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THE 1971 INDIVIDUAL ANNUITY MORTALITY TABLE

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ABSTRACT

This paper proposes a new mortality table as a minimum reserve standard for individual annuities—the 1971 Individual Annuity Mortality Table (1971 IAM Table). The new table is an outgrowth of the efforts of the Joint Actuarial Committee of the ALC-LIAA to relieve the problem of surplus strain experienced by companies which offer annuities at premium rates based on "new money" interest rates but which must set up reserves based on the maximum valuation interest rate allowed by law, generally $3\frac{1}{2}$ per cent. The ALC-LIAA, in seeking an increase in the maximum valuation interest rate for annuities, recognized that the mortality standard would also have to be modernized. The paper describes the construction of the new table and compares it with current valuation standards for individual annuities. Also discussed are (a) provision for future decreases in mortality, (b) variation in mortality by type of annuity, and (c) calculation of joint life annuity values.

I. INTRODUCTION

VER the past several years, the problem of surplus strain arising from the sale of individual and group annuities has become increasingly serious for many life insurance companies. This strain comes about because companies base their premium rates for certain types of annuity business on "new money" interest rates, with the result that, under current conditions, gross premiums are lower than the first-year reserves. The maximum valuation interest rate in most states is $3\frac{1}{2}$ per cent, while the interest rates used in determining premium rates for many types of annuities have been significantly higher than $3\frac{1}{2}$ per cent for some time.

The Joint Actuarial Committee of the American Life Convention and the Life Insurance Association of America has been considering the question of an increase in the maximum valuation interest rate for a number of years. With respect to annuities, the committee recognized that, if the maximum valuation interest rate were raised, it would probably be necessary to modernize the mortality assumptions, since interest margins to offset possible mortality losses would be reduced. Accordingly, in

April, 1970, a subcommittee of the Joint Actuarial Committee was formed for the purpose of constructing two new annuity mortality tables for valuation purposes, one for individual and one for group annuities. Two of the subcommittee members, Charles M. Sternhell and C. Norman Peacor, directed the construction of the individual mortality table, while the other two members, Harold R. Greenlee and William C. Prouty, directed the construction of the group mortality table.

This paper is a report on the valuation mortality table for individual annuities developed by the subcommittee. This table is referred to as the 1971 Individual Annuity Mortality Table (1971 IAM Table).

It should be emphasized that the 1971 IAM Table was developed as a proposed minimum valuation standard and, in general, is not intended to be used directly as a basis for establishing premium rates for individual annuities.

Section II of the paper discusses some of the basic decisions that were made with respect to the construction of the new table. Section III describes the construction of the experience table underlying the 1971 IAM Table, and Section IV describes the construction of the 1971 IAM Table itself. Section V compares mortality rates and annuity values under the 1971 IAM Table (unprojected) with present valuation standards. Section VI discusses the question of provision for decreases in mortality beyond 1971 and compares annuity values based on the 1971 IAM Table with projection to those based on the a-1949 Ultimate Table with projection. Section VII discusses the variation in mortality by type of annuity and possible optional adjustments in valuation assumptions to reflect these variations. Section VIII discusses the calculation of joint life annuity values.

II. SOME BASIC DECISIONS

In this section we will discuss how some of the basic decisions were made with respect to the construction of the proposed new valuation mortality table. The actual construction of the table is described in Sections III and IV.

Need for a New Mortality Table

The first decision to be made was whether a new valuation mortality table for individual annuities was really necessary and, if so, whether it could be obtained by a simple adjustment of the *a*-1949 Table.

The Reports numbers of the Transactions contain extensive data on the two types of intercompany studies of individual annuities regularly

¹ TSA, XXII, D311.

compiled by the Committee on Mortality under Ordinary Insurances and Annuities of the Society of Actuaries: mortality under individual immediate annuities and mortality under life income settlements and matured deferred annuities. (We will refer to annuities in the former study as "immediate annuities" and in the latter study as "settlement annuities.") The latest study of immediate annuities covers the experience between 1963 and 1967 anniversaries,² and the latest study of settlement annuities covers the experience between 1960 and 1965 anniversaries.³

To get an overview of the trend of annuity mortality experience, we have extracted from the *Reports* (from Tables 10 and 11 of the report on immediate annuities in the 1969 *Reports* and Tables 7 and 13 of the report on settlement annuities in the 1966 *Reports*) the mortality ratios over a period of years for various types of annuities. Table 1 shows these mor-

Trend of Mortality Ratios in Intercompany Studies All Contract Years Combined—by Amount of Annual Income Expected Deaths Based on a-1949 Ultimate Table

TABLE 1

MALE LIVES IMMEDIATE ANNUITIES

Attained		WITHOUT PROJECTION					H Projection B		
Ages	1941-48	1948-53	1953-58	1958-63	1963-67	1953-58	1958-63	1963-67	
Under 60	145	171% 139 124 101	189% 129 116 105	126% 121 111 101	135% 98 94 96	201% 136 120 107	143% 135 120 104	162% 115 105 100	
All ages	130%	117%	113%	107%	96%	116%	112%	104%	
			1	Nonrefund	Annuities	3			
Under 60		106% 108 118 91	180% 115 93 106	189% 93 92 90	101% 77 78 84 82%	193% 122 96 107	214% 104 99 92	121% 90 87 87 87	

² TSA, 1969 Reports, pp. 5-62.

³ TSA, 1966 Reports, pp. 191-247.

TABLE 1-Continued SETTLEMENT ANNUITIES

ATTAINED		Without Projection							
Ages	1940-45	1945-50	1950-55	1955-60	1960-65				
			ns Arising fron ers,* and Death						
Under 60	103% 93 107	79% 110 103	94% 90 100	95% 94 100	115% 84 89				
All ages	100%	105%	95%	98%	88%				
-	No	npayee Electio	ns Arising fron	n Death Claims	5				
Under 60	372% 180 238	185% 209 159	240% 88 169	265% 151 125	172% 143 99				
All ages	245%	180%	166%	172%	128%				
;	M		ed Annuities wi or Refund Pro	-	d				
Under 60	185% 136 108	171% 134 109	124% 118 99	113% 111 104	111% 101 103				
All ages	127%	121%	106%	106%	103%				
	Matured Deferred Annuities without a Guaranteed Period or Refund Provision								
Under 60	98% 85	† 110% 98	† 55% 91	173% 75 92	† 91% 88				
All ages	93%	102%	78%	88%	89%				

Note.—Mortality ratio in italics where 10-49 contracts terminated by death. * Excluding maturities and surrenders under pension trust issues.

[†] Fewer than 10 contracts terminated by death.

TABLE 1—Continued
FEMALE LIVES
IMMEDIATE ANNUITIES

Attained		Witi	ноит Рвој	ROJECTION WITH PROJECTION B				
Ages	1941-48	1948-53	1953-58	1958-63	1963-67	1953-58	1958-63	1963-67
			,					
Under 60		160% 122 116 106	164% 115 113 103	157% 108 107 104	96% 108 84 94	175% 122 117 104	179% 120 115 106	116% 127 94 97
All ages	118%	112%	107%	105%	92%	110%	110%	98%
			<u>'</u>	Nonrefund	1 Annuitie	s		
Under 60	111	106% 104 106 98	103% 87 98 95	160% 102 102 98	301% 63 77 89	110% 92 102 96	182% 114 110 100	362% 73 85 92
All ages	109%	101%	96%	99%	86%	98%	102%	90%

TABLE 1-Continued SETTLEMENT ANNUITIES

Attained	WITHOUT PROJECTION							
Aces	1940-45	1945-50	1950-55	1955-60	1960-65			
			ons Arising from rs,* and Death					
Under 60	137% 110 104	126% 109 102	123% 94 98	105% 90 95	145% 88 93			
All ages	111%	107%	98%	94%	93%			
(Nonpayee Elections Arising from Death Claims							
Under 60	172% 132 119	142% 121 122	153% 111 113	148% 110 111	139% 112 103			
All ages	130%	124%	116%	113%	106%			
- - 	N		ed Annuities wi or Refund Pro		d			
Under 60	161% 133 109	153% 114 110	143% 107 101	123% 97 93	162% 86 91			
All ages	125%	113%	104%	94%	91%			
-	Matured Deferred Annuities without a Guaranteed Period or Refund Provision							
Under 60	† 105% 104	118% 86 95	135% 78 96	127% 68 88	134% 82 92			
All ages	105%	92%	91%	83%	91%			

Note.—Mortality ratio in italics where 10-49 contracts terminated by death. * Excluding maturities and surrenders under pension trust issues.

[†] Fewer than 10 contracts terminated by death.

tality ratios for all contract years combined by amount of annual income. The basis for expected deaths shown in the indicated tables of the *Reports* is the *a*-1949 Ultimate Table (unprojected) and, in the case of the last three studies of immediate annuities, the *a*-1949 Ultimate Table with Projection Scale B as well.

The Reports provide further details on the variation in mortality ratios by number of contracts, by contract year, by amount of annual income group in the case of immediate annuities, and so on, which are not reproduced here.

Table 1 shows that under immediate annuities the margins in the a-1949 Ultimate Table have largely disappeared, even when the mortality rates are projected with Projection Scale B. The only mortality ratio for all ages combined which is above 100 per cent in the latest (1963–67) study is under male refund annuities, and then only when Projection Scale B is used in obtaining the expected deaths. (The state regulations generally specify the *unprojected* table as a minimum standard.) The lowest mortality ratios, of course, are for nonrefund annuities—on an unprojected basis, 82 per cent for males and 86 per cent for females for all ages combined.

Under settlement annuities, mortality ratios for all ages combined are under 100 per cent (based on the unprojected a-1949 Table) in most cases in the latest (1960–65) study. There are still some margins in the a-1949 Ultimate Table rates for settlements under nonpayee elections arising from death claims, particularly for males, and for matured deferred annuities with a refund period for males only.

A minimum valuation standard should apply to all types of annuities, both immediate annuities and settlement annuities. To test the over-all adequacy of current valuation standards, Table 2 shows the ratios of actual to expected deaths for all types of immediate annuities (1963–67 study) and settlement annuities (1960–65 study) combined, for all contract years combined, by amount of annual income. The mortality ratios are shown for quinquennial and decennial age groups from 60 to 99. Expected deaths are based on the a-1949 Ultimate Table (unprojected) and the 1937 Standard Annuity Table, two of the most common mortality tables specified as minimum valuation standards for individual annuities. Also shown are the ratios of actual to expected deaths based on the a-1949 Ultimate Table projected to 1963 with Projection Scale B. The year 1963 was chosen because it is the central year of the combined experience based on a weighting by exposures.

For ages 60-99 of the combined experience, the ratios of actual to expected deaths based on the a-1949 Ultimate Table (unprojected) are

TABLE 2

RATIOS OF ACTUAL TO EXPECTED DEATHS FOR COMBINED
1963-67 IMMEDIATE ANNUITY EXPERIENCE AND
1960-65 SETTLEMENT ANNUITY EXPERIENCE
ALL CONTRACT YEARS COMBINED—
BY AMOUNT OF ANNUAL INCOME

Age Group	a-1949 Ultimate Table (Unprojected)	a-1949 Ultimate Table (Projected by Scale B to 1963)	1937 Standard Annuity Table
		Male Lives	
60-64 65-69 70-74 75-79 80-84 85-89 90-94 95-99	101.0% 94.8 94.3 94.2 94.8 91.2 94.6 97.6	117.4% 108.5 105.6 102.6 100.0 93.2 94.6 97.6	80.2% 77.4 81.3 87.7 96.1 101.0 114.1 126.1 78.0
70–79 80–89 90–99	94.2 93.4 95.1	104.1 97.4 95.1	84.3 97.9 116.1
60-99	94.4	103.0	87.1
		Female Lives	
60-64 65-69 70-74 75-79 80-84 85-89 90-94 95-99	110.9% 87.4 86.0 91.6 96.6 98.4 97.8 77.6	128.9% 100.0 96.3 99.6 101.8 100.4 97.8 77.6	65.0% 58.6 67.2 84.1 104.7 125.4 144.4 130.7
60–69	94.0 89.2 97.4 93.3	108.0 98.2 101.2 93.3	60.6 76.1 113.0 141.7
60–99	93.4		89.1

about 94 per cent for both males and females. Even with projection to 1963, the margin in the a-1949 Table is very thin for males (ratio of actual to expected equals 103 per cent) and nonexistent for females (ratio of actual to expected equals 100 per cent). Of course, the 1937 Standard Annuity Table is even more deficient as a valuation standard, with overall mortality ratios below 90 per cent for both males and females.

The variation in mortality ratios by age group is also of some interest. In the case of the a-1949 Table (unprojected) there is still some margin in the low sixties, especially for females. At ages 65 and over, mortality ratios are consistently below 100 per cent. For males the mortality ratios at ages 65 and over are fairly level, while for females there is a certain amount of fluctuation by age group. This indicates that a graduated mortality table representative of the combined experience would require changes in the slope of the mortality curve, in addition to an over-all decrease in mortality rates, as compared to the a-1949 Table.

The effect on the mortality ratios of projecting the a-1949 Table to 1963 is naturally greatest in the sixties and diminishes with increasing age, since Projection Scale B assumes annual rates of decrease in mortality which decrease by age, with no improvement at ages 90 and over.

After review of these results, it was clear that there is a need for a new minimum reserve standard for individual annuities. The data also suggested that it would be best to construct an entirely new mortality table rather than attempt a simple adjustment of the a-1949 Table.

Source of Experience Data

The objective of the subcommittee was to develop an annuity mortality table which would be "safe," based on current levels of annuitant mortality, for the valuation of all types of individual annuities, including single premium immediate annuities, life income settlements, and matured deferred annuities. This objective had a considerable influence on the approach taken by the subcommittee; the approach would have been quite different if the objective had been, say, to establish a basis for gross premium rates for nonparticipating single premium immediate nonrefund annuities.

Keeping this objective in mind, it was decided to base the new table on the combined experience under individual immediate annuities and under life income settlements and matured deferred annuities. The combined experience includes both refund and nonrefund immediate annuities and settlement annuities and, under the latter classification, includes life income settlements from all sources (death claims whether elected by payee or nonpayee, matured endowments, or cash surrenders).

Of course, this decision represents a marked departure from the "traditional" approach of including experience under nonrefund immediate annuities only, which was followed in the construction of the \$a\$-1949 Table. It is true that with the approach adopted for the new table the mix of the experience data by type of annuity is to some extent arbitrary, in that it is based on the data submitted by the particular companies which contributed to the intercompany studies. It was felt, however, that the combined immediate annuity and settlement annuity experience, with suitable margins, was an appropriate, broad base for developing a mortality table to be used as a minimum valuation standard for all types of individual annuities. As noted in Section VII, in certain situations an actuary may decide to use the 1971 IAM Table with some kind of adjustment, such as an age setback, for annuities of a particular type.

It was decided to use the latest intercompany studies as the starting point for developing the table at the ages for which the data were extensive enough to be significant, say, ages 60 and over. For the younger ages, 50 and under, the rates would be taken from the companion 1971 Group Annuity Mortality Table (1971 GAM) being developed by the other subcommittee members. The table would then be completed by bridging the rates between the lower and higher ages.

Data by Number or Amount

The a-1949 Table was based on number of contracts, since "the excess of mortality by amounts of annual income was rather small and because the experience by amounts showed considerable fluctuation by age." As noted in the latest report on individual immediate annuities, there have been significant increases in the average amount of annual income over the past decade or so. Furthermore, the report showed that ultimate mortality ratios for larger amounts of annual income are significantly lower than those for all amounts combined. (A comparable study of settlement annuities by amount is not currently available, but one will be, when the 1965-70 experience is published.)

Since the financial effect of annuity mortality is measured by the amount of annual income rather than by number of contracts, it was decided to base the new table on amount of annual income. This is consistent with the approach used in developing mortality tables for life insurance, where amount of insurance rather than number of policies is used.

Aggregate, Select, or Ultimate Table

The effect of self-selection on annuity mortality is quite pronounced. It was the conclusion of the report on the 1963–67 immediate annuity experience that selection appears to persist in some degree for at least three to five contract years, depending on type of annuity and age.⁶ Under settlement annuities, selection was pronounced for payee-elected death claim settlements but was not as clearly evident or did not appear to exist under other types of settlements.⁷

Mortality tables for valuation of life insurance policies are normally on an ultimate basis. Obviously, while this is conservative for life insurance, it is not for annuities. The a-1949 Table is actually a one-year select and ultimate table, with the first-year mortality rate equal to 75 per cent of the ultimate mortality rate for males and 50 per cent of the ultimate mortality rate for females. However, it is the "Ultimate" portion of the a-1949 Table which is generally specified as a minimum valuation standard.

It was felt that select mortality under annuities was too important to ignore, that is, that the minimum valuation standard should not be based on an ultimate table. On the other hand, there are practical problems in applying a select and ultimate table in the actual mechanics of the valuation. After consideration of these points, it was decided to base the valuation mortality table on the experience of all policy years combined, that is, to construct an aggregate table.

The use of the aggregate experience as the base for the table is subject to the criticism that the particular mix of data by contract year at each age depends heavily on the incidence of annuity sales experienced by the contributing companies up to the end of the period of exposure. This mix may be different for different companies and may change in future years. It was felt, however, that an aggregate table was a reasonable compromise between the need to establish a conservative valuation standard which reflects select mortality and the need to keep the mechanics of the valuation within practical bounds.

Method of Graduation

It was intended, in the initial stages of developing the 1971 IAM Table, to base the graduation on a Makeham curve. This would simplify the calculation of joint life annuity values and would automatically ensure smoothness. It turned out that a Makeham curve produced an excellent fit for males with one set of constants over the entire range of

⁶ TSA, 1969 Reports, pp. 37-40.

⁷ TSA, 1966 Reports, pp. 216-224.

ages from 60 to 99. However, the fit for females was judged to be too poor to justify the convenience of a Makeham curve. Consequently, we abandoned the Makeham approach and turned to other graduation techniques for both males and females in order to use a consistent method for both sexes.

Another factor which influenced our decision to abandon the Makeham curve was the fact that, with modern computers, it is quite simple to calculate joint life annuity values on an exact basis for any age combination, even on a projected basis. This is discussed further in Section VIII.

Comparison with Development of a-1949 Table

It can be seen that the construction of the 1971 IAM Table differs in a number of important respects from the construction of the a-1949 Table. Additional details are provided in Sections III and IV. For convenience, Appendix A presents a brief summary of the differences in the construction of the two tables and of the underlying experience tables (the 1943 Experience Table in the case of the a-1949 Table and the 1963 Experience Table in the case of the 1971 IAM Table).

III. CONSTRUCTION OF 1963 EXPERIENCE TABLE

In this section we will describe the construction of the experience table underlying the 1971 IAM Table. This table will be referred to as the "1963 Experience Table."

Ages 62 and Over

The data used as the basis for the 1963 Experience Table consist of the combined intercompany mortality studies of immediate annuities from anniversaries in 1963 to anniversaries in 1967, and of settlement annuities from anniversaries in 1960 to anniversaries in 1965, by amount of annual income. The total volume of exposures in the combined male and female experience was 2,804,637 contract-years for \$1,422,660,191 of annual income. There were 109,383 contracts, with an annual income of \$52,678,122 terminated by death. The subdivision of the exposures and deaths by sex and type of annuity (refund or nonrefund) is shown in Table 3.

The central date of the immediate annuity study was taken as July 1, 1965, and the central date of the settlement annuity study as January 1, 1963. When these dates are weighted by exposures in the two intercompany studies, the central date falls in July, 1963. (The ratio of the settlement annuity exposures to the immediate annuity exposures is approximately 3.9 to 1.) Thus the experience table developed from these data was taken as representative of annuity mortality in 1963.

TABLE 3

EXPOSURES AND DEATHS IN INTERCOMPANY 1963-67 STUDY OF IMMEDIATE ANNUITIES AND 1960-65 STUDY OF SETTLEMENT ANNUITIES—ALL AGES AND CONTRACT YEARS COMBINED

				1		
	E	XPOS	GURES]	DEATHS	
	No. of Contracts		Amount of Annual Income	No. of Contracts	Amount of Annual Income	
			Males			
Immediate annuities: Refund Nonrefund	104,239 49,879	\$	68,495,429 38,367,247	6,778 3,554	\$ 3,786,276 2,540,459	
Total	154,118	\$	106,862,676	10,332	\$ 6,326,735	
Settlement annuities: Refund Nonrefund	731,468 37,213	\$	427,415,936 20,451,710	30,080 1,688	\$17,159,803 886,793	
Total	768,681	\$	447,867,646	31,768	\$18,046,596	
Immediate and settlement annuities combined: RefundNonrefund	835,707 87,092 922,799	\$	495,911,365 58,818,957 554,730,322	36,858 5,242 42,100	\$20,946,079 3,427,252 \$24,373,331	
		•	Female	s		
Immediate annuities: Refund Nonrefund	265,806 131,362 397,168	\$	118,846,794 64,857,745 183,704,539	16,070 9,354 25,424	\$ 5,338,789 3,678,887 \$ 9,017,676	
Settlement annuities: Refund Nonrefund	1,334,349 150,321	\$	633,230,805 50,994,525	36,568 5,291	\$17,672,039 1,615,076	
Total	1,484,670	\$	684,225,330	41,859	\$19,287,115	
Immediate and settlement annuities combined: Refund	1,600,155 281,683	\$	752,077,599 115,852,270	52,638 14,645	\$23,010,828 5,293,963	
Total	1,881,838	\$	867,929,869	67,283	\$28,304,791	

TABLE 3-Continued

	Ez	KPOSURES	Deaths		
	No. of Contracts	Amount of Annual Income	No. of Contracts	Amount of Annual Income	
		Males and Female	s Combined		
Immediate annuities: Refund Nonrefund Total Settlement annuities: Refund.	370,045 181,241 551,286 2,065,817	\$ 187,342,223 103,224,992 \$ 290,567,215	22,848 12,908 35,756 66,648	\$ 9,125,065 6,219,346 \$15,344,411 \$34.831.842	
Nonrefund	187,534	71,446,235 \$1,132,092,976	73,627	2,501,869 \$37,333,711	
Immediate and settlement annuities combined:	2,435,862	\$1,247,988,964	89,496	\$43,956,907	
Nonrefund Total	368,775	\$1,422,660,191	19,887	\$52,678,122	

Two adjustment factors were determined before combining the data from the immediate annuity and settlement annuity studies. The first adjustment, the weighting of the data by 5 to 4, reflected the fact that the immediate annuity study encompassed a four-year period, while the settlement annuity study encompassed a five-year period. The second adjustment was made to reflect the fact that some companies that contributed to the immediate annuity study did not contribute to the settlement annuity study, and conversely. This adjustment was based on the assumption that, if the companies that contributed to only one study had contributed to both studies, their exposures for the two types of studies would have been in the same proportion as for the companies that actually contributed to both studies. Coincidentally, the second adjustment offset the first almost exactly (the weighting factors based on the two adjustments combined were 1.007 for immediate annuities and 1.000 for settlement annuities). Accordingly, it was decided to simply combine the data from the two intercompany studies without adjustment.

The data were then grouped in five-year age groups, separately for males and females, and King's formula was applied separately to ex-

posures and deaths to obtain pivotal points for a Jenkins fifth-difference modified osculatory interpolation. Pivotal points were obtained for ages 52, 57, ..., 97, which allowed for the calculation of graduated mortality rates at individual ages 62–87. A cubic was then fitted through the pivotal points at ages 87, 92, and 97 and through a mortality rate of 1 at age 115. Additional pivotal points at ages 102 and 107 were obtained from the cubic, which permitted the continuation of the Jenkins interpolation through age 97. Finally, the table was closed by taking the mortality rates at ages 98–115 directly from the cubic.

Age 115 was chosen as the age at which $q_x = 1$ on the basis of the fact that Makeham curves fitted to the data at ages 60–99 by the method of moments gave mortality rates which did not exceed 0.95 until age 116 for males and age 114 for females.

Ages 50 and Under

Mortality rates for ages 5-50 were based on the 1966 experience table underlying the 1971 GAM Table, the details of which are presented in the paper "The 1971 Group Annuity Mortality Table," by Harold R. Greenlee, Jr., and Alfonso D. Keh (in this issue). The rates from the 1966 Experience Table were adjusted to a 1963 basis by using the same projection factors (Projection Scale D) by means of which the 1966 experience table was obtained from the Ga-1951 Table.

Bridging Younger and Older Ages

The mortality rates at the younger and older ages were bridged by fitting a fourth-degree polynomial through the mortality rates at ages 48, 49, 50, 63, and 64. This actually produced a slight change in the mortality rate at age 62 originally obtained by the Jenkins graduation, resulting in a smoother junction between the two segments of the table.

The Complete Table

The 1963 Experience Table for males and females is shown in Table 4. Table 5 shows the mortality ratios for ages 60-99 in five- and ten-year age groups. Table 6 is an additional test of the fit of the 1963 Experience Table by means of comparing annuity values at $3\frac{1}{2}$ per cent interest based on the crude rates of mortality with those based on the graduated table. It can be seen that the graduated experience table results in a very good fit over the range of ages 60-99 for all types of annuities combined. Variation in the ratios of actual to expected by type of annuity is discussed in Section VII.

With respect to smoothness at ages over 50, first differences are positive throughout and the progression of second differences was judged

TABLE 4 1963 EXPERIENCE TABLE—1,000 q_x

Age	Males	Females	Age	Males	Females	Age	Males	Females
5 6 7 8 9	0.517 0.480 0.457 0.445 0.440	0.286 0.235 0.197 0.174 0.163	45 46 47 48 49	3.311 3.759 4.253 4.790 5.370	1.704 1.854 2.018 2.199 2.401	85 86 87 88 89	122.442 133.247 145.608 159.803 175.943	102.958 115.172 128.843 143.996 160.110
10 11 12 13	0.441 0.449 0.459 0.468 0.478	0.162 0.175 0.190 0.204 0.220	50 51 52 53 54	5.988 6.651 7.365 8.134 8.961	2.624 2.890 3.215 3.613 4.088	90 91 92 93 94	194.100 214.344 236.744 261.329 287.941	176.525 192.583 207.623 221.184 233.581
15 16 17 18 19	0.490 0.503 0.518 0.534 0.550	0.235 0.250 0.266 0.282 0.299	55 56 57 58 59	9.851 10.803 11.817 12.893 14.029	4.644 5.278 5.981 6.740 7.538	95 96 97 98 99	316.384 346.460 377.968 410.717 444.538	245.327 256.932 268.911 281.752 295.853
20 21 22 23 24	0.570 0.592 0.616 0.641 0.670	0.317 0.336 0.356 0.376 0.399	60 61 62 63 64	15.222 16.466 17.757 19.088 20.451	8.352 9.153 9.909 10.582 11.129	100 101 102 103 104	479.272 514.755 550.828 587.329 624.096	311.592 329.343 349.486 372.393 398.443
25 26 27 28 29	0.701 0.736 0.775 0.818 0.865	0.423 0.448 0.475 0.505 0.537	65 66 67 68 69	21.988 23.714 25.647 27.804 30.207	11.735 12.467 13.386 14.551 15.992	105 106 107 108 109	660.968 697.784 734.383 770.603 806.284	428.011 461.475 499.209 541.591 588.996
30 31 32 33	0.916 0.975 1.038 1.108 1.185	0.572 0.609 0.650 0.694 0.742	70 71 72 73 74	32.880 35.845 39.126 42.750 46.759	17.732 19.793 22.199 24.976 28.162	110 111 112 113 114	841.263 875.380 908.473 940.381 970.943	641.801 700.383 765.117 836.380 914.548
35 36 37 38 39	1.271 1.364 1.468 1.582 1.710	0.795 0.852 0.915 0.985 1.060	75 76 77 78 79	51.201 56.124 61.574 67.580 74.102	31.799 35.927 40.587 45.822 51.677	115	1,000.000	1,000.000
40 41 42 43 44	1.849 2.027 2.266 2.561 2.910	1.144 1.236 1.336 1.448 1.569	80 81 82 83 84	81 .080 88 .455 96 .167 104 .221 112 .873	58.199 65.434 73.429 82.249 92.038			

to be reasonably satisfactory for the experience table. For males, second differences are positive except at ages 104 and over. For females, several second differences are negative in the early sixties and in the vicinity of age 90. The results for females appeared to reflect basic characteristics of the underlying data, which was confirmed in part by an independent graduation using a Whittaker-Henderson Type B formula.

TABLE 5

RATIOS OF ACTUAL TO EXPECTED DEATHS BASED ON 1963 EXPERIENCE TABLE
1963-67 IMMEDIATE ANNUITY EXPERIENCE
COMBINED WITH 1960-65 SETTLEMENT ANNUITY EXPERIENCE
ALL CONTRACT YEARS COMBINED—BY AMOUNT OF ANNUAL INCOME

:	;	Male Lives	FEMALE LIVES			
Age Group	Actual Deaths	1 10		Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
65-69 70-74 75-79 80-84 85-89 90-94 95-99 60-69 70-79	\$ 1,071,581 4,291,315 5,622,360 5,388,209 4,068,380 2,355,797 915,017 198,448 5,362,896 11,010,569	\$ 1,029,809 4,271,860 5,587,806 5,399,036 4,023,413 2,355,310 920,043 205,450 5,301,669 10,986,842	104.1% 100.5 100.6 99.8 101.1 100.0 99.5 96.6 101.2 100.2	2,448,738 4,131,802 5,745,112 6,145,444 4,957,796 2,351,194 530,291 3,659,373 9,876,914	\$ 1,154,541 2,467,040 4,126,665 5,730,656 6,128,146 5,013,667 2,324,991 544,239 3,621,581 9,857,321	104.9% 99.3 100.1 100.3 100.3 98.9 101.1 97.4
80–89 90–99 60–99	6,424,177 1,113,465 23,911,107	6,378,723 1,125,493 23,792,727	100.7 98.9 100.5	11,103,240 2,881,485 27,521,012	11,141,813 2,869,230 27,489,945	99.7 100.4 100.1

The 1963 Experience Table was developed as the starting point for the construction of a valuation mortality table representative of the level of mortality in 1971. It may also be useful for analysis and historical purposes.

IV. CONSTRUCTION OF 1971 INDIVIDUAL ANNUITY MORTALITY TABLE

This section describes the method of construction of the 1971 IAM Table from the underlying 1963 Experience Table.

Ages 60 and Over

The first step in constructing the 1971 IAM Table from the 1963 Experience Table was to apply to the latter table factors which reflected

TABLE 6

Comparison of Annuity Values Based on Crude Data and on 1963 Experience Table at 3½ Per Cent Temporary Life Annuities to Age 100

		Males	•		Females	
Age	Crude Data*	1963 Experience Table (2)	(2)÷(1) (3)	Crude Data* (4)	1963 Experience Table (5)	(5) ÷ (4) (6)
			Immediate l	Life Annuity		
60	12.590 10.816 8.957 7.145 5.460 4.012 2.686 1.598	12.631 10.828 8.972 7.152 5.480 4.007 2.662 1.548	100.3% 100.1 100.2 100.1 100.4 99.9 99.1 96.9	14.458 12.590 10.475 8.293 6.239 4.464 3.045 2.102	14.479 12.585 10.479 8.296 6.235 4.445 3.058 2.054	100.1% 100.0 100.0 100.0 100.0 99.9 99.6 100.4 97.7
		Lif	e Annuity with	10 Years Cer	tain	
60	13.389 11.944 10.607 9.547 8.819 8.448 8.317 8.317	13.417 11.953 10.621 9.557 8.828 8.442 8.317 8.317	100.2% 100.1 100.1 100.1 100.1 99.9 100.0 100.0	14.906 13.217 11.513 10.061 9.054 8.544 8.317 8.317	14.919 13.215 11.512 10.052 9.049 8.545 8.317 8.317	100.1% 100.0 100.0 99.9 99.9 100.0 100.0
		Lif	e Annuity with	20 Years Cer	tain	
60	15.509 14.836 14.423 14.252 14.212 14.212	15.522 14.842 14.427 14.251 14.212 14.212	100.1% 100.0 100.0 100.0 100.0 100.0	16.223 15.242 14.589 14.301 14.212 14.212	16.225 15.237 14.587 14.301 14.212 14.212	100.0% 100.0 100.0 100.0 100.0 100.0

^{*&}quot;Crude Data" refers to ungraduated mortality experience by amount of annual income for ages 60-99 from combined 1963-67 immediate annuity and 1960-65 settlement annuity studies.

(1) the assumed decrease in mortality over the eight-year period from 1963 to 1971 and (2) an explicit margin for mortality fluctuations and contingencies. The assumed decrease in mortality was based on the annual percentage decrease (geometric basis) between the two latest intercompany studies, that is, between the 1958-63 and 1963-67 studies of immediate annuities and between the 1955-60 and 1960-65 studies

TABLE 7

AVERAGE ANNUAL DECREASE IN MORTALITY (GEOMETRIC BASIS) BETWEEN
TWO LATEST SOCIETY OF ACTUARIES STUDIES OF IMMEDIATE
ANNUITIES AND SETTLEMENT ANNUITIES
ALL CONTRACT YEARS COMBINED—BY AMOUNT OF ANNUAL INCOME

Attained			FROM	SETTLEMENT ANNUITIES FROM 1955-60 STUDY TO 1960-65 STUDY			COMBINED STUDIES		
Ages	Refund	Non- refund	Com- bined	Refund	efund Non- Com-		Refund	Non- refund	Com- bined
		Male Lives							
60-69 70-79 80 and over	4.6% 3.6 1.1	4.1% 3.6 1.5	4.2% 3.6 1.4	1.4% 1.0 1.6	-2.2% 3.4 -1.9	1.4% 1.0 1.4	1.6% 1.4 1.2	0.7% 3.5 1.0	1.6% 1.6 0.9
				F	emale Live	;			
60-69 70-79 80 and over	0.0% 5.2 2.2	10.1% 6.1 2.1	2.4% 5.6 2.1	0.6% 1.5 0.0	-3.6% -1.2 -0.6	0.2% 1.3 0.0	0.6% 2.1 0.8	1.6% 3.3 1.4	0.6% 2.4 1.1
			Ма	le and Fe	male Lives	Combin	red		
79 and under 80 and over									1.6%

of settlement annuities. These percentages, which are based on amount of annual income for all contract years combined, are shown in Table 7.

Since there was considerable variation in the percentages by type of annuity and age group, it was decided to base the projection factors on very broad groupings of the data. Accordingly, the percentages were determined for all types of annuities and both sexes combined, and ages 79 and under were combined. On the basis of the percentages for these

broad groupings, shown in the lower right-hand portion of Table 7, the annual percentage decrease in mortality between 1963 and 1971 was taken as 1.6 per cent for ages 79 and under, decreasing by 0.1 per cent for each age thereafter and reaching zero at age 95.

An explicit margin for mortality fluctuations and contingencies equal to 10 per cent of the projected mortality rate was then applied, that is, the 1971 mortality rates with margins were taken as 90 per cent of the 1963 rates projected to 1971 by the percentage decrease factors just described. A 10 per cent margin appeared to be reasonable on the basis of the variation in mortality among the companies contributing to the intercompany studies. About 75-80 per cent of the companies contributing to the immediate annuity study had mortality ratios which were not more than 10 percentage points below the all-company mortality ratios. Similarly, a 10 per cent margin would "cover" most of the companies contributing to the study of settlement annuities. The particular percentage of companies that would be covered by a 10 per cent margin varies by type of annuity, that is, refund or nonrefund, payee or nonpayee elected, and so on.

The factors applied to the 1963 Experience Table to project to 1971 and to reflect the 10 per cent margin are shown in Table 8.

Of course, a 10 per cent margin in mortality rates does not generally produce a 10 per cent margin in annuity values. The extent to which the projection factors and the 10 per cent margin increase life annuity values over the corresponding values based on the 1963 Experience Table is shown later in this section.

Ages 5 to 50

The mortality rates for the 1971 IAM Table at ages 5-50 were taken directly from the 1971 GAM Table.

Bridging Younger and Older Ages

As in the case of the 1963 Experience Table, the younger and older ages were bridged by passing a fourth-degree polynomial through the mortality rates at ages 48, 49, 50, 63, and 64. This resulted in very slight changes in the rates at ages 60, 61, and 62, with attendant improvement in smoothness.

The Complete Table

At this point, a complete table of mortality rates for ages 5-115 was at hand. Although the rates for ages 60 and over were inherently smooth, since they were obtained by multiplying the rates from a graduated table (the 1963 Experience Table) by a set of factors grading down by age, it

was felt, after inspection of the differences, that smoothness could be improved slightly. Accordingly, the rates for ages 56 and over were adjusted by a Whittaker-Henderson Type A formula with a=1. In the case of females very slight empirical adjustments were made in the rates at ages 51–55 to further improve smoothness.

The 1971 IAM Table is shown in Table 9. Table 10 shows the ratios of

TABLE 8
FACTORS USED TO DERIVE 1971 IAM TABLE
FROM 1963 EXPERIENCE TABLE

Attained Ages	Annual Decrease in Mortality (Geometric Basis) (1)	Projection Factor for 8-Year Period =(1-Col. 1)8 (2)	Projection Factor with 10 Per Cent Margin =0.9×Col. 2 (3)
60-79	1.6%	0.879	0.791
80. 81. 82. 83.	1.5 1.4 1.3 1.2	0.886 0.893 0.901 0.908 0.915	0.797 0.804 0.811 0.817 0.824
85 86 87 88 89	1.0 0.9 0.8 0.7 0.6	0.923 0.930 0.938 6.945 0.953	0.831 0.837 0.844 0.851 0.858
90	0.5 0.4 0.3 0.2 0.1	0.961 0.968 0.976 0.984 0.992	0.865 0.871 0.878 0.886 0.893
95 and over	0	1.000	0.900

actual to expected based on the 1971 IAM Table for the combined immediate annuity and settlement annuity experience underlying the 1963 Experience Table. Overall, the mortality ratio based on the 1971 IAM Table is 125.1 per cent for males and 123.0 per cent for females, which reflects the 10 per cent margin and the projection from 1963 to 1971.

Table 11 shows values of l_x and d_x , and commutation columns at $3\frac{1}{2}$, 5, 6, and 7 per cent, respectively.

Effect of Projection to 1971 and Introduction of Margins on Annuity Values

Table 12 shows the effect of the projection from 1963 to 1971 and the
effect of the 10 per cent margin on immediate life annuity values with

TABLE 9 1971 Individual Annuity Mortality Table—1,000 q_x

					1	I	1	
Age	Males	Females	Age	Males	Females	Age	Males	Females
5	0.456	0.234	45	2.922	1.397	85	101 . 689	85.770
6	0.424	0.193	46	3.318	1.519	86	111 . 652	96.898
7	0.403	0.162	47	3.754	1.654	87	123 . 048	109.338
8	0.392	0.143	48	4.228	1.802	88	136 . 123	122.978
9	0.389	0.134	49	4.740	1.967	89	151 . 070	137.508
10	0.390	0.132	50	5.285	2.151	90	168.040	152.472
11	0.397	0.143	51	5.860	2.371	91	187.147	167.370
12	0.405	0.155	52	6.461	2.641	92	208.457	181.776
13	0.413	0.167	53	7.088	2.966	93	231.885	195.386
14	0.422	0.180	54	7.740	3.351	94	257.146	208.071
15	0.433	0.193	55	8.417	3.791	95	283 .841	219.896
16	0.444	0.205	56	9.119	4.284	96	311 .565	231.097
17	0.457	0.218	57	9.850	4.826	97	340 .214	242.211
18	0.471	0.231	58	10.613	5.409	98	369 .769	253.823
19	0.486	0.245	59	11.411	6.017	99	400 .194	266.452
20	0.503	0.260	60	12.249	6.628	100	431.413	280.535
21	0.522	0.275	61	13.133	7.219		463.312	296.449
22	0.544	0.292	62	14.073	7.773		495.756	314.535
23	0.566	0.309	63	15.083	8.285		528.599	335.121
24	0.591	0.327	64	16.185	8.775		561.692	358.537
25	0.619	0.347	65	17.405	9.290	105	594.884	385.122
26	0.650	0.368	66	18.767	9.888	106	628.022	415.238
27	0.684	0.390	67	20.290	10.622	107	660.949	449.274
28	0.722	0.414	68	21.992	11.536	108	693.503	487.649
29	0.763	0.440	69	23.890	12.664	109	725.521	530.787
30	0.809	0.469	70	26.000	14.029	110	756.852	579.040
31	0.860	0.499	71	28.341	15.651	111	787.390	632.529
32	0.916	0.533	72	30.933	17.548	112	817.125	690.903
33	0.978	0.569	73	33.801	19.742	113	846.198	753.081
34	1.046	0.608	74	36.976	22.256	114	874.915	817.218
35 36 37 38 39	1.122 1.204 1.295 1.397 1.509	0.651 0.698 0.750 0.807 0.869	75 76 77 78 79	40.494 44.393 48.715 53.500 58.787	25.120 28.369 32.050 36.225 40.975	115	1,000.000	1,000.000
40 41 42 43	1.633 1.789 2.000 2.260 2.569	0.938 1.013 1.094 1.186 1.286	80 81 82 83 84	64.599 70.902 77.668 84.941 92.874	46.386 52.513 59.409 67.160 75.899			
				_	<u> </u>		ļ	

annual payments at $3\frac{1}{2}$ per cent interest for ages 60 and over. Column 4 shows that the projection to 1971 has the effect of producing an increase in male life annuity values without refund of 4.0 per cent at age 60, which rises to a peak of 5.6 per cent at age 75 and declines to 0 per cent at age 95 (since the annual decrease in mortality was assumed to be zero at ages 95 and over). The corresponding increase in female life annuity

TABLE 10

RATIOS OF ACTUAL TO EXPECTED DEATHS BASED ON 1971 IAM TABLE 1963-67 IMMEDIATE ANNUITY EXPERIENCE COMBINED WITH 1960-65 SETTLEMENT ANNUITY EXPERIENCE ALL CONTRACT YEARS COMBINED—BY AMOUNT OF ANNUAL INCOME

		Male Lives		FEMALE LIVES			
Age Group	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths	
65-691 70-7475-7980-8485-8990-9495-99	5,622,360 5,388,209 4,068,380 2,355,797 915,017 198,448 5,362,896 11,010,569 6,424,177	\$ 817,953 3,379,697 4,418,222 4,273,712 3,249,210 1,984,393 806,463 184,657 4,197,650 8,691,934 5,233,603	131.0% 127.0 127.3 126.1 125.2 118.7 113.5 107.5	2,448,738 4,131,802 5,745,112 6,145,444 4,957,796 2,351,194 530,291 3,659,373 9,876,914 11,103,240	\$ 908,791 1,955,369 3,262,485 4,530,598 4,962,385 4,242,532 2,030,252 489,244 2,864,160 7,793,083 9,204,917	133.2% 125.2 126.6 126.8 123.8 116.9 115.8 108.4 127.8 126.7 120.6	
90 - 99	1,113,465 23,911,107	991,120	112.3	2,881,485 27,521,012	2,519,496 22,381,656	114.4	

values without refund is 2.8 per cent at age 60, rising to a peak of 4.5 per cent at age 80 and declining to 0 per cent at age 95.

Column 5 shows that the 10 per cent margin further increases annuity values by a percentage which increases with age, ranging from 3.3 per cent at age 60 to 13.7 per cent at age 95 for males and from 2.5 per cent at age 60 to 12.1 per cent at age 95 for females.

It can be seen from column 6 that the combined effect of the projection from 1963 to 1971 and the 10 per cent margin on the male life annuity values without refund runs from 7.4 per cent at age 60 to 13.7 per cent at age 95. For females the percentage increase runs from 5.3 per cent at

TABLE 11
1971 INDIVIDUAL ANNUITY MORTALITY TABLE
ELEMENTARY VALUES

MALE LIVES

Age x	l_x	d_x	Age x	l_x	d_x
5	10,000.0000	4.5600	61	8,628.8188	113.3223
6	9,995.4400	4.2381	62	8,515.4965	119.8386
7	9,991.2019	4.0264	63	8,395.6579	126.6317
8	9,987.1755	3.9150	64	8,269.0262	133.8342
9	9,983.2605	3.8835	65	8,135.1920	141.5930
10	9,979.3770	3.8919	66	7,993.5990	150.0159
11	9,975.4851	3.9603	67	7,843.5831	159.1463
12	9,971.5248	4.0385	68	7,684.4368	168.9961
13	9,967.4863	4.1165	69	7,515.4407	179.5439
14	9,963.3698	4.2046	70	7,335.8968	190.7333
15	9,959.1652	4.3123	71	7,145.1635	202.5011
16	9,954.8529	4.4200	72	6,942.6624	214.7574
17	9,950.4329	4.5473	73	6,727.9050	227.4099
18	9,945.8856	4.6845	74	6,500.4951	240.3623
19	9,941.2011	4.8314	75	6,260.1328	253.4978
			76		
20	9,936.3697	4.9980 5.1842	77	6,006.6350	266.6525
21	9,931.3717			5,739.9825	279.6232
22	9,926.1875	5.3999	78	5,460.3593	292.1292
23	9,920.7876	5.6151	79	5,168.2301	303.8247
24	9,915.1725	5.8599	80	4,864.4054	314.2357
25	9,909.3126	6.1339	81	4,550.1697	322.6161
26	9,903.1787	6.4370	82	4,227.5536	328.3456
27	9,896.7417	6.7694	83	3,899.2080	331.2026
28	9,889.9723	7.1406	84	3,568.0054	331.3749
29	9,882.8317	7.5406	85	3,236.6305	329.1297
30	9,875.2911	7.9891	86	2,907.5008	324.6283
31	9,867.3020	8.4858	87	2,582.8725	317.8173
32	9,858.8162	9.0307	88	2,265.0552	308.3261
33	9,849.7855	9.6331	89	1,956.7291	295.6031
34	9,840.1524	10.2928	90	1,661.1260	279.1356
35	9,829.8596	11.0291	91	1,381.9904	258.6354
36	9,818.8305	11.8219	92	1,123.3550	234.1712
37	9,807.0086	12.7001	93	889.18380	206.18839
38	9,794.3085	13.6826	94	682.99541	175.62954
39	9,780.6259	14.7590	95	507.36587	144.01124
1 0	9,765.8669	15.9476	96	363.35463	113.20859
41	9,749.9193	17.4426	97	250.14604	85.10318
1 2	9,732.4767	19.4650	98	165.04286	61.02773
1 3	9,713.0117	21.9514	99	104.01513	41.62623
14	9,691.0603	24.8963	100	62.388900	26.915383
15	9,666.1640	28.2446	101	35.473517	16.435306
16	9,637.9194	31.9786	102	19.038211	9.438307
17	9,605.9408	36.0607	103	9.5999040	5.0744997
18	9,569.8801	40.4614	104	4.5254043	2.5418834
19	9,529.4187	45.1695	105	1.9835209	1.1799648
50	9,484.2492	50.1242	106	0.80355610	0.50465091
51	9,434.1250	55.2840	107	0.29890519	0.19756109
52	9,378.8409	60.5967	108	0.10134410	0.07028244
53	9,318.2442	66.0477	109	0.031061660	0.022535887
54	9,252.1965	71.6120	110	0.008525773	0.006452748
55	9,180.5845	77.2730	111	0.002073025	0.001632279
6	9,103.3115	83.0131	112	0.000440746	0.000360145
7	9,020.2984	88.8499	113	0.000080601	0.000068204
8	8,931.4485	94.7895	114	0.000012397	0.000010846
9	8,836.6590	100.8351	115	0.00001257	0.00001551

1971 Individual Annuity Mortality Table Elementary Values

FEMALE LIVES

Age x	l_x	d_x	Age x	l_x	d_x
5	10,000.0000	2.3400	61	9,314.4714	67.2412
6	9,997.6600	1.9295	62	9,247.2302	71.8787
7	9,995.7305	1.6194	63	9,175.3515	76.0178
8	9,994.1111	1.4291	64	9,099.3337	79.8467
9	9,992.6820	1.3390	65	9,019.4870	83.7910
10	9,991.3430	1.3189	66	8,935.6960	88.3562
11	9,990.0241	1.4286	67	8,847.3398	93.9764
12	9,988.5955	1.5482	68	8,753.3634	100.9788
13	9,987.0473	1.6678	69	8,652.3846	109.5738
14	9,985.3795	1.7974	70	8,542.8108	119.8471
15	9,983.5821	1.9268	71	8,422.9637	131.8278
	9,981.6553	2.0463	72	8,291.1359	
16					145.4929
17	9,979.6090	2.1755	73	8,145.6430	160.8113
18	9,977.4335	2.3048	74	7,984.8317	177.7104
19	9,975.1287	2.4439	75	7,807.1213	196.1149
20	9,972.6848	2.5929	76	7,611.0064	215.9166
21	9,970.0919	2.7418	77	7,395.0898	237.0126
22	9,967.3501	2.9105	78	7,158.0772	259.3013
23	9,964.4396	3.0790	79	6,898.7759	282.6773
24	9,961.3606	3.2573	80	6,616.0986	306.8943
25	9,958.1033	3.4555	81	6,309.2043	331.3152
26	9,954.6478	3.6633	82	5,977.8891	355.1404
27	9,950.9845	3.8809	83	5,622.7487	377.6238
28	9,947.1036	4.1181	84	5,245.1249	398.0997
29	9,942,9855	4.3749	85	4,847.0252	415.7294
30	9,938.6106	4.6612	86	4,431.2958	429.3837
31	9,933.9494	4.9571	87	4,001.9121	437.5611
32	9,928.9923	5.2921	88	3,564.3510	438.3368
33	9,923.7002	5.6466	89	3,126.0142	429.8520
34	9,918,0536	6.0302	90	2,696.1622	411.0892
35	9,912.0234	6.4527	91	2,285.0730	382.4527
36	9,905.5707	6.9141	92	1,902.6203	345.8507
37	9,898.6566	7.4240	93	1,556.7696	304.1710
38	9,891.2326	7.9822	94	1,252.5986	260.6294
39	9,883.2504	8.5885	95	991.96920	218.13006
	9,874.6619	9.2625	96	773.83914	178.83190
40	9,865.3994	9.2023	97	595.00724	144.11730
	9,855.4058	10.7818	98		114.44624
42				450.88994	
43	9,844.6240	11.6758	99	336.44370	89.64610
44	9,832.9482	12.6451	100	246.79760	69.23536
45	9,820.3031	13.7190	101	177.56224	52.63815
46	9,806.5841	14.8962	102	124.92409	39.29300
47	9,791 6879	16.1955	103	85.631090	28.696777
48	9,775.4924	17.6154	104	56.934313	20.413058
49	9,757.8770	19.1937	105	36.521255	14.065139
50	9,738.6833	20.9479	106	22.456116	9.324633
51	9,717.7354	23.0408	107	13.131483	5.899634
52	9,694.6946	25.6037	108	7.2318490	3.5266039
53	9,669.0909	28.6785	109	3.7052451	1.9666959
54	9,640.4124	32.3050	110	1 7385492	1.0066895
55	9,608.1074	36.4243	111	0.73185970	0.46292248
	9,571.6831	41.0051	112	0.26893722	0.18580953
56		45.9951	113	0.083127690	0.062601884
	9,530.6780	マン・ファンコ			
57 58	9,530.6780 9,484.6829	51.3027	114	0.020525806	0.016774058
57 58			5 I	0.020525806 0.003751748	0.016774058 0.003751748
57	9,484.6829	51.3027	114		

1971 INDIVIDUAL ANNUITY MORTALITY TABLE STANDARD COMMUTATION COLUMNS

MALE LIVES-3; PER CENT INTEREST

Age	D_x	N_x	Age	D_x	N_x
x		-``*	x		1 ***
5	8,419.7318	224,526.3856	61	1,058.2542	15,037.7944
6	8,131.2970	216,106.6538	62	1,009.0398	13,979.5402
7	7,852.9946	207,975.3568	63	961.19764	12,970.50039
8	7,584.3767	200,122.3622	64	914.68589	12,009.30275
		192,537.9855			
9	7,325.0277	192,337.9633	65	869.45092	11,094.61686
10	7,074.5684	185,212.9578	66	825.42814	10,225.16594
11	6,832.6660	178,138.3894	67	782.54814	9,399.73780
12	6,598.9888	171,305.7234	68	740.74420	8,617.18966
13	6,373.2524	164,706.7346	69	699.95532	7,876.44546
14	6,155.1887	158,333.4822	70	660.12887	7,176.49014
15	5,944.5326	152,178.2935	71	621.22273	6,516.36127
16	5,741.0228	146,233.7609	72	583.20449	5,895.13854
17		140,400,7000	72		
17	5,544.4192	140,492.7381	73	546.05239	5,311.93405
18	5,354.4786	134,948.3189	74	509.75389	4,765.88166
19	5,170.9726	129,593.8403	75	474.30457	4,256.12777
20	4.993.6807	124,422.8677	76:	439.70829	3,781.82320
21	4,822.3855	119,429.1870	77	405.97906	3,342.11491
22	4,656.8775	114,606.8015	78	373.14183	2,936.13585
23	4,496.9509	109,949.9240	79	341.23550	2,562.99402
23		105,949.9240			
24	4,342.4209	105,452.9731	80	310.31429	2,221.75852
25	4,193.0962	101,110.5522	81	280.45246	1,911.44423
26	4,048.7929	96,917.4560	82	251.75635	1,630.99177
27	3,909.3345	92,868.6631	83	224.35067	1,379.23542
28	3,774.5512	88,959.3286	84	198.35179	1,154.88475
29	3,644.2763	85,184.7774	85	173.84547	956.53296
30	3,518.3534	81,540.5011	86	150.88628	782.68749
	3,396.6252	78,022.1477	87	129.50679	
31					631.80121
32	3,278.9412	74,625.5225	88	109.73066	502.29442
33	3,165.1572	71,346.5813	89	91.588210	392.563762
34	3,055.1320	68,181.4241	90	75.122684	300.975552
35	2,948.7308	65,126.2921	91	60.385573	225.852868
36	2,845.8187	62,177.5613	92	47.424727	165.467295
37	2,746.2728	59,331.7426	93	36.269286	118.042568
38	2,649.9675	56,585.4698	94	26.916892	81.773282
39	2,556.7783	53,935.5023	95	19.319150	54.856390
			06		
40	2,466.5895	51,378.7240	96	13.367713	35.537240
41	2,379.2865	48,912.1345	97	8.8915956	22.1695267
42	2,294.7149	46,532.8480	98	5.6681647	13.2779311
43	2,212.6817	44,238.1331	99	3.4514524	7.6097664
44	2,133.0251	42,025.4514	100.	2.0001951	4.1583140
45	2,055.5994	39,892.4263	101	1.0988260	2.1581189
46	1,980.2830	37,836.8269	102	0.56978427	1.05929285
47	1,906.9686	35,856.5439	103	0.27759450	0.48950858
48	1,835.5650	33,949.5753	104	0.12643316	0.21191408
49	1,765.9945	32,114.0103	105	0.053542673	0.085480915
	1,698.1871	30,348.0158	106	0.033342073	
50					0.031938242
51	1,632.0888	28,649.8287	107	0.007532099	0.010980759
52	1,567.6568	27,017.7399	108	0.002467406	0.003448660
53	1,504.8581	25,450.0831	109	0.000730679	0.000981254
54	1,443.6634	23,945.2250	110	0.000193774	0.000250575
55	1,384.0478	22,501.5616	111	0.000045522	0.000056801
56	1,325.9887	21,117.5138	112	0.000009351	0.000011279
57	1,269.4657	19,791.5251	113.	0.000001652	0.000001928
	1 214 4555				
58	1,214.4555	18,522.0594	114	0.000000246	0.000000276
59	1,160.9338	17,307.6039	115	0.000000030	0.000000030
60	1,108.8757	16,146.6701			
ı	, '	l l	I	t	l

1971 INDIVIDUAL ANNUITY MORTALITY TABLE STANDARD COMMUTATION COLUMNS

FEMALE LIVES-31 PER CENT INTEREST

Age	_		Age	_	
x	D_x	N_x	x	D_x	N_x
5	8,419.7318	229,287.3992	61	1,142.3439	18,159.6820
6	8,133.1030	220,867.6674	62	1,095.7462	17,017.3381
	7,856.5540		63	1,050.4628	15,921.5919
7	7 500 6427	212,734.5644			
8	7,589.6437	204,878.0104	64	1,006.5311	14,871.1291
9	7,331.9405	197,288.3667	65	963.96019	13,864.59795
10	7,083.0512	189,956.4262	66	922.71015	12,900.63776
11]	6,842.6245	182,873.3750	67	882.69216	11,977.92761
12	6,610.2860	176,030.7505	68	843.78378	11,095.23545
13	6,385.7598	169,420.4645	69	805.84530	10,251.45167
14	6,168.7859	163,034.7047	70	768.73438	9,445.60637
15	5,959.1068	156,865.9188	71	732.31865	8,676.87199
16	5,756.4799	150,906.8120	72	696.48032	7,944.55334
17	5,730.4799	145 150 2221	72		
17	5,560.6762	145,150.3321	73	661.11930	7,248.07302
18	5,371.4628	139,589.6559	74	626.15216	6,586.95372
19 [5,188.6203	134,218.1931	75	591.51354	5,960.80156
20	5,011.9315	129,029.5728	76	557.15432	5,369.28802
21	4,841.1869	124,017.6413	77	523.04195	4,812.13370
22	4,676.1889	119,176.4544	78	489.15793	4,289.09175
23	4,516.7377	114,500.2655	79	455.49583	3,799.93382
24	4,362.6493	109,983.5278	80	422.05980	3,344.43799
25	4,213.7418	105,620.8785	81	388.87163	2,922.37819
			82	355.99113	2,533.50656
26	4,069.8354	101,407.1367			
27	3,930.7611	97,337.3013	83	323.51889	2,177.51543
28	3,796.3556	93,406.5402	84	291.58585	1,853.99654
29	3,666.4579	89,610.1846	85	260.34278	1,562.41069
30	3,540.9128	85,943.7267	86	229.96443	1,302.06791
31	3,419.5672	82,402.8139	87	200.65829	1,072.10348
32	3,302.2810	78,983.2467	88	172.67509	871 44519
33	3,188.9091	75,680.9657	89	146.31869	698.77010
34	3,079.3185	72,492.0566	90	121.93111	552.45141
35	2,973.3780	69,412.7381	91	99.845443	430.520302
36	2,870.9588	66,439,3601	92:	80.323005	330.674859
37	2,771.9371	63,568.4013	93	63.499720	250.351854
38	2,676.1915	60,796.4642	94	49.364989	186.852134
39	2,583.6056	58,120.2727	95	37.771563	137.487145
40	2,494.0681	55,536.6671	96	28.469321	99.715582
		53,042.5990	97	21.149900	71.246261
41	2,407.4673				
12	2,323.6991	50,635.1317	98	15.485180	50.096361
43	2,242.6637	48,311.4326	99	11.163947	34.611181
14	2,164.2550	46,068.7689	100	7.9123584	23.4472337
45	2,088.3785	43,904.5139	101	5.5001595	15.5348753
4 6	2,014.9382	41,816.1354	102	3.7387852	10.0347158
47	1,943.8430	39,801.1972	103	2.4761414	6.2959306
48 [1,875.0028	37,857.3542	104	1.5906613	3.8197892
19	1,808.3324	35,982.3514	105	0.98584574	2.22912785
50	1,743.7444	34,174.0190	106	0.58567619	1.24328211
51	1,681.1530	32,430.2746	107	0.33089968	0.65760592
52	1,620.4512	30,749.1216	108	0.17607252	0.32670624
53	1,561.5184	29,128.6704	109	0.087160320	0.150633717
54	1,504.2386	27,567.1520	110	0.039513774	0.063473397
55	1,448.5004	26,062.9134		0.039313774	0.003473337
			111		
56	1,394.2117	24,614.4130	112	0.005706000	0.007888397
57	1,341.2936	23,220.2013	113	0.001704065	0.002182397 0.000478332
			114	0.000406537	L D DUD4/8552
58	1,289.6817	21,878.9077			
	1,289.0817 1,239.3293 1,190.2147	20,589.2260 19,349.8967	115	0.000071795	0.000071795

1971 INDIVIDUAL ANNUITY MORTALITY TABLE STANDARD COMMUTATION COLUMNS

MALE LIVES-5 PER CENT INTEREST

Age x	D_x	N_x	Age x	D_{x}	N_x
5	7,835.2618	157,529.8523	61	439.95079	5,472.92174
6	7,458.7514	149,694.5905	62	413.49802	5,032.97095
7	7,100.5608	142,235.8391	63	388.26558	4,619.47293
8	6,759.7136	135,135.2783	64	364.19940	4,231.20735
9	6,435.2989	128,375.5647	65	341.24270	3,867.00795
10	6,126.4720	121,940.2658	66	319.33654	3,525.76525
11	5,832.4597	115,813.7938	67	298.42243	3,206.42871
12	5,552.5183	109,981.3341	68	278.44518	2,908.00628
13	5,285.9710	104,428.8158	69	259.35392	2,629.56110
	5,032.1790	99,142.8448	70	241.10281	
14					2,370.20718
15	4,790.5290	94,110.6658	71	223.65156	2,129.10437
16	4,560.4330	89,320.1368	72	206.96481	1,905.45281
17	4,341.3411	84,759.7038	73	191.01216	1,698.48800
18	4,132.7211	80,418.3627	74	175.76739	1,507.47584
19	3,934.0711	76,285.6416	75	161.20782	1,331.70845
20	3,744.9134	72,351.5705	76	147.31417	1,170.50063
21	3,564.7902	68,606.6571	77	134 . 07090	1,023.18646
22	3,393.2661	65,041.8669	78	121 . 46633	889.11556
23	3,229.9240	61,648.6008	79	109.49322	767 . 64923
24	3,074.3770	58,418.6768	80	98.148991	658.156011
25	2,926.2477	55,344.2998	81	87.436823	560.007020
26	2,785.1775	52,418.0521	82	77.368931	472.570197
27	2,650.8258	49,632.8746	83	67.961754	395.201266
28	2,522.8692	46,982.0488	84	59.227634	327.239512
29	2,400.9978	44,459.1796	85	51.168502	268.011878
30	2,284.9199	42,058.1818	86	43.776408	216.843376
31	2,174.3537	39,773.2619	87	37.036842	173.066968
32	2,069.0321	37,598.9082	88	30.932888	136.030126
33	1,968.7018	35,529.8761	89	25.449724	105.097238
34	1,873.1204	33,561.1743	90	20.576223	79.647514
35	1,782.0582	31,688.0539	91	16.303424	59.071291
36	1,695.2941	29,905.9957	92	12.621225	42.767867
37	1,612.6218	28,210.7016	93	9.5145166	30.1466419
38	1,533.8414	26,598.0798	94	6.9602313	20.6321253
39	1,458.7606	25,064.2384	95	4.9242244	13.6718940
40	1,387.1994	23,605.4778	96	3.3585977	8.7476696
41	1,318.9849	22,218.2784	97	2.2020725	5.3890719
42	1,253.9288	20,899.2935	98	1.3837111	3.1869994
43	1,191.8294	19,645.3647	99	0.83053112	1.80328828
44	1,132.5104	18.453.5353	100	0.47443577	0.97275716
45	1,075.8104	17,321.0249	101	0.25691238	0.49832139
46	1,021.5875	16,245.2145	102	0.13131600	0.24140901
47	969.71230	15,223.62704	103	0.063062196	0.110093014
48	920.06860	14,253.91474	104	0.028311982	0.047030818
49	872.55100	13,333.84614	105	0.011818446	0.018718836
50	827.06200	12,461.29514	106	0.004559849	0.006900390
51	783.51511	11,634.23314	107	0.001615394	0.002340541
52	741.83209	10,850.71803	108	0.000521620	0.002340341
53	701.94201	10,830.71803	109.	0.000321020	0.000723147
54				0.000132202	0.000203327
54	663.77776	9,406.94393 8,743.16617	110		
55 56	592.37764		111	0.000009217 0.000001866	0.000011462
		8,115.88986			0.000002245
57	559.02452	7,523.51222	113	0.000000325	0.000000379
58	527.16012	6,964.48770	114	0.000000048	0.000000054
59 60	496.72893	6,437.32758	115	0.000000006	0.000000006
a) i	467.67691	5,940.59865	1		

1971 INDIVIDUAL ANNUITY MORTALITY TABLE STANDARD COMMUTATION COLUMNS

FEMALE LIVES-5 PER CENT INTEREST

Age x	D_x	N_x	Age x	D_x	N_x
5	7,835.2617	159,491.1491	61	474.90962	6,525.62680
6	7,460.4078	151,655.8874	62	449.02976	6,050.71718
7	7,103.7790	144,195.4796	63	424.32329	5,601.68742
8	6,764.4078 6,441.3719	137,091.7006 130,327.2928	64 65	400.76931 378.33576	5,177.36413
9 10	6,133.8179	123,885.9209	1	356.97241	4,776.59482 4,398.25906
11	5,840.9602	117,752.1030	67	336.61206	4,041.28665
12	5,562.0237	111,911.1428	68	317.17768	3,704.67459
13	5,296.3444	106,349.1191	69	298.58926	3,387.49691
14	5,043.2952	101,052.7747	70	280.76945	3,088.90765
15	4,802.2737	96,009.4795	71	263.64813	2,808.13820
16	4,572.7113	91,207.2058	72	247.16359	2,544.49007
17	4,354.0704	86,634.4945	73	231.26321	2,297.32648
18	4,145.8297	82,280.4241	74	215.90248	2,066.06327
19	3,947.4972	78,134.5944	75	201.04510	1,850.16079
20	3,758.6000	74,187.0972	76	186.66176	1,649.11569
21	3,578.6884	70,428.4972	77	172.72986	1,462.45393
22	3,407.3373	66,849.8088	78	159.23226	1,289.72407
23	3,244.1356	63,442.4715	79	146.15626	1,130.49181
24	3,088.6983 2,940.6555	60,198.3359	80	133.49286	984.33555
25 26	2,799.6525	57,109.6376 54,168.9821	81	121.23873	850.84269 729.60396
27	2,665.3545	51,369.3296	83	109.40202 98.002432	620.201937
28	2,537.4428	48,703.9751	84	87.067227	522.199505
29	2,415.6117	46,166.5323	85	76.627535	435, 132278
30	2,299.5704	43,750.9206	86	66.719229	358.504743
31	2,189.0399	41,451.3502	87	57.385018	291.785514
32	2,083.7596	39,262.3103	88	48.676814	234.400496
33	1,983.4752	37,178.5507	89	40.657749	185.723682
34	1,887.9491	35,195.0755	90	33.397126	145.065933
35	1,796.9536	33,307.1264	91	26.957143	111.668807
36	1,710.2702	31,510.1728	92	21.376501	84.711664
37	1,627.6919	29,799.9026	93	16.657872	63.335163
38	1,549.0201	28,172.2107	94	12.764912	46.677291
39	1,474.0667	26,623.1906	95	9.6275277	33.9123792
40	1,402.6531	25,149.1239	96	7.1528313	24.2848515
41 42	1,334.6070 1,269.7667	23,746.4708 22,411.8638	97 98	5.2379366 3.7802388	17.1320202 11.8940836
43	1,207.9787	21,142.0971	99	2.6864069	8.1138448
44	1,149.0914	19,934.1184	100	1.8767699	5.4274379
45	1,092.9655	18,785.0270	101	1.2859717	3.5506680
46	1,039.4653	17,692.0615	102	0.86166349	2.26469630
47	988.46320	16,652.59624	103	0.56251443	1.40303281
48	939.83646	15,664.13304	104	0.35619431	0.84051838
49	893.46940	14,724.29658	105	0.21760521	0.48432407
50	849.24948	13,830.82718	106	0.12742920	0.26671886
51	807.06928	12,981.57770	107	0.070967381	0.139289662
52	766.81497	12,174.50842	108	0.037222458	0.068322281
53	728.37125 691.62943	11,407.69345 10,679.32220	109	0.018162823 0.008116412	0.031099823 0.012937000
54 55	656.48741	9,987.69277	111	0.003110412	0.004820588
56	622.85587	9,331.20536	112	0.003233980	0.004520368
57	590.65482	8,708.34949	113	0.000335239	0.001300002
58	559.81363	8,117.69467	114	0.000078835	0.00092558
59	530.27200	7,557.88104	115	0.000013723	0.000013723
60	501.98224	7,027.60904			
		<u> </u>			<u> </u>

1971 INDIVIDUAL ANNUITY MORTALITY TABLE STANDARD COMMUTATION COLUMNS

MALE LIVES-6 PER CENT INTEREST

		I	1		<u> </u>
Age x	D_x	N_x	Age x	D_{x}	N_x
5	7,472.5818	128,564.7581	61	246,77069	2,829.74093
6	7,046.3908	121,092.1763	62	229.74514	2,582.97024
7	6,644,7200	114,045.7855	63	213.69050	2,353.22510
8	6,266.0775	107,401.0655	64	198.55416	2,139.53460
9	5,909.0766	101,134.9880	65	184.28355	1,940.98044
10	5,572,4321	95,225.9114	66	170.82650	1,756.69689
11	5,254.9612	89,653.4793	67	158.13264	1,585.87039
12	4.955.5424	84,398.5181	68	146.15484	1.427.73775
13	4,673.1466	79,442,9757	69	134.84963	1,281.58291
14	4,406.8081	74,769.8291	70	124.17742	1,146.73328
15	4,155.6118	70,363.0210	71	114.10265	1,022.55586
16	3,918.6909	66,207.4092	72	104.59327	908.45321
17	3,695.2368	62,288.7183	73	95.620649	803.859936
18	3,484.4793	58,593.4815	74	87.159034	708.239287
19	3,285.6964	55,109.0022	75	79.185133	621.080253
20	3,098.2071	51,823.3058	76	71.677935	541.895120
21	2,921.3667	48,725.0987	77	64.618808	470.217185
22	2,754.5677	45,803.7320	78	57.991418	405.598377
23	2,597.2351	43,049.1643	79	51.781960	347.606959
24	2,448.8350	40,451.9292	80	45.979108	295.824999
25	2,308.8563	38,003.0942	81	40.574437	249.845891
26	2,176.8181	35,694.2379	82	35.563801	209.271454
27	2,052.2671	33,517.4198	83	30.944936	173.707653
28	1,934.7768	31,465.1527	84	26.713625	142.762717
29	1,823.9433	29,530.3759	85	22.860966	116.049092
30	1,719.3883	27,706.4326	86	19.373827	93.188126
31	1,620.7522	25,987.0443	87	16.236510	73.814299
32	1,527.6965	24,366.2921	88	13.432679	57.577789
33	1,439.9030	22,838.5956	89	10.947342	44.145110
34	1,357.0705	21,398.6926	90	8.7674783	33.1977684
35	1,278.9161	20,041.6221	91	6.8813126	24.4302901
36	1,205.1709	18,762.7060	92	5.2768824	17.5489775
37	1,135.5848	17,557.5351	93	3.9404522	12.2720951
38	1,069.9191	16,421.9503	94	2.8553967	8.3316429
39	1,007.9475	15,352.0312	95	2.0010781	5.4762462
40	949.45900	14,344.08372	96	1.3519718	3.4751681
41	894.25330	13,394.62472	97	0.87806103	2.12319626
42	842.12600	12,500.37142	98	0.54653999	1.24513523
43	792.86950	11,658.24542	99	0.32494949	0.69859524
44	746.29970	10,865.37592	100	0.18387420	0.37364575
45	702 . 24760	10,119.07622	101	0.098630641	0.189771551
46	660.56190	9,416.82862	102	0.049937625	0.091140910
47	621 10390	8,756.26672	103	0.023755423	0.041203285
48	583.74740	8,135.16282	104	0.010564462	0.017447862
49	548 37680	7,551.41542	105	0.004368385	0.006883400
50	514.88440	7,003.03862	106	0.001669531	0.002515015
51	483.17282	6,488.15422	107	0.000585876	0.000845484
52	453.15229	6,004.98140	108	0.000187398	0.000259608
53	424.74006	5,551.82911	109	0.000054186	0.000072210
54	397.85803	5,127.08905	110	0.000014031	0.000018024
55	372.43265	4,729.23102	111	0.000003218	0.000003993
56	348.39423	4,356.79837	112	0.000000646	0.000000775
57	325.67662	4,008.40414	113	0.000000111	0.000000129
58	304.21576	3,682.72752	114	0.000000016	0.000000018
59	283.95011	3,378.51176	115	0.000000002	0.000000002
60	264.82072	3,094.56165			
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1971 INDIVIDUAL ANNUITY MORTALITY TABLE STANDARD COMMUTATION COLUMNS

FEMALE LIVES-6 PER CENT INTEREST

Age	D_x	N_x	Age	D_x	N_x
x	ν_x	27 2	x	ν_z	17#
5	7,472.5818	129,715.9365	61	266.37928	3,348.80001
6	7,047.9559	122,243.3547	62	249.48706	3,082.42073
7	6,647.7317	115,195.3988	63	233.53566	2,832.93367
8	6,270.4290	108,547.6671	64	218.49133	2,599.39801
9	5,914.6532	102,277.2381	65	204.31516	2,380.90668
10	5,579.1138	96,362.5849	66	190.95950	2,176.59152
11	5,262.6202	90,783.4711	67	178.36915	1,985.63202
12	4,964.0260	85,520.8509	68	166.48539	1,807.26287
13	4,682.3176	80,556.8249	69	155.24982	1,640.77748
14	4,416.5430	75,874.5073	70	144.60730	1,485.52766
15	4,165.8001	71,457.9643	71	134.50812	1,340.92036
16	3,929.2416	67,292.1642	72	124.90843	1,206.41224
17	3,706.0718	63,362.9226	73	115.77031	1,081.50381
18	3,495.5319	59,656.8508	74	107.06111	965.73350
19	3,296,9099	56,161.3189	75	98.753167	858.672389
20	3,109.5303	52,864.4090	76	90.823101	759.919222
21	2,932,7565	49,754.8787	77	83.251454	669.096121
22	2,765.9905	46,822.1222	78	76.021929	585.844667
23	2,608.6631	44,056.1317	79	69.120788	509.822738
24	2,460.2425	41,447.4686	80	62.536382	440.701950
25	2,320.2245	38,987.2261	81	56.259971	378.165568
26	2,188.1315	36,667.0016	82	50.288294	321.905597
27	2,063.5153	34,478.8701	83	44.623318	271.617303
28	1,945.9534	32,415.3548	84	39.270203	226.993985
29	1,835.0450	30,469.4014	85	34.235505	187.723782
30	1,730.4129	28,634.3564	86	29.527476	153.488277
31	1,631.6993	26,903.9435	87	25.156908	123.960801
32	1,538.5709	25,272.2442	88	21.138021	98.803893
33	1,450.7083	23,733.6733	89	17.489159	77.665872
34	1,367.8140	22,282.9650	90	14.230434	60.176713
35	1,289.6060	20,915.1510	91	11.378011	45.946279
36	1,215.8175	19,625.5450	92	8.9374272	34.5682678
37 :	1,146.1970	18,409.7275	93	6.8988844	25.6308406
38	1,080.5069	17,263.5305	94	5.2367348	18.7319562
39	1,018.5235	16,183.0236	95	3.9123796	13.4952214
40	960.03630	15,164.50013	96	2.8793047	9.5828418
41	904.84510	14,204.46383	97	2.0885906	6.7035371
42	852.76270	13,299.61873	98	1.4931236	4.6149465
43	803.61300	12,446.85603	99	1.0510702	3.1218229
44	757.22630	11,643.24303	100	0.72736835	2.07075273
45	713.44580	10,886.01673	101	0.49369442	1.34338438
46	672.12180	10,172.57093	102	0.32767850	0.84968996 0.52201146
47	633.11400	9,500.44913	103	0.21189824 0.13291197	0.32201140
48	596.28950	8,867.33513	104	0.080432181	0.31011322
49 50	561.52350 528.69720	8,271.04563 7,709.52213	105	0.046656583	0.177201240
51	497.69805	7,180.82493	107	0.025738676	0.050112482
52	468.41322	6,683.12688	108	0.013372601	0.034373806
53	440.73220	6,214.71366	109	0.006463647	0.011001205
54	414.55188	5,773.98146	110	0.002861158	0.004537558
55	389.77615	5,359.42958	111	0.001136258	0.001676400
56	366.31935	4,969.65343	112	0.000393907	0.000540142
57	344.10381	4,603.33408	113	0.000114864	0.000146235
58	323.05958	4,259.23027	114	0.000026757	0.000031371
59	303.12467	3,936.17069	115	0.000004614	0.000004614
60	284.24601	3,633.04602			
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1971 INDIVIDUAL ANNUITY MORTALITY TABLE STANDARD COMMUTATION COLUMNS

MALE LIVES-7 PER CENT INTEREST

Age x	D_x	N_x	Age x	D_x	N_z
5	7,129.8620	107,129.0064	61	139.16849	1,478.83922
5	6,660.3840	99,999.1444	62	128.35588	1,339.67073
7	6,222.0187	93,338.7604	63	118.27058	1,211.31485
3	5,812.6274	87,116.7417	64	108.86608	
9	5,430.2326	81,304.1143		100.09728	1,093.04427
			65		984.17819
10	5,073.0096	75,873.8817	66	91.920638	884.080912
11	4,739.2814	70,800.8721	67	84.294919	792.160274
12	4,427.4766	66,061.5907	68	77.181846	707.865355
13	4,136.1528	61,634.1141	69	70.546227	630.683509
14	3,863.9669	57,497.9613	70	64.355960	560.137282
15	3,609.6602	53,633.9944	71	58.581968	495.781322
16	3,372.0535	50,024.3342	72	53.197847	437.199354
17	3,150.0526	46,652.2807	73	48.179698	384.001507
18	2,942.6290	43,502.2281	74	43.505773	335.821809
19	2,748.8253	40,559.5991	75	39.156172	292.316036
20	2,567.7471	37,810.7738	76	35.112693	253.159864
21	2,398.5566	35,243.0267	77	31.358818	218.047171
22	2,240.4715	32,844.4701	78	27.879601	186.688353
23	2,092.7596	30,603.9986	79	24.661722	158.808752
24	1,954.7431	28,511.2390	80	21.693396	134.147030
25	1,825.7830		81	18.964509	
		26,556.4959			112.453634
26	1,705.2830	24,730.7129	82	16.467185	93.489125
27	1,592.6866	23,025.4299	83	14.194590	77.021940
28	1,487.4740	21,432.7433	84	12.139147	62.827350
29	1,389.1589	19,945.2693	85	10.291342	50.688203
30	1,297.2888	18,556.1104	86	8.6400242	40.3968606
31	1,211.4386	17,258.8216	87	7.1732227	31.7568364
32	1,131.2119	16,047.3830	88	5.8790391	24.5836137
33	1,056.2390	14,916.1711	89	4.7465110	18.7045746
34	986.17380	13,859.93210	90	3.7658462	13.9580636
35	920.69380	12,873.75830	91	2.9280687	10.1922174
36	859.49600	11,953.06450	92	2.2243825	7.2641487
37	802.30020	11,093.56850	93	1.6455089	5.0397662
38 '	748.84220	10,291.26830	94	1.1812523	3.3942573
39	698.87490	9,542.42610	95	0.82009163	2.21300497
40	652.16850	8,843.55120	96	0.54889342	1.39291334
41	608.50790	8,191.38270	97	0.35315650	0.84401992
42	567.68160	7,582.87480	98	0.21776422	0.49086342
43	529.48250	7,015.19320	99	0.12826333	0.27309922
14	493.72510	6,485.71070	100	0.071900113	0.144835890
45	460.23990	5,991.98560	101	0.038206979	0.072935777
46	428.87390	5,531.74570	102	0.019163764	0.07293377
1 0,	399.48690	5,102.87180	103	0.009031041	0.015565034
48	371.95060	4,703.38490	104	0.003978730	0.006533993
40	346.14770	4,331.43430		0.001629822	
49			105		0.002555263
50	321.96910	3,985.28660	106	0.000617072	0.000925441
51	299.31533	3,663.31750	107	0.000214521	0.000308369
52	278.09471	3,364.00217	108	0.000067975	0.000093848
53	258.22237	3,085.90746	109	0.000019471	0.000025873
54	239.61878	2,827.68509	110	0.000004995	0.000006402
55 :	222.20947	2,588.06631	111	0.000001135	0.00000140
56	205.92442	2,365.85684	112	0.000000226	0.000000272
57 <i>.</i>	190.69775	2,159.93242	113	0.000000039	0.000000040
58	176.46671	1,969.23467	114	0.000000006	0.00000000
59 :	163.17184	1,792.76796	115	0.000000001	0.000000001
60	150.75690	1,629.59612			

1971 INDIVIDUAL ANNUITY MORTALITY TABLE STANDARD COMMUTATION COLUMNS

FEMALE LIVES-7 PER CENT INTEREST

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x x x x 5. 7,129, 8618 107, 829, 1586 61. 150, 22692 1,738, 22918 6. 6,661, 8630 100, 699, 2968 62. 139, 38545 1,588, 00226 7. 6,224, 8386 94,037, 4338 63. 129, 25422 1,448, 61681 8. 5,816, 6637 87,812, 5952 64. 119, 79752 1,319, 36259 9. 5,435, 3569 81,995,9315 65. 110, 97785 1,199, 56507 10. 5,079,0921 76,560,5746 66. 102,75408 1,088,58722 11. 4,746,1885 71,481,4825 67. 95,082283 985,833140 12. 4,435,0559 667,352,2940 68. 87,918056 890,750857 13. 4,144,2696 62,300,2381 69. 81,218536 890,750857 14. 3,872,5023 58,155,9685 70. 74,943910 721,614265 15. 3,618,5096 47,283,8246 73. 53,330466 577,611923	Age	n	A7	Age	ח	A7
6. 6, 661, 8630	x	ν_{z}	11'x	x	ν_x	1 * z
6. 6, 661, 8630				<u> </u>		
6. 6,661.8330 100,699.2968 62. 139,38545 1,588.00226 7. 6,224.8386 94,037.4338 63. 129.25422 1,448.61681 8. 5,816.6637 87,812.5952 64. 119,79752 1,319.36259 9. 5,435.3569 81,995.9315 65. 110.97785 1,199.56507 10. 5,079.0921 76,560.5746 66. 102.75408 1,088.58722 11. 4,746.1885 71,481.4825 67. 95.082283 985.833140 12. 4,435.0559 66,735.2940 68. 87.918056 890.750857 13. 4,144.2696 62,300.2381 69. 81.218536 802.832801 14. 3,872.5023 58,155.9685 70. 74. 943910 721.614265 15. 3,618.5096 54,283.4662 71. 69.058432 646.670355 16. 3,381.1320 50,664.9566 72. 63.530466 577.611923 17. 3,159.2886 47,283.8246 73. 58.332367 514.081457 18. 2,951.9625 44,124.5360 74. 53.439971 455.749090 19. 2,758.2062 441,172.5735 75. 48.832348 402.309119 19. 2,758.2062 441,172.5735 75. 48.832348 402.309119 20. 2,577.1313 38,414.3673 76. 44.491289 353.476771 21. 2,407.9077 35,837.2360 77. 40.401042 308.985482 22. 2,249.7621 33,429.3283 78. 36.547840 268.584440 268.584440 268.584440 268.584440 268.584440 268.584440 269.177.14.1455 25.278.9778 80. 29.505281 199.117072 25. 1,834.7723 27,113.7501 81. 26.295934 169.611791 25.14.1455 25.278.9778 80. 29.505281 199.117072 25. 1,834.7723 27,113.7501 81. 26.295934 169.611791 25.14.846.0664 21,963.4167 84. 17.845080 99.561828 91.397.6140 20.477.3503 85. 15.411829 99.51828 23.2036600 30.139.5666 19.079.7363 86. 13.168183 66.304919 31. 1,219.6008 17,774.1297 87. 11.114218 33.136736 32. 1,139.2638 16,554.5089 88. 9.2514120 42.0225177 34. 993.98083 14.351.08009 90. 6.1123191 25.1882161 33.157364 44.500.93364 69.2773 99.95.57119 95. 1.6033005 5.4196894 42.0225177 34. 993.98083 14.351.08009 90. 6.1123191 25.1882163 31. 1,219.6008 17,774.1297 87. 11.114218 33.136736 44. 993.98083 14.351.08009 90. 6.1123191 25.1882163 31. 1,219.6008 17,774.1297 87. 11.114218 33.136736 44. 993.98083 14.351.08009 90. 6.1123191 25.1882163 49. 44. 993.98083 14.351.08009 90. 6.1123191 25.1882163 49. 93.98083 14.351.08009 90. 6.1123191 25.1882163 49. 93.98083 14.351.08009 90. 6.1123191 95. 182064344 900.00000000000000000000000000000000	5	7,129.8618	107,829.1586	61	150.22692	1,738.22918
7. 6,224,8386 94,037,4338 63 129,25422 1,448,61681 87,812,5952 64 119,79752 1,319,36259 9. 5,435,3569 81,995,9315 65. 110,97785 1,199,56507 10. 5,079,021 76,560,5746 66. 102,75408 1,985,6507 11. 4,746,1885 71,481,4825 67,95,082283 985,833140 12. 4,435,0559 66,735,2940 68. 87,918056 890,750857 144,144,2696 62,300,2381 69 81,218536 890,750857 144,144,2696 62,300,2381 69 81,218536 890,750857 14,441,2696 62,300,2381 69 81,218536 890,750857 14,441,2696 62,300,2381 69 81,218536 890,750857 14,331,1320 50,664,9566 72. 63,530466 577,611923 17,3159,2886 47,283,8246 73. 58,332367 514,081457 18,251,951,9625 44,124,5360 74,53,439971 455,740909 19,2,758,2602 41,172,5735 75,48,832348 402,309119 19,2,758,2602 41,172,5735 75,48,832348 402,309119 19,2,758,2602 41,172,5735 75,48,832348 402,309119 12,2,577,1313,38,414,3673 76,44,491289 333,476771 21,2,407,9077 35,837,2360 77,40,401042 308,985482 12,249,7621 33,429,3283 78,36,547840 268,584440 268,584440 238,94522 2,249,7621 33,429,3283 78,36,547840 268,584440 268,584440 238,148,148,148,148,148,148,148,148,148,14	6			62	139.38545	1,588,00226
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		187.39746	2,261.63246			
60 161.81532 1,900.04450				115	0.000001567	0.000001567
	60	161.81532	1,900.04450			
		<u> </u>		<u> </u>		<u> </u>

TABLE 12

COMPARISON OF ANNUITY VALUES AT 3½ PER CENT INTEREST 1963 EXPERIENCE TABLE VERSUS 1971 IAM TABLE WITH AND WITHOUT MARGINS

MALE LIVES

Age	1963 Experience Table	1971 IAM TABLE		EFFECT OF PROJECTING	EFFECT OF ADDING	COMBINED EFFECT OF PROJECTING			
		Without 10 Per Cent Margin	With 10 Per Cent Margin	FROM 1963 TO 1971 = $(2) \div (1)$	10 PER CENT MARGIN TO 1971 TABLE =(3)÷(2)	TO 1971 AND 10 PER CENT MARGIN = (3) ÷ (1)			
	(1)	(2)	(3)	(4)	(5)	(6)			
	Immediate Life Annuity								
60 65	12.631 10.829	13.131 11.320	13.561 11.760	104.0% 104.5	103.3% 103.9	107.4% 108.6			
70	8.973	9.436	9.871	105.2	104.6	110.0			
75	7.154	7.557	7.973	105.6	105.5	111.4			
80	5.483	5.777	6.160	105.4	106.6	112.3			
85	4.013	4.175	4.502	104.0	107.8	112.2			
90	2.677	2.735	3.006	102.2	109.9	112.3			
95	1.617	1.617	1.839	100.0	113.7	113.7			
	Immediate Life Annuity with 10 Years Certain								
60	13.418	13.827	14.194	103.0%	102.7%	105.8%			
65	11.954	12.319	12.667	103 1	102.8	106.0			
70	10.623	10.909	11.213	102.7	102.8	105.6			
75 80	9.559 8.831	9.734 8.900	9.967 9.045	101.8 100.8	102.4 101.6	104.3 102.4			
85	8.448	8.460	8.521	100.8	101.0	102.4			
90	8.332	8.332	8.346	100.1	100.7	100.9			
95	8.318	8.318	8.319	100.0	100.0	100.0			
	Immediate Life Annuity with 20 Years Certain								
60	15.523	15.726	15.936	101.3%	101.3%	102.7%			
65	14.843	14.962	15.112	100:8	101.0	101.8			
70	14.428	14.473	14.554	100.3	100.6	100.9			
75	14.253	14.261	14.287	100.1	100.2	100.2			
		44 045	1 44 040 1	400 0	1 100 0	4000			
80 85	14.215 14.212	14.215 14.212	14.219 14.212	100.0 100.0	100.0 100.0	100.0			

TABLE 12—Continued
FEMALE LIVES

										
Age	1963 Experience Table (1)	Without 10 Per Cent Margin (2)	With 10 Per Cent Margin (3)	EFFECT OF PROJECTING FROM 1963 TO 1971 = (2) ÷ (1)	EFFECT OF ADDING 10 PER CENT MARGIN TO 1971 TABLE = (3) ÷ (2) (5)	COMBINED EFFECT OF PROJECTING TO 1971 AND 10 PER CENT MARGIN =(3)÷(1) (6)				
	Immediate Life Annuity									
60. 65. 70. 75. 80. 85. 90.	14.485 12.593 10.489 8.309 6.254 4.479 3.138 2.356	14.885 12.995 10.883 8.670 6.533 4.644 3.201 2.356	15.257 13.383 11.287 9.077 6.924 5.001 3.531 2.640	102.8% 103.2 103.8 104.3 104.5 103.7 102.0 100.0	102.5% 103.0 103.7 104.7 106.0 107.7 110.3 112.1	105.3% 106.3 107.6 109.2 110.7 111.7 112.5 112.1				
	Immediate Life Annuity with 10 Years Certain									
60. 65. 70. 75. 80. 85. 90.	14.925 13.223 11.522 10.066 9.068 8.578 8.397 8.335	15.273 13.552 11.801 10.252 9.151 8.598 8.399 8.335	15.607 13.887 12.119 10.518 9.337 8.700 8.444 8.350	102.3% 102.5 102.4 101.8 100.9 100.2 100.0 100.0	102.2% 102.5 102.7 102.6 102.0 101.2 100.5 100.2	104.6% 105.0 105.2 104.5 103.0 101.4 100.6 100.2				
	Immediate Life Annuity with 20 Years Certain									
60	16.231 15.244 14.597 14.314 14.231 14.214	16.439 15.380 14.657 14.329 14.233 14.214	16.667 15.563 14.772 14.381 14.249 14.217	101.3% 100.9 100.4 100.1 100.0 100.0	101.4% 101.2 100.8 100.4 100.1 100.0	102.7% 102.1 101.2 100.5 100.1 100.0				

age 60 to 12.1 per cent at age 95. Of course, the effect of projection to 1971 and of the introduction of a margin is less pronounced for annuities with a refund period.

V. COMPARISON OF 1971 IAM TABLE WITH CURRENT VALUATION STANDARDS

This section compares mortality rates and life annuity values based on the 1971 IAM Table (unprojected) with those based on the tables most commonly specified as minimum valuation standards for individual annuities, the *a*-1949 Ultimate Table (unprojected) and the 1937 Standard Annuity Table. Comparisons are also made with the *a*-1949 Ultimate Table projected to 1971 with Projection Scale B.

We will use the terminology "projected to year z" or "with projection to year z" to indicate that annuity values are based on a static table representing the projected level of mortality in year z, with no provision for decreases in mortality after that year. Thus the a-1949 Ultimate Table projected to 1971 is intended to represent the level of mortality in 1971 without allowance for mortality decreases after 1971, and in this respect it is directly comparable to the 1971 IAM Table (unprojected).

We will use the terminology "with projection for year of valuation z" (or "year of entry z") to indicate that annuity values are on a fully projected basis; for example, if the attained age in year z is x, the mortality rate for age x is projected to year z, the mortality rate for age x+1 is projected to year z+1, and so on. In this section no comparisons are made on a fully projected basis. Such comparisons are made in Section VI.

Comparison of Mortality Rates

Table 13 compares the mortality rates under the 1971 IAM Table with those under (1) the a-1949 Ultimate Table (unprojected), (2) the a-1949 Ultimate Table with Projection Scale B to 1971, and (3) the 1937 Standard Annuity Table. From column 5 it can be seen that unprojected mortality rates under the 1971 IAM Table are lower at all ages than the mortality rates under the a-1949 Ultimate Table for both males and females. For males the 1971 IAM rates are about 80 per cent of the a-1949 rates through age 50, dipping to about 75 per cent at ages 65–85 and rising again to about 93 per cent at age 100 or higher. For females the 1971 IAM rates are close to 70 per cent of the a-1949 rates through age 50. Thereafter there is a fluctuating relationship between the two tables, with the rates under the 1971 IAM Table ranging roughly from 60–90 per cent of the a-1949 rates at various ages.

 ${\it TABLE~13}$ Comparison of 1,000 $q_{\rm e}$ —1971 Individual Annuity Table versus a-1949 Table and 1937 Standard Annuity Table

1971 IAM Table (1)	a-1949 Ultimate (without Projection) (2)	a-1949 Ultimate with Pro- jection B to 1971 (3)	1937 Standard Annuity Table (4)	(1)÷(2) (5)	(1)÷(3)	(1)÷(4)
0.456 0.390 0.433 0.503 0.619 0.809 1.122 1.633 2.922 5.285 8.417 12.249 17.405 26.000 40.494 64.599 101.689 168.040 283.841 431.413 594.884 756.852	0.566 0.483 0.537 0.624 0.768 1.004 1.391 2.025 3.625 6.557 10.565 15.662 23.066 35.092 54.501 85.503 134.178 208.485 316.834 463.415 638.956	0.435 0.371 0.412 0.479 0.590 0.771 1.068 1.555 2.783 5.035 8.164 12.155 18.285 28.718 46.531 76.960 127.307 208.485 316.834 463.415 638.956	1.234 1.257 1.262 1.331 1.561 2.065 2.981 4.356 6.362 9.288 13.554 19.753 28.751 41.758 60.464 87.161 124.837 177.138 248.059 362.122 610.442	80.6% 80.8 80.6 80.6 80.6 80.6 80.6 80.6 80.6	104.8% 105.1 105.1 105.0 104.9 105.1 105.0 105.0 105.0 105.0 105.0 105.0 105.2 90.5 87.0 83.9 79.9 80.6 89.6 93.1	37.0% 31.0 34.3 37.8 39.7 39.2 37.6 37.5 45.9 62.1 62.0 60.5 62.3 67.0 74.1 81.5 94.9 114.4 119.1
			Females			
0. 234 0. 132 0. 193 0. 260 0. 347 0. 469 0. 651 0. 938 1. 397 2. 151 3. 791 6. 628 9. 290 14. 029 25. 120 46. 386 85. 770 152. 472 219. 896 280. 535 385. 122 579. 040	0.339 0.191 0.278 0.376 0.501 0.677 0.942 1.355 2.019 3.109 4.705 7.504 12.406 20.964 35.829 61.415 104.760 176.161 288.153 449.400 649.459	0.260 0.147 0.213 0.289 0.385 0.520 0.723 1.040 1.550 2.387 3.636 5.824 9.835 17.156 30.590 55.279 99.395 176.161 288.153 449.400 649.459	1. 190 1. 234 1. 257 1. 262 1. 331 1. 561 2. 065 2. 981 4. 356 6. 362 9. 288 13. 554 19. 753 28. 751 41. 758 60. 464 87. 161 124. 837 177. 138 248. 059 362. 122 610. 442	69.0% 69.1 69.4 69.2 69.3 69.2 69.2 69.2 80.6 88.3 74.9 66.9 70.1 75.5 81.9 86.6 76.3 62.4 59.3	90.0% 89.8 90.6 90.0 90.1 90.2 90.1 104.3 113.8 94.5 81.8 82.1 83.9 86.3 86.3 86.6 76.3 62.4 59.3	19.7% 10.7 15.4 20.6 26.1 30.0 31.5 32.1 33.8 40.8 48.9 47.0 48.8 60.2 76.7 98.4 122.1 124.1 113.1 106.4 94.9
	1AM Table (1) 0.456 0.390 0.433 0.503 0.619 0.809 1.122 1.633 2.922 5.285 8.417 12.249 17.405 26.000 40.494 64.599 101.689 168.040 283.841 431.413 594.884 756.852 0.234 0.132 0.193 0.260 0.347 0.469 0.651 0.938 1.397 2.151 3.791 6.628 9.290 14.029 25.128 9.290 14.029 25.129.896 85.770 1529.472 219.896 85.770 1529.472 219.896 85.770 1529.472 219.896 85.770 1529.472 219.896 85.770 1529.472 219.896 85.770 1529.472 219.896 85.770 1529.472 219.896 85.770 1529.472 219.896 85.770 1529.472 219.896 85.770 1529.472 219.896	1971	1971 IAM Table Ultimate (without Projection) (1) (2) (3) (3) (2) (3) (3) (2) (3) (3) (2) (3) (3) (3) (3) (2) (3)	1971 IAM Table With Projection Control Contr	1971 TAM	1971 1AM Table Ultimate (without Projection B) 1097 (3)

The fact that the 1971 IAM mortality rates are an almost constant percentage of the a-1949 rates through age 50 is not an accident. The 1971 IAM rates at ages 50 and under were taken directly from the 1971 GAM Table, which in turn was based on applying a constant factor under Projection Scale D to the Ga-1951 Table. The rates under the Ga-1951 Table were based on a projection of the a-1949 rates at these ages.

Male mortality rates at ages 60 and under are actually higher (less conservative) on the 1971 IAM Table than on the a-1949 Ultimate Table with Projection Scale B to 1971, but are lower at the higher ages. Rates on the a-1949 Table for ages 90 and over are the same whether on a projected or an unprojected basis, since the annual rate of mortality decrease under Projection Scale B is zero at these ages. Mortality rates for females in the mid-fifties to the early sixties are higher under the 1971 IAM Table than under the a-1949 Table projected to 1971, but are lower at the other ages.

The 1971 IAM Table has mortality rates significantly lower than those of the 1937 Standard Annuity Table, except at the very high ages (from the nineties to over 100 for males and from the late eighties to over 105 for females).

Comparison of Annuity Values

Table 14 shows annuity values for immediate life annuities with annual payments and with certain periods of 0, 10, and 20 years. These values are shown at the ages of most importance for individual annuities, ages 60 and over. The values based on current valuation mortality standards, the a-1949 Ultimate Table and the 1937 Standard Annuity Table, are shown at $3\frac{1}{2}$ per cent, the current maximum valuation interest rate in most states. Since it is expected that there will be a great deal of discussion in the immediate future about the question of raising the maximum valuation interest rate, the values based on the 1971 IAM Table are shown at $3\frac{1}{2}$, 5, 6, and 7 per cent.

Although the 1937 Standard Annuity Table is often specified as a minimum valuation standard, it is doubtful whether this table is used to value a significant amount of in-force individual annuities. We have made comparisons of basic mortality rates and annuity values under the 1971 IAM Table with the 1937 Standard Annuity Table mainly because they may be of historical interest. The balance of the paper limits comparisons of the 1971 IAM Table to the *a*-1949 Ultimate Table.

Table 15 shows the ratios of the annuity values on the 1971 IAM Table at the indicated interest rates to the corresponding values on the a-1949 Ultimate Table at $3\frac{1}{2}$ per cent, unprojected and projected to 1971. Also

TABLE 14

Annuity Values on a-1949 Ultimate Table and 1937 Standard Annuity Table at $3\frac{1}{2}$ Per Cent and on 1971 IAM Table (Unprojected) at Various Interest Rates

MALE LIVES

Age	G-1949 ULTIMATE TABLE (UNPRO- JECTED) AT 31 PER CENT	a-1949 Ultimate Table WITH PROJECTION B TO 1971 AT 31 PER CENT	1937 STANDARD ANNUITY TABLE AT 3½ PER CENT	197 ————————————————————————————————————	1 IAM TABL	e (Unprojec 6%	7%		
		Immediate Life Annuity							
60	12.398 10.563 8.704 6.901 5.235 3.777 2.574 1.640	13.104 11.192 9.217 7.267 5.444 3.852 2.574 1.640	11.818 10.124 8.470 6.906 5.473 4.206 3.120 2.200	13.561 11.760 9.871 7.973 6.160 4.502 3.006 1.839	11.702 10.332 8.831 7.261 5.706 4.238 2.871 1.776	10.685 9.533 8.235 6.843 5.434 4.076 2.786 1.737	9.809 8.832 7.704 6.465 5.184 3.925 2.706 1.698		
		Imm	ediate Life A	nnuity with	10 Years Ce	rtain			
60	13.212 11.750 10.446 9.421 8.754 8.431 8.332 8.317	13.754 12.171 10.718 9.549 8.788 8.435 8.332 8.317	12.821 11.532 10.415 9.530 8.911 8.545 8.372 8.320	14.194 12.667 11.213 9.967 9.045 8.521 8.346 8.319	12.275 11.152 10.045 9.067 8.324 7.893 7.746 7.723	11.221 10.301 9.372 8.537 7.891 7.512 7.382 7.361	10.313 9.553 8.771 8.056 7.494 7.159 7.043 7.025		
		Imm	ediate Life A	nnuity with	20 Years Ce	rtain			
60 65 70 75 80 85	15.410 14.762 14.390 14.246 14.215 14.212	15.628 14.865 14.420 14.250 14.215 14.212	15.328 14.777 14.440 14.278 14.223 14.212	15.936 15.112 14.554 14.287 14.219 14.212	13.659 13.097 12.707 12.516 12.467 12.462	12.413 11.976 11.667 11.514 11.474 11.470	11.340 10.998 10.752 10.630 10.597 10.594		

TABLE 14—Continued FEMALE LIVES

Age	a-1949 Ultimate Table (Unpro- jected) at 34	a-1949 ULTIMATE TABLE WITH PROJECTION B TO 1971	1937 Standard Annuity Table at 3½	1971 IAM TABLE (UNPROJECTED)								
	PER CENT	AT 3} PER CENT	PER CENT	31/8	5%	6%	7%					
		Immediate Life Annuity										
60	14.248 12.250 10.169 8.094 6.128 4.376 2.919 1.795	14.776 12.747 10.596 8.413 6.317 4.445 2.919 1.795	13.507 11.818 10.124 8.470 6.906 5.473 4.206 3.120	15.257 13.383 11.287 9.077 6.924 5.001 3.531 2.640	13.000 11.625 10.002 8.203 6.374 4.679 3.344 2.522	11.781 10.653 9.273 7.695 6.047 4.483 3.229 2.449	10.742 9.809 8.629 7.239 5.749 4.302 3.121 2.380					
		Imm	ediate Life A	nnuity with	10 Years Ce	rtain						
60	14.683 12.963 11.337 9.967 9.004 8.501 8.341 8.317	15.122 13.332 11.597 10.101 9.043 8.506 8.341 8.317	14.212 12.821 11.532 10.415 9.530 8.911 8.545 8.372	15.607 13.887 12.119 10.518 9.337 8.700 8.444 8.350	13.316 12.081 10.752 9.505 8.559 8.039 7.828 7.750	12.077 11.079 9.975 8.914 8.095 7.640 7.454 7.385	11.020 10.209 9.287 8.382 7.671 7.272 7.108 7.046					
		Immed	liate Life Ani	nuity with 20	0 Years Cert	ain						
60 65 70 75 80 85	16.103 15.160 14.551 14.282 14.218 14.212	16.318 15.276 14.589 14.288 14.218 14.212	16.084 15.328 14.777 14.440 14.278 14.223	16.667 15.563 14.772 14.381 14.249 14.217	14.157 13.410 12.860 12.583 12.489 12.465	12.800 12.221 11.788 11.567 11.491 11.473	11.642 11.191 10.849 10.672 10.611 10.596					

TABLE 15

RATIOS OF ANNUITY VALUES ON 1971 IAM TABLE (UNPROJECTED) AT VARIOUS INTEREST RATES TO ANNUITY VALUES ON a-1949 ULTIMATE TABLE AT $3\frac{1}{2}$ PER CENT

MALE LIVES

	a-1949	ULTIMATE	Table wi	тноит Рк	DJECTION	a-1949 Ultimate Table with Projection B to 1971							
Age	Ratio of Annuity Values—1971 IAM Table (Unprojected) at Indicated Interest Rate to a-1949 Ultimate Table at 3½ Per Cent				"Break- even" Interest	IAM Indicat	-1971 d) at a-1949 r Cent	"Break- even" Interest					
	31/%	5%	6%	7%	Rate	31%	5%	6%	7%	Rate			
		Immediate Life Annuity											
65 70 75 80 85	113.4 115.5 117.7 119.2	94.4% 97.8 101.5 105.2 109.0 112.2 111.5 108.3	86 2% 90 2 94 6 99 2 103 8 107 9 108 2 105 9	79.1% 83.6 88.5 93.7 99.0 103.9 105.1 103.5	4.4% 4.7 5.2 5.9 6.8 8.1 8.8 8.6	103.5% 105.1 107.1 109.7 113.2 116.9 116.8 112.1	89.3% 92.3 95.8 99.9 104.8 110.0 111.5 108.3	81.5% 85.2 89.3 94.2 99.8 105.8 108.2 105.9	74.9% 78.9 83.6 89.0 95.2 101.9 105.1 103.5	3.8% 4.1 4.4 5.0 6.0 7.5 8.8 8.6			
			Imi	nediate L	ife Annuit	y with 10	Years Cer	rtain					
65 70 75 80 85	105.8 103.3	92.9% 94.9 96.2 96.2 95.1 93.6 93.0 92.9	84.9% 87.7 89.7 90.6 90.1 89.1 88.6 88.5	78.1% 81.3 84.0 85.5 85.6 84.9 84.5 84.5	4.3% 4.4 4.5 4.4 4.1 3.7 3.5 3.5	103.2% 104.1 104.6 104.4 102.9 101.0 100.2 100.0	89.2% 91.6 93.7 95.0 94.7 93.6 93.0 92.9	81.6% 84.6 87.4 89.4 89.8 89.1 88.6 88.5	75.0% 78.5 81.8 84.4 85.3 84.9 84.5 84.5	3.8% 4.0 4.1 4.2 4.0 3.7 3.5 3.5			
			Imn	nediate Li	fe Annuity	with 20	Years Cer	tain					
65 70	103.4% 102.4 101.1 100.3 100.0 100.0	88.6% 88.7 88.3 87.9 87.7	80.6% 81.1 81.1 80.8 80.7 80.7	73.6% 74.5 74.7 74.6 74.5 74.5	3.8% 3.7 3.6 3.5 3.5 3.5	102.0% 101.7 100.9 100.3 100.0 100.0	87.4% 88.1 88.1 87.8 87.7 87.7	79.4% 80.6 80.9 80.8 80.7 80.7	72.6% 74.0 74.6 74.6 74.5 74.5	3.7% 3.7 3.6 3.5 3.5 3.5			

TABLE 15—Continued
FEMALE LIVES

 -	a-1949	ULTIMATE	Table wi	тноит Ркс	JECTION	a-1949 Ultimate Table with Projection B to 1971						
Age	Ratio of Annuity Values-1971 IAM Table (Unprojected) at Indicated Interest Rate to a-1949 Ultimate Table at 34 Per Cent			"Break- even" Interest Rate	IAM Indicat	Ratio of Annuity Values—1971 IAM Table (Unprojected) at Indicated Interest Rate to a-1949 Ultimate Table at 3½ Per Cent						
	31/2%	5%	6%	7%	Rate	31/%	5%	6%	7%	Rate		
	Immediate Life Annuity											
60 65 70 75 80 85 90	111.0 112.1 113.0 114.3 121.0	91.2% 94.9 98.4 101.3 104.0 106.9 114.6 140.5	82.7% 87.0 91.2 95.1 98.7 102.4 110.6 136.4	75.4% 80.1 84.9 89.4 93.8 98.3 106.9 132.6	4.1% 4.4 4.8 5.2 5.7 6.6 9.1	103.3% 105.0 106.5 107.9 109.6 112.5 121.0 147.1	88.0% 91.2 94.4 97.5 100.9 105.3 114.6 140.5	79.7% 83.6 87.5 91.5 95.7 100.9 110.6 136.4	72.7% 77.0 81.4 86.0 91.0 96.8 106.9 132.6	3.8% 4.0 4.3 4.6 5.2 6.2 9.1		
			Imr	nediate Li	fe Annuit	y with 10	Years Cer	tain				
60 65 70 75 80 85 90	106.9 105.5 103.7 102.3 101.2	90.7% 93.2 94.8 95.4 95.1 94.6 93.8 93.2	82.3% 85.5 88.0 89.4 89.9 89.9 89.4 88.8	75.1% 78.8 81.9 84.1 85.2 85.5 85.2 84.7	4.1% 4.2 4.3 4.3 4.1 3.9 3.7 3.6	103.2% 104.2 104.5 104.1 103.3 102.3 101.2 100.4	88.1% 90.6 92.7 94.1 94.6 94.5 93.8 93.2	79.9% 83.1 86.0 88.2 89.5 89.8 89.4 88.8	72.9% 76.6 80.1 83.0 84.8 85.5 85.2 84.7	3.8% 3.9 4.0 4.1 4.0 3.9 3.7 3.6		
	Immediate Life Annuity with 20 Years Certain											
65 70 75 80	103.5% 102.7 101.5 100.7 100.2 100.0	87.9% 88.5 88.4 88.1 87.8 87.7	79.5% 80.6 81.0 81.0 80.8 80.7	72.3% 73.8 74.6 74.7 74.6 74.6	3.8% 3.8 3.7 3.6 3.5 3.5	102.1% 101.9 101.3 100.7 100.2 100.0	86.8% 87.8 88.1 88.1 87.8 87.7	78.4% 80.0 80.8 81.0 80.8 80.7	71.3% 73.3 74.4 74.7 74.6 74.6	3.7% 3.7 3.6 3.6 3.5 3.5		

^{*} Greater than 10.0 per cent.

shown is the "break-even" interest rate, that is, the interest rate at which a given annuity value on the 1971 IAM Table would be equal to the corresponding value on the a-1949 Table at $3\frac{1}{2}$ per cent.

Results in the Aggregate

Obviously, the over-all effect on reserves of changing from the a-1949 Ultimate Table at $3\frac{1}{2}$ per cent to the 1971 IAM Table at various interest rates depends on the distribution of business by plan, sex, and age. Also, it is clear that the results could be quite different if the new table were applied to new issues only or, on a voluntary basis, to in-force business as well.

TABLE 16

PERCENTAGE DISTRIBUTION BY AMOUNT OF ANNUAL INCOME
OF ONE YEAR'S ISSUES OF SINGLE PREMIUM IMMEDIATE
ANNUITIES USED FOR TEST VALUATIONS

	MA	LES	Fem.	ALES	Combined			
Age	Refund (10 Years Certain)	Non- refund	Refund (10 Years Certain)	Non- refund	Refund (10 Years Certain)	Non- refund	Refund and Non- refund	
60	4.5% 13.0 4.5 2.5 2.5 2.0	1.0% 1.5 1.5 2.5 3.0 2.0	5.5% 16.0 7.0 7.0 5.0 2.0	0.5% 2.0 5.5 4.5 3.0 1.5	10.0% 29.0 11.5 9.5 7.5 4.0	1.5% 3.5 7.0 7.0 6.0 3.5	11.5% 32.5 18.5 16.5 13.5 7.5	
Total	29.0%	11.5%	42.5%	17.0%	71.5%	28.5%	100.0%	

To get an idea of how aggregate reserves on new issues of single premium immediate annuities might be affected, we valued a model office based on the distribution of single life business sold by one large company in 1970. For the purpose of simplifying the model office, all instalment refund annuities were tabulated as annuities with a 10-year certain period. The resulting distribution of issues is shown in Table 16. The annuity values in Table 14 were applied to the distribution in Table 16, with the results shown in Table 17 for one year's sales with a total annual income of \$1,000,000.

Naturally, as the block of business represented by one year's issues matures, the relative amount of reserves on the different valuation bases illustrated changes. An inspection of Table 15 shows that, as the business matures (with consequent increases in attained ages and decreases in

remaining certain period), the ratio of aggregate reserves on the 1971 IAM Table at a given interest rate to reserves on the a-1949 Table tends to increase. (The actual pattern of the relationship will depend not only on the initial distribution of the business but also on how the actual incidence of mortality affects the distribution of in-force after issue.)

The aggregate reserves based on the progress of a number of years of issues can be tested by a company using its own distribution of business and assumed mortality.

TABLE 17
TEST VALUATION OF ONE YEAR'S ISSUES OF
SINGLE PREMIUM IMMEDIATE ANNUITIES

	PREMIUM WITH TOT \$1,000 AS SE	S AT ISSUE IMMEDIATE AL ANNUAL),000 DISTR IOWN IN TA 1 \$1,000 UN	RATIO TO a-1949 ULTIMATE TABLE (UNPRO-	RATIO TO a-1949 ULTIMATE TABLE WITH PROJEC-	
	Males	Females	Total	JECTED)	тіом В то 1971
a -1949 Ultimate (unprojected) at $3\frac{1}{2}$ per cent	\$4,033	\$6,482	\$10,515	100.0%	97.2%
a-1949 Ultimate projected to 1971 at 3½ per cent	4,170	6,652	10,822	102.9	100.0
3½ per cent	3,889	6,962 6,150 5,694 5,292	11,332 10,039 9,310 8,666	107.8 95.5 88.5 82.4	104.7 92.8 86.0 80.1

VI. PROVISION FOR FUTURE DECREASES IN MORTALITY

The 1971 IAM Table was designed to be a "safe" table for the valuation of all types of individual annuities, based on current (1971) levels of annuity mortality. We are proposing the 1971 IAM Table without projection as the new minimum valuation standard, consistent with the designation of the a-1949 Ultimate Table without projection as the current valuation standard. We feel that the minimum valuation standard should continue to allow flexibility with regard to provision for future decreases in mortality, since there are wide differences of opinion as to how future mortality levels will change over a long period of years. Some authorities believe that mortality rates will be relatively stable over the next couple of decades, while others believe that there will be moderate decreases in mortality levels. Past experience may not be a reliable guide

as to how mortality levels will change in the future. The judgment of different companies with respect to provision for future mortality improvements will vary not only because of differences of opinion with regard to average long-term trends but also because of differences in the nature and composition of annuity business sold by specific companies and differences in their actual past experience. Of course, many companies currently provide in their valuation for future decreases in mortality, and, undoubtedly, many will continue to do so. The balance of this section discusses what reasonable assumptions might be made for future mortality decreases under individual annuities for companies wishing to make such provision in their valuations.

Past Decreases in Mortality

It has often been noted that past decreases in mortality can be very deceptive as a guide in determining how mortality levels will change in the future. The key factors that led to mortality improvements in the past, such as control of communicable diseases and general improvements in the standard of living, have probably reached the limits of their effect on increasing longevity. On the other hand, if significant progress is made in conquering cancer and cardiovascular-renal diseases, improvement in mortality at ages in the fifties and higher would be more pronounced than in the past.

There is another factor which could lead to radical changes in patterns of mortality decreases, a development that would represent a "quantum jump" from the mortality experience of the past. We are referring to the possible discovery of methods of directly arresting the aging process itself, as distinguished from the discovery of methods of preventing or curing specific diseases. While this may appear to be a wild speculation, it is interesting to note that articles on this subject have appeared occasionally not only in popular magazines but in professional journals as well. For example, in an address delivered to the Faculty of Actuaries (Edinburgh) in February, 1970, entitled "Gerontology-Implications for Future Mortality Experience," Dr. Alex Comfort discusses with utmost seriousness a program for the conducting of experiments with human beings designed to discover methods of slowing down the biological clock or clocks by which we grow old.8 Dr. Comfort predicts that such experiments will begin by 1975, probably in more than one country. He foresees a potential increase in the human life span of up to 20 per cent, resulting from discoveries made within the next twenty years. He suggests that "we need to get used to the idea" and that "we might do well

⁸ Transactions of the Faculty of Actuaries, XXXII, 157-67.

to start considering its psychological, political, business and demographic implications—just in case."9

Of course, projection scales of the magnitude used by actuaries thus far would be completely useless in providing for this kind of development, one which would obviously have profound implications for the life insurance and annuity business. While the arresting of the aging process may seem a farfetched idea, it illustrates the kind of new factor that could radically change the pattern of mortality decreases from what it has been in the past.

Having said all this, we will now present some statistics on past mortality decreases, since this is the best starting point we have for gauging future mortality decreases.

As noted in Section IV, the projection of the 1963 Experience Table to 1971 was based on the decrease in mortality between the two latest Society of Actuaries studies for all policy years combined. This was consistent with the fact that the basic mortality table was developed on an aggregate basis. However, in reviewing past long-term mortality decreases, we felt that it would be more appropriate to look at the ultimate experience, thereby largely eliminating the varying impact of selection from one study to the next over a long period of time. Also, the factors used for the short-range projection from 1963 to 1971 were based on amount of annual income, again consistent with the basic table. For long-range trends, data by number of contracts were felt to be a better guide.

Table 18 shows the average annual percentage decrease (geometric basis) between successive intercompany studies of immediate annuities and settlement annuities, and between a given study and the latest study, for contract years 6 and over, by number of contracts. The table is based on the mortality ratios in Tables 10-12 of the report on immediate annuities in the 1969 *Reports* number and the mortality ratios in Tables 7 and 13 of the report on settlement annuities in the 1966 *Reports* number.

There is considerable variation in the annual percentage decreases by type of annuity, age group, and period over which the percentages are calculated. As an over-all indication of mortality trends, the percentage decreases between the earliest and latest studies, about a twenty-year period, are perhaps of most interest. We show these percentages in Table 19 for all types of immediate annuities combined and for all types of settlement annuities, the

⁹ Ibid., pp. 166-67.

TABLE 18—AVERAGE ANNUAL DECREASE IN MORTALITY (GEOMETRIC BASIS) UNDER IMMEDIATE ANNUITIES AND SETTLEMENT ANNUITIES CONTRACT YEARS 6 AND OVER—BY NUMBER OF CONTRACTS

MALE LIVES

IMMEDIATE ANNUITIES

Age	Age							Non	REFUND				REFUND	and No	NREFUND	
GROUP	1941-	48	1948-53	1953-58	1958	8-63	1941-48	1948-53	195	3-58	1958-63	1941-48	1948-	53	195358	1958–63
					Avera	ge Annu	al Decrease in	Mortality	from Stu	dy Indica	ated to Follo	wing Study				
Under 60 60-69 70-79 80 and over	2.4 1.0 -0.1 1.9)"	-0.9% -1.8 0.3 -0.8	5.3% 3.2 1.7 0.2	_ [2	.7% .7 .2	3.8% 5.8 -0.3 2.2	-4.9% -0.4 1.3 -1.1	-	2.0% 1.3 1.0 1.2	11.4% 7.9 2.1 3.2	2.6% 2.1 -0.3 1.9	-1.6 -1.5 0.5 -0.8	7	1.1% 2.3 1.4 -0.2	0.7% 3.8 1.5 0.4
		Average Annual Decrease in Mortality from Study Indicated to 1963-67 Study														
Under 60 60-69 70-79 80 and over	1.0 1.3 0.7 0.2	3	0.4% 1.4 1.1 -0.5	1.29 3.0 1.5 -0.3	* 2	.7% .7 .2	-0.1% 3.1 0.9 0.8	-1.8% 1.9 1.4 0.2	-	0.2% 3.2 1.5 0.9	11.4% 7.9 2.1 3.2	0.8% 1.6 0.8 0.4	0.1 1.4 1.2 -0.2	1	0.9% 3.0 1.5 0.1	0.7% 3.8 1.5 0.4
	SETTLEMENT ANNUITIES															
Age Group		NDERS,*	ION ARISIN MATURITH H CLAIMS		Nonpay		tions Arisin h Claims	IG PROM	Ma		eferred Ant	NUITIES	Мато		FERRED AN UT REPUND	
GROOF (1940-45	1945-50	1950-55	1955-60	1940-45	1945-5	0 1950-55	1955–60	1940-4	1945-	-50 1950-55	1955-60	1940-45	1945-50	0 1950-55	1955-60
					Avera	ge Annu	al Decrease in	Mortality	from Stu	ly Indica	ited to Follo	wing Study			·	
Under 60 60–69 70 and over	0.6% 0.5	3.5% 1.4 1.3	-1.2% -0.4 0.2	-0.8% 2.7 1.4	-2.5% 1.9	5.7% 7.8 -9.1	$\begin{bmatrix} -0.4\% \\ 1.1 \\ 3.2 \end{bmatrix}$	6.3% -1.1 -0.5	-16.99 3.7		-1.3	6.0% -1.2 0.4	-1.7% -0.4	19.5%	-4.8% 2.2	2.0%
	Average Annual Decrease in Mortality from Study Indicated to 1960-65 Study															
Under 60 60–69 70 and over	1.1%	0.5% 1.2 1.0	-1.0% 1.2 0.8	-0.8% 2.7 1.4	2.4% 2.5	3.99 2.7 -2.0	3.0% 0.0 1.4	6.3% -1.1 -0.5	- 2.29 1.3	7.0 2.3 0.5	-1.3	$\begin{vmatrix} 6.0\% \\ -1.2 \\ 0.4 \end{vmatrix}$	4.2% 0.4	6.1%	-1.4% 1.0	2.0%

^{*} Excluding maturities and surrenders under pension trust issues.

TABLE 18-Continued

FEMALE LIVES

REFUND

1953-58

1958-63

1941-48

1948-53

AGE GROUP

1941-48

IMMEDIATE ANNUITIES

1948-53

Nonrefund

Average Annual Decrease in Mortality from Study Indicated to Following Study

1953-58

1958-63

REFUND AND NONREFUND

1953-58

1958-63

1948-53

1941-48

Under 60	0.3	1.4% 2.0 0.7 1.1	-1.3% -1.2 1.1 -0.2	4.3% 3.3 1.8 0.7	2.0% 1.9 0.6 1.4		3.8% 2.3 1.5	1.7° -4.7 -0.8 -1.2	% -	-1.3% 3.9 1.7 2.5	0.1% 0.7 0.6 0.9	0.6% 2.1 0.9 1.0	-2	.6	3.6% 3.3 1.8 1.3
		Average Annual Decrease in Mortality from Study Indicated to 1963-67 Study													
Under 60	1.0% 1.0 1.0 0.5	1.4% 1.3 1.2 0.5	1.4% 1.0 1.4 0.2	4.3% 3.3 1.8 0.7	-0.2% 0.9 0.7 0.9		1.1% 0.5 0.8 0.7	0.3° -0.5 0.4 0.6	% -	-1.3% 3.9 1.7 2.5	0.7% 0.9 0.9 0.7	1.0% 1.1 1.6 0.6	1 0	2%).5 .1 .3	3.6% 3.3 1.8 1.3
	SETTLEMENT ANNUITIES														
Age Group	SURRENDI	LECTION ARIS ERS,* MATURI DEATH CLAIM	TIES,* AND	Nonpayee Elections Arising from Death Claims			MATURED DEFERRED ANNUITIES WITH REFUND			MATURED DEFERRED ANNUITE WITHOUT REFUND					
	1940-45 194	5-50 1950-5	5 1955-60	1940-45	1945-50	1950-55	1955–60	1940-45	1945-50	1950-55	1955–60	1940–45	1945-50	1950-55	1955-60
				Average Ar	nual Decre	ase in Mo	rtality fr	om Study	Indicate	d to Follov	ving Study				
Under 60 60–69 70 and over	0.9 2.	0.9° 2.5° 0.6° 0.9° -0.8 1.8 0.5 1.3 2.3% 1.2 1.5 2.3 10.3% 0.6% 3.6% -2.9%													
	Average Annual Decrease in Mortality from Study Indicated to 1960-65 Study														
Under 60	0.3% 0. 1.2 1. 0.9 1.		6 -3.7% 0.9 0.0	1.0% 0.7 0.7	0.0% 1.2 1.5	1.4% 0.9 0.7	1.4% 1.3 2.0	1.8%	3.9% 1.7 1.5	-0.1% 1.9 1.2	-0.2% 2.3 1.7	3.0%	0.5%	0.4%	-2.9% -1.1
* Excluding n	* Excluding maturities and surrenders under pension trust issues.														

percentages for the combined experience were obtained by weighting the percentages in Table 18 by the exposures for the four types of settlement annuities illustrated.) For comparison, the Projection Scale B factors at the indicated age are also shown.

Roughly speaking, Projection Scale B is a fairly good representation of over-all mortality decreases in the last twenty years under immediate

TABLE 19

AVERAGE ANNUAL DECREASE IN MORTALITY (GEOMETRIC BASIS)
FOR ALL TYPES OF ANNUITIES SHOWN IN TABLE 18 COMBINED
CONTRACT YEARS 6 AND OVER—BY NUMBER OF CONTRACTS

Age Group	Males	Females	Projection Scale B (Age in Parentheses)
	Immediate Annu	ities: 1941-48	Study to 1963-67 Study
Under 60	1.6	0.7% 0.9 0.9 0.7	1.225% (55) 1.1 (65) 0.75 (75) 0.25 (85)
	Settlement Annu	ities: 1940–45 S	Study to 1960–65 Study
Under 60	2.4% -1.0 1.2	0.8% 1.4 1.1	1.225% (55) 1.1 (65) 0.5 (80)

annuities and settlement annuities, although there is considerable variation by type of annuity, age, and sex. Of course, the caution already mentioned about using past experience to determine future trends should be kept in mind.

Projection Scale for Future Decreases in Mortality

After review of the past experience on individual annuities and consideration of the factors that may affect future decreases in mortality, we have concluded that Projection Scale B (or the modification of Scale B mentioned below) provides a reasonable set of assumptions, based on our present knowledge, for use in reflecting future mortality decreases for annuities over a long period of time, say the next twenty years. This conclusion was reached by Mr. Edward A. Lew in a paper presented to

the International Congress of Actuaries in June, 1968. 10 Mr. Lew made an extensive review of recent trends in mortality, not only of individual and group annuity experience but also of life insurance and population experience in the United States and other countries. After consideration of these recent trends, Mr. Lew presented an excellent discussion of the outlook for future mortality decreases:

The short-term outlook on mortality at ages 55 and older must necessarily be based on recent trends and current developments. The long-term decline in death rates lost its momentum during the 1950's when the cardiovascular-renal diseases, cancer and other chronic conditions, emerged as a virtually stationary core of the total death rate, while mortality from influenza and pneumonia, tuberculosis, and other infectious diseases, which had registered sharp decreases during the 1940's, either ceased to contribute in a major way to the total death rate or leveled off. Recent studies show no indication of any significant changes in the core of the total death rate; some components such as heart diseases of infectious origin and cancer of the female genital organs are still decreasing while others such as cancer of the lung are distinctly on the rise. As previously noted, pneumonia death rates have increased at the older ages and so has the death rate from diabetes; particularly dramatic has been the upward trend of mortality at these ages attributed to the chronic respiratory diseases, which is closely related to growing urbanization and air pollution.

Major reductions in mortality at ages 55 and older can come only from major advances in the prevention and treatment of the cardiovascular-renal diseases and cancer. Although the prospect of eventual breakthroughs in the understanding and control of these diseases is good, the outlook for any appreciable lowering of mortality from these causes in the near future is not favorable. Furthermore, the immediate consequences of rising population densities, rapid technological changes, and more complex urban ways of life exacerbate the problems of air, water, and food pollution, psychological tensions and growing resort to drugs. It does not, therefore, appear reasonable to anticipate a major downtrend in mortality during the next decade, since it will take time to muster the forces needed to control the adverse effects of current developments.

In estimating the prospects of declines in mortality over a longer period of years there is a large element of personal judgment, particularly as to the irreducible biological limits of the human organism. Nevertheless, a comparison of death rates by age and sex for the various countries with low mortality suggests that modest decreases in mortality are fully within the realm of feasibility for the United States. Reductions in mortality comparable to those experienced in the past are highly unlikely because improvements in sanitation, control of communicable diseases, elimination of major environmental hazards, improved personal hygiene, better conditions of work, and a general rise in living standards

¹⁰ Transactions of the 18th International Congress of Actuaries, pp. 377-98.

have probably already attained near their maximum effects on mortality. We must now look chiefly to advances in medicine and surgery for further decreases in mortality, and it is believed that these cannot produce reductions in mortality at ages in the sixties and seventies as large as those recorded in the recent past. There is mounting evidence, moreover, that persons surviving to the more advanced ages today represent to an increasing degree a less selected group physically than in the past, so that expectations of significant declines in mortality past age 90 have little more than wishful thinking to recommend them. Paul Vincent in his authoritative investigation of the mortality of the aged in France, Netherlands, Switzerland, and Sweden found no evidence of any decreases in death rates at ages 90 and over in seven out of the ten sets of data studied.

For these reasons, the author is inclined to stay with Projection Scale B as a plausible set of assumptions for mortality decreases over a longer period of years.¹¹

The actuary who feels that Projection Scale B does not represent his judgment about the trend of future mortality can choose from a number of different projection scales that have been proposed over the years for use with annuity mortality tables, or he can devise modifications of these scales. The most well known are Projection Scales A (a "retrospective" scale) and B (a "prospective" scale), proposed by Messrs. Jenkins and Lew, and Projection Scale C, proposed by Mr. Peterson. Messrs. Sternhell and Page proposed a modified Scale B which provided for higher levels of improvement at the older ages. Mr. Two additional scales, Projection Scales D and E, are being proposed in connection with the 1971 GAM Table. Comparison of these scales is shown in Table 20.

In the balance of this section we will use Projection Scale B exclusively in making comparisons of the 1971 IAM Table with the *a*-1949 Ultimate Table.

Comparison of 1971 IAM Table and a-1949 Ultimate Table with Projection

Table 21 shows annuity values based on the a-1949 Ultimate Table at $3\frac{1}{2}$ per cent and on the 1971 IAM Table at $3\frac{1}{2}$, 5, 6, and 7 per cent, both with Projection Scale B, for year of valuation (or "year of entry")1971. The projected annuity values were calculated from first principles, using the basic mortality rates and the projection scale factors. For example,

¹¹ Ibid., pp. 394-95.

¹³ TSA, IV, 272.

¹² TSA, I, 417.

¹⁴ TSA, XIII, 131.

the immediate life annuity values for year of valuation 1971 based on the 1971 IAM Table were calculated by the following formula:

$$1971 \ a_x = v(1-q_x) + v^2(1-q_x)[1-q_{x+1}(1-s_{x+1})]$$

$$+ v^3(1-q_x)[1-q_{x+1}(1-s_{x+1})][1-q_{x+2}(1-s_{x+2})^2]$$

$$+ \dots$$

$$+ v^t \prod_{j=0}^{t-1} [1-q_{x+j}(1-s_{x+j})^j] + \dots ,$$

where q_x is the mortality rate from the 1971 IAM Table without projection and s_x is the projection factor at attained age x on Projection Scale B.

For values on the 1971 IAM Table in a different year of valuation, say 1981, the q_{x+t} terms are multiplied by the appropriate $(1 - s_{x+t})$ terms. For example, q_x is multiplied by the factor $(1 - s_x)^{10}$, q_{x+1} by $(1 - s_{x+1})^{11}$, and so on.

TABLE 20

COMPARISON OF PROJECTION SCALES A, B, B (MODIFIED), C, D, AND E
(Rates of Decrease for Intermediate Ages Obtained by Linear Interpolation)

AGE			<u> </u>			· · · · · ·	
AGE	Α	В	B (Modified)	С	D (Male)	D (Female)	E (Male)
0	2.80%	1.25%	1.25%	1.25%	0.65%	1.30%	0.65%
0	2.40	1.25	1.25	1.25	0.65	1.30	0.65
0	2.00	1.25	1.25	1.25	0.65	1.30	0.65
0	1.60	1.25	1.25	1.25	0.65	1.30	0.65
0	1.20	1.20	1.20	1.25	0.65	1.30	0.65
3			<i></i>		0.65	1.30	0.65
5	1.00	1.10	1.10	1.25			<i></i>
8		<i></i>			0.60	1.25	0.45
0	0.80	0.95	0.95	1.25			
3			 		0.50	1.15	0.45
5	0.60	0.75	0.75	1.00			
8			l		0.40	1.00	0.45
0	0.40	0.50	0.50	0.663			
3		1]		0.30	0.80	0.45
5	0.20	0.25	0.50	$0.33\frac{1}{3}$			
8	l		ł		0.20	0.50	0.45
0	0	0	0.50*	0			
2		l					0.45
6		1				0.02	
7		I			0.02	0	
8 <i></i>					0		
07			1				0

^{*} Through 108; rate at age 109 is zero.

One point that should be noted in calculating projected annuity values is that the a-1949 Table is considered to represent the level of mortality in 1950. Thus, under the a-1949 Table, the life annuity value for year of valuation 1971 is based on a factor of $(1 - s_x)^{1971-1950} = (1 - s_x)^{21}$ applied to q_x , $(1 - s_{x+1})^{22}$ applied to q_{x+1} , and so on. The 1971 IAM

TABLE 21

ANNUITY VALUES ON a-1949 ULTIMATE TABLE WITH PROJECTION SCALE B
AT 3½ PER CENT FOR 1971 YEAR OF VALUATION AND ON 1971 IAM TABLE
WITH PROJECTION SCALE B AT VARIOUS INTEREST RATES
FOR 1971 YEAR OF VALUATION

MALE LIVES

	<i>a</i> -1949 Ultimate		1971 IA	M Table	
Age	TABLE AT 3 PER CENT	3}%	5%	6%	7%
		Imn	nediate Life Ann	uity	
60. 65. 70. 75. 80. 85. 90.	13.392 11.396 9.344 7.331 5.468 3.856 2.574 1.640	13.847 11.963 9.998 8.038 6.183 4.506 3.006 1.839	11.915 10.490 8.933 7.315 5.726 4.241 2.871 1.776	10.861 9.667 8.324 6.892 5.453 4.080 2.786 1.737	9.955 8.947 7.782 6.510 5.201 3.928 2.706 1.698
		Immediate Life	Annuity with 10	O Years Certain	
60	14.020 12.347 10.812 9.583 8.793 8.435 8.332 8.317	14.458 12.843 11.310 10.005 9.051 8.521 8.345 8.318	12.468 11.286 10.121 9.098 8.329 7.893 7.746 7.723	11.379 10.413 9.438 8.564 7.897 7.512 7.382 7.361	10.442 9.647 8.827 8.079 7.498 7.159 7.043 7.025
		Immediate Life	: Annuity with 2	0 Years Certain	
60	15.775 14.926 14.433 14.251 14.215 14.212	16.089 15.181 14.572 14.289 14.219 14.213	13.765 13.146 12.720 12.518 12.467 12.462	12.496 12.014 11.677 11.515 11.474 11.470	11.404 11.028 10.760 10.631 10.597 10.594

TABLE 21—Continued
FEMALE LIVES

	<i>a-</i> 1949 Ultimate	i					
Age	TABLE AT 3 Per Cent	34%	5%	6%	7%		
		Imr	nediate Life Ann	uity			
60	15.023 12.929 10.714 8.475 6.340 4.449 2.919 1.795	15.491 13.556 11.401 9.138 6.947 5.005 3.531 2.640	13.169 11.757 10.093 8.254 6.394 4.682 3.344 2.522	11.919 10.764 9.352 7.741 6.066 4.487 3.229 2.449	10.855 9.903 8.697 7.279 5.766 4.305 3.121 2.380		
		Immediate Life	Annuity with 10) Years Certain			
60	15.356 13.496 11.691 10.139 9.049 8.506 8.341 8.317	15.829 14.045 12.212 10.558 9.345 8.700 8.444 8.350	13.475 12.199 10.826 9.537 8.565 8.039 7.828 7.749	12.206 11.178 10.038 8.942 8.101 7.640 7.454 7.385	11.124 10.291 9.340 8.406 7.675 7.272 7.108 7.046		
		Immediate Life	Annuity with 20	Years Certain			
60	16.467 15.345 14.607 14.290 14.218 14.213	16.815 15.637 14.794 14.384 14.250 14.217	14.257 13.461 12.875 12.586 12.489 12.466	12.878 12.262 11.800 11.569 11.492 11:473	11.703 11.224 10.858 10.674 10.612 10.596		

Table is considered to represent the level of mortality in 1971; hence, as indicated above, the factor applied to q_x to obtain the annuity value for year of valuation 1971 is $(1 - s_x)^{1971-1971} = 1$, the factor applied to q_{x+1} is $(1 - s_{x+1})$, and so on.

Table 22, which can be derived directly from Table 21, shows ratios of annuity values based on the 1971 IAM Table with Projection Scale B at $3\frac{1}{2}$, 5, 6, and 7 per cent to those based on the *a*-1949 Ultimate Table with Projection Scale B at $3\frac{1}{2}$ per cent, for year of valuation 1971. We

TABLE 22

RATIOS OF ANNUITY VALUES ON 1971 IAM WITH PROJECTION SCALE B
AT VARIOUS INTEREST RATES TO ANNUITY VALUES ON a-1949 Table
WITH PROJECTION SCALE B AT 3½ PER CENT
YEAR OF VALUATION 1971

		N	AALE LIVE	s			F	MALE LIVI	×s	
Age	Interest	Rate for	1971 IAN	1 Table	''Break- even''	Interest	Rate for	1971 IAM	I Table	"Break- even"
	31/2%	5%	6%	7%	Interest Rate	31%	5%	6%	7%	Interest Rate
				In	mediate I	ife Annui	ty			
60 65 70 80 85 90	103.4% 105.0 107.0 109.6 113.1 116.9 116.8 112.1	89.0% 92.0 95.6 99.8 104.7 110.0 111.5 108.3	81.1% 84.8 89.1 94.0 99.7 105.8 108.2 105.9	74.3% 78.5 83.3 88.8 95.1 101.9 105.1 103.5	3.8% 4.0 4.4 4.9 5.9 7.5 8.8 8.6	103.1% 104.9 106.4 107.8 109.6 112.5 121.0 147.1	87.7% 90.9 94.2 97.4 100.9 105.2 114.6 140.5	79.3% 83.3 87.3 91.3 95.7 100.9 110.6 136.4	72.3% 76.6 81.2 85.9 90.9 96.8 106.9 132.6	3.8% 4.0 4.3 4.6 5.2 6.2 9.1
			Imn	nediate Li	fe Annuity	with 10	Years Cer	tain		
60 65 70 75 80 85 90 95	103.1% 104.0 104.6 104.4 102.9 101.0 100.2 100.0	88.9% 91.4 93.6 94.9 94.7 93.6 93.0 92.9	81.2% 84.3 87.3 89.4 89.8 89.1 88.6 88.5	74.5% 78.1 81.6 84.3 85.3 84.9 84.5	3.8% 3.9 4.1 4.2 4.0 3.7 3.5 3.5	103 1% 104 1 104 5 104 1 103 3 102 3 101 2 100 4	87.8% 90.4 92.6 94.1 94.7 94.5 93.8 93.2	79.5% 82.8 85.9 88.2 89.5 89.8 89.4 88.8	72.4% 76.3 79.9 82.9 84.8 85.5 85.2 84.7	3.8% 3.9 4.0 4.1 4.0 3.9 3.7 3.6
	Immediate Life Annuity with 20 Years Certain									
65 70	102.0% 101.7 101.0 100.3 100.0 100.0	87.3% 88.1 88.1 87.8 87.7 87.7	79.2% 80.5 80.9 80.8 80.7 80.7	72.3% 73.9 74.6 74.6 74.5 74.5	3.7% 3.7 3.6 3.5 3.5 3.5	102.1% 101.9 101.3 100.7 100.2 100.0	86.6% 87.7 88.1 88.1 87.8 87.7	78.2% 79.9 80.8 81.0 80.8 80.7	71.1% 73.1 74.3 74.7 74.6 74.6	3.7% 3.7 3.6 3.6 3.5 3.5

^{*} Greater than 10 per cent.

also calculated the annuity values for years of valuation 1981 and 1991, but, since the ratios of the projected annuity values under the 1971 IAM Table to those under the *a*-1949 Ultimate Table are quite similar regardless of year of valuation, we are not showing the 1981 and 1991 values herein.

We have also valued the model office of one year's issues of single premium immediate annuities, described in Section V, on the a-1949 Ultimate Table at $3\frac{1}{2}$ per cent and the 1971 IAM Table at $3\frac{1}{2}$, 5, 6, and 7 per cent, both with Projection Scale B, for year of valuation 1971. The results are shown in Table 23. Other relationships, such as that between

TABLE 23
TEST VALUATION OF ONE YEAR'S ISSUES OF SINGLE PREMIUM IMMEDIATE ANNUITIES
YEAR OF VALUATION 1971

	RESER PREMIU WITH T \$1,	RATIO TO a-1949 ULTIMATE TABLE AT 3½ PER CENT		
	Males	Females	Total	
a-1949 Ultimate Table with Projection B at 3½ per cent 1971 IAM Table with Projec- tion B:	\$4,220	\$6,716	\$10,936	100.0%
3½ per cent	4,420 3,927 3,648 3,402	7,023 6,197 5,733 5,326	11,443 10,124 9,381 8,728	104.6 92.6 85.8 79.8

the 1971 IAM Table with Projection Scale B and the a-1949 Ultimate Table without projection, can be obtained by comparing the results in this section with those presented in Section V.

In practice, the actuary may want to use a convenient approximate method of reflecting future decreases in mortality, for example, an age setback on the unprojected table varying by year of birth.

VII. ADJUSTMENTS FOR VARIOUS KINDS OF ANNUITIES

The 1971 IAM Table is based on the combined intercompany experience under immediate annuities and settlement annuities and hence is generally suitable as a valuation standard for the mix of a company's annuity business, without further adjustment. However, in specific situa-

tions an actuary may judge that adjustments should be made in valuation assumptions for certain types of annuities.

The actuary can determine the variation in mortality under different types of annuities by a review of the *Reports* numbers of the *Transactions* containing the reports on the intercompany studies. To give an indication of the extent of variation in mortality, Table 24 shows the ratios of actual to expected based on the 1963 Experience Table for the experience underlying that table, separately for immediate annuities and settlement annuities and for refund and nonrefund annuities.

As expected, nonrefund immediate annuities exhibit the lowest mortality ratios, 87.6 per cent overall for males and 90.0 per cent for females. Nonrefund settlement annuities also have lower-than-average mortality ratios, although not as low as under immediate annuities (92.4 per cent for males and 98.2 per cent for females). Under immediate annuities with a refund period, mortality ratios for males are practically the same as under settlement annuities with a refund period, about 102 per cent in both cases. For female refund annuities, the mortality ratios are 98.8 per cent and 103.2 per cent for immediate annuities and settlement annuities, respectively.

If the 1971 IAM Table (unprojected) is applied to the same experience, the mortality ratio for all ages combined is over 100 per cent for each of the classifications just discussed. This is seen in Table 25. The mortality ratios range from a low of 106.3 per cent for males and 108.1 per cent for females for nonrefund immediate annuities to a high of 128.9 per cent for males and 127.9 per cent for females for refund settlement annuities. For all annuities combined, the mortality ratio is 125.1 per cent for males and 123.0 per cent for females. In the interpretation of these results, it should be remembered that the 1971 IAM Table reflects not only the 10 per cent margin but also the projection from 1963 to 1971.

If a company decides to make any adjustments at all in its valuation assumptions for a given type of annuity, one likely candidate is non-refund immediate annuities. A common method of adjusting valuation assumptions that has been used in the past is the age setback approach. As a test of this approach, Table 26 shows the mortality ratios resulting from 0-, 1-, and 2-year setbacks of the 1963 Experience Table applied to the nonrefund immediate annuity experience in the 1963-67 intercompany study.

The table shows that an age setback on the 1963 Experience Table of about 1.5 years for males and about 1 year for females would produce a mortality ratio of 100 per cent. The setback to produce a mortality ratio of 100 per cent actually varies by age as well as sex, but for practical

TABLE 24

RATIOS OF ACTUAL TO EXPECTED DEATHS BY TYPE OF ANNUITY BASED ON 1963 EXPERIENCE TABLE

ALL CONTRACT YEARS COMBINED—BY AMOUNT OF ANNUAL INCOME

Age		Males			FEMALES	
GROUP	Refund	Nonrefund	Combined	Refund	Nonrefund	Combined
		1963-	67 Study of I	nmediate Anr	nuities	
60–64. 65–69. 70–74. 75–79. 80–84. 85–89. 90–94. 95–99.	113.2% 99.9 101.2 99.3 109.3 99.4 100.7 90.7	72.8% 83.3 83.4 82.2 85.5 88.0 101.7 99.6	103.6% 95.8 95.0 92.2 98.1 93.2 101.2 95.4	127.5% 110.1 85.8 99.0 96.2 98.1 101.0 110.5	100.1% 56.8 80.6 87.5 92.3 90.3 97.3 86.3	121.1% 94.4 84.0 94.5 94.6 94.5 99.2 98.2
60-69 70-79 80-89 90-99	103.1 100.1 105.0 98.6	80.8 82.6 86.7 101.2	97.7 93.3 95.8 100.0	115.8 94.3 97.1 103.0	68.2 85.4 91.2 94.8	102.7 90.9 94.5 99.0
60–99	102.4	87.6	95.8	98.8	90.0	95.0
		1960-	65 Study of Se	ettlement Ann	uities	
60–64. 65–69. 70–74. 75–79. 80–84. 85–89. 90–94. 95–99.	102.7% 101.8 102.6 103.2 102.7 109.2 93.8 106.4	148.6% 83.2 82.4 83.6 107.7 113.2 100.3 51.5	104.1% 101.1 101.6 101.9 103.1 109.5 94.4 104.1	103.9% 100.9 104.0 102.6 104.4 102.8 104.2 95.5	80.8% 90.7 101.2 98.3 94.5 106.8 107.4 89.4	102.3% 100.1 103.7 102.2 103.4 103.2 104.5 95.1
60–69 70–79 80–89 90–99	102.0 102.8 104.6 95.2	93.6 83.0 109.3 97.9	101.7 101.7 104.9 95.4	101.8 103.2 103.7 103.0	87.8 99.6 99.4 105.5	100.8 102.9 103.3 103.3
60–99	102.8	92.4	102.3	103.2	98.2	102.7
			Combine	l Studies		
60–64. 65–69. 70–74. 75–79. 80–84. 85–89. 90–94. 95–99.	104.1% 101.6 102.4 102.6 104.5 105.2 97.9 94.5	103.2% 83.3 83.0 82.7 89.8 90.4 101.6 99.0	104.1% 100.5 100.6 99.8 101.1 100.0 99.5 96.6	106.6% 101.9 101.5 102.0 102.2 101.0 102.6 104.9	87.6% 77.8 91.5 91.9 93.0 93.3 98.3 86.4	104.9% 99.3 100.1 100.3 100.3 98.9 101.1 97.4
60–69	102.1 102.5 104.7 97.3	87.2 82.8 90.1 101.1	101.2 100.2 100.7 98.9	103.4 101.8 101.7 103.0	80.5 91.7 93.1 95.8	101.0 100.2 99.7 100.4
60-99	102.7	88.8	100.5	102.1	92.4	100.1

TABLE 25

RATIOS OF ACTUAL TO EXPECTED DEATHS BY TYPE OF ANNUITY BASED ON 1971 IAM TABLE

ALL CONTRACT YEARS COMBINED—BY AMOUNT OF ANNUAL INCOME

Age		Males			FEMALES	
GROUP	Refund	Nonrefund	Combined	Refund	Nonrefund	Combined
		1963-	67 Study of I	nmediate Ann	uities	
60-64 65-69 70-74 75-79 80-84 85-89 90-94 95-99	142.6% 126.3 128.0 125.4 135.2 117.9 114.8 100.8	91.7% 105.3 105.5 103.8 105.6 104.3 115.9 110.8	130.5% 121.1 120.2 116.5 121.2 110.5 115.4 106.1	161.9% 138.9 108.5 125.2 118.7 115.8 115.6 122.9	127.2% 71.7 102.0 110.7 113.9 106.6 111.4 96.0	153.9% 119.1 106.2 119.5 116.7 111.5 113.5 109.2
60–69 70–79 80–89 90–99	130.2 126.5 127.6 111.9	102.0 104.4 105.0 114.8	123.4 118.0 116.2 113.4	146.5 119.3 117.2 117.2	86.2 108.0 109.8 107.8	129.8 115.0 113.9 112.5
60–99	125.7	106.3	117.1	119.6	108.1	114.7
		1960-	65 Study of Se	ettlement Ann	uities	
60-64. 65-69. 70-74. 75-79. 80-84. 85-89. 90-94. 95-99.	129.3% 128.7 129.7 130.3 127.3 129.7 107.1 118.4	187.1% 105.2 104.2 105.6 133.4 134.4 114.6 57.2	131.1% 127.8 128.4 128.8 127.8 130.1 107.8 115.9	132.0% 127.3 131.5 129.8 129.0 121.6 119.5 106.3	102.7% 114.4 128.0 124.4 116.8 126.3 123.2 99.5	130.0% 126.2 131.2 129.3 127.8 122.0 119.9 105.8
60–69 70–79 80–89 90–99	128.8 130.0 128.0 108.4	118.2 104.9 133.7 111.7	128.4 128.6 128.5 108.7	128.8 130.6 126.0 117.7	111.0 125.9 120.7 120.7	127.4 130.2 125.5 117.9
60-99	128.9	115.5	128.2	127.9	121.5	127.3
			Combine	d Studies	1	
60-64. 65-69. 70-74. 75-79. 80-84. 85-89. 90-94. 95-99.	131.1% 128.5 129.5 129.6 129.4 124.9 111.8 105.1	130.0% 105.2 104.9 104.4 111.0 107.2 115.8 110.2	131.0% 127.0 127.3 126.1 125.2 118.7 113.5 107.5	135.4% 128.6 128.4 129.0 126.2 119.4 117.5 116.7	111.3% 98.1 115.7 116.2 114.7 110.2 112.6 96.1	133.2% 125.2 126.6 126.8 123.8 116.9 115.8 108.4
70–79 80–89 90–99	129.6 127.9 110.6	104.6 109.2 114.7 108.6	126.7 122.7 112.3 125.1	128.7 123.3 117.4 125.8	116.0 112.3 109.0 111.9	126.7 120.6 114.4 123.0

reasons the actuary might decide to use the same setback for both males and females at all ages. In the case of nonrefund immediate annuities, a 1-year setback appears to be reasonable. It is also reasonable to assume that the setback determined on the basis of the 1963 Experience Table can be applied to the valuation table itself, the 1971 IAM Table, to reflect the differential between average mortality under all types of annuities and mortality under nonrefund immediate annuities.

TABLE 26

MORTALITY RATIOS UNDER NONREFUND IMMEDIATE ANNUITIES
IN 1963-67 INTERCOMPANY STUDY

BASED ON 1963 EXPERIENCE TABLE WITH 0-, 1-, AND 2-YEAR SETBACKS IN AGE

ALL CONTRACT YEARS COMBINED—BY AMOUNT OF ANNUAL INCOME

A		Males		Females		
Age	No	1-Year	2-Year	No	1-Year	2-Year
Group	Setback	Setback	Setback	Setback	Setback	Setback
60-64	72.8%	78.4%	84.7%	100.1%	107.7%	117.3%
65-69	83.3	90.1	97.2	56.8	61.3	65.6
70-74	83.4	91.1	99.3	80.6	90.4	100.9
75-79	82.2	90.1	98.8	87.5	98.8	111.6
80-84	85.5	93.0	101.3	92.3	103.6	116.5
85-89	88.0	96.0	104.5	90.3	100.9	112.9
90-94	101.7	112.2	123.8	97.3	105.4	115.3
95-99	99.6	109.0	119.8	86.3	90.5	95.1
60–69 70–79 80–89 90–99	80.8 82.6 86.7 101.2 87.6	87.3 90.5 94.5 111.6 95.7	94.2 99.0 102.9 122.9	68.2 85.4 91.2 94.8	73.5 96.2 102.1 102.0 99.8	79.0 108.2 114.5 110.5

A similar analysis could be made for any other type of annuity for which the actuary might want to vary valuation assumptions, for example, nonrefund matured deferred annuities or payee elections of settlement annuities.

VIII. JOINT LIFE ANNUITY VALUES

Since the 1971 IAM Table is not a Makehamized table, the question of how to best calculate joint life annuity values for unequal ages arises. Furthermore, whether the basic table is Makehamized or not, the calculation of joint life annuity values on a projected basis is a problem.

Undoubtedly, approximation techniques could be devised to make

these kinds of calculations. For example, the "as if" approach (calculating joint life values as if the tables were Makehamized, using appropriate constants) could probably be applied successfully. Similarly, projected annuities could be approximated by applying the ratios of projected to unprojected single life annuities to unprojected joint life annuities, or by an age setback on the unprojected table. However, with the availability of high-speed computers, it is probably just as easy to calculate joint life annuities on an exact basis from first principles, working directly with the mortality rates and the projection scale.

Appendix B presents an example of a program designed to calculate joint life annuity values. This program was written by Jonathan L. Wooley, an Associate of the Society, in FORTRAN IV and was tested and debugged on the IBM 1130.15 The appendix describes how the program can be used to calculate values for various age combinations with any certain period on a projected or an unprojected basis.

The calculation of projected joint life annuity values for a given year of valuation directly from the mortality rates and the projection scale is based on the following formula:

Annuity value in year of valuation z

$$= a_{\overline{n}|} + v^{n+1} {}_{n+1}^{z} p_{xy} + v^{n+2} {}_{n+2}^{z} p_{xy} + v^{n+3} {}_{n+3}^{z} p_{xy} + \ldots,$$

where

$$\prod_{n+1}^{z} p_{xy} = \prod_{j=0}^{n} [1 - q_{x+j} (1 - s_{x+j})^{z+j-b}] [1 - q_{y+j} (1 - s_{y+j})^{z+j-b}]$$

and successive terms of the form $_{n+t+1}^{z}p_{xy}$ can be obtained by the recursion formula

$$n_{t+1}^{z} p_{xy} = \sum_{n+t}^{z} p_{xy} \left[1 - q_{x+n+t} (1 - s_{x+n+t})^{z+n+t-b} \right] \left[1 - q_{y+n+t} (1 - s_{y+n+t})^{z+n+t-b} \right],$$

where n is the certain period; x is the attained age of the first life in the year of valuation; y is the attained age of the second life in the year of valuation; q_{x+j} , q_{y+j} are the mortality rates in the "base year" at attained ages x+j, y+j for the applicable sex; s_{x+j} , s_{y+j} are the projection factors at attained ages x+j and y+j; and b is the "base year" of the mortality table—for example, 1971 for 1971 IAM Table or 1950 for a-1949 Table.

If annuity values based on a static table projected to a specified year z were desired, the above formula would be modified so that the exponent

¹⁶ The program was also easily adapted for a successful run on an IBM 360 computer.

for all (1-s) terms is a constant z-b. If unprojected values were desired, all (1-s) terms would be set equal to 1. Samples of input and output for the FORTRAN program are shown in Appendix B.

To give some indication of how joint life annuity values based on the new table compare with those based on the a-1949 Table, Table 27 shows

TABLE 27

JOINT LIFE ANNUITY VALUES BASED ON 1971 IAM TABLE (UNPROJECTED)

AND a-1949 ULTIMATE TABLE AT 3½ PER CENT

IMMEDIATE LIFE ANNUITIES—ONE MALE AND ONE FEMALE

Male		Female Age						
Age	60	65	70	75	80	85	90	95
			1971 I	AM Table	(Unproje	cted)		
60. 65. 70. 75. 80. 85. 90.	11.658 10.469 9.043 7.470 5.871 4.348 2.932 1.807	10.774 9.824 8.617 7.216 5.732 4.279 2.902 1.794	9.525 8.834 7.902 6.750 5.461 4.138 2.838 1.767	7.973 7.518 6.868 6.014 4.993 3.875 2.710 1.711	6.281 6.008 5.601 5.033 4.307 3.456 2.492 1.611	4.648 4.497 4.263 3.920 3.457 2.877 2.161 1.447	3.337 3.253 3.121 2.920 2.636 2.263 1.766 1.228	2.523 2.473 2.392 2.266 2.080 1.827 1.468 1.051
			a-194	19 Table (Unproject	ed)		
60	10.483 9.279 7.894 6.420 4.966 3.636 2.505 1.608	9.557 8.595 7.429 6.129 4.798 3.546 2.460 1.587	8.368 7.662 6.755 5.684 4.530 3.398 2.384 1.552	6.974 6.503 5.861 5.054 4.128 3.165 2.262 1.493	5.484 5.199 4.792 4.246 3.574 2.823 2.072 1.397	4.036 3.881 3.649 3.322 2.888 2.366 1.799 1.252	2.755 2.678 2.561 2.387 2.142 1.824 1.447 1.049	1.723 1.689 1.635 1.553 1.432 1.264 1.048 0.797
	a-1949 Table (with Projection B to 1971)							
60	11.256 9.982 8.476 6.842 5.216 3.738 2.519 1.615	10.274 9.269 8.005 6.558 5.060 3.659 2.482 1.598	8.980 8.260 7.290 6.102 4.798 3.521 2.415 1.567	7.435 6.971 6.304 5.423 4.380 3.291 2.299 1.511	5.774 5.505 5.098 4.520 3.778 2.934 2.108 1.417	4.168 4.029 3.809 3.477 3.014 2.439 1.821 1.266	2.789 2.723 2.616 2.447 2.195 1.853 1.447 1.049	1.738 1.710 1.662 1.583 1.461 1.282 1.048 0.797

a grid of values for annual payment immediate annuities without a certain period, one male and one female, based on the 1971 IAM Table (unprojected), on the a-1949 Ultimate Table (unprojected), and on the a-1949 Ultimate Table projected to 1971, at $3\frac{1}{2}$ per cent interest. Table

TABLE 28

JOINT LIFE ANNUITY VALUES BASED ON 1971 IAM TABLE AND a-1949

ULTIMATE TABLE AT 3½ PER CENT WITH PROJECTION B

FOR YEAR OF VALUATION 1971

IMMEDIATE LIFE ANNUITIES—ONE MALE AND ONE FEMALE

Male				FEMALE	Age			
Age	60	65	70	75	80	85	90	95
	197	1 IAM Ta	ble (with	Projection	B for Ye	ar of Valu	ation 197	1)
60	11.948 10.695 9.194 7.552 5.904 4.356 2.934 1.807	11.012 10.023 8.759 7.298 5.768 4.289 2.904 1.795	9.695 8.987 8.021 6.827 5.497 4.149 2.841 1.768	8.074 7.615 6.952 6.073 5.024 3.886 2.713 1.712	6.328 6.057 5.646 5.068 4.328 3.464 2.495 1.612	4.664 4.515 4.281 3.935 3.466 2.881 2.162 1.447	3.342 3.260 3.128 2.926 2.640 2.264 1.766 1.228	2.526 2.477 2.396 2.269 2.083 1.828 1.468 1.051
	0	7-1949 Tabl	le (with P	rojection l	B for Year	of Valua	tion 1971)	
60	11.543 10.203 8.622 6.920 5.248 3.745 2.521 1.615	10.509 9.464 8.143 6.637 5.094 3.668 2.483 1.598	9.145 8.408 7.405 6.174 4.831 3.531 2.417 1.567	7.530 7.063 6.383 5.478 4.409 3.301 2.301 1.512	5.815 5.549 5.140 4.553 3.797 2.941 2.110 1.418	4.181 4.043 3.824 3.490 3.023 2.442 1.822 1.266	2.792 2.727 2.621 2.451 2.198 1.854 1.447 1.049	1.739 1.711 1.663 1.585 1.462 1.282 1.048 0.797

28 shows joint life annuity values for the same "cells" as Table 27 on a fully projected basis for year of valuation 1971.

IX. ACKNOWLEDGMENTS

I would like to thank the many people at a number of companies—both members and nonmembers of the Society—who contributed to the preparation of this paper. My special thanks go to Thomas R. Huber, who directed the preparation of the extensive tables.

APPENDIX A

COMPARISON OF APPROACH USED TO DEVELOP a-1949 TABLE WITH APPROACH USED TO DEVELOP 1971 INDIVIDUAL ANNUITY MORTALITY TABLE

	a-1949 Table (and Underlying 1943 Experience Table)	1971 Individual Annuity Mortality Table (and Underlying 1963 Experience Table)
Type of experience included	Immediate nonrefund an- nuities only	Combined nonrefund and refund immediate annuities, life income settlements, and matured deferred annuities
2. Period of exposure	1941-46 (central year taken as 1943)	1963-67 for immediate an- nuities; 1960-65 for life income settlements and matured deferred annui- ties (central year for combined experience tak- en as 1963)
3. Policy years of experience included	Second and subsequent policy years included, resulting in ultimate table; first-year select rates taken as 75 per cent of ultimate for males, 50 per cent for females	All policy years included (aggregate table)
4. Experience taken by number or amount?5. Method of graduation used to obtain experience table	By number of contracts Makeham curve fitted by method of moments	By amount of annual income Jenkins fifth-difference modified osculatory interpolation formula (pivotal values based on King's formula applied separately to exposures and deaths in 5-year age groups); cubic used to close table
6. Age at which mor- tality rate is taken	109	115
as 1 7. Basis of mortality rates for younger ages	At ages 55 and under for males and 50 and under for females, based on intercompany active lives experience under group annuity contracts covering predominantly clerical employees, by lives, for calendar years 1939, 1940, 1946, and 1947, with adjustment for ill-health terminations; graphical graduation used to bridge gap between group annuity experience and Makehamized nonrefund immediate annuity experience at ages 60 and over	Mortality rates at ages 50 and under based on 1966 experience table for group annuities, adjusted to 1963 basis; fourth-degree polynomial used to bridge gap between rates at younger and older ages

APPENDIX A-Continued

	a-1949 Table (and Underlying 1943 Experience Table)	1971 Individual Annuity Mortality Table (and Underlying 1963 Experience Table)
8. Basis for developing final table from experience table	Mortality rates projected to 1949 using conservative estimates (varying by sex) of decreases in annuity mortality rates, based on review of annual decreases in mortality (geometric basis) over a similar period under various types of experience	Mortality rates at ages 60 and over projected to 1971 based on annual decreases in mortality (geometric basis) between two latest intercompany studies, males and females and all types of annuities and life income settlements combined; mortality rates at ages 50 and under taken directly from 1971 GAM Table; fourth-degree polynomial used to bridge gap between younger and older ages
 Method of provid- ing margins for contingencies, etc., in final table 	Included implicitly in con- servative projection factors described in item 8 above	10 per cent margin includ- ed explicitly
10. Method of gradu- ating final table	Makeham curve, with constant A varying for younger ages	Whittaker-Henderson Type A formula with a=1 for ages 56 and over
 Basis for providing for improvements in mortality be- yond base year of final table 	Projection Scales A ("retrospective" scale) and B ("prospective" scale) devised, based on (a) review of statistics on long-term mortality decreases, (b) assumptions used by others, and (c) informed opinion	No new projection scales devised; Projection Scale B, or modification there- of, suggested as reason- able basis for those wishin to make provision in re- serves for future im- provements in mortality

APPENDIX B

FORTRAN PROGRAM FOR CALCULATION OF JOINT LIFE ANNUITY VALUES

This program calculates immediate life annuities with annual payments on a joint life basis for selected interest rates, age combinations, and periods certain and, as desired, (a) on an unprojected basis; (b) on the basis of projection to a specified year (but not beyond that year, i.e., a static table based on the projected level of mortality in the specified year); or (c) with full projection for

a given year of valuation. The input consists of mortality rates for males and females (rates may range from 0 to 119), a projection scale, and any number of "specification cards." Each specification card results in a set of rates for all age combinations in the range requested, in accordance with the parameters on the card.

This program is presented as an example of how to apply the direct method of calculating joint life annuity values. An individual company might want to include certain variations in its own program to meet its particular needs, for example:

- Calculation of entire sets of values (e.g., by specified age combinations, interest rates, years certain, range of valuation years, and so on) as predetermined by logic within the program rather than by input "specification cards."
- 2. Calculation of mean reserve rates from terminal and initial reserves.
- Rounding to more or fewer decimal places. (This program rounds printed values to three decimal places.)
- 4. Allowance for the calculation of single life annuity values within the same program. (This could also be done by running the program "as is" and inputting zeros or blanks as the mortality rates for one of the sexes.)
- 5. Calculation of joint-and-survivor annuity values for any age combination.
- 6. Allowance for different projection factors for males and females.
- 7. Calculation of annuity values for frequency of payment other than annual.

INDIT

The program, which is listed at the end of this appendix, is read in first, followed by a number of input cards whose general format is specified in Table B1. Also shown in Table B1 is the format of a specific set of input cards for a sample run of joint life annuity values with a 10-year certain period, one male and one female, age range 60-80, based on the 1971 IAM Table at 6 per cent with Projection Scale B, year of valuation 1975. The output of the sample run is shown just after the description of the input.

All entries in the input are right-adjusted in the columns shown, except that the name of the mortality table and the projection scale may be left-adjusted.

OUTPUT

The output resulting from the sample input is shown in Tables B2 and B3. The first part of the output consists of the input mortality rates and projection scale (interpolated if required). These rates are printed out only once in any run, regardless of the number of sets of values which follow. This is done for control purposes, that is, to make available a permanent record of the rates actually inputted.

TABLE B1 FORMAT OF INPUT CARDS

Column	General Format	Format for Sample Run						
	Card Type 1 (This card instructs the IBM 1130 to begin executing the program; for other computers or systems the method of initiating the program will be different)							
1 2 4 5	/ (Slash) / (Slash) X E Q	/ / X E Q						
	Card Type (Description of mortality							
1–44	Name of mortality table	1971 INDIVIDUAL ANNUITY MORTALITY TABLE						
47-50 53-55 58-60	Base year of table Lowest age inputted (N_1) Highest age inputted (N_2)	1971 5 115						
	Card Type (Mortality rates: there will be N ₃ -							
2–9 11–18	Mortality rate for males in format x.xxx xxx Mortality rate for females	.000456 [Rates in first card .000234] for age 5]						
	Card Type (Description of projectio							
1–28 33–35	Name of projection scale Number of projection factor cards	PROJECTION SCALE B 8						
	Card Type 5 (Projection factors; there will be as many cards as type 4 card indicates [factors may be supplied for every age or at any desired intervals, in which case factors for ages between those supplied will be obtained by linear interpolation; the factor on the first card will be used for all ages below the age on that card, and the factor on the last card will be used for all ages above the age on that card])							
3-5 8-13	Attained age Projection factor (x.xxxx)	* *						

^{*} The eight cards of type 5 are coded as follows:

Age in	Projection	Age in	Projection
Cols.	Factor in	Cols.	Factor in
4 and 5	Cols. 9-13	4 and 5	Cols. 9-13
50	.0125	75	.0075
	.0120	80	.0050
	.0110	85	.0025
	.0095	90	.0000

TABLE B1-Continued

Column	General Format	Format for Sample Run							
	Card Type 6 (These cards specify the parameters for a set of number of specification cards, one for each se								
13-15 18-20 23-25	Interest rate (.xxxx) Sex combination (1=2 males; 2=2 females; 3=1 male and 1 female) Lowest age desired Highest age desired Certain period Projection method code (1=unprojected; 2=projected to year in cols. 32-35 (static table); 3=with projection for year of valuation in cols. 32-35) Year for code 2 or code 3 in col. 30	.0600 3 60 80 10 3							
	Card Type 7 (This card is blank)								

TABLE B2

MORTALITY RATES AND PROJECTION SCALE

MALE MORTALITY RATES—1971 INDIVIDUAL ANNUITY MORTALITY TABLE

									l	Ï
Ages:										
0–9	0.000000	0.000000	0.000000	0.000000	0.000000	0.000456	0.000424	0.000403	0.000392	0.000389
10-19	0.000390	0.000397	0.000405	0.000413	0.000422	0.000433	0.000444	0.000457	0.000471	0.000486
20-29	0.000503	0.000522	0.000544	0.000566	0.000591	0.000619	0.000650	0.000684	0.000722	0.000763
30-39	0.000809	0.000860	0.000916	0.000978	0.001046	0.001122	0.001204	0.001295	0.001397	0.001509
40–49	0.001633	0.001789	0.002000	0.002260	0.002569	0.002922	0.003318	0.003754	0.004228	0.004740
50-59	0.005285	0.005860	0.006461	0.007088	0.007740	0.008417	0.009119	0.009850	0.010613	0.011411
60-69	0.012249	0.013133	0.014073	0.015083	0.016185	0.017405	0.018767	0.020290	0.021992	0.023890
70–79	0.026000	0.028341	0.030933	0.033801	0.036976	0.040494	0.044393	0.048715	0.053500	0.058787
80-89	0.064599	0.070902	0.077668	0.084941	0.092874	0.101689	0.111652	0.123048	0.136123	0.151070
90-99	0.108040	0.187147	0.208457	0.231885	0.257146	0.283841	0.311565	0.340214	0.369769	0.400194
100~109	0.431413	0.463312	0.495756	0.528599	0.561692	0.594884	0.628022	0.660949	0.693503	0.725521
110-119	0.756852	0.787390	0.817125	0.846198	0.874915	1.000000	0.000000	0.000000	0.000000	0.000000
			İ						İ	

FEMALE MORTALITY RATES-1971 INDIVIDUAL ANNUITY MORTALITY TABLE

						1	l	1		
Ages:	i '					ł	!			
0-9	0.000000	0.000000	0.000000	0.000000	0.000000	0.000234	0.000193	0.000162	0.000143	0.000134
10-19	0.000132	0.000143	0.000155	0.000167	0.000180	0.000193	0.000205	0.000218	0.000231	0.000245
20–29	0.000260	0.000275	0.000292	0.000309	0.000327	0.000347	0.000368	0.000390	0.000414	0.000440
30-39	0.000469	0.000499	0.000533	0.000569	0.000608	0.000651	0:000698	0.000750	0.000807	0.000869
40–49	0.000938	0.001013	0.001094	0.001186	0.001286	0.001397	0.001519	0.001654	0.001802	0.001967
50–59	0.002151	0.002371	0.002641	0.002966	0.003351	0.003791	0.004284	0.004826	0.005409	0.006017
60-69	0.006628	0.007219	0.007773	0.008285	0.008775	0.009290	0.009888	0.010622	0.011536	0.012664
70–79		0.015651	0.017548	0.019742	0.022256	0.025120	0.028369	0.032050	0.036225	0.040975
80–89	0.046386	0.052513	0.059409	0.067160	0.075899	0.085770	0.096898	0.109338	0.122978	0.137508
90–99	0.152472	0.167370	0.181776	0.195386	0.208071	0.219896	0.231097	0.242211	0.253823	0.266452
100-109	0.280535	0.296449	0.314535	0.335121	0.358537	0.385122	0.415238	0.449274	0.487649	0.530787
110-119	0.579040	0.632529	0.690903	0.753081	0.817218	1.000000	0.000000	0.000000	0.000000	0.000000

TABLE B2—Continued PROJECTION SCALE B

Ages: 0-9. 0.000000 10-19. 0.012500 20-29. 0.012500 30-39. 0.012500 40-49. 0.012500	0.012500 0.012500 0.012500	0.000000 0.012500 0.012500 0.012500	0.000000 0.012500 0.012500 0.012500	0.000000 0.012500 0.012500 0.012500	0.012500 0.012500 0.012500 0.012500	0.012500 0.012500 0.012500 0.012500	0.012500 0.012500 0.012500 0.012500	0.012500 0.012500 0.012500	0.012500 0.012500 0.012500
10–19 0.012500 20–29 0.012500 30–39 0.012500	0.012500 0.012500 0.012500	0.012500 0.012500 0.012500	0.012500 0.012500 0.012500	0.012500 0.012500	0.012500 0.012500	0.012500 0.012500	0.012500 0.012500	0.012500 0.012500	0.012500 0.012500
20–290.012500 30–390.012500	0.012500 0.012500	0.012500 0.012500	0.012500 0.012500	0.012500	0.012500	0.012500	0.012500	0.012500	0.012500
				0.012500	0.012500	0.012500			0.0
40 40 10 012504	1 0 012500 1	0.012500					0.012300	0.012500	0.012500
		0.012500	0.012500	0.012500	0.012500	0.012500	0.012500	0.012500	0.012500
50-59 0.012500		0.012400	0.012350	0.012300	0.012250	0.012200	0.012150	0.012100	0.012050
60–69 0.012000		0.011600	0.011400	0.011200		0.010700	0.010400	0.010100	0.009800
70-79 0.009500		0.008700	0.008300	0.007900	0.007500	0.007000	0.006500	0.006000	0.005500
80-89 0.005000		0.004000	0.003500	0.003000	0.002500	0.002000	0.001500	0.001000	0.000500
90–99 0.000000		0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
100-109 0.000000 110-119 0.000000		0.000000 0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
110-119 0.00000	, i o.ooooo	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE B3 1971 INDIVIDUAL ANNUITY MORTALITY TABLE WITH PROJECTION SCALE B AT 6.00 PER CENT YEAR OF VALUATION 1975 LIFE ANNUITIES WITH 10 YEARS CERTAIN—ONE MALE AND ONE FEMALE

Male Age	FEMALE AGE													
	60	61	62	63	64	65	66	67	68	69	70	71		
60	10.493	10.406	10.313	10.212	10.104	9.987	9.864	9.733	9.595	9.453	9.306	9.15		
61	10.375	10.295	10.207	10.112	10.010	9.900	9.782	9.658	9.527	9.390	9.250	9.10		
62	10.252	10.177	10.096	10.007	9.911	9.807	9.696	9.578	9.454	9.324	9.189	9.05		
63	10.124	10.055	9.979	9.897	9.807	9.710	9.605	9.494	9.376	9.253	9.125	8.99		
64	9.991	9.927	9.857	9.781	9.697	9.607	9.509	9.405	9.294	9.177	9.056	8.93		
65	9.853	9.795	9.731	9.660	9.583	9.499	9.408	9.311	9.207	9.098	8.984	8.86		
66	9.711	9.658	9.600	9.535	9.465	9.387	9.303	9.213	9.116	9.014	8.907	8.79		
67	9.566	9.518	9.465	9.407	9.342	9.272	9.194	9.111	9.021	8.927	8.827	8.72		
68	9.419	9.376	9.328	9.275	9.217	9.153	9.082	9.006	8.924	8.836	8.744	8.64		
69	9.270	9.232	9.189	9.142	9.089	9.031	8.967	8.898	8.823	8.743	8.658	8.57		
70	9.122	9.088	9.050	9.007	8.960	8.908	8.851	8.788	8.720	8.647	8.570	8.48		
71	8.973	8.943	8.910	8.873	8.831	8.785	8.733	8.677	8.616	8.550	8.480	8.40		
72	8.827	8.801	8.772	8.739	8.702	8.661	8.616	8.566	8.511	8.453	8.390	8.32		
73	8.684	8.661	8.635	8.607	8.575	8.539	8.499	8.455	8.407	8.355	8.299	8.24		
74	8.544	8.524	8.502	8.478	8.450	8.419	8.384	8.346	8.304	8.258	8.209	8.15		
75	8.409	8.392	8.374	8.352	8.329	8.302	8.272	8.239	8.203	8.163	8.120	8.07		
76	8.279	8.265	8.250	8.232	8.212	8.189	8.164	8.135	8.104	8.070	8.033	7.99		
77	8.156	8.145	8.131	8.117	8.100	8.081	8.059	8.035	8.009	7.980	7.949	7.91		
78	8.041	8.031	8.020	8.008	7.994	7.978	7.960	7.941	7.918	7.894	7.868	7.83		
79	7.933	7.925	7.917	7.907	7.895	7.882	7.868	7.851	7.833	7.813	7.791	7.76		
80	7.835	7.829	7.822	7.814	7.805	7.794	7.782	7.769	7.754	7.738	7.721	7.70		

TABLE B3—Continued

Male Age	Female Age												
	72	73	74	75	76	77	78	79	80				
)	9.005	8.854	8.705	8.559	8.418	8.284	8.158	8.041	7.936				
l	8.960	8.815	8.670	8.529	8.392	8.261	8.139	8.025	7.923				
?	8.912	8.772	8.632	8.496	8.364	8.237	8.118	8.008	7.909				
B	8.860	8.726	8.592	8.461	8.333	8.211	8.096	7.990	7.893				
l	8.804	8.676	8.549	8.423	8.300	8.183	8.073	7.970	7.877				
5	8.745	8.624	8.502	8.382	8.265	8.153	8.047	7.949	7.859				
5	8.683	8.568	8.453	8.339	8.228	8.121	8.020	7.926	7.840				
'	8.618	8.510	8.401	8.294	8.189	8.087	7.991	7.901	7.819				
	8.549	8.449	8.347	8.246	8.147	8.051	7.960	7.876	7.798				
	8.478	8.385	8.290	8.196	8,104	8.014	7.928	7.848	7.773				
	8.405	8.319	8.232	8.145	8.058	7.975	7.895	7.820	7.75				
	8.330	8.252	8.172	8.091	8.012	7.934	7.860	7.791	7.72				
	8.255	8.183	8.110	8.037	7.964	7.893	7.825	7.761	7.702				
	8.178	8.114	8.048	7.982	7.916	7.851	7.789	7.730	7.676				
<u> </u>	8.102	8.045	7.986	7.926	7.867	7.809	7.752	7.699	7.649				
S <i></i>	8.026	7.976	7.924	7.871	7.818	7.766	7.715	7.667	7.623				
i	7.952	7.908	7.862	7.816	7.769	7.723	7.678	7,636	7.596				
	7.879	7.841	7.802	7.762	7.722	7.681	7.642	7.605	7.570				
	7.809	7.777	7.744	7.710	7.675	7.640	7.607	7.574	7.544				
	7.743	7.716	7.688	7.659	7.630	7.601	7.572	7.545	7.519				
	7.681	7.659	7.636	7.612	7.588	7.563	7.539	7.516	7.49				

THE PROGRAM

```
// FOR
*EXTENDED PRECISION
*LIST ALL
*ONE WORD INTEGERS
*IOCS(CARD,1403 PRINTER)
    DIMENSION A(11), B(7),QX(120),QY(120),SX(120),SAVE(40,12),IBAGE
   1 (12)
    READ(2,5) A, MBASE, LLOWR, LUPER
  5 FORMAT(11A4,1X,3I5)
    DO 21 I = 1,120
    QX(I) = 0
    OY(I) = 0
 21 SX(I) = 0
    LP = LLOWR + 1
    LPX = LUPER + 1
    DO 3 I = LP,LPX
  3 \text{ READ}(2.87)QX(I), QY(I)
 87 FORMAT(2F9.6)
    READ(2,95)B,NCARD
 95 FORMAT(7A4,2X,15)
    READ(2,7) IAGE, SXY
    NAGE = IAGE
  7 FORMAT(15,F8.4)
    LPA = IAGE + 1
    DO 9 I = LP,LPA
  9 SX(I) = SXY
     MCARD = NCARD - 1
    DO 10 I = 1, MCARD
    READ(2,7) IAGE, SX(IAGE + 1)
    DELTA = (SX(IAGE + 1) - SX(NAGE + 1))/(IAGE - NAGE)
    IL = NAGE + 1
    LM = IAGE + 1
    DO 11 I = IL.LM
    JXJXJ = (SX(NAGE+1) + (J-NAGE-1)*DELTA)*100000. + 0.5
 11 SX(J) = JXJXJ / 100000.
 10 \text{ NAGE} = IAGE
    LMP = LM
    DO 13 I = LMP,LPX
 13 \text{ SX}(I) = \text{SX}(LMP)
    WRITE(5,456) A,QX
456 FORMAT(1H1,25X, 'MALE MORTALITY RATES - ',11A4///1H, 3X,
   1 'AGES'//1H,' 0 - 9',10F10.6/1H,' 10 - 19',10F10.6/1H,' 20 - 29',10F10.6/
   1 1H, '30 - 39',10F10.6/1H, '40 - 49',10F10.6/1H, '50 - 59',10F10.6/1H, '60
   1 - 69',10F10.6/1H, '70 - 79',10F10.6/1H, '80 - 89',10F10.6/1H, '90 - 99',
   1 10F10.6/1H, '100 - 109', 10F10.6/1H, '110 - 119', 10F10.6)
    WRITE(5,457) A,OY
457 FORMAT(/// 26X, 'FEMALE MORTALITY RATES - ',11A4///1H ,3X,
```

```
1 'AGES'//1H,' 0 - 9',10F10.6/1H,'10 - 19',10F10.6/1H,'20 - 29',10F10.6/
   1 \text{ 1H}, 30 - 39', 10\text{F}10.6/1\text{H}, 40 - 49', 10\text{F}10.6/1\text{H}, 50 - 59', 10\text{F}10.6/1\text{H},
   1 '60 - 69',10F10.6/1H ,'70 - 79',10F10.6/1H ,'80 - 89',10F10.6/1H ,'90-
       99',10F10.6/1H, '100 - 109', 10F10.6/1H, '110 - 119',10F10.6)
    WRITE(5,458) B,SX
458 FORMAT(//,47X,7A4//1H ,3X,'AGES'//1H ,' 0 - 9',10F10.6/1H ,' 10 -
       19',10F10.6/1H, 20 - 29',10F10.6/1H, 30 - 39',10F10.6/1H, 40 - 49'
   1 10F10.6/1H, '50 - 59',10F10.6/1H, '60 - 69',10F10.6/1H, '70 - 79',10F10.6/
   1 1H, '80 - 89',10F10.6/1H, '90 - 99',10F10.6/1H, '100 - 109', 10F10.6/1H,
   1 '110 - 119',10F10.6)
 17 FORMAT(F5.4,6I5)
 16 READ(2,17) RATE,IXS, LAGE,MAGE,ICERT,ITYPE,IYEAR
    IF(RATE) 50,50,49
 49 \text{ IRGE} = \text{IYEAR} - \text{MBASE}
    IYS = IXS
    IF(IXS - 2) 401,401,402
402 IXS = 1
    IYS = 2
401 CONTINUE
    IMAGE = MAGE + 1
    INTX = LAGE
 45 \text{ INT} = \text{INTX} + 1
    INTX = INT + 39
    KNTX = LAGE
    IB = 0
    IF(INTX - IMAGE) 61,61,20
 20 \text{ INTX} = \text{IMAGE}
 61 \text{ DO } 128 \text{ INZ} = 1,40
    DO 128 \text{ INY} = 1.12
128 \text{ SAVE}(INZ,INY) = 0
    KNT = KNTX + 1
    KNTX = KNT + 11
    IF(KNTX-IMAGE) 226,226,32
 32 \text{ KNTX} = \text{IMAGE}
226 DO 25 IX = INT,INTX
    IKNT = KNT
    IF(IX-IKNT) 927,927,926
926 \text{ IKNT} = \text{IX}
927 IF(IX-KNTX) 26,26,25
 26 DO 25 IY = IKNT,KNTX
    SA = 0
    SB = 0
    W = 0
    PXY = 1
    V1 = 1
    PVXY = 0
    IXF = LPX - IX
    DO 80 \text{ IT} = 1.\text{IXF}
```

```
I = IX + IT - 1
          K = IY + IT - 1
          IF(K-LPX+1) 29,29,70
  29 IF(IXS-1) 30,132,30
  30 \text{ RX} = \text{OY(I)}
           GO TO 34
132 RX = QX(I)
  34 IF(IYS-1) 53,54,53
  54 \text{ RY} = OX(K)
           GO TO 55
  53 \text{ RY} = \text{QY(K)}
  55 GO TO (76,77,78),ITYPE
  76 PXY = (1-RY)*(1-RX)*PXY
           GO TO 79
  77 PXY = (1 - RY^*(1 - SX(K)))^*IRGE)^*(1 - RX^*(1 - SX(I)))^*IRGE)^*PXY
          GO TO 79
  78 PXY = (1 - RY*(1 - SX(K)))**(IRGE + IT - 1))*(1 - RX*(1 - SX(1)))**(IRGE + IT - 1))**(1 - RX*(1 - SX(1)))**(1        1 IT-1))*PXY
  79 V1 = (1/(1+RATE))*V1
          W = V1 + W
          PVXY = PXY*V1 + PVXY
  70 IF(ICERT-IT) 180,41,180
180 IF(IXF - ICERT) 181,80,80
181 \text{ IF}(IT - IXF) 80,41,80
  41 \text{ SA} = PVXY
          SB = (1. - (1./(1. + RATE))**ICERT)/RATE
  80 CONTINUE
          JXJXJ = (PVXY - SA + SB) * 1000 + 0.5
          IF(IB) 71,71,72
  71 KT = IX - INT + 1
          KP = IY - KNT + 1
          GO TO 73
  72 KT = IY - KNT + 1
          KP = IX - INT + 1
  73 SAVE(KT,KP) = JXJXJ / 1000.
  25 CONTINUE
          JXJXJ = (100.*RATE) * 100 + 0.5
          IF(IYS- IXS) 81,82,81
  81 IA = IYS
          IYS = IXS
          IXS = IA
          IA = KNT
          KNT = INT
          INT = IA
          IA = KNTX
          KNTX = INTX
          INTX = IA
          IF(IB) 1226,1226,82
```

```
1226 \text{ IB} = 1
    GO TO 226
 82 PRATE = JXJXJ / 100.
    GO TO (896,897,898), ITYPE
896 WRITE(5,296) A, PRATE
296 FORMAT(1H1,25X,11A4,'(UNPROJECTED) AT ',F5.2,' 0/0')
    GO TO 301
897 WRITE(5,297) A,B,PRATE,IRGE,IYEAR
297 FORMAT(1H1,15X,11A4,' WITH ',7A4,' AT',F5.2,' 0/0'/1H ,43X,'PROJEC
   1 TED FOR ',14,' YEARS TO ',16)
    GO TO 301
898 WRITE(5,298) A,B,PRATE,IYEAR
298 FORMAT(1H1,15X,11A4,' WITH ',7A4,' AT',F5.2,' 0/0'/1H ,48X,'YEAR OF
   1 VALUATION ',16)
301 \text{ IF}(IXS - IYS) 303,302,303
302 GO TO (305,306),IXS
303 WRITE(5,461) ICERT
461 FORMAT(1H, 28X, LIFE ANNUITIES WITH ',12,' YEARS CERTAIN
   1 ONE MALE AND ONE FEMALE'/// 1H, 'MALE',51X,'FEMALE'/1H,
   1 'AGE', 54X,'AGE')
    GO TO 307
305 WRITE(5,462) ICERT
462 FORMAT(1H ,35X,'LIFE ANNUITIES WITH ',12,' YEARS CERTAIN
        TWO MALES'///1H, 'AGE',54X,'AGE')
    GO TO 307
306 WRITE(5,463) ICERT
463 FORMAT(1H, 34X, 'LIFE ANNUITIES WITH ', 12, 'YEARS CERTAIN
   1 TWO FEMALES'///1H, 'AGE',54X,'AGE')
307 \text{ DO } 308 \text{ I} = 1.12
308 \text{ IBAGE}(I) = KNT + I - 2
    NTXK = KNTX - KNT + 1
    WRITE(5,309) (IBAGE(IH),IH=1,NTXK)
309 FORMAT(1H, I13,11I9)
    DO 310 I = INT,INTX
    IH = I - INT + 1
    ICAGE = I - 1
310 WRITE(5,311) ICAGE,(SAVE(IH,KH), KH = 1,NTXK)
311 FORMAT(1H ,15,12F9.3)
    IB = 0
    IF(KNTX- IMAGE) 61,40,40
 40 IF(INTX-IMAGE) 45,16,16
 50 CALL EXIT
    END
```

DISCUSSION OF PRECEDING PAPER

MONTE J. HOPPER:

I view the prospect of higher annuity valuation interest rates as exciting; the 1971 Individual Annuity Mortality (IAM) Table is a well-constructed table and one which should satisfy the needs of the industry in general.

Our annuitant mortality is lighter than the table. For example, our latest mortality experience on single premium immediate annuities is running at approximately 80 per cent of the *a*-1949 Table projected to 1970 using the Sternhell-Page modification. This would translate to approximately 90 per cent of the 1971 IAM Table.

As expected, our worst experience is on nonrefund annuities and our best is on instalment refunds, followed by ten-year certain and life, and cash refunds.

J. ALAN LAUER:

Column 5 of Table 13 of the paper shows the ratios of mortality rates on the 1971 IAM Table (unprojected) to the corresponding rates on the a-1949 Ultimate Table (unprojected). In discussing Table 13, Mr. Cherry notes that the ratios for males are about 80 per cent through age 50, dipping to about 75 per cent at ages 65-85 and rising thereafter. For females a fluctuating relationship is noted, although inspection of Table 13 does reveal high ratios at ages 60 and 90, with a dip in between. This dip is probably attributable to the fact that the 1971 IAM Table is based on experience of all contract years and that select mortality is more significant today than at the time of the construction of the a-1949Ultimate Table, which excluded experience of the first contract year. (Note Mr. Cherry's statement that "[i]t was the conclusion of the report on the 1963-67 immediate annuity experience that selection appears to persist . . . for at least three to five contract years" and the statement of Messrs. Jenkins and Lew in TSA, I, 375, that "the Joint Mortality Committee showed that initial selection under immediate nonrefund annuities affects mortality rates significantly for two or three years at most.") Of course, the bulk of the exposures in the select period occurs between attained ages 60 and 90.

In Section VII the discussion of Table 24 notes that "nonrefund settlement annuities . . . have lower-than-average mortality ratios, although not as low as under immediate annuities." Mortality ratios under im-

mediate annuities with a refund period are observed to be for males practically the same as under settlement annuities with a refund period and for females slightly lower than under settlement annuities with a refund period. While these observations are accurate with regard to Table 24, one must be careful in drawing conclusions from them. Reference to Table 1 indicates that mortality ratios for payee elections arising from maturities, surrenders, and death claims (which include only settlements with a guaranteed period or refund provision) are closer to the ratios for nonrefund annuities than to the ratios for refund annuities. especially when adjustment is made for differences in the periods of observation (i.e., ratios from the 1960-65 study of payee elections should be compared with the means of the ratios from the 1958-63 and 1963-67 studies of immediate annuities). Table 1 also indicates that mortality ratios under matured deferred annuities with a guaranteed period or refund provision are higher than the ratios under payee elections arising from maturities, surrenders, and death claims but are still generally lower (except for males in the most recent study) than the ratios under refund annuities. Again, the mortality ratios under matured deferred annuities without a guaranteed period or refund provision are generally lower, with the exception of the most recent study, than the ratios under nonrefund annuities. The relationships which Mr. Cherry observed in Table 24 appear to be influenced by (a) the difference in observation period for the 1963-67 study of immediate annuities and the 1960-65 study of settlement annuities and (b) the inclusion in the settlement annuity study of experience of nonpayee elections arising from death claims.

The author suggests in Section VII that a one-year setback in age would be reasonable when the 1971 IAM Table is used to value nonrefund immediate annuities. The setback in question is designed to reflect the lower mortality experienced under nonrefund immediate annuities and is completely apart from any setback which might be used to provide for future improvements in mortality. I do not agree that such a setback to reflect lower mortality on nonrefund immediate annuities would be appropriate under current conditions except for a company with a very high proportion of annuities (including settlement annuities) in force on the nonrefund basis. The construction of the 1971 IAM Table took into account experience under immediate annuities and settlement annuities, annuities with a refund provision and annuities without a refund provision, and annuities within the select period and annuities in the ultimate period. While the mix of business included in the mortality studies underlying the 1971 IAM Table may not be exactly that of any particular

company, it is probably representative of the mix of business for many companies. Thus the mortality rates under the 1971 IAM Table and the resulting annuity values are weighted to reflect a typical proportion of nonrefund immediate annuity business. There seems to be ample margin in the annuity values under the 1971 IAM Table without adding more margin by using an age setback for nonrefund immediate annuities (except, as mentioned previously, for a company with a significantly higher proportion of nonrefund business than that in the studies underlying the 1971 IAM Table).

While the paper illustrates annuity values based on various reserve interest rates, it does not go into the question of what interest rate would be an appropriate standard for annuity valuation, since the author's intention was only to describe the 1971 IAM Table and its construction and to propose the 1971 IAM Table as a mortality standard for annuity valuation. It seems appropriate, however, to discuss the considerations involved in selecting an interest rate to be used in combination with the new mortality table for annuity valuation.

"New money" interest rates are currently at very high levels, so that interest rates of 6 or 7 per cent are reasonable assumptions for annuity rates and values at this time. Interest rates may not always be at the current high levels, and this raises two concerns in selecting an interest rate as a standard for annuity valuation. The first concern, which relates to future issues of annuities, is that state legislatures should not be expected to change the standard with excessive frequency. Thus the interest standard should not be set so high as to make it likely that it would become inappropriate within an unduly short period.

The second concern relates to current issues of annuities and is that funds received today may have to be reinvested at some later date at much lower yields than are available currently. The reinvestment problem is much less for immediate annuities than for life insurance because the fund for an immediate annuity decreases with duration and because the potential duration of the annuity contract is shorter because of the higher ages at which annuities are usually purchased. Nevertheless, the reinvestment problem does exist. Theoretically, the valuation standard for immediate annuities should involve a high interest rate for perhaps the first 10–20 contract years and a conservative interest rate such as $3\frac{1}{2}$ per cent for later contract years. Such a split interest standard may or may not be practicable. If such a split interest standard is not adopted, some thought should be given to what would be done twenty years from now about valuation of annuities issued in 1972 if interest rates in twenty years were down to the $3-3\frac{1}{2}$ per cent level.

In Tables 15 and 22 of the paper the ratios of life annuity values on the 1971 IAM Table at 6 or 7 per cent interest to the corresponding life annuity values on the a-1949 Table at $3\frac{1}{2}$ per cent are greater than 1 at the higher attained ages. One who is not fully informed might feel that it would be proper to assume for valuation purposes that the high interest rates currently available can be earned for the lifetime of an annuity contract on the grounds that annuity values at the high attained ages are higher on the new table at a high interest rate than on the a-1949 Table at $3\frac{1}{2}$ per cent interest. This would be false reasoning. The higher annuity value on the new table at a high interest rate results from much lower assumed mortality rates. If these lower mortality rates are a reflection of actual experience (and, from the paper, this appears to be so), it is necessary to combine them with interest assumptions that are not unconservative.

The break-even interest rates shown in Table 15 are obviously of interest, but it is unlikely that the author intended to imply that these break-even interest rates should be the criteria used in setting a valuation standard. If the interest rate for the valuation standard were to be determined so that reserves on the new standard equaled (or bore some fixed relationship to) those on the old standard, there would be no point in having a new standard. The 1971 IAM Table has been developed as a possible mortality basis for annuity valuation because it reasonably represents, with appropriate margins, actual annuitant mortality and not because it produces reserves which bear any particular relationship to reserves on any other mortality table. The interest rate used for annuity valuation should also be based on the interest rates, with appropriate margins, which can be earned by the companies on the funds involved and should not be selected with an eye to producing any particular amount of reserves.

Messrs. Sternhell and Peacor have done a fine job in constructing the 1971 IAM Table, and Mr. Cherry has done equally well in preparing this paper. While there are many worthy aspects of the paper, I would particularly like to commend Mr. Cherry for Appendix A. Appendix A should be of great interest not only to actuaries directly concerned with immediate annuities but also to students (or at least to the Part 5 Examination Committee).

EDWARD A. LEW:

Mr. Cherry has done a commendable job in developing a new mortality table for the valuation of individual annuities and life income settlements. Even though the paper repeatedly speaks of the new table as having been

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designed for the computation of reserves, it is desirable to post a warning that the table should not be used for the calculation of premiums on individual annuities. Mortality tables for the calculation of premiums on individual annuities must make adequate provision at each age for select mortality and possible future decreases in mortality rates. In a discussion of the paper "The 1960 Modification of the a-1949 Table with Projection," I indicated that in the case of nonrefund annuities at ages over 70 a reasonable allowance for select mortality carries greater weight than provision for possible future decreases in mortality based on Projection B.

In developing mortality tables for valuation purposes it may be sufficient as a practical matter to make provision for mortality fluctuations and possible future decreases in mortality that are adequate in the aggregate.

In constructing the new valuation table, Mr. Cherry proceeded first to determine the expected mortality rates and then included a margin, presumably for adverse fluctuations in the future. This responsible approach recognizes the primacy of solvency for insurance companies and may be contrasted with the concept of reserves as expected values which has been proposed by the accounting fraternity. The computation of a margin for adverse fluctuations involves a great deal of judgment, but the actuary must lean to the side of caution.

I appreciate that, in developing a valuation table for individual annuities to be used by companies with different levels of annuitant mortality, there is much to be said for a relatively simple contingency loading, such as that produced by a 10 per cent reduction in mortality rates at all ages. However, a more desirable approach would have varied this loading by age, inasmuch as the likelihood of adverse fluctuations is greater at the younger than at the older ages, merely because at the younger ages there is a much longer time exposure to adverse fluctuations than at the older ages. Considerations of risk theory which take into account fluctuations in death rates and in the amounts of reserve released on death would also have resulted in larger percentage margins at the younger than at the older ages. It is noteworthy that in Finland the supervisory authorities have specifically required insurance companies to put up a special reserve on nonlife coverages to protect the company against fluctuations in claims and in the basic probabilities of the claims, calculated on the basis of risk-theory considerations (see ASTIN Bulletin, Vol. IV, Part III, "Magnitude Control of Technical Reserves in Finland," by Errki Pesonen).

A loading based on a 10 per cent reduction in mortality rates at all ages produces margins which increase with advance in age as a proportion of the annuity value (or of the life expectancy), as indicated in Table 12 of Mr. Cherry's paper and in Table 1 below.

Aside from variations in the level of annuitant mortality in different companies and aside from some provision for adverse fluctuations in death rates, we also need to take into account the likelihood that mortality rates in the years to come may be lowered primarily in the forties, fifties, and sixties and not much in the eighties or nineties. Even Alex Comfort,

TABLE 1

LOADINGS AS A PER CENT OF ANNUITY VALUES OR LIFE EXPECTANCIES PRODUCED BY A 10 PER CENT REDUCTION IN DEATH RATES

		VALUE AT 5 AL ANNUITY I	Margin					
Age	With 10 Per Cent Margin		Without 10 Per Cent Margin		Absolute Value		As Per Cent of Annuity Value without Margin	
	Male	Female	Male	Female	Male	Female	Male	Female
55	12.94	14.21	12.63	13.98	0.31	0.23	2.5%	1.6%
60	11.70	13.00	11.37	12.73	0.33	0.27	2.9	2.1
65	10.33	11.63	9.99	11.33	0.34	0.30	3.4	2.6
70	8.83	10.00	8.48	9.68	0.35	0.32	4.1	3.3
75	7.26	8.20	6.91	7.87	0.35	0.33	5.1	4.2
80.	5.71	6.37	5.37	6.04	0.34	0.33	6.3	5.5
85 90	$\frac{4.24}{2.87}$	4.68 3.34	3.94 2.62	4.36 3.05	0.30	0.32	7.6 9.5	7.3

Age		EXPECTANCY NNUITY MOR	Margin					
	With 10 Per Cent Margin		Without 10 Per Cent Margin		Absolute Value		As Per Cent of Life Expectancy without Margin	
	Male	Female	Male	Female	Male	Female	Male	Female
55 60	24.70 20.83	28.61 24.25	23.73 19.93	27.71 23.39	0.97 0.90	0.90 0.86	4.1%	3.2%
65	17.17	24.23	16.36	19.31	0.90	0.79	5.0	4.1
70	13.76	16.08	13.03	15.35	0.73	0.73	5.6	4.8
75	10.67	12.34	10.04	11.68	0.63	0.66	6.3	5.7
80	7.99	9.08	7.47	8.50	0.52	0.58	7.0	6.8
85	5.74	6.43	5.32	5.94	0.42	0.49	7.9	8.2
90	3.87	4.55	3.54	4.14	0.33	0.41	9.3	9.9

whose optimistic prognostications Mr. Cherry asks us to ponder, has more recently gone on record (*Playboy*, November, 1971) to the effect that "at the moment the odds seem fair that a man or woman of 50 can today expect some benefit [from gerontological research], provided we waste no time," but "the gain will be wholly in the productive and ... non-dependent years."

If the loading based on a 10 per cent reduction in mortality rates at all ages is intended to cover, *inter alia*, both possible future decreases in mortality and adverse mortality fluctuations, then the remarks of Alex Comfort just cited point to the advisability of higher contingency loadings at the younger ages and lower loadings at the advanced ages, and not to a percentage loading that increases with age, such as that produced by the assumption of a uniform 10 per cent reduction in death rates at all ages. A reduction in mortality rates decreasing uniformly from, say, 15 per cent at ages 50 and under to $2\frac{1}{2}$ per cent at ages 75 and older would have resulted in a scale of loadings more nearly consonant with the outlook for future decreases in mortality as well as in a more plausible provision for adverse mortality fluctuations.

HAROLD WIEBKE:

Mr. Cherry points out that the problem of surplus strain, as related to the sale of individual and group annuities based on "new money" interest rates, has become quite serious for many companies. The relief sought is to permit higher interest rates under minimum reserve requirements, at the same time recognizing that the resultant loss in interest margins suggests a more conservative mortality basis.

Curiously enough, and also most unfortunately, a shift in this direction in minimum reserve requirements creates or aggravates surplus strain problems under variable annuities, in the absence of explicit recognition of basic differences between reserve requirements for variable and for fixed annuities.

One basic difference between fixed and variable annuities that should be recognized in valuation standards is that the assumed interest rate (AIR) used in determining the initial income is essentially only a parameter affecting incidence of variable annuity payments. It does not reflect either liberality or conservatism in the basis for determining income, because the company does not stand to gain or lose if actual experience is different. The equivalence of the reserves for level payments at the AIR and the reserves for nonlevel payments at the valuation interest rate, where the payments are those that would result if the net investment return exactly equaled the valuation interest rate, is a manifestation of this

property. Thus a change in the permitted valuation interest rate has no effect on the level of the variable annuity reserve, while a change to a more conservative mortality table increases the reserve.

Another characteristic of variable annuities that must be recognized is the mortality risk charge, expressed as an annual percentage (such as 0.5 per cent) of separate account assets supporting the contracts. Investment performance reflected in the annuity payments is net of this charge. Thus the adequacy of the mortality assumption is a function not only of the mortality table but also of the mortality risk charge. Putting it another way, the company has "reserved" an interest margin, equal to that charge, for possible adverse mortality.

The impact of the mortality risk charge is to offset to a degree a need for conservatism in the mortality table. Consequently, if a mortality table with considerable margin for conservatism is used for valuation without recognition of the risk charge, a substantial redundancy results, and, to the extent that such redundancy contributes to surplus strain, the strain is totally undesirable and unnecessary. It seems to me that it is important for valuation requirements to take into account the mortality risk charge in defining minimum standards.

(AUTHOR'S REVIEW OF DISCUSSION)

HAROLD CHERRY:

I wish to thank Messrs. Hopper, Lauer, Lew, and Wiebke for their valuable additions to this paper and for the expression of differing points of view on some questions. I will comment on the discussions in alphabetical order.

Mr. Hopper notes that the over-all mortality ratio for his company, the Connecticut General, is about 90 per cent on the 1971 IAM Table for the latest experience (1963–67) on single premium immediate annuities. Mr. Hopper was kind enough to furnish me with the details of his company's experience on immediate annuities and also of its 1965–70 experience on settlement annuities.

The mortality ratio of 90 per cent cited by Mr. Hopper is based on amount of annual income. By number of contracts, the mortality ratio is about 105 per cent on the 1971 IAM Table. The differential of 15 percentage points between experience by number and experience by amount for Mr. Hopper's company is rather high compared with the corresponding differential of about 5 percentage points for all companies combined in the 1963–67 study of immediate annuities. It is difficult to say whether these results are due to chance fluctuation or basically reflect very low mortality

for higher-amount annuities in Mr. Hopper's company. In this regard, the Connecticut General experience consists of about 10,000 contract-years of exposure and less than 600 deaths in the period from 1963 to 1967, which represents less than 2 per cent of the total intercompany experience for the same period.

Since the 1971 IAM Table is being proposed as a valuation standard for all individual annuities, including settlement annuities, it would be of interest to examine the Connecticut General's mortality ratio under settlement annuities for the 1965–70 period. Their mortality ratio is about 120 per cent by amount for this experience based on the 1971 IAM Table. When their settlement annuity experience is combined with their experience on immediate annuities, the over-all mortality ratio is about 110 per cent. Thus, when the combined annuity experience is considered in this case, the 1971 IAM Table is a satisfactory valuation standard. By number of contracts, the mortality ratio for the settlement annuities is about 135 per cent and the mortality ratio for the combined experience is about 120 per cent on the 1971 IAM Table. This again illustrates the higher-than-average differential between mortality experience by number and by amount for Mr. Hopper's company.

The margins built into the 1971 IAM Table, as noted in the paper, were designed to cover the mortality experience of a substantial majority of companies with annuity business. Thus it is to be expected that the mortality ratios for a particular company, or for a segment of that company's business, will occasionally be less than 100 per cent on the new table, especially if the volume of exposures is small. By the same token, there are some companies whose mortality ratios will be considerably greater than 100 per cent under the new table. This would be true under any reasonable valuation standard. It is a difficult matter to decide on a level of margins to be included in a valuation table that will satisfy the requirement of conservatism but at the same time will meet the requirement that reserves are not unduly high for most companies. I feel that the margins included in the 1971 IAM Table strike a proper balance between these two requirements.

Mr. Lauer comments on the ratios of the mortality rates under the 1971 IAM Table (unprojected) to the corresponding mortality rates under the *a*-1949 Ultimate Table (unprojected) shown in column 5 of Table 13. He notes that there is a dip in these ratios between ages 60 and 90, particularly in the cases of females. Mr. Lauer feels that this dip is probably attributable to the fact that the 1971 IAM Table is an aggregate table, while the *a*-1949 Ultimate Table excludes the experience of the first contract year. He points out that the effect of selection is more pronounced

today than it was when the a-1949 Table was constructed, so that we might expect the kind of results shown in Table 13.

Actually, the relationship between the two female mortality tables noted by Mr. Lauer does not appear to be attributable to the fact that an aggregate table is being compared with an ultimate table. I tested this by constructing a special ultimate 1971 table based on the experience of contract years 6 and over, with the same techniques used in constructing the 1971 IAM Table itself. I then compared these ultimate 1971 mortality rates with the rates on the *a*-1949 Ultimate Table. The results are shown in Table 1 below, along with the comparison of the aggregate 1971 table (i.e., the 1971 IAM Table) with the *a*-1949 Ultimate Table.

This table indicates that the dip noted by Mr. Lauer is even more pronounced after the effects of selection are substantially removed from

TABLE 1

RATIO OF MORTALITY RATE UNDER INDICATED FEMALE TABLE
TO CORRESPONDING RATE UNDER FEMALE a-1949

ULTIMATE TABLE (UNPROJECTED)

Age	1971 IAM Table (from Col. 5 of Table 13)	Special Ultimate 1971 Table	Age	1971 IAM Table (from Col. 5 of Table 13)	Special Ultimate 1971 Table
60	88.3% 74.9 66.9 70.1	101.7% 81.8 71.1 72.0	80	75.5% 81.9 86.6 76.3	78.5% 84.0 88.1 77.6

the 1971 IAM Table. Thus the dip cannot be attributed to the effects of selection, as suggested by Mr. Lauer. These results indicate that there has been a basic change in the shape of the mortality curve, reflecting the fact that mortality improvements in the last twenty years have not been uniform by age. This change in the shape of the mortality curve was noted in Section II of the paper under the caption "Need for a New Mortality Table," and because of this change it was concluded that "it would be best to construct an entirely new mortality table rather than to attempt a simple adjustment of the a-1949 Table."

Another factor that should be considered in comparing the a-1949 Ultimate Table and the 1971 IAM Table is that the latter table is based on the experience under all types of annuities—immediate annuities and settlement annuities, refund annuities and nonrefund annuities—while the a-1949 Table is based on the experience under nonrefund immediate

annuities only. However, the dip in question persists in varying degrees even when a 1971 ultimate table is constructed separately for each type of annuity.

Mr. Lauer correctly calls attention to the fact that one must be careful in drawing conclusions from Table 24 of Section VII of the paper. The mortality ratios for settlement annuities shown in Table 24 are based on the combined data for payee and nonpayee elections and for matured deferred annuities, and the observations made at that point in the paper are valid only for the combined data. Mr. Lauer refers to Table 1 of the paper to emphasize the importance of the source of proceeds as a factor in determining mortality under settlement annuities.

Mr. Lauer says that I suggest in Section VII that a one-year setback in age would be reasonable when the 1971 IAM Table is used to value nonrefund immediate annuities. He further states that he does not agree that a setback would be appropriate except for a company with a very high proportion of nonrefund immediate annuities in force, since the 1971 IAM Table is based on a mix of business which is probably representative of the mix for many companies. Actually, I did not intend to suggest that adjustments in the 1971 IAM Table for valuing different types of annuities should be the general rule. I tried to make this clear in the opening paragraph of Section VII, which reads as follows: "The 1971 IAM Table is based on the combined intercompany experience under immediate annuities and settlement annuities and hence is generally suitable as a valuation standard for the mix of a company's annuity business, without further adjustment. However, in specific situations an actuary may judge that adjustments should be made in valuation assumptions for certain types of annuities."

Mr. Lauer gives an example of the kind of "specific situation" that I had in mind—namely, a company with a very high proportion of non-refund immediate annuities in force. It is only under such exceptional circumstances that I suggest in Section VII that the actuary may want to consider adjustments in the 1971 IAM Table. Thus there does not appear to be any basic difference of opinion between Mr. Lauer and myself on this point.

Mr. Lauer discusses two concerns in selecting an interest rate to be used with the new table. His first concern is that the interest standard should not be set so high as to make it likely that it would become inappropriate within an unduly short period of time. As mentioned at the close of this reply to the discussions, the American Life Convention-Life Insurance Association of America is proposing a 6 per cent maximum valuation

interest rate for use with the new table. I think that this rate satisfies Mr. Lauer's first concern.

Mr. Lauer's second concern is with reinvestment. Mr. Lauer points out that this problem is much less for immediate annuities than for life insurance. However, he feels that the problem still exists and that, theoretically, the valuation standard for immediate annuities should involve a high interest rate for perhaps the first 10-20 contract years and a conservative rate such as $3\frac{1}{2}$ per cent thereafter.

Our studies indicate that, for a typical cross-section of issues of immediate annuities, the sum of investment turnover and investment income just about offsets the benefit payments and expenses under the contracts in each year. The precise results depend upon the nature of the investments (i.e., the rate of return and rate of turnover of principal), the distribution of issues by age and plan, the actual mortality and rate of expense, and so on, but the important point is that the reinvestment problem is virtually nonexistent for immediate annuities. Thus, even if the "new money" rate on 1972 issues declines from, say, 7 per cent to $3-3\frac{1}{2}$ per cent over a period of years, the rate of return on the funds backing up these issues will remain very close to the original 1972 rate of 7 per cent.

One point that should be clarified is that, if the assets backing the immediate annuities actually produce negative funds for reinvestment, then, under the investment-year method of allocating investment income, a decline in "new money" interest rates from the original rate at issue would be advantageous to the immediate annuities in question-Ironically, under the investment-year method, an increase in "new money" interest rates would lead to a reinvestment problem in such a case.

To sum up, I feel that the reinvestment problem is of such small consequence for immediate annuities that it has very little bearing on the question of an appropriate maximum valuation interest rate for current issues. Thus I do not agree with Mr. Lauer that a split interest rate for valuation purposes is justifiable from the theoretical point of view.

Mr. Lauer is correct in his assumption that I did not intend to imply that the break-even interest rates shown in Table 15 of the paper should be the criteria used in setting a valuation standard. These break-even interest rates, together with the other data in Table 15, were intended merely as a convenient aid to the reader for gauging the effect of changing from the current minimum valuation basis to the proposed mortality standard at various interest rates.

Mr. Lew feels that, although the paper repeatedly speaks of the new table as having been designed for valuation purposes, it is desirable to post a warning that the table should not be used for the calculation of premiums on individual annuities. I agree with Mr. Lew that this important point, although it was made in the paper, merits repetition.

Mr. Lew then discusses the question of margins. He feels that a more desirable approach than a constant 10 per cent margin at all ages would be to vary the margins by age, with larger percentage margins at the younger ages than at the older ages.

The 10 per cent reduction in mortality rates at ages 60 and over applied to the experience mortality in constructing the 1971 IAM Table was primarily to provide for variations in mortality among different companies. This margin is also available to provide for fluctuations in mortality in a given company. The 10 per cent margin was not intended to cover possible future decreases in mortality, since our objective was to construct a valuation table based on current (1971) levels of mortality. Any provision for future decreases in mortality is to be made separately by means of an appropriate projection scale. The over-all 10 per cent margin at ages 60 and over for variation in mortality by company was chosen because this margin appeared to be reasonable on the basis of the variation in mortality experienced by companies contributing to the intercompany immediate annuity and settlement annuity studies. These variations are shown in the 1966 and 1969 Reports for all ages combined. but we were able to obtain similar data by age group for the 1963-67 immediate annuity study. (Unfortunately, data by age group could not be obtained for the 1960-65 settlement annuity study.)

Table 2 below shows the variation in mortality ratios among the twenty-two companies contributing to the 1963-67 intercompany study of immediate annuities, separately for age groups 60-69, 70-79, and 80 and over and for ages 60 and over combined. The results are shown for refund and nonrefund annuities and all contract years combined, which is consistent with the experience underlying the 1971 IAM Table. A summary of the companies with mortality ratios which were more than 10 percentage points below the all-company mortality ratios has been prepared from Table 2 below and is shown in the tabulation on page 564.

It can be seen that, for each age group, about the same number of companies have mortality ratios which are more than 10 percentage points below the all-company ratio. Also, an inspection of the proportion of deaths represented by these companies does not indicate any clear-cut trend by age group. Accordingly, I feel that a flat 10 per cent margin at ages 60 and over to provide for variations in mortality among different companies is quite satisfactory.

It might be mentioned that, in one sense, Mr. Lew's suggestion that the

Age Group	Ale-Company Mortality	COMPANIES WITH MORTALITY RATIOS MORE THAN 10 PER- CENTAGE POINTS BELOW ALL- COMPANY MORTALITY RATIO			
	Ratio	Number of Companies	Proportion of Deaths		
	Males				
60-69	93% 88 90	6 8 6	13.1% 23.6 15.1		
Ages 60 and over	90%	6	14.4%		
	Females				
60-69	95% 81 92	8 6 6	18.1% 9.7 10.1		
Ages 60 and over	89%	6	11.0%		

percentage reductions in the experience mortality rates should decrease by age has been carried out in constructing the 1971 IAM Table. The total percentage reduction in the experience mortality rates is based on the combined effect of (1) the 10 per cent margin to provide for variation in mortality among companies and (2) the reduction to provide for assumed decreases in mortality between 1963 and 1971. The latter reduction was taken as 1.6 per cent per annum for ages 60-79, decreasing to zero at ages 95 and over. The combined effect of these two reductions is shown in column 3 of Table 8 of the paper. It can be seen that the total percentage reduction in the experience mortality rates to obtain the 1971 IAM Table is 20.9 per cent at ages 60-79, decreasing to 10 per cent at ages 90 and over. Furthermore, the greatest percentage reductions in the experience mortality rates implicit in the construction of the 1971 IAM Table occur at ages 50-59. As noted in the paper, the mortality rates at these ages were not based on the relatively small intercompany experience but rather were obtained by a mathematical formula bridging the rates at ages 50 and under with those at ages 60 and over. The resulting 1971 IAM mortality rates at ages 50-59 represent a greater percentage reduction in the experience mortality than for the older ages. (The level of the inter-

TABLE 2.—Variation in Mortality Ratios in 1963-67 Intercompany Study of Immediate Annuities by Amount of Annual Income

BASED ON a-1949 ULTIMATE TABLE

DASED ON 4-1949 OLITMATE TABLE

REFUND AND NONREFUND ANNUITIES COMBINED

	Ages	60–69	Ages	70-79	Ages 80 A	AND OVER	Ages 60	and Over
MORTALITY RATIOS	Number of Companies	Proportion of Actual Deaths	Number of Companies	Proportion of Actual Deaths	Number of Companies	Proportion of Actual Deaths	Number of Companies	Proportion of Actual Deaths
				Ma	ales			
Percentage points below average: > 20	6	13.1% 8.3 11.1 %)* 23.8 9.4	3 5 2 3 (889)	6.4% 17.2 15.0 5.5 %)*	3 3 2 4 (900)	3.5% 11.6 15.9 26.9 %)*	2 4 2 3 (909	3.4% 11.0 9.1 31.7 %)*
11-20	3 5	18.7 15.6	1 4	4.3 13.8	3 4	17.3 11.2	3 4	12.8 11.3
				Fen	nales			
Percentage points below average: > 20. 11-20. 6-10. 1-5.	2 3	16.4% 1.7 21.0	4 2 3 (819)	7.4% 2.3 22.0	2 4 1 4 (929	1.3% 8.8 8.7 32.8	2 4 4 (899	2.5% 8.5 37.7
Percentage points above average: 0-5	1	9.1 31.0 20.8	6 3 2 2	38.4 13.7 6.0 10.2	5 1 4 1	23.2 6.5 14.0 4.7	3 4 5	13.5 18.1 19.7

^{*} Figures in parentheses indicate all-company mortality ratios.

company mortality at ages 50–59 was moderately elevated in comparison with the level at neighboring ages. This characteristic has been noted in the past for other annuity experiences, including the experience underlying the a-1949 Table.) Table 3 below shows the actual deaths, expected deaths, and mortality ratios based on the 1971 IAM Table in 10-year age groups from 50 to 99. The data for ages 60–99 are taken from Table 10 of the paper. Also shown is the average percentage reduction in experience mortality for each age group. This was calculated as 1 minus the reciprocal of the mortality ratio.

It can be seen that the total percentage reductions in mortality rates in

TABLE 3

RATIOS OF ACTUAL TO EXPECTED DEATHS BASED ON 1971 IAM TABLE 1963-67 IMMEDIATE ANNUITY EXPERIENCE COMBINED WITH 1960-65 SETTLEMENT ANNUITY EXPERIENCE ALL CONTRACT YEARS COMBINED—BY AMOUNT OF ANNUAL INCOME

Age Group	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths	Average Percentage Reduction in Experience Mortality Represented by 1971 IAM Table = 1-1/ (Col. 3) (4)
		Males	I	
50–59	\$ 403,606 5,362,896 11,010,569 6,424,177 1,113,465	\$ 267,513 4,197,650 8,691,934 5,233,603 991,120	150.9% 127.8 126.7 122.8 112.3	34% 22 21 19
50–99	\$24,314,713	\$19,381,820	125.5%	20%
		Females		
50–59. 60–69. 70–79. 80–89. 90–99.	\$ 591,644 3,659,373 9,876,914 11,103,240 2,881,485 \$28,112,656	\$ 353,487 2,864,160 7,793,083 9,204,917 2,519,496 \$22,735,143	167.4% 127.8 126.7 120.6 114.4	40% 22 21 17 13

column 4 of Table 3 decrease over the age range from 50 to 99, although the percentages are fairly level for a wide span of ages in the middle of this range. Thus the total percentage reduction in experience mortality implicit in the construction of the 1971 IAM Table follows a pattern by age group which is closer to that suggested by Mr. Lew than might appear at first glance.

Mr. Wiebke notes that a change in the maximum valuation interest rate has no effect on the level of reserves for variable annuities, while a change to a more conservative mortality table increases the reserves. He points out that, under variable annuities, a mortality risk charge based on a percentage of the separate account assets is collected and accumulated to provide for possible adverse mortality experience. He feels that a substantial redundancy results if a mortality table with a considerable margin for conservatism is used for valuing variable annuities, in light of the fact that the mortality risk charge is also being accumulated. He therefore suggests that the risk charge be taken into account when setting minimum reserve standards for variable annuities, implying that the 1971 IAM Table, while it may be appropriate for fixed-dollar annuities, is not appropriate for variable annuities.

I feel that the 1971 IAM Table is an appropriate minimum valuation standard for variable as well as for fixed-dollar annuities, for the following reasons:

- 1. The conservatism built into the 1971 IAM Table is at a level appropriate for a valuation table, that is, sufficient to cover the experience of most companies in most years but by no means all companies in all years. Thus I do not feel that reserves are substantially redundant under the proposed table, even under variable annuities where a mortality risk charge is accumulated.
- 2. For fixed-dollar annuities there is an interest margin of 1 per cent or more in reserves, based on current "new money" rates, even if the maximum valuation interest rate is raised to 6 per cent. Furthermore, if "new-money" rates drop, a company would normally maintain some interest margin on fixed-dollar annuities by setting its valuation interest rate somewhat below the rate implicit in its pricing structure. Under variable annuities there is no such interest margin in reserves, which is a reflection of the fact noted by Mr. Wiebke that the company does not stand to gain or lose if the actual investment return is different from the AIR. Hence the additional amount accumulated in the mortality fluctuation fund does not result in a degree of conservatism for variable annuities which is greater than that inherent in the reserves for fixed-dollar annuities.
 - 3. For fixed-dollar annuities general surplus is available to be used if

necessary to provide for adverse mortality experience. For variable annuities in the separate account, there are restrictions on the use of general surplus for this purpose. Hence other funds (e.g., the mortality fluctuation fund) should be accumulated in addition to the regular reserves to serve the same purpose as surplus in the fixed-dollar account.

4. The variable nature of the benefit payments under variable annuities means that the potential mortality loss is greater than that under fixed-dollar annuities. This is another reason for accumulating a mortality fluctuation fund under variable annuities in addition to the regular reserves.

At this point I would like to summarize the status, as of this writing (January, 1972), of the proposal to change the minimum valuation standards for annuities. At the annual meeting of the National Association of Insurance Commissioners on November 30, 1971, Mr. John M. Bragg, chairman of the Joint Actuarial Committee of the ALC-LIAA, presented a recommendation from these associations to raise the maximum statutory interest rate to 6 per cent for all group annuities and individual single premium immediate annuities. At the same time, the 1971 Group Annuity Mortality Table and the 1971 Individual Annuity Mortality Table would become the statutory minimum mortality standards for the applicable class of business. (The recommendation also contains a proposal to raise the general maximum valuation interest rate from $3\frac{1}{2}$ to $4\frac{1}{4}$ per cent.) These proposals were discussed by the (C3) Life Insurance Subcommittee of the NAIC, which decided that the ALC-LIAA should furnish a copy of its recommendation and proposed tables to each state. Each jurisdiction is to study the proposals and submit written recommendations to the (C3) subcommittee not later than May 1, 1972. An NAIC technical task force will be appointed to analyze the reports from the states.

The ALC-LIAA intends to furnish the insurance department of each state with the necessary material in February, 1972. The two associations hope that by working co-operatively with the (C3) subcommittee and the technical task force, it may be possible for the NAIC to take affirmative action on these proposals at its June, 1972, meeting.