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MORTALITY OF THE AGED

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ABSTRACT

This paper presents a new analysis of mortality at the older ages in the United States. The analysis is based principally on data obtained for calendar years 1968–69 in the operations of the Medicare program. In addition, the author develops mortality rates by single ages at the very high ages (85 and over), using the "extinct cohort" method based on deaths recorded through the vital statistics system in calendar years 1951–68.

The Medicare data used, which include virtually all the aged persons residing in the United States, indicate that the Bureau of the Census estimate of the aged population as of July 1, 1968, may be too low by about 2.2 per cent. This estimated understatement is found to be more significant for females than for males and to vary according to age.

Comparisons with vital statistics death data issued by the National Center for Health Statistics show these data to be of acceptable accuracy, except at the extreme older ages (100 and over). The social security mortality rates for nonwhite persons are shown to be significantly different from the vital statistics rates. The mortality rates of white and nonwhite persons are found to be closer together in the social security data. On the other hand, mortality differentials by sex according to social security data are slightly higher than those obtained from vital statistics data.

From the substantial and reliable social security data, it is established that mortality rates tend to increase with age at a decelerated rate after age 85 or age 90, contrary to the general assumption of constant geometric increases. This observation is corroborated by the age pattern of the death rates computed using the "extinct cohort" method.

A comparison with group annuity tables shows those tables to contain mortality rates that are higher at the very high ages than those for the total population of the United States, after the mortality margins that are included in such tables are taken into account.

INTRODUCTION

One of the least known mortality patterns in the United States is that of the aged population. Most of the published mortality data at the higher ages are combined in age group 75 and over, or 85 and over, and, in the few cases in which the data are subdivided into smaller age groups, no claim is made about their reliability. In fact, most textbooks in the demographic field include specific sections wherein the unreliability of the basic mortality data at the higher ages is discussed. These deal with problems of coverage—that is, the completeness of registration of deaths and enumeration of population, problems of misstatement of age (generally overstatement of age by the less educated groups), and problems of comparability of census data with death registration data.

The degree of lack of confidence of many demographers and actuaries in the quality of data at the older ages is such that most official life tables are ended at the upper ages by a purely mathematical procedure or by substituting a set of known rates for the actual experience. As an example of this, all the official United States 1959–61 Life Tables were ended by using the Union Civil War veterans experience[1] at ages 95 and over, and by blending the rates for Union Civil War veterans with those for the actual data from age 85 to age 94. This follows one of the procedures that is recommended in the Society of Actuaries textbook *Elements of Graduation*, by Morton D. Miller. Other methods recommended in the book include fitting a cubic to the last three graduated values and setting $q_w = 1$ at an arbitrary limiting age, and also extending the graduated series of q_x's by assuming a geometric progression with a ratio of about 1.1.

In some instances it has been possible to secure highly reliable data, but such information has been rather scanty and covers too wide a period in time. Myers and Shudde,[2] for example, based their mortality study on highly accurate data comprising 515 life-years of exposure and 209 deaths, covering fiscal years 1946–54.

Over many years the social security program has developed a substantial quantity of reliable data on the mortality of the aged. Some of

[1] Robert J. Myers and Louis O. Shudde, "Mortality Experience of Union Civil War Veterans," *TSA*, VII, 63–68.

[2] *Ibid.*

these data have been analyzed and published,[3] but they pertain only to those receiving social security benefits. It was not until Medicare began operations in 1966 that the program had extensive data on almost the totality of the aged population residing in the United States, since initially Medicare was designed to include virtually the entire group of aged population who were not "insured." Thus it is possible to obtain a relatively clear picture of the mortality patterns of the older population in the United States from Medicare statistics. This paper presents the first analysis of such statistics.

SOURCES AND CHARACTERISTICS OF THE DATA

The present mortality study is based on data gathered in the operations of the Medicare program. The observation period covers the two calendar years 1968 and 1969. All the data used in the study refer to the update of January 1, 1971. Events occurring during the observation period but recorded after January 1, 1971, are not included in the data. It is believed that the number of these events is small and that they would not have a significant effect on the analysis in this study.

The Medicare data come from the two separate programs—Hospital Insurance, generally covering hospitalization costs, and Supplementary Medical Insurance, generally covering physician fees. A person is included in these data if he is covered under either program or both. These two programs cover a geographic area that consists of the fifty states, the District of Columbia, and Puerto Rico and other outlying areas. The data for this study, however, were limited to residents of the fifty states and the District of Columbia, which area is hereafter referred to as "the United States." Not all the residents of the above area aged 65 and over are entitled to Medicare. Besides the residence requirement, other requirements must be met.

For Hospital Insurance entitlement the individual must be (1) entitled to social security monthly cash benefits or (2) entitled to railroad retirement benefits or (3) a citizen or an alien admitted for permanent residence (who has resided in the United States for five or more years), with at least three quarters of coverage under social security for every year elapsed after 1966 and before the year of attainment of age 65 (thus, for persons who attained age 65 before 1968, no quarters of cover-

[3] See Robert J. Myers and Francisco Bayo, "Mortality of Workers Entitled to Old-Age Benefits under OASDI," *TSA*, XVII, 417–31, and Francisco Bayo, "Mortality and Remarriage Experience for Widow Beneficiaries under OASDI," *TSA*, XXI, 59–80.

age are required). Excluded are those federal employees who meet only the last requirement and who could have enrolled under the health insurance plans for government employees.

For Supplementary Medical Insurance enrollment the requirements for entitlement are the same as indicated above, except that (1) the eligible individuals must elect to enroll, (2) all federal employees are eligible to enroll, and (3) there is no quarter-of-coverage requirement.

From the above it will be observed that all individuals aged 65 and over residing in the United States are entitled to Medicare except for the following:

1. Federal employees who could have enrolled under the health insurance plans for government employees who are not entitled to cash benefits under social security or railroad retirement, and who have elected not to enroll in the Supplementary Medical Insurance program (estimated at about 120,000 persons on July 1, 1968).
2. Aliens not entitled to social security or railroad retirement benefits who have not been admitted for permanent residence or who have been admitted for permanent residence but have resided in the United States for less than five years (estimated at about 50,000 persons on July 1, 1968).
3. Citizens, or aliens admitted for permanent residence who have resided in the United States for five or more years, who are not social security or railroad retirement beneficiaries, who have less than three quarters of coverage under social security for every year elapsed after 1966 and before the year of attainment of age 65, and who have elected not to enroll in the Supplementary Medical Insurance program (estimated to be less than 5,000 individuals on July 1, 1968).

The estimates given above of persons excluded from the Medicare data, when added to the number of persons entitled to Medicare, provide an acceptable estimate of the population aged 65 and over residing in the United States. Such an estimate, as of July 1, 1968, is presented in Table 1. The total estimate of 19,564,000 is 430,000 higher than the comparable estimate published by the Bureau of the Census in *Current Population Reports,* Series P-25, No. 441. This represents a difference of about 2.2 per cent, which is lower than most of the previously published estimates of the net 1960 census undercount of the aged population.

A comparison by age and sex of the Medicare-entitled population and the census-estimated number of residents as of July 1, 1968, is shown in Table 2. In analyzing this comparison, it should be remembered that, according to our best estimates, there were about 170,000 aged persons in the United States who were not entitled to Medicare as of that date. The table shows that the difference between census and social security data is greater for females than for males. Regarding differences by age, it should be observed that the census estimates of male population are higher than social security data indicate at all ages except at ages 70–74, where there is a substantially lower census estimate. This substantially lower census estimate at ages 70–74 is also evident for females. We do not have a full explanation for this unexpected difference; it might be due partially to the smoothing procedures used by the Bureau of the Census in the preparation of population estimates.

TABLE 1

ESTIMATED UNITED STATES POPULATION AGED 65 AND OVER ON JULY 1, 1968, ACCORDING TO ELIGIBILITY FOR SOCIAL SECURITY BENEFITS

(In Thousands)

Benefit Eligibility	Number of Persons
Entitled to OASDI or railroad retirement monthly benefits	17,146
Entitled to special monthly benefits for persons aged 72 and over	923
Not entitled to any of foregoing monthly benefits	1,325
Total eligible for Medicare benefits	19,394
Ineligible for Medicare benefits	170
Total population	19,564

TABLE 2

COMPARISON OF ESTIMATED UNITED STATES POPULATION ON JULY 1, 1968, CENSUS ESTIMATE AND SOCIAL SECURITY DATA

(In Thousands)

Age Last Birthday	Census Estimate*		Social Security Data†		Ratio	
	Male	Female	Male	Female	Col. (2) to Col. (4)	Col. (3) to Col. (5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
65–69	3,027	3,624	2,967	3,608	1.020	1.004
70–74	2,210	2,947	2,315	3,114	0.955	0.946
75–79	1,599	2,264	1,581	2,307	1.011	0.981
80–84	898	1,340	853	1,364	1.053	0.982
85 and over	467	758	444	837	1.052	0.906
Total	8,201	10,933	8,160	11,230	1.005	0.974

* From *Current Population Reports*, Series P-25, No. 441.

† Estimated by interpolation from data for the beginning and end of calendar year 1968, excluding 4,000 cases of unknown sex.

The significantly higher census estimates for males at ages 80–84 and at ages 85 and over could conceivably be due to the tendency of aged persons to overstate their age. However, this is not the case for females, particularly at ages 85 and over, where the census estimate is considerably lower than social security data would indicate.

A further idea of the closeness of the social security data to the overall United States experience can be obtained from a comparison of the number of deaths recorded through the general vital statistics system

TABLE 3

COMPARISON OF NUMBER OF DEATHS OCCURRING IN THE UNITED STATES IN CALENDAR YEAR 1968, NATIONAL CENTER FOR HEALTH STATISTICS DATA AND SOCIAL SECURITY DATA

Age Last Birthday	NCHS Data*		Social Security Data†		Ratio	
	Male	Female	Male	Female	Col. (2) to Col. (4)	Col. (3) to Col. (5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
65–69	128,517	79,964	123,644	73,250	1.039	1.092
70–74	142,419	102,646	139,878	101,860	1.018	1.008
75–79	137,575	124,223	138,870	126,476	0.991	0.982
80–84	108,424	122,108	111,344	126,312	0.974	0.967
85–89	63,854	90,230	65,651	92,380	0.973	0.977
90–94	25,203	43,412	25,721	43,879	0.980	0.989
95–99	5,725	11,922	5,757	11,983	0.994	0.995
100–104	711	1,616	649	1,605	1.096	1.007
105 and over	128	220	70	150	1.829	1.467
Total	612,556	576,341	611,584	577,895	1.002	0.997

* Obtained from National Center for Health Statistics (to be published in the 1968 *Vital Statistics of the United States*).

† Estimated by interpolation from data by calendar age at death, excluding 173 cases of unknown sex.

and those recorded under Medicare. Such a comparison is made in Table 3 for deaths occurring in calendar year 1968.

According to the National Center for Health Statistics data, there were 1,188,897 recorded deaths at ages 65 and over in the United States in 1968. Social security data show the total to be 1,189,479, or about 0.05 per cent higher. This difference would be higher if we took into account the deaths among the estimated 170,000 persons who are not entitled to Medicare benefits.

A comparison by age shows the ratio of vital statistics deaths to social security deaths to be a relatively smooth U-shaped curve for both males and females. In this case the data for the age group 70–74 do not fall out of line as they do for the living population. The social security deaths were interpolated by using the formula

$$D_x = \tfrac{1}{16}(3\theta_{x-1} + 5\theta_x + 5\theta_{x+1} + 3\theta_{x+2}) , \qquad (1)$$

where D_x represents the estimated number of deaths at age x last birthday and θ_x represents the observed number of deaths at calendar age x (difference between the calendar year of death and calendar year of birth). It should be observed that, according to this comparison, except for age group 65–69 there is no significant overstatement of age in the case of vital statistics deaths until we get into the extreme older ages, around 100 or over. Also, it will be noted that, except for the high value at ages 65–69, the ratios for females are at about the same level as for males.

As has been indicated previously, one of the basic problems in the analysis of mortality at the older ages is the unreliability of the data regarding stated age.[4] This difficulty is, to a large extent, eliminated from the social security data. The bulk of the social security data relate to individuals who have had to "prove" their date of birth in order to become entitled to benefits. This is particularly true for persons who begin to draw monthly cash benefits before age 65 who comprise a large majority of the beneficiaries. However, this is not the case for those who become entitled to Medicare without becoming entitled to monthly cash benefits, who have simply to prove they are old enough, nor is it the case for beneficiaries who become entitled to monthly cash benefits well after an age affected by the question of the retirement test. In general, however, we believe that the social security data are reliable to about age 90 or age 95 and that they become progressively less reliable thereafter.

A somewhat different picture arises in regard to the data by race. At the present time a substantial quantity of social security data lacks any information regarding race of the beneficiary. According to data as of January 1, 1968, the population was made up of 89.2 per cent whites, 7.7 per cent nonwhites, and 3.1 per cent persons of unknown color. For the purpose of this study it was originally planned to combine data for persons of unknown color with those for the whites or the nonwhites,

[4] See Monroe G. Sirken and Jacob S. Siegel, "Errors in Postcensal Population Estimates due to Inconsistent Age Reporting on Death Certificate and Census Records" (paper presented at the 1969 annual meeting of the Population Association of America); see also Thea Z. Hambright, *Comparability of Age on Death Certificate and Matching Census Records* (U.S. Public Health Service, National Center for Health Statistics, Ser. 2, No. 29).

but it was found that the mortality level and pattern of this group is different from those for either whites or nonwhites. Both level and pattern are somewhere between those for whites and those for nonwhites, but generally closer to those for whites. For this reason it was decided to include the data for persons of unknown color only in the calculations dealing with total persons or with the total population.

COMPARISON OF CENTRAL DEATH RATES

The death rates published by the National Center for Health Statistics in the volumes of *Vital Statistics of the United States* are a composite of the number of deaths as compiled by the Center from the death registration data and the population estimates prepared by the Bureau of the Census on the basis of census counts. The two basic sources of information are different in nature and are subject to different errors. When data from the two are combined, the calculated death rates are subject to the errors of both sources. This is not the case when both deaths and population are obtained from the same source. The resulting death rates could still be subject to some other errors, but the errors due to noncomparability of the data would be eliminated. Since the Medicare data contain information on both death and population, they do not include this type of error.

Table 4 contains values of the central death rates by sex and color for calendar year 1968, according to National Center for Health Statistics estimates and according to social security data. The central death rates for the social security data were computed by using the formula

$$ {}_5m_x^z = \frac{\sum_{i=0}^{4} D_{x+i}^z}{\sum_{i=0}^{4} P_{x+i}^{z+1/2}}, \tag{2} $$

where ${}_5m_x^z$ is the estimated central death rate at ages x to $x + 4$ in calendar year z, D_x^z is the number of deaths at age x last birthday in calendar year z estimated according to formula (1), and $P_x^{z+1/2}$ is the midyear population at age x last birthday estimated as the average of the population at calendar age x on January 1 of years z and $z + 1$. The comparison shows that the two different sources of data yield death rates that are fairly close for white persons but significantly different for nonwhite persons. This raises some questions about the validity of the nonwhite death rates at older ages that are being obtained from vital statistics. The death rates estimated by the National Center for

Health Statistics are generally higher than those from social security data at ages 65–69 and ages 70–74 and are lower at the older ages. The differences between the two sets of data are substantially larger for nonwhite persons than for white persons. According to social security data, the differential in mortality by color is much lower than that shown by vital statistics. This applies where the nonwhite mortality is higher as

TABLE 4

COMPARISON OF ESTIMATED CENTRAL DEATH RATE, ${}_nm_x$, FOR CALENDAR YEAR 1968, NATIONAL CENTER FOR HEALTH STATISTICS ESTIMATE AND SOCIAL SECURITY DATA

Age Last Birthday	NCHS Estimate*		Social Security Data†		Ratio	
	Male	Female	Male	Female	Col. (2) to Col. (4)	Col. (3) to Col. (5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Total Persons					
65–69	0.0426	0.0221	0.0417	0.0203	1.022	1.089
70–74	0.0645	0.0349	0.0604	0.0327	1.068	1.067
75–79	0.0865	0.0549	0.0879	0.0548	0.984	1.002
80–84	0.1207	0.0909	0.1305	0.0926	0.925	0.982
85 and over	0.2037	0.1917	0.2203	0.1792	0.925	1.070
	White Persons					
65–69	0.0410	0.0202	0.0413	0.0195	0.993	1.036
70–74	0.0629	0.0335	0.0599	0.0318	1.050	1.053
75–79	0.0869	0.0549	0.0874	0.0540	0.994	1.017
80–84	0.1235	0.0923	0.1307	0.0926	0.995	0.997
85 and over	0.2156	0.2001	0.2223	0.1813	0.970	1.104
	Nonwhite Persons					
65–69	0.0598	0.0440	0.0464	0.0283	1.289	1.555
70–74	0.0859	0.0526	0.0659	0.0427	1.303	1.232
75–79	0.0820	0.0546	0.0914	0.0652	0.897	0.837
80–84	0.0904	0.0722	0.1290	0.0956	0.701	0.755
85 and over	0.1156	0.1099	0.1962	0.1562	0.589	0.704

* Data obtained from the National Center for Health Statistics (to be published in the 1968 *Vital Statistics of the United States*).

† Based on midyear population estimated by interpolation from data for the beginning and end of calendar year 1968. The rates for white persons and nonwhite persons are based on data recorded by color. The rates for total persons include these data and also data for cases of nonrecorded color.

well as where it is lower. In brief, the mortality of the nonwhites is closer to that of the whites than the official death rates would indicate. This conclusion is more readily observable from Table 5, in which the central death rates are compared by color.

Table 5 also shows that the color differential in mortality is lower for males than for females. This is indicated by both National Center for Health Statistics data and social security data. In addition, it should be noted that, according to social security data, the crossover point (the point at which white mortality begins to be higher than nonwhite mortality) occurs at a higher age than vital statistics would indicate.

TABLE 5

COMPARISON OF THE RATIO OF NONWHITE TO WHITE ESTIMATED CENTRAL DEATH RATES FOR CALENDAR YEAR 1968, NATIONAL CENTER FOR HEALTH STATISTICS ESTIMATE AND SOCIAL SECURITY DATA*

AGE LAST BIRTHDAY	NCHS ESTIMATE		SOCIAL SECURITY DATA	
	Male	Female	Male	Female
(1)	(2)	(3)	(4)	(5)
65–69	1.459	2.178	1.123	1.451
70–74	1.366	1.570	1.100	1.343
75–79	0.944	0.995	1.046	1.207
80–84	0.732	0.782	0.986	1.032
85 and over	0.536	0.549	0.883	0.862

* These ratios are based on the rates in Table 4.

A comparison of death rates by sex is presented in Table 6. According to the ratios shown in the table, the sex differentials in mortality obtained from social security data are fairly close to those obtained from vital statistics. However, social security data indicate slightly higher ratios—that is, they indicate a wider gap between male and female mortality than vital statistics data show.

In general, the above analysis of central death rates demonstrates that, according to social security data, the vital statistics death rates at the older ages are acceptable for white persons and are of questionable validity for nonwhite persons. It also demonstrates that the mortality differentials by color are significantly smaller than vital statistics show and that the mortality differentials by sex are slightly larger.

MORTALITY RATES BY SINGLE YEARS OF AGE

The following analysis of mortality by single years of age is based on approximately 2.3 million deaths that, according to Medicare data, occurred in the United States in calendar years 1968-69. All data used in the analysis were tabulated by calendar age—that is, the difference between the calendar year of reference (year of death or end year of tabulation for population) and the calendar year of birth. This resulted in the following exposure formula, which is of the type referred to by Gershenson[5] as "Case 1":

$$E_x^z = \tfrac{1}{2}(P_{x-1}^z + P_x^{z+1} + \theta_x^z) , \tag{3}$$

TABLE 6

COMPARISON OF THE RATIO OF MALE TO FEMALE ESTIMATED CENTRAL DEATH RATES FOR CALENDAR YEAR 1968, NATIONAL CENTER FOR HEALTH STATISTICS ESTIMATE AND SOCIAL SECURITY DATA*

Age Last Birthday	NCHS Estimate			Social Security Data		
	Total	White	Nonwhite	Total	White	Nonwhite
(1)	(2)	(3)	(4)	(5)	(6)	(7)
65-69	1.928	2.030	1.359	2.054	2.118	1.640
70-74	1.848	1.878	1.633	1.847	1.884	1.543
75-79	1.576	1.583	1.502	1.604	1.619	1.402
80-84	1.328	1.338	1.252	1.409	1.334	1.349
85 and over	1.064	1.077	1.052	1.229	1.189	1.256

* These ratios are based on the rates in Table 4.

where E_x^z is the exposure at tabulated age x in calendar year z, P_x^z is the population at tabulated age x on January 1 of year z, and θ_x^z is the number of deaths at tabulated age x in calendar year z.

The mortality rates at tabulated age x which were obtained using this formula are equivalent, on the average, to the usual mortality rates at exact age $x - \frac{1}{2}$. This created a difficulty in comparing the rates with other known mortality rates, since most of them are published at exact integral ages. In order to make the comparisons more meaningful, the social security mortality rates were interpolated, wherever needed, to

[5] See Harry Gershenson, *Measurement of Mortality* (Chicago: Society of Actuaries, 1971), p.122.

yield rates at the exact integral ages. These rates were computed as the geometric average of the two contiguous half-age rates.

The observed mortality rates for calendar ages 66–102 were graduated using the Whittaker-Henderson Type A formula with values for the a coefficient that varied between 1 and 2 in relation to the number of deaths included in the ungraduated curves. These low values of a were used in order to give preference to fit over smoothness in the graduation and to give recognition to the large quantity of available data.

Table 7 presents the observed and graduated rates for the total population. Because of the sizable quantity of data, the crude observed rates form a relatively smooth progression of increasing mortality. In all instances these rates differ by less than 1 per cent from the graduated rates.

The mortality analysis had to be limited to calendar ages 66 and over, since the data were tabulated by calendar age and eligibility to Medicare benefits starts at exact age 65. The development of mortality rates at age 65 would have required the use of special formulas involving some adjustments to take into account peculiarities of social security data at that crucial age. At the other end of the range, calendar age 102 was selected as the last point for graduation purposes because of the significant variations in mortality rates after that age.

A comparison of the social security graduated rates for the total population with those in the official United States Life Tables for 1959–61 is shown in Table 8. The latter rates are about 5–8 per cent higher than the social security rates at the younger ages, but this differential later increases with age, attaining values of over 25 per cent by age 100. The higher mortality rates in the official tables cannot be fully explained by the difference in the observation periods covered by the two sets of data, since there was no significant improvement in mortality during the 1960's. It is believed that the significantly large differential in mortality at the very high ages is due to the way in which the official life tables were ended. As was indicated earlier, after age 85 the official life tables are a progressive blending of the Union Civil War veterans experience into the observed United States rates for 1959–61. After age 94, the official rates are exactly those of the Union Civil War veterans experience.

Table 9 compares the Union Civil War veterans experience with the rates for social security total male persons at very high ages. It should be noted that Union Civil War veterans rates are about 9–16 per cent higher than the social security male rates. It is not known to what extent this differential in mortality may be due to the difference in the time

TABLE 7

COMPARISON OF OBSERVED AND GRADUATED MORTALITY RATES FOR TOTAL POPULATION, ACCORDING TO SOCIAL SECURITY DATA FOR CALENDAR YEARS 1968–69

Calendar Age* (1)	Observed Rates (2)	Graduated Rates (3)	Ratio Col. (2) to Col. (3) (4)
66	0.02516	0.02508	1.003
67	0.02753	0.02754	1.000
68	0.03016	0.03003	1.004
69	0.03268	0.03256	1.004
70	0.03510	0.03522	0.997
71	0.03803	0.03808	0.999
72	0.04132	0.04124	1.002
73	0.04447	0.04474	0.994
74	0.04874	0.04866	1.002
75	0.05298	0.05297	1.000
76	0.05800	0.05770	1.005
77	0.06279	0.06286	0.999
78	0.06837	0.06856	0.997
79	0.07455	0.07489	0.995
80	0.08231	0.08189	1.005
81	0.08933	0.08946	0.999
82	0.09762	0.09768	0.999
83	0.10684	0.10656	1.003
84	0.11592	0.11608	0.999
85	0.12604	0.12634	0.998
86	0.13730	0.13738	0.999
87	0.14931	0.14914	1.001
88	0.16081	0.16154	0.995
89	0.17533	0.17454	1.005
90	0.18810	0.18788	1.001
91	0.19986	0.20155	0.992
92	0.21642	0.21562	1.004
93	0.22986	0.22958	1.001
94	0.24408	0.24321	1.004
95	0.25417	0.25635	0.991
96	0.27039	0.26916	1.005
97	0.28099	0.28108	1.000
98	0.29216	0.29192	1.001
99	0.30415	0.30150	1.009
100	0.30813	0.30970	0.995
101	0.31759	0.31730	1.001
102	0.32349	0.32455	0.997

* Refers to the difference between the calendar year of death and the calendar year of birth.

TABLE 8

COMPARISON OF GRADUATED MORTALITY RATES FOR TOTAL POPULATION, NATIONAL CENTER FOR HEALTH STATISTICS ESTIMATE FOR CALENDAR YEARS 1959–61 AND SOCIAL SECURITY DATA FOR CALENDAR YEARS 1968–69

Exact Age (1)	NCHS Estimate* (2)	Social Security Data† (3)	Ratio Col. (2) to Col. (3) (4)
66	0.02828	0.02628	1.076
67	0.03053	0.02876	1.062
68	0.03301	0.03127	1.056
69	0.03573	0.03386	1.055
70	0.03866	0.03662	1.056
71	0.04182	0.03963	1.055
72	0.04530	0.04296	1.054
73	0.04915	0.04666	1.053
74	0.05342	0.05077	1.052
75	0.05799	0.05529	1.049
76	0.06296	0.06023	1.045
77	0.06867	0.06565	1.046
78	0.07535	0.07165	1.052
79	0.08302	0.07831	1.060
80	0.09208	0.08559	1.076
81	0.10219	0.09348	1.093
82	0.11244	0.10202	1.102
83	0.12195	0.11122	1.096
84	0.13067	0.12110	1.079
85	0.14380	0.13174	1.092
86	0.15816	0.14314	1.105
87	0.17355	0.15522	1.118
88	0.19032	0.16791	1.133
89	0.20835	0.18109	1.151
90	0.22709	0.19460	1.167
91	0.24598	0.20847	1.180
92	0.26477	0.22249	1.190
93	0.28284	0.23630	1.197
94	0.29952	0.24969	1.200
95	0.31416	0.26268	1.196
96	0.32915	0.27506	1.197
97	0.34450	0.28645	1.203
98	0.36018	0.29667	1.214
99	0.37616	0.30557	1.231
100	0.39242	0.31347	1.252
101	0.40891	0.32090	1.274

* Refers to the official United States Life Tables for 1959–61.

† Obtained by interpolation of graduated rates by single years of calendar age.

period covered by the two studies or to the difference in the type of individuals included in the two groups.

"EXTINCT COHORT" METHOD

Another possible source of mortality data at the older ages is related to the "extinct cohort" method.[6] According to this method, once a cohort of individuals becomes extinct, it is possible to reconstruct the population

TABLE 9

COMPARISON OF GRADUATED MORTALITY RATES FOR UNION CIVIL WAR VETERANS EXPERIENCE AND SOCIAL SECURITY DATA*

Calendar Age† (1)	Union Civil War Veterans Experience (2)	Social Security Data (3)	Ratio Col. (2) to Col. (3) (4)
90	0.23891	0.21313	1.121
91	0.25163	0.22742	1.106
92	0.26479	0.24143	1.097
93	0.27838	0.25462	1.093
94	0.29239	0.26676	1.096
95	0.30681	0.27813	1.103
96	0.32161	0.28932	1.112
97	0.33678	0.30008	1.122
98	0.35230	0.31070	1.134
99	0.36813	0.32122	1.146
100	0.38426	0.33255	1.155
101	0.40064	0.34563	1.159
102	0.41725	0.35925	1.161

* The rates for Union Civil War veterans experience were calculated on the basis of the logistic curve fitted by Robert J. Myers and Louis O. Shudde. The rates for social security refer to the experience for male persons in calendar years 1968–69.

† Refers to the difference between the calendar year of death and the calendar year of birth.

from the death records. As an example of the method, a good estimate of the population aged 90 in 1950 could be obtained by adding together the number of deaths at ages 90 in 1950, 91 in 1951, 92 in 1952, and so on, to the last year of available data plus an estimate of possible future deaths.

For the present paper, the "extinct cohort" method was applied to the death data by single ages for ages 85 and over, published in the 1951–68 annual volumes of *Vital Statistics of the United States*. In order to keep

[6] Paul Vincent, "La Mortalité des vieillards," *Population*, VI (1951), 181–204.

the estimates of future deaths as a small percentage of the population, it was decided to limit the study to those persons who were over age 80 in 1951. The size of each cohort was estimated by using the following formula:

$$
\begin{aligned}
{}^{y}P_x^z = \{[D_x^z + D_{x+1}^{z+1} + D_{x+2}^{z+2} + \ldots + D_{x+y-z}^{y}] \\
+ (D_{x+y-z+1}^{y} + D_{x+y-z+2}^{y} + \ldots + D_{\omega}^{y})\} \, ,
\end{aligned}
\tag{4}
$$

where ${}^{y}P_x^z$ is the estimated number of persons age x in the year z, according to data as of the year y, and D_x^z is the number of deaths at age x in the year z.

It should be observed that the portion of the above formula in square brackets represents the deaths pertaining to the cohorts that have actually been recorded, while the portion in parentheses represents the estimated number of future deaths for the cohort. This estimate is based on the deaths recorded in year y at all ages above the age of the cohort in that year.

It should be made clear that the number of future deaths estimated as above is used only to develop the exposure needed for the mortality rate and that those estimated deaths are not included in the number of observed deaths needed as a numerator in the calculation of the mortality rate. It is believed that the number of future cohort deaths tends to be slightly understated by this method because of the fact that, generally, later cohorts are larger than earlier cohorts. This, in turn, results in a slight understatement of the exposure, which produces a slight overstatement of the mortality rates.

The mortality rates computed for the total population by using the "extinct cohort" method are compared in Table 10 with those observed from social security data. The comparison shows that the "extinct cohort" rates are about 5 per cent higher than the social security rates (a fact that could be due mostly to the different observation periods covered by the two sources of data). It is of interest to observe that after age 100 the "extinct cohort" mortality rates are lower than those observed under social security.

Although the observed social security rates were graduated only up to calendar age 102, in this comparison the mortality rates are shown up to exact age 106. It is of interest to observe that neither the "extinct cohort" method nor the social security data indicate a tendency for the mortality rates at ages over 100 to increase with age. We do not believe that this could be due to statistical fluctuations, since the quantity of data involved in these rates is substantial. The number of deaths varies from 7,189 at age 101 to 984 at age 106 for the "extinct cohort" method and from 1,392 at calendar age 101 to 118 at calendar age 106 for social security data. However, it should be remembered that, as we move into

TABLE 10

COMPARISON OF OBSERVED MORTALITY RATES FOR TOTAL POPULATION, "EXTINCT COHORT" METHOD FOR CALENDAR YEARS 1951–68 AND SOCIAL SECURITY DATA FOR CALENDAR YEARS 1968–69

Exact Age (1)	"Extinct Cohort" Method* (2)	Social Security Data† (3)	Ratio Col. (2) to Col. (3) (4)
85	0.14196	0.13155	1.079
86	0.15361	0.14318	1.073
87	0.16502	0.15495	1.065
88	0.17380	0.16791	1.035
89	0.18854	0.18160	1.038
90	0.20790	0.19389	1.072
91	0.20989	0.20797	1.009
92	0.23019	0.22304	1.032
93	0.24769	0.23686	1.046
94	0.26232	0.24907	1.053
95	0.27662	0.26215	1.055
96	0.29227	0.27564	1.060
97	0.30767	0.28652	1.074
98	0.31635	0.29809	1.061
99	0.31174	0.30613	1.018
100	0.33827	0.31283	1.081
101	0.30295	0.32053	0.945
102	0.30919	0.31419	0.984
103	0.28829	0.30132	0.957
104	0.28645	0.30219	0.948
105	0.30067	0.30591	0.983
106	0.26359	0.26030	1.013

* See text for description of the method and the data used.

† Obtained by interpolation of observed rates by single years of calendar age.

higher ages, the reliability of both sets of data decreases. The author believes that the extent to which mortality rates become constant after age 100 should still be regarded as an unknown subject, but that it is clear that a tendency for deceleration in the rate of increase in mortality after approximately age 85 or age 90 is evident from the data presented. A brief analysis in this respect is included in the next section.

MORTALITY RATES BY SEX

Table 11 compares the social security graduated mortality rates for females with those for males. As was to be expected, the female rates are considerably lower than the male rates. This differential in mortality becomes smaller with age but is still about 10 per cent around age 100.

TABLE 11

COMPARISON OF GRADUATED MORTALITY RATES BY SEX FOR TOTAL PERSONS, ACCORDING TO SOCIAL SECURITY DATA FOR CALENDAR YEARS 1968–69

Calendar Age* (1)	Female Rates (2)	Male Rates (3)	Ratio Col. (2) to Col. (3) (4)
66	0.01630	0.03509	0.465
67	0.01835	0.03834	0.479
68	0.02042	0.04163	0.491
69	0.02254	0.04499	0.501
70	0.02481	0.04848	0.512
71	0.02729	0.05216	0.523
72	0.03009	0.05609	0.536
73	0.03325	0.06029	0.552
74	0.03683	0.06486	0.568
75	0.04083	0.06983	0.585
76	0.04528	0.07523	0.602
77	0.05022	0.08108	0.619
78	0.05575	0.08742	0.638
79	0.06193	0.09430	0.657
80	0.06878	0.10177	0.676
81	0.07627	0.10984	0.694
82	0.08442	0.11856	0.712
83	0.09326	0.12794	0.729
84	0.10282	0.13797	0.745
85	0.11319	0.14870	0.761
86	0.12436	0.16015	0.777
87	0.13627	0.17233	0.791
88	0.14877	0.18527	0.803
89	0.16177	0.19898	0.813
90	0.17513	0.21313	0.822
91	0.18890	0.22742	0.831
92	0.20318	0.24143	0.842
93	0.21773	0.25462	0.855
94	0.23227	0.26676	0.871
95	0.24642	0.27813	0.886
96	0.25988	0.28932	0.898
97	0.27229	0.30008	0.907
98	0.28337	0.31070	0.912
99	0.29303	0.32122	0.912
100	0.30098	0.33255	0.905
101	0.30754	0.34563	0.890
102	0.31352	0.35925	0.873

* Refers to the difference between the calendar year of death and the calendar year of birth.

Before this study was conducted, the author felt that the male and female mortality curves would cross over somewhere at the very high ages. Most Gompertz curves that could be fitted would indicate this to be the case, but social security data show that the Gompertz curve is not an acceptable representation of mortality at the very high ages. A geometrically increasing mortality rate like the Gompertz curve would be acceptable up to age 85 or age 90, but above those ages mortality increases at a progressively smaller rate (not at a constant rate, as required by the Gompertz curve). This change in the shape of the mortality curves is more clearly observable from Table 12, which compares mortality rates with those at the next lower age. The mortality curves for males and females, as well as for the total population (also the case for all sex-color combinations), tend toward a flattening-out at the extreme old ages.

The falling-off of the mortality curves from the exponential, as well as the flattening-out at extreme old ages, have been observed in other mortality experience. Redington[7] discussed the possibility of two distinct mortality patterns—the environmental pattern, which would produce exponentially increasing rates up to about age 90, and the genetic pattern, which would have a tendency to make the curves flatter after that age.

MORTALITY BY COLOR

As was indicated earlier, the social security data demonstrate that the mortality differentials among the aged by color are not as large as the official life tables show. According to the comparison in Table 13' nonwhite females start with a mortality at age 66 that is about 47 per cent higher than for white females. The differential decreases rapidly with age, and there is a crossover around ages 82–83. Thereafter, nonwhite females are shown to be subject to a mortality that is progressively lower than that for the white female; around age 100 their mortality is about 33 per cent lower.

For males the picture is about the same, except that mortality differentials by color are much smaller than for females, particularly at the early old ages. The crossover for males occurs about a year earlier than for females.

[7] F. M. Redington, "An Exploration into Patterns of Mortality," *JIA*, XCV, 243–98.

COMPARISON WITH ANNUITANTS EXPERIENCE

Tables 14 and 15 compare the graduated social security mortality rates for males and females with rates from the Group Annuity Table for 1951[8] and from an estimated Group Annuity Table for 1969.

TABLE 12

RATIO OF GRADUATED MORTALITY RATE TO THAT AT PREVIOUS AGE, ACCORDING TO SOCIAL SECURITY DATA FOR CALENDAR YEARS 1968–69

Calendar Age* (1)	Total Population (2)	Females (3)	Males (4)
67	1.098	1.126	1.093
68	1.090	1.113	1.086
69	1.084	1.104	1.081
70	1.082	1.101	1.078
71	1.081	1.100	1.076
72	1.083	1.103	1.075
73	1.085	1.105	1.075
74	1.088	1.108	1.076
75	1.089	1.109	1.077
76	1.089	1.109	1.077
77	1.089	1.109	1.078
78	1.091	1.110	1.078
79	1.092	1.111	1.079
80	1.093	1.111	1.079
81	1.092	1.109	1.079
82	1.092	1.107	1.079
83	1.091	1.105	1.079
84	1.089	1.103	1.078
85	1.088	1.101	1.078
86	1.087	1.099	1.077
87	1.086	1.096	1.076
88	1.083	1.092	1.075
89	1.080	1.087	1.074
90	1.076	1.083	1.071
91	1.073	1.079	1.067
92	1.070	1.076	1.062
93	1.065	1.072	1.055
94	1.059	1.067	1.048
95	1.054	1.061	1.043
96	1.050	1.055	1.040
97	1.044	1.048	1.037
98	1.039	1.041	1.035
99	1.033	1.034	1.034
100	1.027	1.027	1.035
101	1.025	1.022	1.039
102	1.023	1.019	1.039

* Refers to the difference between the calendar year of death and the calendar year of birth.

[8] Ray M. Peterson, "Group Annuity Mortality," *TSA*, IV, 246–307.

Since the annuity tables refer to a select group of lives—namely, workers who retired on or after the normal retirement date—their rates (after allowing for the margin of 10 per cent for males and $12\frac{1}{2}$ per cent for females) should be expected to be lower than those observed in social security data pertaining to the total population. However, this is not the case. At the very high ages social security rates are lower.

TABLE 13

COMPARISON OF GRADUATED MORTALITY RATES BY COLOR, ACCORDING TO SOCIAL SECURITY DATA FOR CALENDAR YEARS 1968–69

CALENDAR AGE*	FEMALE		MALE		RATIO	
	Nonwhite	White	Nonwhite	White	Col. (2) to Col. (3)	Col. (4) to Col. (5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
66	0.02263	0.01536	0.03852	0.03472	1.473	1.109
67	0.02536	0.01741	0.04228	0.03793	1.457	1.115
68	0.02808	0.01948	0.04603	0.04118	1.441	1.118
69	0.03079	0.02164	0.04976	0.04451	1.423	1.118
70	0.03355	0.02394	0.05350	0.04799	1.401	1.115
71	0.03644	0.02646	0.05732	0.05167	1.377	1.109
72	0.03955	0.02927	0.06130	0.05560	1.351	1.103
73	0.04292	0.03243	0.06549	0.05982	1.323	1.095
74	0.04658	0.03599	0.06996	0.06440	1.294	1.086
75	0.05057	0.03999	0.07473	0.06939	1.265	1.077
76	0.05489	0.04448	0.07978	0.07482	1.234	1.066
77	0.05951	0.04951	0.08512	0.08071	1.202	1.055
78	0.06443	0.05515	0.09075	0.08711	1.168	1.042
79	0.06958	0.06145	0.09674	0.09407	1.132	1.028
80	0.07496	0.06846	0.10310	0.10164	1.095	1.014
81	0.08059	0.07619	0.10985	0.10982	1.058	1.000
82	0.08647	0.08468	0.11691	0.11869	1.021	0.985
83	0.09267	0.09395	0.12426	0.12828	0.986	0.969
84	0.09922	0.10403	0.13197	0.13856	0.954	0.952
85	0.10624	0.11493	0.14000	0.14957	0.924	0.936
86	0.11372	0.12666	0.14817	0.16135	0.898	0.918
87	0.12157	0.13914	0.15643	0.17390	0.874	0.900
88	0.12974	0.15226	0.16498	0.18724	0.852	0.881
89	0.13819	0.16586	0.17374	0.20139	0.833	0.863
90	0.14659	0.17995	0.18227	0.21611	0.815	0.843
91	0.15469	0.19449	0.19005	0.23120	0.795	0.822
92	0.16215	0.20950	0.19671	0.24642	0.774	0.798
93	0.16882	0.22478	0.20218	0.26135	0.751	0.774
94	0.17481	0.24013	0.20651	0.27581	0.728	0.749
95	0.18071	0.26969	0.20980	0.28998	0.670	0.724
96	0.18723	0.28322	0.21220	0.30430	0.661	0.697
97	0.19457	0.29562	0.21422	0.31811	0.658	0.673
98	0.20242	0.30687	0.21704	0.33103	0.660	0.656
99	0.21021	0.31684	0.22060	0.34257	0.663	0.644
100	0.21773	0.32605	0.22504	0.35310	0.668	0.637
101	0.22477	0.32605	0.23040	0.36390	0.689	0.633

* Refers to the difference between the calendar year of death and the calendar year of birth.

TABLE 14

COMPARISON OF GRADUATED MORTALITY RATES FOR MALES, GROUP ANNUITY TABLES FOR 1951 AND 1969 AND SOCIAL SECURITY DATA FOR CALENDAR YEARS 1968-69

EXACT AGE (1)	GROUP ANNUITY TABLE FOR 1951 (2)	ESTIMATED GROUP ANNUITY TABLE FOR 1969* (3)	SOCIAL SECURITY DATA† (4)	RATIO: Col. (2) to Col. (4) (5)	RATIO: Col. (3) to Col. (4) (6)
66	0.02719	0.02174	0.03668	0.741	0.593
67	0.03011	0.02408	0.03995	0.754	0.603
68	0.03299	0.02638	0.04328	0.762	0.610
69	0.03594	0.02874	0.04670	0.770	0.615
70	0.03930	0.03143	0.05029	0.781	0.625
71	0.04318	0.03484	0.05409	0.798	0.644
72	0.04748	0.03866	0.05815	0.817	0.665
73	0.05208	0.04278	0.06253	0.833	0.684
74	0.05708	0.04730	0.06730	0.848	0.703
75	0.06243	0.05219	0.07248	0.861	0.720
76	0.06835	0.05784	0.07810	0.875	0.741
77	0.07513	0.06435	0.08419	0.892	0.764
78	0.08269	0.07167	0.09079	0.911	0.789
79	0.09095	0.07977	0.09796	0.928	0.814
80	0.09968	0.08844	0.10573	0.943	0.836
81	0.10871	0.09764	0.11411	0.953	0.856
82	0.11798	0.10726	0.12316	0.958	0.871
83	0.12744	0.11725	0.13286	0.959	0.883
84	0.13707	0.12760	0.14324	0.957	0.891
85	0.14685	0.13831	0.15432	0.952	0.896
86	0.15684	0.14955	0.16613	0.944	0.900
87	0.16712	0.16129	0.17868	0.935	0.903
88	0.17779	0.17366	0.19201	0.926	0.904
89	0.18892	0.18672	0.20594	0.917	0.907
90	0.20059	0.20059	0.22016	0.911	0.911
91	0.21256	0.21256	0.23432	0.907	0.907
92	0.22516	0.22516	0.24794	0.908	0.908
93	0.23852	0.23852	0.26062	0.915	0.915
94	0.25276	0.25276	0.27238	0.928	0.928
95	0.26802	0.26802	0.28367	0.945	0.945
96	0.28446	0.28446	0.29465	0.965	0.965
97	0.30222	0.30222	0.30535	0.990	0.990
98	0.32152	0.32152	0.31592	1.018	1.018
99	0.34253	0.34253	0.32684	1.048	1.048
100	0.36546	0.36546	0.33903	1.078	1.078
101	0.39054	0.39054	0.35237	1.108	1.108

* These rates were developed by the author on the basis of Projection Scale C suggested by Peterson in his paper. The projected reductions in mortality for ages that are a multiple of 5 were interpolated linearly to obtain reductions at other ages.

† Obtained by interpolation of graduated rates by single years of calendar age.

TABLE 15

COMPARISON OF GRADUATED MORTALITY RATES FOR FEMALES, GROUP ANNUITY TABLES FOR 1951 AND 1969 AND SOCIAL SECURITY DATA FOR CALENDAR YEARS 1968–69

Exact Age (1)	Group Annuity Table for 1951 (2)	Estimated Group Annuity Table for 1969* (3)	Social Security Data† (4)	Ratio: Col. (2) to Col. (4) (5)	Ratio: Col. (3) to Col. (4) (6)
66	0.01499	0.01199	0.01729	0.867	0.693
67	0.01646	0.01316	0.01936	0.850	0.680
68	0.01820	0.01455	0.02145	0.848	0.678
69	0.02035	0.01627	0.02365	0.860	0.688
70	0.02310	0.01847	0.02602	0.888	0.710
71	0.02653	0.02141	0.02866	0.926	0.747
72	0.03047	0.02481	0.03163	0.963	0.784
73	0.03478	0.02857	0.03500	0.994	0.816
74	0.03941	0.03266	0.03878	1.016	0.842
75	0.04431	0.03704	0.04299	1.031	0.862
76	0.04951	0.04190	0.04768	1.038	0.879
77	0.05511	0.04720	0.05291	1.042	0.892
78	0.06109	0.05295	0.05876	1.040	0.901
79	0.06746	0.05916	0.06527	1.034	0.906
80	0.07415	0.06579	0.07243	1.024	0.908
81	0.08111	0.07285	0.08024	1.011	0.908
82	0.08837	0.08034	0.08873	0.996	0.905
83	0.09594	0.08827	0.09792	0.980	0.901
84	0.10390	0.09672	0.10788	0.963	0.897
85	0.11233	0.10580	0.11864	0.947	0.892
86	0.12130	0.11566	0.13018	0.932	0.888
87	0.13088	0.12631	0.14238	0.919	0.887
88	0.14119	0.13791	0.15513	0.910	0.889
89	0.15230	0.15053	0.16832	0.905	0.894
90	0.16433	0.16433	0.18188	0.904	0.904
91	0.17714	0.17714	0.19591	0.904	0.904
92	0.19110	0.19110	0.21033	0.909	0.909
93	0.20634	0.20634	0.22488	0.918	0.918
94	0.22303	0.22303	0.23924	0.932	0.932
95	0.24134	0.24134	0.25306	0.954	0.954
96	0.26145	0.26145	0.26601	0.983	0.983
97	0.28358	0.28358	0.27777	1.021	1.021
98	0.30795	0.30795	0.28816	1.069	1.069
99	0.33481	0.33481	0.29698	1.127	1.127
100	0.36443	0.36443	0.30424	1.198	1.198
101	0.39710	0.39710	0.31051	1.279	1.279

* These rates were developed by the author on the basis of Projection Scale C suggested by Peterson in his paper. The projected reductions in mortality for ages that are a multiple of 5 were interpolated linearly to obtain reductions at other ages.

† Obtained by interpolation of graduated rates by single years of calendar age.

For males, on the basis of the table projected to 1969, the group annuity mortality is lower up to about age 85 (after allowing for the 10 per cent margin). Thereafter, the ratio is above 90 per cent, which implies that the social security mortality rates are lower than those underlying the group annuity tables. For females the social security mortality rates are higher until about age 75. Thereafter, the estimated mortality rates for the 1969 Group Annuity Table are higher.

The above comparison raises some questions regarding the applicability of the Group Annuity Table for 1951 and of its projections in the analysis of recent mortality experience.