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VALUATION OF ANCILLARY BENEFITS

Teaching Session

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I. General

The purpose of an actuarial valuation of a pension plan is to estimate the cost of the benefits promised by the plan. A mathematical model of the population of plan beneficiaries is projected into the future, reflecting emerging eligibilities for promised benefits and the actuary's assumptions as to probable future experience.

In this teaching session we will review the treatment of ancillary benefits (i.e., other than retirement) benefits in such a valuation. It is not meant as an exhaustive treatment of the subject.

II. Definitions

The following definitions will apply. The ages and values refer to those of a single participant being costed.

v_1 = The discount factor based on the valuation interest rate assumed for active participants.

v_2 = The discount factor based on the valuation interest rate assumed for non-active participants.

JE = Entry age.

JY = Attained age.

JZ = Assumed retirement age, or, if probabilities are used, the first age at which this participant's probability of retiring is 100%.

$l^{(aa)}_{JY}$ = Pre-retirement multiple-decrement service table "number living".

$q^{(aK)}_{JI}$ = The probability of exiting at age JI from the active life status from cause K, where cause K may be any of the following:

Retirement
Death before retirement
Disablement
Other termination

Any of the these may represent a family of probabilities. For instance, death and disability may be service-connected (or line-of-duty) or not. One or more of them may be select and ultimate.

When absolute rates are furnished, they must be converted into probabilities. See "The Construction of a Multiple Decrement Table from the Associated Single Decrement Tables--General Case" by Franklin C. Smith, Proceedings of the Conference of Actuaries in Public Practice, XXV, page 450.

$d^{(aK)}_{JI}$ = Number exiting active status at age JI.

$= {}_{1}^{(aa)}_{JI} \; q^{(aK)}_{JI}$

Note that $l^{(aa)}_{JI+1} = l^{(aa)}_{JI} - \sum_{K} d^{(aK)}_{JI}$

Note further that replacing the superscripts (aa) and (aK) with (r) or (i) describes the single decrement table for post-retirement mortality or post-disablement mortality, respectively.

$BFT_{JI,K,t}$ = The benefit if the participant should leave the active status from decrement cause K at age JI. The benefit is unique to this participant, and may be a function of salary history (actual or estimated), accrued service, service under a prior plan, social security integration criteria -- in short, a benefit of any desired complexity.

More than one benefit can arise from cause K at age JI. For example, a participant who becomes disabled at age 50 might receive benefit 1 to age 62, benefit 2 from age 62 to age 65 and benefit 3 from age 65 on. The "t"-th such benefit from cause K at age JI of "n" such benefits will be denoted as $BFT_{JI,K,t}$.

$MV_{JI,K,t}$ = The maturity value associated with the corresponding BFT. The mortality underlying the maturity value may be post-retirement or post-disablement. It may be the mortality expected to be experienced by the spouse rather than the participant, or it may be based on the joint and survivor mortality structure. Which it is depends on the expected mortality basis of the entity scheduled to receive the benefit.

More detail will be given on the maturity values in a later section of this note.

PVFB = Present value of all future benefits for the participant.

PVPB = Present value of all <u>past</u> benefits for the participant.

NC = Entry age normal cost for all past and future benefits for the participant. It may be level in dollars or a constant percent of pay.

PVFNC = Present value of future entry age normal costs for all past and future benefits for the participant.

AL = Accrued liability under the entry age normal cost method for all past and future benefits for the participant.

= PVFB - PVFNC

s_{JI} = Salary scale factor at age JI.

$D_{JI}^{(aa)} = v_1^{JI} \; l_{JI}^{(aa)}$

$s_{N_{JI}}^{(aa)} = \sum_{j=JI} s_j \; D_j^{(aa)}$

III. Formulas

The formulas herein produce the accrued liability and normal cost for the participant under the entry age normal cost method. Other cost methods can readily be determined from the models.

The formula for the present value of future benefits is:

$$PVFB = \sum_{JI=JY}^{JZ-1} v_1^{JI-JY} \sum_{K} (d_{JI}^{(aK)} / l_{JY}^{(aa)}) \sum_{t=1}^{n} (BFT_{JI,K,t} \; MV_{JI,K,t})$$

The formula for the present value of past benefits (PVPB) is identical to this formula except that the first summation is over the range JI = JE to JY-1 instead of JY to JZ-1.

The formulas for the entry age normal cost and the present value of future entry age normal costs, both at age JY, are:

$$NC = (PVFB + PVPB) \; (s_{JY} \; D_{JY}^{(aa)}) \; / \; (s_{N_{JE}}^{(aa)} - s_{N_{JZ}}^{(aa)})$$

$$PVFNC = NC \; (s_{N_{JY}}^{(aa)} - s_{N_{JZ}}^{(aa)}) \; / \; (s_{JY} \; D_{JY}^{(aa)})$$

The accrued liability, of course, is simply PVFB - PVFNC.

IV. Maturity Values

It is reasonable to assume that ancillary decrements will occur on the average half-way through each future plan year. Thus, the probability that a participant will quit during the year of age JI implies that he will remain active until age JI+½ at which precise moment he will (perhaps) quit.

If our participant should quit in the year of age JI with a benefit deferred to age 65, the formula for the maturity value, using post-retirement mortality and discounted at interest from age JI+½ to age JI, would be:

$$MV_{JI,quit} = v_1^{½} \; \ddot{a}_{65}^{(12)} \; v_2^{65-JI-½} \; l_{65}^{(r)} \; / \; l_{JI+½}^{(r)}$$

or, preferably, the more readily computable

$$= F \; \ddot{a}_{65}^{(12)} \; D_{65}^{(r)} \; / \; (\; ½ \; v_2^{JI} \; (\; l_{JI}^{(r)} + l_{JI+1}^{(r)} \;))$$

where F is the following constant.

$$F = (v_1^{½} \; / \; v_2^{½})$$

The formula for the maturity value will vary according to the characteristics of the benefit promised. The mortality (if any) used in computing the maturity value will depend on the expected mortality of the recipient, be it healthy or disabled participant, spouse or joint lifetime.