

NET COST COMPARISON OF DISSIMILAR LIFE INSURANCE CONTRACTS: THE STANDARD MORTALITY COST METHOD

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ABSTRACT

This paper describes a method of comparing net costs of dissimilar life insurance contracts. The principles of the method are consistent with those of the interest-adjusted method recommended by the Moorhead committee. Results of the application of the proposed standard mortality cost method to representative contracts of leading companies are summarized. The significance of these results, possible applications of the method, and some concerns, reservations, and conclusions are discussed.

I. INTRODUCTION

THOUGHTFUL life insurance buyers and sellers are deeply indebted to the Moorhead committee for proposing, in a workmanlike and authoritative manner, a practical, understandable, and usable method of comparing net costs of similar life insurance contracts.¹ It is generally recognized that the committee's proposed interest-adjusted method is not suitable for comparing dissimilar plans, because it does not take into account differences in the net amount at risk. The committee report states:

An ideal method should also be general enough that it would be applicable to any comparison the buyer has logical reason to make . . . and a really satisfactory price illustration method should . . . be applicable to the following:

- a. Comparison of dissimilar contracts offered by the same insurance company.
- b. Comparison of contracts not necessarily similar offered by competing insurance companies.²

To these two situations could be added an important third: comparison of contracts issued in different years (specifically, a new contract proposed to replace one which has been in force for a number of years),

¹ Joint Special Committee on Life Insurance Costs, Report to American Life Convention, Institute of Life Insurance, Life Insurance Association of America, May 4, 1970.

² *Ibid.*, p. 24.

whether similar or dissimilar and whether offered by the same or by different companies. The standard mortality cost method, described in this paper, should be valid and useful in all three of these situations.

The committee report stresses the suitability of the interest-adjusted method for five reasons:

- (1) It takes the time of payment into account.
- (2) Of all the methods that take time of payment into account, it is the easiest to understand.
- (3) It is possible to use this method without having recourse to advanced mathematics.
- (4) It does not suggest a degree of accuracy that is beyond that justified by the circumstances.
- (5) It is sufficiently similar to the Traditional Method so that transition could be accomplished with minimum confusion.³

By and large, the standard mortality cost method has these same qualities, although the second point would have to read: "Of all the methods that take time of payment and net amount at risk into account, it is the easiest to understand." When applied to policies with identical amounts at risk, it produces cost rankings identical with those produced by the interest-adjusted method. In addition, the standard mortality cost method has four qualities relating to the comparison of dissimilar plans:

1. It takes differing net amounts at risk into account.
2. It takes differing economic values of the net amount at risk into account.
3. It takes the incidence (or time of occurrence) of these differing economic values into account.
4. It takes into account the salvage value (or net surrender value) of one or both contracts at the beginning of the period studied. In other words, it provides a valid method for the economic analysis of replacement proposals.

As an extension of the technique of the interest-adjusted method to the comparison of dissimilar plans, it is hoped that the standard mortality cost method will be considered a valid addition to the excellent groundwork laid by the Moorhead committee.

II. DESCRIPTION OF THE METHOD

The committee report describes the five steps required to apply the interest-adjusted method to a policy with level annual premiums.⁴ Since these same steps are embodied in the standard mortality cost method, they will be stated in the following description of the latter method, but without repetition of the excellent discussion material appended to the

³ *Ibid.*, p. 21.

⁴ *Ibid.*

description of each step in the committee report. Accordingly, the reader is urged to review the committee report before attempting to use or evaluate the standard mortality cost method.

The steps required to apply the standard mortality cost method to a policy with level annual premiums are as follows:

1. Select the period over which the analysis is to be made.
2. Select the interest rate to be used.
3. Accumulate the annual dividends, if any, at interest to the end of the selected period, and add to them the cash value (and terminal dividend, if any) available at the end of the period.
4. Divide the result of step 3 by an interest factor that converts it into a level annual amount accruing over (i.e., a sinking fund factor for) the selected period.
5. Subtract the result of step 4 from the annual premium. This remainder is the interest-adjusted cost.
6. Determine the net amount at risk each year by subtracting the cash value from the face amount and adding the lump-sum benefit of any term rider.
7. Multiply each year's net amount at risk by the standard mortality rate for the age at issue plus the policy year minus one. Because of its availability, the probability of dying, taken from the 1958 CSO Mortality Table, may be most suitable. Consideration might be given to the 1958 CSO Basic Table of Mortality.
8. Accumulate the products from step 7 to the end of the selected period (in the same manner as the dividends in step 3).
9. Divide the result of step 8 by the same interest factor used in step 4. The result is the standard mortality cost.
10. Divide the interest-adjusted cost by the result of step 9, to obtain the cost per dollar of standard mortality cost.

The modification of the procedure for policies with varying premiums is, of course, identical with that for the interest-adjusted cost method as described in the committee report.⁵

III. INTERPRETATION OF THE RESULT

In common with the interest-adjusted cost method, any consideration based on a comparison of other quantitative cost measures must be surrounded by all the usual caveats concerning the nonguaranteed nature of dividends, the nonmathematical aspects of the contract, the value of the agents' services, and other considerations. Subject to these caveats, the cost per dollar of standard mortality cost may be viewed by the policyholder as an index of his prospective cost of carrying the proposed program, with the following guidelines: (1) an index of 1.00 indicates no

⁵ *Ibid.*

expense cost (assuming that the mortality table used in step 7 is a good representation of his probable death rates), and (2) the lower the index value, the more attractive the policy is to the buyer, all other considerations being equal.

Strong arguments may be raised for replacing step 10 with the following alternative: Subtract the standard mortality cost from the interest-adjusted cost to obtain the level expense cost. This is a tempting proposal, since it isolates the nonbenefit, or expense, element of the premium in terms of a level annual charge expressed in terms directly comparable to the gross premium or to the interest-adjusted cost. On further reflection, however, this apparent advantage seems outweighed by the following potential danger: in comparing two policies, there would be a tendency to choose the one with the lower level expense cost. This choice would represent the better buy only if the standard mortality costs of the two policies were equal or very nearly so. In most cases this will not be true, since the method will be used to compare dissimilar policies.

Note that the two costs are accumulated to a common date before the ratio is taken. This is no mere mathematical convenience but rather an essential procedure to reflect the economic value of the year-by-year incidence of each cost.

The cost per dollar of standard mortality cost might be thought of as the ratio of premiums to claims after the removal of the investment element of the contract. As such, it might furnish a rough guide to actuaries in multiline companies in broad pricing considerations. This is not to say that the ratio should be the same on all lines (or even on all policies within the individual life line), since significant differences may be justified by actual expense considerations.

To make maximum use of the cost per dollar of standard mortality cost, the buyer should be encouraged to understand the concept. For the more thoughtful buyers (presumed to be those who seek a comparison in the first place) this should not be an impractical goal, if taken in easy instalments.

1. The cash value (together with any other termination values), can be viewed as having an economic benefit equivalent to the level annual payment obtained in step 4. If the savings account analogy is used to clarify this concept, care must be taken to point out that it is not suggested as a practical alternative, since the benefits of the life insurance contract are not severable. If there are dividends, they can be thought of as being left to accumulate. Again, this is not to be confused with the recommended course.
2. The excess of the gross premium over this other level figure (this excess is the interest-adjusted cost) can be thought of as the amount paid for protection and other services rendered.

3. The cost per dollar of standard mortality cost may then be thought of as the amount paid for all services for each dollar's worth of "pure protection."
4. Emphasis should be placed on the selection of an interest rate which is personally applicable to the buyer and is a net rate (after income taxes), a long-term rate, and a near-riskless rate.
5. With proper emphasis, then, the cost per dollar of standard mortality cost becomes a personalized, valid decision-making element in selecting the appropriate contract—not a method of comparing company performance or desirability.

The net interest rate for near-riskless, long-term money for different buyers can differ significantly. Compare, for example, two buyers, both with large mortgage loans payable over a long term, one with a $4\frac{1}{2}$ per cent rate, the other with an 8 per cent rate. Or consider one with a high-rate mortgage loan and another with no indebtedness. Or consider one buyer in a 20 per cent tax bracket and another in a 60 per cent bracket. For a really dramatic comparison, consider the relative positions of the high-bracket taxpayer with no mortgage and the low-bracket taxpayer with a high loan rate—both situations being virtually the norm for each economic class.

It is vital, then, to emphasize the personalized nature of the application of the method and the importance of choosing a valid interest rate. The different conclusions to which such a method will lead two different buyers considering the same two policies are dramatized by Tables 4-9. Truly, in the purchase of life insurance, "one man's meat is another's poison." Valid use of the method will reinforce the conclusion, already reached by a minority of students of life insurance buying, that "buy term and invest the difference" is not as appropriate for the more affluent as it is for the middle-income buyer burdened with a large mortgage loan at a high interest rate, especially when proper emphasis is given to the importance of using a near-riskless rate in making the comparisons.

IV. ACTUARIAL PROOF

In the first paragraph of Section III two guidelines for interpretation were given. The properties of the standard mortality cost method which are implied in the first guideline might be restated as follows: In a policy containing no loading or expense charge, the cost per dollar of standard mortality cost equals \$1.00; that is, the interest-adjusted cost equals the standard mortality cost. This statement is capable of mathematical proof, given the following contract conditions:

1. The gross premium equals the net level reserve premium.
2. The reserve interest rate equals the selected interest rate.

3. The reserve mortality table is the standard mortality table used in step 7, Section II.
4. The cash value equals the reserve.

With these conditions, we have, for an n -year term policy,

Standard mortality cost (SMC) =

$$\frac{q_x(1+i)^{n-1}(1 - {}_1V_{x:\overline{n}|}^1) + q_{x+1}(1+i)^{n-2}(1 - {}_2V_{x:\overline{n}|}^1) + \dots + q_{x+n-1}}{\ddot{s}_{\overline{n}|}};$$

$$\text{Interest-adjusted cost (IAC)} = P_{x:\overline{n}|}^1;$$

Cost per dollar of SMC = 1.00 if IAC = SMC.

The mathematical portion of an inductive form of proof is shown in Appendix B.

Under the same conditions, we have, for an n -year endowment policy,

SMC =

$$\frac{q_x(1+i)^{n-1}(1 - {}_1V_{x:\overline{n}|}) + q_{x+1}(1+i)^{n-2}(1 - {}_2V_{x:\overline{n}|}) + \dots + q_{x+n-1} \cdot 0}{\ddot{s}_{\overline{n}|}};$$

$$\text{IAC} = P_{x:\overline{n}|} - \frac{1}{\ddot{s}_{\overline{n}|}}.$$

Again the statement is satisfied if IAC = SMC; and again the mathematical portion of the proof is shown in Appendix B.

V. EXPLANATION OF THE TABLES

Table 1 shows the interest-adjusted cost, the standard mortality cost, and the cost per dollar of standard mortality cost for one plan, age, interest rate, and period for policies issued by twenty companies. It also shows the company rank by both measures. Its primary purpose is to help the reader get the feel of these measures. The insignificant difference in ranking between the two measures shows that the additional effort required by the standard mortality cost method as compared with the interest-adjusted method is not justified when similar plans are compared.

Table 2 shows the cost per dollar of standard mortality cost for one interest rate, age, and period but for three dissimilar plans of insurance. It also shows the company rankings for each plan of insurance. It appears to show that for a young person to whom near-riskless, long-term money is worth about 4 per cent, net after taxes, the whole life plan is the best choice (out of these three plans), other considerations being equal, in

five out of nine participating policies. On the nonparticipating basis the term to age 65 plan looks superior in ten out of eleven companies.

Table 3 is similar to Table 2, but for a different age-period combination. For age 55 at issue the best choice among participating policies is evenly divided between the term to age 65 and the whole life policy. On the nonparticipating basis the choice is seven to three in favor of the term policy. In general, the life paid at age 65 plan shows up as a poor third.

Other considerations being equal (and it may be argued validly that they never are, precisely), it may be assumed that a buyer would consider the plan with the lowest cost factor to be the "best buy," provided that

TABLE 1
 TWENTY-YEAR SURRENDERED COSTS
 \$10,000 WHOLE LIFE POLICY
 MALE, AGE 35 AT ISSUE, 4 PER CENT INTEREST
 1958 CSO BASIC MORTALITY TABLE

COMPANY NUMBER	INTEREST-ADJUSTED COST (IAC)	STANDARD MORTALITY COST (SMC)	COST PER \$1 OF STANDARD MORTALITY COST (C/SMC)	COMPANY RANK	
				By IAC	By C/SMC
Participating Policies					
1.....	\$ 68	\$32	\$2.13	6	6
2.....	55	31	1.79	2	2
3.....	72	31	2.33	8	8
4.....	61	31	1.95	5	5
5.....	58	31	1.90	3	4
6.....	69	31	2.22	7	7
7.....	55	31	1.78	1	1
8.....	59	32	1.88	4	3
9.....	77	31	2.51	9	9
Nonparticipating Policies					
1.....	\$ 66	\$31	\$2.10	1	1
2.....	82	32	2.55	8	8
3.....	76	32	2.38	6	6
4.....	70	31	2.24	2	2
5.....	76	33	2.33	5	4
6.....	112	31	3.67	11	11
7.....	91	32	2.89	10	10
8.....	74	31	2.36	4	5
9.....	73	32	2.31	3	3
10.....	85	32	2.66	9	9
11.....	80	32	2.49	7	7

it lay within the range of plans that fitted his needs for protection and his ability to pay the premium. Table 4 shows, for participating contracts, the number of companies for which each of the three plans studied is the "best buy" over a twenty-year period. (The quotation marks are used to keep us reminded that there are other considerations in determining what is truly the best buy.) The figures show most dramatically that the "best buy" is a dynamic function of the value of near-riskless money to the buyer and, to a lesser degree, a function of his age. Tables 5-9 show

TABLE 2
 TWENTY-YEAR SURRENDERED COSTS
 \$10,000 POLICY
 MALE, AGE 35 AT ISSUE, 4 PER CENT INTEREST
 1958 CSO BASIC MORTALITY TABLE
 COST PER DOLLAR OF STANDARD MORTALITY COST

COMPANY NUMBER	COST PER DOLLAR OF STANDARD MORTALITY COST			RANK		
	Term to Age 65	Whole Life	Life Paid at Age 65	Term to Age 65	Whole Life	Life Paid at Age 65
Participating Policies						
1.....	\$2.04	\$2.13	\$2.65	5	6	7
2.....	2.48	1.79	2.14	8	2	5
3.....	2.28	2.33	2.87	6	8	8
4.....	2.31	1.95	2.08	7	5	4
5.....	1.96	1.90	2.20	3	4	6
6.....	1.94	2.22	2.03	2	7	3
7.....	1.84	1.78	1.96	1	1	2
8.....	1.99	1.88	1.37	4	3	1
9.....	2.52	2.51	2.91	9	9	9
Nonparticipating Policies						
1.....	\$2.03	\$2.10	\$2.80	1	1	3
2.....	2.12	2.55	2.78	2	8	2
3.....	2.21	2.38	2.70	3	6	1
4.....	2.95	2.24	3.26	11	2	9
5.....	2.24	2.33	2.93	4	4	6
6.....	2.38	3.67	4.40	8	11	11
7.....	2.66	2.89	3.59	10	10	10
8.....	2.26	2.36	2.87	5	5	4
9.....	2.27	2.31	2.88	6	3	5
10.....	2.51	2.66	3.04	9	9	8
11.....	2.30	2.49	3.02	7	7	7

similar information for participating and nonparticipating policies for different periods.

VI. OBSERVATIONS AND INFERENCES

Every reference in this paper to the selection of an interest rate has emphasized that it must be long term, net after taxes, and near-riskless. The third aspect must be constantly emphasized. The use of an expected or average yield for a common stock portfolio, for example, is totally inappropriate unless the yield rate is adjusted for possible losses of principal. Since there is no practical way to calculate such an adjustment directly, we can only revert to the rate paid on certificates of deposit (for the small buyer) or high-grade bonds (for the larger buyer). There is

TABLE 3
 TEN-YEAR SURRENDERED COSTS
 \$10,000 POLICY
 MALE, AGE 55 AT ISSUE, 4 PER CENT INTEREST
 1958 CSO BASIC MORTALITY TABLE
 COST PER DOLLAR OF STANDARD MORTALITY COST

COMPANY NUMBER	COST PER DOLLAR OF STANDARD MORTALITY COST			RANK		
	Term to Age 65	Whole Life	Life Paid at Age 65	Term to Age 65	Whole Life	Life Paid at Age 65
Participating Policies						
1.....	\$1.47	\$1.40	\$2.00	6	3	2
2.....	1.39	1.50	3.00	5	6	5
3.....	1.17	1.43	1.84	1	5	1
4.....	1.32	1.30	2.15	3	2	3
5.....	1.34	1.43	3.56	4	4	6
6.....	1.30	1.28	2.47	2	1	4
Nonparticipating Policies						
1.....	\$1.20	\$1.34	\$2.75	2	1	8
2.....	1.54	1.56	2.44	7	8	6
3.....	1.38	1.44	2.49	4	5	7
4.....	1.75	1.43	2.43	10	4	5
5.....	1.34	1.39	2.19	3	3	2
6.....	1.43	1.85	3.17	5	10	10
7.....	1.56	1.71	2.95	8	9	9
8.....	1.54	1.47	2.36	6	6	4
9.....	1.60	1.52	2.30	9	7	3
10.....	1.19	1.36	2.07	1	2	1

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one important exception: the middle- or lower-income buyer saddled with a mortgage loan or other long-term debt may appropriately use his loan rate (net after income taxes), since his most profitable near-riskless "investment" is in the prepayment of his indebtedness (without penalty), with the consequent saving in interest costs. This aspect has taken on increasing importance to domestic financial considerations in recent years as the

TABLE 4
PARTICIPATING POLICIES OF NINE COMPANIES
MALE LIFE, \$10,000 FACE AMOUNT
1958 CSO BASIC MORTALITY TABLE
"BEST BUY" BASED ON 20-YEAR COST PER DOLLAR
OF STANDARD MORTALITY COST

AGE AT ISSUE	SELECTED INTEREST RATE	NUMBER OF COMPANIES IN WHICH "BEST BUY" IS		
		Term to Age 65	Whole Life	Life Paid at Age 65
25.....	3%	0	6	3
25.....	4	2	5	2
25.....	5	7	1	1
25.....	6	8	1	0
25.....	7	9	0	0
35.....	3	0	6	3
35.....	4	3	5	1
35.....	5	7	1	1
35.....	6	9	0	0
35.....	7	9	0	0
45.....	3	0	7	2
45.....	4	8	1	0
45.....	5	9	0	0
45.....	6	9	0	0
45.....	7	9	0	0

typical amount of loan in relation to income, the interest rate, and the loan term have all increased to startling levels which would have been considered incredible not too many years ago.

No inference should be drawn from the previous paragraph concerning the relative merits of fixed-dollar investments vis-à-vis common stocks or other equity media. In comparing life insurance contracts, however, emphasis must be placed, and kept, on the necessity of selecting a rate which is based on "investments of comparable quality." Of course no one can predict the value of riskless money to the buyer over the next twenty

years, since it depends not only on the investment climate but also on the buyer's tax bracket and whether he is in the position of a debtor, an investor, or both. This fact, however, should not (and does not) deter the thoughtful buyer from considering this important factor and making a best estimate.

The use of a standard mortality table for all buyers may be questioned.

TABLE 5
 NONPARTICIPATING POLICIES OF ELEVEN COMPANIES
 MALE LIFE, \$10,000 FACE AMOUNT
 1958 CSO BASIC MORTALITY TABLE
 "BEST BUY" BASED ON 20-YEAR COST PER DOLLAR
 OF STANDARD MORTALITY COST

AGE AT ISSUE	SELECTED INTEREST RATE	NUMBER OF COMPANIES IN WHICH "BEST BUY" IS		
		Term to Age 65	Whole Life	Life Paid at Age 65
25.....	3%	1	10	0
25.....	4	5	6	0
25.....	5	7	4	0
25.....	6	7	4	0
25.....	7	8	3	0
35.....	3	2	9	0
35.....	4	10	1	0
35.....	5	10	1	0
35.....	6	10	1	0
35.....	7	11	0	0
45.....	3	4	7	0
45.....	4	10	1	0
45.....	5	10	1	0
45.....	6	10	1	0
45.....	7	11	0	0

In consideration of individual differences in applicable mortality rates, the method will not be invalid just because the rates in the standard mortality table do not represent the absolute level of mortality assumed to be applicable to the individual. It is important, however, that these standard rates be in reasonable proportion, year by year, to the assumed applicable rates. Consider the theoretical example of the person truly subject to 500 per cent of the standard rate at every age. When worked on the standard table, his "cost per dollar of standard mortality cost" will, in every case, be exactly five times what it would be when worked on

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an "appropriate" table. Since this five-times ratio holds true in every case, the rankings will not be affected regardless of the types of policies being considered.

The foregoing consideration makes it evident that the 1958 CSO Basic Table provides a better standard than the 1958 CSO Table. The fact that insurance cannot be purchased at these rates is irrelevant, since the method is not to be used to determine whether to buy or not to buy.

TABLE 6
 PARTICIPATING POLICIES OF NINE COMPANIES
 MALE LIFE, \$10,000 FACE AMOUNT
 1958 CSO BASIC MORTALITY TABLE
 "BEST BUY" BASED ON 15-YEAR COST PER DOLLAR
 OF STANDARD MORTALITY COST

AGE AT ISSUE	SELECTED INTEREST RATE	NUMBER OF COMPANIES IN WHICH "BEST BUY" IS		
		Term to Age 65	Whole Life	Life Paid at Age 65
25.....	3%	1	6	2
25.....	4	3	5	1
25.....	5	7	1	1
25.....	6	8	1	0
25.....	7	9	0	0
35.....	3	1	6	2
35.....	4	2	6	1
35.....	5	8	1	0
35.....	6	9	0	0
35.....	7	9	0	0
45.....	3	1	8	0
45.....	4	6	3	0
45.....	5	9	0	0
45.....	6	9	0	0
45.....	7	9	0	0

The method is technically valid for comparing contracts providing coverages for different periods as long as coverage is continuous during the period studied. Adequate emphasis must be given, however, to the suitability of the plan, particularly as to the length of coverage available in relation to that needed.

Although this paper takes no position on the merits of financed insurance, the use of the proposed method to evaluate financed insurance proposals deserves some comment. By accumulation of the proposed net cash payment at the selected interest rate and division by the interest

factor from step 4, the appropriate substitute for the gross premium is found. Dividends do not need to be taken into account, since their disposition is already implied by the calculation of the net cash payment. The net amount at risk remains unchanged by the loan transactions, but it should be adjusted for any application of dividends to buy additional protection. The loan balance at the end of the period is, of course, deducted from the cash value in step 3. The buyer must be alerted to any

TABLE 7
 NONPARTICIPATING POLICIES OF ELEVEN COMPANIES
 MALE LIFE, \$10,000 FACE AMOUNT
 1958 CSO BASIC MORTALITY TABLE
 "BEST BUY" BASED ON 15-YEAR COST PER DOLLAR
 OF STANDARD MORTALITY COST

AGE AT ISSUE	SELECTED INTEREST RATE	NUMBER OF COMPANIES IN WHICH "BEST BUY" IS		
		Term to Age 65	Whole Life	Life Paid at Age 65
25.....	3%	6	5	0
25.....	4	7	4	0
25.....	5	8	3	0
25.....	6	10	1	0
25.....	7	10	1	0
35.....	3	3	8	0
35.....	4	7	4	0
35.....	5	10	1	0
35.....	6	10	1	0
35.....	7	10	1	0
45.....	3	3	8	0
45.....	4	9	2	0
45.....	5	10	1	0
45.....	6	10	1	0
45.....	7	10	1	0

difference in his options to continue coverage beyond the end of the period studied, if that period does not coincide with his need for coverage.

VII. UTILIZATION OF THE METHOD

If the proposed method is generally accepted in principle, the pattern of its implementation will depend on two factors: the sources of demand and the sources of supply. The potential demand sources include federal and state government agencies, life insurance home offices, insurance agents, brokers, individual buyers, financial planning services, and

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leaders of consumer movements. Within each source type the demand level will vary, even to the point of being strongly negative. Although these demand levels and the degree of acceptance of the method are difficult to predict, it seems safe to say that the general demand for improved information about the economics of life insurance buying will continue to increase.

Home offices may supply these cost measures. This information would be a simple and valuable addition to the automated programming and ledger-sheet services already being supplied by many companies. Alternative sources would appear to be agencies, brokers, consulting actuaries, accountants, and financial planning services. Although the calculations

TABLE 8
 PARTICIPATING POLICIES OF NINE COMPANIES
 MALE LIFE, \$10,000 FACE AMOUNT
 1958 CSO BASIC MORTALITY TABLE
 "BEST BUY" BASED ON 10-YEAR COST PER DOLLAR
 OF STANDARD MORTALITY COST

AGE AT ISSUE	SELECTED INTEREST RATE	NUMBER OF COMPANIES IN WHICH "BEST BUY" IS		
		Term to Age 65	Whole Life	Life Paid at Age 65
25.....	3%	1	7	1
25.....	4	2	7	0
25.....	5	6	3	0
25.....	6	7	2	0
25.....	7	9	0	0
35.....	3	2	7	0
35.....	4	3	6	0
35.....	5	8	1	0
35.....	6	8	1	0
35.....	7	8	1	0
45.....	3	3	4	2
45.....	4	6	2	1
45.....	5	8	1	0
45.....	6	8	1	0
45.....	7	8	1	0
55.....	3	3	3	0
55.....	4	3	3	0
55.....	5	6	0	0
55.....	6	6	0	0
55.....	7	6	0	0

NOTE.—Only six companies appear at age 55 because of the lack of published data at this age.

can be made on a desk calculator, the more enterprising of these organizations will make use of the greater cost effectiveness of computers for this purpose. The proposed method seems particularly well suited to calculation by time-sharing and minicomputer facilities. Where such facilities are used to produce ledger sheets or other sales aids, the additional expense of including the interest-adjusted cost and the cost per dollar of standard mortality cost in the printout would be *de minimis*. Thus a conscientious agent, already using such services, would be under no added financial burden to provide this valuable additional information. The satisfactions, both financial and other, in providing this important additional service should be very real.

TABLE 9
 NONPARTICIPATING POLICIES OF ELEVEN COMPANIES
 MALE LIFE, \$10,000 FACE AMOUNT
 1958 CSO BASIC MORTALITY TABLE
 "BEST BUY" BASED ON 10-YEAR COST PER DOLLAR
 OF STANDARD MORTALITY COST

AGE AT ISSUE	SELECTED INTEREST RATE	NUMBER OF COMPANIES IN WHICH "BEST BUY" IS		
		Term to Age 65	Whole Life	Life Paid at Age 65
25.....	3%	6	5	0
25.....	4	7	4	0
25.....	5	8	3	0
25.....	6	10	1	0
25.....	7	10	1	0
35.....	3	2	9	0
35.....	4	8	3	0
35.....	5	9	2	0
35.....	6	10	1	0
35.....	7	10	1	0
45.....	3	3	8	0
45.....	4	9	2	0
45.....	5	10	1	0
45.....	6	10	1	0
45.....	7	10	1	0
55.....	3	5	5	0
55.....	4	7	3	0
55.....	5	9	1	0
55.....	6	9	1	0
55.....	7	9	1	0

NOTE—Only ten companies appear at age 55 because of the lack of published data at this age.

A slight additional sophistication in the computer program would allow the input of the buyer's selected interest rate, along with amount, age and sex, and a list of suitable alternate plans of insurance. The computer would then select the "best buy" (or two or three if desired) and print the ledger sheet only on this plan. The cost per dollar of standard mortality cost would be printed for all plans on the list, however, so that the agent and the buyer could determine the relative significances of the choices.

VIII. REPLACEMENT OF POLICIES

There has been some demand for, and much discussion about, a suitable method for analyzing the economic virtue of any proposal to replace an existing policy with a new one. The standard mortality cost method would seem eminently suited to this purpose, again with emphasis on the selection of an interest rate appropriate for the individual buyer. To apply the method to an old policy, the procedure for policies with varying premiums is used, and the termination value of the old policy at the outset of the period under study is treated as an additional premium. Studies (comparable to those on new policies reported in this paper) in this area lie outside the scope of this paper. Perhaps they will be undertaken by an energetic discussant.

IX. CONCERNS AND RESERVATIONS

There is an ever present danger that any emphasis on net cost comparisons may eclipse important nonquantitative considerations in the selection of a suitable life insurance contract. Conscientious agents and agency officials will continue to stress the importance of these other considerations. Where this is done, it would seem that only good, not harm, can come from substituting improved cost measures for the traditional ones.

It is difficult to say how much insurance has been sold (that otherwise would not have been sold) by presentation of the "profit" and unrealistically low cost figures exhibited in traditional surrendered net cost illustrations. At any rate, agents who have relied on this approach will view with regret its replacement by more meaningful cost measures. The rewards, however, for those agents who make the effort to appreciate and use the newer concepts should be substantial.

There is a risk that the misguided and unscrupulous few who unfortunately find pleasure or profit in misrepresenting the life insurance industry to the public may find some diabolic way to use the new measures in the furtherance of their ends. This risk will be well worthwhile

if the newer measures displace the traditional ones, thereby relieving these people of at least a partially justified existing complaint. The standard mortality cost method ignores the probability of termination by death or withdrawal during the observation period. It is possible to caution the nontechnical person on this by pointing out that the method answers the question "What will happen if I live to the end of the period and keep the policy in force?"

It may be argued that the standard mortality cost method is deficient because it provides only a relative index rather than an absolute measure. This is true, but only partially so. To the extent that the selected mortality table represents a good measure of the buyer's probability of dying, the measure becomes absolute. More important, a relative measure would seem to have more usefulness, since it need not be so heavily qualified or so complicated as to reduce its usefulness. Moreover, the buyer is used to making economic decisions on the basis of relative comparisons in almost every other service he buys. This is particularly true when the decision is which product to buy rather than whether to buy or not.

There may be concern that a buyer may calculate (or be furnished) information showing the excess of the interest-adjusted cost over the standard mortality cost and that the size of this figure may discourage him from making any purchase. This might well happen. For every such case, however, there should be several enthusiastic new buyers who see in the newer measures a solution at last to the problem in buying life insurance which has so long plagued them: "Which plan is the best buy for me?" Moreover, the technique of offering the buyer alternate solutions (rather than a choice to buy or not to buy), so strongly implied in this new measure, is a time-honored potent sales psychology ("Do you prefer the brush with the red handle or the one with the green handle?").

Some may feel that the buyer's "net amount at risk" is not the excess of the face amount over the cash value but rather the excess of the gross death benefit over the accumulated value of the gross premiums. This would be true if the prospect were deciding whether to buy or not. In such a case the newer cost measures are of little or no objective value to him. When the decision is which plan to buy, however, the measure comes into sharp focus. For, having bought a policy, the buyer continually faces the decision whether to pay another premium or to take his cash value. Having paid another premium, he is insured during that period for the contract amount and becomes the owner of the increased cash value at the end of the period. His net insurance benefit, then, is the excess of

what his beneficiary would receive if he died over what he can receive at the end of the premium period if he lives.

Life insurance company executives who have been concerned about the overemphasis on traditional net cost selling may fear that the proposal in this paper is a further step in the wrong direction. It would seem, to the contrary, that any effort to substitute improved cost measures and to place the emphasis on which plan to buy is a step in the right direction.

There might also be concern in the same quarters that use of the new measures would lead to the purchase of lower-premium plans. With one significant class exception (namely, the high-interest-rate debtor) life insurance buyers cannot command a sufficiently high interest rate on near-riskless money to make term plans attractive to them. This is shown dramatically in Tables 4-9. The danger is that insufficient emphasis may be placed on the necessity of using a near-riskless rate. This aspect must be stressed unceasingly.

To some readers the proposed method may seem unfairly to identify all the contract expenses with the mortality element, with no "loading" attributed to the investment element. This was done intentionally in order to meet the financial marketplace on its own terms. Almost without exception, the usual avenues of near-riskless investment available to the buyer (savings accounts, certificates of deposit, bonds, repayment of indebtedness) have their yields quoted in net terms. Moreover, buyers in general implicitly accept premium-to-claim ratios in other lines of insurance which are comparable to the costs per dollar of standard mortality cost.

Objection has been expressed at various times to the use of mortality rates in analyses from the standpoint of an individual buyer. This would appear to be an objection to the use of a probability model. Without becoming involved in that discussion, it seems safe to say that the objection does not appear to apply to the standard mortality cost method, since no probability model is implied.

X. CONCLUSIONS

The standard mortality cost method would appear to be a valid method for comparing the economic costs of dissimilar life insurance plans in a manner which recognizes the individual nature of the buyer's financial situation. It appears to meet both the theoretical and the practical guidelines set down by the report of the Moorhead committee. The concepts are straightforward and can be mastered by anyone who can pass the first LUTC examination. The availability of computer facil-

ities, including time-sharing and the newer minicomputers, provide cost-effective means of producing these new cost measures and making them available to the buyer. The benefits of the new measures seem to outweigh substantially any potential side effects. It remains to be seen how quickly and extensively they will be accepted by those who sell life insurance and those who buy it.

XI. ACKNOWLEDGMENTS

This paper would not be complete without the expression of grateful thanks and acknowledgments to Messrs. R. M. Collins, Jr., R. E. Hunstad, E. J. Moorhead, and C. L. Trowbridge. Their thoughtful comments and questions did much to clarify the author's thinking and expression, although they do not necessarily agree with all of the ideas expressed in the paper.

APPENDIX A

SOURCE AND PROCESSING OF DATA

Information was taken from the Diamond Life Bulletins, as of November, 1970. Only those companies were selected for which data were available on all three plans used for comparison. Actual dividends and cash values were used for years 1-5, 10, 15, and 20. Figures for the intervening years were obtained by straight-line interpolation. Where the waiver of premium benefit is included without specific extra charge, the premiums have been suitably adjusted. Although, for easy reading, the interest-adjusted cost and standard mortality cost are shown to the nearest dollar, the divisions to obtain the cost per dollar of standard mortality cost were made with figures accurate to 28 bits.

The data were first punched on paper tape and then loaded on the Honeywell Timesharing System using a Dura 1051 terminal. Use of the terminal was kindly furnished by The Minnesota Mutual Life Insurance Company.

The cost factors were calculated by a modified version of a program which calculates these factors in the process of producing ledger-sheet illustrations. The program operates in three overlays and would require about 25,000 words of memory if it were all loaded at once. Of course much of the program is extraneous to the pure process of calculating cost factors. An auxiliary program was used to summarize and tabulate the data for Tables 2-9. It required about 3,000 words of memory, mostly devoted to data storage. The source programs are in FORTRAN IV and contain approximately 1,000 and 200 statements, respectively. About 6,000 seconds of computer time and 25 hours of connect time were used for all phases of the process, including data entry and correction, program development and modification, and listing of the very extensive intermediate results.

APPENDIX B

It is desired to prove that, under the conditions stated in Section IV above, the interest-adjusted cost (IAC) equals the standard mortality cost (SMC).

The proof is of the inductive form. Although not so rigorous as the classical inductive proof, since it omits the demonstration that what is true for the n th function is also true for the $(n + 1)$ st, this proof should provide adequate credibility as to the actuarial soundness of the proposition.

Consider first a one-year term policy:

$$\text{SMC} = q_x / \ddot{s}_{\overline{1}|} = vq_x = P_{x:\overline{1}|}^1.$$

But $\text{IAC} = P_{x:\overline{1}|}^1$, since there is no termination value. Therefore $\text{IAC} = \text{SMC}$.

Now consider a two-year term policy:

$$\text{SMC} = \frac{q_x(1+i)(1 - {}_1V_{x:\overline{2}|}) + q_{x+1}}{\ddot{s}_{\overline{2}|}},$$

$$\text{IAC} = P_{x:\overline{2}|}^1.$$

$\text{IAC} = \text{SMC}$ if

$$q_x(1+i)(1 - {}_1V_{x:\overline{2}|}) + q_{x+1} = P_{x:\overline{2}|}^1(1+i)^2 + P_{x:\overline{2}|}^1(1+i)$$

or if

$$\frac{C_x}{v^2 D_x} (1 - A_{x+1:\overline{1}|}^1 + P_{x:\overline{2}|}^1) + \frac{C_{x+1}}{v D_{x+1}} = P_{x:\overline{2}|}^1 [(1+i)^2 + (1+i)]$$

or if

$$\frac{C_x}{D_x} \left(1 - \frac{C_{x+1}}{D_{x+1}} \right) + \frac{v C_{x+1}}{D_{x+1}} = P_{x:\overline{2}|}^1 \left(1 + v - \frac{C_x}{D_x} \right)$$

or if

$$C_x D_{x+1} - C_x C_{x+1} + v C_{x+1} D_x = P_{x:\overline{2}|}^1 [D_x D_{x+1} (1+v) - C_x D_{x+1}]$$

or if

$$\begin{aligned} (D_x + D_{x+1})(C_x D_{x+1} - C_x C_{x+1} + v C_{x+1} D_x) \\ = (C_x + C_{x+1})[D_x D_{x+1} (1+v) - C_x D_{x+1}] \end{aligned}$$

or if

$$(D_x + D_{x+1})(C_x D_{x+1} + C_{x+1} D_{x+1}) = (C_x + C_{x+1})(D_x D_{x+1} + D_{x+1} D_{x+1})$$

or if

$$(D_x + D_{x+1})(C_x + C_{x+1}) D_{x+1} = (C_x + C_{x+1})(D_x + D_{x+1}) D_{x+1},$$

since

$$v D_x - C_x = D_{x+1}.$$

Next consider a two-year endowment policy:

$$\text{SMC} = [q_x(1+i)(1 - {}_1V_{x:\overline{2}|}) + q_x(1 - {}_2V_{x:\overline{2}|})] \div \ddot{s}_{\overline{2}|},$$

$$\text{IAC} = [P_{x:\overline{2}|}(1+i)^2 + P_{x:\overline{2}|}(1+i) - 1] \div \ddot{s}_{\overline{2}|}.$$

IAC = SMC if

$$q_x(1+i)(1 - {}_1V_{x:\overline{2}|}) + q_x(1 - {}_2V_{x:\overline{2}|}) \\ = P_{x:\overline{2}|}(1+i)^2 + P_{x:\overline{2}|}(1+i) - 1$$

or if

$$\frac{C_x}{D_x} \left(1 - P_{x:\overline{2}|} \frac{D_x}{D_{x+1}} + \frac{C_x}{D_{x+1}} \right) = P_{x:\overline{2}|}(1+i) - v^2$$

or if

$$\frac{C_x}{D_x} \left(1 + \frac{C_x}{D_{x+1}} \right) = P_{x:\overline{2}|} \left(1 + v + \frac{C_x}{D_{x+1}} \right) - v^2$$

or if

$$C_x D_{x+1} + C_x C_x = P_{x:\overline{2}|} (D_x D_{x+1} + v D_x D_{x+1} + C_x D_x) - v^2 D_x D_{x+1}$$

or if

$$(C_x D_{x+1} + C_x C_x)(D_x + D_{x+1}) \\ = (C_x + v D_{x+1})(D_x D_{x+1} + C_x D_{x+1} + D_{x+1} D_{x+1} + C_x D_x)$$

or if

$$C_x D_x D_{x+1} + C_x D_{x+1} D_{x+1} + C_x C_x D_x + C_x C_x D_{x+1} \\ = C_x D_x D_{x+1} + C_x C_x D_{x+1} + C_x D_{x+1} D_{x+1} + C_x C_x D_x \\ + v D_x D_{x+1} D_{x+1} + v C_x D_{x+1} D_{x+1} + v D_{x+1} D_{x+1} D_{x+1} + v C_x D_x D_{x+1}$$

or if

$$C_x(C_x + D_{x+1})(D_x + D_{x+1}) \\ = (C_x + v D_{x+1})(C_x + D_{x+1})(D_x + D_{x+1}) - v^2 D_x D_{x+1}(D_x + D_{x+1})$$

or if

$$0 = v D_{x+1}(C_x + D_{x+1}) - v^2 D_x D_{x+1}(D_x + D_{x+1})$$

or if

$$0 = C_x + D_{x+1} - v D_x \quad \text{Q.E.D.}$$

Finally, consider a two-year term policy studied for only the first year:

$$\text{SMC} = q_x(1 - {}_1V_{x:\overline{2}|}^1) \div \ddot{s}_{\overline{1}|} = v q_x(1 - {}_1V_{x:\overline{2}|}^1),$$

$$\text{IAC} = P_{x:\overline{2}|}^1 - {}_1V_{x:\overline{2}|}^1 \div \ddot{s}_{\overline{1}|}.$$

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IAC = SMC if

$$\frac{C_x}{D_x} \left(1 - \frac{C_{x+1}}{D_{x+1}} + P_{x:\overline{2}|}^1 \right) = P_{x:\overline{2}|}^1 - \frac{vC_{x+1}}{D_{x+1}} + vP_{x:\overline{2}|}^1$$

or if

$$C_x D_{x+1} - C_x C_{x+1} + vC_{x+1} D_x = P_{x:\overline{2}|}^1 [(1+v)D_x D_{x+1} - C_x D_{x+1}]$$

or if

$$C_x D_{x+1} - C_{x+1} D_{x+1} = P_{x:\overline{2}|}^1 [(1+v)D_x D_{x+1} - C_x D_{x+1}]$$

or if

$$C_x - C_{x+1} = P_{x:\overline{2}|}^1 (1+v)D_x$$

or if

$$(D_x + D_{x+1})(C_x + C_{x+1}) = (C_x + C_{x+1})(D_x + D_{x+1}) .$$

DISCUSSION OF PRECEDING PAPER

ERNEST J. MOORHEAD:

One of the widely recognized weaknesses of the traditional method of policy net cost comparison is that almost invariably the result by that method implies that any higher-premium plan is more attractive to the buyer than any lower-premium plan in the same company.

When the interest-adjusted method is substituted for the traditional method, this large discrepancy disappears. The tendency is for a variation in the opposite direction, but of very much lesser magnitude. That is, within a company the net cost of a higher-premium plan more frequently than not appears to exceed that of an ordinary life policy.

For example, using the National Underwriter 1970 publication *Cost Facts on Life Insurance—Interest-adjusted Method*, a comparison among seventy-three companies issuing participating policies at the \$10,000 level, male age 35, twenty-year value, shows that in all but ten of these the index for the life paid up at 65 plan exceeds that for ordinary life. The median value of this excess, allowing for the negative values, is 37 cents per thousand.

A corresponding comparison among fifty-one companies issuing non-participating policies shows that in all but two the index for the life paid up at 65 plan exceeds that for ordinary life. The median value of this net excess is \$1.03 per thousand. The standard mortality cost method set forth by Mr. Hill gives a means for determining whether this excess is genuine or whether it is an aberration of the interest-adjusted method.

The small sample displayed in Tables 2 and 3 of the paper indicates that the excess may be modest but genuine. Admittedly the validity of this conclusion depends upon acceptance of the 4 per cent interest rate used; but the subsequent tables of the paper indicate that the conclusion would hold true even if an interest rate as high as 7 per cent were used.

If this difference is genuine, that is not to say that it is improper. However, while emphasis naturally has been upon the use of the interest-adjusted method, and in Mr. Hill's paper on the standard mortality cost method, for the enlightenment of the life insurance buyer I believe that we should not overlook the value of such analyses as these within a company's actuarial department. These methods can be employed to give us insight into comparative cost patterns to an extent that was impossible without large internal effort as long as the only published figures were on the traditional method.

Turning to another matter, Mr. Hill seems to be quite right in emphasizing that the interest rate applicable to one buyer may not be the same as that suitable for another. This is true even though it must not be forgotten that the financial conditions that make a particular interest rate seem appropriate to the circumstances of any one person may change radically throughout a ten-year period, let alone a twenty-year period.

At the risk of stating the obvious, it may be worth stressing that whenever the interest-adjusted method, the standard mortality cost method, or any other method that involves an interest rate is used for intercompany ranking, it is absolutely imperative that one and the same interest rate be used in all cases.

MONTE J. HOPPER:

This paper is an invaluable addition to the Moorhead committee report. In the designing of an illustration program for the use of our producers, I would certainly recommend that the cost per dollar of standard mortality cost be calculated in addition to the traditional and the interest-adjusted net cost.

While it would be redundant to comment on the many advantages and disadvantages that Mr. Hill has mentioned in his paper, there is another advantage for the stock company which deserves mention. When a guaranteed cost plan is compared with a participating plan, the costs per dollar of standard mortality cost for the two plans will generally be closer together than either the traditional or the interest-adjusted net costs. This will be especially true if the guaranteed cost plan is a low-premium, low cash value plan and the participating plan is a high-premium, high cash value plan. This method is also extremely useful to the stock company that issues both participating and guaranteed cost plans for comparison within its own portfolio.

PETER L. J. RYALL:

Mr. Hill has made an interesting contribution to the literature on methods of comparing life insurance policyholder costs.

This discussion will dispute three aspects of the author's application of his method. The first is the use, at a time of historically high interest rates, of current illustrative dividends to compare, over as long a period as twenty years, costs of policies with widely differing investment components. The second is the omission of any illustration of the effect of income tax on the twenty-year cost, in the event that a participating policy on a permanent plan is surrendered at the end of the period. The

third concerns the inability of the author's choice of mortality table to give fair values of standard mortality costs, in accordance with the interpretation given these costs in the paper.

Use of Dividend Illustrations

In comparisons of twenty-year interest-adjusted costs of participating policies on the same plan, the expectation that the excess interest rates underlying the dividend scales will, ten to twenty years hence, most likely be less than at present need not be recognized, since such a change will have a similar effect on all the costs. However, this is not true when widely different plans are compared. This may be shown for a term to age 65 and a whole life policy issued at age 35, with present and future excess interest rates related as shown in Table 1.

TABLE 1

Policy Year	Addition to Present Excess Interest Rate
1-8.	+0.25%
9-11.	0.00
12-14.	-0.25
15-17.	-0.50
18-20.	-0.75

If the excess interest is calculated on 1958 CSO 3 per cent net level premium curtate reserves, the increases in twenty-year 4 per cent interest-adjusted costs on the term to age 65 and whole life policies are, per \$1,000 face amount, \$0.11 and \$0.37, respectively. The corresponding increases in the cost per dollar of standard mortality cost (on the 1958 CSO Basic Table, with cash values equal to the above reserves) are \$0.03 and \$0.12. The data for participating policies given in Table 2 show that the long-deferred and gradual decline in interest rates postulated above would cause the whole life plan to be "best choice" in two, rather than five, of the nine companies.

The author states that the buyer should not be deterred from making a best estimate of the value of riskless money to him, and the investment climate is mentioned as one factor that bears on this estimate. It is inconsistent not to recognize that the future investment climate also affects the company's ability to maintain the relative cost position of low- and high-premium plans of participating insurance. At a time of historically high interest rates, and shortly after new money rates have peaked, it is not sufficient for the prospective buyer to be merely informed "concerning the nonguaranteed nature of dividends." If a "best estimate" of the

company's future excess interest rates shows that these rates may be expected to fall appreciably within the next twenty years, then comparisons between twenty-year costs under high- and low-premium plans of participating insurance should utilize dividend scales consistent with this estimate. If this is not deemed to be feasible, the only honest alternative is not to make the comparison (for an extended period) at all.

Effect of Income Tax

There is a significant amount of tax payable on surrender after twenty years of permanent plans of participating insurance issued at ages up to 40. To illustrate this, twenty-year 4 per cent interest-adjusted costs were determined, using 1971 data, for issue ages 25, 30, 35, and 40 for policies

TABLE 2

Issue Age	Average Cost Increase per 50 Per Cent Tax Rate	Maximum Cost Increase per 50 Per Cent Tax Rate
25.....	32%	59%
30.....	27	48
35.....	19	32
40.....	10	18

with face amount \$25,000 on the whole life or other long-term plans of twenty large companies, chosen on the basis of the amount of insurance they issued on participating life and endowment plans in the United States in 1970. The increase in each of these costs caused by a 50 per cent tax levy on the net proceeds on surrender at the end of the twenty years was calculated. The average and maximum proportions by which the twenty-year interest-adjusted costs (and hence the costs per dollar of standard mortality cost) are increased for each issue age are as shown in Table 2.

The author emphasizes that the interest rate underlying the interest-adjusted cost should be a net rate, after income taxes, but he nowhere recognizes the need to let the prospect see the effect on cost comparisons of the tax on net surrender proceeds. Hardly any prospective policyholder can rule out altogether the possibility of his terminating the policy after it has been in force for many years. It should be the prospect's prerogative to decide, in the light of his knowledge of his own circumstances and plans, and the likelihood of changes therein, what chance

there is that his policy will be surrendered after an extended period and hence what weight he should attach to the tax-included and tax-free cost comparisons. The tax rate used should be the prospect's best estimate of his marginal tax rate twenty years hence.

Choice of Mortality Table

The choice of the mortality table to be used in applying a policyholder cost comparison method depends on the purpose which the mortality rates are intended to serve. In this paper the purpose is set out clearly, for example: "The cost per dollar of standard mortality cost might be thought of as the ratio of premiums to claims after the removal of the investment element of the contract." It is further suggested that "the buyer should be encouraged to understand the concept." Thus it is not intended that the cost per dollar of standard mortality cost be presented to the prospect as an arbitrary price index, without meaning in itself. This being the case, it is vital that the standard mortality cost be calculated in a manner that accords with the interpretation given to the policyholder. He will realize that the "claims," or rates employed, cannot, as a practical matter, be based on a level of mortality taken to be applicable to him individually. He will assume, rather, that the claim rates used closely accord with those recently experienced by the industry for his premium rate classification.

Cost comparisons between policies are rarely made for periods longer than twenty years. The 1958 CSO Basic Table, the mortality basis suggested in the paper, is not representative of recent average company experience during the first ten or twenty policy years. To illustrate this, ten- and twenty-year standard mortality costs for issue age 35, and ten-year standard mortality costs for issue age 55, were calculated for the whole life and term to age 65 plans,¹ both on the 1958 CSO Basic Table and on rates derived from the intercompany experience between 1968 and 1969 anniversaries under standard issues on male lives.² The percentage

¹ The cash values were taken to equal the 1958 CSO net level premium curtate 3 per cent reserves, except that for term to age 65 issued at age 55 the cash values were taken to be zero.

² To obtain rates during the fifteen-year select period, the medical and nonmedical amounts were combined for the exposed to risk and actual deaths in each policy year of the issue age groups 30-34, 35-39, 50-54, and 55-59 (see *TSA, 1970 Reports* number, pp. 90-95, 102-7). The rates for each policy year for issue ages 35 and 55 were obtained by interpolation between the rates for the issue age groups, utilizing the "central age" for each group. The experience for policy years 16-20 for issue age 35 was derived by applying the 92.6 per cent mortality ratio given for attained age group 50-54 of the

increases in the cost per dollar of mortality cost resulting when the latter basis is used instead of the former were then obtained.

For issue age 35, the effect of computing the standard mortality cost on the whole life (or term to age 65) plan on the 1968-69 intercompany experience, instead of on the 1958 CSO Basic Table, is to increase the ten-year cost per dollar of standard mortality cost by 22 per cent (or 21 per cent) and the twenty-year cost per dollar of standard mortality cost by 15 per cent (or 14 per cent). For issue age 55, the corresponding increases for the ten-year costs on these two plans are 73 per cent (or 70 per cent).

The percentages quoted above are increases in the total cost. The implied understatement of the expense component of the interest-adjusted cost is larger. For example, the average of the twenty-year costs per dollar of standard mortality cost given in Table 2 of the paper for nine participating whole life policies issued at age 35 is \$2.05, whereas this average would, on the basis of 1968-69 experience, be \$2.35. Here use of the 1958 CSO Basic Table causes the expense component to appear more than 20 per cent less than it would be, on the average. For ten-year costs at issue age 35, or for costs at higher issue ages, the understatement of expenses involved in the use of the 1958 CSO Basic Table is larger. For example, use of the 1958 CSO Basic Table to calculate the ten-year costs given in Table 3 of the paper for six participating whole life policies issued at age 55 makes the expense component per dollar of standard mortality cost appear, on the average, to be \$0.39 instead of the \$1.41 disclosed when the standard mortality cost is computed on the basis of the 1968-69 experience.

From the examples given above, it is evident that only (to quote the author) "the misguided and unscrupulous few who find pleasure or profit in misrepresenting the life insurance industry to the public" will "use the new measure in the furtherance of their ends" by quoting costs per dollar of standard mortality cost derived from the 1958 CSO Basic Table, or indeed from any table that fails to reflect fully the effects of selection on recent intercompany experience.

ultimate experience (see *TSA, 1970 Reports* number, p. 71) to the 1955-60 Ultimate Basic Table rates. (This tends to overstate the experience standard mortality cost, as it is to be expected that a lower ratio would apply during the first five years of the ultimate period.)

The volume of claims entering into the calculation of the costs was as follows: issue age 35: ten-year costs, \$74,933,000, and twenty-year costs, \$187,237,000; issue age 55: ten-year costs, \$55,443,000.

Since it is anticipated that the costs will be obtained by computer, the mortality rates could be derived from a set of ultimate rates (or, preferably, rates based on experience in policy years 16-20, since costs are not normally computed for longer than twenty years) and ratios, for a few pivotal issue ages, of select to ultimate rates for each of the first fifteen policy years. The data would be based on recent standard (combined medical and nonmedical) intercompany experience and would be held separately for each sex. Mortality rates would be computed for specific issue ages as required by first obtaining the relevant ratios of select to ultimate mortality rates for each policy duration by interpolation from the pivotal ratios and then applying these interpolated ratios to the ultimate rates.

It should be noted that the result proved in Appendix B of the paper follows directly from equation (5.18) given on page 107 of the second edition of the Society's textbook *Life Contingencies*, by C. W. Jordan.

ZEHMAN I. MOSESSON:

My discussion is confined to the purely technical details of Section IV and Appendix B of Mr. Hill's paper. As a former mathematician, I was disturbed by the omission of the basic induction from the inductive proof in Appendix B, and as one with some experience in teaching Part 4, I was fascinated by the complexity of the proofs given in Appendix B for three of the four very simple special cases considered. It occurred to me that there must be some simpler method of proving the statement in Section IV that under certain conditions the interest-adjusted cost equals the standard mortality cost. There is, in fact, a simpler method. The complete proof of the statement follows immediately from equation (5.18) on page 107 of the second edition of Jordan's textbook on *Life Contingencies*.

CECIL J. NESBITT:

With regard to Appendix B of Mr. Hill's paper, I believe that a quite short, general proof can be given for the equality of IAC and SMC under the conditions of Section IV.

One starts with the equation

$$P = vq_{x+t}(1 - {}_{t+1}V) + (v {}_{t+1}V - {}_tV),$$

appearing on page 107 of Jordan's *Life Contingencies* (2d ed.). Multiplication of the equation through by $(1 + i)^{n-t}$ yields

$$P(1 + i)^{n-t} = q_{x+t}(1 - {}_{t+1}V)(1 + i)^{n-t-1} + \Delta[(1 + i)^{n-t} {}_tV].$$

Summation over the range $t = 0, 1, \dots, n - 1$ gives

$$P\ddot{s}_{\overline{n}|} = \sum_{t=0}^{t=n-1} q_{x+t}(1 - {}_{t+1}V)(1 + i)^{n-t-1} + {}_nV.$$

On dividing through by $\ddot{s}_{\overline{n}|}$, one obtains

$$P = \text{SMC} + {}_nV/\ddot{s}_{\overline{n}|},$$

or, under the conditions of Section IV,

$$\text{IAC} = P - {}_nV/\ddot{s}_{\overline{n}|} = \text{SMC}.$$

This proof holds for any n within the premium payment period, and for any level premium, level amount of insurance plan. I believe it could be adapted to even more general situations.

HAROLD M. NUDELMAN:

The following is a more mathematically satisfying proof that $\text{IAC} = \text{SMC}$, which is proved by the "inductive method" in Appendix B of Mr. Hill's paper:

$$\begin{aligned}({}_tV + P)(1 + i) &= {}_{t+1}V + q_{x+t}(1 - {}_{t+1}V), \\vq_{x+t}(1 - {}_{t+1}V) &= P - v {}_{t+1}V + {}_tV, \\ \ddot{s}_{\overline{n}|}\text{SMC} &= \sum_{t=1}^n q_{x+t-1}(1 + i)^{n-t}(1 - {}_tV) \\ &= \sum_{t=1}^n (P - v {}_{t+1}V + {}_{t-1}V)(1 + i)^{n-t+1} \\ &= P\ddot{s}_{\overline{n}|} - {}_nV.\end{aligned}$$

For a n -year term insurance policy ${}_nV$ will equal 0, and therefore SMC will be equal to $P_{x:\overline{n}|}/\ddot{s}_{\overline{n}|} = \text{IAC}$. For a n -year endowment policy ${}_nV$ will equal 1, and SMC will be equal to $P_{x:\overline{n}|} - 1/\ddot{s}_{\overline{n}|} = \text{IAC}$. Therefore, the above proves that $\text{SMC} = \text{IAC}$ under the conditions described in Mr. Hill's paper.

STEPHEN H. FRANKEL, CLAIR A. LEWIS, AND JAMES J. MURPHY:

The standard mortality cost (SMC) method is presented as an approach which can be used to compare dissimilar policies. The topic is certainly both important and timely, and the author has suggested a new solution to an old problem. We discussed this paper among ourselves at

great length and concluded that certain of its implications required comment. There are also some time-worn precautions which we feel need to be produced one more time, lest someone believe that all aspects of the problem have been solved.

First, a few comments on the method itself. We do not question the fact that the index is equal to unity for all plans under net level premium assumptions. On the other hand, we do not accept this as sufficient proof that the proposed index necessarily leads one to proper conclusions. As it stands, this index is simply a ratio of two mathematical expressions.

Normally, one expects such a solution to have its genesis in certain axiomatic assumptions and to arise from a series of mathematical operations applied thereto. In spite of the author's general description of the index, we have been unable to find any realistic set of axioms which can be used to substantiate it. In trying to assess its worth and establish its meaning, we have experienced considerable frustration and confess that we still do not know what the ratio actually measures when net assumptions do not apply. In addition, it has certain characteristics which do nothing to alleviate our fears.

The denominator involves an amount at risk based on the yearly cash values. Presumably, this is the policyholder's "cost of insurance." We fail to see, from the policyholder's point of view, how this concept of an amount at risk can even arise unless and until he voluntarily terminates. Prior to that time, he could care less what his cash values are (assuming that policy loans are not a consideration). Thus the author's statement that the index presumes termination at the end of n years and the fact that it depends on the first $n - 1$ cash values appear to us to be notably contradictory.

The index contains another inconsistency which makes it difficult to accept as a standard measure. Consider two policies being compared over an n -year period with identical premiums, dividends, and n th-year cash value, but with one having higher cash values for the first $n - 1$ years. Under the SMC method the policy with the higher early cash values will have the poorer index, even though it obviously provides the better value. If, in a given situation, an agent found this index operating to his disadvantage, could he not easily discredit it?

The most perplexing problem associated with the author's method is the interpretation of the index itself. If, for example, one SMC index is 1.25 and another is 1.50, what is the significance of the difference? What dollar amount are we talking about?

Another puzzling point is the statement made in the paper that the

choice of mortality assumptions is not material as long as the rates are proportional to those actually expected. This is true if one accepts the index at face value; however, our intuition balks at blanket acceptance of the idea that one can ignore his present state of health while making insurance decisions.

All in all, we feel that the author has asked his readers to accept a great deal on faith. We urge him to take a specific situation and discuss it in detail. For example, consider a prospect who needs \$100,000 of immediate protection and is trying to decide between term to 65 and endowment at 65. A step-by-step account of how he can determine the most beneficial choice and a discussion of the considerations involved would, in our estimation, be very enlightening. In particular, we are curious as to why the author bypassed a complete actuarial approach in which (1) the possible contingencies for a given course of action are determined, (2) each contingency is assigned its value, and (3) each such value is weighted by both the probability that the contingency will occur and the time value of money.

The SMC method exemplifies the recent tendency to conduct insurance policy analysis by splitting out the "amount at risk." This tendency is puzzling because the basic whole life insurance contract is an integral property. It is not available in the marketplace in component parts, it is not used in component parts, and it is not priced in component parts. The buyer pays a level premium to cover an increasing risk of death. Cash values arise because part of each earlier premium is reserved for payment of the face amount in later years when the premium alone would not be adequate. These cash values are available only upon voluntary termination or through a policy loan, and they do not otherwise influence the financial status of the policyholder.

The contrary view is expressed in the paper: "Having paid another premium, he is insured during that period for the contract amount and becomes the owner of the increased cash value at the end of the period. His net insurance benefit, then, is the excess of what his beneficiary would receive if he dies over what he can receive at the end of the premium period if he lives."

Fortunately, whichever view is taken, the beneficiary will receive the face amount upon the death of the insured. However, we fail to see the added advantage of this "term and side fund" approach in a prospective buyer's analysis. If one wishes to buy term and invest the difference, this is a separate course of action which must be evaluated separately on its own merits, the results being compared to those determined for the alternative action of buying permanent life insurance.

The author may have felt that a "split" was an essential component of a practical index for comparing dissimilar policies, but we wonder whether the result will stand the burden that such an index must bear. An insurance buyer's first task is to select that type of policy best suited to his circumstances. A published index which purports to compare dissimilar policies may lead the buyer to make his choice solely on the basis of that index. Thus the important factors of need, ability to pay, and additional policy benefit such as conversion or renewal of term, guaranteed settlement option rates, and guaranteed policy loans might be overlooked. This makes the accuracy of any index a rather critical factor.

The safest use of indexes is found in the comparison of similar policies. However, even in comparing similar policies, there is the danger that decisions might be based on the index alone. We suggest that, as a minimum, the following information should be furnished to help the prospective buyer to keep his feet on the ground:

1. The premium for the basic policy.
2. The premium for supplementary benefits shown separately, so that the buyer can measure the prices of those benefits against their values.
3. Dividends on the current scale, individually for at least the first five years and ten- and twenty-year totals.
4. Cash values for at least the first five years and for the tenth and twentieth year.

Efforts toward greater disclosure of such basic information may serve the consumer better than emphasis on development of a single index.

We again commend the author for this able presentation of his view on a particularly timely subject. We thank him, too, for the stimulation of thought which he has given to all of us and for this opportunity to express our own views.

VICTOR E. HENNINGSSEN:

Mr. Hill is indeed to be commended for following up so promptly and thoroughly on a shortcoming of the interest-adjusted method which he pointed out at one of the early discussions at a Society meeting, following the release of the report of the Joint Special Committee on Life Insurance Costs.

For my own part, I wish that Mr. Hill had combined with his assiduity in the pursuit of improved methodology an equal diligence with respect to terminology. It is, of course, axiomatic that the cost of a given policy cannot be determined at the time of the purchase. The real cost depends upon the actual payments made prior to the termination of the policy compared with the benefits received upon termination (death, maturity,

surrender for cash value, or lapse without value), whenever that event may occur. This point of meaningful nomenclature for the emerging results of computations should be of greater concern than passing reference to caveats. In moving away from traditional computational methods, we would also do well to move away from traditional and misleading terminology. Very specifically, should we not be eliminating the word "cost" as a label for these computations, both old and new?

The joint committee itself introduces the term "cost index." Possibly "comparative index of performance" might have been a better choice. Certainly the emphasis in their studies was on comparison methods. Viewed in the very specific situation in which an individual is considering the purchase of similar plans of insurance in several companies, the interest-adjusted method does provide a better comparative index than the traditional method. However, reported speeches, editorials, and even advertisements could lead readers to conclude that the committee came up with a meaningful absolute measure of "cost of insurance" standing by itself and removed from the context of comparisons. Such a conclusion would be not only misleading but erroneous and would far overshadow the merit of using the interest-adjusted method in comparisons of similar plans of insurance. Possibly terminology is at least partially at fault.

Mr. Hill states that some may regard the figure coming from his methodology as being "deficient because it provides only a relative index rather than an absolute measure." But isn't this a distinct advantage of Mr. Hill's work, rather than a deficiency?

Mr. Hill accepts one of the statements of the joint committee without question and then proceeds to build on it. This statement is made on page 21 of the report in connection with the description given by the committee of its selection of a 4 per cent interest rate:

Select the interest rate to be used. Although it must be recognized that an interest rate that is appropriate for one individual may be inappropriate for another, a reasonable choice for general use is a rate close to the after-tax rate readily obtainable over a period of years on accounts in savings institutions.

In my opinion, this particular statement with respect to the selection of the interest rate plays into the concept that there is an imputed or forgone interest cost which should be taken into account in determining the total cost of insurance.

The objective of this calculation under the interest-adjusted method is to recognize the incidence of payments in order that a fairer comparison may be made between a company with a fairly flat dividend scale and another company with a much steeper scale. Interest should be brought in

through the concept of the time value of money as a weighting factor in the calculation. To me it is not a question of what a given individual can earn on the same money invested elsewhere but rather what weight should be placed, for example, on a twentieth-year dividend as compared with a tenth-year dividend, as compared with a fifth-year dividend, and so on. Varying relationships emerge in comparing indexes of performance according to whether the weights are at 0 per cent, 4 per cent, 6 per cent, or some other rate.

Possibly the weighting concept would be clearer if a discounting rather than an accumulating approach had been used. The figures would be identical. Again, possibly the descriptive title should have been "interest-weighted method."

Whether the use of interest is regarded in the light of the time value of money or through the concept of forgone interest is not of primary concern when the resulting figures are used in terms of comparative indexes of performance. When these figures are regarded as absolute costs, however, with interest becoming an element of cost, whether that was the intent or not, we set the stage for some of the uses which Mr. Hill conjures up with the term "diabolic."

Because Mr. Hill is concerned with indexes, he can bring in various interest rates—as he does in Tables 4-9, leading to the following helpful conclusion: "With one significant class exception (namely, the high-interest-rate debtor) life insurance buyers cannot command a sufficiently high interest rate on near-riskless money to make term plans attractive to them"—without at the same time aiding and abetting the advocates of replacing existing permanent insurance with term combined with some outside speculative investment, as may erroneously be concluded from the use of an absolute measurement of cost which incorporates interest. Then the higher the rate of interest, the greater the distortion.

Mr. Hill concerns himself with a comparison of dissimilar life insurance contracts. "Dissimilar" obviously involves a comparison. But the interest-adjusted method which he commends so strongly in his introduction in terms of "comparing net costs of similar policies" will not, and cannot in the real world, be used solely in that context. It is that situation which, as developed in this discussion, causes me not to be an advocate of the interest-adjusted method which would be contrary to Mr. Hill's views.

(AUTHOR'S REVIEW OF DISCUSSION)

J. STANLEY HILL:

I am grateful to Messrs. Moorhead, Hopper, and Ryall for their thoughtful discussions. Their discussions, particularly the additional ca-

veats, will be of value to any potential user of the method. Messrs. Moseson, Nesbitt, and Nudelman have cured a glaring omission by providing a more rigorous and elegant mathematical proof. It has been called to my attention that the cost per dollar of standard mortality cost can be developed from certain functions described by Professor Joseph M. Belth in his paper entitled "The Relationship between Benefits and Premiums in Life Insurance," which appeared in the March, 1969, issue of the *Journal of Risk and Insurance*.

Despite the extensive writings on this subject, there would still appear to be a fertile opportunity for someone to do some rigorous research on the application of the standard mortality cost method to replacements, and to report on the results of his or her research.