortality fluctuations, and so forth, are assumed to be absorbed by the company's free surplus. I do not believe that this is an unreasonable assumption, as long as we are dealing with a block of equity-based endowments which is considerably small in relation to the size of the company. On the other hand, the problem of risk-fund deficiencies could be quite a serious one if the block of these contracts is allowed to grow too large in relation to the company's free surplus.