

**ESTIMATING THE COST OF VESTING
IN PENSION PLANS**

HOWARD E. WINKLEVOSS* AND ARNOLD F. SHAPIRO

ABSTRACT

This paper investigates a fundamental approach to the development of the expected cost of vesting in pension plans. This investigation is accomplished in three stages. First, a general pension cost function is developed that includes both a retirement and a vesting cost component. This function is then analyzed for changes in its underlying factors. The second stage includes an analysis of the relative cost of vesting as measured by the ratio of the vesting cost component to the retirement cost component. This analysis is then extended by including in the numerator of the ratio a function representing the cost of vesting associated with vested terminated participants, this approach being analogous to the relative cost of vesting for the plan as a whole. The third stage provides a rationale for the general pension cost function by showing how it might be used to investigate the retirement and vesting costs, as well as the relative cost of vesting, under several actuarial cost methods.

INTRODUCTION

THE subject of vesting in pension plans is an important and timely issue. Not only are vesting provisions continually being liberalized, but several legislative proposals, if enacted, would require a minimum level of vesting in all qualified pension plans. These developments require that the cost of vesting be analyzed carefully, since vesting costs are becoming as much a part of pension plans as retirement-related costs.

The objective of this paper is to analyze the cost of vesting from several points of view through the use of a general pension cost model. The first section of the paper develops the basic pension plan cost model and investigates both retirement-related and vesting-related pension costs with respect to attained-age values and for changes in the underlying assumptions. A later section analyzes the relative cost of vesting

* Dr. Winklevoss, not a member of the Society, is assistant professor of insurance and actuarial science, Department of Insurance, Wharton School, University of Pennsylvania.

for an active participant at an attained age¹ and for a participant at an attained age who has both an expected vested termination status and an expected active status.

Subsequent sections of the paper deal with pension costs under various actuarial cost methods. In these sections the general pension cost model developed and analyzed in the earlier sections is used as an aid in investigating the retirement-related pension cost, the vesting-related pension cost, and the relative cost of vesting for each actuarial cost method's normal cost and accrued liability.

The following notation is used:

- a = Age at entry into employment, coterminous with age at entry into the pension plan;
- x = Attained age, $x \geq a$;
- z = Age at which initial vesting occurs;
- r = Normal retirement age;
- r' = First age at which a particular active employee qualifies for early retirement, $r' \leq r$;
- s = Age at which a vested terminated employee becomes separated from employment, $z \leq s < r'$;
- ${}_n B_x$ = Annual pension benefits earned for service rendered from age x to age $x + n$, n not necessarily integral, based on an entrant at age a ;
- ${}_n g_z$ = Proportion of the accrued pension benefit vested at age $z + n$, $0 \leq {}_n g_z \leq 1$ (subsequently referred to as the grading function)
= 0 for $n < 0$;
- ${}_n p_x$ = Probability that a nonactive employee aged x will survive to age $x + n$, based on the decremental force of mortality only;

and, extending Jordan's notation,

- ${}_n p_x^{aa}$ = Probability that an active employee aged x will survive in employment to age $x + n$;

¹ Two previous articles in the *Transactions* have made significant contributions to developing and analyzing attained-age vesting cost ratios: Daniel F. McGinn, "Indices to the Cost of Vested Pension Benefits," *TSA*, XVIII, 187, and William F. Marples, "Cost of Vesting in Pension Plans," *TSA*, XVIII, 277. McGinn develops attained-age vesting cost ratios for the normal cost and accrued liability under the individual projected benefit cost method with supplemental cost, while Marples develops a vesting cost ratio without regard to a particular costing technique. The vesting cost ratios formulated by these authors are referred to subsequently in the development of this paper.

p_x^{aw} = Probability that an active employee aged x will be alive in a terminated status at age $x + 1$, having terminated for reasons other than retirement or disability

$$= \int_0^1 {}_t p_x^{aa} \mu_{x+t}^{aw} {}_{1-t} p_{x+t} dt \quad (a \leq x < r')$$

$$\equiv 0 \quad (x \geq r').$$

PRESENT VALUE OF EXPECTED BENEFITS

In this section a participant's pension costs are analyzed independently of the way in which such costs are funded or accounted for in a particular plan. This approach allows pension costs to be analyzed in terms of one basic value as opposed to a normal cost and an accrued liability value under an actuarial cost method.

A single-valued measure of pension costs may be established by determining an active employee's attained-age present value of expected benefits (PVEB), which consists of two components: (i) the present value associated with expected retirement benefits (RB) and (ii) the present value associated with expected vested termination benefits (VB). The RB component for an entrant aged a who will retire at age r and whose attained age is x ($a \leq x < r$) is

$${}_{r-x} | B \ddot{a}_x^{ar} = {}_{r-a} B_a {}_{r-x} p_x^{aa} v^{r-x} \ddot{a}_r. \tag{1}^2$$

If we define

$${}_{r-x} | B v \ddot{a}_{x:\overline{y-x}}^{aw} = \sum_{k=x}^{y-1} {}_{k-z} g_z {}_{k-a} B_a {}_{k-x} p_x^{aa} p_k^{aw} {}_{r-k-1} p_{k+1} v^{r-x} \ddot{a}_r, \tag{2}$$

the corresponding VB component under the PVEB method, since ${}_{k-z} g_z \equiv 0$ for $k < z$ and $p_k^{aw} \equiv 0$ for $k \geq r'$, becomes

$${}_{r-x} | B v \ddot{a}_x^{aw} = \sum_{k=m}^{r'-1} {}_{k-z} g_z {}_{k-a} B_a {}_{k-x} p_x^{aa} p_k^{aw} {}_{r-k-1} p_{k+1} v^{r-x} \ddot{a}_r, \tag{3}$$

where $m = \text{maximum } \{x, z\}$.

² Throughout this paper it is assumed that all employees retire at the exact same age r , where $r \geq r'$. A more general assumption would be that employees may retire at any age in the interval r' to r'' , where r'' is the mandatory retirement age. Under this assumption the RB component would be given by

$$\int_{k=r'}^r v^{k-x} {}_{k-a} B_a {}_{k-x} p_x^{aa} \mu_k^r {}_{r-k} | \ddot{a}_k dk + \int_{k=r}^{r''} v^{k-x} {}_{k-a} B_a {}_{k-x} p_x^{aa} \mu_k^r \ddot{a}_k dk,$$

which assumes an actuarial reduction for retirement ages less than r .

In addition to active employees, a pension plan may have retired employees and vested terminated employees as participants. Although a vested terminated employee may or may not have attained a retired status as defined by the plan's normal retirement age, only persons who have separated from the plan by reason of retirement are classified herein as retired employees. Vested terminated employees, on the other hand, are persons who separated from the plan subsequent to initial vesting and prior to qualification for early retirement for reasons other than death or disability. It is assumed throughout this paper that vested terminated employees receive a pension once they attain the normal retirement age of the plan. Thus, vested terminated employees are subsequently referred to as being in a preretirement status if $x < r$ and in a retired status if $x \geq r$.

The PVEB for a retired employee who entered the plan at age a , retired at age r , and is currently aged x ($x \geq r$) is

$$r-a B_a \ddot{a}_x . \tag{4}$$

The corresponding value for a vested terminated employee who entered at age a , terminated at age s , and is in a preretirement status ($s \leq x < r$) is

$$s-z g_x s-a B_a r-x | \ddot{a}_x \tag{5}$$

and during his retirement status ($x \geq r$) becomes

$$s-z g_x s-a B_a \ddot{a}_x . \tag{6}$$

The subsequent three sections of this paper deal extensively with the RB and VB cost components of an active employee's PVEB. The first two sections analyze the absolute value of the employee's RB and VB functions for different attained ages and for changes in the underlying assumptions, while the third section is concerned with analyzing the ratio of the employee's VB cost function to his RB cost function. As with the functions themselves, the relative cost analysis considers the effect of different attained ages and of changes in the underlying assumptions. At this point, it seems appropriate to make some observations regarding the PVEB of retired employees and vested terminated employees.

The PVEB for a retired employee and a vested terminated employee, when the latter is in a retired status, decreases with age;³ however, for a

³ The PVEB for these two groups of pension plan members decreases at a decreasing rate if $\Delta^2 \ddot{a}_x > 0$ for $x \geq r$. For the Ga-1951 "graduated" mortality table projected with Scale C to 1970, for example, the second difference in the annuity function is slightly positive from retirement age 65 to age 100 at a 3½ per cent interest rate.

vested terminated employee in a preretirement status the PVEB value increases with age up to age r as a result of a greater probability of persisting to age r from each successive attained age and a shorter interest accumulation interval.

The absolute level of a retired employee's or a vested terminated employee's PVEB for any given attained age is inversely related to interest and future mortality and directly related to the earned pension benefits upon separation from the active plan population. The accrued benefit, of course, may be a function of service, in which case the ages at entry and separation would affect the attained-age PVEB value. To the extent that benefits are a function of compensation, the rate of increase in each participant's salary would also have a bearing on the absolute size of this function.

RB Pension Cost Function

An active employee's RB pension cost, ${}_{r-x}|^B\ddot{a}_x^{ar}$, increases with age as a result of an increasing probability of persisting in employment to the retirement age and a decreasing interest accumulation period.⁴ For a given attained age, however, the RB pension cost is inversely related to withdrawal rates and mortality rates beyond the attained age x and the interest assumption, whereas it is directly related to the magnitude of the projected retirement benefit.

In general, a change in the entry age may have an impact on each of the components of ${}_{r-x}|^B\ddot{a}_x^{ar}$. For the general case where the retirement age may be a function of the entry age, this impact is observable by investigating

$$\begin{aligned} \Delta_a {}_{r-x}|^B\ddot{a}_x^{ar} &= {}_{r-a}B_a {}_{r-x}p_{[a]+x-a}^{aa} v^{r-a-x} \Delta_a \ddot{a}_r \\ &\quad + {}_{r-a}B_a {}_{r-x}p_{[a]+x-a}^{aa} (\Delta_a v^{r-a-x}) \ddot{a}_{r_{a+h}} \\ &\quad + {}_{r-a}B_a (\Delta_a {}_{r-x}p_{[a]+x-a}^{aa}) v^{r_{a+h}-x} \ddot{a}_{r_{a+h}} \\ &\quad + (\Delta_a {}_{r-a}B_a) {}_{r_{a+h}-x}p_{[a+h]+x-a-h}^{aa} v^{r_{a+h}-x} \ddot{a}_{r_{a+h}} \end{aligned} \tag{7}$$

($\Delta_a = h$). Whether or not the retirement age is a function of the entry age, it is clear that $\Delta_a v^{ra}$ and $\Delta_a \ddot{a}_{r_a}$ are nonpositive. In general, $\Delta_a {}_{r-a}p_{[a]+x-a}^{aa}$

⁴ This function increases at an increasing, constant, or decreasing rate accordingly as $\Delta_a {}_{r-x}|^B\ddot{a}_x^{ar}$ is greater than, equal to, or less than zero, which occurs if and only if

$$p_x^{aa} + i^2 + (1+i)^2 (q_{x+1}^{aa} / p_{x+1}^{aa}) \geq 1 \quad \text{for } a \leq x < r.$$

and $\Delta_{a \ r_a - a} B_a$ are also nonpositive, again irrespective of the entry age—retirement age dependency. This implies that the RB pension cost for younger entrants generally is at least as large as that for older entrants at the same age. Furthermore, other things being equal, it is clear that if the ultimate portion of the service table has been reached and the pension benefit is strictly a function of duration, younger entrants will indeed generate a higher RB pension cost.

It is possible for either $\Delta_{a \ r_a - x} p_{[a]+x-a}^{aa}$ or $\Delta_{a \ r_a - a} B_a$ to be positive, in which case the above implication need not hold. For example, $\Delta_{a \ r_a - x} p_{[a]+x-a}^{aa}$ could be positive if the withdrawal function were ill-behaved in the sense that a younger entrant has a larger probability of withdrawing prior to retirement than an older entrant at the same attained age, a situation that may be induced through a years-of-service vesting provision. On the other hand, if $\Delta_{a \ r_a - a} B_a$ were not strictly a function of service—for example, if it were based on the employee's final average salary—then $\Delta_{a \ r - x} |^B \ddot{a}_x^{ar}$ could also be positive. Even in these cases, however, if the retirement age is independent of the entry age, the RB pension cost among entrants of the same attained age is easily determinable and in direct proportion to each employee's expected retirement benefit.

VB Pension Cost Function

Like the RB pension cost, the VB pension cost of an active employee, $r_{-x} |^B v \ddot{a}_x^{av}$, increases with age until the initial vesting age z . At some attained age at or beyond z , however, the employee's VB pension cost attains a maximum value and decreases thereafter. The maximum value occurs at some age, not necessarily integral, after the last attained age x such that

$$i \sum_{k=x+1}^{r'-1} {}_{k-a} B_a {}_{k-x} p_x^{aa} p_k^{av} > {}_{x-a} B_a p_x^{av} . \tag{8}^5$$

⁵ This criterion is derived from the exact inequality which specifies the interval during which $\Delta_x r_{-x} |^B v \ddot{a}_x^{av}$ first becomes less than zero. The maximum value occurs in the last interval x to $x + 1$ such that

$$\sum_{k=x+1}^{r'-1} {}_{k-z} g_x {}_{k-a} B_a {}_{k-x} p_x^{aa} p_k^{av} {}_{r-k-1} p_{k+1} > \left[\frac{v p_x^{aa}}{1 - v p_x^{aa}} \right] {}_{x-z} g_x {}_{x-a} B_a p_x^{av} {}_{r-x-1} p_{x+1} .$$

By use of the relationships

$$\frac{1}{i} = \sum_{l=1}^{\infty} v^l > \sum_{l=1}^{\infty} (v p_x^{aa})^l = \frac{v p_x^{aa}}{1 - v p_x^{aa}} ,$$

Clearly, the greater the interest rate and the greater the convexity of the benefit function, the further will be the critical point from age z . It should be noted that the above criterion will be less sensitive if the plan includes a grading provision, and, in this case, the grading function should be included.

The dollar value of $r_{-x}|^{Bv}\ddot{a}_x^{aw}$ is inversely related to the interest factor and is importantly a function of the plan's vesting provision, early retirement provision, and benefit schedule. The vesting provision specifies the employee's initial vesting age and the grading function. Other things being equal, the younger the initial vesting age and the steeper the slope in the grading function, during the grading interval, the greater the cost of vesting. On the other hand, the younger the employee's early retirement qualification age, as specified in the plan's retirement provision, the smaller the dollar cost of vesting, since termination rates beyond age r' are zero. Finally, the greater the magnitude of the benefit function or the steeper its slope, the greater the VB pension cost, other things being equal.

A second group of important attained-age VB cost determinants are prevesting and postvesting mortality and termination rates. A change in the mortality or termination rates at attained age $y - 1$ reduces ${}_{k-x}\dot{p}_x^{aa}$ to $(l'_y{}^{aa}/l_y{}^{aa})_{k-x}\dot{p}_x^{aa}$ for $x < y \leq k$, where $l'_y{}^{aa} = l_y{}^{aa} - \Delta d_{y-1}^{aa}$. Hence $r_{-x}|^{Bv}\ddot{a}_x^{aw}$ is inversely related to a change in mortality or termination rates after attained age x but prior to the vesting age. This relationship holds also for changes in postvesting mortality rates.

In general, an employee's VB pension cost varies directly with changes in postvesting termination; however, this need not be the case. When an increase in a termination rate occurs beyond age z or age x , whichever is greater, the change in $r_{-x}|^{Bv}\ddot{a}_x^{aw}$ will be positive, zero, or negative according to whether the incremental cost resulting from the increase in the termination rate exceeds, equals, or is exceeded by the reduction in cost resulting from the decreased persistency rates.⁶ Thus a significant aspect

$${}_{k-z}g_z \geq {}_{x-z}g_x, \quad \text{and} \quad r_{-k-1}\dot{p}_{k+1} \geq r_{-x-1}\dot{p}_{x+1},$$

the above criterion was developed.

⁶ More precisely, for $x, z < y < r'$, an increase in termination rates at age $y - 1$ will increase \dot{p}_{y-1}^{aw} to $\dot{p}_{y-1}^{aw} + \Delta\dot{p}_{y-1}^{aw}$ and reduce ${}_{k-x}\dot{p}_x^{aa}$ to $(l'_y{}^{aa}/l_y{}^{aa})_{k-x}\dot{p}_x^{aa}$, $k \geq y$, with the result that the change in $r_{-x}|^{Bv}\ddot{a}_x^{aw}$ will be positive, zero, or negative accordingly as

$$\begin{aligned} & {}_{y-1-z}g_x \quad {}_{y-1-a}B_a \quad {}_{y-1-x}\dot{p}_x^{aa} (\Delta\dot{p}_{y-1}^{aw}) r_{-y}\dot{p}_y \\ & \geq \left(1 - \frac{l'_y{}^{aa}}{l_y{}^{aa}}\right) \sum_{k=y}^{r'-1} {}_{k-z}g_z \quad {}_{k-a}B_a \quad {}_{k-x}\dot{p}_x^{aa} \dot{p}_k^{aw} \quad r_{-k-1}\dot{p}_{k+1}. \end{aligned}$$

of an employee's VB pension cost is the incidence of termination beyond age z . For example, if the probability of surviving to retirement is held constant, an increase in termination rates at older ages in conjunction with a decrease at younger ages will increase the cost of vesting, this effect being amplified the greater the convexity in the benefit function.

A final factor to be considered in analyzing the value of $r_{-x}|^{Bo}\ddot{a}_x^{aw}$ is the employee's age at entry. It makes little difference whether this variable is considered as a factor in and of itself which affects the cost of vesting or simply as a catalyst which interacts with some or all of the factors mentioned above. For this presentation the latter approach has been selected, and the subsequent discussion describes the manner in which the entry age generally interacts with the variables affecting the cost of vesting.

A change in the entry age may affect the magnitude of $r_{-x}|^{Bo}\ddot{a}_x^{aw}$ as a consequence of its interaction with the vesting age and, hence, the grading function, the early and normal retirement ages, the pension benefit, and, because of selection, the rates of termination. If the VB pension cost is rewritten as

$$r_{-x}|^{Bo}\ddot{a}_x^{aw} = \sum_{k=x}^{r'_a-1} k-z g_{z_a} k-a B_{a-k-x} \dot{p}_{[a]+x-a}^{aa} \dot{p}_{[a]+k-a}^{aw} r_{a-k-1} \dot{p}_{k+1} v^{r'_a-x} \ddot{a}_{r'_a}, \quad (9)$$

the impact of this interrelationship is emphasized.

For an attained-age comparison of the impact of a change in the entry age, only the interval $a + h \leq x < r'$ ($\Delta a = h$) is pertinent. During this interval, if the retirement benefit is strictly a function of duration, and the ultimate portion of the service table has been reached for the oldest entrant, younger entrants tend to generate a greater VB pension cost than older entrants at the same attained age, as was the case for RB pension costs. In addition, if the retirement age is a function of the entry age, the incremental cost of younger entrants is further increased, since the persistency factor ($r_{a-k-1}\dot{p}_{k+1}$), the interest factor (v^{ra}), and the annuity factor (\ddot{a}_{ra}) increase with younger entry ages. The grading function also augments the cost of the younger entrant at a given attained age during the grading interval if the vesting age is a function of duration. On the other hand, during the select period and to the extent that the withdrawal function is well behaved in the sense that younger entrants will have a smaller probability of withdrawal at a given attained age, the withdrawal function will tend to lower the VB pension cost for the younger entrant up until the time that the ultimate portion of the service table is reached. Thus, younger entry ages bring about

conflicting forces on the VB pension cost during the select period, the net effect of which depends on the characteristics of a given pension plan.

Relative Vesting Cost Function

The relative cost of vesting is important in at least two respects. First, relative cost indexes provide a convenient method by which the RB pension costs associated with a given participant can be adjusted to reflect a particular vesting provision. Second, a relative measure of the cost of vesting allows one to compare readily the impact of a vesting provision on the pension costs associated with different participants in the plan.

By defining

$$(VCR)_{x:|y-x|} = \frac{r-x | Bg_{x:y-x}^{aw} }{r-x | B\ddot{a}_x^{ar} } , \tag{10a}$$

we obtain the relative cost of vesting for an active employee or, alternatively, his vesting cost ratio (VCR) under the PVEB method,

$$(VCR)_z = \frac{\sum_{k=z}^{r'-1} k-z g_z k-a B_a k-z p_x^{aa} p_k^{aw} r-k-1 p_{k+1}}{r-a B_a r-z p_z^{aa}} , \quad a \leq x < z , \tag{10b}^7$$

and

$$(VCR)_x = \frac{\sum_{k=x}^{r'-1} k-z g_z k-a B_a k-x p_x^{aa} p_k^{aw} r-k-1 p_{k+1}}{r-a B_a r-z p_x^{aa}} , \quad z \leq x < r' , \tag{10c}$$

$$= 0 , \quad x \geq r' .$$

Clearly, the VCR is attained age independent for ages less than z . Beyond these ages, however, the numerator of the $(VCR)_x$ decreases while the denominator increases. As a consequence, the VCR reduces as the attained age increases and becomes zero at and beyond age r' .⁸

It may be observed that the relative cost of vesting is independent of postretirement mortality and the interest assumption used in deter-

⁷ Equations (10b) and (10c) are essentially equivalent to eq. (11a) developed in Marples, *op. cit.*

⁸ Beyond age z the VCR decreases at an increasing rate if $\Delta^2(VCR)_x < 0$. This occurs over the interval x to $x + 2$ if

$$x+1-z g_z x+1-a B_a p_x^{aa} p_{x+1}^{aw} > x-z g_z x-a B_a p_x^{aw} p_{x+1} .$$

A sufficient, but perhaps less sensitive, criterion is

$$p_x^{aa} p_{x+1}^{aw} > p_x^{aw} p_{x+1} .$$

mining the pension costs under the PVEB approach. Prevesting termination and prevesting mortality likewise have no effect on the VCR, since they have no effect on ${}_k-zp_x^{aa}$ for $x \geq z$. Except for these factors, however, equation (10c) shows that the VCR is a function of the other factors which were previously specified as affecting the dollar cost of vesting. It should be noted that variations in some of these components affect only the numerator of the VCR, causing the ratio to increase or decrease according to whether the dollar cost of vesting increases or decreases. Included in this category of components are the employee's initial vesting age, the slope of the grading function, and the early retirement qualification age.

Changes in the remaining factors have a more complicated effect on the VCR. For example, the denominator of the VCR receives the full impact of the ultimate increase in the employee's projected pension benefit, while the numerator does not. Thus the greater the convexity of the benefit function, the smaller the VCR. Moreover, to the extent that the accrued pension benefits increase beyond age z , changes in postvesting/preretirement mortality have a greater impact on the denominator than on the numerator. Thus, in general, the greater the level of mortality during the vested interval (age z to age r'), the greater the VCR. It should be pointed out that for a flat benefit plan, that is, where vested terminated employees and retired employees receive equal pension benefits, the VCR is invariant to mortality over all ages, including the vested interval.⁹

Changes in postvesting termination rates have a complex effect on the VCR. Clearly, any increase in the probability of terminating during the vested interval reduces the denominator of the VCR, that is, the expected retirement benefits. As discussed previously, it does not always follow that such changes in the probability of terminating beyond age z will increase the expected vested benefits (the numerator of the VCR). To the extent that the dollar cost of vesting does increase, however, the VCR may be quite sensitive to changes in the termination assumption beyond age z , since the numerator and denominator are affected in opposite directions.

A final consideration with respect to the VCR is the entry age of the participant. It was previously observed that the attained-age value of both the RB and the VB pension cost (the denominator and numerator of the VCR, respectively) is greater for younger entrants at ages beyond the select period of the withdrawal function, provided that benefits are a function of duration. Thus the VCR will be greater for younger entrants

⁹ Implicit in this conclusion is the requirement that ${}_lq_x^{aa} = {}_lq_x$, $0 < l \leq 1$, which may not necessarily be satisfied.

at ages beyond the select period, provided that the rate of growth of the VB pension cost is less than the rate of growth of the RB pension cost.¹⁰ During the select period, however, the impact of younger entrants depends on the characteristics of the plan under consideration.

Complete Attained-Age VCR Function

The vesting provision of a pension plan establishes a third category of pension plan members in addition to active employees and retired employees, namely, vested terminated employees. In this section an attained-age VCR is developed which includes in its numerator the cost associated with vested terminated employees. This ratio is subsequently referred to as the *complete* VCR. Although this ratio would not be used to adjust RB pension costs for the introduction of a vesting provision, as in the case of the previous VCR, it would be used to analyze the long-run incremental cost of vesting for any attained age.

It makes little difference whether this analysis is based on a plan population being supported by l_a new entrants each year with the survivorship group consisting of active employees, retired employees, and vested terminated employees or is based on a participant who has both an expected active employee status and an expected vested terminated employee status for ages $a \leq x < r$. For ages beyond r the participant would have both an expected retired employee status and an expected retired vested terminated employee status. The following presentation assumes the latter approach.

In order to develop the complete attained-age VCR using the probabilistic approach, one must determine the probability that a pension plan participant at a given attained age is in either an active employee status or a vested terminated employee status. The probability that a plan participant aged x is in an active status is ${}_{x-a}p_a^{aa}$, and associated with this status are (1) an RB cost equal to ${}_{r-x}|B\ddot{u}_x^{ar}$ and (2) a VB cost equal to ${}_{r-x}|B\theta\ddot{u}_x^{aw}$. The probability that the same participant is in a vested terminated employee status, having terminated at age z , for example, is ${}_{z-a}p_a^{aa}p_z^{aw}{}_{x-z-1}p_{z+1}$, and the present value of his vested benefit is ${}_aB_a{}_{r-x}p_xv^{r-x}\ddot{u}_r$. Thus the expected pension cost associated with the

¹⁰ Here the rates of growth with respect to the entry age of ${}_{r-x}|B\theta\ddot{u}_x^{aw}$ and ${}_{r-x}|B\ddot{u}_x^{ar}$ are defined as

$$\frac{{}_a\Delta_{r-x}|B\theta\ddot{u}_x^{aw}}{{}_{r-x}|B\theta\ddot{u}_x^{aw}} \quad \text{and} \quad \frac{{}_a\Delta_{r-x}|B\ddot{u}_x^{ar}}{{}_{r-x}|B\ddot{u}_x^{ar}},$$

respectively.

expected vested terminated status is

$${}_0g_z {}_{z-a}B_a {}_{z-a}p_a^{aa} p_z^{aw} {}_{r-z-1}p_{z+1}v^{r-z} \ddot{u}_r. \tag{11a}$$

Since a vested termination can occur at any age prior to age x but subsequent to age $z - 1$, the expected cost associated with the expected participant, whose attained age is x , is

$$\sum_{k=z}^{x-1} {}_{k-z}g_z {}_{k-a}B_a {}_{k-a}p_a^{aa} p_k^{aw} {}_{r-k-1}p_{k+1}v^{r-x} \ddot{u}_r. \tag{11b}$$

To summarize, the attained-age RB cost for the expected active status is

$${}_{x-a}p_a^{aa} {}_{r-x} | B \ddot{u}_x^{ar}, \tag{12a}$$

for $a \leq x < \mu$, which can be expressed as

$${}_{r-a}B_a {}_{r-a}p_a^{aa} v^{r-x} \ddot{u}_r. \tag{12b}$$

The expected attained-age VB cost is

$${}_{x-a}p_a^{aa} {}_{r-x} | Bg \ddot{u}_x^{aw}, \tag{13}$$

for $a \leq x < z$, and

$${}_{x-a}p_a^{aa} {}_{r-x} | Bg \ddot{u}_x^{aw} + \sum_{k=z}^{x-1} {}_{k-z}g_z {}_{k-a}B_a {}_{k-a}p_a^{aa} p_k^{aw} {}_{r-k-1}p_{k+1}v^{r-x} \ddot{u}_r \tag{14}$$

for $x \geq z$, both of which reduce to

$$\sum_{k=z}^{r'-1} {}_{k-z}g_z {}_{k-a}B_a {}_{k-a}p_a^{aa} p_k^{aw} {}_{r-k-1}p_{k+1}v^{r-x} \ddot{u}_r. \tag{15}$$

It can be seen that for different attained ages both the expected retirement benefits and the expected vested benefits differ only by the interest factor v^{r-x} . Since this factor is common to both the RB and VB cost components, it cancels out for the complete attained-age VCR, as does the probability of persisting to age z and the retirement age annuity, yielding

$$\frac{\sum_{k=z}^{r'-1} {}_{k-z}g_z {}_{k-a}B_a {}_{k-z}p_z^{aa} p_k^{aw} {}_{r-k-1}p_{k+1}}{{}_{r-a}B_a {}_{r-z}p_z^{aa}}. \tag{16}$$

This ratio is identical with the VCR for an active employee whose attained age is less than z , $(VCR)_z$. This is a convenient and interesting relationship, for it allows one to assess quickly the long-run incremental cost of vesting for all attained ages prior to age r , including the interval r' to r .

If the PVEB method were used to develop VCR's at the time a vesting provision is introduced, the ratios beyond age z would be less than their

ultimate values, since the active employee statuses would not as yet have a corresponding vested terminated component. In fact, the relative cost of vesting as given by the complete VCR would not materialize until a period of time had elapsed equal to the difference between the attained age and the initial vesting age.

It is possible to develop a complete attained-age VCR for ages beyond age r . The numerator would consist of the expected pension cost associated with the expected vested terminated status, that is,

$$\sum_{k=z}^{r'-1} k-z g_z k-a B_a k-a p_a^{aa} p_k^{aw} r-k-1 p_{k+1} x-r p_r \ddot{u}_r, \tag{17}$$

and the denominator may be expressed as

$$r-a B_a r-a p_a^{aa} x-r p_r \ddot{u}_x. \tag{18}$$

This analysis shows that beyond age r the complete VCR is once again equal to $(VCR)_z$. Thus $(VCR)_z$ is applicable to all attained ages from a to ω .

As stated previously, $(VCR)_z$ is invariant on interest, prevesting termination, and prevesting/postretirement mortality, other things being equal. Moreover, since $(VCR)_z$ is attained age independent, the VCR for a single entry age pension plan would exhibit these invariant properties also. It should not be concluded, however, that the incremental cost of vesting for a plan which is not restricted to a single entry age is invariant to these factors. In order to illustrate this point, the remainder of this section is devoted to the development of two complete VCR's that are more general in nature.

The first ratio to be considered is an attained-age ratio which is developed under a multiple entry age assumption. In this case, associated with each entry age is the proportion

$$l_a^{aa} / \sum_{t=0}^n l_{a_t}^{aa},$$

which defines the relative likelihood that an expected status age x entered at age a , $a_0 \leq a \leq a_n$, where a_0 and a_n are the youngest and oldest entry ages, respectively. If $\sum_{a_t}^{aa}$ is canceled out in numerator and denominator, the multiple entry age complete VCR becomes

$$\frac{\sum_{a=a_0}^{a_n} l_a^{aa} \sum_{k=z_a}^{r'_a-1} k-z_a g_{z_a} k-a B_a k-a p_{[a]}^{aa} p_{[a]+k-a}^{aw} r_0-k-1 p_{k+1} v^{r_0-x} \ddot{u}_{r_a}}{\sum_{a=a_0}^{a_n} l_a^{aa} r_a-a B_a r_a-a p_{[a]}^{aa} v^{r_0-x} \ddot{u}_{r_a}}. \tag{19}$$

If the retirement age is a function of the entry age, this ratio cannot be reduced in the same manner as the single entry age VCR. The persistency function, $z_{a-a}p_{[a]}^{aa}$, which accounts for mortality and termination prior to vesting, does not cancel, nor does the interest factor or the annuity factor. However, even in the case of multiple entry ages, if the retirement age is not a function of the age at entry, the complete VCR will be independent of interest and postretirement mortality. In addition, regardless of the entry age-retirement age interrelationship, the complete multiple entry age VCR is attained age independent.

The second ratio, which is analogous to a VCR of a mature pension plan for all members less than age r , is obtained by summing the numerator and denominator in the preceding ratio over the attained ages a to $r - 1$. This can be expressed as

$$\frac{\sum_{a=a_0}^{a_n} l_a^{aa} \sum_{x=a}^{r_a-1} \sum_{k=z_a}^{r'_a-1} k-z_a g_{z_a} k-a B_a k-a p_{[a]}^{aa} p_{[a]}^{aw} + k-a r_a - k-1 p_{k+1} v^{r-a-x} \ddot{u}_{r_a}}{\sum_{a=a_0}^{a_n} l_a^{aa} \sum_{x=a}^{r_a-1} r_a - a B_a r_a - a p_{[a]}^{aa} v^{r-a-x} \ddot{u}_{r_a}} \quad (20)$$

If the retirement age is independent of the entry age, this complete VCR is independent of postretirement mortality; however, in all cases it is dependent on interest. Moreover, except for a mature population assumption which is implied in equation (20), the complete VCR is dependent on the plan's attained-age distribution. In the general case, then, the incremental cost of vesting for the plan as a whole is dependent on all factors.

INDIVIDUAL PROJECTED BENEFIT COST METHOD
WITH SUPPLEMENTAL COST

Normal Cost

The individual projected benefit cost method (PBCM) with supplemental cost is one actuarial cost method from a family of projected benefit cost methods. The normal cost associated with the RB and VB pension costs can be expressed as

$$\frac{r-a | B \ddot{u}_a^{ar}}{\ddot{u}_{a:r-a}^{aa}} = \frac{r-a B_a r-a p_a^{aa} v^{r-a} \ddot{u}_r}{\ddot{u}_{a:r-a}^{aa}} \quad (21)$$

and

$$\frac{r-a | B g \ddot{u}_a^{aw}}{\ddot{u}_{a:r-a}^{aa}} = \frac{\sum_{k=z}^{r'-1} k-z g_{z} k-a B_a k-a p_a^{aa} p_k^{aw} r-k-1 p_{k+1} v^{r-a} \ddot{u}_r}{\ddot{u}_{a:r-a}^{aa}} \quad (22)$$

respectively, which are simply the RB and VB pension costs under the PVEB method at the attained age a divided by an $(r - a)$ -year temporary active life annuity.

Absolute RB and VB Normal Cost Analysis

The RB and VB normal cost values under this actuarial cost method, like the net annual premium of a standard insurance contract, are attained age independent and importantly a function of the entry age. In addition to the obvious interrelationship between the normal cost of the PBCM and the pension costs of the PVEB method, the following characteristics should be noted.

The impact of a change in the termination rate can be seen by considering a change at age $y - 1$. By rewriting $\ddot{a}_{a:r-a}^{aa}$ as

$$\sum_{k=a}^{y-1} v^{k-a} {}_{k-a}p_a^{aa} + \sum_{k=y}^{r-1} v^{k-a} {}_{k-a}p_a^{aa},$$

we note that the first term is not affected by the change, whereas the second term is, causing the denominator of the normal cost function to be reduced to a lesser extent than the numerator, since the latter does not have an equivalent invariant interval. Consequently, the RB normal cost is inversely related to termination rates. A similar argument holds for the inverse relationship with mortality. In addition, the RB normal cost is inversely related to the interest rate.¹¹

The VB normal cost is inversely related to prevesting termination and to mortality over all ages. As discussed previously, the effects of changes in postvesting termination on the PVEB method's VB pension cost (the numerator of the VB normal cost) is indeterminable. However, to the extent that changes in postvesting terminations do not reduce this value, the VB normal cost will be directly related to termination rates. Finally, it can be shown in a manner similar to the method discussed for the RB normal cost that the VB normal cost is also inversely related to the interest rate.

Accrued Liability

The accrued liability associated with the RB pension cost under the projected benefit actuarial cost method with supplemental cost is

¹¹ This obtains, since the RB normal cost can be rewritten as $r-a B_a \ddot{a}_r / \ddot{s}_{a:r-a}^{aa}$, and it can be shown that the rate of growth of $\ddot{s}_{a:r-a}^{aa}$, with respect to the interest rate, exceeds the rate of growth of \ddot{a}_r . For numerical estimates of the inverse relationship of RB normal costs to the interest assumption see Warren Adams, "The Effects of Interest on Pension Contributions," *TSA*, XIX, 170.

$${}_{r-x} | B \ddot{a}_x^{ar} - \frac{{}_{r-a} | B \ddot{a}_a^{ar}}{\ddot{a}_{a:r-a}^{aa}} \ddot{a}_{x:r-x}^{aa}, \tag{23}$$

and the VB accrued liability is

$${}_{r-x} | B \ddot{a}_x^{av} - \frac{{}_{r-a} | B \ddot{a}_a^{av}}{\ddot{a}_{a:r-a}^{aa}} \ddot{a}_{x:r-x}^{aa}. \tag{24}$$

It will be observed that the first terms in the RB and VB accrued liability equations are the attained-age RB and VB costs under the PVEB method. The second terms, of course, represent the present value of future normal costs.

Absolute RB and VB Accrued Liability Analysis

Since the accrued liability has a close relationship to the pension cost under the PVEB method, it seems reasonable to compare the accrued liability to the latter function. Subtracting the RB accrued liability from the RB cost of the PVEB method, one obtains $({}_{r-a} | B \ddot{a}_a^{ar} / \ddot{a}_{a:r-a}^{aa}) \ddot{a}_{x:r-x}^{aa}$. This shows that the difference in these two functions at age a is, in fact, equal to the RB pension cost of the PVEB method evaluated at age a . This difference decreases steadily for each successive attained age until age r , at which time the difference is zero. Alternatively, the RB accrued liability may be rewritten as

$${}_{r-x} | B \ddot{a}_x^{ar} \frac{\ddot{a}_{a:x-a}^{aa}}{\ddot{a}_{a:r-a}^{aa}}, \tag{25}$$

which, when expressed as a ratio to the RB cost of the PVEB method, simplifies to

$$\ddot{a}_{a:x-a}^{aa} / \ddot{a}_{a:r-a}^{aa}. \tag{26}$$

Examining this ratio, one finds that it is directly related to changes in termination and mortality, which implies that the impact of these changes is not uniform for both the RB accrued liability and the RB pension cost under the PVEB method. Such changes, however, do affect both of these functions in the same direction.

Subtracting the VB accrued liability from the VB pension cost of the PVEB method, one obtains $({}_{r-a} | B \ddot{a}_a^{av} / \ddot{a}_{a:r-a}^{aa}) \ddot{a}_{x:r-x}^{aa}$. Again it can be seen that the difference at age a is simply the VB cost under the PVEB method, and that the difference declines steadily through age r . For

$a \leq x < r'$, the VB accrued liability is a concave function which changes direction at some age beyond age z , as was the case with the PVEB function. Since the VB cost of the PVEB method exceeds the VB accrued liability, and their difference forms a monotonically decreasing function, the VB accrued liability turns down at some age beyond the age which maximizes the PVEB function.

The VB accrued liability, unlike the VB cost of the PVEB method, which is zero at age r' , is negative at this age if $r' < r$. In this case the negative value is

$$-\frac{r-a \left| \frac{B\theta \ddot{a}_a^{av}}{\ddot{a}_{a:r-a}^{aa}} \right| \ddot{a}_{x:r-x}^{aa}}{\ddot{a}_{a:r-a}^{aa}} \tag{27}$$

This follows, since the present value of future vested benefits, $r-x \left| B\theta \ddot{a}_x^{av} \right|$, at and beyond age r' is zero, while the present value of future VB normal costs is positive. Clearly, for ages r' to r the negative VB accrued liability decreases in value to zero by age r . It is important to point out, however, that the VB accrued liability will be negative at some age prior to age r' and that r' is the last age at which it can obtain its minimum value. The negative VB accrued liability could be eliminated by costing an employee's expected vested benefit to some age less than r , and in most cases less than r' .

It should be noted that an employee's total accrued liability attains a maximum value at age r ; however, because of the negative VB accrued liability that occurs at some attained ages, the total accrued liability may not increase monotonically from age a to age r .

The VB accrued liability for $a < x < z$ may be expressed as

$$r-x \left| \frac{B\theta \ddot{a}_x^{av} \ddot{a}_{a:x-a}^{aa}}{\ddot{a}_{a:r-a}^{aa}} \right| \tag{28}$$

The ratio of this liability to the VB cost under the PVEB method, upon simplification, yields once again $\ddot{a}_{a;x-a}^{aa} / \ddot{a}_{a:r-a}^{aa}$. This ratio, as noted in the discussion of the RB accrued liability, implies an impact which is in the same direction but is nonuniform for changes in termination and mortality on the VB accrued liability and the VB pension cost of the PVEB method. The VB accrued liability for $z \leq x < r$ can be expressed as

$$\frac{r-x \left| \frac{B\theta \ddot{a}_x^{av} \ddot{a}_{a:x-a}^{aa}}{\ddot{a}_{a:r-a}^{aa}} \right|}{\ddot{a}_{a:r-a}^{aa}} - \frac{r-a \left| \frac{B\theta \ddot{a}_{a:x-a}^{av} \ddot{a}_{x:r-x}^{aa}}{\ddot{a}_{a:r-a}^{aa}} \right|}{\ddot{a}_{a:r-a}^{aa}} \tag{29}$$

This function, like its original form given in equation (24), is not readily analyzed for changes in the underlying factors. It is interesting to note,

however, that when the expected vested terminated status is incorporated into the VB accrued liability, a ratio of the *complete* accrued liability to the VB cost of the PVEB method yields $(1 + i)^{x-a} \ddot{u}_{a:x-a}^{aa} / \ddot{u}_{a:r-a}^{aa}$. Consequently, similar conclusions hold for changes in terminations and mortality as before.

Relative Vesting Cost Analysis

The relative cost of vesting for the normal cost under the projected benefit cost method is

$$\frac{\sum_{k=z}^{r-1} k-z g_z k-a B_a k-z p_z^{aa} p_k^{aw} r-k-1 p_{k+1}}{r-a B_a r-z p_z^{aa}} \tag{30}^{12}$$

Thus the relative cost of vesting for the normal cost of the PBCM is equivalent to $(VCR)_z$.

The relative cost of vesting for the accrued liability for $x < z$ is given by

$$\frac{r-x | B g \ddot{u}_x^{aw} - (r-a | B g \ddot{u}_a^{aw} / \ddot{u}_{a:r-a}^{aa}) \ddot{u}_{x:r-x}^{aa}}{r-x | B \ddot{u}_x^{ar} - (r-a | B \ddot{u}_a^{ar} / \ddot{u}_{a:r-a}^{aa}) \ddot{u}_{x:r-x}^{aa}} = \frac{r-x | B g \ddot{u}_x^{aw}}{r-x | B \ddot{u}_x^{ar}} \tag{31}$$

This quantity, like the normal cost under this method, also reduces to $(VCR)_z$.

The corresponding relative cost of vesting for the accrued liability during the interval z to r is given by

$$\frac{r-x | B g \ddot{u}_x^{aw} \ddot{u}_{a:x-a}^{aa} / \ddot{u}_{a:r-a}^{aa} - r-a | B g \ddot{u}_a^{aw} \ddot{u}_{x:r-x}^{aa} / \ddot{u}_{a:r-a}^{aa}}{r-x | B \ddot{u}_x^{ar} - (r-a | B \ddot{u}_a^{ar} / \ddot{u}_{a:r-a}^{aa}) \ddot{u}_{x:r-x}^{aa}} \tag{32a}$$

which reduces to

$$(VCR)_x = (VCR)_{z:x-z} \frac{\ddot{u}_{x:r-x}^{aa}}{\ddot{s}_{a:x-a}^{aa}} \tag{32b}$$

If one were to proceed for this actuarial cost method as under the PVEB method, that is, incorporating the vesting costs associated with the expected vested terminated status, he would find that the attained-age VCR for the accrued liability given in equation (32b) also reduces to $(VCR)_z$. In fact, just as under the PVEB method, $(VCR)_z$ is applicable

¹² This vesting cost ratio, as well as the one in eq. (32b) yet to be discussed, in the form $1 + (VCR)$, is analyzed under a wide set of assumptions for the special case where $k-a B_a = k-a$ in McGinn, *op. cit.*

to the accrued liability for all ages from a to ω when the complete VCR is desired. As under the PVEB method, however, an aggregate VCR which includes a multiple entry age assumption does not have the invariance of $(VCR)_z$ for either the normal cost or the accrued liability.

The individual PBCM with supplemental cost, in its strictest form, may not be the most logical approach to costing a participant's future vested benefits at the time vesting is introduced into the plan. As noted previously, this approach develops a negative VB accrued liability over some attained ages, and, consequently, if the participant's age at the time vesting is introduced falls into this age range, his total accrued liability would be less than this RB accrued liability prior to vesting. One approach to eliminating the initial negative impact for this participant is to use only his future VB pension cost, rather than his entry age VB pension cost, to calculate the normal cost under the PBCM with supplemental cost. Even under this approach, however, the employee's VB accrued liability may still attain a negative value at some age prior to r' and continue through age r . As previously mentioned, this eventuality may be eliminated by costing the participant's entry age or attained VB cost, whichever is selected, to some age prior to r .

ACCRUED BENEFIT COST METHOD

The RB and VB pension cost components under the accrued benefit cost method (ABCM) are exactly the same as their PVEB counterparts except for the benefit function. For the normal cost values under this actuarial cost method only the unit of benefit earned by the employee for his current year of service is considered, vis-à-vis his projected benefit under the PVEB method; hence ${}_{r-a}B_a$ in ${}_{r-x}|{}^B\ddot{u}_x^{ar}$ and ${}_{k-a}B_a$ in ${}_{r-x}|{}^{B_0}\ddot{u}_x^{av}$ are replaced by $\Delta_x{}_{x-a}B_a$ to form the RB and VB normal cost values, respectively. The accrued liability, on the other hand, considers all previously earned units of benefit. Consequently, in this case the benefit functions in the RB and VB cost of the PVEB method are replaced by ${}_{x-a}B_a$ to form the corresponding RB and VB accrued liabilities.

A convenient way to analyze the RB and VB normal cost and accrued liability functions is to regard the accrued benefit cost method as being equal to the difference of two PVEB functions. Considering first the RB normal cost and accrued liability, the two PVEB functions become

$${}_{r-x}|{}^B\ddot{u}_x^{ar} - {}_{r-x}|{}^{B'}\ddot{u}_x^{ar}, \tag{33a}$$

where ${}_{r-a}B'_a = {}_{r-a}B_a - \Delta_x{}_{x-a}B_a$ for the RB normal cost and where ${}_{r-a}B'_a = {}_{r-a}B_a - {}_x{}_{-a}B_a$ for the RB accrued liability. For the VB normal

cost and accrued liability, on the other hand, the two PVEB functions are

$${}_{r-x}|B\ddot{a}_x^{aw} - {}_{r-x}|B'\ddot{a}_x^{aw}, \tag{33b}$$

where ${}_{k-a}B'_a = {}_{k-a}B_a - \Delta_x {}_{x-a}B_a$ for the VB normal cost and where ${}_{k-a}B'_a = {}_{k-a}B_a - {}_{x-a}B_a$ for the VB accrued liability. Since the RB and VB pension cost functions under the PVEB method have been analyzed previously for changes in the underlying assumptions, the behavior of the RB and VB normal cost and accrued liability functions is available when expressed in terms of these two PVEB functions.

The normal cost and accrued liability VCR for the ABCM is

$$\frac{{}_{r-x}|B\ddot{a}_x^{aw} - {}_{r-x}|B'\ddot{a}_x^{aw}}{{}_{r-x}|B\ddot{a}_x^{ar} - {}_{r-x}|B'\ddot{a}_x^{ar}} = \frac{\sum_{k=x}^{r'-1} {}_{k-x}g_z {}_{k-x}\dot{p}_x^{aa} \dot{p}_k^{aw} {}_{r-k-1}\dot{p}_{k+1}}{{}_{r-x}\dot{p}_x^{aa}}, \tag{34}$$

where B' is defined as before, which is the $(VCR)_x$ evaluated at ${}_{k-a}B_a = {}_{r-a}B_a$ for all $k \geq x$. It follows, therefore, that since ${}_{k-a}B_a$ normally is less than ${}_{r-a}B_a$ for $k < r$, the VCR under the accrued benefit cost method generally will exceed $(VCR)_x$ until age r' , at which time they both vanish.

The complete VCR under this actuarial cost method, using the approach previously presented, is

$$\left(\sum_{k=x}^{z-1} {}_{k-x}g_z {}_{k-a}B_a {}_{k-a}\dot{p}_a^{aa} \dot{p}_k^{aw} {}_{r-k-1}\dot{p}_{k+1} + \sum_{k=x}^{r'-1} {}_{k-x}g_z {}_{x-a}B_a {}_{k-a}\dot{p}_a^{aa} \dot{p}_k^{aw} {}_{r-k-1}\dot{p}_{k+1} \right) / {}_{x-a}B_a {}_{r-a}\dot{p}_a^{aa}. \tag{35}$$

It can be shown that, if $\Delta_x {}_{x-a}B_a > 0$ for $x < r$, which is generally the case, the following hold:

$$\begin{aligned} (VCR)_z &< {}^{ABCM}(VCR)_x < (VCR)_z \frac{{}_{r-a}B_a}{{}_{x-a}B_a}, \quad z \leq x < r, \tag{36} \\ (VCR)_z &= {}^{ABCM}(VCR)_r, \end{aligned}$$

where ${}^{ABCM}(VCR)_x$ is the complete VCR for the accrued liability under the accrued benefit cost method. The first inequality points out that for $z \leq x < r$ the complete VCR under the accrued benefit cost method exceeds $(VCR)_z$, the latter being the complete VCR for the PVEB method and the projected benefit cost method. The second inequality provides an upper bound on ${}^{ABCM}(VCR)_x$. In addition, ${}^{ABCM}(VCR)_x$ and $(VCR)_z$ converge as the attained age approaches r . Beyond age r , of course, the complete $(VCR)_z$ is applicable for all three costing methods.

TERMINAL BENEFIT COST METHOD¹³

A pension plan participant's pension cost under the terminal benefit cost method (TBCM) occurs at the inception date of his pension and is equal to the present value of his future pension benefits. The pension cost for a retiring active employee, which would be classified as an RB cost under the plan, is ${}_{r-a}B_a\ddot{a}_r$. Similarly, the pension cost for a retiring vested terminated employee, which would be classified as a VB cost under the plan, is ${}_{s-z}g_z {}_{s-a}B_a\ddot{a}_r$. The accrued liability under this cost method exists for each subsequent age in retirement ($x > r$) and is equal to the cost under the PVEB method as given in equations (4) and (6), respectively, for retired employees and vested terminated employees in a retired status.

A VCR does not exist for a particular participant under the TBCM, since each participant generates either an RB or a VB cost component of the plan. However, by developing a single entry age complete VCR, the VB pension cost associated with the participant's expected retiring vested terminated employee status to the RB pension cost associated with his expected retiring active employee status yields once again $(VCR)_z$. Moreover, a similar ratio of the VB accrued liability to the RB accrued liability for all ages $x > r$ also results in $(VCR)_z$.

CONCLUSION

The PVEB method provides a pension costing technique for assessing a participant's pension costs without regard to a particular actuarial cost method. As has been shown, this method is fundamental to the normal cost and accrued liability under the individual projected benefit cost method with supplemental cost, the accrued benefit cost method, and the terminal benefit cost method.

The vesting cost ratio given by $(VCR)_z$ has wide applicability in measuring the relative cost of vesting associated with active employees. In addition to being applicable to the PVEB method for ages less than z , it also applies to the normal cost over all ages under the projected benefit cost method with supplemental cost and its corresponding accrued liability for ages less than z . A somewhat simpler version of $(VCR)_z$ is applicable to the normal cost and accrued liability under the accrued benefit cost method. $(VCR)_z$ was found to be invariant with prevesting mortality and termination, postretirement mortality, and interest.

¹³ This approach to pension costs is generally referred to as the terminal funding method; however, the phrase "terminal benefit cost method" is used here to maintain consistency with the other two methods discussed, namely, the projected and accrued benefit cost methods.

A *complete* vesting cost ratio was developed to take into account the cost associated with a vested terminated status. Modifying $(VCR)_z$ to recognize this liability produces $(VCR)_z$ for all ages greater than or equal to z for the pension costs under the PVEB method, for the accrued liability under the PBCM with supplemental cost, and for the pension costs of the terminal benefit cost method. Finally, $(VCR)_z$ is also applicable to the relative cost of vesting as measured in terms of the plan's long-run expected benefit outlays, that is, its pay-as-you-go pension costs.

ACKNOWLEDGMENTS

The authors would like to express their appreciation for the comments and suggestions of Mr. William F. Marples, Dr. Dan M. McGill, and Mr. Charles L. Trowbridge.

DISCUSSION OF PRECEDING PAPER

[NOTE.—The simultaneous presentation of two papers dealing with the cost of vesting in pension plans elicited discussion generally addressed to the content of both papers. Therefore, instead of separate discussions appearing after each paper, all discussion of both papers will be found on the pages following Mr. C. L. Trowbridge's paper, the second of the two on the subject.]

