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A UNIFORM APPROACH TO ACCOUNTING FOR BOND AND COMMON STOCK INVESTMENTS

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ABSTRACT

This paper considers two principles which at present are given little weight in accounting for investments: (1) the reported earnings of a business entity should not change solely because an investment is sold at a capital gain or loss and the proceeds are reinvested; and (2) the accounting treatment of common stock investments should not vary according to the portion of the issuer's outstanding common stock which the investment represents. The paper explores some of the implications of making the above two principles paramount in accounting for: (*a*) bonds, (*b*) common stocks accounted for by the equity or consolidation methods, (*c*) marketable common stocks, and (*d*) business combinations.

The paper makes no attempt to appraise the relative merits of various principles which might be given prime importance in accounting for investments.

The paper does not deal with accounting for investments held in life insurance company separate accounts or in other accounts of a similar nature.

Although the discussion is not necessarily limited to generally accepted accounting principles (GAAP), it is set within a GAAP frame of reference.

INTRODUCTION

THE development by the American Institute of Certified Public Accountants (AICPA) of an audit guide for stock life insurance companies has stirred considerable interest in life insurance circles in "generally accepted accounting principles" (GAAP). One aspect of GAAP which is highly significant for insurance companies is the treatment of bond and stock investments. During the last two or three years there have been extended discussions of accounting for marketable common stocks held by life insurance companies in general account portfolios or by companies other than life insurers. These discussions have thus far been inconclusive. Recently there have also been intense discussions of whether, and, if so, how, to choose be-

tween two significantly different ways of accounting for business combinations effected through an exchange of common stock. In addition, there are continuing discussions of the method of amortizing the goodwill item which arises in connection with many large common stock transactions. Meanwhile, two quite different methods of accounting for bonds are currently permitted by the AICPA for use in certain circumstances by banks. This paper will explore certain aspects of accounting for bonds and common stocks which relate to all the above-mentioned areas of discussion or difference.

This paper will not deal with life insurance company separate accounts or other accounts of a similar nature.

BONDS IN GOOD STANDING

Let us define "bonds in good standing" roughly as bonds on which it appears that interest and principal will be paid when due. Let us assume that such bonds are carried at amortized value.

For some years many observers have felt that if (1) an investment-grade bond is sold before its maturity and a capital gain or loss is incurred (perhaps because the prevailing market interest rate for bonds of comparable quality has changed since the bond was bought) and (2) the proceeds of the sale are reinvested in a new bond of comparable quality at the then prevailing market yield, the gain or loss at sale and the offsetting lower or higher subsequent annual yield from the new bond represent artificial distortions of the income pattern which would have emerged if the original bond had not been sold—that is, if the bond had not been "rolled over." These persons feel that the mere exchange of one bond for another does not change the holder's situation in such a way that a gain or loss should be recognized at the moment of the exchange.

Without taking a position on the above viewpoint, let us see what adjustment would have to be made to a company's income at and after the date of rollover of a bond in order to remove the effect on income which would otherwise be introduced by the rollover. Table 1 illustrates the effects of several approaches. For this example it is assumed that a company purchases for $100,000 a bond which pays $3,000 at the end of each year for ten years and matures on the tenth anniversary for $100,000. As each coupon is received, it is immediately reinvested in a similar 3 per cent bond of the same quality, purchased at par and maturing ten years from the date of its purchase. During the third year from purchase of the $100,000 bond, however, the prevailing market interest rate for bonds of the given quality changes from 3 to 5 per cent. Accordingly, all coupons received on or after the third anniversary are reinvested in

5 per cent ten-year bonds, similarly purchased at par. When bonds mature, they are immediately reinvested in 5 per cent bonds purchased at par. Commissions, taxes, and other expenses are ignored.

Table 1 covers the first fifteen years from the purchase date of the $100,000 bond. Column 1 shows the income which would be reported for

TABLE 1

INCOME FROM $100,000 INVESTED IN TEN-YEAR BOND AT 3 PER CENT, WITH PROCEEDS (INCLUDING COUPON INCOME) REINVESTED IN TEN-YEAR BONDS AT CURRENT YIELD RATE, IF YIELD RATE CHANGES TO 5 PER CENT AT DURATION 3

YEAR t	ORIGINAL BOND RETAINED TO MATURITY, THEN REINVESTED (1)	ORIGINAL BOND SOLD FOR $88,427 AT DURATION 3 AND PROCEEDS REINVESTED IN A 5% TEN-YEAR BOND				
		Loss Recognized at Duration 3 (2)	Loss Amortized at 3% over 7 Years* (3)	Loss Amortized at 5% over 10 Years* (4)	Loss Amortized at 5% over 7 Years* (5)	Replacement Bond Booked at $100,000* (6)
1	$ 3,000	$ 3,000	$ 3,000	$ 3,000	$ 3,000	$ 3,000
2	3,090	3,090	3,090	3,090	3,090	3,090
3	3,183	−8,390	3,183	3,183	3,183	3,183
4	3,342	4,764	3,254	3,844	3,342	3,773
5	3,509	5,002	3,446	4,036	3,510	3,979
6	3,685	5,252	3,650	4,237	3,685	4,193
7	3,869	5,514	3,863	4,449	3,868	4,420
8	4,062	5,790	4,090	4,672	4,063	4,657
9	4,266	6,080	4,329	4,906	4,266	4,908
10	4,479	6,384	4,581	5,150	4,479	5,173
11	6,703	6,703	6,703	5,409	6,703	5,450
12	7,098	7,098	7,098	5,738	7,098	5,801
13	7,514	7,514	7,514	6,087	7,514	6,174
14	7,890	7,890	7,890	7,890	7,890	7,890
15	8,285	8,285	8,285	8,285	8,285	8,285
Total	$73,975	$73,976	$73,976	$73,976	$73,976	$73,976

* Explanation of cols. 3–6: Each year income is (*a*) charged with a level amount which is sufficient to pay off over the amortization period the amount to be amortized plus interest on the outstanding balance and (*b*) credited with interest on the outstanding balance. The quantity in (*a*) is equal to $11,573 $\div a_{\overline{n}|i}$, where n is the number of years in the amortization period and i is the amortization interest rate. In col. 6, $n = 10$ and $i = 3.43$ per cent, which is the effective yield of a bond purchased for $100,000, bearing coupons of 5 per cent of $88,427, and maturing in ten years for $88,427.

each year if all bonds were held to maturity. Column 2 shows the income which would be reported if the $100,000 bond were rolled over at duration 3 and no adjustment were made. The remaining columns show the results of adjusting the column 2 figures in various ways. The desired adjustment is the one which reproduces column 1.

It may be noted that there is a discontinuity in the column 1 income

curve between durations 10 and 11. Espousal of an approach which adjusts column 2 in such a way as to reproduce column 1 would imply acceptance of the discontinuity in column 1 itself. In this connection it may be noted that the discontinuity at durations 10 and 11 does not result from a management decision to realize a capital gain or loss.

When the $100,000 bond is sold at duration 3, the proceeds equal $\$3{,}000a_{\overline{7}|0.05} + \$100{,}000v^{7}_{0.05} = \$88{,}427$. Accordingly, the company incurs a capital loss of $11,573 at duration 3 in column 2. It is clear that the adjustment we seek must involve an addition of $11,573 to income at duration 3 and deductions from income at the next seven durations. Column 3 shows the income which results when the $11,573 is charged off, or amortized, over seven years at 3 per cent interest, in the way in which a 3 per cent mortgage would be paid off. The results do not reproduce column 1. Column 4 involves an amortization over ten years at 5 per cent. Although a ten-year amortization is easily ruled out by anyone who has columns 1 and 2 in front of him, column 4 is included here as a matter of interest; it can be described as amortizing the gain over the lifetime of the new bond, at the effective interest rate of the new bond.

Column 5 involves a seven-year amortization at 5 per cent. The results reproduce column 1 except for a few rounding differences. Column 6, also included as a matter of interest, involves entering the new bond on the books at $100,000 and amortizing it over its lifetime at the interest rate which equals the effective yield from a bond bearing coupons of $4,421 (=5 per cent × $88,427) annually, purchased for $100,000 and maturing in ten years for $88,427.

The AICPA audit guide for banks, dated 1968, permits either no adjustment (i.e., col. 2 in the above illustration) or, in certain circumstances, a deferral and amortization of the gain or loss over the remaining period of the bond which was sold—that is, the approach of columns 3 and 5 in Table 1. The AICPA audit guide does not specify the interest rate to be used in the amortization. Table 1 shows that using the old interest rate will not always give the desired result and that use of the new interest rate may always give that result. The Appendix to this paper offers a demonstration that, for any case of the type illustrated above, where each bond runs for n years from its date of purchase and the interest rate changes from i to i' during the rth year, the approach which reproduces the nonrollover income involves amortization at the new interest rate over the remaining period of the bond sold.

If analysis of more general cases were to lead to results less simple than the above, the above would, perhaps, still serve as an acceptable practical approach. Also, since there is relatively little difference between columns 3

and 5 in Table 1, approximations to the theoretically correct interest rate would, presumably, give satisfactory results. For discussion purposes this paper will proceed on the assumption that whenever one bond is exchanged for another and the sale and purchase are based on the same new effective interest rate, amortization of the gain or loss at approximately that new interest rate over the remaining period of the bond sold is an appropriate way to adjust the income, if the ideal goal is to reproduce the income that would have emerged if the bond had not been rolled over. It should be noted that this assumption is for discussion purposes only and is not a logical prerequisite to the steps which follow.

If we proceed on the above premise, the next question is what to do if the new bond is bought to yield an effective interest rate different from the effective interest rate which is implied by the price at which the old bond is sold. For example, suppose that the old bond was originally bought to yield 3 per cent. It is sold, before maturity, at a time when bonds of comparable quality are yielding 5 per cent. In its place the company buys a bond which is expected to yield 6 per cent.

In order to analyze the above situation, let us first suppose that, instead of using the proceeds from the sale of a previous investment, the company bought the new bond with newly obtained money or with proceeds from the maturity of, or income from, other investments. If the company bought, with money from such sources, a 6 per cent bond instead of a 5 per cent bond, its future income would be greater each year by an easily determined amount related to the 1 per cent differential between the two yields. There seems to be no reason why the company's future income should differ, as between a new 5 per cent bond and a new 6 per cent bond, in different ways depending on whether it uses investment sales proceeds or money from other sources to buy the bond which it has selected. Accordingly, if we are adjusting the company's investment income after a sale and reinvestment, let us make the same adjustment after purchase of a 6 per cent bond that we would have made after purchase of a 5 per cent bond. If we do that, we will reflect the effect of the company's choice between the two new bonds in the way in which the choice would have been reflected if money from other sources had been used.

The above approach requires, in the case where a 6 per cent bond is bought, amortization of the deferred capital loss at 5 per cent. In general, it requires that amortization be at the interest rate implicit in the sales price of the bond sold, regardless of the yield obtained on the new investment. This rule applies even when the sales proceeds are placed in a cash account bearing no interest or when they are invested in common stocks

or something else. In each case the company's choice of what to do with the proceeds will be reflected in its future income in the way in which a choice of investment of money from other sources would be reflected.

A limitation on the foregoing discussion is the fact that it contemplates that the proceeds of every sale are reinvested in some way (in bonds, stocks, cash, or other assets). This situation might not hold in a reporting period in which the company experienced a negative cash flow. If it did not hold, then a proportionate part of the capital gain or loss arising during the period would be recognized in the income for the current period, and the remaining part would be deferred and amortized. The amount to be recognized currently would equal (1) the portion of the negative cash flow that could not be met from sources (e.g., maturities) other than sales of investments during the period multiplied by (2) the ratio of the aggregate realized capital gain or loss arising in the period to the total proceeds from sales of investments in the period.

COMMON STOCKS ACCOUNTED FOR BY THE EQUITY OR CONSOLIDATION METHODS

When a company holds a rather large percentage of the outstanding shares of common stock of another corporation (the conditions are described in *Opinion No. 18* of the Accounting Principles Board of the AICPA), the company is required under GAAP to account for its holding by the "equity method" (also described in *APB Opinion No. 18*). This method involves crediting to the holder's income any earnings reported by the issuer on the shares held and charging to the holder's income any reported losses. The value of the shares on the holder's books is periodically increased by the amount of any earnings and decreased by the amount of any losses and any dividends paid to the holder. If, upon acquiring the shares, the holder has paid more for them than their then book value (based on the value of the issuer's tangible and identifiable intangible assets less liabilities), he generally will have entered them on his own books at their full acquisition cost to him. However, he will have to amortize the excess of acquisition cost over book value (which excess we shall call "goodwill") over a period of years. Hence in a typical (reasonably good) year the holder credits his income with (1) dividends received plus (2) earnings in excess of the dividends, charges income with (3) the year's piece of the amortization of goodwill, and increases or decreases the value of the stock on his books (the "carrying amount") by item 2 minus item 3. If the holder sells any of the shares, he includes in his income for the period in which he sold the shares any excess of sales proceeds over carrying amount, or vice versa, as a capital gain or loss.

The idea behind the equity method is to be consistent with the consolidation method, under which a proportionate share of each item of a subsidiary's financial statement is included in the consolidated statement.

If capital gains and losses on bonds are regarded by some observers as distorting the natural earnings pattern of the holder, then it seems that gains and losses arising from the sale of common stock which is accounted for by the equity method might be similarly regarded. Let us consider an example. Suppose that Company S is formed with an investment of $1 million, of which $200,000 is put up by Company A. Company A receives 20 per cent of the common stock of Company S, books it at $200,000, and accounts for it by the equity method. During the first few years, because of starting-up expenses, Company S loses money. Soon, however, it begins to earn money, and after a few more years its net worth is back up to $1 million. At this point Company A sells its 20 per cent of Company S (with carrying amount $200,000) to Company B for $300,000 and includes in its income the capital gain of $100,000.

Now let us suppose that on the following day both Company A and Company B change their minds, and the Company S shares are sold back to Company A for the same price, $300,000. Most observers would probably agree that in this situation Company A should not now book the Company S shares at $300,000. That is, Company A has not become $100,000 richer just because the Company S shares passed out of its hands for a day, with no ultimate change in Company A's cash position. It seems clear that the Company S shares should be booked at $200,000. The accounting entry needed to accomplish this result would, presumably, be either an immediate writeoff of goodwill or a reversal of the prior transaction.

But let us now suppose that, instead of repurchasing the Company S shares, Company A spends the $300,000 for 20 per cent of the common stock of Company T, and that the Company T shares have a book value of $200,000. One might question whether Company A should book the Company T shares at $300,000 (their acquisition cost) if it would have booked the Company S shares at $200,000. It might seem that any such difference in treatment would have to be based on a judgment that the Company T shares were worth more than the Company S shares, even though the two blocks had the same market value and the same book value. Yet acquisition cost is the figure at which shares of a newly acquired subsidiary would normally be booked under GAAP.

As a way out of the above dilemma, a person who feels that capital gains and losses on bonds should be deferred and amortized might be inclined to seek a similar way of treating the capital gain which arises

from the sale of the Company S shares in the above example. Let us refine the above example to make it more closely analogous to our earlier example on bonds. Suppose that Company A sells the Company S shares and buys the Company T shares on the same day and that from that day on the earnings of Company S and Company T are identical. Let us now account for Company A's holdings in such a way that Company A's income will be the same as it would have been if it had kept its Company S shares and not bought the Company T shares.

Suppose that Company A books the Company T shares at $300,000 and amortizes the $100,000 goodwill over twenty years, as might be done in a reasonably typical case under the equity method. If we are to reproduce Company A's income as it would have been without an exchange, we must defer the $100,000 capital gain from sale of the Company S shares and accrue it over twenty years on a curve which exactly matches the writeoff curve of the goodwill. This looks like a very simple solution to the simple problem we have posed.

In real life, of course, there may not be a new block of stock which so closely resembles the old. There may not even be a new block at all. Hence there may not be any writeoff of goodwill against which to match the accrual of the capital gain. In our discussion of the deferral of capital gains and losses on bonds, we concluded that the deferral and amortization procedure should depend only on the bond sold and not in any way on the type of reinvestment that was made. If we were to treat the case of Company A by analogy, we would similarly make the deferral and accrual of its $100,000 capital gain depend only on the characteristics of the Company S shares.

In the case of bonds, the appropriate amortization period turned out to be, apparently, the remaining lifetime of the bond. In the case of a block of common stock, there is no fixed lifetime. The analogous treatment for the block of stock might seem to be amortization (or accrual, in the typical case of a capital gain) over an indefinite period. However, if our objective is to reproduce the income which Company A would have reported if it had not sold the stock, it would seem reasonable to accrue the capital gain over the period of years during which it was anticipated that Company S would continue to report favorable earnings on the shares. By "favorable" earnings is meant earnings exceeding those which might be regarded as a reasonably satisfactory return on the company's net worth. If it were felt that favorable earnings could not reasonably be anticipated from Company S (or, indeed, from any company) in more than, say, twenty years from the present date, then the accrual period would be limited to twenty years. The amount of each year's

accrual would equal the "favorable" (excess over satisfactory) portion of the earnings anticipated at the time of sale for that year. The present value of these excess earnings as of the time of sale would have to bear the same ratio to the present value of total earnings anticipated in the indefinite future as the capital gain at sale bore to the total sales proceeds.

Of course, the earnings which would be "anticipated" for future years for purposes of accruing the capital gain would not be regarded as reliable estimates. However, their pattern would have to be reasonable, and their present value at time of sale would have to bear a reasonable relationship to the sales proceeds, on the basis of representative price-earnings ratios then prevailing or any other available measures.

If Company A experienced a capital loss instead of a capital gain, it would defer and amortize the loss over the period of years in which Company S was expected either to lose money or to earn less than was considered a reasonably satisfactory return on net worth. The amount to be amortized in a given year would be the amount by which the earnings anticipated at the time of sale for that year fell short of a reasonably satisfactory figure.

At this point we might take a look at the way in which Company B should amortize the goodwill which it acquired when it bought the Company S shares from Company A. Since there has been no newly invested capital in Companies A and B in the aggregate, it would seem that their aggregate assets and net worth should not change because of the transaction. The way to keep their aggregate assets and net worth from changing solely as a result of the transaction would be to require Company B to amortize its $100,000 of goodwill at the rate at which Company A accrued its $100,000 capital gain. Such an approach would require, of course, that Companies A and B agree on the accrual/amortization schedule at the time of the sale.

Of course, an important reason for the sale might be that Company B thought it could obtain more earnings as holder of, perhaps, a controlling block of shares of Company S stock than Company A thought it (Company A) could obtain. The accrual/amortization schedule should be tailored to Company A's expectations as holder rather than to Company B's expectations as holder. If Company B then did well as holder, its subsequent income would reflect that fact.

The foregoing approach to the amortization of goodwill differs from the approach currently required under GAAP. Under GAAP, goodwill is amortized in relation to the duration of its estimated value to Company B (in our example) in terms of management skills, markets, sales force, and the like which may become available to Company B by virtue of its

owning a substantial portion of the outstanding common stock of Company S. The amortization period, under GAAP, may in no event exceed forty years, although there are some who argue that there should be no such arbitrary limit. Under the approach outlined in this paper, amortizing goodwill would be merely a means of neutralizing certain effects which a transaction would otherwise have on the assets and earnings of the parties. It may be noted that under the approach outlined here, if a capital loss were to arise and be amortized by Company A, Company B would book a negative goodwill item and accrue it instead of booking and amortizing positive goodwill. Clearly some name other than "goodwill" would have to be found for such accounting items.

It could happen that the carrying amount of a block of shares at the time of its sale included an amount of unamortized goodwill remaining from the purchase of the shares at one or more previous times. If the capital gain at time of sale were taken as simply the sales proceeds minus the carrying amount, then the size of the offsetting item which the purchaser would amortize in its accounts would depend on the price or prices which the seller had originally paid for the shares. There might even be situations in which a company that wished to acquire a block of stock had a choice between two blocks of the same issue which were held by two different companies and were being carried at different amounts by the two companies (a situation which can occur as well under the equity method as it is presently prescribed under GAAP). In such a situation it would seem unreasonable to make the size of the goodwill-type item to be set up by the purchaser depend on which company it chose to buy the block from. A better approach would seem to be to base the capital gain or loss to be deferred and the offsetting goodwill-type item on the difference between the sales proceeds and the book value excluding the seller's unamortized goodwill (which is how the goodwill item is presently determined under GAAP). If capital gains and losses were determined in this way in following the approach outlined in this paper, then any unamortized or unaccrued goodwill (positive or negative) at time of sale would remain on the books of the seller and continue to be amortized or accrued according to the schedule which had been set up when the seller originally acquired the shares.

In summary, the approach outlined here for common stock accounted for by the equity method involves deferring and accruing (or amortizing) a capital gain (or loss) over the anticipated future favorable or unfavorable earnings period of the issuer of the stock sold and amortizing (or accruing) an offsetting item in the buyer's accounts by the same schedule. In determining the amount of capital gain or loss to be deferred (and

offset), unamortized or unaccrued amounts of goodwill-type items on the books of the seller are excluded from the carrying amount of the shares sold.

The same approach could be followed in the case of common stock holdings accounted for on a consolidated basis.

MARKETABLE COMMON STOCKS

"Marketable" common stocks are considered, under GAAP, to be common stock holdings which do not constitute such a large portion of the issuer's outstanding common stock that they must be accounted for by the equity method or on a consolidated basis.

Over the last two years or so there have been extended discussions, participated in by the life insurance business, of accounting for marketable common stocks. Four principal approaches have been discussed:

1. Carry common stocks at cost. Include dividends and realized capital gains and losses in income. Under this approach, unrealized gains and losses are not recognized in either income or surplus.
2. Carry common stocks at market value. Include dividends and both realized and unrealized capital gains and losses in income (and in surplus) as they arise.
3. Carry common stocks at market value. Include dividends in income. Carry realized and unrealized capital gains and losses directly to surplus (in the year in which they arise) without recognizing them in income.
4. Include dividends in income. Recognize realized and unrealized capital gains and losses in both income and surplus on a long-term yield basis (e.g., by recognizing each year's capital gain or loss in equal pieces over an n-year period or by some other method which does not recognize the entire gain or loss in the year in which it arises). Carry common stocks at market value adjusted by the amount needed to recognize gains and losses on the long-term yield basis.

Under each of the above four approaches, dividends, rather than reported earnings, would be included in income as they were incurred.

In the discussions of marketable common stocks, each of the above four approaches has been supported by some persons and opposed by others. The approaches have each received extensive discussion, which this paper will not attempt to summarize. Instead, let us explore an approach based on criteria other than those which have been given primary emphasis by supporters of one or another of the above four approaches.

Let us first note that none of the four approaches is consistent with the equity and consolidation methods, since none of them is based on recognizing as income the earnings on the common stock as reported by the issuer. It might seem reasonable to seek an approach for marketable

common stocks which is consistent with the method of accounting for larger blocks of stock. Such an approach would avoid discontinuities between holdings on either side of the dividing line and would obviate decisions as to which approach should be followed for a given holding.

It appears that the approach outlined in the section of this paper on common stocks accounted for by the equity or consolidation methods could, in theory, be followed for marketable common stocks as well. Earnings reported by the issuer would be included in income. The carrying amount of the holding would be increased by the excess of earnings reported over dividends paid. Capital gains and losses would be deferred and accrued or amortized over a period of years, and there would be offsetting entries on the books of the other party to the transaction, if the other party were a business entity. Unamortized or unaccrued amounts of goodwill-type items on the books of the seller would be excluded in determining the amounts of capital gains or losses.

One practical problem would be obtaining up-to-date earnings figures for the issuers. This problem is dealt with at the present time, in one way or another, in connection with common stocks which are accounted for by the equity method. In the case of marketable common stocks, the problem would be magnified. If estimated up-to-date earnings (rather than the latest reported earnings) were used, there should probably be a mechanism for ensuring that estimates used by various holders of each issue were the same.

It would not, of course, be feasible to make the accrual/amortization schedule a matter of negotiation between the parties to each transaction. Some rule would have to be devised for determining a reasonable schedule on the basis of readily obtainable facts concerning the stock issue involved.

The approach outlined here would, in essence, treat common stock not as an inventory item but as a share in the assets and liabilities of the issuing corporation. This is how it is treated under the equity and consolidation methods. If this approach were followed for marketable common stocks, the holder's financial statements should, perhaps, disclose the aggregate market value of those common stocks for which market value could be determined with reasonable accuracy and should also disclose the difference between market value and carrying amount for those stocks.

BUSINESS COMBINATIONS

As stated in *APB Opinion No. 16*, "A business combination occurs when a corporation and one or more incorporated or unincorporated businesses are brought together into one accounting entity. The single entity carries on the activities of the previously separate, independent enterprises."

Under GAAP, a business combination must be accounted for by the purchase method or by the pooling of interests method, depending on the circumstances. The purchase method accounts for a combination as an acquisition. Under it the acquiring corporation records at its cost the acquired assets less liabilities assumed. A portion of the acquisition cost is, in the typical case, recorded as goodwill. Such goodwill must be amortized in the same way as is done in connection with the equity method. Under the pooling of interests method, on the other hand, the recorded assets and liabilities of the combining ownership interests are carried forward to the combined corporation at their recorded amounts. One condition for use of this method is that the business combination be effected by exchange of equity securities.

As an example, let us suppose that Company X has 100,000 outstanding shares of common stock and a net worth of $2,000,000, but that its market value is considered to be $3,000,000. Company Y has a net worth of $4,000,000, and the market value of its 200,000 outstanding shares of common stock is $6,000,000. A combination of the two companies is effected, with Company Y the surviving entity. To effect the combination, Company Y issues 100,000 shares of common stock, which are given to the shareholders of Company X on the basis of one Company Y share in return for one Company X share. The Company X shares are then retired, and Company Y has 300,000 outstanding shares of common stock.

If the purchase method of accounting is used, the cost of acquiring Company X is $3,000,000, which was the market value of Company X at the time of acquisition and is also the fair value of the 100,000 newly issued Company Y shares which were used to acquire Company X, since there are now 300,000 Company Y shares outstanding, and the market value of the combined companies is, presumably, $9,000,000. The combination is accounted for by booking the entire $3,000,000 acquisition cost as an asset and amortizing over a period of time the $1,000,000 goodwill (which equals the $3,000,000 acquisition cost minus the $2,000,000 book value of the shares acquired). Hence the net worth of the combined corporation immediately after the acquisition is $7,000,000 ($4,000,000 plus $3,000,000), of which $1,000,000 must be amortized over future years.

If, on the other hand, the pooling of interests method is used, the net worth of the combined company will be only $6,000,000, and there will be no goodwill to amortize. (We are assuming throughout that no goodwill was included in the assets of either company before the combination.)

In view of the significant differences which can arise between the results of following these two methods, it is not surprising that there has been much discussion of how to determine which method should be used in a given case. One is tempted to wish that a single method could be settled

upon as appropriate for all business combinations. Let us see what would happen if in the above example we followed the approach which was outlined in the section of this paper on common stocks accounted for by the equity or the consolidation method.

Under the approach which we are going to explore, one of the combining companies has to be considered the acquiring company. Let us select Company Y as the acquiring company. It acquires Company X shares at a price of $3,000,000 and books them at that figure. $1,000,000 of the amount is booked as goodwill and is amortized over a period of years.

Meanwhile, Company X can be thought of as having sold its stock for $3,000,000, for a capital gain of $1,000,000. This capital gain must be deferred and accrued over a period of years. If Company X can be thought of as surviving in Company Y, the $1,000,000 capital gain will be deferred and accrued in Company Y's accounts. The offsetting $1,000,000 goodwill will be amortized over the same period.

The net worth of the combined entity immediately after the transaction will be $6,000,000, which equals Company Y's previous net worth, $4,000,000, plus the acquisition cost of $3,000,000, minus the $1,000,000 deferral of the capital gain. Future earnings of the combined entity will be its book earnings without adjustment, since the accrual and amortization of the $1,000,000 capital gain and goodwill items will offset each other. Hence in this example the approach of this paper is the same as the pooling of interests method, if the capital gain can be handled as described above.

In general, if the acquisition price is P (in equities or cash), and the book values of Companies X and Y before the transaction are B_x and B_y, respectively, then the net worth of the combined entity is $B_y + P - (P - B_x)$, where $P - B_x$ is the capital gain being deferred. Hence the net worth equals $B_x + B_y$, and the method of this paper is equivalent to the pooling method. In other words, if the purchase method is modified by treating capital gains and losses as outlined in this paper, it is the same as the pooling method, and there is no longer any need to choose between two ways of accounting for business combinations.

It may be of interest to see what would happen if capital gains were deferred in an exchange of equities which did not constitute a business combination. Suppose, for example, that Companies X and Y of our above illustration decided merely to exchange 20,000 of each other's common shares. Each would be giving away shares having a market value of $600,000. Accordingly, each would book its newly acquired shares at $600,000. Since the acquired shares had had a book value of only $400,000,

$200,000 of the $600,000 would be considered goodwill and would be amortized over a period of years. At the same time, each company would have experienced a capital gain of $200,000, since it had received a consideration of $600,000 for shares of its own having a book value of only $400,000. Each company would, accordingly, set up a deferred credit of $200,000, which would be accrued over a period of years. Hence the net worth of each company immediately after the transaction would be the same as immediately before it. The accrual of the deferred credit and the amortization of the goodwill might or might not exactly offset each other within each company, since each such schedule would exactly offset the offsetting amortization or accrual in the other company's accounts.

BONDS NOT IN GOOD STANDING

Let us define "bonds not in good standing" roughly as bonds on which it appears that interest and principal will, quite possibly, not all be paid when due.

One way to treat such bonds might be to carry them at market value. If this were done, no capital gain or loss would arise from the sale of such a bond. In this case the sale of such a bond would not involve any of the considerations which have been discussed in this paper.

It would be theoretically possible to treat bonds not in good standing in a way which is analogous to the approach which has been outlined in this paper for bonds in good standing and common stocks. Under this approach, at the moment when it was determined that a bond was no longer in good standing, an estimate would be made of the chance that each future scheduled payment would be made and of the time at which it might be made. The amounts, times, and probabilities of payment would be combined to produce a new schedule of expected payments. For example, the new schedule might indicate no coupon payments for the next three years, then payments of coupons at the full rate annually until the originally scheduled maturity date, and then payment on the maturity date of the three remaining coupons in full plus one-half the originally scheduled maturity value.

The present value of this new schedule of payments would be determined, based on the interest rate which the bond had originally been bought to yield. This new value would serve as the starting point for a new amortization schedule, which would be based on the original interest rate. Each year the amount of amortization or accrual for the year under the new schedule would be charged or credited to income, and any interest payment received would also be credited to income. If circumstances changed materially, the chances of receiving future scheduled payments

would be re-evaluated, a new present value would be determined (still on the basis of the original interest rate), and a new amortization schedule would be established.

At the time of the original writedown of a bond which was being reclassified from good standing to not-good standing there would be an unrealized capital loss. At the time of any revaluation of a bond not in good standing or of a restoration of a bond to good standing, there would be a similar unrealized gain or loss. These unrealized gains or losses would be credited or charged to income for the periods in which they arose or, perhaps, treated in some other way.

At the time of sale of a bond not in good standing there would, generally, be a capital gain or loss arising from the sale. This gain or loss would be treated in the way described in this paper for bonds in good standing.

The effect of this approach for bonds in general would be to regard the scheduled interest payments and amortization charges (or accrual credits) as the "earnings" from the bond. Writedowns or writeups resulting from changes in the assumed quality of the bond would be in the nature of extraordinary charges or credits. Capital gains and losses arising from sales would be treated as artificial distortions of the natural earnings pattern, which would be neutralized by means of deferral and accrual or amortization.

The estimation of the chance and time of payment of each scheduled future payment from a bond would present obvious difficulties. Whatever techniques were used, the resulting present value should bear a reasonable relation to the then market value, if known, of the bond, with due regard to any difference between the original interest rate involved in the revaluation and the current market yield for bonds of quality comparable to that of the bond at the time it was purchased. The question of whether and when to revalue a bond which was already not in good standing would involve judgment of, presumably, the same kind which would be involved in judgments of whether and when to reclassify a bond from good standing to not-good standing.

This section of the paper is intended merely to show that there exists a method for treating bonds not in good standing which is consistent with the approach of this paper for bonds in good standing and for common stocks. As everywhere in this paper, no attempt has been made to assess the relative merits of various possible approaches.

Nothing in this section is intended to suggest that it is not possible or desirable to combine with the approach which has been described here a valuation reserve such as the mandatory securities valuation reserve, which might provide a convenient way to differentiate among bonds of

slightly differing qualities without having to evaluate probabilities of payment.

CONCLUSIONS

This paper has explored some of the implications of adopting two principles as paramount: (1) the reported earnings of a business entity should not change solely because an investment is sold at a capital gain or loss and the proceeds are reinvested, and (2) the accounting treatment of common stock investments should not vary according to the portion of the issuer's outstanding common stock which the investment represents. Two conclusions are now drawn: (1) if the first principle is followed, a capital gain or loss arising through sale and reinvestment should be deferred and accrued or amortized in a way which depends only on the investment sold and not on the investment which replaces it, and (2) it is possible, subject to practical limitations, to account for bond and common stock investments of all types by a consistent method which follows the above two principles.

Although this paper has not discussed mortgages, preferred stocks, real estate, and other types of investments made by life insurance companies, it would seem that the principles would apply to them in similar fashion.

This paper has made no attempt to appraise the relative merits of various principles which might be given prime importance in accounting for investments.

ACKNOWLEDGMENT

The author gratefully acknowledges the help of Grace V. Dillingham, who simplified the derivations in the Appendix, performed the calculations for Table 1, and offered helpful comments on the text of the paper.

APPENDIX

Assume that an amount A is invested in a bond which will yield a coupon in the amount of iA at the end of each year for n years and will mature for an amount A at the end of the n-year period. Assume that each coupon is reinvested in a bond of the same quality maturing n years from its purchase date for a value equal to its purchase price and bearing annual coupons in the amount of i times its purchase price. Coupons from the additional bonds are similarly reinvested. Further assume, however, that during the rth year from the date of purchase of the original bond, where r is less than n, the prevailing interest rate for bonds of the same quality changes from i to i', with the result that investments made on or after the rth anniversary are made in bonds yielding coupons in the amount of i' times their purchase prices, while still in every case maturing for their purchase amounts.

Consider the situation where all bonds are held to maturity, at which

time the proceeds are reinvested at then current yields in the manner described above. Call this situation "Situation Stay." Now consider a situation where on the rth anniversary the original bond is sold at a fair market price (in terms of the then prevailing interest rate i'), and the proceeds R are reinvested in a bond of the same quality, bearing coupons of $i'R$ for n years and maturing at the end of n years (i.e., $n + r$ years from the date of the original investment) for an amount R. In all other respects this situation is similar to Situation Stay. Call this situation "Situation Roll Over."

Ignore commissions, taxes, and other expenses throughout.

It is desired to prove that the income on each anniversary under Situation Stay equals the income under Situation Roll Over adjusted as follows: On the rth anniversary an amount equal to $(A - R)$ is added to income. On each anniversary t after r, to and including the nth anniversary of the purchase of the original bond, there is (1) deducted from income an amount equal to $(A - R) \div a_{\overline{n-r}|i'}$ and (2) added to income the amount $i'[(A - R)/a_{\overline{n-r}|i'}]\, a_{\overline{n-t+1}|i'}$.

Consider first the period beginning at duration 0 and ending at duration r. If we disregard for the moment the rollover which may occur at duration r, the amount which is invested at each duration after duration 0 is the amount of income received at the particular duration. At duration 1 this amount is iA. At duration 2 it is iA (representing the second coupon from the original bond) plus $i \cdot iA$ (representing the first coupon on the second bond), for a total of $iA(1 + i)$. At duration 3 it is iA (from the original bond) plus $i \cdot iA$ (from the investment at duration 1) plus $i \cdot iA(1 + i)$ (from the investment at duration 2), for a total of $iA(1 + i)^2$. Now prove by mathematical induction that the investment I_t at duration t equals $iA(1 + i)^{t-1}$ as follows:

If

$$I_0 = A$$

and

$$I_s = iA(1 + i)^{s-1} \qquad \text{for every } s, \quad 1 \leq s \leq t, \quad t < r,$$

then

$$I_{t+1} = i\sum_{0}^{t} I_s = i\left[A + \sum_{s=1}^{t} iA(1 + i)^{s-1}\right]$$

$$= i\left[A + iA\,\frac{(1 + i)^t - 1}{i}\right]$$

$$= iA - iA + iA(1 + i)^t$$

$$= iA(1 + i)^t.$$

Hence

$$I_t = iA(1 + i)^{t-1}, \qquad t \leq r. \tag{1}$$

It may be noted that the same result can be obtained by viewing the investments as a bank account drawing compound interest, in which case

$$\begin{aligned} I_t &= A(1+i)^t - A(1+i)^{t-1} \\ &= A(1+i)^{t-1}(1+i-1) \\ &= iA(1+i)^{t-1} . \end{aligned}$$

Now consider the period of years beginning at duration r and ending at duration n. Investments made on or after duration r yield coupons at rate i'. For Situation Stay we have

$$\begin{aligned} I_{r+1} &= i\sum_{s=0}^{r-1} I_s + i' I_r \\ &= iA(1+i)^{r-1} + i' \cdot iA(1+i)^{r-1} \end{aligned}$$

(by reference to the derivation of formula [1]), or

$$I_{r+1} = iA(1+i)^{r-1}(1+i') .$$

Now, ignoring the reinvestment at duration n, prove by induction that I_t equals $iA(1+i)^{r-1}(1+i')^{t-r}$ as follows:

If

$$I_0 = A ,$$

$$I_s = iA(1+i)^{s-1} \qquad \text{for every } s, \quad 1 \le s \le r ,$$

and

$$I_s = iA(1+i)^{r-1}(1+i')^{s-r} , \qquad r < s \le t , \quad t < n ,$$

then

$$\begin{aligned} I_{t+1} &= i\sum_{s=0}^{r-1} I_s + i'\sum_{s=r}^{r+t} I_s \\ &= iA(1+i)^{r-1} + i'\left[iA(1+i)^{r-1}\sum_{s=0}^{t-r}(1+i')^s\right] \\ &= iA(1+i)^{r-1} + i'\left[iA(1+i)^{r-1}\frac{(1+i')^{t-r+1}-1}{i'}\right] \\ &= iA(1+i)^{r-1} + iA(1+i)^{r-1}(1+i')^{t-r+1} - iA(1+i)^{r-1} \\ &= iA(1+i)^{r-1}(1+i')^{t-r+1} . \end{aligned}$$

Hence

$$I_t = iA(1+i)^{r-1}(1+i')^{t-r} , \qquad r < t \le n . \tag{2}$$

For Situation Roll Over the income is decreased through the withdrawal of I_0 and increased through the additional investment of R. Hence we have

$$\begin{aligned} I_{r+1} &= i\sum_{s=1}^{r-1} I_s + i' I_r \\ &= i[A(1+i)^{r-1} - A] + i'[iA(1+i)^{r-1} + R] \\ &= iA(1+i)^{r-1}(1+i') + i'R - iA\,. \end{aligned}$$

Prove by induction that

$$I_t = iA(1+i)^{r-1}(1+i')^{t-r} + (i'R - iA)(1+i')^{t-r-1}$$

as follows:

If

$$I_0 = A\,,$$

$$I_s = iA(1+i)^{s-1} \qquad \text{for every } s\,,\quad 1 \leq s < r\,,$$

$$I_r = iA(1+i)^{r-1} + R\,,$$

and

$$I_s = iA(1+i)^{r-1}(1+i')^{s-r} + (i'R - iA)(1+i')^{s-r-1} \qquad \text{for every } s,\quad r < s < t\,,\quad t < n\,,$$

then

$$\begin{aligned} I_{t+1} &= i\sum_{s=1}^{r-1} I_s + i' I_r + i' \sum_{s=r+1}^{t} I_s \\ &= i[A(1+i)^{r-1} - A] + i'[iA(1+i)^{r-1} + R] \\ &\quad + i' \sum_{s=r+1}^{t} [iA(1+i)^{r-1}(1+i')^{s-r} + (i'R - iA)(1+i')^{s-r-1}] \\ &= iA(1+i)^{r-1}(1+i') + i'R - iA \\ &\quad + i'\Big[iA(1+i)^{r-1}(1+i')\,\frac{(1+i')^{t-r}}{i'} \\ &\quad + (i'R - iA)\,\frac{(1+i')^{t-r} - 1}{i'}\Big] \\ &= iA(1+i)^{r-1}(1+i') + i'R - iA + iA(1+i)^{r-1}(1+i')^{t-r+1} \\ &\quad - iA(1+i)^{r-1}(1+i') + (i'R - iA)(1+i')^{t-r} - (i'R - iA) \\ &= iA(1+i)^{r-1}(1+i')^{t-r+1} + (i'R - iA)(1+i')^{t-r}\,. \end{aligned}$$

Hence

$$I_t = iA(1+i)^{r-1}(1+i')^{t-r} + (i'R - iA)(1+i')^{t-r-1}, \qquad r < t < n. \tag{3}$$

It will be seen from a comparison of formulas (2) and (3) that from durations $r+1$ to n, inclusive, if no adjustment is made to the Situation Roll Over income, the Situation Roll Over income will exceed the Situation Stay income by the amount $(i'R - iA)(1+i')^{t-r-1}$ at each duration t (formula [4]).

Finally, consider the period beginning at duration n and continuing indefinitely. If all the bonds continued to pay interest forever, instead of maturing at the end of n years, with the proceeds being reinvested, formulas (2) and (3) would describe the incomes from Situations Stay and Roll Over, respectively, for an indefinite period of years. The difference between them would be described by formula (4) for an indefinite period. However, there will be some further reinvestments under the assumptions given in this Appendix.

At durations $n+1$ to $n+r-1$, inclusive, the amounts which were invested at durations 1 through $r-1$ will be reinvested to yield i'. The effect of these reinvestments will be the same under Situation Roll Over as it is under Situation Stay. Hence these reinvestments will give rise to no additional difference between the incomes under the two situations. The only additional difference which will arise during this period stems from the reinvestment at duration n of the original amount A under Situation Stay. This reinvestment will give rise to additional income of $(i'-i)A$ at each duration after n, with these additional amounts being reinvested in the usual way. By analogy with formula (1), which refers to a basic income stream of iA per year, the aggregate additional income in this third period will be $(i'-i)A(1+i')^{t-n-1}$ (formula [5]). This formula is directly analogous to formula (4), which could have been derived in the same simple way.

It now remains to compare the differences in income between the two situations, as derived above, with the adjustment described in the fourth paragraph of this Appendix. At duration r the addition of $(A-R)$ exactly offsets the capital loss which occurred when the original investment A was sold for the amount R. At durations $r+1$ through n, it is desired (from formula [4] and the fourth paragraph) to show that $(i'R - iA)(1+i')^{t-r}$ plus $i'[(A-R)/a_{\overline{n-r}|i'}]a_{\overline{n-t+1}|i'}$ minus $(A-R)/a_{\overline{n-r}|i'}$ equals zero. Now R represents the amount which was obtained for the original bond at duration r on the basis that it should yield i' to the purchaser. Hence $R = iAa_{\overline{n-r}|i'} + v'^{n-r}A$. Hence we set

$$[i'(iAa_{\overline{n-r}|i'} + v'^{n-r}A) - iA](1 + i')^{t-r-1}$$

$$+ \frac{A - (iAa_{\overline{n-r}|i'} + v'^{n-r}A)}{a_{\overline{n-r}|i'}} (i'a_{\overline{n-t+1}|i'} - 1) \stackrel{?}{=} 0 ;$$

$$\left[i'\left(i\frac{1 - v'^{n-r}}{i'} + v'^{n-r}\right) - i\right](1 + i')^{t-r-1}$$

$$+ \frac{1 - [i(1 - v'^{n-r})/i' + v'^{n-r}]}{(1 - v'^{n-r})/i'}\left(i'\frac{1 - v'^{n-t+1}}{i'} - 1\right) \stackrel{?}{=} 0 ;$$

$$(i - iv'^{n-r} + i'v'^{n-r} - i)(1 + i')^{t-r-1}$$

$$+ \frac{(1 - i/i')(1 - v'^{n-r})}{(1 - v'^{n-r})/i'} (1 - v'^{n-t+1} - 1) \stackrel{?}{=} 0 ;$$

$$(i' - i)v'^{n-r}(1 + i')^{t-r-1} - (i' - i)v'^{n-t+1} \stackrel{?}{=} 0 ;$$

$$(i' - i)v'^{n-t+1} - (i' - i)v'^{n-t+1} = 0 . \qquad (Q.E.D.)$$

At durations $n + 1$ and higher, the fourth paragraph of this Appendix calls for no adjustment to be made to the Situation Roll Over income. Hence it is desired (from formulas [4] and [5]) to show that $(i'R - iA)(1 + i')^{t-r-1}$ minus $(i' - i)A(1 + i')^{t-n-1}$ equals zero. Hence we set

$$[i'(iAa_{\overline{n-r}|i'} + v'^{n-r})A - iA](1 + i')^{t-r-1} - (i' - i)A(1 + i')^{t-n-1} \stackrel{?}{=} 0 ;$$

$$\left\{i'\left[\frac{i(1 - v'^{n-r})}{i'} + v'^{n-r}\right] - i\right\}(1 + i')^{t-r-1}$$

$$- (i' - i)(1 + i')^{t-n-1} \stackrel{?}{=} 0 ;$$

$$(i - iv'^{n-r} + i'v'^{n-r} - i)(1 + i')^{t-r-1} - (i' - i)(1 + i')^{t-n-1} \stackrel{?}{=} 0 ;$$

$$(i' - i)v'^{n-r}(1 + i')^{t-r-1} - (i' - i)(1 + i')^{t-n-1} \stackrel{?}{=} 0 ;$$

$$(i' - i)(1 + i')^{t-n-1} - (i' - i)(1 + i')^{t-n-1} = 0 . \qquad (Q.E.D.)$$

DISCUSSION OF PRECEDING PAPER

GRACE V. DILLINGHAM:

As one who was privileged to work with Mr. Case on his paper "A Uniform Approach to Accounting for Bond and Common Stock Investments," I early became interested in providing an analysis of the general case for capital gain or loss on the sale of bonds. Only the pressure of time prevented me from submitting such an analysis as part of the assistance acknowledged in the paper. Mr. Case's assistance and encouragement in the preparation of this addendum to his paper are gratefully acknowledged.

The general case requires a slight modification of the approach suggested in the paper, for it develops that accrual or amortization of the entire gain or loss at the interest rate implicit in the sale price reproduces the nonrollover income only when the bond that is sold has been carried at its maturity value. When the bond has been carried at a premium or a discount, amortization of that premium or accrual of that discount should continue at the original interest rate, and it is the balance of the gain or loss, that is, the gain or loss measured by the maturity value, rather than the book value, of the bond which is sold, which should be accrued or amortized at the rate implicit in the sale price. It will be shown that, if a bond is sold and (*a*) the proceeds of the sale and all subsequent coupons and maturities are invested to yield the rate of interest implicit in the sale price, (*b*) any capital gain or loss (exclusive of amounts attributable to different valuations of accrued coupons) is deferred, (*c*) any difference between the price received for accrued coupons and the value at which accrued coupons were carried is recognized immediately, (*d*) any premium or discount unamortized or unaccrued at the time of the sale continues to be amortized or accrued according to the original schedule, and (*e*) the balance of the gain or loss (positive or negative) is amortized or accrued over the period of the original amortization or accrual schedule at the rate of interest implicit in the sale price, then reported income will be the same as if the bond had not been sold and all subsequent coupons and maturities were invested to yield the rate of interest implicit in the sale price.

The continued amortization or accrual, according to the original schedule, of premium or discount on bonds has an interesting parallel in the suggestion that any unamortized or unaccrued "good will" at the

time of sale of common stocks should remain on the books of the seller and continue to be amortized or accrued according to the original schedule. In each case there is an established schedule of amortization or accrual which is to be continued by the seller and which has no effect on the buyer, and in each case there is an amount which is to be amortized (or accrued) by the buyer at the same rate and over the same period as it is accrued (or amortized) by the seller. In the case of the bond, the respective amortization and accrual schedules are fully determined by the terms of the bond and the price at which the transaction takes place. In the case of the stock, the schedules would be a matter for negotiation between the buyer and the seller, if conditions were such as to make this feasible, or else would be determined by some rule yet to be devised. Such a rule for marketable holdings, when devised, might well become an important, although not necessarily a determining, consideration in the negotiations between buyer and seller regarding the accrual and amortization schedules for stocks accounted for by the equity method.

For the seller of the stock or the bond, the combination of the accrual or amortization schedule which is reflected by the buyer's amortization or accrual schedule with the original accrual or amortization schedule should reproduce the income which would have been received if the sale had not taken place and conditions at the time of the sale had continued. This will now be proved in the case of a bond carried at a premium or a discount, with coupons payable m times a year at dates which do not necessarily include the end of the year, sold at a time which is not necessarily either a coupon date or the end of a year.

Assume that a bond A_a, with coupons J_a/m_a payable m_a times a year and maturity value ${}_{\alpha}A_{\alpha}$ at time α, is purchased at time a to yield i_a. Assume, further, that each of the coupons of A_a is invested in a new bond of the same quality and that each coupon of each new bond and all maturities are similarly invested. Let A_k represent the bond bought at time k, and let

Year k = Calendar or other fiscal year in which k occurs;
k' = End of year k;
κ = Maturity date of A_k;
${}_{\kappa}A_k$ = Maturity value of A_k;
m_k = Number of coupons of A_k payable in each year other than the year of acquisition or maturity;
n_k = Number of coupons of A_k payable in year κ;
p_k = Number of coupons payable in year of acquisition or forgone in year of sale;

J_k = Annual coupon of A_k;

J_k/m_k = Amount of each coupon;

${}_sf_k$ = Fraction of a coupon period of A_k between time s and preceding coupon date;

c_k = Fraction of a coupon period of A_k between end of year and preceding coupon date $(={}_{s'}f_k)$;

$d_k = {}_sf_k$ when s is the date of purchase or sale of A_k;

i_k = Effective yield rate at time k;

$i_k^{(m)}/m$ = Rate per period, where $(1 + i_k^{(m)}/m)^m = 1 + i_k$;

$v_k = (1 + i_k)^{-1}$;

${}_sv_k = (1 + i_k^{(m_s)}/m_s)^{-1}$;

$j_k = J_k/{}_\kappa A_k$;

${}_sA_k$ = Value of A_k at time s, excluding any accrued coupon

$$= \frac{J_k}{m_k}\, a_{\overline{m_k(\kappa-s)}|}^{i_k^{(m_k)}/m_k} + {}_\kappa A_k\, {}_k v_k^{m_k(\kappa-s)},$$

where $m_k(\kappa - s)$ need not be integral;

${}_kA_k$ = Value of A_k at time of purchase, excluding accrued coupon;

${}_sB_k$ = Value of A_k at time s, including any accrued coupon

$= {}_sA_k$ if s is a coupon date

$$= (1 + i_k^{(m_k)}/m_k)^{{}_sf_k}\, {}_{s-{}_sf_k}A_k \text{ if } s \text{ is not a coupon date}$$

$$= {}_kv_k^{-{}_sf_k}\left(\frac{J_k}{m_k}\, a_{\overline{m_k(\kappa-s)+{}_sf_k}|}^{i_k^{(m_k)}/m_k} + {}_\kappa A_k\, {}_k v_k^{m_k(\kappa-s)+{}_sf_k}\right)$$

(note the assumption that $\kappa - s$ is a mixed number involving fractions of a year, while ${}_sf_k$ is a fraction of a coupon period)

$$= \frac{J_k}{m_k}\, \frac{{}_kv_k^{-{}_sf_k} - {}_kv_k^{m_k(\kappa-s)}}{i_k^{(m_k)}/m_k} + {}_\kappa A_k\, {}_k v_k^{m_k(\kappa-s)}$$

$$= \frac{J_k}{m_k}\, s_{\overline{{}_sf_k}|}^{i_k^{(m_k)}/m_k} + \frac{J_k}{m_k}\, a_{\overline{m_k(\kappa-s)}|}^{i_k^{(m_k)}/m_k} + {}_\kappa A_k\, {}_k v_k^{m_k(\kappa-s)}$$

$$= \frac{J_k}{m_k}\, s_{\overline{{}_sf_k}|}^{i_k^{(m_k)}/m_k} + {}_sA_k;$$

${}_kB_k$ = Cost of A_k.

Now assume that at time b it is decided to sell A_a at a price

$${}_bP_a = \frac{J_a}{m_a}\, s_{\overline{d_a}|}^{i_b^{(m_a)}/m_a} + \frac{J_a}{m_a}\, a_{\overline{m_a(a-b)}|}^{i_b^{(m_a)}/m_a} + {}_aA_a\, v_b^{m_a(a-b)},$$

resulting in a capital gain (if positive) or loss (if negative) of

$$
\begin{aligned}
{}_bC_a &= {}_bP_a - {}_bB_a \\
&= {}_bP_a - \frac{J_a}{m_a} s_{\overline{d_a}|}^{i_a^{(m_a)}/m_a} - {}_bA_a .
\end{aligned}
$$

Excluding the portion of ${}_bC_a$ attributable to different valuations of the accrued coupon, we have

$$
\begin{aligned}
{}_bC'_a &= {}_bC_a - \frac{J_a}{m_a} \left(s_{\overline{d_a}|}^{i_b^{(m_a)}/m_a} - s_{\overline{d_a}|}^{i_a^{(m_a)}/m_a} \right) \\
&= \frac{J_a}{m_a} a_{\overline{m_a(a-b)}|}^{i_b^{(m_a)}/m_a} + {}_aA_a \, {}_av_b^{m_a(a-b)} - {}_bA_a .
\end{aligned}
$$

In practice, the approximation d_a will probably be used for both $s_{\overline{d_a}|}^{i_b^{(m_a)}/m_a}$ and $s_{\overline{d_a}|}^{i_a^{(m_a)}/m_a}$. In developing theory, however, we should treat the difference as significant.

We wish to prove that, if the sale price ${}_bP_a$ is invested to yield i_b, the difference in accrued coupon,

$$
\frac{J_a}{m_a} \left(s_{\overline{d_a}|}^{i_b^{(m_a)}/m_a} - s_{\overline{d_a}|}^{i_a^{(m_a)}/m_a} \right) ,
$$

is recognized immediately and the balance of the capital gain or loss, ${}_bC'_a$, is deferred, and the unaccrued discount or unamortized premium on A_a, ${}_aA_a - {}_bA_a$, is accrued or amortized over the period $a - b$ at the original rate of interest, i_a, while the balance,

$$
{}_bP_a - \frac{J_a}{m_a} s_{\overline{d_a}|}^{i_b^{(m_a)}/m_a} - {}_aA_a ,
$$

is accrued or amortized over the same period at the rate of interest, i_b, implicit in the sale price, then reported income will be the same as if A_a had not been sold and all coupons and maturities after time b had been invested to yield i_b.

Coupons and other income from bonds other than A_a bought prior to time b will not be affected by the sale and can therefore be ignored. We are concerned only with the difference in income and amounts available for investment which result from the rollover on the assumption that there is no further change in the interest rate.

The first such difference is the inclusion of ${}_bC_a$ in income and the availability of ${}_bP_a$ for investment. Assume that a new bond, A_b, with maturity value ${}_gA_b$ and coupons J_b/m_b, is purchased for a price

$$
\begin{aligned}
{}_bB_b = {}_bP_a &= \frac{J_b}{m_b}\, s_{\overline{d_b}|}^{i_b^{(m_b)}/m_b} + {}_bA_b \\
&= \frac{J_b}{m_b}\, s_{\overline{d_b}|}^{i_b^{(m_b)}/m_b} + \frac{J_b}{m_b}\, a_{\overline{m_b(\beta-b)}|}^{i_b^{(m_b)}/m_b} + {}_\beta A_b\, {}_bv_b^{m_b(\beta-b)} .
\end{aligned}
$$

At time $b - d_b/m_b + 1/m_b$, the first coupon date of A_b following time b, coupon proceeds of J_b/m_b will be received and will be invested in a bond A_{b_1}. Let us assume, temporarily at least, that A_{b_1} is bought at par and has the same coupon frequency and dates as A_b, as well as the same yield rate, i_b. Then, at time $b + (2 - d_b)/m_b$, coupons of $J_b/m_b + (i_b^{(m_b)}/m_b) \times (J_b/m_b)$ will become available for investment, and we have

$$
{}_{b_2}A_{b_2} = \frac{J_b}{m_b} + \frac{i_b^{(m_b)}}{m_b}\frac{J_b}{m_b} = \frac{J_b}{m_b}\left(1 + \frac{i_b^{(m_b)}}{m_b}\right);
$$

$$
\begin{aligned}
{}_{b_3}A_{b_3} &= \frac{J_b}{m_b} + \frac{i_b^{(m_b)}}{m_b}\frac{J_b}{m_b} + \frac{i_b^{(m_b)}}{m_b}\frac{J_b}{m_b}\left(1 + \frac{i_b^{(m_b)}}{m_b}\right) \\
&= \frac{J_b}{m_b}\left(1 + \frac{i_b^{(m_b)}}{m_b}\right)^2 ;
\end{aligned}
$$

and, in general,

$$
{}_{b_k}A_{b_k} = \frac{J_b}{m_b}\left(1 + \frac{i_b^{(m_b)}}{m_b}\right)^{k-1} .
$$

Coupon receipts during year b will amount to

$$
\sum_{k=1}^{p_b} \frac{J_b}{m_b}\left(1 + \frac{i_b^{(m_b)}}{m_b}\right)^{k-1} = \frac{J_b}{m_b}\, s_{\overline{p_b}|}^{i_b^{(m_b)}/m_b} .
$$

To this must be added the part of the next payment which has accrued by the end of the year,

$$
\frac{J_b}{m_b}\left(1 + \frac{i_b^{(m_b)}}{m_b}\right)^{p_b} s_{\overline{c_b}|}^{i_b^{(m_b)}/m_b} ,
$$

and from it must be subtracted the accrued coupon included in the purchase price, $(J_b/m_b)s_{\overline{d_b}|}^{i_b^{(m_b)}/m_b}$. Accrual of discount or amortization of premium on A_b must also be taken into account. For year b, this amounts to

$$
\begin{aligned}
{}_{b'}A_b - {}_bA_b = \frac{J_b}{m_b}\, a_{\overline{m_b(\beta-b)+d_b-p_b-c_b}|}^{i_b^{(m_b)}/m_b} &+ {}_\beta A_b\, {}_bv_b^{m_b(\beta-b)+d_b-p_b-c_b} \\
&- \frac{J_b}{m_b}\, a_{\overline{m_b(\beta-b)}|}^{i_b^{(m_b)}/m_b} - {}_\beta A_b\, {}_bv_b^{m_b(\beta-b)}
\end{aligned}
$$

$$= \frac{J_b}{m_b}\frac{1 - {}_bv_b^{m_b(\beta-b)+d_b-p_b-c_b} - 1 + {}_bv_b^{m_b(\beta-b)}}{i_b^{(m_b)}/m_b}$$

$$+ {}_\beta A_b\left({}_bv_b^{m_b(\beta-b)+d_b-p_b-c_b}\right) - {}_bv_b^{m_b(\beta-b)}$$

$$= \frac{i_b^{(m_b)} - j_b}{m_b}\,{}_\beta A_b\,{}_bv_b^{m_b(\beta-b)}\,s\overline{\left|\frac{i_b^{(m_b)}/m_b}{p_b+c_b-d_b}\right|}.$$

Thus the income from A_b during year b is

$$\frac{J_b}{m_b}\left[s\overline{\left|\frac{i_b^{(m_b)}/m_b}{p_b}\right|} + \left(1+\frac{i_b^{(m_b)}}{m_b}\right)^{p_b} s\overline{\left|\frac{i_b^{(m_b)}/m_b}{c_b}\right|} - s\overline{\left|\frac{i_b^{(m_b)}/m_b}{d_b}\right|}\right]$$

$$+ \frac{i_b^{(m_b)} - j_b}{m_b}\,{}_\beta A_b\,{}_bv_b^{m_b(\beta-b)}\,s\overline{\left|\frac{i_b^{(m_b)}/m_b}{p_b+c_b-d_b}\right|}$$

$$= \frac{J_b}{m_b}\left[\left(1+\frac{i_b^{(m_b)}}{m_b}\right)^{p_b} - 1 + \left(1+\frac{i_b^{(m_b)}}{m_b}\right)^{p_b+c_b} - \left(1+\frac{i_b^{(m_b)}}{m_b}\right)^{p_b}\right.$$

$$\left. - \left(1+\frac{i_b^{(m_b)}}{m_b}\right)^{d_b} + 1\right] \Big/ \frac{i_b^{(m_b)}}{m_b}$$

$$+ \frac{i_b^{(m_b)} - j_b}{m_b}\,{}_\beta A_b\,{}_bv_b^{m_b(\beta-b)}\,s\overline{\left|\frac{i_b^{(m_b)}/m_b}{p_b+c_b-d_b}\right|}$$

$$= \frac{J_b}{m_b}\left[\frac{(1+i_b^{(m_b)}/m_b)^{p_b+c_b} - (1+i_b^{(m_b)}/m_b)^{d_b}}{i_b^{(m_b)}/m_b}\right]$$

$$+ \frac{i_b^{(m_b)} - j_b}{m_b}\,{}_\beta A_b\,{}_bv_b^{m_b(\beta-b)}\,s\overline{\left|\frac{i_b^{(m_b)}/m_b}{p_b+c_b-d_b}\right|}$$

$$= \frac{J_b}{m_b}\left(1+\frac{i_b^{(m_b)}}{m_b}\right)^{d_b} s\overline{\left|\frac{i_b^{(m_b)}/m_b}{p_b+c_b-d_b}\right|} + \frac{i_b^{(m_b)} - j_b}{m_b}\,{}_\beta A_b\,{}_bv_b^{(\beta-b)}\,s\overline{\left|\frac{i_b^{(m_b)}/m_b}{p_b+c_b-d_b}\right|}$$

$$= \left[\frac{J_b}{m_b}\left(1+\frac{i_b^{(m_b)}}{m_b}\right)^{d_b} - \frac{J_b}{m_b} + \frac{J_b}{m_b} + \frac{i_b^{(m_b)} - j_b}{m_b}\,{}_\beta A_b\,{}_bv_b^{(\beta-b)}\right] s\overline{\left|\frac{i_b^{(m_b)}/m_b}{p_b+c_b-d_b}\right|}$$

$$= \frac{i_b^{(m_b)}}{m_b}\left(\frac{J_b}{m_b}\,s\overline{\left|\frac{i_b^{(m_b)}/m_b}{d_b}\right|} + {}_bA_b\right) s\overline{\left|\frac{i_b^{(m_b)}/m_b}{p_b+c_b-d_b}\right|}$$

$$= \frac{i_b^{(m_b)}}{m_b}\,{}_bP_a\,s\overline{\left|\frac{i_b^{(m_b)}/m_b}{p_b+c_b-d_b}\right|}.$$

We now must subtract from the income attributable to the purchase of A_b and subsequent reinvestments the income which would have been received if A_a had not been sold.

Coupon receipts during year b, accumulated at the rate i_b, would have amounted to $(J_a/m_a)s\overline{p_a|}^{i_b^{(m_a)}/m_a}$, and an additional amount equal to

$$\frac{J_a}{m_a}\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{p_a} s\overline{c_a|}^{i_b^{(m_a)}/m_a}$$

would have accrued by the end of the year. The accrued coupon included in the sale price, $(J_a/m_a)s\overline{d_a|}^{i_a^{(m_a)}/m_a}$, must be subtracted. (Note that this accrual is at the rate i_a.) Adding the accrual of discount or amortization of premium, the income which would have been received if A_a had not been sold amounts to

$$\frac{J_a}{m_a}\left[s\overline{p_a|}^{i_b^{(m_a)}/m_a}+\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{p_a} s\overline{c_a|}^{i_b^{(m_a)}/m_a}-s\overline{d_a|}^{i_a^{(m_a)}/m_a}\right]$$

$$+\frac{i_a^{(m_a)}-j_a}{m_a}\,{}_aA_a\,{}_av_a^{m_a(a-b)}\,s\overline{p_a+c_a-d_a|}^{i_a^{(m_a)}/m_a}.$$

Thus, for year b, net income attributable to the rollover is

$${}_bC_a+\frac{i_b^{(m_b)}}{m_b}\,{}_bP_a\,s\overline{p_b+c_b-d_b|}^{i_b^{(m_b)}/m_b}-\frac{J_a}{m_a}\left[s\overline{p_a|}^{i_b^{(m_b)}/m_b}+\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{p_a} s\overline{c_a|}^{i_b^{(m_a)}/m_a}\right.$$

$$\left.-s\overline{d_a|}^{i_a^{(m_a)}/m_a}\right]-\frac{i_a^{(m_a)}-j_a}{m_a}\,{}_aA_a\,{}_av_a^{m_a(a-b)}\,s\overline{p_a+c_a-d_a|}^{i_a^{(m_a)}/m_a}$$

$$={}_bC_a'+\frac{i_b^{(m_b)}}{m_b}\,{}_bP_a\,s\overline{p_b+c_b-d_b|}^{i_b^{(m_b)}/m_b}-\frac{J_a}{m_a}\left[s\overline{p_a|}^{i_b^{(m_a)}/m_a}+\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{p_a} s\overline{c_a|}^{i_b^{(m_a)}/m_a}\right.$$

$$\left.-s\overline{d_a|}^{i_a^{(m_a)}/m_a}\right]-\frac{i_a^{(m_a)}-j_a}{m_a}\,{}_aA_a\,{}_av_a^{m_a(a-b)}\,s\overline{p_a+c_a-d_a|}^{i_a^{(m_a)}/m_a}$$

$$={}_bC_a'+\frac{i_b^{(m_b)}}{m_b}\,{}_bP_a\,s\overline{p_b+c_b-d_b|}^{i_b^{(m_b)}/m_b}-\frac{J_a}{m_a}\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{d_a} s\overline{p_a+c_a-d_a|}^{i_b^{(m_a)}/m_a}$$

$$-\frac{i_a^{(m_a)}-j_a}{m_a}\,{}_aA_a\,{}_av_a^{m_a(a-b)}\,s\overline{p_a+c_a-d_a|}^{i_a^{(m_a)}/m_a}.$$

Expanding the terms involving interest at the effective rate i_b, we have

$$\frac{i_b^{(m_b)}}{m_b}\,{}_bP_a\,s_{\overline{p_b+c_b-d_b}|}^{i_b^{(m_b)}/m_b} - \frac{J_a}{m_a}\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{d_a} s_{\overline{p_a+c_a-d_a}|}^{i_b^{(m_a)}/m_a}$$

$$= \frac{i_b^{(m_b)}}{m_b}\,{}_bP_a\,\frac{(1+i_b^{(m_b)}/m_b)^{p_b+c_b-d_b}-1}{i_b^{(m_b)}/m_b} - \frac{J_a}{m_a}\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{d_a}\frac{(1+i_b^{(m_a)}/m_a)^{p_a+c_a-d_a}-1}{i_b^{(m_a)}/m_a}$$

$$= \frac{1}{i_b^{(m_a)}}\left\{i_b^{(m_a)}\,{}_bP_a\left[\left(1+\frac{i_b^{(m_b)}}{m_b}\right)^{p_b+c_b-d_b}-1\right] - J_a\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{d_a}\left[\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{p_a+c_a-d_a}-1\right]\right\}$$

$$= \frac{1}{i_b^{(m_a)}}\left[i_b^{(m_a)}\,{}_bP_a - J_a\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{d_a}\right]\left[(1+i_b)^{(p_a+c_a-d_a)/m_a}-1\right]$$

(since $(p_a+c_a-d_a)/m_a$ and $(p_b+c_b-d_b)/m_b$ represent the same fraction of a year)

$$= \frac{1}{m_a}\left[i_b^{(m_a)}\left(\frac{J_a}{m_a}\,s_{\overline{d_a}|}^{i_b^{(m_a)}/m_a} + \frac{J_a}{m_a}\,a_{\overline{m_a(a-b)}|}^{i_b^{(m_a)}/m_a} + {}_aA_a\,{}_av_b^{m_a(a-b)}\right) - J_a\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{d_a}\right] s_{\overline{p_a+c_a-d_a}|}^{i_b^{(m_a)}/m_a}$$

$$= \frac{1}{m_a}\left[J_a\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{d_a} - J_a + J_a - J_a\,{}_av_b^{m_a(a-b)} + i_b^{(m_a)}\,{}_aA_a\,{}_av_b^{m_a(a-b)} - J_a\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{d_a}\right] s_{\overline{p_a+c_a-d_a}|}^{i_b^{(m_a)}/m_a}$$

$$= \frac{i_b^{(m_a)}-j_a}{m_a}\,{}_aA_a\,{}_av_b^{m_a(a-b)}\,s_{\overline{p_a+c_a-d_a}|}^{i_b^{(m_a)}/m_a}.$$

If we had assumed that A_b was bought at par plus accrued interest and that coupon frequencies and dates of A_a and A_b were the same, so that $m_b = m_a$, $p_b = p_a$, $c_b = c_a$, $d_b = d_a$, and $j_b = i_b^{(m_b)} = i_b^{(m_a)}$, we would have found that income for year b from the investments at the rate i_b was

$$\frac{J_b - J_a}{m_a}\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{d_a} s_{\overline{p_a+c_a-d_a}|}^{i_b^{(m_a)}/m_a}.$$

Under these assumptions,

$$J_b = i_b^{(m_a)}\,{}_{\beta}A_b = i_b^{(m_a)}\,{}_{b}A_b = i_b^{(m_a)}\,{}_{b-d_a}A_b$$

$$= i_b^{(m_a)}\,{}_{a}v_b^{d_a}\,{}_{b}B_b = i_b^{(m_a)}\,{}_{a}v_b^{d_a}\,{}_{b}P_a$$

$$= i_b^{(m_a)}\,{}_{a}v_b^{d_a}\left(\frac{J_a}{m_a}\,s_{\overline{d_a}|}^{i_b^{(m_a)}/m_a} + \frac{J_a}{m_a}\,a_{\overline{m_a(a-b)}|}^{i_b^{(m_a)}/m_a} + {}_{a}A_a\,{}_{a}v_b^{m_a(a-b)}\right)$$

$$= i_b^{(m_a)}\,{}_{a}v_b^{d_a}\left\{\frac{J_a}{m_a}\left[\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{d_a} - 1 + 1 - {}_{a}v_b^{m_a(a-b)}\right]\Big/\frac{i_b^{(m_a)}}{m_a} + {}_{a}A_a\,{}_{a}v_b^{m_a(a-b)}\right\}$$

$$= J_a + (i_b^{(m_a)} - j_a)\,{}_{a}A_a\,{}_{a}v_b^{m_a(a-b)+d_a}$$

and

$$\frac{J_b - J_a}{m_a}\left(1+\frac{i_b^{(m_a)}}{m_a}\right)^{d_a} s_{\overline{p_a+c_a-d_a}|}^{i_b^{(m_a)}/m_a} = \frac{i_b^{(m_a)} - j_a}{m_a}\,{}_{a}A_a\,{}_{a}v_b^{m_a(a-b)}\,s_{\overline{p_a+c_a-d_a}|}^{i_b^{(m_a)}/m_a},$$

the result we obtained before. Thus it appears that we can make these assumptions without loss of generality, and we can simplify our notation by dropping the subscripts from m, p, c, and d. Net income for year b can now be written

$${}_{b}C_a' + \frac{i_b^{(m)} - j_a}{m}\,{}_{a}A_a\,{}_{a}v_b^{m(a-b)}\,s_{\overline{p+c-d}|}^{i_b^{(m)}/m} - \frac{i_a^{(m)} - j_a}{m}\,{}_{a}A_a\,{}_{a}v_a^{m(a-b)}\,s_{\overline{p+c-d}|}^{i_a^{(m)}/m}.$$

The proposed accrual or amortization of the gain or loss on the sale of A_a will add to income each year an amount equal to

$$\frac{{}_{a}A_a - {}_{b}A_a}{a_{\overline{a-b}|}^{i_a}} + \frac{{}_{b}P_a - (J_a/m)\,s_{\overline{d}|}^{i_b^{(m)}/m} - {}_{a}A_a}{a_{\overline{a-b}|}^{i_b}}$$

less interest at the respective rates on the previously unaccrued or unamortized balances. For year b, when interest accrues for only part of a year, the accrual or amortization amounts to

$$\frac{{}_{a}A_a - {}_{b}A_a}{a_{\overline{a-b}|}^{i_a}}\left(1 - i_a a_{\overline{a-b}|}^{i_a}\right)s_{\overline{(p+c-d)/m}|}^{i_a}$$

$$+ \frac{{}_{b}P_a - (J_a/m)\,s_{\overline{d}|}^{i_b^{(m)}/m} - {}_{a}A_a}{a_{\overline{a-b}|}^{i_b}}\left(1 - i_b a_{\overline{a-b}|}^{i_b}\right)s_{\overline{(p+c-d)/m}|}^{i_b}$$

$$= \frac{{}_aA_a - (J_a/m)\, a\overline{\frac{i_a^{(m)}/m}{m(a-b)|}} - {}_aA_a\, {}_av_a^{m(a-b)}}{(1 - v_a^{a-b})/i_a} [1 - (1 - v_a^{a-b})]$$

$$\times \frac{(1 + i_a)^{(p+c-d)/m} - 1}{i_a} + \frac{(J_a/m)\, a\overline{\frac{i_b^{(m)}/m}{m(a-b)|}} + {}_aA_a\, {}_av_b^{m(a-b)} - {}_aA_a}{(1 - v_b^{a-b})/i_b}$$

$$\times [1 - (1 - v_b^{a-b})] \frac{(1 + i_b)^{(p+c-d)/m} - 1}{i_b}$$

$$= \frac{1}{1 - {}_av_a^{m(a-b)}} \left[{}_aA_a(1 - {}_av_a^{m(a-b)}) - \frac{J_a}{m} \frac{1 - {}_av_a^{m(a-b)}}{i_a^{(m)}/m} \right]$$

$$\times v_a^{a-b} \left[\left(1 + \frac{i_a^{(m)}}{m}\right)^{p+c-d} - 1 \right]$$

$$+ \frac{1}{1 - {}_av_b^{m(a-b)}} \left[\frac{J_a}{m} \frac{1 - {}_av_b^{m(a-b)}}{i_b^{(m)}/m} - {}_aA_a(1 - {}_av_b^{m(a-b)}) \right]$$

$$\times v_b^{a-b} \left[\left(1 + \frac{i_b^{(m)}}{m}\right)^{p+c-d} - 1 \right]$$

$$= \frac{i_a^{(m)} - j_a}{i_a^{(m)}} {}_aA_a\, {}_av_a^{m(a-b)} \left[\left(1 + \frac{i_a^{(m)}}{m}\right)^{p+c-d} - 1 \right]$$

$$- \frac{i_b^{(m)} - j_a}{i_b^{(m)}} {}_aA_a\, {}_av_b^{m(a-b)} \left[\left(1 + \frac{i_b^{(m)}}{m}\right)^{p+c-d} - 1 \right]$$

$$= \frac{i_a^{(m)} - j_a}{m} {}_aA_a\, {}_av_a^{m(a-b)} s\overline{\frac{i_a^{(m)}/m}{p+c-d|}} - \frac{i_b^{(m)} - j_a}{m} {}_aA_a\, {}_av_b^{m(a-b)} s\overline{\frac{i_b^{(m)}/m}{p+c-d|}}.$$

The adjustment for year b also includes deferral of the gain or loss, exclusive of any amount attributable to different valuation of accrued coupon.

Adding the pieces together, we find that, for year b, reported income attributable to the rollover is

$${}_bC_a' + \frac{i_b^{(m)} - j_a}{m} {}_aA_a\, {}_av_b^{m(a-b)} s\overline{\frac{i_b^{(m)}/m}{p+c-d|}} - \frac{i_a^{(m)} - j_a}{m} {}_aA_a\, {}_av_a^{m(a-b)} s\overline{\frac{i_a^{(m)}/m}{p+c-d|}}$$

$$+ \frac{i_a^{(m)} - j_a}{m} {}_aA_a\, {}_av_a^{m(a-b)} s\overline{\frac{i_a^{(m)}/m}{p+c-d|}} - \frac{i_b^{(m)} - j_a}{m} {}_aA_a\, {}_av_b^{m(a-b)} s\overline{\frac{i_b^{(m)}/m}{p+c-d|}} - {}_bC_a'$$

$$= 0. \qquad Q.E.D.$$

For the period from time b to the end of some year $b + t$, $0 < t < a - b$, coupon income attributable to the rollover is

$$\frac{i_b^{(m)} - j_a}{m} \, {}_aA_a \, {}_av_b^{m(a-b)} s_{\overline{mt+p+c-d}|}^{i_b^{(m)}/m},$$

and during year $b + t$ coupon income is

$$\frac{i_b^{(m)} - j_a}{m} \, {}_aA_a \, {}_av_b^{m(a-b)} (s_{\overline{mt+p+c-d}|}^{i_b^{(m)}/m} - s_{\overline{m(t-1)+p+c-d}|}^{i_b^{(m)}/m})$$

$$= \frac{i_b^{(m)} - j_a}{m} \, {}_aA_a \, {}_av_b^{m(a-b-t)+d-p-c} a_{\overline{m}|}^{i_b^{(m)}/m}.$$

Accrual of discount or amortization of premium on A_a is

$${}_{b'+t}A_a - {}_{b'+t-1}A_a = \frac{i_a^{(m)} - j_a}{m} \, {}_aA_a \, {}_av_a^{m(a-b-t)+d-p-c} a_{\overline{m}|}^{i_a^{(m)}/m},$$

and the adjustment amounts to

$$\frac{{}_aA_a - {}_bA_a}{a_{\overline{a-b}|}^{i_a}} (1 - i_a a_{\overline{a-b-t+1-(p+c-d)/m}|}^{i_a})$$

$$+ \frac{{}_bP_a - (J_a/m) s_{\overline{d}|}^{i_b^{(m)}/m} - {}_aA_a}{a_{\overline{a-b}|}^{i_b}} (1 - i_b a_{\overline{a-b-t+1-(p+c-d)/m}|}^{i_b})$$

$$= \frac{i_a^{(m)} - j_a}{m} \, {}_aA_a \, v_a^{a-b-t+1-(p+c-d)/m} - \frac{i_b^{(m)} - j_a}{m} \, {}_aA_a \, v_b^{a-b-t+1-(p+c-d)/m}$$

$$= \frac{i_a^{(m)} - j_a}{m} \, {}_aA_a \, {}_av_a^{m(a-b-t+1)+d-p-c} s_{\overline{m}|}^{i_a^{(m)}/m}$$

$$- \frac{i_b^{(m)} - j_a}{m} \, {}_aA_a \, {}_av_b^{m(a-b-t+1)+d-p-c} s_{\overline{m}|}^{i_b^{(m)}/m}$$

$$= \frac{i_a^{(m)} - j_a}{m} \, {}_aA_a \, {}_av_a^{m(a-b-t)+d-p-c} a_{\overline{m}|}^{i_a^{(m)}/m}$$

$$- \frac{i_b^{(m)} - j_a}{m} \, {}_aA_a \, {}_av_b^{m(a-b-t)+d-p-c} a_{\overline{m}|}^{i_b^{(m)}/m},$$

so that, for year $b + t$, reported income attributable to the rollover is

$$\frac{i_b^{(m)} - j_a}{m}\,{}_aA_a\,{}_av_b^{m(a-b-t)+d-p-c}\,a_{\overline{m}|}^{i_b^{(m)}/m} - \frac{i_a^{(m)} - j_a}{m}\,{}_aA_a\,{}_av_a^{m(a-b-t)+d-p-c}\,a_{\overline{m}|}^{i_a^{(m)}/m}$$

$$+ \frac{i_a^{(m)} - j_a}{m}\,{}_aA_a\,{}_av_a^{m(a-b-t)+d-p-c}\,a_{\overline{m}|}^{i_a^{(m)}/m}$$

$$- \frac{i_b^{(m)} - j_a}{m}\,{}_aA_a\,{}_av_b^{m(a-b-t)+d-p-c}\,a_{\overline{m}|}^{i_b^{(m)}/m}$$

$$= 0\,. \qquad Q.E.D.$$

For the period in year a prior to time a, coupon income attributable to the rollover is

$$\frac{i_b^{(m)} - j_a}{m}\,{}_aA_a\,{}_av_b^{m(a-b)}\left(s_{\overline{m(a-b)}|}^{i_b^{(m)}/m} - s_{\overline{m(a-b)-n_a+c}|}^{i_b^{(m)}/m}\right)$$

$$= \frac{i_b^{(m)} - j_a}{m}\,{}_aA_a\,a_{\overline{n_a-c}|}^{i_b^{(m)}/m}\,,$$

and accrual of discount or amortization of premium on A_a is

$$\frac{i_a^{(m)} - j_a}{m}\,{}_aA_a\,a_{\overline{n_a-c}|}^{i_a^{(m)}/m}\,.$$

Since interest again accrues for only part of a year, the adjustment is

$$\frac{{}_aA_a - {}_bA_a}{a_{\overline{a-b}|}^{i_a}}\left(1 - i_a a_{\overline{(n_a-c)/m}|}^{i_a}\right)s_{\overline{(n_a-c)/m}|}^{i_a}$$

$$+ \frac{{}_bP_a - (J_a/m)s_{\overline{d}|}^{i_b^{(m)}/m} - {}_aA_a}{a_{\overline{a-b}|}^{i_b}}\left(1 - i_b a_{\overline{(n_a-c)/m}|}^{i_b}\right)s_{\overline{(n_a-c)/m}|}^{i_b}$$

$$= \frac{i_a^{(m)} - j_a}{i_a^{(m)}/i_a}\,{}_aA_a\,v_a^{(n_a-c)/m}\,\frac{(1+i_a)^{(n_a-c)/m} - 1}{i_a}$$

$$- \frac{i_b^{(m)} - j_a}{i_b^{(m)}/i_b}\,{}_aA_a\,v_b^{(n_a-c)/m}\,\frac{(1+i_b)^{(n_a-c)/m} - 1}{i_b}$$

$$= \frac{i_a^{(m)} - j_a}{m}\,{}_aA_a\,a_{\overline{n_a-c}|}^{i_a^{(m)}/m} - \frac{i_b^{(m)} - j_a}{m}\,{}_aA_a\,a_{\overline{n_a-c}|}^{i_b^{(m)}/m}\,,$$

and reported income attributable to the rollover is

$$\frac{i_b^{(m)} - j_a}{m}\,{}_aA_a\,a\overline{\vert}_{n_a-c\vert}^{i_b^{(m)}/m} - \frac{i_a^{(m)} - j_a}{m}\,{}_aA_a\,a\overline{\vert}_{n_a-c\vert}^{i_a^{(m)}/m}$$

$$+ \frac{i_a^{(m)} - j_a}{m}\,{}_aA_a\,a\overline{\vert}_{n_a-c\vert}^{i_a^{(m)}/m} - \frac{i_b^{(m)} - j_a}{m}\,{}_aA_a\,a\overline{\vert}_{n_a-c\vert}^{i_b^{(m)}/m} = 0. \qquad Q.E.D.$$

At time a, if A_a had not been sold, its coupons would cease and its maturity value, ${}_aA_a$, would become available for reinvestment in a new bond A_a with coupons $J_a/m = (i_b^{(m)}/m)\,{}_aA_a$. For the balance of year a, the difference in coupon income attributable to the maturity would be

$$\frac{J_a - J_a}{m}\,s\overline{\vert}_{m-n_a+c\vert}^{i_b^{(m)}/m} = \frac{i_b^{(m)} - j_a}{m}\,{}_aA_a\,s\overline{\vert}_{m-n_a+c\vert}^{i_b^{(m)}/m},$$

and the coupon income attributable to the rollover is

$$\frac{i_b^{(m)} - j_a}{m}\,{}_aA_a\,{}_av_b^{m(a-b)}\left(s\overline{\vert}_{m(a+1-b)-n_a+c\vert}^{i_b^{(m)}/m} - s\overline{\vert}_{m(a-b)\vert}^{i_b^{(m)}/m}\right) - \frac{i_b^{(m)} - j_a}{m}\,{}_aA_a\,s\overline{\vert}_{m-n_a+c\vert}^{i_b^{(m)}/m}$$

$$= \frac{i_b^{(m)} - j_a}{m}\,{}_aA_a\,\frac{m}{i_b^{(m)}}\left[(1 + i_b^{(m)}/m)^{m-n_a+c} - {}_av_b^{m(a-b)} - 1\right.$$

$$\left. + {}_av_b^{m(a-b)} - (1 + i_b^{(m)}/m)^{m-n_a+c} + 1\right]$$

$$= 0.$$

It is easy to see that the same result obtains for years after a. There is, of course, no further accrual of discount or amortization of premium on A_a and no further adjustment. Hence reported income attributable to the rollover equals zero for any period after time a.

Maturity of any of the bonds bought at or after time b would not affect income attributable to the rollover, since it is assumed that the maturity value of any such bond would be promptly reinvested to yield i_b, as before.

Therefore, if proceeds from the sale of a bond are invested to yield the rate of interest implicit in the sale price, and all coupons and maturities arising from that investment are similarly invested, and if any capital gain or loss exclusive of amounts atrributable to accrued coupons is deferred, while amounts attributable to accrued coupons are recognized immediately, and an amount equal to any unaccrued discount or unamortized premium on the bond that is sold is accrued or amortized over the same period at the rate of interest implicit in the sale price, then reported income will be the same as if the bond had not been sold and all

subsequent coupons and maturities had been invested to yield the rate of interest implicit in the sale price. *Q.E.D.*

J. ALAN LAUER:

This paper suggests that realized capital gains and losses on bonds in good standing be amortized over the remaining lifetimes of the bonds sold. Like the author, I am hesitant to discuss the relative merits of this approach and other methods of accounting for capital gains and losses on bonds in good standing. The general concepts presented by the author do seem to me to be logical and worthy of serious consideration. In this discussion, I would like to touch on a theoretical point and a practical consideration, both in regard to bonds in good standing.

A Theoretical Point

Mr. Case sets up a hypothetical situation in which a bond is purchased at par. A few years after this purchase, the bond is sold at a price based on a different interest rate and the proceeds are invested in a new bond at par. Case analyzes this situation and concludes that it would be appropriate to amortize the capital gain or loss realized on sale of the bond over the remaining lifetime of the original bond. By calculating the amortization on the basis of the interest rate used to calculate the sale price, the effect of the sale and reinvestment is neutralized in that the yearly income of the investor is unaffected by the transaction.

The author recognizes that analysis of more general cases might lead to results less simple than in his hypothetical situation. I have set up a more general situation, although still not a completely general situation, and formulas for asset values and amounts of income under this more general situation are appended to this discussion. Derivations of these formulas are not given, but for the most part the derivations are not difficult. Also, it is a relatively simple matter to extend the formulas for durations beyond those indicated.

My situation is more general in that it allows for either or both of the bonds involved (bond 1 is the bond originally purchased, while bond 2 is the bond purchased with the proceeds from the sale of bond 1) to be purchased at a discount or at a premium. The principal conclusion from analysis of this situation is that the capital gain or loss realized on the sale of bond 1 should be amortized in two pieces over the remaining lifetime of bond 1 if the effect of the sale is to be neutralized so that the yearly income of the investor is unaffected by the transaction. The capital loss on the transaction is equal to the excess of (*a*) the par value of the bond minus the sale price over (*b*) the par value of the bond minus the book

value of the bond at the time of sale. If this difference is negative, there is a capital gain rather than a capital loss. The quantity in (*a*) should be amortized at the rate of interest used to determine the sale price of bond 1, while the quantity in (*b*) should be amortized at the interest rate used to determine the purchase price of bond 1. This is illustrated in Examples I and II (Tables 1 and 2).

TABLE 1

EXAMPLE I: 10-YEAR BOND WITH $100,000 PAR VALUE AND 3 PER CENT ANNUAL COUPON PURCHASED FOR $84,556 TO YIELD 5 PER CENT; PROCEEDS (INCLUDING COUPON INCOME) REINVESTED IN 10-YEAR BONDS AT CURRENT YIELD RATE; (REINVESTMENTS AT PAR) YIELD RATE CHANGES TO 7 PER CENT AT DURATION 3

Year t	Book Value of Bond B_t	Book Value of Reinvestments R_t	Total Assets B_t+R_t	Cash Income from Bond and Reinvestments I_t	Accrual of Discount on Bond D_t	Total Income I_t+D_t
1	$ 85,784	$ 3,000	$ 88,784	$ 3,000	$1,228	$ 4,228
2	87,073	6,150	93,223	3,150	1,289	4,439
3	88,427	9,458	97,885	3,308	1,354	4,662
4	89,848	12,997	102,845	3,539	1,421	4,960
5	91,341	16,784	108,125	3,787	1,493	5,280
6	92,908	20,836	113,744	4,052	1,567	5,619
7	94,553	25,171	119,724	4,335	1,645	5,980
8	96,281	29,810	126,091	4,639	1,728	6,367
9	98,095	34,774	132,869	4,964	1,814	6,778
10	100,000	40,085	140,085	5,311	1,905	7,216
11	.....	149,768	149,768	9,683	.....	9,683
12	.....	160,189	160,189	10,421	.....	10,421
13	.....	171,402	171,402	11,213	.....	11,213
14	.....	183,400	183,400	11,998	.....	11,998
15	.....	196,238	196,238	12,838	.....	12,838

In Example I (Table 1) a ten-year bond with $100,000 par value and a 3 per cent annual coupon is purchased to yield 5 per cent and is held to maturity. All cash income, including the proceeds of the bond at maturity, are reinvested at par in ten-year bonds at the current yield rate, which changes from 5 per cent to 7 per cent at duration 3. The total assets consist of the book value of the bond (this is bond 1) plus the book value of reinvestments. The book value of the bond is the amortized value by the usual method and is equal to the purchase price plus the total of the accrual of discount on the bond to date. The book value of reinvestments is equal to the total cash income to date plus, for years 11 and later, the

maturity value of bond 1. The total book income is the cash income plus the accrual of discount for the year.

Example II (Table 2) is similar to Example I except that bond 1 is sold at duration 3 with the sale price determined on the basis of a 7 per cent yield rate, and the proceeds from sale are reinvested in bond 2.

TABLE 2

EXAMPLE II: SAME AS EXAMPLE I EXCEPT ORIGINAL BOND SOLD AT DURATION 3 AND PROCEEDS OF $78,443 REINVESTED IN 10-YEAR BOND WITH $99,384 PAR VALUE AND 4 PER CENT ANNUAL COUPON TO YIELD 7 PER CENT

Year t	Book Value of Bond B_t	Book Value of Reinvestments R_t	Deferred Capital Loss C_t	Total Assets $B_t+R_t+C_t$	Cash Income from Bond and Reinvestments I_t	Accrual of Discount on Bond D_t	Amortization of Deferred Capital Loss L_t	Net Income $I_t+D_t-L_t$
1	$85,784	$ 3,000	.....	$ 88,784	$ 3,000	$1,228	.....	$ 4,228
2	87,073	6,150	.....	93,223	3,150	1,289	.....	4,439
3	78,443	9,458	$9,984	97,885	3,308	1,354	.....	4,662
4	79,959	13,972	8,914	102,845	4,514	1,516	$1,070	4,960
5	81,581	18,802	7,742	108,125	4,830	1,622	1,172	5,280
6	83,316	23,971	6,457	113,744	5,169	1,735	1,285	5,619
7	85,173	29,501	5,050	119,724	5,530	1,857	1,407	5,980
8	87,160	35,418	3,513	126,091	5,917	1,987	1,537	6,367
9	89,286	41,750	1,833	132,869	6,332	2,126	1,680	6,778
10	91,560	48,525	.....	140,085	6,775	2,274	1,833	7,216
11	93,994	55,774	.....	149,768	7,249	2,434	.....	9,683
12	96,598	63,591	.....	160,189	7,817	2,604	.....	10,421
13	99,384	72,018	.....	171,402	8,427	2,786	.....	11,213
14	.....	183,400	.....	183,400	11,998	.....	.....	11,998
15	.....	196,238	.....	196,238	12,838	.....	.....	12,838

COMPONENTS OF L_t

Year	Accrual at 7% of $21,557 Discount in Sale Price	Accrual at 5% of $11,573 Unaccrued Discount on Sale Date	Year	Accrual at 7% of $21,557 Discount in Sale Price	Accrual at 5% of $11,573 Unaccrued Discount on Sale Date
4	$2,491	$1,421	8	$3,265	$1,728
5	2,665	1,493	9	3,494	1,814
6	2,852	1,567	10	3,738	1,905
7	3,052	1,645			

Bond 2 is a ten-year bond with a 4 per cent annual coupon. From the purchase price, the coupon rate, and the yield rate the par value of bond 2 is determined to be $99,384. The capital loss realized on the sale of bond 1 is amortized over the following seven years.

The total assets in Example II are the book value of the bond (bond 1 or bond 2, whichever is held at the time) plus the book value of reinvestments plus the deferred (unamortized) capital loss. At durations 1 and 2 the bond held is bond 1, so that the book value of the bond is the same as in Example I. At durations 3 and over, the bond held is bond 2, and the book value of the bond is the purchase price of bond 2 plus the total accrual to date of the discount on bond 2. The book value of reinvestments is the total cash income to date plus, for durations 14 and over, the maturity value of bond 2. The proceeds from the sale of bond 1 are not included in the reinvestments because they have been assumed to be applied to purchase bond 2. The deferred capital loss is that part of the capital loss which has not been taken into income. At duration 3 the deferred capital loss is the entire loss of $9,984 realized at the sale of bond 1. This amount is the difference between the $78,443 sale price of bond 1 and the $88,427 amortized value of bond 1 at the date of sale. This figure is also equal to the excess of (*a*) the $21,557 excess difference between the par value of bond 1 and the sale price of bond 1 over (*b*) the $11,573 unaccrued discount on bond 1 at the time of sale.

The net income in Example II is equal to the cash income plus the accrual of discount on the bond (bond 1 or bond 2, whichever is held during the particular year) minus the amortization of deferred capital loss during the current year. For years 1–3 the bond held during the year is bond 1 and the accrual of discount is the same as in Example I. For years 4 and over, the accrual of discount is that for bond 2 by the usual method. The amortization of the deferred capital loss is broken down into its two components at the bottom of Example II. The amortization for the year of the deferred capital loss is equal to the accrual for the year of the discount in the sale price on bond 1 minus the accrual for the year of the unaccrued discount in the purchase price of bond 1.

It will be noted that the total assets in Example II are identical with the total assets in Example I, and the net income in Example II is equal to the total income in Example I. This result occurs because the capital loss on the sale of bond 1 has been amortized in the manner described.

A Practical Consideration

In the paper Case suggests that capital gain or loss on sale of a bond in good standing be amortized over the remaining lifetime and that the

change from one year to the next in amortized value of deferred gain and loss be included in income (positively or negatively, as appropriate). Amortization would be on the basis of the yield rate consistent with the sale price of the bond. Earlier in this discussion I have suggested that it would be theoretically preferable, or at least more elegant, to split the capital gain or loss into two pieces (if the bond was purchased at par, one piece is null) and to amortize the two pieces separately at different interest rates.

One is led to wonder whether it would not be preferable, as a practical matter, to amortize deferred gains and losses on a straight-line basis, that is, on the basis of an interest rate of zero. It seems likely that strict adherence to the theory presented in the paper and earlier in this discussion would lead to a large, unwieldy portfolio of deferred capital gains and losses requiring individual calculations of amount of amortization each year. Another disadvantage of the strictly theoretical approach is that, while income is maintained at the same amount each year as if the bonds had not been sold, the amounts of gain or loss recognized (i.e., amortized) in the years immediately following sale are not as large as the amounts recognized in years near the end of the amortization period. Use of straight-line amortization would reduce the number of items in the portfolio of deferred gains and losses to one for each year of maturity represented by bonds sold, would greatly simplify the calculation of the yearly amount of amortization, and would result in recognition of as much gain or loss on a particular bond in the year following sale as in the year preceding the end of the amortization period.

APPENDIX

PRESENTATION OF A GENERAL SITUATION INVOLVING BONDS IN GOOD STANDING, AND FORMULAS FOR ASSETS AND INCOME APPLICABLE TO THAT SITUATION

Bond 1 carries an annual coupon at rate j and matures for its par value of 1 in n years. Bond 1 is purchased to yield rate i. Bond 2 carries an annual coupon at rate h and matures for its par value of P in m years (after purchase of bond 2). Bond 2 is purchased r years after the purchase of bond 1, and the purchase price of bond 2 is equal to the sale price of bond 1, both prices being calculated on the basis of yield rate k.

All cash income, as well as all proceeds at maturity, is reinvested at par at the current yield rate. The current yield rate is i at the end of the first $r - 1$ years after purchase of bond 1 and is k from the rth year on. It is assumed that $m \geq n - r$ and that all reinvestments from bond 1 mature at least n years after the purchase of bond 1. Also, t measures duration from purchase of bond 1.

Formulas Applicable to Bond 1 (Assuming Bond 1 Held to Maturity)

B_t = Book value of bond at duration t (note that B_0 represents the purchase price of bond 1)

$= 1 - (i - j)a_{\overline{n-t}|i}$ $\qquad (0 < t < n)$;

R_t = Book value at duration t of reinvestments

$= js_{\overline{t}|i}$ $\qquad (1 \leq t < r)$

$= js_{\overline{r-1}|i} + j(1 + i)^{r-1}s_{\overline{t-r+1}|k}$ $\qquad (r \leq t \leq n)$;

$B_t + R_t$ = Total asset at duration t

$= 1 - (i - j)a_{\overline{n-t}|i} + js_{\overline{r-1}|i} + j(1 + i)^{r-1}s_{\overline{t-r+1}|k}$

$\qquad (r \leq t \leq n)$;

I_t = Cash income at duration t from bond and reinvestments

$= j(1 + i)^{t-1}$ $\qquad (t < r)$

$= j(1 + i)^{r-1}(1 + k)^{t-r}$ $\qquad (r \leq t \leq n)$;

D_t = Accrual of discount for year t on bond (or amortization of premium)

$= (i - j)v_i^{n-t+1}$ $\qquad (1 < t < n)$;

$I_t + D_t$ = Book income at duration t

$= j(1 + i)^{r-1}(1 + k)^{t-r} + (i - j)v_i^{n-t+1}$ $\qquad (r < t < n)$.

Formulas Applicable to Bond 2 (Assuming Bond 1 Sold but Reinvestments from Bond 1 Retained)

B_t = Book value of bond at duration t (note that B_r represents the sale price of bond 1 and the purchase price of bond 2)

$= 1 - (k - j)a_{\overline{n-r}|k}$ $\qquad (t = r)$

$= P[1 - (k - h)a_{\overline{m+r-t}|k}]$ $\qquad (r \leq t \leq n)$;

P = Par value of bond 2

$= [1 - (k - j)a_{\overline{n-r}|k}] \div [1 - (k - h)a_{\overline{m}|k}]$;

R_t = Book value at duration t of reinvestments

$= js_{\overline{r-1}|i} + j(1 + i)^{r-1}s_{\overline{t-r+1}|k} + (hP - j)s_{\overline{t-r}|k}$

$\qquad (r < t \leq n)$;

C_t = Deferred (unamortized) capital loss (or gain) from sale of bond 1 (note C_r represents the amount of capital loss or gain)

$$= (k - j)a_{\overline{n-t}|k} - (i - j)a_{\overline{n-t}|i} \qquad (r \leq t \leq n) ;$$

$B_t + R_t + C_t$ = Total asset at duration t

$$= 1 - (i - j)a_{\overline{n-t}|i} + js_{\overline{r-1}|i} + j(1 + i)^{r-1}s_{\overline{t-r+1}|k} \qquad (r \leq t \leq n) ;$$

I_t = Cash income at duration t from bond and reinvestments

$$= j(1 + i)^{r-1}(1 + k)^{t-r} + (hP - j)(1 + k)^{t-r-1} \qquad (r + 1 \leq t \leq n) ;$$

D_t = Accrual of discount for year t on bond (or amortization of premium)

$$= P(k - h)v_k^{n+r-t+1} \qquad (r + 1 \leq t \leq n) ;$$

L_t = Amortization in year t of deferred capital gain or loss

$$= (k - j)v_k^{n-t+1} - (i - j)v_i^{n-t+1} \qquad (r + 1 \leq t \leq n) .$$

$I_t + D_t - L_t$ = Book income at duration t

$$= j(1 + i)^{r-1}(1 + k)^{t-r} + (i - j)v_i^{n-t+1} \qquad (r + 1 \leq t \leq n) .$$

JOHN H. BIGGS:

Mr. Case proposes a consistent and comprehensive solution to a major accounting difficulty that currently exists not only in life insurance company financial statements but in the statements of all business entities. Although the method described in this paper presents many new complexities in financial reporting, it is an interesting and significant alternative solution that should be considered seriously by actuaries and by those accountants responsible for developing accounting principles.

Even putting aside the obvious problems of the present procedures for computing capital gains and losses on investments for stockholder financial reports, the present treatment for mutual companies and for internal management of all life companies includes the following more subtle distortions:

1. Gains and losses at present are credited or charged to the mandatory securities valuation reserve (MSVR) and will continue to be so credited for most companies into the foreseeable future. Since such gains (losses) are there-

fore restricted in the balance sheet, most actuaries are very hesitant about reflecting even part of such items in pricing. With ever larger fractions of investment portfolios in equity securities, and more accurate crediting to policyholders required (as evidenced, for example, by the use of the investment year method), there is much pressure on actuaries to credit, by dividends or otherwise, the results of capital appreciation as they actually arise rather than deferring them as a reserve in our statutory statements.

2. The first time that any earnings on capital appreciation of a life company's portfolio are reflected in its income is when interest or dividends are subsequently earned on the realized gains from the portfolio. This might well be to a generation of policyholders following that which made the funds available to the company for investment.

3. When, as often occurs, only the gain from operations, and not surplus items, of a life insurance company is considered, the distortion resulting from the sale of a bond is even more serious than that shown in Mr. Case's example of the $100,000 bond. The loss on the sale of the 3 per cent bond is not reflected in income (it is charged against surplus and most probably is offset there by a release from the MSVR), and the result in the operating statement is as shown in the accompanying tabulation (using Mr. Case's example).

Year t	Original Bond Retained to Maturity, Then Re-invested	Bond Rolled Over at Duration 3
2.....	$3,090	$3,090
3.....	3,183	3,183 ($11,573 loss to surplus)
4.....	3,342	4,764
5.....	3,509	5,002

4. Even with the best reporting of financial results, the problem of quantifying surplus "needs" is extraordinarily difficult for a life insurance company, and this problem is compounded by an accounting approach which requires that virtually all capital items be included in a surplus-type fund, the MSVR.

The need for a change from a system which includes so many perverse incentives makes the complexities of Mr. Case's method seem more acceptable. Described below are two of the obvious complexities, with some suggestions as to how they might be overcome:

1. Mr. Case has not discussed the effects of federal income taxes on his method of accounting for capital items. Perhaps at some future date a method such as the one described in his paper might become the basis for taxing capital items. Whether or not this does come about, it appears that some knotty tax problems would exist. Since most companies tend to take an equal amount of gains and

losses in a year to avoid taxes on gains, it might be argued that the tax could be ignored for companies in that position. Otherwise the resulting tax effect could also be amortized or accrued on the same basis as the capital item.

2. The mechanical problems of setting values for bonds not in good standing and the earnings to be credited on common stocks under the equity method could be solved for life insurance companies by use of the NAIC annual releases on valuation of securities. If the method were applied to all corporations by the Accounting Principles Board or its successor organization, or by SEC fiat, it could be made a requirement for a company declaring a dividend that it also declare at the same time the current ratio of earnings to that dividend. That ratio could then be used to adjust the carrying value of the stock on the holder's books.

Mr. Case's method is comprehensive, and it is interesting that he can demonstrate its consistency even in the question of merger accounting by showing that the purchase basis, using his approach, becomes equivalent to the pooling of interests basis.

One last, somewhat facetious remark: Since negative "goodwill" clearly must be "ill will," which no company would wish to show in its statements, perhaps an item called the "asset adjustment account" should include the net total of all "goodwill" and "ill will," and when positive, and "good," be treated as an asset, and when negative, and "ill," be treated as a liability.

GARY CORBETT:

Mr. Case has presented an excellent paper, which illustrates the application of actuarial thought to a problem somewhat removed from traditional actuarial practice, at least in North America. It is to be hoped that this paper receives wide circulation, specifically among accountants and investment analysts. Mr. Case has presented theories of accounting for investment results that provide an excellent basis for further discussion of the concepts by all interested parties. The theories may not be capable of implementation in all respects but, if accepted, could form a basis for approximate methods yielding substantially the same results.

The remainder of this discussion deals with only the "Bonds in Good Standing" section of the paper. Because of my involvement with the problem of reporting the earnings of life insurance companies in accord with generally accepted accounting principles, I early came up against the rather illogical effect on reported earnings of the traditional methods of accounting for the exchange of one bond for another when the underlying yield rates have changed in the interim. In order to demonstrate these distortions in earnings, a model, much like Mr. Case's, was constructed. However, because the invested assets were matched against a

type of ten-year single premium pure endowment policy in order to show the resulting earnings distortions, the interest earned on the bonds was assumed to be reinvested in bonds with the same maturity date as the original investment. Also, at the time of rollover, the entire portfolio, including the bonds purchased from interest proceeds, was assumed to be reinvested in a new bond with the same maturity date as the original investment. Interest from this new bond was similarly invested year by year in bonds with the same maturity date. Thus the model was closed at the end of ten years.

Correspondence with Mr. Case and others confirmed that Mr. Case's suggested solution of spreading the capital gain or loss on the sale of the original bond(s) over the remaining lifetime of the original investment at the yield rate on the new investment produced the best results for my model as well as for his. Mr. Case has now analyzed the problem further and is correct when he states that the amortization schedule should be "at the interest rate implicit in the sales price of the bond sold, regardless of the yield obtained on the new investment."

Unfortunately, in writing the portion of the "Response of the Joint Actuarial Committee on Financial Reporting to the August, 1972 Exposure Draft of 'Audits of Stock Life Insurance Companies' " that deals with this problem, I made an error. The last line on page 25 of our response, in the section dealing with "Valuation of Investments and Recognition of Realized and Unrealized Gains (Losses) Thereon," states that the capital gain or loss should be spread "at the yield rate of the original investment." This approach, as illustrated by column 3 of Mr. Case's Table 1, clearly does not produce results as logical as using "the interest rate implicit in the sales price of the bond sold." The response should be corrected. The main purpose in writing this discussion is to remove this apparent disagreement and to express my full agreement with the method suggested in Mr. Case's paper.

RAYMOND A. BIERSCHBACH:

Daniel Case has presented an interesting proposal for the handling of capital gains and losses on both stocks and bonds. It is well written and thought-provoking. My decision to write this discussion was the result of reading Mr. Case's paper and subsequently discussing it with Mr. Al Colles, C.P.A., vice-president and chief accounting officer of my company.

Let me first discuss the matter of the bonds. The paper indicates that it does not question the validity of any present "principles" of accounting but almost starts with the premise that it is improper to report a capital loss on a bond if a sale has been made in order to reinvest in higher-yield

bonds and thus artificially inflate future years' investment income earnings. Mr. Case proposes an interesting manner of accounting for such transactions which would, if adopted, remove the illusory benefit that currently arises from such transactions. However, the paper is "set with a GAAP frame of reference," and the illusory investment income would therefore disappear only in the reporting of earnings on a GAAP basis. Furthermore, the current AICPA audit guide would have to be revised in order to allow for the treatment suggested in the paper.

Even if the audit guide were revised, I doubt that much benefit would be gained by making the accounting change only in the GAAP earnings reports. It is a fact of life that statutory earnings are reviewed for stock companies who also report earnings on a GAAP basis, and mutual companies currently report only statutory earnings. Even after issuances of the AICPA audit guide, many stock companies will probably continue to report only on a statutory basis. Therefore, unless the proposal made by the author is also used in the statutory statements, the "benefits" realized by companies which make such transactions would still be gained. Companies would still be able to inflate the ratio of net investment income to mean assets shown in Exhibit II of the Annual Statement and their statutory gain from operations, since capital gains and losses are excluded therein.

Would it then make sense to adopt Mr. Case's proposed method for bonds in both the statutory statements and the GAAP statements? Even though the AICPA audit guide for banks permits a deferral or amortization of bond gains or losses over the remaining period of the bond which was sold, I know that some C.P.A.'s find this somewhat unpalatable. The paper makes mention of the fact that the AICPA audit guide for banks has allowed this approach, but it fails to point out that the method was rejected by regulatory authorities and is not now used by any bank, to our knowledge. Further, the major reason for much of the bond rollover in banks was a tax benefit which has subsequently disappeared, along with probably most of the previous rollover.

As an actuary, I cannot find anything immoral in the amortization of a capital loss taken on a bond, since, if that bond is replaced by a higher-yielding bond, there is really no economic loss to be realized immediately. Therefore, I could accept the method proposed by Mr. Case in order to remove the possibility of creating artificial investment income.

But one cannot look at bonds alone. If a company takes a capital loss on a bond, it is quite likely that they will simultaneously take a capital gain on a stock in order to offset that capital loss. To do otherwise would

create a *real* loss, in that the capital loss would not be tax-affected in that year because of the limits on deductibility of capital losses. Therefore, acceptance of the method proposed by the author for amortization of bond capital losses almost forces one into amortizing the stock capital gains used to offset bond capital losses. However, does it necessarily follow that all stock capital gains should be amortized? If our chief investment officer feels that the price of a particular stock is currently at a high point and that if he holds that stock any longer it will depreciate in value, then I feel that he is justified and exercising good business sense in selling the stock. Furthermore, I also feel that the realized capital gain should be recognized in the year in which it is taken rather than spread over future years, so that credit due present management for good business judgment will not also be deferred to future management. If my position is accepted, then one is almost forced into the illogical position of amortizing some stock capital gains while taking immediate credit for others.

It should also be pointed out that the methods proposed in Case's paper will not accomplish the goal that is being sought. He proposes that, if a company takes a capital loss on a bond and simultaneously takes a like capital gain on a stock, that company should use different methods for amortizing the bond capital loss and the stock capital gain. This will cause distortions in future years' earnings, which, after all, is the effect that the paper is trying to avoid.

Is this a dilemma to which there is no solution? I think not. If all realized capital gains and losses were taken into earnings in the year in which they occurred, there would not be as much incentive to take capital losses on bonds in order to inflate future years' investment income, because those capital losses would impact the current year's earnings. It is true that a company could take the capital loss on a bond and offset it with a capital gain on a stock in order to have no effect on the current year's earnings, with the result that future years' investment income would be higher. But, after all, that reflects the actual economic decisions made by management, and the increased future interest on the new bonds would be real investment income.

Mr. Case seems to have some trouble in accepting the theory that, although certain stocks may be treated one way in accounting, others (perhaps of the same company) may be treated in another way. Take a practical example, in order to see the reasoning behind the theory. One investor has a significant equity position in MGM and recently acquired, through a tender offer, several hundred thousand more shares. Both be-

fore and after the latest acquisition he would seem to have owned enough shares to have a significant voice in the management of the company. His influence on management decisions of the company should have a significant impact on its profitability. Such a large equity position is reflected in an accounting sense through the percentage of the company's net assets owned at a particular date—that is, by the equity method. Also reflected is the fact that his shares are not "marketable" in the usual sense—that is, by normal trades on a stock exchange.

On the other hand, the former owners of the shares tendered to this investor truly owned marketable securities. Individually, they could make a "sell" decision and expect it to be executed on the exchange almost immediately, so that they can be deemed to have held "marketable" securities. The accounting, then, is designed to differentiate between holders of enough of a company's shares to exercise "significant control" over its operations and those holders who have little or no control in this sense and make and execute buy-and-sell decisions based upon their opinion of whether they think a stock is undervalued or fully valued in the marketplace. In the example, all those who tendered their shares apparently decided that they were fully valued and took advantage of the tender offer. The chances are that many of these shareholders also owned shares of many other corporations. It would be highly impractical to force them to use the equity method for valuing all their shares.

Dan Case's paper makes one fact abundantly clear. Actuaries in this country are beginning to branch out into areas that have previously not been looked upon as the actuaries' domain. This paper could just as appropriately have appeared in the *Journal of Accountancy*, and Dan has done us a service by contributing it to our *Transactions*.

RUSSELL M. COLLINS, JR.:

Mr. Case is to be congratulated on his excellent presentation of a rational and consistent approach to the asset valuation problem, a subject which has received all too little attention from our profession. The paper is quite timely, since this subject has been a very controversial one in accounting and insurance circles in recent months.

The author's suggestion for treatment of capital gains and losses on bonds in good standing is a very interesting one and has much to recommend it. It has appeal on theoretical grounds and seems to be practical in application.

I have more difficulty with the author's suggested treatment of common stocks. The suggested approach is based on two principles stated by

the author at the beginning of his paper. The author deliberately refrains from making an attempt to evaluate the merits of these principles, but it is difficult to discuss the paper without at least expressing an opinion on the matter. For reasons elaborated on below, I would not accept the first principle as being axiomatic, and I really can find very little reason to accept the second principle. It seems to me that the principles on which current treatment of common stock investments is based are both reasonable and preferable.

I believe that the suggested method would also be very difficult to apply. Some of the problems are outlined below:

1. What is the definition of "favorable earnings"? What earnings may be regarded as a "reasonably satisfactory return on the company's net worth"? How do we determine the period of years during which it is anticipated that a company would continue to report favorable earnings on shares of common stock? It seems to me that the answers to these questions—which are fundamental to the application of the method—will often be of a speculative nature and arbitrary at best.
2. The method requires co-ordination between the seller and the buyer. In this respect, why shouldn't the difference in value of a given investment as between the seller and the buyer—a difference which may be very real—be reflected in the assets and net worth before and after the exchange? It seems to me that this difference may very well reflect a real difference in value, even though there has been no newly invested capital in the aggregate.
3. There are additional obvious problems, one of which has been cited by the author, when the method is applied to marketable common stocks as the term is defined by the author.

In discussing the treatment of marketable common stocks, the author cites four principal approaches which have been discussed and then suggests a fifth possibility. The first approach contemplates inclusion of dividends and realized capital gains or losses in income. To include realized gains or losses in net income and to exclude unrealized gains or losses from net income would appear to permit earnings to be "managed," and this treatment would seem to invite unfair reporting. This very problem is the same one on which the author bases his objection with respect to current treatment of bonds. To include both realized and unrealized capital gains or losses in net income obviously leads to confusing, if not ridiculous, results. It is clearly for this reason that the fourth approach cited by the author is under consideration. The fourth approach involves recognizing realized and unrealized capital gains and losses in income and surplus on a formula basis. While this spreading formula obviously

damps the violent swings in income from year to year which would result from no spreading, I believe that a formula basis is not fair reporting for the following reasons:

1. It does not represent fairly the actual results during the accounting period. Capital losses are deferred, and the method fails to disclose promptly materially adverse changes in investment gains or losses.
2. Unrealized investment gains or losses are not income within the normal meaning of the term. They have not been realized and may never be realized. Therefore, to include even a portion of them in net income would impair the significance and meaning of the term.
3. If the formula basis were optional and the choice of formula up to each company, at best the reported results would be difficult to compare and at worst earnings could still be "managed."
4. There is a risk that the government, in its search for revenues, will come to regard the gains and losses taken in net income as real earnings and hence as taxable income on the same basis.
5. Including investment gains or losses on a formula basis is no more or less rational than spreading gains or losses from mortality or other items subject to high fluctuation.

Some of the above criticisms could be applied to the author's proposed method, in addition to those criticisms mentioned earlier.

This leaves us with the third "principal approach" cited by the author, that is, to carry common stocks at market value, to include dividends in income, and to carry realized and unrealized capital gains and losses as a separate item in the year in which they arise.

I agree with the statement of the Joint Committee on Financial Reporting Principles that "there is a real difference between capital gains and losses and the results of basic insurance and investment operating items" and that "capital gains and losses need to be in some way shown separately." In my opinion, the only satisfactory way to accomplish this is to keep the two items entirely separate in financial statements. This can best be accomplished by reporting two statements: (1) a statement of income and (2) a statement of investment gains or losses. I believe that many of the problems we are encountering in discussion on this subject today result from the attempt to determine one figure—"net income per share"—to be used to evaluate and compare company performance. I do not believe that there is such a "magic number." Historical financial data should include *both* earnings and investment gains or losses on a per share basis, so that this information can be fairly appraised by the investor.

There is good evidence that, even if the industry and the accounting profession insist on including investment gains and losses in income on any

basis, the financial community will still insist on the separation of operating income and investment gains or losses. Financial analysts have long been considered authorities on what constitutes fair reporting, and I believe that we should give careful consideration to their viewpoint. As evidence of how many financial analysts feel about realized capital gains and losses, many bank stock analysts, in reporting on comparative performance of bank earnings, consider operating earnings rather than so-called "net income" (which includes realized capital gains and losses) the most important indicator of current performance, and look at investment gains and losses separately. It is quite possible that even if a spreading formula were adopted by the accountants and the industry, the financial analysts might very well adopt the same practice in reporting on comparative performance for insurance companies' earnings as they do for banks.

If the separate treatment of operating income and investment gains or losses is adopted for financial reporting, it would seem that current methods of accounting for bond and common stock investments are appropriate.

FREDERICK S. TOWNSEND, JR.:

There are no theoretical considerations that I would like to discuss in connection with Mr. Case's paper, but I would like to discuss briefly two practical considerations.

The first consideration is also related to the paper presented by Mr. Kayton and Mr. Tookey.[1] Mr. Kayton and Mr. Tookey deal with the separation of interests of the continuing policyholder (owner) and the new policyholder (owner) of the mutual life insurance company. Mr. Case discusses business combinations as they affect the accounting for stock life insurance companies. When it comes to the method of accounting for business combinations, I would like to point out that the interests of continuing stockholders and prospective new stockholders are entirely different.

In the case of a continuing stockholder, if a cash acquisition is made, some of his accumulated capital funds have been expended to enhance corporate earning power. However, the loss of some of his capital funds should be charged against the income which it is helping to produce. Therefore, there is a basic rationale for amortizing goodwill against future earnings from the point of view of a continuing stockholder.

[1] Howard H. Kayton and Robert C. Tookey, "Merger of Mutual Life Insurance Companies" (see p. 261 of this volume).

From the point of view of a prospective new stockholder, however, the proportionate share of the company which he is purchasing is the new, lower absolute level of capital funds (excluding goodwill as an asset). He is purchasing his stock on the basis of the company's current earning power before any charge for goodwill. At the time at which he is purchasing his stock in the company, the amount of surplus expended in goodwill has already disappeared and there is no reason to charge such amounts against earnings accruing for his benefit.

A similar argument might even be made in favor of the continuing stockholder. In the case of a cash purchase, the continuing stockholder has expended assets in exchange for goodwill. A loss of those assets results in a loss of investment income. If earnings have been penalized in this fashion, why is it necessary to amortize goodwill? (Or, in the case of an acquisition in exchange for common stock, the issuance of additional shares of common stock for goodwill results in dilution of earnings on a per share basis. Is it then necessary to further reduce future earnings by a charge against net income for the amortization of goodwill?)

All of this discussion leads to the related areas of purchase accounting versus pooling of interests. Take the case of United States Life Insurance Company and its successor organization, USLIFE Corporation. The life insurance company reported earnings of $1.71 per share at year end 1965. Following a series of acquisitions, at year end 1969, reported earnings were $1.76 per share, an increase of $0.05 per share over the four-year period. From the point of view of a continuing stockholder, earnings increased 3 per cent from 1965 to 1969. The company restated earnings, however, on a pooling of interests basis from the $1.71 originally reported in 1965 to a figure of $1.08 per share for 1965 restated in 1969. On a restated basis, earnings increased 65 per cent in four years. From the point of view of a continuing stockholder, his earnings had grown at less than 1 per cent per year for four years, while a prospective new stockholder reading the company's annual report was presented with a restated earnings history indicating an annual growth rate in excess of 13 per cent per year.

In summary, the first practical consideration which I raise is that the position and viewpoint of the continuing stockholder differ from those of a prospective new stockholder.

A second area of discussion centers around the reaction of management and the investment community to the inclusion of both realized and unrealized capital gains in reported net income.

At the present time most life insurance companies have a relatively

small proportion of their assets invested in common stocks. This is because of the combination of legal requirements, long-term guarantees included in the life insurance contract, and a lack of investment expertise in the stock market. Within recent years there has been growing emphasis on increasing the proportion of industry assets invested in common stocks.

If the accounting profession decides that both realized and unrealized capital gains are to be included in net income, the effect may be to discourage this type of investment. Conglomerate holding companies, mutual funds, bond portfolio managers, and preferred stock portfolio managers all avoid taking capital losses on their assets. Conglomerates tend to include realized capital gains in net income but exclude unrealized gains and therefore do not realize losses. Mutual funds like to build an attractive record of distributions to fund holders and therefore are hesitant to realize capital losses. Many investment departments in life insurance companies hesitate to deplete surplus by taking bond losses or by taking losses on preferred stocks (which may be carried at the higher of cost or market value).

The recognition of losses, or the amortization of losses, which have not been realized would reduce the attractiveness of investing in common stocks for conglomerate accounting operations and for conservative independent life insurance companies. The desire to avoid cyclical earnings might result in the avoidance of common stocks as an investment vehicle and therefore tend to depress security values. Pressure for higher investment income might effectively limit investment in common stocks to higher-yielding stocks with less investment risk and correspondingly slower growth potential. Even in investment in common stocks, a policy of emphasis on yield would tend to depress security values.

In summary, the possibilities outlined in Mr. Case's paper will be receiving the attention of the new Financial Accounting Standards Board in the near future. Their recommendations as to proper accounting principles will have a significant effect on the method of reporting to shareholders and on corporate investment policies.

(AUTHOR'S REVIEW OF DISCUSSION)

DANIEL F. CASE:

I wish to thank Miss Dillingham and Messrs. Bierschbach, Biggs, Collins, Corbett, Lauer, and Townsend for their discussions of my paper.

Miss Dillingham and Mr. Lauer have discovered, and Miss Dillingham has demonstrated, an important refinement of the amortization method

described in the paper for bonds in good standing. According to the refined procedure, the amortization or accrual schedule which was being followed for the bond which is sold should continue to be followed, on the seller's books, until the maturity date of the bond sold. In addition, a new schedule is set up, by which an initial amount equal to the difference between the maturity value (which is not necessarily equal to the par value) and the selling price of the bond sold is accrued or amortized over the remaining lifetime of the bond sold, at the interest rate implicit in the sale price of the bond sold. If the bond sold was originally purchased at a price equal to its maturity value, the situation is the simpler one which I analyzed in the paper, and the amortization is as described in the paper.

The discovery of the above refinement is important not only because of the greater accuracy to which it could lead in accounting for bonds but also because it establishes a closer parallel between the treatment of bonds and the paper's treatment of common stocks. For bonds, with the Dillingham-Lauer refinement, we have (1) continuation of the original amortization schedule plus (2) initiation of a new schedule to amortize the difference between maturity value and selling price of the bond sold. For common stocks, from the paper, we have (1) continuation of the original schedule of amortizing any goodwill-type item which may have arisen at the original purchase of the stock being sold plus (2) initiation of a new schedule to amortize the difference between the present book value (excluding the unamortized portion of the original goodwill-type item, if any) and the sale price. The parallel exists if we consider the book value (excluding unamortized goodwill) of a common stock to be analogous to the maturity value of a bond. Such an analogy may not seem close, but it may be noted that these two values are the only meaningful values of the bond and the stock which can be objectively determined—that is, which do not depend on present or past market values or some other similarly judgmental evaluation.

Hence we have, for bond and stock alike, the difference between an objective value and the sale price being amortized and, in addition, the remaining portion of any previously arisen difference between an objective value and a sale price continuing to be amortized. Furthermore, in each case the buyer amortizes the difference between the objective value and the sale price, by means of a schedule which mirrors the schedule newly set up by the seller.

Just as the case analyzed by Mr. Lauer is more general than mine, so is Miss Dillingham's more general than Mr. Lauer's. Yet I venture to say that even hers is not fully general. One appropriate area of further ex-

ploration might involve bonds which are callable at various times at specified prices.

Mr. Lauer makes the constructive suggestion that the computational work be reduced through the adoption of a single amortization pattern for each year of maturity represented by bonds sold. Presumably, any simplification which would not materially distort the theoretically correct earnings pattern would be acceptable. I am not sure, however, that it would be appropriate to use straight-line, or zero-interest-rate amortization. It seems to me that if deferral and amortization of realized capital gains and losses on bonds were to be considered theoretically correct, then there would be no theoretical justification for making looser approximations in amortizing deferred capital gains and losses than in amortizing premiums or discounts which arise at purchase. Viewed the other way around, if straight-line amortization could be justified for deferred capital gains and losses, then I would think it could be justified for amortization of premiums and discounts arising at purchase. To my knowledge, such an approximation is not now considered appropriate for the amortization of premiums and discounts. However, I agree that some degree of approximation is probably desirable for both premiums and discounts and realized capital gains and losses. In arriving at a justifiable approximation, it would be necessary to keep in mind that an approximation in the amortization of a realized capital gain or loss might or might not be offset, in whole or in part, by a similar approximation in the amortization of the premium or discount that might be involved in the purchase of a new bond in which the sales proceeds might be reinvested.

As a second reason for suggesting straight-line amortization, Mr. Lauer presents the argument that it would recognize as much gain or loss in the first year following sale as in the year preceding the end of the amortization period. In making this point, Mr. Lauer seems to be indicating that he is not entirely comfortable with the basic goal of neutralizing the effect of the transaction on the company's reported income. Straight-line amortization would amount to a compromise between the approach of the paper and recognition of realized capital gains and losses on bonds in their entirety as they arise. While I am not prepared to debate fully the pros and cons of the paper's approach, I would like to point out that it is also true in the case of amortization of premiums or discounts involved in a purchase that the greatest amount of amortization takes place in the final year of the amortization period. Hence the implications of the paper's approach may not be as greatly counterintuitive as they might seem at first.

I think that Miss Dillingham and Mr. Lauer, in pointing out the important refinement of the paper's findings for bonds, have added greatly to the value of the paper.

I wish to thank Messrs. Biggs and Corbett, as well as Mr. Lauer, for expressing the opinion that the paper's approach should be given serious consideration. Mr. Biggs points out that there are areas other than GAAP reporting where current procedures raise problems. He presents some good examples of such problems in the present statutory accounting for life insurance companies. He also points out that the paper's approach could be applied in taxation. The tax implication of the paper's approach is that realized capital gains could be taxed, at rates applicable to ordinary income, as they were taken into a business entity's income over a period of years. For private individuals, a practical alternative might be to tax the capital gains as they arose, but at lower rates designed to give approximate effect to the difference in the timing of the tax payments. The rates might be close to the present capital gains tax rates. No distinction would be made between short-term and long-term capital gains. It must be noted, however, that the paper contemplates carrying bonds at amortized values and common stocks on the equity basis. This would, presumably, not be feasible for private persons.

Mr. Corbett, in taking pains to correct what could be described as a minor oversight in the "Response of the Joint Actuarial Committee on Financial Reporting to the August, 1972 Exposure Draft of 'Audits of Stock Life Insurance Companies,'" has exhibited the same scientific spirit that has characterized all his contributions to the study of GAAP accounting for stock life companies since his pioneering essay on natural reserves. Incidentally, Mr. Corbett's own model for studying the effect on earnings of the rollover of a bond portfolio was more ambitious than mine, in that it was designed to study the matching of income with related costs. My paper treats only the income side of the matching process. In choosing what principles to follow in determining GAAP earnings where investment income is a material factor, the accountants will have to take both sides of the picture into account.

Mr. Bierschbach points out that the deferral and amortization approach to bond capital gains and losses, which is permitted under certain circumstances by the AICPA audit guide for banks, was rejected by regulatory authorities. I was unaware of this fact when I wrote my paper, and I apologize for the omission. I hope that the rejection by the bank authorities and by some C.P.A.'s in the past will not preclude a reconsideration of the method.

Mr. Bierschbach questions the appropriateness of deferring realized capital gains on common stocks in all instances. He feels that company management deserves credit for disposing of a stock at an opportune time. I feel that there is no objective basis for saying at once that someone has exercised good judgment in disposing of a stock if he has simply sold it at an agreed-upon price in an arm's-length transaction. It seems to me that the caliber of the decision will become evident only gradually over the years, as the stock which was sold performs less well or better than people expected it to when it was sold. Under the approach of my paper, if the company reinvests the proceeds in a better-earning stock, or some other better-earning investment, the difference in earning performance will be taken into income over the years in which it becomes evident. It is, perhaps, unfortunate that the management which made the decision will not always be the one to get credit for it; but I think it is quite typical of accounting, and not inappropriate, that the wisdom of a decision does not appear immediately in the financial statements.

I do not understand Mr. Bierschbach's assertion that the methods outlined in my paper will not accomplish the goal that is being sought. The paper's goal is to neutralize the effect of each sale of a bond or a stock on the company's reported earnings. If a bond and a stock are sold simultaneously and the effect of each sale is neutralized by means of deferral and amortization, then the combined effect of the two sales will be neutralized, whether or not the two amortizations are similar. I find no problem in the fact that the amortizations will, generally, not be similar.

Mr. Bierschbach feels that if realized capital gains and losses are taken into earnings as they occur, which is his preference, then there will not be as much incentive (as under the present statutory accounting, presumably) to take capital losses on bonds in order to inflate future years' investment income. I agree, but I feel that the immediate recognition of realized capital losses constitutes an artificial incentive against sale.

Mr. Bierschbach offers reasons for treating marketable common stock holdings differently from controlling blocks of common stock. It is not clear to me what sort of treatment he is trying to justify for the marketable type. His explanation seems to imply that marketable common stocks should be carried at their market values, yet the earlier portions of his discussion suggest that he would favor carrying them at cost, realized capital gains and losses being included in income as they arise. Although I may not be responding to Mr. Bierschbach's comments, I should like to make three remarks. First, I feel that carrying securities at market value, except at the moment after they have been purchased, would be

inconsistent with historical-cost accounting as I understand it. While current-value accounting may offer some advantages, I cannot see how it could be meaningfully combined with (as distinguished from supplementing) historical-cost accounting. Second, I do have trouble seeing why the presence or absence of control through a common stock holding should influence the accounting treatment of that holding. The presence of control may influence the market value placed on the shares, but, as I have stated, I feel that the market value cannot be made the valuation basis within the context of the historical-cost accounting which is in general use today. Third, I hope that Mr. Bierschbach did not think I was suggesting that private individuals, as distinguished from business entities, account for their common stock holdings by the equity method.

I thank Mr. Bierschbach for his appreciative comment about this paper's appearance in a publication of our Society.

Mr. Collins makes three points in criticizing the paper's approach to common stocks. With regard to his first point, I must agree that the choice of assumptions would be difficult and, perhaps, arbitrary. Regarding the second point, it seems to me that reflecting the difference in value of a given investment as between the seller and the buyer before and after the exchange would constitute current-value accounting, which, as I have explained above, I feel cannot be combined in a meaningful way with historical-cost accounting.

Mr. Collins discusses the four principal approaches that have been described and presents criticisms of three of them and of the approach of my paper. I will comment briefly on the criticisms which relate to my paper. First, regarding Mr. Collins' paragraph 1, I commented on the deferral of realized capital gains and losses in my review of Mr. Bierschbach's discussion. As for unrealized gains and losses, Mr. Collins agrees that they should not be reflected in income. Second, regarding paragraph 4, the tax implications of the paper's approach may not be unfavorable, as is suggested in my review of Mr. Biggs's discussion.

Mr. Collins recommends the approach under which realized capital gains and losses are accounted for separately from income. This approach seems to me to be a way out if no more specific approach can be found. In effect, it says that we cannot determine what portion of the realized and unrealized capital gains and losses should be regarded as income; hence let us just say that income includes some amount between none of the gains and losses and all of them. If this "separate statement" approach were adopted for income presentation, then it would seem reasonable to adopt it for balance-sheet presentation as well. That is, realized and un-

realized capital appreciation, say, would be presented separately from assets. The real assets and net worth would be considered to include some unspecified portion of the appreciation. To avoid absurdly large appreciation items, and for a degree of consistency with accounting for controlling holdings, the amount taken into income each year should be the issuer's reported earnings rather than dividends paid. I see nothing wrong with the separate statement approach, except that a more specific approach, if a sound one is available, would seem to be more informative.

Mr. Collins' remarks about financial analysts are very interesting. Financial analysts are definitely necessary to provide the information and judgments that accounting cannot be expected to provide. Even if we get current-value accounting at some future date, one can hardly expect a balance sheet to furnish the value which the market will actually place on the stock. There will always be intangibles, such as the caliber of the management and, yes, the degree of control which can be exercised through the holding, which cannot be appropriately evaluated in the actual balance-sheet figures of a company.

Mr. Townsend offers many interesting insights into the thinking of many types of investors. I am not sure that differing interests of continuing and prospective stockholders should be reflected in a corporation's financial statements. Regarding the pooling of interests method, it seems eminently reasonable that the recent past histories of the two previous entities should be combined for comparison with the present performance of the merged entity. If, however, the management of one of the previous entities occupies the top positions of the merged entity, then investors will be interested in recent financial statements of the previous entity involved. In any event, the advice of a good financial analyst is especially valuable in merger situations.

Mr. Townsend points out some probable effects of adopting the approach which would recognize realized and unrealized gains and losses as income. As for my paper's approach, I have not studied the effect that it would have on reported earnings (and hence, no doubt, on investment choices). I have developed an example of the application of my paper's approach to common stocks, which I would have included in the paper itself if there had been time. The example is given in Table 1. For it I chose a 10 per cent rate of return on book value as being "reasonable," because it made the calculations easier. Choice of a higher rate would have implied a lower selling price than the $90 figure used in the illustration.

One oversight in the paper of which I have become aware is the fact that, under GAAP, when a company acquires a controlling block of stock,

TABLE 1

ILLUSTRATIVE ACCRUAL/AMORTIZATION SCHEDULE RESULTING FROM SALE OF STOCK*

Year	Tentative Assumed Book Value B.O.Y.† Reported by Issuer $=[(1)+(2)-(3)]_{-1}$ (1)	Tentative Assumed Earnings Reported by Issuer (2)	Assumed Dividends Paid by Issuer (3)	Tentative Assumed "Reasonable" Return on Book Value $=0.1\times(1)$ (4)	Tentative Assumed Excess over "Reasonable" Return $=(2)-(4)$ (5)	Accrual/ Amortization Schedule $=(5)\times 60\div 60.52$ (6)	Assumed Earnings Reported by Issuer $=(2)+(6)-(5)$ (7)	Buyer's Assumed Book Value B.O.Y.† $=[(8)+(7)-(3)-(6)]_{-1}$ (8)	Resulting Reported Earnings for Buyer as Per Cent of Book $=[(7)-(6)]\div(8)$ (9)
1	$ 30.00	$ 4.00	$2.00	$ 3.00	$ 1.00	$ 0.99	$ 3.99	$ 90.00	3.3%
2	32.00	4.50	2.25	3.20	1.30	1.29	4.49	91.00	3.5
3	34.25	5.00	2.50	3.43	1.57	1.56	4.99	91.95	3.7
4	36.75	5.50	2.75	3.68	1.82	1.80	5.48	92.88	4.0
5	39.50	6.00	3.00	3.95	2.05	2.03	5.98	93.81	4.2
6	42.50	6.50	3.25	4.25	2.25	2.23	6.48	94.76	4.5
7	45.75	7.00	3.50	4.58	2.42	2.40	6.98	95.76	4.8
8	49.25	7.50	3.75	4.93	2.57	2.55	7.48	96.84	5.1
9	53.00	8.00	4.00	5.30	2.70	2.68	7.98	98.02	5.4
10	57.00	8.50	4.25	5.70	2.80	2.78	8.48	99.32	5.7
11	61.25	9.00	4.50	6.13	2.87	2.85	8.98	100.77	6.1
12	65.75	9.50	4.75	6.58	2.92	2.89	9.47	102.40	6.4
13	70.50	10.00	5.00	7.05	2.95	2.92	9.97	104.23	6.8
14	75.50	10.50	5.25	7.55	2.95	2.92	10.47	106.28	7.1
15	80.75	11.00	5.50	8.08	2.92	2.89	10.97	108.58	7.4

* Stock with $30 book value (excluding any unamortized goodwill on seller's books at time of sale) sold for $90. Seller's "reasonable" rate of return on book value taken as 10 per cent. Accrual/amortization period taken as thirty years.

† Beginning of year.

TABLE 1—*Continued*

Year	Tentative Assumed Book Value B.O.Y.† Reported by Issuer $=[(1)+(2)-(3)]_{-1}$ (1)	Tentative Assumed Earnings Reported by Issuer (2)	Assumed Dividends Paid by Issuer (3)	Tentative Assumed "Reasonable" Return on Book Value $=0.1\times(1)$ (4)	Tentative Assumed Excess over "Reasonable" Return $=(2)-(4)$ (5)	Accrual/ Amortization Schedule $=(5)\times 60\div 60.52$ (6)	Assumed Earnings Reported by Issuer $=(2)+(6)-(5)$ (7)	Buyer's Assumed Book Value B.O.Y.† $=[(8)+(7)-(3)-(6)]_{-1}$ (8)	Resulting Reported Earnings for Buyer as Per Cent of Book $=[(7)-(6)]\div(8)$ (9)
16	$ 86.25	$11.50	$5.75	$ 8.63	$ 2.87	$ 2.85	$11.48	$111.16	7.8%
17	92.00	12.00	6.00	9.20	2.80	2.78	11.98	114.04	8.1
18	98.00	12.50	6.25	9.80	2.70	2.68	12.48	117.24	8.4
19	104.25	13.00	6.50	10.43	2.57	2.55	12.98	120.79	8.6
20	110.75	13.50	6.75	11.08	2.42	2.40	13.48	124.72	8.9
21	117.50	14.00	7.00	11.75	2.25	2.23	13.98	129.05	9.1
22	124.50	14.50	7.25	12.45	2.05	2.03	14.48	133.80	9.3
23	131.75	15.00	7.50	13.18	1.82	1.80	14.98	139.00	9.5
24	139.25	15.50	7.75	13.93	1.57	1.56	15.49	144.68	9.6
25	147.00	16.00	8.00	14.70	1.30	1.29	15.99	150.86	9.7
26	155.00	16.60	8.25	15.50	1.10	1.09	16.59	157.56	9.8
27	163.35	17.20	8.50	16.34	0.86	0.85	17.19	164.81	9.9
28	172.05	17.80	8.75	17.21	0.59	0.58	17.79	172.65	10.0
29	181.10	18.50	9.00	18.11	0.39	0.39	18.50	181.11	10.0
30	190.60	19.20	9.80	19.06	0.14	0.14	19.20	190.22	10.0
31	200.00	20.00‡	.....	20.00	.....	.....	.....	199.48	.....
Total	.....	.....	.....	.....	$60.52	$60.00	.....	.....	.....

† Beginning of year.

‡ Thereafter 10 per cent of book value at beginning of year.

it allocates portions of the purchase price to the tangible assets which it has acquired. This allocation reduces the amount of goodwill and other intangibles which must be amortized under the purchase method of accounting. It will be seen that this allocation process would not be a part of the approach of my paper. The paper's stated objective is to reproduce for the seller (subject to the performance of the seller's reinvestments) the income which the seller expects it would have reported if it had not sold the stock. Allocations on the buyer's books would, therefore, not influence the seller's accrual schedule. Meanwhile, the principle that the combined assets of the two entities should not change if no new money has been invested (which should, perhaps, have been stated as a third basic premise of the paper) requires that the buyer's amortization schedule be the mirror of the seller's accrual schedule. Of course, the buyer could go through the motions of allocating portions of the purchase price to various tangible assets. The mirror principle would then require the buyer to modify its amortization schedule to eliminate the effect of the allocations.