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THE 1971 GROUP ANNUITY MORTALITY TABLE

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ABSTRACT

The Joint Actuarial Committee of the ALC-LIAA has been considering the steps necessary to obtain relief from the annual statement surplus strain caused by new group and individual annuity business. The strain results from the low purchase rates available for new business, a result of recent and apparently continuing high rates of interest relative to the maximum rate of interest permitted for determining minimum annual statement reserves.

The committee decided that, in addition to considering possible interest rate changes for valuation purposes, there should be an investigation of recent group and individual annuitant mortality. This paper reports the results of the investigation of recent group annuitant mortality.

As a result of the investigation, new group annuity mortality tables have been prepared for both males and females. Also, two new projection scales, Projection Scales D and E, have been prepared for males, and one new projection scale, Projection Scale D, has been prepared for females.

The paper concludes that, if a new mortality table is adopted for valuation purposes, a substantial increase in the maximum rate of interest would be needed to obtain any appreciable relief. It also concludes that, if the male table is used for female lives, the appropriate uniform age setback is six years.

INTRODUCTION

CURRENT high interest rates have enabled insurance companies to establish very attractive purchase rates for single-sum group and individual annuity contracts. It appears that relatively high rates of interest may be available for some time to come. The new annuity business that companies have been able to attract because of the favorable purchase rates has been welcome but has caused some problems. Perhaps the greatest single difficulty faced by all companies is the surplus strain which results from the establishment of minimum legal reserves. In 1971 the amount of surplus strain can be about 40 per cent of the considerations received on some classes of business. Naturally, companies

would like to know whether the minimum valuation standard can be relaxed.

Although the obvious way to obtain immediate relief is to raise the maximum statutory valuation interest rate, the Joint Actuarial Committee of the American Life Convention and the Life Insurance Association of America concluded that a reinvestigation of annuitant mortality experience would be an appropriate part of any study of minimum valuation standards. Accordingly, the Subcommittee on Statutory Interest Rates—Annuities was further subdivided into a group annuity section and an individual annuity section. This paper reports the results of the group annuity mortality investigation.

The paper introduces a new group annuity mortality table for 1971, together with new mortality improvement projection scales. The new table is specifically intended to be used for valuation purposes. It is a table based on the estimated mortality rates experienced by persons at all ages in calendar year 1971—that is, a static mortality table—and it includes some margin.

GROUP ANNUITY TABLE FOR 1951

The publication of the Group Annuity Table for 1951,[1] G*a*-1951, introduced the device of mortality projection scales to the pension world. Although the fact that annuitant mortality was steadily improving had long been recognized by actuaries, Mr. Peterson was the first to recommend an explicit means of recognizing this improvement on a continuing basis for group annuity business.

In the years since its publication, the G*a*-1951 Table has become the valuation standard for most companies in the group annuity business and is so designated for new business in the Standard Valuation Law. For valuation purposes some companies are using the G*a*-1951 Table unprojected; other companies are using the G*a*-1951 Table projected to the year of valuation and fully projected thereafter by Projection Scale C—that is, a generation mortality system. A different method of approximating a generation mortality system is also in use. Age ratings for each year or group of years of birth are applied to a mortality table deemed appropriate for a single year of birth. This practice avoids the need for calculating new tables each year.

NEED FOR A NEW TABLE

The group annuity mortality experience published in the annual *Reports* number of the *Transactions of the Society of Actuaries* is sub-

[1] *TSA*, IV, 246.

divided into three major classes of experience: (1) retirement on or after normal retirement date, (2) retirement prior to normal retirement date, and (3) retirement under a plan having no stated retirement date. The investigation of a need for a new mortality table was based primarily on the "retirement on or after normal retirement date" experience, since this group is both the largest and the most homogeneous of the three. Table 1 shows the ratio of actual to expected mortality based on the G*a*-1951 Table, without projection, over four time intervals.

TABLE 1

GROUP ANNUITY MORTALITY RATIOS BY AMOUNT OF ANNUITY INCOME
RETIREMENT ON OR AFTER NORMAL RETIREMENT DATE
COMPARISON WITH G*a*-1951 WITHOUT PROJECTION*

Attained Age	1951–55	1956–60	1961–65	1964–68
	Males			
60 and under	158.4%	125.0%	115.8%	127.4%
61–65	115.9	109.3	100.3	102.7
66–70	108.0	101.1	100.9	99.0
71–75	108.0	103.6	100.7	100.0
76–80	140.2	101.2	100.2	96.1
81–85	99.0	101.4	101.2	97.6
86–90	108.4	104.9	97.4	97.2
91–95	157.2	108.2	89.6	99.7
96 and over	78.3	62.8	78.4	74.0
All ages	107.8%	102.7%	100.5%	98.7%
All ages adjusted†	107.2	102.5	100.5	98.7
	Females			
60 and under	84.1%	80.8%	102.2%	115.1%
61–65	92.1	84.7	81.5	68.7
66–70	97.8	86.4	84.8	76.1
71–75	94.0	87.8	87.7	83.9
76–80	122.0	119.8	99.4	96.2
81–85	138.6	113.7	113.4	102.1
86–90	112.4	123.3	113.0	116.9
91–95	‡	121.3	143.6	140.4
96 and over	‡	‡	‡	91.0
All ages	102.6%	94.1%	91.2%	85.7%
All ages adjusted†	104.1	96.4	92.0	85.7

* Male table set back five years for females.
† Based on 1968 distribution of annuity income by age.
‡ Less than ten deaths—actual or expected.

Since publication of the G*a*-1951 Table, pension mortality rates have continued their steady decline. For both males and females there have been substantial decreases, 9 per cent and 17 per cent, respectively, in the aggregate ratios over the period 1951–55 to 1964–68 after allowance is made for the effect of underreporting of exposures and deaths in 1968. The mortality margin contained in the G*a*-1951 has virtually disappeared at the significant ages. It should be noted that the substantial drop in mortality ratios between the 1951–55 and 1956–60 periods was caused in part by the establishment of the separate category "retirement under a

TABLE 2

DISTRIBUTION OF AMOUNT OF ANNUITY INCOME
RETIREMENT ON OR AFTER NORMAL RETIREMENT DATE

Age Group	1953	1958	1963	1968
	Males			
70 and under...	67.1%	66.4%	59.9%	52.4%
Over 70.....	32.9	33.6	40.1	47.6
	Females			
70 and under....	76.1%	74.3%	69.9%	64.8%
Over 70.....	23.9	25.7	30.1	35.2

plan having no stated retirement date" after 1955; previously these data had been included with the "retirement on or after normal retirement date" data.

It is interesting to note, too, that the distribution of amounts of annuity income in force has been shifting gradually to higher ages. Table 2 shows the shift between ages 70 and under and ages over 70. This gradual shift has contributed to the decrease in aggregate mortality ratios shown for males in Table 1.

Table 3 compares with Projection Scale C the rates of mortality improvement from the period 1956–60 to 1963–1967. Projection Scale C overstates the male rate of mortality improvement substantially at the younger ages and understates it at higher ages. For females the pattern is not so clear, but Scale C does understate the rate of mortality improvement for the high age groups.

Table 4 shows ratios of actual to expected mortality for 1967 and 1968

TABLE 3

RATES OF MORTALITY IMPROVEMENT BASED ON AMOUNT OF ANNUAL INCOME RETIREMENT ON OR AFTER NORMAL RETIREMENT DATE

Age	Mortality Ratio 1956–60 (1)	Mortality Ratio 1963–67 (2)	(2)÷(1) (3)	Rate of Improvement $100\% \times [1-(3)^{1/7}]$ (4)	Projection Scale C (5)
	Males				
60 and under	125.0%	127.0%	1.0160	−0.23%	1.25%
61–65	109.3	104.7	0.9579	+0.61	1.25
66–70	101.1	99.8	0.9871	+0.18	1.25
71–75	103.6	101.3	0.9778	+0.32	1.10
76–80	101.2	98.9	0.9773	+0.33	0.80
81–85	101.4	98.2	0.9684	+0.46	0.47
86–90	104.9	98.4	0.9380	+0.91	0.13
	Females				
60 and under	80.8%	119.1%	1.4740	−5.70%	1.25%
61–65	84.7	73.2	0.8642	+2.06	1.25
66–70	86.4	80.3	0.9294	+1.04	1.25
71–75	87.8	85.4	0.9727	+0.39	1.10
76–80	119.8	97.9	0.8172	+2.84	0.80
81–85	113.7	107.5	0.9455	+0.80	0.47
86–90	123.3	117.9	0.9562	+0.64	0.13

TABLE 4

RATIO OF ACTUAL TO EXPECTED MORTALITY BASED ON *Ga*-1951 PROJECTED BY PROJECTION SCALE C

(Percentages Based on Amount of Annual Income)

Age	1967	1968	Age	1967	1968
	Males			Females	
56–60	147.5%	154.5%	56–60	170.0%	133.8%
61–65	143.1	118.6	61–65	138.1	86.2
66–70	118.2	118.7	66–70	112.1	127.0
71–75	120.2	116.4	71–75	129.5	110.0
76–80	110.4	102.5	76–80	139.6	113.5
81–85	102.3	105.3	81–85	101.9	115.2
86–90	108.4	91.6	86 and over	115.0	158.1
91–95	94.0	89.0			
96 and over	58.5	59.7			

(the most recent experience available), where the actual mortality is "on or after" experience and the expected mortality is based on the G*a*-1951 Table projected to the appropriate year by Scale C. The results indicate that a possible alternative to constructing a new mortality table would have been to construct a new projection scale only. However, the decision was that a new table and a new projection scale together could better reflect both the mortality changes which have occurred since the introduction of the G*a*-1951 Table and the current mortality trends.

CONSTRUCTION OF THE 1971 TABLE (1971 GROUP ANNUITY MORTALITY)

Data

The intercompany experience by amount of annual income for the most recent five-calendar-year group available, 1964–68, was selected as the source of retired life data. Since the "retirement on or after normal retirement date" data not only were the most extensive and the most homogeneous but also exhibited the lowest mortality rates, these data were deemed suitable for developing a valuation mortality table.

As various types of deposit administration contracts have come to dominate the funding of retirement benefits for active lives, deferred group annuity business has steadily declined. Thus the most logical source of active life data upon which to base a group annuity valuation mortality table is drying up. At first, group insurance mortality data seemed to be a likely source of data; however, the data can be split by sex only on an estimated basis. As an alternative, data were obtained on four large deferred group annuity contracts and on one large municipal employee group, excluding persons engaged in hazardous occupations. The data were available by number of lives, and a large portion only by five-year attained age groups. The average exposure year for the data turned out to be 1967. Table 5 summarizes the active life data used in developing the 1971 Group Annuity Mortality (GAM) Table.

Projection Scales D and E

Work on Projection Scale D was performed while the mortality data were being collected and analyzed. Projection Scale E for males was developed later. Since Projection Scale D was used in graduating the data, a discussion of the scales is appropriate at this point.

Projection Scale D was developed after examination of the changes in "on and after" retired life mortality between the periods 1956–60 and 1964–68. Although the work presented in this paper is based on Scale D, the data for males suggest that an even flatter scale could be used to estimate future mortality improvement. Projection Scale E for males is

one such scale. Suitable age ratings of the unprojected 1971 GAM Table might produce results closely approximating results of the 1971 GAM Table projected by Scale E. The authors have not investigated this possibility.

Table 6 presents the two projection scales. As is evident, the annual rate of mortality improvement does not present a uniform pattern by increasing age. The value for males at the young ages was chosen after examining the rates of improvement shown in Tables 3 and 6 at ages

TABLE 5

ACTIVE LIFE DATA BASED ON LIVES

Age	Exposure	Deaths	Age	Exposure	Deaths
	Males			Females	
Under 25..	107,097.5	125	Under 25..	113,763.0	34
25–29.....	155,184.5	123	25–29.....	58,201.0	31
30–34.....	141,317.5	137	30–34.....	40,282.0	39
35–39.....	152,365.5	254	35–39.....	42,748.0	58
40–44.....	168,001.0	408	40–44.....	52,191.0	74
45–49.....	163,797.0	679	45–49.....	59,887.0	139
50–54.....	144,332.5	1,010	50–54.....	55,861.0	174
55–59.....	122,768.5	1,389	55–59.....	45,593.0	208
60–64.....	77,569.0	1,161	60–64.....	23,548.0	150
Total...	1,232,433.0	5,286	Total...	492,074.0	907

61–65. The authors noted that the underreporting of 1968 experience may have caused the annual rates of improvement in Table 6 to be slightly higher than they otherwise might have been. The high experience rates of male retired life mortality at most ages under 60 appeared to be unreasonable, especially when compared with the experience rates at higher ages. Consequently, limited credence was given the annual rate of mortality improvement derived for ages 60 and under when the Projection Scale D and Projection Scale E mortality improvement factors were chosen. The values for females at the younger ages were selected somewhat arbitrarily after considering the rates of improvement shown in Tables 3 and 6 for ages under 66, weighted by 1966 intercompany group annuity income exposed for females at those ages. In view of the apparent improvement in mortality at high ages, Scale D allows for mortality improvement at some higher ages than in Scale C. Scale E provides for substantially more mortality improvement at the higher ages, and less improvement at ages 64–75, than does Scale D.

TABLE 6

ANNUAL CHANGES IN RETIRED LIFE MORTALITY
BETWEEN 1956–60 AND 1964–68

Age	Mortality Ratio (1964–68) ÷ Mortality Ratio (1956–60)	Annual Rate of Mortality Improvement	Projection Scale D (Applicable to Central Age of Age Group)
	Males		
60 and under	1.0192	−0.24%	0.65%
61–65	0.9396	+0.77	0.65
66–70	0.9792	+0.26	0.60
71–75	0.9653	+0.44	0.50
76–80	0.9496	+0.64	0.40
81–85	0.9625	+0.48	0.30
86–90	0.9266	+0.95	0.20
	Females		
60 and under	1.4245	−4.52%	1.30%
61–65	0.8111	+2.58	1.30
66–70	0.8808	+1.57	1.25
71–75	0.9556	+0.57	1.15
76–80	0.8030	+2.70	1.00
81–85	0.8980	+1.34	0.80
86–90	0.9481	+0.66	0.50

PROJECTION SCALE E: MALES

Age	Scale E	Age	Scale E
5–63	0.65%	98	0.27%
64	0.61	99	0.24
65	0.57	100	0.21
66	0.53	101	0.18
67	0.49	102	0.15
68–92	0.45	103	0.12
93	0.42	104	0.09
94	0.39	105	0.06
95	0.36	106	0.03
96	0.33	107–110	0
97	0.30		

Although the rate of decrease of group annuitant mortality rates in the aggregate has been higher than among lives retiring on or after their normal retirement dates, there is no reason to assume that the mortality rates for the total group would ever drop below the rates for the group retiring on or after normal retirement age. As "aggregate" mortality rates approach the "on or after" rates, the corresponding rates of decrease also should draw closer together. Nevertheless, calculations were performed (not illustrated herein) on the assumption that recent rates of decrease in "aggregate" mortality rates could continue indefinitely and so drop below the "on or after" rates. At the most significant ages, and for all ages combined, the crossover point would be after 1991.

Graduation

Several preliminary graduations were performed. One major difficulty with these graduations was that of bridging the very significant discontinuity between active and retired life mortality experience. The male retired life crude mortality rates at many ages under 65 are very high. In all likelihood not any of the retired life data below age 65 are pure "on or after" data; some poor-health early retirements are probably included too.

The next step, after the failure to produce a satisfactory merger of the active and retired life preliminary graduations, was to apply the Scale D rates of mortality decrease to the G*a*-1951 Table mortality rates. For males the results for retired lives were not consistent with the actual experience rates; however, this technique provided reasonably good results in the aggregate for active male lives. For females the results were remarkably consistent with the crude rates, especially for retired lives. Therefore, the 15-year projection of the G*a*-1951 female table by means of Scale D produced the graduated 1966 female experience table.

The graduated 1966 male experience table was obtained by calculating ratios of the crude mortality rates to G*a*-1951 rates for ages 60–92. Since, below age 65, the G*a*-1951 rates projected 16 years by Scale D were reasonably consistent with aggregate active life data, at ages below 60 the ratios were determined as $[1 - (\text{Scale D})]^{15}$. Ratios above age 92, except 110, were taken as $[1 - (\text{Scale D})]^{15}$, using the age 88 Scale D factor. The ratios were graduated by a nine-factor linear compound, minimum smoothing coefficient formula to produce adjusted ratios from age 55 to age 97. When the resulting mortality rates were examined, it was discovered that the formula had lived up to its name: negative second differences appeared at ages 70, 71, 78, 82–84, and 88–90. The mortality

rates produced by the preliminary graduations had negative second differences at ages 69–71. Ray Peterson discussed a similar phenomenon in his paper "Group Annuity Mortality."[2] Consequently, the ratios at ages 56, 57, 80, 83–92, 95, and 96 were adjusted. The adjustments were arbitrary but were selected to make the adjusted ratios more consistent with the apparent general pattern. After adjustment, the resulting fit was only very slightly worse, and the negative second differences had been eliminated at all ages except 70 and 71. The adjusted ratios were then applied to the G*a*-1951 male mortality rates to obtain the graduated 1966 male experience table. Table 7 shows retired life crude mortality rates and ratios of actual deaths to expected deaths calculated on the basis of the graduated 1966 experience tables.

Derivation of the 1971 GAM Table from the 1966 Experience Table Margin

To determine the appropriate margins, the standard deviations of crude mortality rates, by lives, were computed. Table 8 shows the results. Two standard deviations correspond to a 97.7 per cent level of confidence on a one-tail normal curve. Table 8 shows that, theoretically, the margin should vary by age, since both the mortality rates and the exposures affect the standard deviation. However, a uniform percentage is more practical. A suitable margin for males was deemed to be an 8 per cent reduction; a 10 per cent reduction was chosen for female rates.

Underreporting of 1968 Exposures and Deaths

Intercompany mortality data are adjusted each year for errors and late reported deaths by any of the contributing companies. Although the major differences generally have been reported the year after a given year's experience is initially reported, there are subsequent adjustments. The effect of the adjustments has always been to increase the crude mortality rates. The reporting procedures being adopted for 1969 and later should reduce the magnitude of these subsequent adjustments; however, 1968 data were reported according to the old procedures. An analysis of the differences for the period 1956–60 as originally published, and as shown in the 1969 *Reports*, indicated that the crude rates should be increased by approximately 1 per cent to adjust for underreporting. An adjustment of about this magnitude was also indicated by information obtained concerning 1968 underreporting. The 1 per cent increase in mortality rates was introduced by reducing the 8 per cent and 10 per cent margins to 7 per cent and 9 per cent, respectively.

[2] *TSA*, IV, 292.

TABLE 7

1964–68 EXPERIENCE BY AMOUNT OF ANNUAL INCOME
RETIREMENT ON OR AFTER NORMAL RETIREMENT DATE

MALES

Age	Crude Data 1,000 q_x	1966 Experience Table 1,000 q_x	Expected Deaths	Actual Deaths	Actual÷ Expected
55	8.860	9.464	$ 9,806	$ 9,180	0.936
56	18.254	10.289			
57	26.972	11.152			
58	28.701	12.097	176,874	242,338	1.370
59	12.199	13.247			
60	14.782	14.574			
61	19.059	16.041			
62	20.329	17.622			
63	24.074	19.344	3,901,543	4,151,285	1.064
64	27.176	21.302			
65	23.674	23.594			
66	26.268	26.226			
67	29.122	29.176			
68	32.009	32.344	20,077,039	20,141,205	1.003
69	35.576	35.906			
70	41.243	39.929			
71	42.987	44.200			
72	49.498	48.370			
73	51.602	52.359	21,246,427	21,179,928	0.997
74	56.817	56.418			
75	60.634	60.841			
76	65.840	66.029			
77	71.726	72.394			
78	80.860	79.639	14,201,132	14,166,444	0.998
79	87.431	87.337			
80	93.847	95.723			
81	105.470	104.392			
82	108.251	113.297			
83	123.831	122.583	6,917,697	7,017,610	1.014
84	145.406	132.071			
85	145.934	141.727			
86	151.124	151.612			
87	159.357	161.678			
88	160.919	172.130	2,211,517	2,219,441	1.004
89	177.122	183.049			
90	237.124	194.510			
91	179.615	206.267			
92	227.073	218.167			
93	289.466	230.166	540,002	555,641	1.029
94	282.568	244.562			
95	244.319	260.096			
96	450.883	276.040			
97	137.760	293.282			
98	160.798	312.003	111,677	92,912	0.832
99	143.285	332.393			
100	251.896	354.650			
101	176.969	378.984			
102	0	405.613			
103	72.544	436.780	19,739	7,313	0.370
104	0	474.728			
105	269.939	521.701			
106	0	579.939	3,273	2,499	0.764
107	892.819	651.687			
Total	.....	.....	$69,406,928	$69,776,616	1.005

TABLE 7—*Continued*

FEMALES

Age	Crude Data 1,000 q_x	1966 Experience Table 1,000 q_x	Expected Deaths	Actual Deaths	Actual÷ Expected
55	14.421	3.820	$ 2,500	$ 9,437	3.775
56	4.259	4.193			
57	10.614	4.632			
58	8.274	5.148	61,604	106,128	1.723
59	9.452	5.750			
60	11.327	6.440			
61	8.015	7.222			
62	6.869	8.093			
63	8.050	9.048	458,302	462,338	1.009
64	12.261	10.094			
65	11.004	11.208			
66	11.197	12.376			
67	14.807	13.607			
68	15.933	15.069	1,206,051	1,217,307	1.009
69	16.326	16.905			
70	19.847	19.243			
71	19.270	22.167			
72	23.881	25.537			
73	29.346	29.239	1,264,815	1,216,489	0.962
74	32.441	33.286			
75	38.673	37.592			
76	39.056	42.198			
77	46.101	47.181			
78	47.298	52.544	948,948	923,563	0.973
79	66.468	58.371			
80	60.969	64.547			
81	73.817	71.042			
82	80.169	77.871			
83	90.246	85.053	488,966	494,335	1.011
84	74.695	92.950			
85	111.976	101.400			
86	123.262	110.491			
87	139.912	120.311			
88	118.236	130.962	179,091	206,975	1.156
89	190.160	142.552			
90	215.525	155.209			
91	206.800	168.829			
92	288.687	183.780			
93	133.470	200.237	54,426	68,567	1.260
94	380.237	218.391			
95	195.782	238.457			
96	251.581	260.667			
97	123.688	283.581			
98	464.185	307.953	12,216	10,120	0.828
99	331.377	334.812			
100	31.167	364.429			
101	116.618	397.100			
102	0	433.150			
103	492.659	472.930	3,580	782	0.218
104	0	518.156			
Total	.....	.....	$4,680,499	$4,716,041	1.008

Projection to 1971

The Scale D factors were used to project the mortality rates from 1966 to 1971. The following formulas were used to convert the graduated 1966 rates to 1971 GAM tabular rates.

For males:

$$1971\ q_x = [1 - (\text{Scale D})]^5(0.93)(1966\ q_x)\ .$$

For females:

$$1971\ q_x = [1 - (\text{Scale D})]^5(0.91)(1966\ q_x)\ .$$

1971 GROUP ANNUITY MORTALITY TABLE

Table 9 summarizes and compares the sources and methods of constructing the G*a*-1951 and 1971 GAM tables. Table 10 shows G*a*-1951 Table, 1966 graduated experience table, and 1971 GAM Table mortality probabilities, and Projection Scale D improvement factors. Tables 11–16

TABLE 8

STANDARD DEVIATION OF CRUDE MORTALITY RATES BASED ON LIVES

Age x	Crude q_x (1)	Number of Lives (n_x) (2)	$\sigma_x = \sqrt{p_x q_x / n_x}$ (3)	$(2\sigma_x/q_x)$ ×100% (4)
	Males			
63	0.028746	6,087.79	0.002142	14.9%
68	0.034494	106,742.12	0.000559	3.2
73	0.055795	83,161.14	0.000796	2.8
78	0.085962	44,356.71	0.001331	3.1
83	0.125671	16,089.64	0.002613	4.2
88	0.179646	3,384.43	0.006599	7.3
Weighted* average of col. 4				4.5%
	Females			
63	0.009276	9,486.81	0.000984	21.2%
68	0.017242	28,476.50	0.000771	8.9
73	0.028449	18,875.62	0.001210	8.5
78	0.048383	8,949.35	0.002268	9.4
83	0.088725	2,806.41	0.005368	12.1
88	0.148683	578.41	0.014793	19.9
Weighted* average of col. 4				12.0%

* Weighted by annual income exposed in the five-year age group to which the indicated age is central.

provide commutation functions at several rates of interest for the unprojected 1971 GAM Table.

Table 17 shows male annuity values on several mortality bases at several rates of interest. The relationships are about as one would expect considering the reduction of mortality rates since 1951 and the overstatement at the younger ages and understatement at older ages of the Scale C mortality improvement rates.

Although a minimum valuation standard should be based on the unprojected 1971 GAM Table, actuaries undoubtedly will wish to make use of projection factors or generation mortality techniques in valuing some

TABLE 9

COMPARISON OF CONSTRUCTION OF G*a*-1951 AND 1971 GAM TABLES

	G*a*-1951	1971 GAM
1. Active life data..	Used *a*-1949 Table	Experience by lives on four large deferred annuity groups and one large municipal employee group used for reference when adjusting G*a*-1951 Table
2. Retired life data.	1946–50 Intercompany group annuity experience for retirements on and after normal retirement date, adjusted for retirements prior to normal retirement date, by lives	1964–68 Intercompany group annuity experience for retirements on and after normal retirement date, by amount of annual income
3. Graduation method.....	Retired lives: Whittaker-Henderson Type B formula, with smoothness represented by a function constraining first differences toward a geometric series; active lives: used *a*-1949 Table	Males: for retired lives a nine-factor linear compound, minimum smoothing coefficient formula was used, and Scale D applied to G*a*-1951 Table was used at low and high ages; females: Scale D applied to G*a*-1951 Table used at all ages
4. Projection of basic table.....	Retired lives: 1948 Experience table projected three years by Projection Scale B; active lives: *a*-1949 Table projected one year by Projection Scale B	1966 Experience table projected 5 years by Scale D
5. Margin.....	Male rates reduced 10 per cent; female rates reduced 12½ per cent	Male rates reduced 8 per cent; female rates reduced 10 per cent; rates then increased 1 per cent to adjust for underreporting of data
6. Projection scales.	B and C	Males: D and E; females: D

TABLE 10

PROBABILITY OF MORTALITY AND PROJECTION SCALE D

MALES

Age x	G*a*-1951 q_x	1966 Experience Table q_x	1971 GAM q_x	Projection Scale D	Age x	G*a*-1951 q_x	1966 Experience Table q_x	1971 GAM q_x	Projection Scale D
5	.000559	.000507	.000456	.65%	61	.016866	.016041	.014440	.65%
6	.000519	.000471	.000424	.65	62	.018353	.017622	.015863	.65
7	.000494	.000448	.000403	.65	63	.020068	.019344	.017413	.65
8	.000481	.000436	.000392	.65	64	.022067	.021302	.019185	.64
9	.000476	.000432	.000389	.65	65	.024418	.023594	.021260	.63
10	.000477	.000433	.000390	.65	66	.027193	.026226	.023643	.62
11	.000486	.000441	.000397	.65	67	.030112	.029176	.026316	.61
12	.000496	.000450	.000405	.65	68	.032986	.032344	.029188	.60
13	.000506	.000459	.000413	.65	69	.035943	.035906	.032435	.58
14	.000517	.000469	.000422	.65	70	.039303	.039929	.036106	.56
15	.000530	.000481	.000433	.65	71	.043183	.044200	.040008	.54
16	.000544	.000493	.000444	.65	72	.047476	.048370	.043827	.52
17	.000560	.000508	.000457	.65	73	.052084	.052359	.047489	.50
18	.000577	.000523	.000471	.65	74	.057077	.056418	.051221	.48
19	.000595	.000540	.000486	.65	75	.062427	.060841	.055293	.46
20	.000616	.000559	.000503	.65	76	.068347	.066029	.060068	.44
21	.000640	.000580	.000522	.65	77	.075132	.072394	.065924	.42
22	.000666	.000604	.000544	.65	78	.082687	.079639	.072595	.40
23	.000693	.000628	.000566	.65	79	.090946	.087337	.079692	.38
24	.000724	.000657	.000591	.65	80	.099679	.095723	.087431	.36
25	.000758	.000687	.000619	.65	81	.108706	.104392	.095445	.34
26	.000796	.000722	.000650	.65	82	.117979	.113297	.103691	.32
27	.000838	.000760	.000684	.65	83	.127437	.122583	.112303	.30
28	.000885	.000803	.000722	.65	84	.137073	.132071	.121116	.28
29	.000935	.000848	.000763	.65	85	.146852	.141727	.130102	.26
30	.000991	.000899	.000809	.65	86	.156836	.151612	.139315	.24
31	.001054	.000956	.000860	.65	87	.167120	.161678	.148714	.22
32	.001122	.001017	.000916	.65	88	.177787	.172130	.158486	.20
33	.001198	.001086	.000978	.65	89	.188919	.183049	.168709	.18
34	.001281	.001162	.001046	.65	90	.200594	.194510	.179452	.16
35	.001374	.001246	.001122	.65	91	.212555	.206267	.190489	.14
36	.001475	.001338	.001204	.65	92	.225161	.218167	.201681	.12
37	.001587	.001439	.001295	.65	93	.238524	.230166	.212986	.10
38	.001711	.001552	.001397	.65	94	.252765	.244562	.226535	.08
39	.001849	.001677	.001509	.65	95	.268025	.260096	.241164	.06
40	.002000	.001814	.001633	.65	96	.284455	.276040	.256204	.04
41	.002192	.001988	.001789	.65	97	.302223	.293282	.272480	.02
42	.002450	.002222	.002000	.65	98	.321515	.312003	.290163	0
43	.002769	.002511	.002260	.65	99	.342526	.332393	.309125	0
44	.003147	.002854	.002569	.65	100	.365462	.354650	.329825	0
45	.003580	.003246	.002922	.65	101	.390538	.378984	.352455	0
46	.004065	.003686	.003318	.65	102	.417979	.405613	.377220	0
47	.004599	.004170	.003754	.65	103	.450096	.436780	.406205	0
48	.005180	.004697	.004228	.65	104	.489201	.474728	.441497	0
49	.005807	.005266	.004740	.65	105	.537605	.521701	.485182	0
50	.006475	.005872	.005285	.65	106	.597619	.579939	.539343	0
51	.007187	.006517	.005867	.65	107	.671554	.651687	.606069	0
52	.007938	.007198	.006480	.65	108	.761722	.739187	.687444	0
53	.008731	.007917	.007127	.65	109	.870434	.844683	.785555	0
54	.009563	.008672	.007806	.65	110	.999999	.999999	.999999	0
55	.010436	.009464	.008519	.65					
56	.011346	.010289	.009262	.65					
57	.012298	.011152	.010039	.65					
58	.013302	.012097	.010889	.65					
59	.014379	.013247	.011924	.65					
60	.015555	.014574	.013119	.65					

TABLE 10—*Continued*

FEMALES

Age x	Ga-1951 q_x	1966 Experience Table q_x	1971 GAM q_x	Projection Scale D
5	.000335	.000275	.000234	1.30%
6	.000275	.000226	.000193	1.30
7	.000231	.000190	.000162	1.30
8	.000204	.000168	.000143	1.30
9	.000191	.000157	.000134	1.30
10	.000189	.000155	.000132	1.30
11	.000205	.000168	.000143	1.30
12	.000222	.000182	.000155	1.30
13	.000239	.000196	.000167	1.30
14	.000257	.000211	.000180	1.30
15	.000275	.000226	.000193	1.30
16	.000292	.000240	.000205	1.30
17	.000311	.000256	.000218	1.30
18	.000330	.000271	.000231	1.30
19	.000350	.000288	.000245	1.30
20	.000371	.000305	.000260	1.30
21	.000393	.000323	.000275	1.30
22	.000416	.000342	.000292	1.30
23	.000440	.000362	.000309	1.30
24	.000467	.000384	.000327	1.30
25	.000495	.000407	.000347	1.30
26	.000524	.000432	.000368	1.30
27	.000556	.000457	.000390	1.30
28	.000591	.000486	.000414	1.30
29	.000628	.000516	.000440	1.30
30	.000669	.000550	.000469	1.30
31	.000712	.000585	.000499	1.30
32	.000760	.000625	.000533	1.30
33	.000812	.000667	.000569	1.30
34	.000868	.000713	.000608	1.30
35	.000930	.000764	.000651	1.30
36	.000997	.000819	.000698	1.30
37	.001071	.000880	.000750	1.30
38	.001152	.000947	.000807	1.30
39	.001240	.001019	.000869	1.30
40	.001338	.001100	.000938	1.30
41	.001446	.001188	.001013	1.30
42	.001563	.001284	.001094	1.30
43	.001694	.001392	.001186	1.30
44	.001836	.001509	.001286	1.30
45	.001994	.001639	.001397	1.30
46	.002169	.001782	.001519	1.30
47	.002361	.001940	.001654	1.30
48	.002573	.002114	.001802	1.30
49	.002809	.002308	.001967	1.30
50	.003070	.002523	.002151	1.30
51	.003319	.002727	.002324	1.30
52	.003597	.002956	.002520	1.30
53	.003908	.003212	.002738	1.30
54	.004257	.003498	.002982	1.30
55	.004648	.003820	.003256	1.30
56	.005102	.004193	.003574	1.30
57	.005637	.004632	.003948	1.30
58	.006265	.005148	.004388	1.30
59	.006997	.005750	.004901	1.30
60	.007837	.006440	.005489	1.30
61	.008788	.007222	.006156	1.30%
62	.009848	.008093	.006898	1.30
63	.011010	.009048	.007712	1.30
64	.012264	.010094	.008608	1.29
65	.013597	.011208	.009563	1.28
66	.014991	.012376	.010565	1.27
67	.016457	.013607	.011621	1.26
68	.018198	.015069	.012877	1.25
69	.020354	.016905	.014461	1.23
70	.023098	.019243	.016477	1.21
71	.026527	.022167	.019000	1.19
72	.030468	.025537	.021911	1.17
73	.034779	.029239	.025112	1.15
74	.039413	.033286	.028632	1.12
75	.044309	.037592	.032385	1.09
76	.049512	.042198	.036408	1.06
77	.055108	.047181	.040769	1.03
78	.061093	.052544	.045472	1.00
79	.067459	.058371	.050616	0.96
80	.074146	.064547	.056085	0.92
81	.081114	.071042	.061853	0.88
82	.088374	.077871	.067936	0.84
83	.095943	.085053	.074351	0.80
84	.103904	.092950	.081501	0.74
85	.112328	.101400	.089179	0.68
86	.121295	.110491	.097468	0.62
87	.130885	.120311	.106452	0.56
88	.141188	.130962	.116226	0.50
89	.152300	.142552	.126893	0.44
90	.164331	.155209	.138577	0.38
91	.177144	.168829	.151192	0.32
92	.191099	.183780	.165077	0.26
93	.206341	.200237	.180401	0.20
94	.223029	.218391	.197349	0.14
95	.241336	.238457	.216129	0.08
96	.261451	.260667	.236970	0.02
97	.283581	.283581	.258059	0
98	.307953	.307953	.280237	0
99	.334812	.334812	.304679	0
100	.364429	.364429	.331630	0
101	.397100	.397100	.361361	0
102	.433150	.433150	.394167	0
103	.472930	.472930	.430366	0
104	.518156	.518156	.471522	0
105	.570545	.570545	.519196	0
106	.631813	.631813	.574950	0
107	.703676	.703676	.640345	0
108	.787851	.787851	.716944	0
109	.886054	.886054	.806309	0
110	.999999	.999999	.999999	0

TABLE 11

1971 GAM COMMUTATION FUNCTIONS AT 3½ PER CENT

MALES

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
5.....	10,000.0000	4.5600	8,419.7318	223,383.7490	219,524.7054
6.....	9,995.4400	4.2381	8,131.2970	214,964.0172	211,237.1729
7.....	9,991.2019	4.0264	7,852.9946	206,832.7202	203,233.4311
8.....	9,987.1755	3.9150	7,584.3767	198,979.7256	195,503.5531
9.....	9,983.2605	3.8835	7,325.0277	191,395.3489	188,038.0477
10.....	9,979.3770	3.8919	7,074.5684	184,070.3212	180,827.8109
11.....	9,975.4851	3.9603	6,832.6660	176,995.7529	173,864.1144
12.....	9,971.5248	4.0385	6,598.9888	170,163.0869	167,138.5504
13.....	9,967.4863	4.1165	6,373.2524	163,564.0980	160,643.0241
14.....	9,963.3698	4.2046	6,155.1887	157,190.8456	154,369.7176
15.....	9,959.1652	4.3123	5,944.5326	151,035.6569	148,311.0796
16.....	9,954.8529	4.4200	5,741.0228	145,091.1243	142,459.8223
17.....	9,950.4329	4.5473	5,544.4192	139,350.1015	136,808.9095
18.....	9,945.8856	4.6845	5,354.4786	133,805.6823	131,351.5464
19.....	9,941.2011	4.8314	5,170.9726	128,451.2037	126,081.1746
20.....	9,936.3697	4.9980	4,993.6807	123,280.2310	120,991.4608
21.....	9,931.3717	5.1842	4,822.3855	118,236.5503	116,076.2904
22.....	9,926.1875	5.3999	4,656.8775	113,464.1648	111,329.7627
23.....	9,920.7876	5.6151	4,496.9509	108,807.2873	106,746.1849
24.....	9,915.1725	5.8599	4,342.4209	104,310.3365	102,320.0603
25.....	9,909.3126	6.1339	4,193.0962	99,967.9156	98,046.0799
26.....	9,903.1787	6.4370	4,048.7929	99,774.8194	93,919.1228
27.....	9,896.7417	6.7694	3,909.3345	91,726.0265	89,934.2483
28.....	9,889.9723	7.1406	3,774.5512	87,816.6920	86,086.6895
29.....	9,882.8317	7.5406	3,644.2763	84,042.1408	82,371.8476
30.....	9,875.2911	7.9891	3,518.3534	80,397.8645	78,785.2859
31.....	9,867.3020	8.4858	3,396.6252	76,879.5111	75,322.7246
32.....	9,858.8162	9.0307	3,278.9412	73,482.8859	71,980.0379
33.....	9,849.7855	9.6331	3,165.1572	70,203.9448	68,753.2478
34.....	9,840.1524	10.2928	3,055.1320	67,038.7876	65,638.5188
35.....	9,829.8596	11.0291	2,948.7308	63,983.6556	62,632.1540
36.....	9,818.8305	11.8219	2,845.8187	61,034.9248	59,730.5913
37.....	9,807.0086	12.7001	2,746.2728	58,189.1061	56,930.3978
38.....	9,794.3085	13.6826	2,649.9675	55,442.8334	54,228.2650
39.....	9,780.6259	14.7590	2,556.7783	52,792.8659	51,621.0092
40.....	9,765.8669	15.9476	2,466.5895	50,236.0876	49,105.5675
41.....	9,749.9193	17.4426	2,379.2865	47,769.4982	46,678.9919
42.....	9,732.4767	19.4650	2,294.7149	45,390.2117	44,338.4674
43.....	9,713.0117	21.9514	2,212.6817	43,095.4967	42,081.3510
44.....	9,691.0603	24.8963	2,133.0251	40,882.8151	39,905.1786
45.....	9,666.1640	28.2446	2,055.5994	38,749.7900	37,807.6403
46.....	9,637.9194	31.9786	1,980.2830	36,694.1905	35,786.5609
47.....	9,605.9408	36.0607	1,906.9686	34,713.9075	33,839.8803
48.....	9,569.8801	40.4614	1,835.5650	32,806.9389	31,965.6383
49.....	9,529.4187	45.1695	1,765.9945	30,971.3739	30,161.9598
50.....	9,484.2492	50.1242	1,698.1871	29,205.3794	28,427.0437
51.....	9,434.1250	55.3501	1,632.0891	27,507.1923	26,759.1515
52.....	9,378.7749	60.7744	1,567.6460	25,875.1032	25,156.5988
53.....	9,318.0005	66.4094	1,504.8190	24,307.4572	23,617.7485
54.....	9,251.5911	72.2179	1,443.5692	22,802.6382	22,141.0023
55.....	9,179.3732	78.1991	1,383.8655	21,359.0689	20,724.7973
56.....	9,101.1741	84.2951	1,325.6776	19,975.2034	19,367.6012

TABLE 11—*Continued*

MALES—*Continued*

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
57.....	9,016.8790	90.5204	1,268.9847	18,649.5258	18,067.9079
58.....	8,926.3586	97.1991	1,213.7637	17,380.5411	16,824.2328
59.....	8,829.1595	105.2789	1,159.9488	16,166.7775	15,635.1343
60.....	8,723.8806	114.4486	1,107.3600	15,006.8287	14,499.2887
61.....	8,609.4320	124.3202	1,055.8768	13,899.4688	13,415.5252
62.....	8,485.1118	134.5994	1,005.4396	12,843.5919	12,382.7655
63.....	8,350.5124	145.4074	956.0293	11,838.1524	11,399.9723
64.....	8,205.1050	157.4150	907.6154	10,882.1231	10,466.1327
65.....	8,047.6900	171.0939	860.0993	9,974.5077	9,580.2956
66.....	7,876.5961	186.2263	813.3465	9,114.4084	8,741.6246
67.....	7,690.3698	202.3798	767.2623	8,301.0619	7,949.4000
68.....	7,487.9900	218.5594	721.8078	7,533.7996	7,202.9710
69.....	7,269.4306	235.7840	677.0432	6,811.9918	6,501.6803
70.....	7,033.6466	253.9569	632.9307	6,134.9486	5,844.8554
71.....	6,779.6897	271.2418	589.4474	5,502.0179	5,231.8545
72.....	6,508.4479	285.2457	546.7293	4,912.5705	4,661.9862
73.....	6,223.2022	295.5337	505.0897	4,365.8412	4,134.3417
74.....	5,927.6685	303.6211	464.8343	3,860.7515	3,647.7025
75.....	5,624.0474	310.9705	426.1111	3,395.9172	3,200.6163
76.....	5,313.0769	319.1459	388.9373	2,969.8061	2,791.5432
77.....	4,993.9310	329.2199	353.2122	2,580.8688	2,418.9799
78.....	4,664.7111	338.6347	318.7701	2,227.6566	2,081.5536
79.....	4,326.0764	344.7537	285.6319	1,908.8864	1,777.9718
80.....	3,981.3227	348.0910	253.9800	1,623.2546	1,506.8471
81.....	3,633.2317	346.7738	223.9365	1,369.2746	1,266.6370
82.....	3,286.4579	340.7761	195.7129	1,145.3381	1,055.6363
83.....	2,945.6818	330.8089	169.4872	949.6251	871.9435
84.....	2,614.8729	316.7029	145.3655	780.1379	713.5121
85.....	2,298.1700	298.9966	123.4390	634.7724	578.1962
86.....	1,999.1734	278.5148	103.7482	511.3334	463.7822
87.....	1,720.6586	255.8860	86.2749	407.5852	368.0426
88.....	1,464.7726	232.1460	70.9610	321.3103	288.7865
89.....	1,232.6266	207.9552	57.6953	250.3494	223.9057
90.....	1,024.6714	183.8793	46.3397	192.6540	171.4150
91.....	840.7921	160.1617	36.7381	146.3143	129.4760
92.....	680.6304	137.2702	28.7342	109.5762	96.4064
93.....	543.3602	115.7281	22.1634	80.8420	70.6838
94.....	427.6321	96.8736	16.8530	58.6786	50.9543
95.....	330.7585	79.7671	12.5944	41.8256	36.0532
96.....	250.9914	64.1230	9.2339	29.2312	24.9990
97.....	186.6864	51.0503	6.6359	19.9973	16.9559
98.....	135.8181	39.4094	4.6645	13.3614	11.2235
99.....	96.4087	29.8023	3.1991	8.6969	7.2307
100.....	66.6064	21.9685	2.1354	5.4979	4.5192
101.....	44.6379	15.7328	1.3827	3.3625	2.7287
102.....	28.9051	10.9036	0.8651	1.9798	1.5833
103.....	18.0015	7.3123	0.5205	1.1147	0.8761
104.....	10.6892	4.7193	0.2986	0.5942	0.4573
105.....	5.9699	2.8965	0.1612	0.2955	0.2217
106.....	3.0734	1.6576	0.0802	0.1344	0.0976
107.....	1.4158	0.8581	0.0357	0.0542	0.0379
108.....	0.5577	0.3834	0.0136	0.0185	0.0123
109.....	0.1743	0.1369	0.0041	0.0050	0.0031
110.....	0.0374	0.0374	0.0008	0.0008	0.0005

TABLE 12

1971 GAM COMMUTATION FUNCTIONS AT 5 PER CENT

MALES

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
5.....	10,000.0000	4.5600	7,835.2618	157,165.1699	153,574.0084
6.....	9,995.4400	4.2381	7,458.7514	149,329.9081	145,911.3139
7.....	9,991.2019	4.0264	7,100.5608	141,871.1568	138,616.7332
8.....	9,987.1755	3.9150	6,759.7136	134,770.5960	131,672.3940
9.....	9,983.2605	3.8835	6,435.2989	128,010.8823	125,061.3705
10.....	9,979.3770	3.8919	6,126.4720	121,575.5834	118,767.6172
11.....	9,975.4851	3.9603	5,832.4597	115,449.1115	112,775.9009
12.....	9,971.5248	4.0385	5,552.5183	109,616.6518	107,071.7476
13.....	9,967.4863	4.1165	5,285.9710	104,064.1334	101,641.3968
14.....	9,963.3698	4.2046	5,032.1790	98,778.1624	96,471.7472
15.....	9,959.1652	4.3123	4,790.5290	93,745.9835	91,550.3245
16.....	9,954.8529	4.4200	4,560.4330	88,955.4545	86,865.2561
17.....	9,950.4329	4.5473	4,341.3411	84,395.0215	82,405.2402
18.....	9,945.8856	4.6845	4,132.7211	80,053.6804	78,159.5166
19.....	9,941.2011	4.8314	3,934.0711	75,920.9593	74,117.8434
20.....	9,936.3697	4.9980	3,744.9134	71,986.8882	70,270.4696
21.....	9,931.3717	5.1842	3,564.7902	68,241.9748	66,608.1126
22.....	9,926.1875	5.3999	3,393.2661	64,677.1845	63,121.9376
23.....	9,920.7876	5.6151	3,229.9240	61,283.9184	59,803.5366
24.....	9,915.1725	5.8599	3,074.3770	58,053.9944	56,644.9050
25.....	9,909.3126	6.1339	2,926 2477	54,979.6174	53,638.4206
26.....	9,903.1787	6.4370	2,785.1775	52,053.3697	50,776.8301
27.....	9,896.7417	6.7694	2,650.8258	49,268.1922	48,053.2304
28.....	9,889.9723	7.1406	2,522.8692	46,617.3664	45,461.0514
29.....	9,882.8317	7.5406	2,400.9978	44,094.4972	42,994.0399
30.....	9,875.2911	7.9891	2,284.9199	41,693.4994	40,646.2445
31.....	9,867.3020	8.4858	2,174.3537	39,408.5795	38,412.0008
32.....	9,858.8162	9.0307	2,069.0321	37,234.2258	36,285.9195
33.....	9,849.7855	9.6331	1,968.7018	35,165.1937	34,262.8721
34.....	9,840.1524	10.2928	1,873.1204	33,196.4919	32,337.9784
35.....	9,829.8596	11.0291	1,782.0582	31,323.3715	30,506.5948
36.....	9,818.8305	11.8219	1,695.2941	29,541.3132	28,764.3035
37.....	9,807.0086	12.7001	1,612.6218	27,846.0192	27,106.9009
38.....	9,794.3085	13.6826	1,533.8414	26,233.3973	25,530.3867
39.....	9,780.6259	14.7590	1,458.7606	24,699.5559	24,030.9573
40.....	9,765.8669	15.9476	1,387.1994	23,240.7953	22,604.9956
41.....	9,749.9193	17.4426	1,318.9849	21,853.5959	21,249.0612
42.....	9,732.4767	19.4650	1,253.9288	20,534.6110	19,959.8937
43.....	9,713.0117	21.9514	1,191.8294	19,280.6823	18,734.4272
44.....	9,691.0603	24.8963	1,132.5104	18,088.8529	17,569.7856
45.....	9,666.1640	28.2446	1,075.8104	16,956.3425	16,463.2627
46.....	9,637.9194	31.9786	1,021.5875	15,880.5320	15,412.3044
47.....	9,605.9408	36.0607	969.7123	14,858.9445	14,414.4930
48.....	9,569.8801	40.4614	920.0686	13,889.2322	13,467.5341
49.....	9,529.4187	45.1695	872.5510	12,969.1636	12,569.2444
50.....	9,484.2492	50.1242	827.0620	12,096.6126	11,717.5425
51.....	9,434.1250	55.3501	783.5152	11,269.5506	10,910.4395
52.....	9,378.7749	60.7744	741.8270	10,486.0354	10,146.0314
53.....	9,318.0005	66.4094	701.9238	9,744.2084	9,422.4934
54.....	9,251.5911	72.2179	663.7344	9,042.2847	8,738.0731
55.....	9,179.3732	78.1991	627.1936	8,378.5502	8,091.0865
56.....	9,101.1741	84.2951	592.2386	7,751.3566	7,479.9139

TABLE 12—*Continued*

MALES—*Continued*

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
57	9,016.8790	90.5204	558.8127	7,159.1179	6,902.9955
58	8,926.3586	97.1991	526.8598	6,600.3052	6,358.8279
59	8,829.1595	105.2789	496.3074	6,073.4455	5,845.9712
60	8,723.8806	114.4486	467.0376	5,577.1380	5,363.0791
61	8,609.4320	124.3202	438.9624	5,110.1004	4,908.9093
62	8,485.1118	134.5994	412.0227	4,671.1380	4,482.2943
63	8,350.5124	145.4074	386.1779	4,259.1153	4,082.1172
64	8,205.1050	157.4150	361.3841	3,872.9375	3,707.3031
65	8,047.6900	171.0939	337.5724	3,511.5534	3,356.8327
66	7,876.5961	186.2263	314.6625	3,173.9810	3,029.7607
67	7,690.3698	202.3798	292.5932	2,859.3186	2,725.2133
68	7,487.9900	218.5594	271.3270	2,566.7253	2,442.3671
69	7,269.4306	235.7840	250.8643	2,295.3983	2,180.4189
70	7,033.6466	253.9569	231.1691	2,044.5340	1,938.5816
71	6,779.6897	271.2418	212.2119	1,813.3650	1,716.1012
72	6,508.4479	285.2457	194.0207	1,601.1531	1,512.2270
73	6,223.2022	295.5337	176.6832	1,407.1325	1,326.1527
74	5,927.6685	303.6211	160.2787	1,230.4493	1,156.9882
75	5,624.0474	310.9705	144.8277	1,070.1706	1,003.7912
76	5,313.0769	319.1459	130.3045	925.3429	865.6200
77	4,993.9310	329.2199	116.6451	795.0384	741.5760
78	4,664.7111	338.6347	103.7671	678.3932	630.8333
79	4,326.0764	344.7537	91.6515	574.6262	532.6192
80	3,981.3227	348.0910	80.3311	482.9746	446.1562
81	3,633.2317	346.7738	69.8168	402.6436	370.6442
82	3,286.4579	340.7761	60.1458	332.8268	305.2599
83	2,945.6818	330.8089	51.3422	272.6809	249.1491
84	2,614.8729	316.7029	43.4060	221.3388	201.4444
85	2,298.1700	298.9966	36.3322	177.9328	161.2805
86	1,999.1734	278.5148	30.1003	141.6006	127.8046
87	1,720.6586	255.8860	24.6732	111.5003	100.1917
88	1,464.7726	232.1460	20.0038	86.8271	77.6587
89	1,232.6266	207.9552	16.0319	66.8233	59.4753
90	1,024.6714	183.8793	12.6925	50.7914	44.9740
91	840.7921	160.1617	9.9189	38.0989	33.5527
92	680.6304	137.2702	7.6471	28.1800	24.6751
93	543.3602	115.7281	5.8141	20.5329	17.8681
94	427.6321	96.8736	4.3579	14.7188	12.7215
95	330.7585	79.7671	3.2102	10.3609	8.8896
96	250.9914	64.1230	2.3200	7.1508	6.0874
97	186.6864	51.0503	1.6434	4.8308	4.0775
98	135.8181	39.4094	1.1387	3.1873	2.6654
99	96.4087	29.8023	0.7698	2.0487	1.6958
100	66.6064	21.9685	0.5065	1.2789	1.0467
101	44.6379	15.7328	0.3233	0.7723	0.6242
102	28.9051	10.9036	0.1994	0.4491	0.3577
103	18.0015	7.3123	0.1183	0.2497	0.1955
104	10.6892	4.7193	0.0669	0.1314	0.1008
105	5.9699	2.8965	0.0356	0.0646	0.0483
106	3.0734	1.6576	0.0174	0.0290	0.0210
107	1.4158	0.8581	0.0077	0.0116	0.0080
108	0.5577	0.3834	0.0029	0.0039	0.0026
109	0.1743	0.1369	0.0009	0.0010	0.0006
110	0.0374	0.0374	0.0002	0.0002	0.0001

TABLE 13

1971 GAM COMMUTATION FUNCTIONS AT 6 PER CENT

MALES

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
5.....	10,000.0000	4.5600	7,472.5818	128,391.8325	124,966.8994
6.....	9,995.4400	4.2381	7,046.3908	120.919.2508	117,689.6551
7.....	9,991.2019	4.0264	6,644.7200	113,872.8599	110,827.3634
8.....	9,987.1755	3.9150	6,266.0775	107,228.1400	104,356.1879
9.....	9,983.2605	3.8835	5,909.0766	100,962.0624	98,253.7358
10.....	9,979.3770	3.8919	5,572.4321	95,052.9858	92,498.9546
11.....	9,975.4851	3.9603	5,254.9612	89,480.5537	87,072.0300
12.....	9,971.5248	4.0385	4,955.5424	84,225.5926	81,954.3024
13.....	9,967.4863	4.1165	4,673.1466	79,270.0502	77,128.1914
14.....	9,963.3698	4.2046	4,406.8081	74,596.9036	72,577.1166
15.....	9,959.1652	4.3123	4,155.6118	70,190.0954	68,285.4401
16.....	9,954.8529	4.4200	3,918.6909	66,034.4837	64,238.4171
17.....	9,950.4329	4.5473	3,695.2368	62,115.7927	60,422.1426
18.....	9,945.8856	4.6845	3,484.4793	58,420.5559	56,823.5030
19.....	9,941.2011	4.8314	3,285.6964	54,936.0766	53,430.1325
20.....	9,936.3697	4.9980	3,098.2071	51,650.3802	50,230.3687
21.....	9,931.3717	5.1842	2,921.3667	48,552.1731	47,213.2134
22.....	9,926.1875	5.3999	2,754.5677	45,630.8064	44,368.2962
23.....	9,920.7876	5.6151	2,597.2351	42,876.2387	41,685.8393
24.....	9,915.1725	5.8599	2,448.8350	40,279.0036	39,156.6209
25.....	9,909.3126	6.1339	2,308.8563	37,830.1686	36,771.9428
26.....	9,903.1787	6.4370	2,176.8181	35,521.3123	34,523.6040
27.....	9.896.7417	6.7694	2,052.2671	33,344.4942	32,403.8718
28.....	9,889.9723	7.1406	1,934.7768	31,292.2271	30,405.4544
29.....	9,882.8317	7.5406	1,823.9433	29,357.4503	28,521.4763
30.....	9,875.2911	7.9891	1,719.3883	27,533.5070	26,745.4541
31.....	9,867.3020	8.4858	1,620.7522	25,814.1187	25,071.2740
32.....	9,858.8162	9.0307	1,527.6965	24,193.3666	23,493.1723
33.....	9,849.7855	9.6331	1,439.9030	22,665.6700	22,005.7145
34.....	9,840.1524	10.2928	1,357.0705	21,225.7670	20,603.7764
35.....	9,829.8596	11.0291	1,278.9161	19,868.6965	19,282.5266
36.....	9,818.8305	11.8219	1,205.1709	18,589.7804	18,037.4104
37.....	9,807.0086	12.7001	1,135.5848	17,384.6095	16,864.1332
38.....	9,794.3085	13.6826	1.069.9191	16,249.0247	15,758.6452
39.....	9,780.6259	14.7590	1,007.9475	15,179.1057	14,717.1297
40.....	9,765.8669	15.9476	949.4590	14,171.1581	13,735.9894
41.....	9,749.9193	17.4426	894.2533	13,221.6991	12,811.8330
42.....	9,732.4767	19.4650	842.1260	12,327.4458	11,941.4714
43.....	9,713.0117	21.9514	792.8695	11,485.3199	11,121.9213
44.....	9,691.0603	24.8963	746.2997	10,692.4503	10,350.3963
45.....	9,666.1640	28.2446	702.2476	9,946.1507	9,624.2872
46.....	9,637.9194	31.9786	660.5619	9,243.9031	8,941.1456
47.....	9,605.9408	36.0607	621.1039	8,583.3412	8,298.6686
48.....	9,569.8801	40.4614	583.7474	7,962.2373	7,694.6864
49.....	9,529.4187	45.1695	548.3768	7,378.4898	7,127.1505
50.....	9,484.2492	50.1242	514.8844	6,830.1131	6,594.1244
51.....	9,434.1250	55.3501	483.1729	6,315.2287	6,093.7745
52.....	9,378.7749	60.7744	453.1491	5,832.0558	5,624.3625
53.....	9,318.0005	66.4094	424.7290	5,378.9067	5,184.2393
54.....	9,251.5911	72.2179	397.8320	4,954.1777	4,771.8381
55.....	9,179.3732	78.1991	372.3835	4,556.3457	4,385.6699
56.....	9,101.1741	84.2951	348.3125	4,183.9622	4,024.3190

TABLE 13—*Continued*

MALES—*Continued*

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
57.....	9,016.8790	90.5204	325.5532	3,835.6497	3,686.4379
58.....	8,926.3586	97.1991	304.0424	3,510.0965	3,370.7438
59.....	8,829.1595	105.2789	283.7091	3,206.0541	3,076.0208
60.....	8,723.8806	114.4486	264.4587	2,922.3450	2,801.1348
61.....	8,609.4320	124.3202	246.2163	2,657.8863	2,545.0372
62.....	8,485.1118	134.5994	228.9254	2,411.6700	2,306.7459
63.....	8,350.5124	145.4074	212.5415	2,182.7446	2,085.3298
64.....	8,205.1050	157.4150	197.0193	1,970.2032	1,879.9027
65.....	8,047.6900	171.0939	182.3014	1,773.1839	1,689.6291
66.....	7,876.5961	186.2263	168.3261	1,590.8825	1,513.7330
67.....	7,690.3698	202.3798	155.0438	1,422.5563	1,351.4946
68.....	7,487.9900	218.5594	142.4185	1,267.5126	1,202.2374
69.....	7,269.4306	235.7840	130.4355	1,125.0941	1,065.3111
70.....	7,033.6466	253.9569	119.0611	994.6586	940.0889
71.....	6,779.6897	271.2418	108.2663	875.5975	825.9754
72.....	6,508.4479	285.2457	98.0517	767.3311	722.3908
73.....	6,223.2022	295.5337	88.4475	669.2794	628.7410
74.....	5,927.6685	303.6211	79.4785	580.8319	544.4042
75.....	5,624.0474	310.9705	71.1392	501.3533	468.7478
76.....	5,313.0769	319.1459	63.4016	430.2141	401.1550
77.....	4,993.9310	329.2199	56.2200	366.8125	341.0450
78.....	4,664.7111	338.6347	49.5413	310.5925	287.8860
79.....	4,326.0764	344.7537	43.3442	261.0512	241.1851
80.....	3,981.3227	348.0910	37.6321	217.7070	200.4589
81.....	3,633.2317	346.7738	32.3980	180.0749	165.2258
82.....	3,286.4579	340.7761	27.6469	147.6769	135.0054
83.....	2,945.6818	330.8089	23.3776	120.0300	109.3153
84.....	2,614.8729	316.7029	19.5775	96.6524	87.6794
85.....	2,298.1700	298.9966	16.2324	77.0749	69.6350
86.....	1,999.1734	278.5148	13.3213	60.8424	54.7369
87.....	1,720.6586	255.8860	10.8164	47.5212	42.5636
88.....	1,464.7726	232.1460	8.6867	36.7047	32.7233
89.....	1,232.6266	207.9552	6.8962	28.0180	24.8573
90.....	1,024.6714	183.8793	5.4083	21.1218	18.6431
91.....	840.7921	160.1617	4.1865	15.7136	13.7948
92.....	680.6304	137.2702	3.1972	11.5270	10.0617
93.....	543.3602	115.7281	2.4079	8.3298	7.2262
94.....	427.6321	96.8736	1.7878	5.9219	5.1025
95.....	330.7585	79.7671	1.3045	4.1341	3.5362
96.....	250.9914	64.1230	0.9339	2.8296	2.4016
97.....	186.6864	51.0503	0.6553	1.8957	1.5953
98.....	135.8181	39.4094	0.4498	1.2404	1.0342
99.....	96.4087	29.8023	0.3012	0.7906	0.6526
100.....	66.6064	21.9685	0.1963	0.4894	0.3995
101.....	44.6379	15.7328	0.1241	0.2931	0.2363
102.....	28.9051	10.9036	0.0758	0.1690	0.1343
103.....	18.0015	7.3123	0.0445	0.0932	0.0728
104.....	10.6892	4.7193	0.0250	0.0487	0.0372
105.....	5.9699	2.8965	0.0131	0.0237	0.0177
106.....	3.0734	1.6576	0.0064	0.0106	0.0076
107.....	1.4158	0.8581	0.0028	0.0042	0.0029
108.....	0.5577	0.3834	0.0010	0.0014	0.0009
109.....	0.1743	0.1369	0.0003	0.0004	0.0002
110.....	0.0374	0.0374	0.0001	0.0001	0.0000

TABLE 14

1971 GAM COMMUTATION FUNCTIONS AT 7 PER CENT

MALES

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
5.....	10,000.0000	4.5600	7,129.8620	107,046.0288	103,778.1756
6.....	9,995.4400	4.2381	6,660.3840	99,916.1668	96,863.4909
7.....	9,991.2019	4.0264	6,222.0187	93,255.7828	90,404.0244
8.....	9,987.1755	3.9150	5,812.6274	87,033.7641	84,369.6433
9.....	9,983.2605	3.8835	5,430.2326	81,221.1367	78,732.2803
10.....	9,979.3770	3.8919	5,073.0096	75,790.9042	73,465.7749
11.....	9,975.4851	3.9603	4,739.2814	70,717.8946	68,545.7240
12.....	9,971.5248	4.0385	4,427.4766	65,978.6131	63,949.3531
13.....	9,967.4863	4.1165	4,136.1528	61,551.1365	59,655.3999
14.....	9,963.3698	4.2046	3,863.9669	57,414.9837	55,643.9989
15.....	9,959.1652	4.3123	3,609.6602	53,551.0168	51,896.5893
16.....	9,954.8529	4.4200	3,372.0535	49,941.3566	48,395.8322
17.....	9,950.4329	4.5473	3,150.0526	46,569.3032	45,125.5291
18.....	9,945.8856	4.6845	2,942.6290	43,419.2506	42,070.5457
19.....	9,941.2011	4.8314	2,748.8253	40,476.6216	39,216.7434
20.....	9,936.3697	4.9980	2,567.7471	37,727.7963	36,550.9122
21.....	9,931.3717	5.1842	2,398.5566	35,160.0492	34,060.7108
22.....	9,926.1875	5.3999	2,240.4715	32,761.4926	31,734.6099
23.....	9,920.7876	5.6151	2,092.7596	30,521.0211	29,561.8397
24.....	9,915.1725	5.8599	1,954.7431	28,428.2615	27,532.3377
25.....	9,909.3126	6.1339	1,825.7830	26,473.5185	25,636.7013
26.....	9,903.1787	6.4370	1,705.2830	24,647.7355	23,866.1475
27.....	9,896.7417	6.7694	1,592.6866	22,942.4524	22,212.4711
28.....	9,889.9723	7.1406	1,487.4740	21,349.7659	20,668.0070
29.....	9,882.8317	7.5406	1,389.1589	19,862.2919	19,225.5941
30.....	9,875.2911	7.9891	1,297.2888	18,473.1330	17,878.5423
31.....	9,867.3020	8.4858	1,211.4386	17,175.8442	16,620.6015
32.....	9,858.8162	9.0307	1,131.2119	15,964.4056	15,445.9335
33.....	9,849.7855	9.6331	1,056.2390	14,833.1937	14,349.0841
34.....	9,840.1524	10.2928	986.1738	13,776.9547	13,324.9583
35.....	9,829.8596	11.0291	920.6938	12,790.7808	12,368.7962
36.....	9,818.8305	11.8219	859.4960	11,870.0871	11,476.1514
37.....	9,807.0086	12.7001	802.3002	11,010.5910	10,642.8701
38.....	9,794.3085	13.6826	748.8422	10,208.2909	9,865.0715
39.....	9,780.6259	14.7590	698.8749	9,459.4486	9,139.1310
40.....	9,765.8669	15.9476	652.1685	8,760.5737	8,461.6632
41.....	9,749.9193	17.4426	608.5079	8,108.4052	7,829.5058
42.....	9,732.4767	19.4650	567.6816	7,499.8973	7,239.7099
43.....	9,713.0117	21.9514	529.4825	6,932.2157	6,689.5362
44.....	9,691.0603	24.8963	493.7251	6,402.7332	6,176.4425
45.....	9,666.1640	28.2446	460.2399	5,909.0081	5,698.0648
46.....	9,637.9194	31.9786	428.8739	5,448.7682	5,252.2010
47.....	9,605.9408	36.0607	399.4869	5,019.8942	4,836.7961
48.....	9,569.8801	40.4614	371.9506	4,620.4074	4,449.9300
49.....	9,529.4187	45.1695	346.1477	4,248.4567	4,089.8057
50.....	9,484.2492	50.1242	321.9691	3,902.3090	3,754.7399
51.....	9,434.1250	55.3501	299.3154	3,580.3399	3,443.1537
52.....	9,378.7749	60.7744	278.0929	3,281.0245	3,153.5653
53.....	9,318.0005	66.4094	258.2157	3,002.9316	2,884.5828
54.....	9,251.5911	72.2179	239.6032	2,744.7159	2,634.8978
55.....	9,179.3732	78.1991	222.1802	2,505.1127	2,403.2801
56.....	9,101.1741	84.2951	205.8761	2,282.9325	2,188.5726

TABLE 14—*Continued*

MALES—*Continued*

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
57	9,016.8790	90.5204	190.6255	2,077.0564	1,989.6863
58	8,926.3586	97.1991	176.3662	1,886.4308	1,805.5963
59	8,829.1595	105.2789	163.0334	1,710.0646	1,635.3410
60	8,723.8806	114.4486	150.5509	1,547.0312	1,478.0287
61	8,609.4320	124.3202	138.8559	1,396.4803	1,332.8381
62	8,485.1118	134.5994	127.8979	1,257.6245	1,199.0046
63	8,350.5124	145.4074	117.6347	1,129.7265	1,075.8106
64	8,205.1050	157.4150	108.0246	1,012.0919	962.5806
65	8,047.6900	171.0939	99.0207	904.0673	858.6828
66	7,876.5961	186.2263	90.5752	805.0466	763.5330
67	7,690.3698	202.3798	82.6484	714.4714	676.5909
68	7,487.9900	218.5594	75.2088	631.8230	597.3523
69	7,269.4306	235.7840	68.2370	556.6142	525.3389
70	7,033.6466	253.9569	61.7044	488.3772	460.0960
71	6,779.6897	271.2418	55.5855	426.6728	401.1961
72	6,508.4479	285.2457	49.8707	371.0873	348.2299
73	6,223.2022	295.5337	44.5655	321.2165	300.7907
74	5,927.6685	303.6211	39.6720	276.6511	258.4681
75	5,624.0474	310.9705	35.1776	236.9790	220.8560
76	5,313.0769	319.1459	31.0584	201.8015	187.5664
77	4,993.9310	329.2199	27.2830	170.7431	158.2384
78	4,664.7111	338.6347	23.8172	143.4601	132.5439
79	4,326.0764	344.7537	20.6432	119.6429	110.1815
80	3,981.3227	348.0910	17.7552	98.9998	90.8620
81	3,633.2317	346.7738	15.1428	81.2446	74.3041
82	3,286.4579	340.7761	12.8014	66.1017	60.2344
83	2,945.6818	330.8089	10.7234	53.3003	48.3854
84	2,614.8729	316.7029	8.8964	42.5769	38.4994
85	2,298.1700	298.9966	7.3074	33.6805	30.3313
86	1,999.1734	278.5148	5.9408	26.3731	23.6503
87	1,720.6586	255.8860	4.7787	20.4323	18.2421
88	1,464.7726	232.1460	3.8019	15.6537	13.9111
89	1,232.6266	207.9552	2.9900	11.8518	10.4813
90	1,024.6714	183.8793	2.3230	8.8617	7.7970
91	840.7921	160.1617	1.7814	6.5388	5.7223
92	680.6304	137.2702	1.3477	4.7574	4.1396
93	543.3602	115.7281	1.0055	3.4096	2.9488
94	427.6321	96.8736	0.7396	2.4041	2.0651
95	330.7585	79.7671	0.5346	1.6645	1.4195
96	250.9914	64.1230	0.3792	1.1299	0.9561
97	186.6864	51.0503	0.2636	0.7507	0.6299
98	135.8181	39.4094	0.1792	0.4871	0.4050
99	96.4087	29.8023	0.1189	0.3079	0.2534
100	66.6064	21.9685	0.0768	0.1891	0.1539
101	44.6379	15.7328	0.0481	0.1123	0.0903
102	28.9051	10.9036	0.0291	0.0642	0.0509
103	18.0015	7.3123	0.0169	0.0351	0.0274
104	10.6892	4.7193	0.0094	0.0182	0.0139
105	5.9699	2.8965	0.0049	0.0088	0.0065
106	3.0734	1.6576	0.0024	0.0039	0.0028
107	1.4158	0.8581	0.0010	0.0015	0.0011
108	0.5577	0.3834	0.0004	0.0005	0.0003
109	0.1743	0.1369	0.0001	0.0001	0.0001
110	0.0374	0.0374	0.0000	0.0000	0.0000

TABLE 15

1971 GAM COMMUTATION FUNCTIONS AT 3½ PER CENT

FEMALES

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
5.....	10,000.0000	2.3400	8,419.7318	229,006.8544	225,147.8108
6.....	9,997.6600	1.9295	8,133.1030	220,587.1226	216,859.4505
7.....	9,995.7305	1.6194	7,856.5540	212,454.0195	208,853.0991
8.....	9,994.1111	1.4291	7,589.6437	204,597.4656	201,118.8790
9.....	9,992.6820	1.3390	7,331.9405	197,007.8219	193,647.3493
10.....	9,991.3430	1.3189	7,083.0512	189,675.8814	186,429.4830
11.....	9,990.0241	1.4286	6,842.6245	182,592.8301	179,456.6274
12.....	9,988.5955	1.5482	6,610.2860	175,750.2057	172,720.4914
13.....	9,987.0473	1.6678	6,385.7598	169,139.9197	166,213.1132
14.....	9,985.3795	1.7974	6,168 7859	162,754.1599	159,926.7998
15.....	9,983.5821	1.9268	5,959.1068	156,585.3740	153,854.1168
16.....	9,981.6553	2.0463	5,756.4799	150,626.2672	147,987.8807
17.....	9,979.6090	2.1755	5,560.6762	144,869.7873	142,321.1441
18.....	9,977.4335	2.3048	5,371.4628	139,309.1111	136,847.1907
19.....	9,975.1287	2.4439	5,188.6203	133,937.6483	131,559.5308
20.....	9,972.6848	2.5929	5,011.9315	128,749.0280	126,451.8928
21.....	9,970.0919	2.7418	4,841.1869	123,737.0965	121,518.2193
22.....	9,967.3501	2.9105	4,676.1889	118,895.9096	116,752.6565
23.....	9,964.4396	3.0790	4,516.7377	114,219.7207	112,149.5493
24.....	9,961.3606	3.2573	4,362.6493	109,702.9830	107,703.4355
25.....	9,958.1033	3.4555	4,213.7418	105,340.3337	103,409.0355
26.....	9,954.6478	3.6633	4,069.8354	101,126.5919	99,261.2508
27.....	9,950.9845	3.8809	3,930.7611	97,056.7565	95,255.1578
28.....	9,947.1036	4.1181	3,796.3556	93,125.9955	91,385.9992
29.....	9,942.9855	4.3749	3,666.4579	89,329.6398	87,649.1800
30.....	9,938.6106	4.6612	3,540.9128	85,663.1819	84,040.2636
31.....	9,933.9494	4.9571	3,419.5672	82,122.2691	80,554.9676
32.....	9,928.9923	5.2921	3,302.2810	78,702.7019	77,189.1565
33.....	9,923.7002	5.6466	3,188.9091	75,400.4209	73,938.8376
34.....	9,918.0536	6.0302	3,079.3185	72,211.5118	70,800.1575
35.....	9,912.0234	6.4527	2,973.3780	69,132.1933	67,769.3951
36.....	9,905.5707	6.9141	2,870.9588	66,158.8152	64,842.9592
37.....	9,898.6566	7.4240	2,771.9371	63,287.8564	62,017.3853
38.....	9,891.2326	7.9822	2,676.1915	60,515.9193	59,289.3316
39.....	9,883.2504	8.5885	2,583.6056	57,839.7279	56,655.5753
40.....	9,874.6619	9.2625	2,494.0681	55,256.1223	54,113.0078
41.....	9,865.3994	9.9936	2,407.4673	52,762.0542	51,658.6318
42.....	9,855.4058	10.7818	2,323.6991	50,354.5869	49,289.5582
43.....	9,844.6240	11.6758	2,242.6637	48,030.8879	47,003.0004
44.....	9,832.9482	12.6451	2,164.2550	45,788.2242	44,796.2740
45.....	9,820.3031	13.7190	2,088.3785	43,623.9692	42,666.7957
46.....	9,806.5841	14.8962	2,014.9382	41,535.5907	40,612.0773
47.....	9,791.6879	16.1955	1,943.8430	39,520.6524	38,629.7244
48.....	9,775.4924	17.6154	1,875.0028	37,576.8094	36,717.4331
49.....	9,757.8770	19.1937	1,808.3324	35,701.8066	34,872.9876
50.....	9,738.6833	20.9479	1,743.7444	33,893.4741	33,094.2580
51.....	9,717.7354	22.5841	1,681.1533	32,149.7297	31,379.2012
52.....	9,695.1513	24.4317	1,620.5278	30,468.5765	29,725.8346
53.....	9,670.7196	26.4785	1,561.7817	28,848.0487	28,132.2321
54.....	9,644.2411	28.7591	1,504.8363	27,286.2670	26,596.5504
55.....	9,615.4820	31.3080	1,449.6124	25,781.4307	25,117.0251
56.....	9,584.1740	34.2538	1,396.0314	24,331.8183	23,691.9706

TABLE 15—*Continued*

FEMALES—*Continued*

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
57	9,549.9202	37.7031	1,344.0019	22,935.7869	22,319.7861
58	9,512.2171	41.7396	1,293.4259	21,591.7850	20,998.9648
59	9,470.4775	46.4149	1,244.2032	20,298.3591	19,728.0993
60	9,424.0626	51.7286	1,196.2371	19,054.1559	18,505.8805
61	9,372.3340	57.6961	1,149.4405	17,857.9188	17,331.0919
62	9,314.6379	64.2524	1,103.7339	16,708.4782	16,202.6002
63	9,250.3855	71.3390	1,059.0535	15,604.7443	15,119.3448
64	9,179.0465	79.0132	1,015.3488	14,545.6908	14,080.3226
65	9,100.0333	87.0236	972.5688	13,530.3420	13,084.5813
66	9,013.0097	95.2225	930.6939	12,557.7732	12,131.2052
67	8,917.7872	103.6336	889.7209	11,627.0793	11,219.2906
68	8,814.1536	113.4998	849.6439	10,737.3585	10,347.9384
69	8,700.6538	125.8202	810.3411	9,887.7146	9,516.3083
70	8,574.8336	141.2875	771.6162	9,077.3735	8,723.7161
71	8,433.5461	160.2374	733.2389	8,305.7573	7,969.6895
72	8,273.3087	181.2765	694.9830	7,572.5184	7,253.9846
73	8,092.0322	203.2071	656.7683	6,877.5355	6,576.5167
74	7,888.8251	225.8728	618.6237	6,220.7672	5,937.2313
75	7,662.9523	248.1647	580.5906	5,602.1435	5,336.0395
76	7,414.7876	269.9576	542.7905	5,021.5529	4,772.7739
77	7,144.8300	291.2876	505.3416	4,478.7624	4,247.1475
78	6,853.5424	311.6443	468.3472	3,973.4207	3,758.7616
79	6,541.8981	331.1247	431.9329	3,505.0735	3,307.1043
80	6,210.7734	348.3312	396.2031	3,073.1407	2,891.5476
81	5,862.4422	362.6096	361.3353	2,676.9376	2,511.3256
82	5,499.8326	373.6367	327.5223	2,315.6023	2,165.4879
83	5,126.1959	381.1378	294.9486	1,988.0800	1,852.8952
84	4,745.0581	386.7269	263.7863	1,693.1314	1,572.2294
85	4,358.3312	388.6717	234.0942	1,429.3451	1,322.0519
86	3,969.6595	386.9147	206.0076	1,195.2509	1,100.8308
87	3,582.7448	381.3904	179.6410	989.2433	906.9078
88	3,201.3544	372.0806	155.0898	809.6022	738.5194
89	2,829.2738	359.0150	132.4293	654.5125	593.8157
90	2,470.2588	342.3211	111.7149	522.0832	470.8806
91	2,127.9377	321.7271	92.9795	410.3683	367.7527
92	1,806.2106	298.1639	76.2529	317.3888	282.4396
93	1,508.0467	272.0531	61.5124	241.1359	212.9428
94	1,235.9936	243.9221	48.7106	179.6236	157.2979
95	992.0715	214.4154	37.7755	130.9130	113.5992
96	777.6561	184.2812	28.6098	93.1375	80.0247
97	593.3749	153.1257	21.0919	64.5277	54.8606
98	440.2492	123.3741	15.1197	43.4359	36.5060
99	316.8751	96.5452	10.5146	28.3161	23.4969
100	220.3299	73.0680	7.0638	17.8015	14.5639
101	147.2619	53.2147	4.5616	10.7377	8.6470
102	94.0472	37.0703	2.8147	6.1761	4.8860
103	56.9769	24.5209	1.6476	3.3614	2.6063
104	32.4560	15.3037	0.9068	1.7139	1.2982
105	17.1523	8.9054	0.4630	0.8071	0.5949
106	8.2469	4.7416	0.2151	0.3441	0.2455
107	3.5053	2.2446	0.0883	0.1290	0.0885
108	1.2607	0.9038	0.0307	0.0407	0.0266
109	0.3569	0.2878	0.0084	0.0100	0.0061
110	0.0691	0.0691	0.0016	0.0016	0.0009

TABLE 16

1971 GAM COMMUTATION FUNCTIONS AT 6 PER CENT

FEMALES

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
5.....	10,000.0000	2.3400	7,472.5818	129,684.2744	126,259.3412
6.....	9,997.6600	1.9295	7,047.9559	122,211.6926	118,981.3796
7.....	9,995.7305	1.6194	6,647.7317	115,163.7367	112,116.8598
8.....	9,994.1111	1.4291	6,270.4290	108,516.0050	105,642.0585
9.....	9,992.6820	1.3390	5,914.6532	102,245.5760	99,534.6934
10.....	9,991.3430	1.3189	5,579.1138	96,330.9228	93,773.8291
11.....	9,990.0241	1.4286	5,262.6202	90,751.8090	88,339.7749
12.....	9,988.5955	1.5482	4,964.0260	85,489.1889	83,214.0104
13.....	9,987.0473	1.6678	4,682.3176	80,525.1628	78,379.1007
14.....	9,985.3795	1.7974	4,416.5430	75,842.8453	73,818.5965
15.....	9,983.5821	1.9268	4,165.8001	71,426.3022	69,516.9773
16.....	9,981.6553	2.0463	3,929.2416	67,260.5022	65,459.5998
17.....	9,979.6090	2.1755	3,706.0718	63,331.2606	61,632.6444
18.....	9,977.4335	2.3048	3,495.5319	59,625.1888	58,023.0700
19.....	9,975.1287	2.4439	3,296.9099	56,129.6568	54,618.5732
20.....	9,972.6848	2.5929	3,109.5303	52,832.7469	51,407.5456
21.....	9,970.0919	2.7418	2,932.7565	49,723.2166	48,379.0366
22.....	9,967.3501	2.9105	2,765.9905	46,790.4601	45,522.7145
23.....	9,964.4396	3.0790	2,608.6631	44,024.4696	42,828.8324
24.....	9,961.3606	3.2573	2,460.2425	41,415.8065	40,288.1954
25.....	9,958.1033	3.4555	2,320.2245	38,955.5641	37,892.1279
26.....	9,954.6478	3.6633	2,188.1315	36,635.3396	35,632.4460
27.....	9,950.9845	3.8809	2,063.5153	34,447.2081	33,501.4303
28.....	9,947.1036	4.1181	1,945.9534	32,383.6927	31,491.7975
29.....	9,942.9855	4.3749	1,835.0450	30,437.7394	29,596.6771
30.....	9,938.6106	4.6612	1,730.4129	28,602.6943	27,809.5885
31.....	9,933.9494	4.9571	1,631.6993	26,872.2815	26,124.4193
32.....	9,928.9923	5.2921	1,538.5709	25,240.5821	24,535.4038
33.....	9,923.7002	5.6466	1,450.7083	23,702.0113	23,037.1033
34.....	9,918.0536	6.0302	1,367.8140	22,251.3029	21,624.3882
35.....	9,912.0234	6.4527	1,289.6060	20,883.4889	20,292.4195
36.....	9,905.5707	6.9141	1,215.8175	19,593.8829	19,036.6333
37.....	9,898.6566	7.4240	1,146.1970	18,378.0654	17,852.7252
38.....	9,891.2326	7.9822	1,080.5069	17,231.8685	16,736.6361
39.....	9,883.2504	8.5885	1,018.5235	16,151.3615	15,684.5383
40.....	9,874.6619	9.2625	960.0363	15,132.8380	14,692.8214
41.....	9,865.3994	9.9936	904.8451	14,172.8017	13,758.0811
42.....	9,855.4058	10.7818	852.7627	13,267.9566	12,877.1071
43.....	9,844.6240	11.6758	803.6130	12,415.1940	12,046.8713
44.....	9,832.9482	12.6451	757.2263	11,611.5810	11,264.5189
45.....	9,820.3031	13.7190	713.4458	10,854.3546	10,527.3587
46.....	9,806.5841	14.8962	672.1218	10,140.9088	9,832.8530
47.....	9,791.6879	16.1955	633.1140	9,468.7870	9,178.6098
48.....	9,775.4924	17.6154	596.2895	8,835.6730	8,562.3737
49.....	9,757.8770	19.1937	561.5235	8,239.3836	7,982.0186
50.....	9,738.6833	20.9479	528.6972	7,677.8600	7,435.5405
51.....	9,717.7354	22.5841	497.6981	7,149.1628	6,921.0512
52.....	9,695.1513	24.4317	468.4353	6,651.4648	6,436.7652
53.....	9,670.7196	26.4785	440.8065	6,183.0294	5,980.9932
54.....	9,644.2411	28.7591	414.7165	5,742.2230	5,552.1446
55.....	9,615.4820	31.3080	390.0753	5,327.5064	5,148.7219
56.....	9,584.1740	34.2538	366.7974	4,937.4311	4,769.3156

TABLE 16—*Continued*

FEMALES—*Continued*

Age x	l_x	d_x	D_x	N_x	$N_x^{(12)}$
57	9,549.9202	37.7031	344.7986	4,570.6337	4,412.6010
58	9,512.2171	41.7396	323.9975	4,225.8351	4,077.3363
59	9,470.4775	46.4149	304.3167	3,901.8376	3,762.3591
60	9,424.0626	51.7286	285.6842	3,597.5209	3,466.5823
61	9,372.3340	57.6961	268.0341	3,311.8367	3,188.9877
62	9,314.6379	64.2524	251.3057	3,043.8026	2,928.6208
63	9,250.3855	71.3390	235.4455	2,792.4969	2,684.5844
64	9,179.0465	79.0132	220.4054	2,557.0514	2,456.0323
65	9,100.0333	87.0236	206.1398	2,336.6460	2,242.1653
66	9,013.0097	95.2225	192.6117	2,130.5062	2,042.2258
67	8,917.7872	103.6336	179.7894	1,937.8945	1,855.4910
68	8,814.1536	113.4998	167.6416	1,758.1050	1,681.2693
69	8,700.6538	125.8202	156.1159	1,590.4634	1,518.9103
70	8,574.8336	141.2875	145.1494	1,434.3475	1,367.8207
71	8,433.5461	160.2374	134.6771	1,289.1981	1,227.4711
72	8,273.3087	181.2765	124.6399	1,154.5210	1,097.3944
73	8,092.0322	203.2071	115.0084	1,029.8811	977.1690
74	7,888.8251	225.8728	105.7739	914.8728	866.3931
75	7,662.9523	248.1647	96.9296	809.0989	764.6729
76	7,414.7876	269.9576	88.4816	712.1693	671.6153
77	7,144.8300	291.2876	80.4341	623.6877	586.8221
78	6,853.5424	311.6443	72.7876	543.2536	509.8926
79	6,541.8981	331.1247	65.5451	470.4660	440.4245
80	6,210.7734	348.3312	58.7052	404.9208	378.0143
81	5,862.4422	362.6096	52.2761	346.2156	322.2558
82	5,499.8326	373.6367	46.2667	293.9395	272.7339
83	5,126.1959	381.1378	40.6826	247.6728	229.0266
84	4,745.0581	386.7269	35.5262	206.9902	190.7074
85	4,358.3312	388.6717	30.7838	171.4640	157.3548
86	3,969.6595	386.9147	26.4514	140.6803	128.5567
87	3,582.7448	381.3904	22.5219	114.2288	103.9063
88	3,201.3544	372.0806	18.9853	91.7069	83.0053
89	2,829.2738	359.0150	15.8290	72.7216	65.4666
90	2,470.2588	342.3211	13.0381	56.8926	50.9168
91	2,127.9377	321.7271	10.5956	43.8545	38.9982
92	1,806.2106	298.1639	8.4846	33.2589	29.3702
93	1,508.0467	272.0531	6.6830	24.7744	21.7113
94	1,235.9936	243.9221	5.1673	18.0914	15.7230
95	992.0715	214.4154	3.9128	12.9241	11.1307
96	777.6561	184.2812	2.8935	9.0113	7.6851
97	593.3749	153.1257	2.0829	6.1178	5.1631
98	440.2492	123.3741	1.4579	4.0349	3.3667
99	316.8751	96.5452	0.9899	2.5770	2.1233
100	220.3299	73.0680	0.6494	1.5871	1.2895
101	147.2619	53.2147	0.4094	0.9377	0.7501
102	94.0472	37.0703	0.2467	0.5283	0.4152
103	56.9769	24.5209	0.1410	0.2816	0.2170
104	32.4560	15.3037	0.0758	0.1406	0.1059
105	17.1523	8.9054	0.0378	0.0648	0.0475
106	8.2469	4.7416	0.0171	0.0271	0.0192
107	3.5053	2.2446	0.0069	0.0099	0.0068
108	1.2607	0.9038	0.0023	0.0031	0.0020
109	0.3569	0.2878	0.0006	0.0007	0.0005
110	0.0691	0.0691	0.0001	0.0001	0.0001

TABLE 17

COMPARISON OF MALE ANNUITY VALUES

A. $_{65-x}|\ddot{a}_x^{(12)}$

AGE x	MORTALITY TABLE*	INTEREST RATE				
		3½%	5%	6%	7%	i%†
25	Ga-1951	2.0922	1.0557	0.6754	0.4352	
	Ga-1971	2.3769	1.1925	0.7603	0.4883	
	1971 GAM	2.2848	1.1471	0.7318	0.4703	3.75
	Ga-1971 Proj. C	2.8791	1.4277	0.9037	0.5765	
	1971 GAM Proj. D	2.5686	1.2796	0.8125	0.5200	
30	Ga-1951	2.4954	1.3531	0.9077	0.6130	
	Ga-1971	2.8322	1.5269	1.0207	0.6871	
	1971 GAM	2.7229	1.4691	0.9827	0.6619	3.77
	Ga-1971 Proj. C	3.3629	1.7934	1.1910	0.7968	
	1971 GAM Proj. D	3.0205	1.6181	1.0778	0.7231	
35	Ga-1951	2.9806	1.7367	1.2216	0.8646	
	Ga-1971	3.3786	1.9573	1.3720	0.9679	
	1971 GAM	3.2490	1.8837	1.3211	0.9326	3.79
	Ga-1971 Proj. C	3.9284	2.2535	1.5702	1.1016	
	1971 GAM Proj. D	3.5552	2.0481	1.4310	1.0067	
40	Ga-1951	3.5684	2.2343	1.6479	1.2224	
	Ga-1971	4.0378	2.5137	1.8475	1.3660	
	1971 GAM	3.8840	2.4199	1.7796	1.3167	3.82
	Ga-1971 Proj. C	4.5939	2.8349	2.0725	1.5249	
	1971 GAM Proj. D	4.1922	2.5971	1.9035	1.4040	
45	Ga-1951	4.2918	2.8877	2.2331	1.7362	
	Ga-1971	4.8427	3.2397	2.4966	1.9348	
	1971 GAM	4.6606	3.1203	2.4060	1.8657	3.86
	Ga-1971 Proj. C	5.3880	3.5769	2.7437	2.1171	
	1971 GAM Proj. D	4.9614	3.3054	2.5413	1.9654	
50	Ga-1951	5.2174	3.7723	3.0588	2.4925	
	Ga-1971	5.8567	4.2103	3.4021	2.7631	
	1971 GAM	5.6415	4.0587	3.2816	2.6670	3.90
	Ga-1971 Proj. C	6.3705	4.5499	3.6620	2.9634	
	1971 GAM Proj. D	5.9243	4.2445	3.4233	2.7759	
55	Ga-1951	6.4499	5.0113	4.2606	3.6387	
	Ga-1971	7.1758	5.5434	4.6967	3.9980	
	1971 GAM	6.9229	5.3521	4.5373	3.8648	3.95
	Ga-1971 Proj. C	7.6347	5.8665	4.9544	4.2048	
	1971 GAM Proj. D	7.1755	5.5286	4.6775	3.9769	
60	Ga-1951	8.1517	6.8059	6.0673	5.4309	
	Ga-1971	8.9439	7.4247	6.5960	5.8846	
	1971 GAM	8.6515	7.1875	6.3890	5.6950	4.01
	Ga-1971 Proj. C	9.3224	7.7070	6.8297	6.0793	
	1971 GAM Proj. D	8.8609	7.3420	6.5162	5.8090	

* Ga-1951: 1951 Group Annuity Table (unprojected); Ga-1971: projected Ga-1951 Table (Scale C) to calendar year 1971 and unprojected thereafter; 1971 GAM: 1971 Group Annuity Mortality Table (unprojected); Ga-1971 Proj. C: projected Ga-1951 Table to 1971 and fully projected thereafter (Scale C); 1971 GAM Proj. D: 1971 Group Annuity Mortality Table fully projected (Scale D).

† Interest rate needed with 1971 GAM to produce Ga-1951, 3½ per cent value.

TABLE 17—*Continued*

B. $\ddot{a}_x^{(12)}$

Age x	Mortality Table*	Interest Rate				
		3½%	5%	6%	7%	i%†
55	G*a*-1951	14.4201	12.4839	11.4293	10.5236	
	G*a*-1971	15.2546	13.1149	11.9583	10.9703	
	1971 GAM	14.9760	12.9005	11.7773	10.8168	3.90
	G*a*-1971 Proj. C	15.7283	13.4512	12.2284	11.1886	
	1971 GAM Proj. D	15.2370	13.0844	11.9244	10.9354	
60	G*a*-1951	12.5676	11.0766	10.2461	9.5213	
	G*a*-1971	13.4002	11.7340	10.8120	10.0111	
	1971 GAM	13.0935	11.4832	10.5920	9.8175	4.00
	G*a*-1971 Proj. C	13.7809	12.0183	11.0478	10.2078	
	1971 GAM Proj. D	13.3042	11.6389	10.7203	9.9240	
65	G*a*-1951	10.6337	9.5405	8.9178	8.3661	
	G*a*-1971	11.4245	10.1914	9.4932	8.8765	
	1971 GAM	11.1386	9.9440	9.2683	8.6718	4.13
	G*a*-1971 Proj. C	11.7009	10.4081	9.6786	9.0357	
	1971 GAM Proj. D	11.2953	10.0650	9.3709	8.7591	
70	G*a*-1951	8.7561	7.9916	7.5470	7.1468	
	G*a*-1971	9.4322	8.5694	8.0699	7.6214	
	1971 GAM	9.2346	8.3860	7.8958	7.4564	4.35
	G*a*-1971 Proj. C	9.6057	8.7116	8.1950	7.7318	
	1971 GAM Proj. D	9.3406	8.4711	7.9698	7.5210	
75	G*a*-1951	7.0011	6.4927	6.1912	5.9163	
	G*a*-1971	7.4893	6.9234	6.5888	6.2837	
	1971 GAM	7.5112	6.9309	6.5892	6.2783	4.82
	G*a*-1971 Proj. C	7.5781	6.9992	6.6572	6.3457	
	1971 GAM Proj. D	7.5765	6.9853	6.6375	6.3214	
80	G*a*-1951	5.4718	5.1460	4.9491	4.7688	
	G*a*-1971	5.7566	5.4045	5.1924	4.9964	
	1971 GAM	5.9329	5.5540	5.3268	5.1175	5.36
	G*a*-1971 Proj. C	5.7894	5.4335	5.2193	5.0212	
	1971 GAM Proj. D	5.9680	5.5841	5.3542	5.1424	
85	G*a*-1951	4.2990	4.0913	3.9629	3.8454	
	G*a*-1971	4.4020	4.1866	4.0548	3.9315	
	1971 GAM	4.6841	4.4390	4.2899	4.1508	5.94
	G*a*-1971 Proj. C	4.4075	4.1917	4.0596	3.9360	
	1971 GAM Proj. D	4.6997	4.4529	4.3027	4.1627	
90	G*a*-1951	3.3821	3.2511	3.1697	3.0929	
	G*a*-1971	3.3821	3.2511	3.1697	3.0929	
	1971 GAM	3.6991	3.5433	3.4471	3.3564	6.71
	G*a*-1971 Proj. C	3.3821	3.2511	3.1697	3.0929	
	1971 GAM Proj. D	3.7039	3.5478	3.4513	3.3604	

* G*a*-1951: 1951 Group Annuity Table (unprojected); G*a*-1971: projected G*a*-1951 Table (Scale C) to calendar year 1971 and unprojected thereafter; 1971 GAM: 1971 Group Annuity Mortality Table (unprojected); G*a*-1971 Proj. C: projected G*a*-1951 Table to 1971 and fully projected thereafter (Scale C); 1971 GAM Proj. D: 1971 Group Annuity Mortality Table fully projected (Scale D).

† Interest rate needed with 1971 GAM to produce G*a*-1951, 3½ per cent value.

TABLE 17—*Continued*

C. ${}_{65-x}E_x \cdot \ddot{a}^{(12)}_{\overline{65:10}|}$

Age x	Mortality Table*	Interest Rate				
		3½%	5%	6%	7%	i%†
25	Ga-1951	2.3228	1.1737	0.7512	0.4842	
	Ga-1971	2.5711	1.2919	0.8240	0.5295	
	1971 GAM	2.5012	1.2579	0.8029	0.5162	3.72
	Ga-1971 Proj. C	3.0042	1.4914	0.9445	0.6029	
	1971 GAM Proj. D	2.7463	1.3705	0.8708	0.5576	
30	Ga-1951	2.7704	1.5043	1.0095	0.6820	
	Ga-1971	3.0637	1.6542	1.1064	0.7451	
	1971 GAM	2.9809	1.6110	1.0781	0.7265	3.73
	Ga-1971 Proj. C	3.5190	1.8793	1.2487	0.8358	
	1971 GAM Proj. D	3.2368	1.7371	1.1578	0.7773	
35	Ga-1951	3.3090	1.9308	1.3586	0.9619	
	Ga-1971	3.6547	2.1205	1.4871	1.0496	
	1971 GAM	3.5567	2.0655	1.4494	1.0237	3.74
	Ga-1971 Proj. C	4.1242	2.3693	1.6518	1.1595	
	1971 GAM Proj. D	3.8191	2.2040	1.5410	1.0847	
40	Ga-1951	3.9617	2.4841	1.8328	1.3599	
	Ga-1971	4.3677	2.7233	2.0025	1.4813	
	1971 GAM	4.2519	2.6535	1.9524	1.4451	3.77
	Ga-1971 Proj. C	4.8399	2.9912	2.1880	1.6107	
	1971 GAM Proj. D	4.5145	2.8018	2.0533	1.5165	
45	Ga-1951	4.7649	3.2105	2.4837	1.9315	
	Ga-1971	5.2384	3.5098	2.7061	2.0980	
	1971 GAM	5.1021	3.4215	2.6397	2.0478	3.80
	Ga-1971 Proj. C	5.6979	3.7886	2.9077	2.2449	
	1971 GAM Proj. D	5.3566	3.5751	2.7505	2.1284	
50	Ga-1951	5.7924	4.1941	3.4021	2.7729	
	Ga-1971	6.3352	4.5613	3.6875	2.9963	
	1971 GAM	6.1759	4.4506	3.6002	2.9272	3.83
	Ga-1971 Proj. C	6.7645	4.8390	3.8970	3.1553	
	1971-GAM Proj. D	6.4133	4.6031	3.7149	3.0140	
55	Ga-1951	7.1607	5.5715	4.7388	4.0481	
	Ga-1971	7.7621	6.0055	5.0907	4.3352	
	1971 GAM	7.5786	5.8688	4.9780	4.2420	3.87
	Ga-1971 Proj. C	8.1424	6.2668	5.2956	4.4970	
	1971-GAM Proj. D	7.7894	6.0123	5.0900	4.3300	
60	Ga-1951	9.0501	7.5668	6.7483	6.0418	
	Ga-1971	9.6747	8.0437	7.1493	6.3810	
	1971 GAM	9.4710	7.8814	7.0095	6.2602	3.90
	Ga-1971 Proj. C	9.9892	8.2721	7.3349	6.5326	
	1971-GAM Proj. D	9.6465	8.0073	7.1111	6.3427	

* Ga-1951: 1951 Group Annuity Table (unprojected); Ga-1971: projected Ga-1951 Table (Scale C) to calendar year 1971 and unprojected thereafter; 1971 GAM: 1971 Group Annuity Mortality Table (unprojected); Ga-1971 Proj. C: projected Ga-1951 Table to 1971 and fully projected thereafter (Scale C); 1971 GAM Proj. D: 1971 Group Annuity Mortality Table fully projected (Scale D).

† Interest rate needed with 1971 GAM to produce Ga-1951, 3½ per cent value.

TABLE 17—*Continued*

D. $\ddot{a}^{(12)}_{x:\overline{10}|}$

Age x	Mortality Table*	Interest Rate				
		3½%	5%	6%	7%	i%†
55	Ga-1951	14.9223	12.9417	11.8577	10.9263	
	Ga-1971	15.6483	13.4738	12.2938	11.2856	
	1971 GAM	15.3953	13.2825	12.1344	11.1524	3.84
	Ga-1971 Proj. C	16.1072	13.7969	12.5515	11.4924	
	1971 GAM Proj. D	15.6479	13.4590	12.2746	11.2645	
60	Ga-1951	13.3222	11.7635	10.8896	10.1262	
	Ga-1971	13.9947	12.2751	11.3185	10.4872	
	1971 GAM	13.7506	12.0812	11.1519	10.3437	3.88
	Ga-1971 Proj. C	14.3530	12.5395	11.5358	10.6666	
	1971 GAM Proj. D	13.9488	12.2257	11.2698	10.4405	
65	Ga-1951	11.8055	10.6071	9.9186	9.3072	
	Ga-1971	12.3580	11.0409	10.2896	9.6253	
	1971 GAM	12.1936	10.9040	10.1684	9.5180	3.95
	Ga-1971 Proj. C	12.6017	11.2283	10.4476	9.7590	
	1971 GAM Proj. D	12.3331	11.0096	10.2566	9.5920	
70	Ga-1951	10.5175	9.5959	9.0540	8.5653	
	Ga-1971	10.8882	9.8944	9.3134	8.7911	
	1971 GAM	10.8531	9.8604	9.2808	8.7601	4.01
	Ga-1971 Proj. C	11.0222	10.0010	9.4053	8.8705	
	1971 GAM Proj. D	10.9395	9.9279	9.3383	8.8093	
75	Ga-1951	9.5666	8.8321	8.4051	7.9892	
	Ga-1971	9.7489	8.9817	8.5232	8.1051	
	1971 GAM	9.8293	9.0440	8.5760	8.1498	4.00
	Ga-1971 Proj. C	9.8007	9.0240	8.5603	8.1377	
	1971 GAM Proj. D	9.8747	9.0804	8.6075	8.1772	

* Ga-1951: 1951 Group Annuity Table (unprojected); Ga-1971: projected Ga-1951 Table (Scale C) to calendar year 1971 and unprojected thereafter; 1971 GAM: 1971 Group Annuity Mortality Table (unprojected); Ga-1971 Proj. C: projected Ga-1951 Table to 1971 and fully projected thereafter (Scale C); 1971 GAM Proj. D: 1971 Group Annuity Mortality Table fully projected (Scale D).

† Interest rate needed with 1971 GAM to produce Ga-1951, 3½ per cent value.

or all classes of business. Assuming the minimum valuation standard for new contracts were to be based on the unprojected 1971 GAM Table, the last column of Table 17 shows the interest rate needed to produce annuity values equal to the present minimum standard, Ga-1951 unprojected at 3½ per cent interest. The changes which occur among these relationships with advancing age are of interest.

Deferred annuities with payments commencing at age 65.—At ages 50 and under, life annuity, no certain period, values based on the 1971 GAM fully projected by Scale D are greater than the values based on the

G*a*-1951 projected to 1971 by Scale C. At ages 55 and over the relationship reverses. For life annuities with a 10-year-certain period the same reversal occurs, but at a higher age. The age at which the reversal occurs depends on the interest rate.

Immediate annuities due.—At ages 70 and under, life annuity, no certain period, values based on the 1971 GAM unprojected and fully projected by Scale D are less than the values based on the G*a*-1951 projected to 1971 by Scale C. At ages 75 and over the relationship reverses (except at the 7 per cent interest level where, with respect to the 1971 GAM unprojected values, the reversal occurs later). At ages 75 and under, life annuity, no certain period, values based on the 1971 GAM unprojected and fully projected by Scale D are less than the values based on the G*a*-1951 projected to 1971 and fully projected thereafter by Scale C. At ages 80 and over the relationship reverses. For life annuities with a 10-year-certain period the same patterns are evident, but the reversals occur at younger ages.

Table 18 demonstrates the relative magnitudes of aggregate retired life reserves under several valuation bases. The tabular values are based on immediate life annuities due, payable monthly, no death benefit, and are based on the distribution of the 1966 intercompany retired life exposures by amount of annual income.

The relatively small variation between the four bases other than the unprojected G*a*-1951 is interesting. Particularly noteworthy is the closeness of the results based on the G*a*-1951 projected 20 years by Scale C and the 1971 GAM fully projected by Scale D. Also, it is evident that a change in interest rates to 5 per cent together with a change to a current mortality basis would not cure the surplus strain problem; a much higher interest rate would be needed before there is noticeable relief.

USE OF MALE AGE SETBACK FOR FEMALES

Although the 1971 GAM includes a female mortality table, most companies probably would prefer to use the male table with an age adjustment for females. This practice is much more convenient, is quicker, and is less expensive than using a completely separate table for females. Thus the G*a*-1951 female table has had very little use and is no longer even being used in the annual intercompany mortality studies. The main reason for constructing the 1971 GAM table for females is to determine a suitable age rating of the male table for females. Table 19 compares male and female annuity values at two interest rates, $3\frac{1}{2}$ per cent and 6 per cent, and on two mortality bases, the fully projected

1971 GAM and the 1971 GAM projected 10 years and fully projected thereafter by Scale D.

From Table 19, a uniform male age setback of 6 years for females seems appropriate. Some companies might prefer to use several age setbacks, depending on attained age. If the 1971 GAM is adopted as an acceptable valuation standard, companies should be allowed reasonable flexibility in this respect.

CONCLUSION

Nearly twenty years have elapsed since Ray Peterson presented his excellent study "Group Annuity Mortality." A tribute to the care with which his study was done is how remarkably well the G*a*-1951 Table

TABLE 18

COMPARISON OF AGGREGATE MALE RETIRED LIFE RESERVES UNDER DIFFERENT VALUATION BASES

$$\frac{\sum_{t=0}^{5} f_{63+5t} \cdot \ddot{a}^{(12)}_{63+5t}\ i.\ \text{Mort. Table}}{\sum_{t=0}^{5} f_{63+5t} \cdot \ddot{a}^{(12)}_{63+5t}\ 3\frac{1}{2}\%,\ Ga\text{-}1951}$$

Mortality Table*	Interest Rate			
	3½%	5%	6%	7%
G*a*-1951	100.00%	91.71%	86.87%	82.51%
G*a*-1971	107.12	97.83	92.43	87.57
1971 GAM	106.14	96.87	91.50	86.68
G*a*-1971 Proj. C	108.86	99.25	93.69	88.68
1971 GAM Proj. D	107.25	97.77	92.29	87.36

* G*a*-1951: 1951 Group Annuity Table (unprojected); G*a*-1971: projected G*a*-1951 Table (Scale C) to calendar year 1971 and unprojected thereafter; 1971 GAM: 1971 Group Annuity Mortality Table (unprojected); G*a*-1971 Proj. C: projected G*a*-1951 Table to 1971 and fully projected thereafter (Scale C); 1971 GAM Proj. D: 1971 Group Annuity Mortality Table fully projected (Scale D).

FREQUENCY DISTRIBUTION OF AMOUNT OF ANNUITY INCOME
(1966 INTERCOMPANY GROUP ANNUITY MATURED LIFE EXPERIENCE FOR RETIREMENT ON OR AFTER NORMAL RETIREMENT DATE)

	Age					
	63	68	73	78	83	88
f_{63+5t}	12.904%	41.896%	27.754%	12.472%	3.895%	1.075%

performed. Nevertheless, the vanishing margins and shift in the age distribution of business have raised doubts as to the continued suitability of the G*a*-1951 Table for valuation purposes. The 1971 GAM Table is intended to correct these distortions.

At the time of adopting a new mortality table for valuation purposes, new interest rates should also be considered. Under present conditions, and considering the outlook for the next few years, a much higher interest rate than the current 3½ per cent appears eminently reasonable for minimum valuation standard purposes. Companies should also be allowed reasonable flexibility with respect to the recognition of mortality improvement and with respect to the valuation of benefits for females.

TABLE 19

COMPARISON OF 1971 GAM PROJECTION D MALE AND FEMALE ANNUITY VALUES

A. ANNUITY VALUES IN CALENDAR YEAR 1971

Age x	Female $\ddot{a}_x^{(12)}$	Male $\ddot{a}_{x-t}^{(12)}$			r*	Frequency† Distribution (Per Cent)
		$t=5$	$t=6$	$t=7$		
		3½ Per Cent Interest Rate				
58	16.7202	15.9738	16.3350	16.6915	7.1	5.4196
63	14.6824	14.0928	14.4798	14.8611	6.5	22.6793
68	12.4916	12.1026	12.5049	12.9055	6.0	38.2396
73	10.2259	10.1044	10.4963	10.8937	5.3	21.1078
78	8.1489	8.2656	8.6154	8.9728	4.7	8.9299
83	6.3389	6.5757	6.9004	7.2357	4.2	2.7778
88	4.7794	5.1668	5.4199	5.6866	3.4	0.8459
					$\bar{r}=5.8$	
		6 Per Cent Interest Rate				
58	12.8489	12.3583	12.5661	12.7682	7.4	5.4196
63	11.6439	11.2233	11.4643	11.6978	6.8	22.6793
68	10.2313	9.9249	10.1951	10.4603	6.1	38.2396
73	8.6451	8.5269	8.8080	9.0897	5.4	21.1078
78	7.0977	7.1667	7.4306	7.6976	4.7	8.9299
83	5.6750	5.8461	6.1055	6.3709	4.3	2.7778
88	4.3869	4.6940	4.9040	5.1238	3.4	0.8459
					$\bar{r}=6.0$	

* Where r is chosen such that female $\ddot{a}_x^{(12)}$ = male $\ddot{a}_{x-r}^{(12)}$; $\bar{r}$ is the weighted average of r values.

† Frequency distribution of amount of annuity income based on 1966 intercompany group annuity female matured life experience for retirement on or after normal retirement date.

TABLE 19—*Continued*

B. ANNUITY VALUES IN CALENDAR YEAR 1981

AGE x	FEMALE $\ddot{a}_x^{(12)}$	MALE $\ddot{a}_{x-t}^{(12)}$			r*	FREQUENCY† DISTRIBUTION (PER CENT)
		$t=5$	$t=6$	$t=7$		
		3½ Per Cent Interest Rate				
58.....	17.0409	16.1770	16.5361	16.8901	7.4	5.4196
63.....	15.0243	14.3016	14.6880	15.0685	6.9	22.6793
68.....	12.8372	12.3071	12.7114	13.1134	6.3	38.2396
73.....	10.5519	10.2905	10.6873	11.0891	5.7	21.1078
78.....	8.4183	8.4189	8.7761	9.1405	5.0	8.9299
83.....	6.5238	6.6922	7.0242	7.3669	4.5	2.7778
88.....	4.8735	5.2456	5.5062	5.7804	3.5	0.8459
					$\bar{r}=6.2$	
		6 Per Cent Interest Rate				
58.....	13.0388	12.4807	12.6855	12.8844	7.8	5.4196
63.....	11.8636	11.3586	11.5973	11.8285	7.2	22.6793
68.....	10.4708	10.0669	10.3366	10.6007	6.5	38.2396
73.....	8.8880	8.6642	8.9473	9.2305	5.8	21.1078
78.....	7.3112	7.2858	7.5542	7.8353	5.1	8.9299
83.....	5.8294	5.9409	6.2054	6.4757	4.6	2.7778
88.....	4.4693	4.7609	4.9767	5.2022	3.6	0.8459
					$\bar{r}=6.4$	

* Where r is chosen such that female $\ddot{a}_x^{(12)}$ = male $\ddot{a}_{x-r}^{(12)}$; $\bar{r}$ is the weighted average of r values.

† Frequency distribution of amount of annuity income based on 1966 intercompany group annuity female matured life experience for retirement on or after normal retirement date.

ACKNOWLEDGMENTS

The authors are greatly indebted to Mr. Peterson for his pioneering work, and in the course of this study they referred to and relied on his study more than might be evident. Also, the authors would like to express their gratitude to the various people in many companies who offered help and encouragement. Particular thanks are due Stephen Margolies, Varnum Abbot, Jr., Harold Moulton, Ernest Heyde, and Jonathan Schwartz, who contributed significantly to this project.

DISCUSSION OF PRECEDING PAPER

HARRISON GIVENS, JR.:

The last major study of group annuity mortality, which produced the G*a*-1951 Table, was conducted almost twenty years ago; the margins in that table have now disappeared; and the rate of mortality improvement currently experienced is both quite different from that of a decade ago and an uncertain guide at best for the future. Hence a current examination is certainly welcome.

The Present

It is certainly regrettable that reporting errors have crept into the Society's reports on group annuity mortality. As a result, the most recent information available is for 1967. This or a combination with 1966 is a natural starting point for representing current experience. The route taken by the authors to reach a "1966 Experience" Table introduces a little uncertainty in allowing quantitatively for the probable under-reporting of 1968 mortality. The 1966 Experience Table (Table 7) is 2 per cent lighter than the reported 1966 experience in aggregate and in the important age ranges 61–75 and 81–85. In part this is because the reported 1968 experience is light, but, if it is no more so than the originally reported 1967 experience (about 2 per cent) or 1966 experience (about 3 per cent), this accounts for only about ½ per cent overall when submerged in a five-year block of experience. Since, after corrections, the reported 1966 experience well represents that for 1964–67, it is natural to wonder how the other 1½ per cent margin crept into the 1966 Experience Table.

As to active employee mortality, we recently made a fairly careful analysis of the intercompany 1960–64 group life mortality experience in order to reach the male component. Compared with the male experience of Table 5, our results would produce 27 per cent more deaths overall and in the important age range 50–59, and 40 per cent more deaths in the age range 60–64. It is natural to feel uneasy at what may be a distinctly rich margin in valuing group deferred annuities.

The Future

In analyzing current rates of improvement, the authors have reasonably put aside the relatively strong improvement experienced in the 1950's in favor of the slight degree of improvement experienced in the

1960's. Different starting and end points in the 1960's would have produced somewhat different scales, but the route taken is reasonable to get a representation of the current trend. It would be helpful to have a clearer measure of how well Projection Scale D reproduces that trend.

In any case, current trends are an uncertain guide for the future. Discussion should be encouraged of the various factors that may change mortality experience significantly in the future. For example, if present standards of medicine and public health are maintained without further change, how much more improvement can be expected from deferred effects? What continuing improvements in medicine and public health are required to continue recent improvements in mortality? Where are improvements likely to come from? For example, will they be more in the area of preventive care than in that of curative care? Thus, at what ages and in what calendar period will their results be observable? To what extent may countertrends be developing because of increasing population density, pollution, rising social unrest, crime, and the psychological pressure of massive social and economic changes amid uncertainty? Finally, it would be helpful to interpret any proffered projection scale in terms of the extent to which it provides financial coverage for various age-specific improvements in the major causes of death after retirement.

Margins

Even from the viewpoint solely of statistical fluctuation, it is difficult to perceive the usefulness of Table 8. First, it is not clear whether the number of lives shown is the exposure for the five years 1964–68 at the single central age shown or the exposure for one year for the five-year age group to which the indicated age is central. In any event, the standard deviation obtained depends upon the number of lives, of course, and a different grouping would have led to different entries in columns 3 and 4. This would not matter if the composite figure for the whole experience were calculated by the usual formula for the variance of an aggregate of independent experiments. In that case two standard deviations for the aggregate of the male experience would be 1.6 per cent, rather than the 4.5 per cent displayed as the weighted average of column 4.

Second, and more important, the relevance of Table 8 is obscure because no single carrier has an experience as large as that of the intercompany study. Do the authors wish to suggest a margin for statistical fluctuation that is determined by the volume of business in force? If so, for a company with 10 per cent of the volume of business in the intercompany study, two standard deviations would be 5.1 per cent rather than 1.6 per cent; for a company with 1 per cent of the intercompany

volume, this would be 16 per cent. The 8 per cent margin chosen by the authors happens to provide a margin of two standard deviations for a company with 4 per cent of the intercompany volume.

Third, and more important still, the variation to be expected by industry, geographical location, character of employment, and so on, is more significant than that caused by purely statistical fluctuation. It would be desirable to have a current appraisal of this at least as developed as that contained in the presentation of the G*a*-1951 Table.

Fourth, and most important, an analysis is needed of the aggregate margin required for all mortality and interest contingencies: this would clearly be less than the sum of the margins needed for independent contingencies. Such a discussion is all too rare in our literature.

Valuation Standard

The omissions pointed out here would be serious in a paper that purported to be a fully developed analysis of present and future group annuity mortality. Such an analysis would indeed be most valuable. In this case, however, the authors have clearly pointed out that the 1971 Group Annuity Mortality Table was developed solely as a valuation standard. For this purpose it probably measures "current" experience well enough if the margins are removed, and these margins are presumably intended to allow a static current table to be used safely as a valuation table for an extended period, as has been the case with the G*a*-1951 Table. Indeed, the apparent 2 per cent margin noted in the 1966 Experience Table compared with the corrected 1966 experience reported, plus the explicit 7 per cent margin built into the 1971 table, produces a 9 per cent margin, which is strikingly close to the 10 per cent margin that was explicitly built into the G*a*-1951 Table.

The paper is most helpful in providing the tools for appraising the financial consequences of using this new table in conjunction with various rates of interest for a representative distribution of business. Table 18 indicates, for example, that the reserves required for male retired lives by the G*a*-1951 Table and $3\frac{1}{2}$ per cent interest, the common minimum standard today, would be about equal to those on the new table and $4\frac{1}{2}$ per cent interest. This is a clear indication of the strong margins introduced in comparison with the G*a*-1951 Table, and it underscores the importance of considering the new table as a valuation standard only in conjunction with the accompanying interest rate. While this is true in any circumstances, it is of the greatest importance in the present environment. It would be a substantial miscarriage of the authors' intent if the new table were mandated without a substantial liberalization in the

required maximum interest rate, since the composite effect is not to begin relief until the interest rate exceeds the 4½ per cent level.

WILLIAM H. CROSSON:

With the disappearance of mortality margins in the present statutory annuity mortality valuation standard, and the importance of obtaining some kind of relief from the present statutory maximum valuation interest rate, it is a highly propitious time for the emergence of a proposed new annuity valuation mortality table.

In reviewing the valuation mortality table proposed by Messrs. Greenlee and Keh, I was impressed by the magnitude of the mortality margin provided, and I was led to consider in a rather fundamental way the question of how margins should be developed for and included in a statutory minimum valuation standard. As a result of this consideration, I conclude that the proposed mortality margins are seriously redundant. (An elaboration of this theme constitutes the main body of this discussion.) Consequently, I believe that substantially more work is required before we can consider that we have a satisfactory proposed valuation standard.

I am also deeply concerned about the possibility that this new mortality table might be imposed without any relief whatever on the interest rate. This would be a terribly unfortunate and completely inappropriate development.

This discussion concludes with brief consideration of a few technical difficulties, and a brief note about female mortality and nomenclature.

Margins

The reserves produced by application of a statutory minimum valuation standard should be larger than the reserves that would be produced by application of an "expected experience valuation standard," by an amount called "margin." The purpose of margin is to compel an insurance company to retain in hand sufficient funds to provide for any and all divergences (in directions that tend to produce larger present values of annuities) of actual experience conditions from expected experience conditions that may reasonably be contemplated. Such divergences may take the form of (1) chance mortality fluctuations producing mortality lighter than expected, (2) mortality improvement at rates greater than expected, (3) chance interest-rate fluctuations producing interest rates lower than expected, and (4) interest rate decreases at rates greater than expected.

In an idealized situation, the margin required to provide for each of these contingencies separately could be determined. The total margin required to provide for all of these contingencies would obviously be larger than any of the separate margin requirements but would certainly be less than the sum of the separate margin requirements (since the possibility that all of the contingencies will materialize simultaneously is quite remote). If the chances of occurrence of each of the contingencies were independent, then the total margin requirement would be the vector sum of the separate margin requirements (the separate margin requirements being viewed as mutually perpendicular vectors). In symbols, if M_1, M_2, M_3, and M_4 are the separate margin requirements, then the total margin requirement M is

$$\sqrt{(M_1)^2 + (M_2)^2 + (M_3)^2 + (M_4)^2}\,.$$

If an "expected experience valuation standard" is composed of (1) a current experience mortality table, (2) a set or sets of mortality improvement factors, (3) a current experience interest rate, and (4) a set or sets of interest rate decrease factors, it would not be correct to derive from this a statutory minimum valuation standard merely by adjusting each of these four elements so as to cover, separately, the margins separately required for each of the corresponding four contingencies. On the other hand, it would be reasonable to modify the adjustments to each of the four elements in such a way that the total margin produced by the resulting statutory minimum valuation standard is equal to the total margin required and not to the sum of the separate margin requirements. (Failure to recognize the propriety of partial offsets of margins for independent contingencies can contribute, and has contributed somewhat, to the justifiable criticism that life insurance company reserves are substantially higher than they should be.)

As a practical example of how these kinds of considerations affect company reserving practices, I merely need to point out that many companies are using, as a reserve basis, one of the present statutory minimum valuation standards (G*a*-1951, $3\frac{1}{2}$ per cent), despite the absence of margin in the mortality table component of that standard, because the interest rate margin in that standard is clearly adequate to cover any reasonable variations in mortality and interest experience that could be contemplated.

In short, the point of this discussion so far is that if the statutory maximum interest rate, even though it may be increased, should continue to provide an interest rate margin that is clearly adequate to cover

the reasonable variations in mortality and interest experience that could be contemplated, then there is no need to have a margin in the mortality basis at all. In the event that the statutory maximum interest rate is increased to such a point as to provide only the required interest rate margins, then the margins in the statutory mortality basis need be sufficient to cover only a small fraction of the total mortality margin requirement. Also, it is impossible to decide whether the mortality margin is right unless we know the interest margins. I believe that it is quite likely that whatever statutory maximum interest rate is permitted, the interest margin will still be somewhat redundant with respect to the total margin requirement, and to have any margin in the mortality standard would be highly redundant. In the absence of knowledge as to what interest margins there will be, I will nevertheless examine the margins in the proposed mortality table and projection scales, keeping in mind that whatever mortality margins are required should be tempered when the interest margins are recognized.

As to the mortality table proposed as the valuation standard, mortality rate margins are proposed as 8 per cent for males and 10 per cent for females. The development of these particular margin needs is not completely laid out in the paper. The description of how the margins were developed would allow one to infer that these margins are intended to provide for mortality fluctuation, and very little margin is provided for mortality improvements. The data presented should lead to the conclusion that the mortality rate margin for fluctuations should be something like 5 per cent for males and 12 per cent for females, or appropriate multiples of these two numbers depending on the desired level of confidence that there be adequate margin. It is difficult to see how we can conclude that 8 per cent and 10 per cent are the right figures, particularly when it is easy to conclude that the proper multiples are 0 per cent and 0 per cent. (The confidence level shown in the paper is appropriate only for an experience as large as the intercompany experience. Any one insurance company will have a smaller experience, so that a company's level of confidence will be smaller than the confidence level shown.)

The paper proposes two mortality improvement projection scales for males and one for females, to be used at the company's option, to recognize expected mortality improvement. Projection Scale D is intended to represent a projection of recent past experience. It is not clear what Scale E is intended to represent. It is also not completely clear how either of these scales was developed. In considering the question of projection scales, it is appropriate and, I would think, essential to examine the

nature of what changes in mortality rates are likely to occur in the future. To consider what sort of margins for mortality improvement should be provided, it is necessary to examine the nature of the changes in mortality rates that could occur in the future with reasonable probability. For example, it would be quite appropriate to develop a projection scale by projecting the mortality improvement that is likely to occur or could reasonably be conceived of as occurring as a result of probable developments and reasonably possible developments in the prevention and treatment of cardiovascular-renal diseases and cancer.

A Few Technical Points

It is difficult to understand how the authors could have concluded, in Table 6, that the ratio of mortality ratios for females at ages 60 and under is 0.8992, when the mortality ratios themselves are 1.151 and 0.808, as presented in Table 1. It is certainly not clear why, in adjusting the graduation of the table, the negative second differences were not eliminated at ages 70 and 71, nor do I understand why the graduation was adjusted at ages 56, 57, 95, and 96 in order to eliminate negative second differences at ages 70–90.

The underreporting of deaths and exposures for 1968, referred to in the paper, presumably results in an overstatement of the improvement rates of Scales D and E. The underreporting was corrected, however, by adjusting the margins in the static table. It is certainly difficult to see why the adjustment was made in this way.

Female Annuity Values

While the paper is quite cogent in suggesting a six-year age setback from the male table for female retired lives, it says nothing about female deferred annuities. If we wish to use the male table, including the male Projection D or Projection E, for female deferred annuities, it is probable that a seven- or eight-year age setback of the male table would be appropriate.

Nomenclature

I will conclude this discussion with a brief note as to nomenclature. Several references to tables derived from the G*a*-1951 Table have appeared in the literature, and these tables have consistently been referred to in terms of the G*a*-1951 Table together with a brief description of the modification. This usage, as a result, reserves the term "G*a*-19XX Table" for the title of a mortality table for the year 19XX that is derived from a substantially independent investigation of mortality rates. In

accordance with this usage, the various tables appearing in the paper should be redesignated, and I suggest the following:

Title of Table in Paper	Proposed Title of Table
G*a*-1971	G*a*-1951, Projection C to 1971
G*a*-1971, Projection C	G*a*-1951, Projection C, age in 1971
1971 GAM	G*a*-1971
1971 GAM, Projection D	G*a*-1971, Projection D, age in 1971

JOHN C. ANTLIFF:

Table 1 of the paper demonstrates that the G*a*-1951 Mortality Table without projection is not suitable as a valuation standard for variable annuities, since the actual intercompany experience in 1964 to 1968 was 98.7 per cent of expected for males and 85.7 per cent of expected for females (rated down five years in G*a*-1951 Male Table). For fixed annuities

TABLE 1

Years of Birth	Generation Reserve Basis			Regulation No. 47 Reserve Basis	
	Male Age Ratedown (Years)	Female Age Ratedown (Years)	Reserve	Central Year of Retirement	Reserve
Up to 1925	3	9	$152.39	1983	$149.97
1926–1940	4	10	156.11	1998	154.68
1941–1955	5	11	159.88	2013	159.14
(etc.)					

the G*a*-1951 Table with the present maximum valuation interest rate of $3\frac{1}{2}$ per cent produces reserves which are overconservative in relation to single-sum annuity purchase rates which have been offered for several years and apparently will continue for some time to come. However, there is no possibility of excess interest earnings to cover mortality losses on variable annuities. This is recognized by the New York Insurance Department in Regulation No. 47. As a valuation standard for group variable annuities, the regulation specifies the G*a*-1951 Table projected to the year of retirement using Projection Scale C or "any other table approved by the Superintendent." In order to avoid using a new set of commutation functions for the new generation of retirees each year, my company is using the G*a*-1951 Male Table without projection but with age ratedowns on a progressive basis according to year of birth and sex, as shown in Table 1. The reserves shown in the table are for $1 per month payable for ten years certain and life to a male at age 65 with an assumed investment result of $3\frac{1}{2}$ per cent. Assuming a central year of

retirement 65 years after the central year of birth in each bracket, we find that our reserves will continue to be slightly more conservative than those specified in Regulation No. 47. The generation reserve basis for group variable annuities is analogous to the Progressive Annuity Table, which is specified in Regulation No. 47 as one of two alternate valuation standards for individual variable annuities.

If the 1971 GAM Table is adopted as the group annuity mortality valuation standard with a much higher interest rate than the present maximum of $3\frac{1}{2}$ per cent, it should be as suitable for variable annuities as for fixed annuities. In other words, the mortality margin of 8 per cent for males and 10 per cent for females should be adequate for variable annuities, just as for fixed annuities, until such time as actual future mortality improvement eliminates most of the margin.

TABLE 2

Years of Birth	Age Ratings I (Years)		Years of Birth	Age Ratings II (Years)	
	Male	Female		Male	Female
Up to 1920.....	1	7	Up to 1925.....	1	7
1921–1935.....	2	8	1926–1940.....	2	8
1936–1950.....	3	9	1941–1955.....	3	9
(etc.)			(etc.)		

Nevertheless, it may be of interest to illustrate a set of age ratedowns which will closely approximate the effect of Projection Scale D on the 1971 GAM Male Table. As indicated by the authors, it might be of greater value to determine a set of age ratings which would approximate Projection Scale E, since Scale E seems to have more merit as an estimate of future mortality improvement than Scale D. However, the necessary commutation functions are available for Scale D but not yet for Scale E. Two possible sets of age ratedowns which are shown below to be close approximations of Projection Scale D are given in Table 2.

Table 3 shows reserves for $1 per year payable monthly for life, based on 6 per cent annual interest (or assumed investment result for a variable annuity) and the 1971 GAM Male or Female Table fully projected to the valuation date and beyond by Projection Scale D. The weights were obtained from the second portion of Table 18 of the paper, assuming a female percentage increasing steadily from 20 per cent at age 63 to 30 per cent at age 88.

The same model-office valuation was also done on the basis of the 1971

GAM Male Table unprojected and the two sets of age ratedowns defined above. Ratios of the resulting average reserves to those shown on the "All ages" line in Table 3 were as follows:

Age Ratings I	1.0250	1.0019	1.0065	1.0133
Age Ratings II	1.0250	0.9980	0.9990	1.0060

This approximation is very close. Similar ratios were obtained using $3\frac{1}{2}$ per cent interest instead of 6 per cent, as follows:

Age Ratings I	1.0293	1.0032	1.0094	1.0180
Age Ratings II	1.0293	0.9982	1.0005	1.0093

The second set of age ratedowns seems to be a closer approximation to Projection Scale D than the first set. By coincidence, the second set involves the same year-of-birth brackets as the generation reserve basis used by my company with the G*a*-1951 Table. However, the work of

TABLE 3

Attained Age	Weight by Amount in Force	Year of Valuation			
		1971	1986	2001	2016
		Males			
63	0.10323	$ 9.9249	$10.1366	$10.3404	$10.5364
68	0.32679	8.5269	8.7319	8.9310	9.1243
73	0.21094	7.1667	7.3448	7.5195	7.6906
78	0.09230	5.8461	5.9880	6.1282	6.2667
83	0.02805	4.6940	4.7942	4.8937	4.9924
88	0.00753	3.7741	3.8319	3.8895	3.9467
		Females			
63	0.02581	$11.6439	$11.9682	$12.2621	$12.5273
68	0.09217	10.2313	10.5857	10.9112	11.2088
73	0.06661	8.6451	9.0053	9.3410	9.6523
78	0.03243	7.0977	7.4151	7.7154	7.9982
83	0.01091	5.6750	5.9051	6.1260	6.3375
88	0.00323	4.3869	4.5099	4.6297	4.7461
All ages (male and female)	1.00000	$8.1481	$8.3691	$8.5807	$8.7831

developing a set of age ratedowns for the 1971 GAM Table with Projection Scale E remains to be done.

JOHN S. MC COY:

The efforts of Messrs. Greenlee and Keh in undertaking the task of updating the study of group pension mortality and producing the 1971 Group Annuity Mortality Table are appreciated. The Group Annuity Table for 1951 has served its turn well, due largely to the painstaking care which Ray Peterson gave to its preparation, but certainly a detailed review of group pensioner mortality experience was long overdue. A major deterrent to would-be successors of Mr. Peterson may well have been the knowledge that their efforts would be judged by comparison with his, and both the diligence of his effort and the longevity of his product made it a difficult act to follow.

The authors have done a workmanlike job, but there are several aspects of their study that I find somewhat questionable. The retired life experience involved in the study has been limited to that taken from the portion of the Intercompany Group Annuity Mortality Study applicable to retirements on and after normal retirement date. In the period 1964–68 the actual deaths among males for retirements in this classification constituted about 60 per cent of the total deaths under all male classifications. Since the effect of adverse mortality experience on retirements prior to normal retirement date and those under plans having no stated retirement date can be expected to disappear from the experience after a few years (the data reported in the intercompany study suggest that after attained age 70 the experience for all categories is becoming homogeneous), it seems that more data could have been used to study the experience at the older ages. In this connection it is well to note that liberalized benefits at early retirement in recent years have improved the caliber of lives coming into the early retirement experience. Although it may be true that the block of experience selected exhibited the lowest mortality rates, is it appropriate to exclude the source of additional mortality experience in making the study?

An important element in the construction of the new table is the scale of mortality improvement factors which has been developed from the intercompany experience of 1956–60 and 1964–68. This experience was also derived solely from the retirements on or after normal retirement date. I have some of the same misgivings here as in the area previously mentioned. It is especially difficult to accept the sparse data available for ages 60 and under as evidence that mortality improvement for all ages under 65 approximates that for ages 61–65 and is about half as much as it was assumed to be under Projection Scale C. This conclusion

is also contrary to some experience which has emerged under deferred annuity plans which we administer.

As a part of our annual reserve reconciliation work, we make comparisons of actual reserves released by death during a calendar year with the expected reserves released on the basis of tabular mortality. Since our group annuity reserves have been based on the Group Annuity Table for 1951 with full Projection C since 1963, these comparisons give some indication of the reliability of Projection Scale C. The figures show some minor fluctuations from year to year, but the ratio of actual to expected reserves released has had a downward trend, and the ratio for 1968 was about 98 per cent of that for 1963. Unfortunately, we develop only aggregate figures, and there is no separation between deferred and matured annuities. However, as the bulk of the reserves are for deferred annuities, it is reasonable to assume that the mortality improvement actually emerging for active lives is not significantly less than that of Projection Scale C.

The other aspect of the study that is difficult to accept with much confidence is the significance of the data used to derive mortality experience for the active life group. Should experience which is based on such limited data be the basis for a minimum valuation standard? This question seems especially pertinent because the annuity values shown in Table 17 indicate that reserves for deferred annuities on, say, the 1971 GAM Table are about 5 per cent weaker than those on the G*a*-1971 Table. Those on the 1971 GAM Projection D Table would appear to be about 8 per cent less than those on the G*a*-1971 Projection C Table. It seems to me that we should be wary of moving to a weaker standard from one which for the first time in history has managed to keep pace with mortality improvement unless there is strong evidence to support such a move.

One final point may bear mentioning. History indicates that mortality tables adopted for minimum valuation standards have a way of becoming used as standards for estimating the costs of pensions in the uninsured private pension plan sector. It would be ironic if a table such as this, which reflects only a portion of the mortality experience under insured group annuity plans, were to be put to use in the uninsured sector—a sector that, according to recent estimates, accounts for about $96 billion of the $135 billion in private pension plan funds. It is obvious that even if the authors had been able to use all the data available from the intercompany study, only by coincidence would their mortality table be applicable in the broader pension field.

The activities of the lawmakers in Washington and other interested parties around the country suggest to me that the time has come when actuaries should gather mortality data from the various private pension plans with which they are involved and carry out a comprehensive mortality study of both active and retired lives to produce mortality statistics to which they can refer with confidence. The authors deserve our appreciation for a paper which draws attention to this subject and may spur long-needed action on a broader front.

BARNET N. BERIN:

I have nothing but admiration for the authors' work in putting together the study and in developing the mortality table. These comments concern a point of interest and of education.

The theoretically correct basis for an annuity mortality study is a comparison of the actual reserves released by death with the expected reserves released by death—not amounts of annual income, and not lives. The problem of preparing a meaningful study of this kind for non-retired employees must be enormous. The problems associated with a study of retired lives only, probably less difficult, are still considerable.

Have the authors any information as to modern mortality studies based on reserves?

(AUTHORS' REVIEW OF DISCUSSION)

HAROLD R. GREENLEE AND ALFONSO D. KEH:

At the outset the authors would repeat an observation made in several of the discussions: the 1971 GAM Table is a valuation mortality table. The major comments on the 1971 GAM Table fall into questions concerning the data used, the margin selected, and the projection scales presented.

Data

In selecting the data, the authors wanted a relatively large volume which would be realistic but conservative, as befits a valuation mortality table. Since the "on and after normal retirement date" mortality rates are lower than those under the other two retirement date categories, and since the "on and after" data were significantly more numerous than the other two sets of data, the "on and after" data for the most recent five-year period available seemed to be a natural choice. A single year's experience was thought to be too small a sample for this undertaking. Mr. McCoy reasons that the authors could have used all the data at the higher ages after the effects of early retirement mortality had worn

off. The argument sounds reasonable; however, the data do not clearly indicate an emerging homogeneous experience even above age 70 (see Table 1).

Mr. Givens has observed that the resulting 1966 Experience Table, before adjustment, has a 2 per cent margin over the 1966 experience in most important retired life age ranges. The authors' investigation showed that the 1966 Experience Table rates should be increased by 0.5 per cent for males and 0.9 per cent for females to account for underreporting of exposures and deaths. A uniform increase of 1 per cent was deemed to be

TABLE 1

MALE MORTALITY RATIOS, 1964–68, BASED ON AMOUNT OF ANNUAL INCOME

AGE GROUP	RETIREMENT CATEGORY		
	On and After	Early	No Stated Date
61–65	102.7	134.3	136.0
66–70	99.0	114.2	104.4
71–75	100.0	109.8	104.8
76–80	96.1	94.9	100.3
81–85	97.6	104.7	92.2
86–90	97.2	106.9	95.8
91–95	99.7	125.6	119.1
96 and over	74.0	*	97.6

* Less than 10 deaths (actual or expected).

suitable for both sexes, and this increase was included in the final loading formula used to derive the 1971 GAM Table from the 1966 Experience Table. Another approach could have been to adjust the experience table first and then derive the 1971 GAM Table. With the 1 per cent adjustment, there would still be a 1 per cent difference between the 1966 Experience Table derived from the 1964–68 combined data and the 1966 data at the ages Mr. Givens mentioned.

The active life data presented a problem; the data were sparse. Mr. Givens suggests that the mortality rates from ages 50 to 64 are too low. As stated in the paper, the reason for not using group life mortality data was that there is no separation by sex of such data other than on an estimated basis. The authors were wary of using such estimates to produce mortality rates for the 1971 GAM Table and finally decided upon the method used as being the most reasonable one available. In struggling with the

problem of the active life rates, one comforting thought was that, with the dominance of various types of deposit administration contracts in today's world, active life mortality rates are much less important than the retired life rates in a valuation mortality table.

Margin

Mr. Crosson has prepared a thoughtful discussion of how margins should be developed. He expresses his concern that the new table might be adopted without any interest rate relief. Everyone in the group annuity business must share this concern, since, under this condition, minimum reserves under the 1971 GAM Table would be substantially higher than under the G*a*-1951 Table. The study was undertaken with the understanding that the table would be combined with realistic interest rates to produce a minimum valuation standard, and, therefore, redundant interest rates should not be considered in establishing a suitable margin. Also as Mr. Antliff notes, a company in the variable annuity business may not be able to rely on any interest rate margin with respect to its variable business. Thus, if there is to be a single valuation mortality table for both fixed-dollar and variable annuities, the margin adopted should consider the mortality fluctuation element only. The authors believe that any margin for mortality improvement at a faster rate than anticipated should be considered in deriving an appropriate mortality improvement projection scale.

Table 8 was developed by the authors to provide some idea of the extent of fluctuations which could occur in intercompany experience. An individual company's experience could, of course, fluctuate more widely because of its smaller exposure and also could vary because of the nature of its business. The number of lives shown in Table 8 is the exposure at the indicated central age for the five-year period 1964–68. The weighted average was determined with the thought that a single margin factor would be applied at all ages rather than a factor varying by age. Furthermore, the assumption was made that, while mortality rates at specific ages can fluctuate independently, a valuation mortality table should provide for the possibility of fluctuation in mortality rates at all ages all in the same direction at the same time, not independently of one another. The step from Table 8 to the 8 per cent margin for males and 10 per cent for females was made after noting that, while Mr. Peterson's 10 per cent margin for males apparently had been a good choice, now 10 per cent seemed overly conservative for males, and perhaps a little light, but satisfactory, for females.

If a company's experience is likely to differ significantly from the intercompany experience because of the nature of its business, it is the company's actuary's responsibility to establish a more conservative reserve basis than the minimum standard if such action is in order. Some companies could find the minimum basis conservative (and will, if the realistic interest rate assumption turns out to be unwarranted), but to set a lower standard to accommodate such companies could lead other companies into unsound practices. It may be of interest to note the effect of the 8 per cent margin on male life annuity values at $3\frac{1}{2}$ per cent and 6 per cent interest (see Table 2).

TABLE 2

MALE LIFE ANNUITY VALUES

MALE AGE	$3\frac{1}{2}$ Per Cent			6 Per Cent		
	1971 GAM (12) $\ddot{a}_x$	Unloaded 1971 GAM (12) $\ddot{a}_x$	Ratio Loaded/ Unloaded	1971 GAM (12) $\ddot{a}_x$	Unloaded 1971 GAM (12) $\ddot{a}_x$	Ratio Loaded/ Unloaded
55.....	14.9760	14.6384	1.0231	11.7773	11.5734	1.0176
60.....	13.0935	12.7391	1.0278	10.5920	10.3623	1.0222
65.....	11.1386	10.7757	1.0337	9.2683	9.0169	1.0279
70.....	9.2346	8.8758	1.0404	7.8958	7.6317	1.0346
75.....	7.5112	7.1715	1.0474	6.5892	6.3258	1.0416
80.....	5.9329	5.6188	1.0559	5.3268	5.0714	1.0504
85.....	4.6841	4.4036	1.0637	4.2899	4.0528	1.0585
90.....	3.6991	3.4561	1.0703	3.4471	3.2350	1.0656

Projection Scales

Mr. Givens has prepared an impressive list of factors which may affect future mortality rates; Mr. Crosson added several more. The authors concluded, while preparing the paper, that it was best to assume that there would be no spectacular breakthroughs in such fields as cancer cure, heart disease, and slowing the aging process in the near future. While substantial progress in one or more of these fields may be a good long-term prospect, the authors did not feel themselves capable of making any reliable estimate of when the advances might come. On the other hand, forces are operating in the opposite direction, as Mr. Givens suggests. Recent group life insurance experience indicates that significant increases in mortality rates from accidents are occurring, particularly at the younger ages. The *New York Times* on November 21, 1971, reported on a recent study published by the National Center for Health Statistics.

It states that male death rates for most American men rose in the late 1960's. This information was not available to the authors at the time of preparing the table. The article mentions the higher death rate from accidents at the younger ages (war deaths are excluded), increased incidence of lung cancer, circulatory diseases, and cirrhosis of the liver, apparently at most or all male ages. In view of the information the authors did have when preparing the paper, the decision was made to consider recent experience in deriving mortality improvement projection scales. Scale E, in particular, follows this experience. The authors believe that the improvement factor at the younger ages is adequate in the light of the information now emerging from group life insurance experience and the above-mentioned report. Any data that Mr. McCoy could present in this area would be most welcome. With respect to older age mortality improvement, Mr. Cherry presents an interesting discussion in his companion paper, "The 1971 Individual Annuity Mortality Table." While he concludes that Projection Scale B is appropriate for individual annuity purposes, one can infer from his discussion that use of recent experience to determine a suitable projection scale for retired lives is a reasonable approach at this time. The actuary who does not agree with the authors' approach can use one of the other existing scales or develop a new scale which he may deem to be a better representation of probable future experience.

Additional Comments

The authors share Mr. Crosson's difficulty in understanding how they arrived at the Table 6 entry he questions; suffice it to say that a correction has been made.

As mentioned in the paper, negative second differences near male age 70 seemed to be a feature of these data as well as of Mr. Peterson's. The authors could have eliminated the negatives but decided not to do so. The adjustments at ages 56, 57, 95, and 96 were made to remove an anomaly induced by the graduation process in the ratios at these end points rather than to eliminate negative second differences, although without the adjustment there would have been a negative second difference at age 95 too.

The authors are not aware of any table based on a comparison of actual reserves released by death with expected reserves released by death. In view of the difficulties encountered in gathering active life data, it is doubtful whether any such active life study could be made. Many companies might have to revise their retired life systems to produce suitable retired life data for the study Mr. Berin suggests.

Conclusion

The authors would like to thank Mr. Antliff for his fine analysis of an age rating system which will reproduce the effects of Projection Scale D. It is indeed a valuable addition to the paper. Thanks, too, to those who remarked kindly on the authors' efforts. Finally, thanks to the loyal opposition who, through their questions and discussions, not only have contributed to the value of the paper but also have pointed out several possible areas of further research.