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BASIC FUNDING METHODS AND ACTUARIAL ASSUMPTIONS

Teaching Session

WILLIAM H. CROSSON III

A pension plan is a formalized structure for providing retirement benefits to employee groups, and there are many different methods of determining the funding costs of those retirement benefits. Some of them are acceptable under ERISA (the Employee Retirement Income Security Act of 1974) and some of them are not acceptable. I do not plan to discuss any of the non-acceptable funding methods. I will concentrate on five principal funding methods, all of which are acceptable under ERISA.

Those methods are the accrued benefit unit credit method, the entry age normal cost method, the frozen initial liability cost method, the aggregate cost method, and the attained age normal cost method. Under any of these cost methods, the primary objective is to determine funding in such a way that when a life retires the funds on hand are adequate to support his retirement benefit. This funding is accomplished, not all at once at the time of retirement, but over a period of years, usually over the entire period of participation of the employee in the pension plan. So the primary objective is to tell the employer how much he should set aside each year so that when each individual reaches his retirement age there will be adequate funds on hand to provide his benefits.

Now for something to which many actuaries object, the balance sheet. I call this an "actuarial balance summary", because there are so many contingent assets and liabilities in it that it is not really what a balance sheet should be. There is always one item in any balance sheet or balance summary which is the "fudge factor", the factor that is needed to make the balance sheet balance. Here is a generalized actuarial balance summary:

A Generalized Actuarial Balance Summary

Present Value of Pension Plan Resources				
1. Accumulated plan assets				XX
2. Present value of future funding				
a. By employees			XX	
b. By the employer				
i. Unfunded supplemental present value		XX		
ii. Future normal costs				
(I) Current year normal cost	XX			
(II) Future years normal costs	XX			
(III) Total future normal costs		XX		
iii. Total future funding by the employer			XX	
c. Total future funding				XX
3. Total present value of plan resources				XX

Present Value of Pension Plan Obligations (and Actuarial Surplus)			
4. Present value of plan obligations			
a. for Accrued benefits			
i. for Accrued vested benefits	XX		
ii. for Accrued non-vested benefits	XX		
iii. total for Accrued benefits		XX	
b. for Non-accrued benefits			
i. for Benefits to be accrued in the current year	XX		
ii. for Benefits to be accrued after the current year	XX		
iii. total for Non-Accrued benefits		XX	
c. Total for all benefits			XX
5. Actuarial surplus			XX
6. Total obligations and surplus			XX

This summary is somewhat more extensive and has much more detail than you are ever likely to report to the plan sponsor. Now I am looking at the balance sheet from the point of view of the plan, not the point of view of the employer. Accordingly any contributions by the employer that, under the cost method adopted, are expected to be paid at some time in the future, while they represent a liability to the employer, they really represent an asset or a resource to the pension plan. This generalized balance summary can apply to all cost methods. All the cost method does is tell you how to separate the present value of future funding by the employer into two portions. One is called the Unfunded Supplemental Present Value and the other is called the Present Value of Future Normal Costs. No matter what cost method you use, the sum of these two portions is fixed by the data and by your actuarial assumptions and does not depend on the actuarial cost method. If the cost method says you determine one portion in some particular way then the balance sheet principle tells you what the other portion is. In order to be specific in our discussion of actuarial cost methods, I have taken a very simple pension plan with very simple data and I have run two successive annual valuations of that plan on each of the five cost methods that we are discussing. Here is a summary description of the plan:

Illustrative Company Pension Plan

Plan Description	
Established	79-1-1
Eligibility	Age 25 and 1 year of service
Employee Contribution	None
Accrued Annual Benefit:	1.5% of salary rate as of 79-1-1 for each year of service prior thereto, after age 25 and 1 year of service +2% of aggregate salary accrued after 79-1-1 after age 25 and 1 year of service.
Normal Retirement:	Age 65, Life Annuity, Accrued Annual Benefit
Early Retirement:	None

Disability Retirement: Disabled, age 35 or over, 10 years of service, Immediate Life Annuity, Accrued Annual Benefit.

Death Benefit: None

Withdrawal Benefit: Withdrawn, age 32 or over, 10 years of service, Life Annuity deferred to age 65, Accrued Annual Benefit.

Income Benefits payable continuously

And here are the actuarial assumptions and the data:

Illustrative Company Pension Plan
Valuations as of 79/1/1 and 80/1/1

Actuarial Assumptions		Constant Annual Force
Interest		.06
Mortality:	Non-disabled, prior to 65	.01
	Non-disabled, after 65	.04
	Disabled	.10
Withdrawal		.05
Salary scale		.045
Disablement		.02

Provision for expenses: Zero
Salary assumed payable continuously and increases continuously

Data as of 79-1-1

Plan assets Zero

Employee census:

Actives:

Name	DOB	DOE	Salary Rate	Accrued Benefit
W. T. Door	34-1-1	56-1-1	$10,000	$3,000
Total 1			$10,000	$3,000

Disabled, Terminated Vested, Retired: None

Data as of 80-1-1

Contributions Paid 79-1-1 $1,730.50

Assets 1,893.47

Employee census:

Actives:

Name	DOB	DOE	Salary Rate	Accrued Benefit
W. T. Door	34-1-1	56-1-1	$11,051.71	$3,210.34
Total 1			$11,051.71	$3,210.34

Disabled, Terminated Vested, Retired: None

Applying the plan and the actuarial assumptions to the data, I have calculated all of the actuarial present values needed to perform the ten valuations we are discussing. Here are the results of these calculations:

Illustrative Company Pension Plan

Actuarial Values for W. T. Door	As of 79-1-1	As of 80-1-1
Age	45	46
Plan Entry Age	25	25
Credited Service	20	21
Present Value of Accrued Benefits:		
Retirement Benefits	$1,824.30	$2,245.58
Disability Benefits	2,515.69	2,665.88
Withdrawal Benefits	3,981.15	4,460.74
Total	$8,321.14	$9,372.20
Present Value of Non-accrued Benefits		
Retirement Benefits	$3,944.82	$4,643.01
Disability Benefits	1,246.34	1,321.15
Withdrawal Benefits	2,726.74	2,999.00
Total	$7,917.90	$8,963.16
Present Value of All Future Benefits	$16,239.04	$18,335.36
Present Value of Benefits to be Accrued in One Year		
Retirement Benefits	$124.40	$158.14
Disability Benefits	159.24	174.15
Withdrawal Benefits	259.06	299.42
Total	$542.70	$631.71
Present Value of Future Salary	$89,519.09	$97,199.86
Present Value of Salary to be Accrued in One Year	9,539.69	10,542.99
Present Value at Plan Entry Age of Future Benefits:		
Retirement Benefits	$337.42	$356.50
Disability Benefits	474.44	501.26
Withdrawal Benefits	603.25	637.36
Total	$1,415.11	$1,495.12
Present Value at Plan Entry Age of Future Salary	$41,839.41	$44,205.04

Now you will notice two items here called "Present Value at Plan Entry Age of Future Benefits" and "Present Value at Plan Entry Age of Future Salary". These items require a little bit of explanation. They are needed for an entry age normal cost valuation. Basically, you determine what the employee's age at plan entry would have been if the current plan had always been in effect, and you calculate these values as of that hypothetical entry age if the current plan had always been in effect. For a salary related plan such as this one, it is necessary to obtain the salary rate at the date of hypothetical plan entry, so as to have an appropriate foundation for projecting future benefits and salaries from that date. This information is usually not available, so the usual practice, which I have used here, is to retroject the current salary back to the hypothetical plan entry date, in accordance with the actuarial assumption as to salary scale.

Now for a discussion of the accrued benefit unit credit cost method. Under this cost method the supplemental present value is defined as the present value of accrued benefits. Thus, after you deduct the assets, you have the unfunded supplemental present value. The present value of future normal costs is the balancing amount. However, under the accrued benefit method it is not really necessary to calculate the present value of future benefits, so this calculation is rarely performed. As a result, you can hardly ever determine the present value of future normal costs, and so you cannot set up the full balance summary that I described above. You determine the present value of accrued benefits and you determine the present value of benefits to be accrued in the forthcoming year. This latter item is the normal cost, often called the current service cost. So the balance summary takes a slightly different form from the generalized form I discussed above:

Actuarial Balance Summary	
Resources	
1. Accumulated plan assets	XX
2. Unfunded supplemental present value	XX
3. Total resources	XX
Obligations (and Surplus)	
4. Accrued benefits	XX
5. Actuarial surplus	XX
6. Total obligations and surplus	XX

Now we get to the details of the valuation report. Here is a display of the 1979 valuation results for our particular plan.

Accrued Benefit Unit Credit
January 1, 1979 Valuation

A. Summary of Results

1. Current service cost (Section F)	$542.70
2. Unfunded supplemental present value (Section E)	8,321.14
3. 10-year amortization base (Section G)	8,321.14
4. Funding standard account (Initial value)	0

B. Annual Costs

	If paid 79-1-1	79-12-31
1. Maximum deductible contribution (Section H)	$1,616.72	$1,716.69
2. Minimum required contribution (Section J)	1,123.25	1,192.71

C. Actuarial Balance Summary

1. Present value of plan resources	
a. Unfunded supplemental present value (Section E)	$8,321.14
b. Total	$8,321.14
2. Present value of plan obligations	
a. Present value of accrued benefits (Section D)	$8,321.14
b. Total	$8,321.14

D. Present Value of Accrued Benefits

1. Retirement benefits	$1,824.30
2. Disability benefits	2,515.69
3. Withdrawal benefits	3,981.15
4. Total	$8,321.14

E. Unfunded Supplemental Present Value

1. Present value of accrued benefits (Supplemental present value), (Section D)	$8,321.14
2. Plan assets	0
3. Unfunded supplemental present value, (1)-(2)	$8,321.14

F. Current Service Cost

1. Retirement benefits	$124.40
2. Disability benefits	159.24
3. Withdrawal benefits	259.06
4. Total	$542.70

G. 10-year Amortization Base and Limit Adjustment

1. Date established	79-1-1
2. Base (Section E)	$8,321.14
3. Unamortized portion (Section E)	8,321.14
4. Limit adjustment, (2)x.129071, (3)	1,074.02

H. Maximum Deductible Contribution

1. Current service cost (Section F)	$ 542.70
2. Limit adjustment (Section G)	1,074.02
3. Total	$1,616.72
4. Full funding limitation (Section I)	8,863.84
5. Lesser of (3),(4)	$1,616.72
6. Minimum required contribution (Section J)	1,123.25
7. Maximum deductible contribution, Greater of (5),(6)	$1,616.72

I. Full Funding Limitation

1. Current service cost (Section F)		$ 542.70
2. Supplemental present value (Section E)		8,321.14
3. Assets		
a. Valuation basis	0	
b. Market value	0	
c. Lesser		0
4. Full funding limitation (1)+(2)-(3c)		$8,863.84

J. Minimum Required Contribution

1. Current service cost (Section F)	$ 542.70
2. Amortization charge (Section K)	580.55
3. Total	$1,123.25
4. Full funding limitation (Section I)	8,863.84
5. Minimum required contribution, Lesser of (3),(4)	$1,123.25

K. Amortization Schedule for the Funding Standard Account

Date Established	Type	Balance	Period	Factor	Amortization Charge	Net Balance	Interest	End of Year Balance
79-1-1	Initial	$8,321.14	30	.069768	$580.55	$7,740.59	$78.65	$8,219.24

I have laid out exactly the kind of information and displays that I show my clients in the annual valuation report. Now I have in my reports quite a bit of narrative and description and explanation of what the cost method is and an explanation of what the various items mean, which I have dispensed with here,but these are the numerical results pages of my valuation report.

(To save space, I am not showing any more full valuation displays for the remaining nine valuations to be discussed. I will merely show any sections that will be specially discussed.)

Under Section A, above, we see the 10-year amortization base. In May, 1978, the Treasury Department issued proposed regulation 1.404(a)-14, which lays out how you determine the ten-year amortization base, the limit adjustment, the maximum deduction and the maximum deductible contribution. The ten-year amortization base, in effect, is the initial amount of unfunded supplemental present value and it remains the same until something happens, and the things that can happen are plan changes, assumption changes, gains, losses, or a change in the actuarial cost method. Since this is the first year of the plan, the base is equal to the unfunded.

ERISA established minimum funding standards. The mechanism for keeping track and making sure that each pension plan is funded in accordance with the law is called the "funding standard account". The funding standard account starts at zero and you add to it contributions and deduct minimum charges as specified in ERISA and you also add interest or subtract interest as the case may be. So long as the funding standard account shows a positive balance (a credit balance) or a zero balance then the plan has satisfied the minimum funding standards of ERISA. Since this is the very first day of the plan you start off with a zero funding standard account.

Section B shows a maximum deduction, maximum deductible contribution and a minimum required contribution. Now usually I show some other things too at this point in my valuation reports. I show the annual cost on various amortization schedules where you contribute normal cost plus amortization of the unfunded over some number of years. Occasionally I will show normal cost plus amortization on the basis of future salaries so that the amortization payment is calculated to be a level percentage of expected future salaries. Sometimes the plan sponsor has specified a funding program, either in the plan or by separate resolution of the plan sponsor, in which case I will report my calculation of what the funding program contribution should be.

In Section G, the factor of .129071 used in determining the limit adjustment is the 10-year amortization factor on the assumption of annual payment in advance under a constant force of interest of 6%.

Textbooks often fail in describing a cost method in that they tell you how to do one year but they do not tell you how to do next year. That is a very important thing because next year is where you first see a gain or a loss and the treatment of gains and losses is a very fundamental feature of an actuarial cost method.

Now we proceed to the 1980 valuation under the accrued benefit unit credit method. Here are some of the calculation displays for this valuation:

Accrued Benefit Unit Credit
January 1, 1980 Valuation

A. Summary of Results

1.	Current service cost (Section F)	$631.71
2.	Unfunded supplemental present value (Section E)	7,478.73
3.	10-year amortization base (Section G)	8,225.43
4.	Funding standard account (Section J)	644.80
5.	Actuarial gain for the previous year (Section I)	95.71
6.	Carry-forward deduction into the current year (Section H)	113.78

B. Annual Costs

		If Paid	80-1-1	80-12-31
1.	Maximum deduction (Section K)		$1,693.38	$1,798.09
2.	Maximum deductible contribution (Section K)	1,579.60		1,677.28
3.	Minimum required contribution (Section M)	558.07		592.58

G. 10-year Amortization Base and Limit Adjustment

		79-1-1	80-1-1	Total
1.	Date established			
2.	Bases	$8,321.14	$-95.71	$8,225.43
3.	Unamortized portion, 79-1-1	8,321.14		8,321.14
4.	Limit adjustment, 79-1-1	1,074.02		1,074.02
5.	(3) w/interest to 80-1-1	8,835.69		8,835.69
6.	Deductible contributions less current service cost, w/interest to 80-1-1 (Section H), allocated per (4)	1,140.43		1,140.43
7.	Unamortized portion, 80-1-1, (5)-(6), (and Section I)	7,695.26	-95.71	7,599.55
8.	Limit adjustment, 80-1-1, (2)x.129071, ≯ in magnitude than (7)	1,074.02	-12.35	1,061.67

H. Deductible Contribution and Carry-forward Deduction

		(A) Amount	(B) w/Interest to 80-1-1
1.	Carry-forward deduction, previous year	0	0
2.	Contributions, previous year	$1,730.50	$1,837.51
3.	Total	$1,730.50	$1,837.51
4.	Maximum deduction, previous year	1,616.72	1,716.69
5.	Deductible contributionn, previous year, (5B)=(3B) ≯ (4B); (5A)=(5B)x(3A)÷(3B)	$1,616.72	$1,716.69
6.	Carry-forward deduction, current year (3)-(5)	$113.78	xxx
7.	Current service cost, previous year	542.70	576.26
8.	Deductible contribution less current service cost, (5)-(7)	xxx	$1,140.43

I. Actuarial Gain

		Amount	w/Interest to 80-1-1
1.	Unfunded supplemental present value, previous year	$8,321.14	$8,835.69
2.	Current service cost, previous year	542.70	576.26
3.	Contributions, previous year	1,730.50	1,837.51

Accrued Benefit Unit Credit (Continued)
January 1, 1980 Valuation (Continued)

	Amount	w/Interest to 80-1-1
4. Interest	441.10	-
5. Expected unfunded, current year, (1)+(2)-(3)+(4)	$7,574.44	$7,574.44
6. Unfunded supplemental present value (Section E)		7,478.73
7. Actuarial gain, (5)-(6)		$95.71

J. Funding Standard Account

	Amount	w/Interest to 80-1-1
1. Credits		
a. Balance, Previous year	0	0
b. Contributions	$1,730.50	$1,837.51
c. Interest on credits	107.01	-
d. Total credits	$1,837.51	$1,837.51
2. Charges		
a. Current service cost	$ 542.70	$ 576.26
b. Amortization charge	580.55	616.45
c. Interest on charges	69.46	-
d. Total charges	$1,192.71	$1,192.71
3. Credit balance, (1d)-(2d)	$ 664.80	$ 664.80

K. Maximum Deduction and Maximum Deductible Contribution

1. Current service cost (Section F)	$ 631.71
2. Limit adjustment (Section G)	1,061.67
3. Total	$1,693.38
4. Full funding limitation (Section L)	8,224.22
5. Lesser of (3),(4)	$1,693.38
6. Minimum required contribution (Section M)	558.07
7. Maximum deduction, Greater of (5),(6)	$1,693.38
8. Carry-forward deduction (Section H)	113.78
9. Maximum deductible contribution, (7)-(8)	$1,579.60

L. Full Funding Limitation

1. Current service cost (Section F)		$ 631.71
2. Supplemental present value (Section E)		9,372.20
3. Assets		
a. Valuation basis	$1,893.47	
b. Market value	1,893.47	
c. Lesser		1,893.47
4. Full funding limitation for the funding standard account, (1)+(2)-(3c)		$8,110.44
5. Carry-forward deduction (Section H)		113.78
6. Full funding limitation, (4)+(5)		$8,224.22

M. Minimum Required Contribution

1. Current service cost (Section F)	$631.71
2. Amortization charge (Section N)	580.55
3. Amortization credit (Section N)	9.39
4. Funding standard account (Section J)	644.80
5. (1)+(2)-(3)-(4)	$558.07
6. Full funding limitation (Section L)	8,110.44
7. Minimum required contribution, Lesser of (5),(6)	$558.07

N. Amortization Schedule for the Funding Standard Account

Date Established	Type	Balance	Period	Factor	Amortization	Net Balance	Interest	End of Year Balance
	Amortization Charge							
79-1-1	Initial	$8,219.24	29	.070633	$580.55	$7,638.69	$472.35	$8,111.04
	Amortization Credit							
80-1-1	Gain	$95.71	15	.098134	$9.39	$86.32	$5.34	$91.66

O. Analysis of Actuarial Gain

In Section A we see a carry-forward deduction into the current year, which means the plan sponsor contributed so much that he could not take a deduction for the full amount of contribution, and of the total amount he contributed, $113.78 was not deductible in 1979 and is a carry-forward as a possible deduction in 1980 or some later year. In Section B we have a maximum deduction which is larger than the maximum deductible contribution by this $113.78.

In Section G, since we have an actuarial gain during 1979 of $95.71, in the ten-year amortization base we set up that gain as an additional base of minus $95.71. If you had more than one portion of the base from the previous year with an unamortized balance you would allocate the excess of deductible contributions over normal cost in proportion to the limit adjustment from last year. This is not shown because it would have necessitated showing three successive valuations. In determining the limit adjustment the "not greater" sign means "not greater in magnitude", or "not greater in absolute value". Section H analyzes the contribution between deductible and non-deductible. This is a very simple matter in this case because the contributions happen to have been made on the valuation date. But if the contributions are made off the valuation date it becomes a tricky little problem to make this analysis. The scheme shown here represents my own solution to the problem.

Section I shows the development of the actuarial gain. Based on last years valuation, on the assumption that all of your actuarial assumptions are realized and on the actual plan contributions you calculate what the expected unfunded should be. Compare that expected unfunded with the actual unfunded and the difference is the gain or loss. If the actual unfunded is less than your expected you have a gain. If the actual unfunded is larger then expected you have a loss.

Section J displays the funding standard account. The amount column should agree with the information reported on Schedule B (Form 5500).

Section K shows the maximum deduction and the maximum deductible contribution as distinct items, because of the carry-forward deduction.

Section L recognizes that the full funding limitation to be used in determining the full funding limitation credit to the funding standard account is different from the full funding limitation to be used in determining the maximum deduction, due to the carry-forward deduction.

In Section M we now have an amortization credit arising from the actuarial gain, with the amortization schedule shown in Section N.

Section O would normally present a source analysis of actuarial gain. I have omitted this analysis as being beyond the scope of the lecture.

Now for the entry age normal cost method. In this method you concentrate on the normal cost. You determine what the cost is, life by life, as a level amount from hypothetical plan entry to termination of service. For salary related plans it is usual to relate the normal cost, not as a level amount per life, but as a level percentage of salary.

Since this is a salaried plan I determine a normal cost ratio for each employee, which is the value at his plan entry of future benefits less future employee contributions divided by the value at his plan entry of future salary. That calculation is done life by life so that for each life you have a normal cost ratio. I then apply each life's normal cost ratio to his current salary for current normal cost life by life. Then I total those normal costs and I get the total normal cost for the plan. I relate that total normal cost to the total salary for the plan to obtain an average normal cost ratio for the plan, which I then apply to the present value of all future salaries to obtain the present value of future normal costs. I do not display the normal cost calculation in this form in my valuation reports because to do so would require listing and showing the calculation for each employee. After determining the average normal cost ratio I apply it back against the total present salary to arrive at the total normal cost (which usually differs due to rounding from the aggregate of the individually calculated normal costs).

If the plan is not related to salary, you do these calculations on the basis of a level normal cost per life so you have a normal cost for each life which is totalled to obtain the total normal cost; divide by the number of lives for the average normal cost per life; and apply that to the present value of $1.00 per year of future service to obtain the present value of future normal costs. Here I display the mathematical formulas corresponding to the foregoing.

$$NCR_i = \frac{PVEFB_i - PVEFEeC_i}{PVEFS_i}$$

$$NC_i = NCR_i \cdot S_i$$

$$NC = \sum_i NC_i$$

$$S = \sum_i S_i$$

$$NCR = \frac{NC}{S}$$

$$PVFNC = NCR \cdot PVFS$$

$$SPV = PVFB - PVFNC - PVFEeC$$

$$USPV = SPV - A$$

$$NC = NCR \cdot S$$

Here is a typical balance summary under the entry age normal cost method:

Actuarial Balance Summary

Resources	
1. Accumulated plan assets (A)	XX
2. Unfunded supplemental present value (USPV)	XX
3. Present value of future normal costs (PVFNC)	XX
4. Present value of future employee contributions (PVFEeC)	XX
5. Total resources	XX

Obligations (and Surplus)	
6. Present value of future benefits (PVFB)	XX
7. Actuarial surplus (AS)	XX
8. Total obligation and surplus	XX

Now we have two successive valuations on entry age normal cost. Here are some of the sections in the 1979 report:

Entry Age Normal
January 1, 1979 Valuation

E. Unfunded Supplemental Present Value	
1. Present value at entry of future benefits	$ 1,415.11
2. Present value at entry of future salaries	41,839.41
3. Normal cost ratio, (1)÷(2)	3.382%
4. Present value of future salaries	$89,519.09
5. Present value of future normal costs, (3)x(4)	$ 3,027.54
6. Present value of future benefits (Section D)	16,239.04
7. Supplemental present value, (6)-(5)	$13,211.50
8. Assets	0
9. Unfunded supplemental present value, (7)-(8)	$13,211.50

F. Normal Cost	
1. Normal cost ratio (Section E)	3.382%
2. Present value of salary to be accrued in one year	$9,539.69
3. Normal cost, (1)x(2)	$322.63

J. Minimum Required Contribution	
1. Normal cost (Section F)	$322.63
2. Amortization charge (Section K)	921.74
3. Total	$1,244.37
4. Full funding limitation (Section I)	13,534.13
5. Lesser of (3),(4)	$1,244.37
6. Per the alternative minimum funding standard (Section L)	8,643.77
7. Minimum required contribution, Lesser of (5),(6)	$1,244.37

L. Minimum Required Contribution Per the Alternative Minimum Funding Standard	
1. Normal cost (Section F)	$322.63
2. Current service cost	542.70
3. Lesser of (1),(2)	$322.63
4. Present value of accrued benefits (Section D)	$8,321.14
5. Market value of assets	0
6. Excess, if any, of (4) over (5)	$8,321.14
7. Minimum required contribution, (3)+(6)	8,643.77

In Section E I actually show the development of the normal cost ratio, since there is only one life. Normally I would start with line 3.

In Section F, normal cost is calculated by applying the normal cost ratio to the present salary.

Now for Section L, and line 6 in Section J. When you use the entry age normal cost method, ERISA says that you have another option for determining the minimum required contribution so that you now have three tests for minimum required contribution. Your regular method is normal cost plus amor-

tization charges less amortization credits, less the balance in the funding standard account. The second test is the full funding limitation, and the third test is the alternative minimum funding standard account. The third test is to allow you to substitute an accrued benefit calculation, with full immediate funding of the unfunded, in place of the entry age calculation if the plan can thereby reduce its minimum required contribution. In my experience, I have found the alternative minimum funding standard to be quite useless, and thus I do not normally determine the minimum required contribution on that basis.

Now here are some of the sections from the 1980 valuation report:

Entry Age Normal
January 1, 1980 Valuation

G. 10-year Amortization Base and Limit Adjustment

1.	Date established	79-1-1	80-1-1	Total
2.	Base	$13,211.50	$621.07	$13,832.57
3.	Unamortized portion, 79-1-1	13,211.50		13,211.50
4.	Limit adjustment, 79-1-1	1,705.22		1,705.22
5.	(3) w/interest to 80-1-1	14,028.45		14,028.45
6.	Contributions less normal cost, w/interest to 80-1-1 (Section H), allocated per (4)	1,494.93		1,494.93
7.	Unamortized portion, 80-1-1, (5)-(6) (and Section H)	$12,533.52	$621.07	$13,154.59
8.	Limit adjustment, (2)x.129071, (7)	1,705.22	80.16	1,785.38

M. Amortization Schedule for the Funding Standard Account

Amortization Charges

Date Established	Type	Balance	Period	Factor	Amortization	Net Balance	Interest	End of Year Balance
79-1-1	Initial	$13,049.72	29	.070633	$921.74	$12,127.98	$749.95	$12,977.93
80-1-1	Loss	621.07	15	.098134	60.95	560.12	34.64	594.76
	Total	$13,670.79			$982.69	$12,688.10	$784.59	$13,472.69

It turns out that there was an actuarial loss, so in Section G you have a plus to the base of $621.07, rather than a minus. In Section M, since there was a loss instead of a gain you have two amortization charges in your amortization schedule.

We now turn to the frozen initial liability method, which is really the frozen initial liability variation of the entry age normal cost method. But it is a very fundamental variation; it is so fundamentally different it is a different cost method completely. Now how does it differ from entry age normal? Entry age normal concentrates on normal cost and lets the unfunded be the balancing item. Frozen initial concentrates on the unfunded, and lets the normal costs be the balancing item. Under the frozen initial liability method, in the first valuation you use the entry age method, and in each subsequent valuation you calculate the expected unfunded, which is what the unfunded would be if there were no actuarial gain nor loss, and you set the unfunded in each year equal to the expected unfunded brought forward from the previous year. Since "gain" or "loss" is defined as the difference between the expected unfunded and the actual unfunded then you have eliminated any direct recognition of gains or losses. Of course, since you keep gains and losses out of your unfunded they automatically appear in the present

value of future normal cost, but not explicitly identified.

As I said, in the first valuation, the method is identical to the entry age normal cost method. It is in the second valuation there is a difference. What you do is you bring forward the unfunded with interest and you deduct the excess of contributions over normal cost with interest and that is your unfunded. To the unfunded you add the assets to obtain the supplemental present value and then the present value of future normal cost is the balancing item. The present value of future benefits less the present value of future employee contributions, less the supplemental present value gives the present value of future normal cost, which is then related to salary for the normal cost ratio, which is applied to current salary to arrive at the current normal cost.

Here is the arithmetic after the first valuation:

$$USPV_1 = (USPV_0)(1+i) - C_0^i - NC_0(1+i)$$

$$SPV = A + USPV$$

$$PVFNC = PVFB - SPV - PVFEeC$$

$$NCR = \frac{PVFNC}{PVFS}$$

$$NC = NCR \cdot {}^{S}$$

The January 1, 1979 valuation is identical to January 1, 1979 valuation under the entry age normal cost method. Here is the January 1, 1980 valuation:

Frozen Initial Liability
January 1, 1980 Valuation

E. Unfunded Supplemental Present Value

		Amount	w/Interest to 80-1-1
1.	Balance, previous year	$13,211.50	$14,028.45
2.	Normal cost, previous year	$ 322.63	$ 342.58
3.	Contribution, previous year	1,730.50	1,837.51
4.	(3)-(2)	$ 1,407.87	$ 1,494.93
5.	Interest	729.89	-
6.	Balance, current year, (1)-(4)+(5)	$12,533.52	$12,533.52

F. Normal cost

1.	Present value of future benefits (Section D)	$18,335.36
2.	Unfunded supplemental present value (Section E)	$12,533.52
3.	Assets	1,893.47
4.	Supplemental present value, (2)+(3)	$14,426.99
5.	Present value of future normal costs, (1)-(4)	$ 3,908.37
6.	Present value of future salaries	97,199.86
7.	Normal cost ratio, (5)÷(6)	4.021%
8.	Present value of salary to be accrued in one year	10,542.99
9.	Normal cost, (7)x(8)	$423.93

J. Full Funding Limitation

1.	Present value at entry of future benefits	$1,495.12
2.	Present value at entry of future salaries	44,205.04
3.	Normal cost ratio (Entry age normal), (1)÷(2)	3.382%

Frozen Initial Liability (Continued)
January 1, 1980 Valuation (Continued)

4.	Present value of salary to be accrued in one year		$10,542.99
5.	Normal cost (Entry age normal), (3)x(4)		$356.56
6.	Present value of future salaries		$97,199.86
7.	Present value of future normal costs (Entry age normal), (3)x(6)		$ 3,287.30
8.	Present value of future benefits (Section D)		18,335.36
9.	Supplemental present value (Entry age Normal), (8)-(7)		$15,048.06
10.	Assets		
	a. Valuation basis	$1,893.47	
	b. Market value	1,893.47	
	c. Lesser		1,893.47
11.	Full funding limitation, (5)+(9)-(10c)		$13,511.15

M. Analysis of Change in Normal Cost From the Previous Valuation Date

		P.V. of Exp. Sal. for 1 Yr.	Normal Cost Ratio	Normal Cost	P.V. of Future Normal Cost
1.	Previous valuation	$ 9,539.69	3.382%	$322.63	$3,027.54
2.	Changes in salaries	+1,003.30	-	+33.93	+259.76
3.	Sub-total	$10,542.99	3.382%	$356.56	$3,287.30
4.	Changes in normal cost ratio (Subject to analysis by cause)		+0.639	+67.37	+621.07
5.	Total		4.021%	$423.93	$3,908.37

Section E looks like a gain or loss calculation, except that now the expected is the actual.

Section F here is the heart of the valuation, and follows the arithmetic displayed above.

Now look at Section J, the full funding limitation. What has been done here is to calculate the full funding limitation as if the cost method is the entry age normal cost method. ERISA says, if I may paraphase, if your cost method is such that you cannot directly determine the unfunded supplemental present value then in determining the full funding limitation you must use the entry age normal method. When I first read that, I said to myself what could be more direct then bringing forward last year's unfunded and adjusting for excess contributions? I have now decided that that is not really what ERISA intends. I believe that the law really means that if you do not use the entry age normal cost method nor the accrued benefit method for determining your costs then you must determine the full funding limitation by using the entry age normal cost method.

A new item here is Section M, the analysis of change in normal cost from the previous valuation date. You have no gain or loss, as such, on frozen initial, but yet your normal cost changes in reflection of the gain or loss and so I attempt to account for the change in normal cost. What I am really doing is an analysis of true gains and losses. Section M shows the scheme I use for displaying the analysis of change in normal cost and it seems to work fairly well. I just lay out the previous valuation results and show the effect of changes in salaries under the artificial assumption

that no matter what change in salary there was there was no actuarial gain or loss. In other words, continuing the same normal cost ratio on the new salary accounts for part of the change in normal cost, that is, the change in normal cost there would have been if there had been no "gain or loss", and then the balance of the change in normal cost is due to gains or losses. Normally I show an analysis of line 4 by source.

The aggregate cost method allocates future employer contribution between unfunded supplemental present value and future normal costs in a very simple way. The aggregate cost method in effect says the unfunded supplemental present value is zero so that you allocate all to the present value of future normal costs. The present value of future normal cost is determined as the present value of future benefits less the assets, less the future employee contributions, of course. In other words, whatever has not been funded yet, or is not going to be funded by the employees, is normal cost or future normal cost under the aggregate method. So the normal cost rate is that present value divided by the present value of future salaries and the normal cost in any one year is obtained by multiplying the normal cost rate by the salaries. Here are the mathematics and the balance summary:

$$PVFNC = PVFB - A - PVFEeC$$

$$NCR = \frac{PVFNC}{PVFS}$$

$$NC = NCR \times S$$

Actuarial Balance Summary

Resources	
1. Accumulated Plan Assets	XX
2. Present Value of Future Normal Costs	XX
3. Present Value of Future Employee Contributions	XX
4. Total Resources	XX
Obligations (and Surplus)	
5. Present value future benefits	XX
6. Actuarial Surplus	XX
7. Total	XX

Here is the valuation report for our illustrative plan.

Aggregate
January 1, 1979 Valuation

A. Summary of Results

1. Normal cost (Section E)	$1,730.50
2. Funding standard account (Initial value)	0

B. Annual Costs

	If paid 79-1-1	79-12-31
1. Maximum deductible contribution (Section F)	$1,730.50	$1,837.51
2. Minimum required contribution (Section H)	1,730.50	1,837.51

Aggregate (Continued)
January 1, 1979 Valuation

C. Actuarial Balance Summary

1.	Present value of plan resources	
	a. Future normal costs (Section F)	$16,239.04
	b. Total	$16,239.04
2.	Present value of plan obligations	
	a. Present value of future benefits (Section D)	$16,239.04
	b. Total	$16,239.04

E. Normal Cost

1.	Present value of future benefits (Section D)	$16,239.04
2.	Assets	0
3.	Present value of future normal costs, (1)-(2)	$16,239.04
4.	Present value of future salaries	89,519.09
5.	Normal cost ratio, (3)÷(4)	18.140%
6.	Present value of salary to be accrued in one year	$9,539.69
7.	Normal cost, (5)x(6)	$1,730.50

G. Full Funding Limitation

1.	Present value at entry of future benefits		$ 1,415.11
2.	Present value at entry of future salaries		41,839.41
3.	Normal cost ratio (Entry age normal), (1)÷(2)		3.382%
4.	Present value of salary to be accrued in one year		9,539.69
5.	Normal cost (Entry age normal), (3)x(4)		$322.63
6.	Present value of future salaries		89,519.09
7.	Present value of future normal costs (Entry age normal), (3)x(6)		$ 3,027.54
8.	Present value of future benefits (Section D)		16,239.04
9.	Supplemental present value (Entry age normal), (8)-(7)		$13,211.50
10.	Assets		
	a. Valuation basis	0	
	b. Market value	0	
	c. Lesser		0
11.	Full funding limitation, (5)+(9)-(10c)		$13,534.13

Section A is quite simple. In Section B, the maximum deductible contribution and the minimum required contribution are exactly the same. The balance summary, Section C, is quite simple because there are no assets at this time. Section E shows the details of the calculation of the normal cost as discussed above. Since the aggregate cost method does not develop an accrued liability in any meaningful sense, you must use the entry age normal method for calculating the full funding limitation, as shown in Section G.

Here is the second valuation on the aggregate method:

Aggregate
January 1, 1980 Valuation

F. Funding Standard Account

		Amount	with Interest to 80-1-1
1.	Credits		
	a. Balance, previous year	0	0
	b. Contributions	$1,730.50	$1,837.51
	c. Interest on credits	107.01	-
	d. Total credits	$1,837.51	$1,837.51

Aggregate (Continued)
January 1, 1980 Valuation (Continued)

	Amount	with Interest to 80-1-1
2. Charges		
a. Normal cost	$1,730.50	$1,837.51
b. Interest on charges	107.01	-
c. Total charges	$1,837.51	$1,837.51
3. Credit balance, (1d)-(2c)	0	0

J. Analysis of Change in Normal Cost from the Previous Valuation Date

	P.V. of Exp. Sal. for 1 yr.	Normal Cost Ratio	Normal Cost	P.V. of Future Normal Cost
1. Previous valuation	$ 9,539.69	18.140%	$1,730.50	$16,239.04
2. Changes in salaries	+1,003.30	-	+182.00	+1,393.01
3. Sub-total	$10,542.99	18.140%	$1,912.50	$17,632.05
4. Changes in normal cost ratio (Subject to analysis by cause)		-1.224	-129.05	-1,190.16
5. Total		16.916%	$1,783.45	$16,441.89

We see in Section F that we have a zero funding standard account. Under the aggregate method, so long as you contribute the minimum and do not go over the maximum you will always have a zero balance in the funding standard account because there are no amortization charges to charge, there are no amortization credits to credit, the contributions always are equal to normal cost with interest so your credits are always going to cancel your charges and vice versa.

In Section J we have the analysis of the change in normal cost from the previous valuation date. Under the aggregate method you cannot really determine the gain or loss which is being reflected in the valuation. Under any of the other methods you can but you cannot under the aggregate, at least in any direct way. For example, in line 4 we have a decrease in the normal cost ratio of 1.224%. This line usually displays the change in the normal cost ratio due to actuarial gain or loss. But under the aggregate method it is not the gain or loss and the reason is this: Under the aggregate method, even if you had no gains or losses, you would still have a non-zero entry at this point. Under the aggregate cost method, in the absence of gains or losses, the aggregate cost ratio will start out high and will decrease as time goes by because you are building up assets and so this line really is not an actuarial gain or loss line.

Now for the attained age normal cost method. Under this method, in the first valuation, you determine the unfunded supplemental present value in accordance with the accrued benefit unit credit method. Having determined the unfunded in the first valuation in this way, from that point on you apply the frozen initial liability method, using the value of the projected benefits. Thus, the present value of all future benefits is recognized in the normal cost while only the value of the accrued benefits is recognized in the unfunded supplemental present value.

Here are the mathematics of this method:

1st Valuation:

$$SPV = PVAB$$
$$USPV = PVAB - A$$
$$PVFNC = PVFB - SPV - PVFEeC$$
$$NCR = \frac{PVFNC}{PVFS}$$

Thereafter:

$$NC = NCR \times S$$
$$USPV_1 = USPV_0(1+i) - \left[C_0^i - NC_0(1+i)\right]$$
$$SPV = USPV+A$$
$$PVFNC = PVFB - SPV - PVFEeC$$
$$NCR = \frac{PVFNC}{PVFS}$$
$$NC = NCR \times S$$

Here are some sections from the first valuation:

Attained Age Normal
January 1, 1979 Valuation

E.	Unfunded Supplemental Present Value	
1.	Present value of accrued benefits (Section D)	$8,321.14
2.	Assets	0
3.	Unfunded supplemental present value, (1)-(2)	$8,321.14
F.	Normal Cost	
1.	Present value of future benefits (Section D)	$16,239.04
2.	Unfunded supplemental present value (Section E)	$8,321.14
3.	Assets	0
4.	Supplemental present value, (2)+(3)	$8,321.14
5.	Present value of future normal costs, (1)-(4)	$7,917.90
6.	Present value of future salaries	89,519.09
7.	Normal cost ratio, (5)÷(6)	8.845%
8.	Present value of salary to be accrued in one year	$9,539.69
9.	Normal cost, (7)x(8)	$843.79
I.	Full Funding Limitation	
1.	Normal cost (Section F)	$843.79
2.	Supplemental present value (Section F)	8,321.14
3.	Assets	
	a. Valuation basis 0	
	b. Market value 0	
	c. Lesser	0
4.	Full funding limitation, (1)+(2)-(3c)	$9,164.93

It is no surprise that the unfunded supplemental present value in the first valuation turns out to be exactly what it was under the accrued benefit unit credit method.

There is a little problem in determining the full funding limitation, a problem of interpretation. Normally you cannot determine the accrued liability or the supplemental present value directly under a frozen initial method such as the attained age normal method. However, in the first valuation under attained age normal you do determine the supplemental present value quite directly by measuring the present value of accrued benefits. Thus,in the first valuation I calculate the full funding limitation on the attained age normal method. In the next valuation, when we come to it, we

cannot apply that reasoning any longer so we are back to using the entry age normal method for the full funding limitation.

Here is the second valuation under the attained age normal method.

Attained Age Normal
January 1, 1980 Valuation
E. Unfunded Supplemental Present Value

		Amount	w/Interest to 80-1-1
1.	Balance, previous year	$8,321.14	$8,835.69
2.	Normal cost, previous year	$843.79	$895.97
3.	Contributions, previous year	1,730.50	1,837.51
4.	(3)-(2)	$886.71	$941.54
5.	Interest	459.72	-
6.	Balance, current year, (1)-(4)+(5)	$7,894.15	$7,894.15

J. Full Funding Limitation

1.	Present value at entry of future benefits		$1,495.12
2.	Present value at entry of future salaries		44,205.04
3.	Normal cost ratio (Entry age normal), (1)÷(2)		3.382%
4.	Present value of salary to be accrued in one year		$10,542.99
5.	Normal cost (Entry age normal), (3)x(4)		$356.56
6.	Present value of future salaries		97,199.86
7.	Present value of future normal costs (Entry age normal), (3)x(6)		$3,287.30
8.	Present value of future benefits (Section D)		18,335.36
9.	Supplemental present value (Entry age normal), (8)-(7)		$15,048.06
10.	Assets		
	a. Valuation basis	$1,893.47	
	b. Market value	1,893.47	
	c. Lesser		1,893.47
11.	Full funding limitation, (5)+(9)-(10c)		$13,511.15

Now we will go on to the second valuation, and there is nothing really startling here. You can see that the unfunded is brought forward in accordance under frozen initial liability principles. The full funding limitation is then calculated using the entry age normal method rather than by using the attained age normal method.

How do you handle a plan change? For the purpose of Schedule B you must determine the effect of the change on the unfunded supplemental present value because you must set up a new portion of an amortization schedule for the minimum funding requirements. On the accrued benefit unit credit method, and on the entry age normal cost method, you do a valuation before the change; you do another one after the change, and the difference in unfunded between those two valuations is the effect of the change.

Under the aggregate method the unfunded before is zero, the unfunded after is zero, so the effect of the change is zero. It is the normal cost that changes to reflect the effect of the change in the plan or assumptions.

For the frozen initial liability variation of the entry age method and the attained age normal method, it is now quite clear that you must calculate

the effect of the change on the unfunded by calculating the unfunded before and the unfunded after on the basic cost method which was used initially on your frozen method. For example, on the frozen initial liability variation of the entry age method you would do new entry age calculations before and after the change and the difference in the unfunded on those two valuations would adjust your accumulated unfunded under the frozen initial method. On the attained age normal you do the same thing except that you use the accrued benefit method to calculate the unfunded before and the unfunded after. You take the difference and add it to your brought-forward unfunded on the attained age normal method.