TRANSACTIONS OF SOCIETY OF ACTUARIES
1972 VOL. 24 PT. 1 NO. 68

SCHEDULE FOR AMORTIZATION OF ACQUISITION COSTS VERSUS AMORTIZATION OF ACQUISITION COSTS BY USE OF NATURAL RESERVE FACTORS

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ABSTRACT

The purpose of this actuarial note is to highlight the similarities of results from the amortization of initial life insurance acquisition costs by means of (1) an acquisition cost amortization schedule similar to that used by accountants and (2) a set of actuarial-type acquisition cost (natural reserve) factors appropriately applied to beginning and ending in-force units.

ACCOUNTANTS are certainly very familiar with amortization schedules. Actuaries are probably more comfortable with either expense natural reserve factors or acquisition cost natural reserve factors—both of which are usually expressed as functions of insurance in force. In the co-ordination of efforts between accountants and actuaries to adjust life insurance company financial statements to a generally accepted accounting principles basis, these two viewpoints probably receive considerable discussion.

The purpose of this actuarial note is to demonstrate that (1) the use of a typical accounting type of acquisition cost amortization schedule and (2) the use of actuarial acquisition cost natural reserve factors produce exactly the same result, if these two approaches are based on the same assumptions and actual experience follows that assumed. It is not a purpose of this note to list the advantages or disadvantages of either of these two techniques.

The methodology used in these demonstrations is first to develop an acquisition cost natural reserve premium. This is followed by derivation of an acquisition cost amortization schedule similar to that which may be used by accountants. Actuarial-type acquisition cost natural reserve factors are then developed from the same theory and assumptions which underlie computation of the acquisition cost natural reserve premium and the amortization schedule. A comparison is made of the amounts of acquisition costs amortized each year by use of the two approaches. Actuarial formulas are then further developed to substantiate the conclusion drawn from a comparison of amortization schedule and actuarial factor acquisition cost amortization results.

From an actuarial orientation or viewpoint, it might be preferable to handle an acquisition cost adjustment through "negative liabilities." The algebraic presentation in this note, however, is made under the assumption that the expense adjustment is handled as an asset rather than as a "negative liability."

TABLE 1

Year (t)	Decrement Factors			Interest Factors		Present Value of 1 (Due at the Beginning of Each Year t) at the Beginning of Year 1 [(3)×(5)]
	q_t* (1)	$1-q_t$* (2)	${}_{t-1}p_0$ (3)	i_t (4)	v^{t-1} (5)	(6)
1....	0.30	0.70	1.000	0.06	1.000	1.000
2....	0.20	0.80	0.700	0.06	0.943	0.660
3....	0.10	0.90	0.560	0.05	0.890	0.498
4....	0.05	0.95	0.504	0.05	0.848	0.427
5....	0.05	0.95	0.479	0.05	0.807	0.387
6....	0.05	0.95	0.455	0.04	0.769	0.350
7....	0.05	0.95	0.432	0.04	0.739	0.319
						3.641

* The symbol q_t is defined in this note to represent both the decrement of withdrawal and the decrement of mortality.

DEVELOPMENT OF ACQUISITION COST NATURAL RESERVE PREMIUM

Suppose that an initial acquisition cost of $18.21 is to be amortized over the relatively short period of seven years. Further, assume that amortization is based on annual rates of decrements and interest outlined in Table 1. On the basis of these decrement and interest assumptions, the present value (at the beginning of year 1) of 1 due at the beginning of each year (or an annuity for the seven-year period) is 3.641. Dividing the assumed initial acquisition cost of $18.21 by this annuity factor gives an acquisition expense natural reserve premium of $5.00. In other words, the equivalent of $18.21 at issue is $5.00 paid at the beginning of each year, allowing for effects of both decrement and interest.

DEVELOPMENT OF ACQUISITION COST AMORTIZATION SCHEDULE

An amortization schedule may be derived, based upon the assumptions outlined above and the acquisition cost natural reserve premium of $5.00.

In column 3 of Table 2 are shown the initial acquisition cost of \$18.21 and subsequent unamortized acquisition costs which are derived by steps outlined in columns 4–7. The acquisition cost amortized each year is shown in column 6 under "Amortization of Principal." The figures in column 8 represent year-end unamortized acquisition costs expressed per unit of business in force at the end of each year.

From a strict accounting viewpoint, interest is charged at the end of

TABLE 2

Year (t)	Expected In Force at Beginning of Year (1)	Interest Rate (2)	Beginning-of-Year Unamortized Acquisition Cost (3)	Acquisition Cost Natural Reserve Premium Payment [(1)×\$5] (4)	Transactions for Year: Amortization of Interest {[(3)−(4)]×(2)} (5)	Transactions for Year: Amortization of Principal [(4)−(5)] (6)	End-of-Year Unamortized Acquisition Cost: Amount [(3)−(6)] (7)	End-of-Year Unamortized Acquisition Cost: Per In Force at End of Year [(7)÷(1)$_{t+1}$] (8)
1	1.000	0.06	\$18.21	\$5.00	\$0.79	\$ 4.21	\$14.00	\$20.00
2	0.700	0.06	14.00	3.50	0.63	2.87	11.13	19.88
3	0.560	0.05	11.13	2.80	0.42	2.38	8.75	17.36
4	0.504	0.05	8.75	2.52	0.31	2.21	6.54	13.65
5	0.479	0.05	6.54	2.40	0.21	2.19	4.35	9.56
6	0.455	0.04	4.35	2.28	0.08	2.20	2.15	4.98
7	0.432	0.04	2.15	2.15*	0.00	2.15	0.00	0.00
						\$18.21		

* Adjusted for rounding.

the year instead of being deducted at the beginning of the year. With some rearrangements of columns, a more traditional accounting presentation is as shown in Table 3.

DEVELOPMENT OF ACQUISITION COST NATURAL RESERVE FACTORS

Acquisition cost natural reserve factors may be derived using asset share type accumulation formulas. A basic actuarial identity is

$$l_{t-1}({}_{t-1}V - P)(1 + i_t) = l_{t\,t}V\,,$$

or

$$l_{t-1}({}_{t-1}V^{\text{acq}} - P^{\text{acq}})(1 + i_t) = l_{t\,t}V^{\text{acq}}$$

if reserve factors and the premium refer solely to initial acquisition costs. For further formula developments see a later section on actuarial formulas in this note.

Acquisition expense natural reserve factors are shown in Table 4 as developed from the same interest and decrement assumptions previously used to develop the acquisition cost natural reserve premium and the acquisition cost amortization schedule. Acquisition cost natural reserve

TABLE 3

Year	Balance at Beginning of Year	Natural Premium Received at Beginning of Year	Principal Balance Held during Year	Interest Added at End of Year	Balance at End of Year	Amount Amortized during Year
1........	$18.21	$5.00	$13.21	$0.79	$14.00	$4.21
2........	14.00	3.50	10.50	0.63	11.13	2.87
3........	11.13	2.80	8.33	0.42	8.75	2.38
4........	8.75	2.52	6.23	0.31	6.54	2.21
5........	6.54	2.40	4.14	0.21	4.35	2.19
6........	4.35	2.28	2.07	0.08	2.15	2.20
7........	2.15	2.15*	0.00	0.00	0.00	2.15

* Adjusted for rounding.

TABLE 4

Year	i_t	$1-q_t$	Acquisition Expense Natural Reserve Factors ${}_tV^{acq}=[({}_{t-1}V^{acq}-\$5)(1+i_t)]\div(1-q_t)$
1..........	0.06	0.70	$20.00*
2..........	0.06	0.80	19.88
3..........	0.05	0.90	17.36
4..........	0.05	0.95	13.66
5..........	0.05	0.95	9.57
6..........	0.04	0.95	5.00
7..........	0.04	0.95	0.00

* The value of ${}_0V^{acq}$ is $18.21.

factors—an actuarial approach—are the same as the unamortized acquisition costs—an accounting technique—per in force as developed from the amortization schedule (except for minor differences due to rounding!).

COMPARISON OF AMOUNTS OF ACQUISITION COST AMORTIZED BY USE OF AN AMORTIZATION SCHEDULE WITH AMOUNTS OF ACQUISITION COST AMORTIZED USING ACQUISITION EXPENSE NATURAL RESERVE FACTORS

The amount of acquisition cost amortized each year using acquisition natural reserve factors may be derived by multiplying the expected in force at the end of each year and taking differences in these products year by year (see Table 5). Acquisition costs amortized in each year using the amortization schedule approach are the same as those derived using the natural reserve acquisition cost factor approach.

Stated another way, the amount of acquisition cost amortized in each year by the actuarial factor approach is equal to the product of (1) the in force at the end of the previous year times the respective acquisition cost natural reserve factor, minus (2) the in force at the end of the current year times the current year's acquisition cost natural reserve factor.

TABLE 5

Year	Expected In Force at End of Year (1)	Acquisition Expense Natural Reserve Factor (2)	(1)×(2) (3)	Amount of Acquisition Cost Amortized* (4)
1.....	0.700	$20.00	$14.00	$ 4.21
2.....	0.560	19.88	11.13	2.87
3.....	0.504	17.36	8.75	2.38
4.....	0.479	13.66	6.54	2.21
5.....	0.455	9.57	4.35	2.19
6.....	0.432	5.00	2.15	2.20
7.....	0.000	0.00	0.00	2.15
				$18.21

* These are col. 3 differences taken year by year, with figure for beginning of year 1 equal to $18.21.

ACTUARIAL FORMULAS

A theoretical justification of the conclusions drawn from the comparisons is shown below, using basic actuarial reserve formula techniques:

$$l_{t-1}({}_{t-1}V^{acq} - P^{acq})(1 + i_t) = l_{t\,t}V^{acq} \quad \text{(a basic identity)} . \quad (1)$$

This is considered a basic identity in that V^{acq} is taken as an asset which is to be reduced over its useful lifetime at the annual rate of P^{acq}.

$$l_{t-1}P^{acq} - i_t l_{t-1}({}_{t-1}V^{acq} - P^{acq}) = \text{Amount of acquisition cost amortized in year } t \text{ (col. 6, Table 2)}; \quad (2)$$

$$l_{t-1\ t-1}V^{acq} - l_{t\ t}V^{acq} = \text{Amount of acquisition cost amortized in year } t \text{ (col. 4, Table 5)} . \quad (3)$$

The question is: Is expression (2) equal to expression (3)?

Assume that expression (3) does equal expression (2):

$$l_{t-1}\,{}_{t-1}V^{\mathrm{acq}} - l_t\,{}_tV^{\mathrm{acq}} = l_{t-1}P^{\mathrm{acq}} - i_t l_{t-1}({}_{t-1}V^{\mathrm{acq}} - P^{\mathrm{acq}}) \,.$$

Then, algebraically,

$$l_{t-1}({}_{t-1}V^{\mathrm{acq}} - P^{\mathrm{acq}})(1 + i_t) = l_t\,{}_tV^{\mathrm{acq}} \,,$$

which is equation (1)—the basic identity.