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ADJUSTED EARNINGS FOR MUTUAL LIFE INSURANCE COMPANIES

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ABSTRACT

The objectives of this paper are to demonstrate the following:

1. Natural reserves are not appropriate for adjusting earnings of mutual life insurance companies under GAAP.

2. Adjusted earnings developed by using statutory net level premium reserves together with a nonadmitted asset equal to the present value of unamortized variable acquisition expenses, and, if material, possibly with the reserves for terminal dividends, should be accepted under GAAP, provided that dividend scales are suitably designed.

3. Other approaches which are essentially equivalent to that in paragraph 2, such as those involving historical asset share funds, should likewise be acceptable.

The design of dividend scales and the plan for developing and conserving surplus prove to be important factors. The relationship to a generalization of the theory of the "release from risk" system with "revenue reserves," developed by the Joint Actuarial Committee on Financial Reporting, is noted briefly. Utilization is made of a rigorous mathematical Annual Statement model which permits mathematical demonstrations to be reduced to simple algebra.

THERE are many important differences between stock life insurance companies and mutual life insurance companies which affect the manner in which generally accepted accounting principles (GAAP) should be applied to mutual life insurance company statements. These differences have led many actuaries to conclude that adjustment of mutual life insurance company earnings by utilization of natural reserves is unreasonable and unnecessary.

I have had the pleasure of an ongoing exchange of ideas with Mr. Robert Posnak, manager of Ernst & Ernst, San Francisco, in connection with his intensive research on the characteristics of mutual life insurance companies with implications for GAAP. As a reasoning device, I have developed a mathematical model for the balance sheet and income statement of a mutual life insurance company.

The model is based upon the generalized equations of equilibrium well known in simpler form in actuarial literature. For the purpose of easy manipulation, I have used the differential calculus form of the equations of equilibrium. On the one hand, this form has complete generality; on the other hand, it reduces the analysis to simplified algebraic processes.

It is unnecessary to derive the integral solutions of the differential equations for purposes of the statement analysis. However, the formats of the solutions can be found in my discussions in *TSA*, XXI, 385–90 and 541–44, where the same technique was applied to the general solution of variable life insurance and variable annuity design. The design of dividend scales (whether on the three-factor formula, the fund account formula, or the asset share formula) and the resultant balance sheets and income statements are also practical and approximate solutions of these differential equations.

I have approached the GAAP problem by seeking an answer to the following question: If earnings have been adjusted by creating a prepaid expense asset equal to variable acquisition expense amortized over an appropriate period, how do earnings emerge in a mutual life insurance company holding statutory net level premium reserves?

The suggested conclusion of the demonstration which I set forth below is this: The annual dividend is an annual return of premium adjustment, not a benefit, and has historically followed current experience. The excess of premium over dividend is new revenue, as is investment income. If the dividend scale reasonably and conservatively reflects investment, mortality, expense, and termination experience, the rate of "profit" shown in the annual statement, adjusted by spreading variable acquisition expenses in an appropriate way, is a reasonable representation of earned income related to the current accounting period. Total revenue, consisting of new revenue and "inside" revenue from the adjusting action of the statutory net level premium reserve and the variable acquisition expense asset, matches cost in the current accounting period.

The emphasis here is on the dividend factors as reasonable representations of current experience, so that profits from each source emerge in controlled manner without significant deferment or anticipation. There are no specified criteria such as the emergence of adjusted earnings in constant proportion to premium if experience follows that assumed in the premiums, or the requirement that each generation of policyholders, in the long run, be in a zero-profit position, or that the dividend be restricted to a three-factor formula or an asset share formula or a fund account formula, as long as profits are reasonably released currently. The demonstration seems to suggest that net level premium reserves are a natural adjunct of any well-designed equitable dividend formula, just as natural reserves may be a natural adjunct of a nonparticipating premium.

Determination of dividends in a mutual life insurance company is a complex affair. It is essentially tied to a long-range plan for the development and conservation of surplus. Surplus is dependent not only upon operating earnings but also upon the realized and unrealized capital gain and loss experience of the investment portfolio, including the effects of stock market fluctuation, bond and mortgage defaults, variations in nonadmitted assets, and the like. Dividends to the various lines of business not only must relate to long-range surplus objectives but also must reflect losses in certain lines like individual and group medical care insurance, the allocation of investment income to the various lines in accordance with the investment-year method, and the allocation of federal income tax among the lines. A further complication is the requirement in most mutual company plans of operation that the distribution of investment income to individual policies in the ordinary lines must not reflect the investment-year method by year of issue. Furthermore, companies in the long bull market have been distributing part of the realized and unrealized capital gains to policyholders. In the group annuity line, distribution of net capital gains is almost universal. Also, many life insurance companies reflect investment income from assets held against non-interest-bearing liabilities and surplus in the investment income factor of their dividend formulas.

In many mutual companies, gross premium scales are determined in connection with a dividend-scale design to assure, among other objectives, that dividends will be payable in the future under a broad spectrum of possible conditions. Asset shares used in conjunction with these determinations, including the cash-value design, are basically developed using current company investment, claim, expense, and termination experience. It is assumed that in the future, as conditions change, dividend scales will be suitably adjusted to retain the earnings inherent in the original asset shares.

The mathematical approach considers a single policy. The balance sheet and income statement are developed by taking all policies in aggregate. For simplicity, the subscripts as to issue age (x) and duration (t) are usually omitted. All functions are on a continuous basis at time t.

The differential equations of equilibrium governing the most general contract design are shown below. These may appear idealized but actually are quite general, because, if Stieltjes integrals are used in the solution, we can define premiums, for instance, as

$$Pdt = 0 \quad \text{for } t \text{ nonintegral},$$

$$Pdt = P \quad \text{for } t \text{ integral},$$

DEFINITIONS

	Actual Experience	Dividend Assumptions	Reserve and Net Premium Assumptions
Gross premium	π	π	....
Net premium	P	P	P
Force of investment income*	δ''	δ'	δ
Acquisition expense	E_0''†	E_0'‡	....
Renewal expense	E''	E'‡	....
Assets	A''	A'	....
Adjusted assets	${}^aA''$	....	....
Force of mortality	μ''	μ'	μ
Reserve	V	V	V
Death benefit	F	F	F
Force of termination	ω''	ω'	....
Dividend	D	D	....
Cash value	C	C	....
Profit§ on adjusted basis	${}^aG$	....	....
Discounted probability of persistency (issue age x)	$v''^t\ {}_tp_x'$	$v'^t\ {}_tp_x'$	....

* Portfolio basis, not investment-year method.

† Variable expenses only.

‡ Asset share expense rates, which allocate to acquisition not only variable expenses but ongoing sales, and issue fixed and overhead expenses (combined in practice as E''', which also contains a general risk charge).

§ Profit in this paper means retained earnings or gain after dividends to policyholders.

and we have a precise statement for an annual premium policy (see *TSA*, XXI, 385–90). However, in the algebraic manipulations, all functions are assumed to be continuous for purposes of simplicity; this assumption does not materially alter the results, which are intended only to illustrate principles.[1]

[1] In addition, the situation in the first policy year or so, where D is held at zero, is not treated; eqs. (6)–(8) of course apply where $D = 0$.

Let us derive, first, the three-factor dividend formula:

$$\frac{dA'}{dt} = \pi + \delta' A' - \mu'(F - A') - \omega'(C - A') - E' - D\,, \quad (1)$$

$$\frac{dV}{dt} = P + \delta V - \mu(F - V)\,. \quad (2)$$

Ideally,

$$C = V - E_0' \frac{a'_{\overline{n-t}|}}{a'_{\overline{n}|}}\,, \quad (3)$$

where a = annuity at interest δ' with decrement of mortality μ' and termination ω', and n = period during which $C \leq V$. The dividend D is first determined to make $A' = C$, in order to derive its basic form so that

$$A' = V - E_0' \frac{a'_{\overline{n-t}|}}{a'_{\overline{n}|}}\,,$$

where $a'_{\overline{n-t}|} = 0$ for $t > n$, and

$$\frac{dA'}{dt} = \frac{dV}{dt} - \frac{E_0'}{a'_{\overline{n}|}} \frac{da'_{\overline{n-t}|}}{dt}\,.$$

Hence equation (1) can be rewritten as

$$\frac{dV}{dt} - \frac{E_0'}{a'_{\overline{n}|}} \frac{da'_{\overline{n-t}|}}{dt} = \pi + \delta' A' - \mu'(F - A') - E' - D\,.$$

Subtract equation (2); the result is

$$B' \frac{E_0'}{a'_{\overline{n}|}} = \pi - P + \delta' A' - \delta V - \mu'(F - A') + \mu(F - V) - E' - D\,,$$

where

$$B' = 1 - (\omega' + \mu' + \delta') a'_{\overline{n-t}|}\,,$$

noting that

$$\frac{da'_{\overline{n-t}|}}{dt} = \frac{d}{dt} \int_t^n v'^{r-t}\, {}_{r-t}p'_{x+t}\, dr = -1 + (\omega' + \mu' + \delta') a'_{\overline{n-t}|}\,,$$

so that

$$D = \pi - P - E' - \frac{E_0'}{a'_{\overline{n}|}} B' + (\delta' - \delta) V + (\mu - \mu')(F - V) - (\delta' + \mu')E_0' \frac{a'_{\overline{n-t}|}}{a'_{\overline{n}|}} ,$$

$$D = \left[\pi - P - E' - \frac{E_0'}{a'_{\overline{n}|}} (1 - \omega' a'_{\overline{n-t}|})\right] + (\delta' - \delta) V + (\mu - \mu')(F - V) . \quad (4)$$

Actually, D is determined from year to year, depending on currently representative mortality, investment, expense, over-all profit-and-loss experience, surplus development philosophy, and asset share tests; the dividend, although retaining the general form of equation (4), is expressed as

$$D = (\pi - P - E''') + (\delta' - \delta) V + (\mu - \mu')(F - V) , \quad (5)$$

where the various factors are formulated as a dividend scale. Other sophisticated dividend designs involving direct asset share and fund account formulations are essentially the same (see eqs. [14] and [15]).

To re-enter the real world, the rate of increase in assets at time t developing in practice is as follows:

$$\frac{dA''}{dt} = \pi + \delta'' A'' - \mu''(F - A'') - \omega''(C - A'') - E'' - D . \quad (6)$$

The rate of profit G at time t is

$$\frac{dA''}{dt} - \frac{dV}{dt} = G \quad \text{on the statutory basis} \quad (7)$$

and is

$$\frac{d\,{}^aA''}{dt} - \frac{dV}{dt} = {}^aG \quad \text{on the adjusted basis ,} \quad (8)$$

where

$${}^aA_0'' = A_0'' + E_0'' \quad (9)$$

and

$${}^aA'' = A'' + \frac{E_0''}{a''_{\overline{n}|}} a''_{\overline{n-t}|} , \quad (10)$$

so that

$$\frac{d\,{}^aA''}{dt} = \frac{dA''}{dt} - \frac{E_0''}{a''_{\overline{n}|}} B'' , \quad (11)$$

where

$$B'' = 1 - (\omega'' + \mu'' + \delta'')a''_{\overline{n-t}|} .$$

The rate of profit on the adjusted basis, using equations (2), (5), (6), and (11), is therefore as follows, where the E_0'' term drops out for $t > n$:

$$
\begin{aligned}
{}^aG = E''' - E'' & \\
- \frac{E_0''}{a''_{\overline{n}|}}[1 - (\omega'' + \mu'' + \delta'')a''_{\overline{n-t}|}] & \quad \text{(expense charge profit)} \\
+ \mu'(F - V) - \mu''(F - A'') & \quad \text{(mortality charge profit)} \\
+ \delta'' A'' - \delta' V & \quad \text{(interest credit profit)} \\
+ \omega''(A'' - C) & \quad \text{(surrender profit)}\ .
\end{aligned}
\tag{12}
$$

If we introduce relationship (10), equation (12) becomes the following:

$$
\begin{aligned}
{}^aG = E''' - E'' - \frac{E_0''}{a''_{\overline{n}|}} & \quad \text{(expense charge profit)} \\
+ \mu'(F - V) - \mu''(F - {}^aA'') & \quad \text{(mortality charge profit)} \\
+ \delta''\, {}^aA'' - \delta' V & \quad \text{(interest credit profit)} \\
+ \omega''({}^aA'' - C) & \quad \text{(surrender profit)}\ .
\end{aligned}
\tag{13}
$$

It should be noted that, although cash values in early policy years may be defined, for competitive reasons, to be somewhat higher than those referred to as ideal in equation (3), cash values in later policy years (e.g., year 10 and later) in nearly every mutual company are equal to statutory net level premium reserves. Thus the statutory net level premium reserve is a basic concept of actuarial policy design, not just a creation of conservative statutes.

Although my background is that of an actuary in mutual companies using the classical contribution dividend formula supported by asset shares, I want to present an interpretation of the changes in the equations, as I perceive them, when a historical asset share dividend scale is used.

Relationships (12) and (13) take a similar form if the dividend scale is based on historical asset shares or fund accounts where $A' = fV$ and f is a function less than unity for a period of years (e.g., ten) and greater thereafter. The function f represents the actuarial appraisal of the desirable level of dividend fund looking into the future. Here the continuous dividend expression would take the form

$$D = \pi + \delta' fV - \mu'(F - fV) - E''' - G' - \frac{d(fV)}{dt}\,, \tag{14}$$

where G' is a contingency charge against unusual mortality fluctuations and the like. D can also be expressed as follows:

$$D = (\pi - P - E''' - G') + (\delta' - \delta)V + (\mu - \mu')(F - V) + (\delta' + \mu')(f - 1)V - \frac{d}{dt}[(f - 1)V] . \tag{15}$$

Under these circumstances, the rate of adjusted earnings takes on the following form, corresponding to equation (12):

$$\begin{aligned} {}^aG = E''' - E'' - \frac{E_0''}{a''_{\overline{n}|}}[1 - (\omega'' + \mu'' + \delta'')a''_{\overline{n-t}|}] \\ + \mu'(F - fV) - \mu''(F - A'') + (1 - f)\mu(F - V) \\ + \delta''A'' - \delta'fV - \delta(1 - f)V \\ + \omega''(A'' - C) \\ + G' + V\frac{df}{dt} - (1 - f)P . \end{aligned} \tag{16}$$

This can be further reduced to the following, corresponding to equation (13):

$$\begin{aligned} {}^aG = E''' - E'' - \frac{E_0''}{a''_{\overline{n}|}} \qquad & \text{(expense charge profit)} \\ + \delta''\,{}^aA'' - \delta'fV \qquad & \text{(interest credit profit)} \\ + \mu'(F - fV) - \mu''(F - {}^aA'') \qquad & \text{(mortality charge profit)} \\ + \omega''({}^aA'' - C) \qquad & \text{(surrender profit)} \\ + G' + \frac{d}{dt}[(f - 1)V] \qquad & \text{(contingency profit)} . \end{aligned} \tag{17}$$

Conceivably, a company using historical dividend funds might appropriately hold policyholder reserves equal to its aggregate dividend funds. Such funds would automatically adjust for amortization of acquisition expenses in the early years; in the later years these funds would normally exceed cash values, so that, on termination, amounts would be released from which terminal dividends might be paid. The result would appear to be similar to the result of holding policyholder reserves equal to statutory net level premium reserves plus any contingent reserves for terminal dividends deemed appropriate, together with an asset for unamortized acquisition expense. (The terminal dividend matter is further discussed below.) The following mathematical formulation illustrates these points.

The emergence of gain, for a company using the historical dividend fund method and holding such funds in the balance sheet, may be developed as follows, using equations (6) and (14):

$$\begin{aligned} G &= \frac{dA''}{dt} - \frac{d(fV)}{dt} \\ &= \pi + \delta''A'' - \mu''(F - A'') - \omega''(C - A'') - E'' \qquad (18) \\ &\quad - \frac{d(fV)}{dt} - \pi - \delta'fV + \mu'(F - fV) + E''' + G' + \frac{d(fV)}{dt} , \end{aligned}$$

$$\begin{aligned} G = {} & E''' - E'' && \text{(expense charge profit)} \\ & + \mu'(F - fV) - \mu''(F - A'') && \text{(mortality charge profit)} \\ & + \delta''A'' - \delta'fV && \text{(interest credit profit)} \qquad (19) \\ & + \omega''(A'' - C) && \text{(surrender profit)} \\ & + G' && \text{(contingency profit)} . \end{aligned}$$

The difference between equations (19) and (17) is as follows:

$$G - {}^aG = \frac{E_0''}{a_{\overline{n}|}''} - \frac{d}{dt}[(f - 1)V] - (\mu'' + \delta'' + \omega'')({}^aA'' - A'') \ . \qquad (20)$$

Equation (20) in policy years $t < n$ expresses the difference in the treatment of acquisition expenses: if statutory net level premium reserves are held in the balance sheet, acquisition expenses are amortized through an asset, but if dividend funds are held, such expenses are amortized in the dividend funds, which are similar to natural reserves.

The situation at issue is interesting. Equation (9) states that

$${}^aA_0'' = A_0'' + E_0'' , \qquad (21)$$

where E_0'' represents variable acquisition expenses. Similarly,

$$f_0V_0 = V_0 - E_0''' , \qquad (22)$$

where E_0''' represents acquisition expenses charged in the dividend fund, equal to variable expenses, possibly plus nonvariable and overhead expenses. Then, using equations (21) and (22), we have

$$\begin{aligned} G_0 - {}^aG_0 &= (A_0'' - f_0V_0) - ({}^aA_0'' - V_0) \\ &= E_0''' - E_0'' , \end{aligned} \qquad (23)$$

that is, the nonvariable and overhead acquisition expenses. Historical dividend funds may, of course, be developed taking $E_0''' = E_0''$. Except for the effect of this possible difference in the definition of acquisition expense, the difference $G - {}^aG$, for $t < n$, ought to be nil.

Equation (20) in later policy years $t > n'$, after acquisition expenses are fully amortized and after f has reached its actuarial objective, becomes the following:

$$G - {}^aG = -\frac{d}{dt}[(f-1)V], \quad t > n' > n. \quad (24)$$

Since in these later policy years $f - 1$ may be defined by some equation like the following:

$$(f-1)V = aV + bF \quad (a, b \text{ constants}), \quad (25)$$

we can calculate

$$G - {}^aG = -a\frac{dV}{dt} < 0, \qquad t > n'. \quad (26)$$

This shows the effect of the dividend fund's eventually exceeding the actuarial reserve. When the dividend fund exceeds the guaranteed cash value and any appropriate termination charges, the excess released on termination is available for the payment of terminal dividends.

If $f \equiv 1$ and $G' \equiv 0$, equations (16) and (17) reduce, respectively, to equations (12) and (13), the "dividend fund" then becoming the statutory net level premium reserve. Similarly, the dividend D of equation (15) reduces to the D of equation (5), the familiar three-factor dividend formula.

Judging by dividend scales and net costs in the competitive market, there seems to be much similarity between the dividend scales of those few companies using historical dividend funds and the dividend scales of the majority of companies using the three-factor dividend formula supported and tested by asset shares based on current operating factors. In other words, by suitable design of dividend factors, the ${}^aG$ of equation (13) can be made similar to the ${}^aG$ of equation (17).

The above discussion has ignored the matter of terminal dividends paid upon termination by surrender, lapse, or death in later policy years. Provision for these dividends can be made by adjusting the death benefits payable and the cash values payable by the amount of the terminal dividend in equations (13) and (17), respectively. The amount of terminal dividends is known to vary from 1.5 per cent of total annual dividend in one company to 8 per cent in another company. Naturally, for a fixed level of margins, the lower the level of terminal dividends in relation to the excess of funds released over cash values paid, the higher the level of gains from terminating policyholders available to increase the level of annual dividends.

In the case of the historical dividend fund approach, the terminal dividend might be defined as $k(A' - V) = k(f - 1)V$, where $k \leq 1$. The factor k would reflect expenses of termination and charges for contingent financial and mortality risks.

The May 14, 1971, Response of the Joint Actuarial Committee on Financial Reporting to the December 10, 1970, AICPA exposure draft of the audit guide sets forth a powerful generalization of "revenue" with a resultant family of "revenue reserves" in a "release from risk" system for stock life insurance companies. Natural reserves are shown to be a special limiting case of revenue reserves.

The extension of the family of revenue reserves to mutual companies introduces a higher order of complexity because the gross premium is replaced by the excess of gross premium over the annual dividend. My own analysis treats the special case in this enlarged family in which the revenue reserve is stipulated to be the statutory net level premium reserve minus the unamortized variable acquisition expense asset. My equations (13) and (17) are derived reductions of the earnings broken down by source, corresponding to that on page A-6 of the committee's response. These equations suggest that the dividend itself acts to "release risk" gradually from the statutory net level premium reserve. The statutory net level premium reserve, unamortized acquisition expense asset, and dividend system as a whole appear to match revenue and cost in the general context of the committee's treatment.

The development and conservation of surplus in a mutual life insurance company are very complex matters, as noted earlier. Furthermore, whatever the design of annual dividends and terminal dividends, they are not guaranteed as to the future. The long bull market and improving portfolio interest rates, since the mid-1940's, have enabled companies to increase dividends continuously in the face of inflation of costs and expansion of markets, lines, and procedures. This situation appears to be coming to an end, and whether current scales of annual dividends and terminal dividends will persist remains to be seen.

For these reasons, it appears doubtful that mutual life insurance company surplus should be split into company share and policyholder share. It is tempting to regard part of the surplus as representing future terminal dividends, but, at least for companies whose terminal dividends are only 1–2 per cent of total dividends, such a split would not seem to be indicated on grounds of materiality. The above mathematical demonstrations would indicate also that annual dividends, as devices for release from risk, require no surplus provision where statutory net level premium reserves are held.

If dividend-scale design is such that profits emerge realistically from each source of profit currently, then GAAP are reasonably realized with statutory net level reserves. Since annual dividends are the mechanism used by a mutual life insurance company to reflect the extent to which it has been "released from risk," the concept of adjusted earnings emerging as a percentage of the premium, when actual experience is as expected, is clearly unrealistic for participating business. Additionally, splitting of statutory reserves between natural reserves and the balance appears to be meaningless; indeed, the determination of natural reserves is impractical because the ingredients of the natural reserve are not always determined at the time a policy series is introduced, and any subsequent determination would be highly arbitrary and theoretical. It would appear that the continual updating of dividend scales to reflect current conditions will control the emergence of profit currently far more directly and realistically than the occasional massive adjustments indicated for the stock company's natural reserve approach.

Finally, it should be noted that, if the dividend-scale design is appropriate and net level premium reserves are accepted under GAAP, the adjustment of an annual statement can be reduced to the simple determination of (*a*) appropriate variable acquisition expenses in aggregate and (*b*) an appropriate amortization rule to charge such expenses in the income statement, for example, in proportion to persisting aggregate premiums over an appropriate period, with (*c*) introduction of an asset equal to the summation of unamortized acquisition expenses for all years of issue in the balance sheet. The latter asset would probably be a not-admitted asset in the statutory statement.

In summary, the mathematical demonstration is intended to suggest a sufficient procedure to qualify statutory net level premium reserves for GAAP in mutual companies. It does not preclude other procedures, such as the fund account procedure demonstrated in equation (17). However, the natural reserve procedure is unnecessary, arbitrary, and impracticable for mutual life insurance companies. Hence no argument emerges for partitioning statutory net level premium reserves into natural reserves (or any other "experience" reserve) and the balance. Also, any extensive partitioning of surplus into company surplus and policyholder surplus may be undesirable on any material grounds, considering the sources, purposes, and uses of surplus.