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TOWARD ADJUSTABLE INDIVIDUAL LIFE POLICIES

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ABSTRACT

This paper develops the direct application of the computer to the creation of individual life policies. A policy at original issue will correspond to a typical ratebook policy, except that a much wider range of plans of insurance may be used. The policy is designed to permit any number of changes in status after issue, subject to appropriate underwriting or exercise of options guaranteeing insurability. In any status the gross premium and amount of insurance are level. When a status is changed, the amount of insurance may be increased or decreased, the premium may be increased or decreased, and the plan of insurance may be changed.

Plans of insurance fall into five categories: term, limited pay whole life, continuous premium endowment, limited pay endowment, and income endowment where the ratio of amount of insurance to maturity value may be varied. Term expirations or endowment maturities may be at any age; limited premium payment periods may run to any age.

When changes are made, policy provisions remain unchanged. A new page for the policy prepared by the computer supplies all figures and data for the new status.

It is obvious that a single original policy may be built up to cover changing needs of the insured with complete flexibility. In this respect it meets the industry concept of a "life-cycle" policy.

In addition to developing the formulas necessary for computer programming and providing numerical illustrations, the paper discusses conformity of the adjustable policy to current insurance law, underwriting, substandard issues, dividends, policy guarantees, valuation, agents' compensation, and sales illustrations. The paper concludes with comment on the adaptability of the policy to long-term service by the agent.

I. INTRODUCTION

THE ideas developed in this paper were first suggested near the beginning of the computer age, in 1947, by Edward A. Rieder in discussing Edmund C. Berkeley's paper "Electronic Machinery for Handling Information and Its Uses in Insurance." "If the new electronic machinery," wrote Mr. Rieder, "with its tremendous computing and memory capacity had been available from the outset, we might have developed life contracts and procedures along entirely different lines . . . it might have been possible to design one policy which would have been flexible enough to meet every policyholder's insurance needs for the rest of his lifetime" (*TASA*, XLVIII, 283).

Seventeen years later, Alfred N. Guertin, in his last "Report of the Actuary" to the American Life Convention, alluded more forcibly to the same idea:

> Our business has had to limit the forms and terms of its policies to certain plan types. One of the reasons for this has been the enormous labor and expense of computations associated with special forms and the more exotic plans. Has not the electronic computer freed us from such limitations of size and form? Can we not foresee custom-made products available to the individual policyholder, independent of policy plan as such? . . . I am referring to the ultimate contract of protection containing a complete program specifically modeled for the individual—something the skillful underwriter does today through a combination of various plans, riders and special settlement options. I would envision a contract of protection susceptible of alteration from time to time, in accordance with the changing needs of the policyholder ["Report of the Actuary," *Proceedings of the Fifty-ninth Annual Meeting of the American Life Convention, October 12–15, 1964*, p. 79].

In the last twelve years, discussion on the same ideas has continued in a number of reports speculating on the future of individual policies. These discussions have used the name "life-cycle policy" to describe the concept introduced by Mr. Rieder and Mr. Guertin.

This paper describes methods adapted to the computer for calculating premiums and nonforfeiture values separately for each policy at its original issue and for as many changes in the policy as may later be desired. It is assumed that all premiums and values are printed directly by the computer on one page of the policy, and whenever a change is made a new page may be substituted giving premiums and values after the change. The policy after a change retains all of the original printed provisions and is considered amended or reissued. In any status the premium and amount of insurance are level.

Defining the principal elements of a policy as amount of insurance, gross premium, and plan of insurance, an original issue involves election of any two of the elements and calculation of the third. Each change after issue involves a change elected for one element, either a change or continuation for a second element, and calculation of the third element.

Elections	Solve for
Amount and premium	Plan
Amount and plan	Premium
Plan and premium	Amount

A change in amount of insurance may be an increase or a decrease. A change in premium may be an increase or a decrease. The premium may also include addition of a nonrepeating premium.

If plan of insurance is to be determined, the category in which the plan falls must be indicated. There are five categories:

1. Continuous premium term to any expiry age.
2. Limited pay life with premiums payable to any age.
3. Continuous premium endowment maturing at any age.
4. Endowment maturing at an indicated age with premiums payable to an earlier age.
5. Continuous premium endowments where the maturity value may be varied in relation to the initial amount of insurance.

If a premium and amount of insurance are elected that will not permit a plan of insurance indicated for one of categories 2–5, the solution for plan of insurance will automatically be for term insurance.

The following basic assumptions apply to the descriptions and illustrations in this paper. Obviously these may be varied in achieving the same objectives:

1. The 1958 Commissioners Standard Ordinary Mortality Table (1958 CSO), curtate, with 3 per cent interest.
2. Reserves by the Commissioners Reserve Valuation Method (CRVM). Cash values are reserves on the full amount of insurance taken to the nearest dollar.
3. All durations of term insurance with positive reserves have cash values.
4. The paid-up nonforfeiture value applicable to term plans is paid-up whole life insurance. Normal paid-up nonforfeiture values apply to whole life and endowment plans of insurance.
5. The extended term insurance nonforfeiture value is not used.
6. The minimum premium-paying period at original issue or after a change is five years, except where a nonrepeating premium is involved.
7. All the policies are participating.

II. NOTATION

m = Designation of the status of a policy ($m = 1$ is status for an original issue, $m = 2$ is status after the first change, etc.);

x_m = Age of the insured at the beginning of the mth status;

z = Age at expiry of a term policy or age at maturity of an endowment policy;

w = Age to which premiums are paid on a limited pay whole life or endowment policy;

k = Ratio of a unit of maturity value to a unit of insurance ($k = 0$ applies to term insurance, $k = 1$ to endowment insurance, $k > 1$ to income endowment insurance maturing for k per unit of insurance, and $0 < k < 1$ to partial endowment insurance maturing for k per unit of insurance);

a = Period during which insurance is in effect on an income endowment plan;

I_{x_m} = Amount of insurance in effect during the mth status;

$I_{x_m}G_{x_m}$ = Gross premium for I_{x_m} insurance during the mth status;

B_{x_m} = A nonrepeating gross premium;

$I_{x_m}\pi_{x_m}$ = CRVM net premium during the mth status;

γB_{x_m} = A nonrepeating net premium;

${}_{\overline{x_m - x_1}}V_{x_1}$ = Terminal reserve at commencement of the mth status;

${}_{\overline{y - x_1}}V_{x_1}$ = Terminal reserve at attained age y;

F_{x_m} = Reserve factor in effect during the mth status;

F'_{x_m} = Reserve factor for durations of income endowment insurance after the reserve exceeds the amount of insurance;

$I_{x_m}\Delta_{x_m}$ = Allowance for statutory expense in the first year of status m;

$\alpha_s, \beta_s, \lambda_s$ = Factors defining the gross premium in terms of the net premium and vice versa;

J = Limiting amount to which the factor λ_s applies.

III. THE GROSS PREMIUM

A plan of premium loading is required which will reproduce fairly closely the gross premium for regular issues.

Gross premium in terms of net premium and loading factors:

$$I_{x_m} < J: \quad I_{x_m}G_{x_m} = \frac{I_{x_m}\pi_{x_m}}{\alpha_s} + I_{x_m}\beta_s + I_{x_m}\lambda_s ; \tag{1a}$$

$$I_{x_m} \geq J: \quad I_{x_m}G_{x_m} = \frac{I_{x_m}\pi_{x_m}}{\alpha_s} + I_{x_m}\beta_s + J\lambda_s . \tag{1b}$$

Net premium in terms of gross premium and loading factors:

$$I_{x_m} < J: \quad I_{x_m}\pi_{x_m} = I_{x_m}G_{x_m}\alpha_s - I_{x_m}\alpha_s\beta_s - I_{x_m}\alpha_s\lambda_s ; \tag{2a}$$

$$I_{x_m} \geq J: \quad I_{x_m}\pi_{x_m} = I_{x_m}G_{x_m}\alpha_s - I_{x_m}\alpha_s\beta_s - J\alpha_s\lambda_s . \tag{2b}$$

The subscript s in the loading factors defines the numerical value of the loading factors for a range of the net premium per 1,000 of insurance. A

	s_1	s_2	s_3
α_s	0.82	0.87	0.92
β_s	0.0015	0.0025	0.0050
λ_s	0.0020	0.0020	0.0020
J	10,000	10,000	10,000
Range of $1{,}000\pi_{x_m}$	Through 14.27	14.28–40.02	40.03 and over
Range of $(I_{x_m}G_{x_m}-I_{x_m}\lambda_s)1{,}000/I_{x_m}$ or $(I_{x_m}G_{x_m}-J\lambda_s)1{,}000/I_{x_m}$	Through 18.90	18.91–48.50	48.51 and over

corresponding range applies to the gross premium reduced by the term involving λ_s. The illustrations in this paper assume that s refers to three ranges and the numerical values of the loading factors shown in the accompanying tabulation. The nonrepeating net premium γB_{xm} is $0.93B_{xm}$. The upper limit of $1{,}000\pi_{x_m}$ for range s_1 is found by the equation

$$1{,}000\left(\frac{\pi_{xm}}{\alpha_1}+\beta_1+\lambda_1\right)=1{,}000\left(\frac{\pi_{xm}}{\alpha_2}+\beta_2+\lambda_2\right)$$

and for range s_2 by

$$1{,}000\left(\frac{\pi_{xm}}{\alpha_2}+\beta_2+\lambda_2\right)=1{,}000\left(\frac{\pi_{xm}}{\alpha_3}+\beta_3+\lambda_3\right).$$

The upper limit of $1{,}000G_{xm}$ less the term involving λ_s is, for range s_1, $1{,}000(\pi_{xm}/\alpha_1+\beta_1)$; for range s_2 it is $1{,}000(\pi_{xm}/\alpha_2+\beta_2)$. The number of constants and number of ranges may be increased to the extent necessary to obtain satisfactory gross premiums.

IV. THE ALLOWANCE FOR STATUTORY EXPENSE

When a policy is originally issued ($m = 1$), the allowance for statutory expense $(I_{x_1}\Delta_{x_1})$ is the same as in a regular policy.

$$\pi_{x1} < {}_{19}P_{x1+1}: \quad I_{x1}\Delta_{x1} = I_{x1}(\pi_{x1} - c_{x1})\,;$$

$$\pi_{x1} \geq {}_{19}P_{x1+1}: \quad I_{x1}\Delta_{x1} = I_{x1}({}_{19}P_{x1+1} - c_{x1})\,.$$

When $m > 1$, the increase in statutory expense in the mth status for a preliminary term policy consists of two parts: (a) the statutory expense for a new issue at age x_m for the amount of insurance and net premium in the mth status, less (b) the expense for a new issue at age x_{m-1} for the amount of insurance and net premium in the (m − 1)st status. For a policy whose net premium exceeds $I_{x_m}\,{}_{19}P_{x_m+1}$ there is a further deduction (c) consisting of the difference in the applicable nineteen-pay life net premiums in the $(m-1)$st and mth statuses applied to the amount of insurance in the $(m-1)$st status. This deduction limits all statutory expense increases to amounts governed by the nineteen-pay life net premiums.

There are four possible conditions, each of which results in a different formula for $I_{x_m}\Delta_{x_m}$:

1. When $\pi_{x_m} < {}_{19}P_{x_m+1}$ and $\pi_{x_{m-1}} < {}_{19}P_{x_{m-1}+1}$,

$$I_{x_m}\Delta_{x_m} = I_{x_m}(\pi_{x_m} - c_{x_m}) - I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}}) \,. \tag{3}$$

In this case

$$I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}}) = \sum_{r=1}^{r=m-1} I_{x_r}\Delta_{x_r} \,.$$

2. When $\pi_{x_m} < {}_{19}P_{x_m+1}$ and $\pi_{x_{m-1}} \geq {}_{19}P_{x_{m-1}+1}$,

$$I_{x_m}\Delta_{x_m} = I_{x_m}(\pi_{x_m} - c_{x_m}) - \sum_{r=1}^{r=m-1} I_{x_r}\Delta_{x_r} \,. \tag{3a}$$

3. When $\pi_{x_m} \geq {}_{19}P_{x_m+1}$ and $\pi_{x_{m-1}} \geq {}_{19}P_{x_{m-1}+1}$,

$$\begin{aligned} I_{x_m}\Delta_{x_m} &= I_{x_m}({}_{19}P_{x_m+1} - c_{x_m}) - I_{x_{m-1}}({}_{19}P_{x_{m-1}+1} - c_{x_{m-1}}) \\ &\quad - I_{x_{m-1}}({}_{19}P_{x_m+1} - {}_{19}P_{x_{m-1}+1}) \\ &= (I_{x_m} - I_{x_{m-1}})\,{}_{19}P_{x_m+1} - I_{x_m}c_{x_m} + I_{x_{m-1}}c_{x_{m-1}} \,. \end{aligned} \tag{4}$$

4. When $\pi_{x_m} > {}_{19}P_{x_m+1}$ and $\pi_{x_{m-1}} < {}_{19}P_{x_{m-1}+1}$,

$$\begin{aligned} I_{x_m}\Delta_{x_m} &= I_{x_m}({}_{19}P_{x_m+1} - c_{x_m}) - I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}}) \\ &\quad - I_{x_{m-1}}({}_{19}P_{x_m+1} - {}_{19}P_{x_{m-1}+1}) \\ &= (I_{x_m} - I_{x_{m-1}})\,{}_{19}P_{x_m+1} + I_{x_{m-1}}({}_{19}P_{x_{m-1}+1} - \pi_{x_{m-1}}) \\ &\qquad - I_{x_m}c_{x_m} + I_{x_{m-1}}c_{x_{m-1}} \,. \end{aligned} \tag{4a}$$

Formula (3a) applies only when the net premiums on a change of status pass from above $I_{x_{m-1}}\,{}_{19}P_{x_{m-1}+1}$ to below $I_{x_m}\,{}_{19}P_{x_m+1}$. Because

$$\sum_{r=1}^{r=m-1} I_{x_r}\Delta_{x_r} < I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}}) \,,$$

the proper deduction in the formula is

$$\sum_{r=1}^{r=m-1} I_{x_r}\Delta_{x_r} .$$

Formula 4(a) applies only when the net premiums on a change pass from below $I_{x_{m-1}}\ {}_{19}P_{x_{m-1}+1}$ to above $I_{x_m}\ {}_{19}P_{x_m+1}$. Formula (4a) differs from formula (4) by the addition of $I_{x_{m-1}}({}_{19}P_{x_{m-1}+1} - \pi_{x_{m-1}})$. This term picks up additional statutory expense at the beginning of the $(m-1)$st status, bringing the total of such expense up to the limit imposed by $I_{x_{m-1}}\ {}_{19}P_{x_{m-1}+1}$. The conditions for the use of formulas (3a) and (4a) will, in practice, occur infrequently, and the extra expense that they provide will be relatively small. Formulas (3a) and (4a) are not included in the developments in the remainder of this paper.

When $I_{x_m}\Delta_{x_m}$ by any of the formulas is negative, it is taken as zero. The total of statutory expenses through the $(m-1)$st status,

$$\sum_{r=1}^{r=m-1} I_{x_r}\Delta_{x_r} ,$$

is treated as the total in the mth status for the purpose of computing $I_{x_{m+1}}\Delta_{x_{m+1}}$.

V. THE NET PREMIUM

The general formula for the net premium when $k \leq 1$ is

$$I_{x_m}\pi_{x_m} = \frac{I_{x_m}(M_{x_m} - M_z + kD_z) + (I_{x_m}\Delta_{x_m} - \gamma B_{x_m} - {}_{\overline{x_m - x_1}}V_{x_1})D_{x_m}}{N_{x_m} - N_w} . \tag{5}$$

The nonrepeating net premium γB_{x_m} is omitted in the formula derivations. When present, it is added to ${}_{\overline{x_m - x_1}}V_{x_1}$.

When $\pi_{x_m} < {}_{19}P_{x_m+1}$ and $I_{x_m}\Delta_{x_m} = I_{x_m}(\pi_{x_m} - c_{x_m}) - I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}})$, the term $I_{x_m}\pi_{x_m}$ appears on both sides of the equation. The solution for $I_{x_m}\pi_{x_m}$ is

$$I_{x_m}\pi_{x_m} = \frac{1}{N_{x_m} - N_w}\{I_{x_m}(M_{x_m} - M_z + kD_z)$$
$$+ [I_{x_m}(\pi_{x_m} - c_{x_m}) - I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}}) - {}_{\overline{x_m - x_1}}V_{x_1}]D_{x_m}\} ;$$

$$I_{x_m}\pi_{x_m}\left(1 - \frac{D_{x_m}}{N_{x_m} - N_w}\right) = \frac{1}{N_{x_m} - N_w}\{I_{x_m}(M_{x_m} - M_z + kD_z)$$
$$- [I_{x_m}c_{x_m} + I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}}) + {}_{\overline{x_m - x_1}}V_{x_1}]D_{x_m}\} .$$

Since $I_{x_m} c_{x_m} D_{x_m} = I_{x_m} C_{x_m}$, $N_{x_m} - D_{x_m} = N_{x_m+1}$, and $M_{x_m} - C_{x_m} = M_{x_m+1}$,

$$I_{x_m}\pi_{x_m}(N_{x_m+1} - N_w) = I_{x_m}(M_{x_m+1} - M_z + kD_z) - [I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}}) + {}_{\overline{x_m-x_1}}V_{x_1}]D_{x_m} ;$$

$$I_{x_m}\pi_{x_m} = \frac{1}{N_{x_m+1} - N_w}\{I_{x_m}(M_{x_m+1} - M_z + kD_z) - [I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}}) + {}_{\overline{x_m-x_1}}V_{x_1}]D_{x_m}\} . \tag{6}$$

When $\pi_{x_m} \geq {}_{19}P_{x_m+1}$,

$$I_{x_m}\Delta_{x_m} = (I_{x_m} - I_{x_{m-1}})\ {}_{19}P_{x_m+1} - I_{x_m}c_{x_m} + I_{x_{m-1}}c_{x_{m-1}} ;$$

$$I_{x_m}\pi_{x_m} = \frac{1}{N_{x_m} - N_w}\{I_{x_m}(M_{x_m} - M_z + kD_z) + [(I_{x_m} - I_{x_{m-1}})\ {}_{19}P_{x_m+1} - I_{x_m}c_{x_m} + I_{x_{m-1}}c_{x_{m-1}} - {}_{\overline{x_m-x_1}}V_{x_1}]D_{x_m}\} . \tag{7}$$

When $I_{x_m}\Delta_{x_m}$ is negative and taken as zero,

$$I_{x_m}\pi_{x_m} = \frac{I_{x_m}(M_{x_m} - M_z + kD_z) - {}_{\overline{x_m-x_1}}V_{x_1}D_{x_m}}{N_{x_m} - N_w} . \tag{8}$$

A summary of the conditions under which formulas (6)–(8) apply is given in Table 1. Formulas of net premiums where $k > 1$ are given in Section VIII.

TABLE 1

CONDITIONS FOR APPLICATION OF FORMULAS (6)–(8)

Category of Insurance	π_{x_m}	$I_{x_m}\Delta_{x_m}$	k	z	w	Formula
Term	No test	Positive	0	Through 100	z	(6)
Term	No test	Negative	0	Through 100	z	(8)
Limited pay life	$<{}_{19}P_{x_m+1}$	Positive	0	100	<100	(6)
Limited pay life	$\geq{}_{19}P_{x_m+1}$	Positive	0	100	<100	(7)
Limited pay life	No test	Negative	0	100	<100	(8)
Continuous premium endowment	$<{}_{19}P_{x_m+1}$	Positive	≤ 1	<100	z	(6)
Continuous premium endowment	$\geq{}_{19}P_{x_m+1}$	Positive	≤ 1	<100	z	(7)
Continuous premium endowment	No test	Negative	≤ 1	<100	z	(8)
Limited pay endowment	$<{}_{19}P_{x_m+1}$	Positive	≤ 1	<100	$<z$	(6)
Limited pay endowment	$\geq{}_{19}P_{x_m+1}$	Positive	≤ 1	<100	$<z$	(7)
Limited pay endowment	No test	Negative	≤ 1	<100	$<z$	(8)

VI. TERMINAL RESERVES

The attained-age valuation method is used for obtaining terminal reserves at the beginning of the mth status and for any attained age y during the mth status. The reserve factor F_{x_m} is computed at the beginning of each status and applies at all attained ages y during the status. An exception when $k > 1$ is discussed in Section VIII.

$$F_{x_m} = \left({}_{\overline{x_m - x_1}}V_{x_1} + \gamma B_{x_m} - I_{x_m}\Delta_{x_m}\right)\frac{D_{x_m}}{10^7} + I_{x_m}\pi_{x_m}\frac{N_{x_m}}{10^7} - I_{x_m}\frac{M_{x_m}}{10^7}\,. \quad (9)$$

The term γB_{x_m} appears only when a nonrepeating premium is paid at age x_m.

The terminal reserve at any attained age y is

$${}_{\overline{y-x_1}}V_{x_1} = I_{x_m}\frac{M_y}{D_y} + F_{x_m}\frac{10^7}{D_y} - I_{x_m}\pi_{x_m}\frac{N_y}{D_y}\,. \quad (10)$$

Tables for calculating reserve factors and terminal reserves for policies where $k \leq 1$ are given in the Appendixes to this paper.

VII. DETERMINATION OF GROSS PREMIUM, AMOUNT OF INSURANCE, OR PLAN OF INSURANCE

Gross Premium

Given the amount of insurance (I_{x_m}) and the plan of insurance identified by k, z, and w, the net premium is calculated by formulas (6)–(8), stopping at the calculation that satisfies the tests:

$\pi_{x_m} < {}_{19}P_{x_m+1}$	$I_{x_m}\Delta_{x_m}$	Correct Formula
Yes	Positive	(6)
No	Positive	(7)
Yes or No	Negative	(8)

$1{,}000 I_{x_m}\pi_{x_m}/I_{x_m}$ is then computed to determine the range s. The gross premium is computed by formula (1a) or formula (1b), whichever is applicable, using loading factors α_s, β_s, and λ_s for the applicable range.

Amount of Insurance

Given the gross premium ($I_{x_m}G_{x_m}$) and plan of insurance identified by k, z, and w, the value of $I_{x_m}\pi_{x_m}$ in terms of $I_{x_m}G_{x_m}$ and loading factors in formulas (1a) and (1b) may be equated to the value of $I_{x_m}\pi_{x_m}$ in formulas (6)–(8) to give formulas for solving for the amount of insurance (I_{x_m}).

$I_{xm} \geq J$, $\pi_{xm} < {}_{19}P_{xm+1}$, and $I_{xm}\Delta_{xm}$ is positive:

$$\begin{aligned} I_{xm} = \{&(I_{xm}G_{xm} - \lambda_s J)\alpha_s(N_{xm+1} - N_w) \\ &+ [I_{xm-1}(\pi_{xm-1} - c_{xm-1}) + {}_{\overline{xm-x1}}V_{x1}]D_{xm}\}/ \\ &\qquad [M_{xm+1} - M_z + kD_z + \beta_s\alpha_s(N_{xm+1} - N_w)]\,. \end{aligned} \tag{11}$$

$I_{xm} \geq J$, $\pi_{xm} \geq {}_{19}P_{xm+1}$, and $I_{xm}\Delta_{xm}$ is positive:

$$\begin{aligned} I_{xm} = \{&(I_{xm}G_{xm} - \lambda_s J)\alpha_s(N_{xm} - N_w) \\ &+ [I_{xm-1}({}_{19}P_{xm+1} - c_{xm-1}) + {}_{\overline{xm-x1}}V_{x1}]D_{xm}\}/[M_{xm} - M_z \\ &\qquad + kD_z + ({}_{19}P_{xm+1} - c_{xm})D_{xm} + \beta_s\alpha_s(N_{xm} - N_w)]\,. \end{aligned} \tag{12}$$

$I_{xm} \geq J$, and $I_{xm}\Delta_{xm}$ is negative:

$$I_{xm} = \frac{(I_{xm}G_{xm} - \lambda_s J)\alpha_s(N_{xm} - N_w) + {}_{\overline{xm-x1}}V_{x1}D_{xm}}{M_{xm} - M_z + kD_z + \beta_s\alpha_s(N_{xm} - N_w)}\,. \tag{13}$$

$I_{xm} < J$, $\pi_{xm} < {}_{19}P_{xm+1}$, and $I_{xm}\Delta_{xm}$ is positive:

$$\begin{aligned} I_{xm} = \{&I_{xm}G_{xm}\alpha_s(N_{xm+1} - N_w) + [I_{xm-1}(\pi_{xm-1} - c_{xm-1}) \\ &+ {}_{\overline{xm-x1}}V_{x1}]D_{xm}\}/[M_{xm+1} - M_z + kD_z \\ &\qquad + (\beta_s + \lambda_s)\alpha_s(N_{xm+1} - N_w)]\,. \end{aligned} \tag{14}$$

$I_{xm} < J$, $\pi_{xm} \geq {}_{19}P_{xm+1}$, and $I_{xm}\Delta_{xm}$ is positive:

$$\begin{aligned} I_{xm} = \{&I_{xm}G_{xm}\alpha_s(N_{xm} - N_w) + [I_{xm-1}({}_{19}P_{xm+1} - c_{xm-1}) \\ &+ {}_{\overline{xm-x1}}V_{x1}]D_{xm}\}/[M_{xm} - M_z + kD_z \\ &\qquad + ({}_{19}P_{xm+1} - c_{xm})D_{xm} + (\beta_s + \lambda_s)\alpha_s(N_{xm} - N_w)]\,. \end{aligned} \tag{15}$$

$I_{xm} < J$, and $I_{xm}\Delta_{xm}$ is negative:

$$I_{xm} = \frac{I_{xm}G_{xm}\alpha_s(N_{xm} - N_w) + {}_{\overline{xm-x1}}V_{x1}D_{xm}}{M_{xm} - M_z + kD_z + \alpha_s(\beta_s + \lambda_s)(N_{xm} - N_w)}\,. \tag{16}$$

If the calculation of a trial I_{xm} is made by formula (11) and then used in formula (2a) or formula (2b), as indicated by the amount of I_{xm}, a trial $I_{xm}\pi_{xm}$ is obtained. The trial values will determine whether $I_{xm}\Delta_{xm}$ is positive or negative, as well as the probable correct range of s. The formula for a second trial, if the first trial fails, will be one of the formulas (12)–(16). If a second trial fails to satisfy the three tests for the formula and the correct range of s, the error will be small, and a third trial will undoubtedly satisfy the tests.

Plan of Insurance

Given the amount of insurance, the gross premium, and the category of insurance, the computation is to find the plan of insurance. The net premium is obtained from the gross premium by formula (2a) or formula (2b), whichever is applicable. The range s is determined by $(I_{xm}G_{xm} - I_{xm}\lambda_s)1{,}000/I_{xm}$ or $(I_{xm}G_{xm} - \lambda_s J)1{,}000/I_{xm}$, depending on whether $I_{xm} < 10{,}000$ or $\geq 10{,}000$. $I_{xm}\Delta_{xm}$ is obtained from formula (3) or formula (4), whichever is applicable.

If the factor F_{xm} is negative when $m = 1$, the category of insurance is usually term. A small partial endowment may have a negative factor.

The reserve at age z for term insurance may be expressed by formula (10) as

$$I_{xm}\frac{M_z}{D_z} + F_{xm}\frac{10^7}{D_z} - I_{xm}\pi_{xm}\frac{N_z}{D_z} \geq 0\,,$$

so that

$$\frac{F_{xm}}{I_{xm}\pi_{xm}} \geq \frac{N_z}{10^7} - \frac{I_{xm}M_z}{I_{xm}\pi_{xm}\times 10^7}\,; \tag{17}$$

z is the highest age that satisfies formula (17). The plan of insurance is continuous premium term to age z.

The reserve at age w for limited pay life plans may be expressed by formula (10) as

$$I_{xm}\frac{M_w}{D_w} + \frac{F_{xm}\times 10^7}{D_w} - I_{xm}\pi_{xm}\frac{N_w}{D_w} \geq I_{xm}\frac{M_w}{D_w}\,,$$

so that

$$\frac{F_{xm}}{I_{xm}\pi_{xm}} \geq \frac{N_w}{10^7}\,; \tag{18}$$

w is the lowest age that satisfies formula (18). The plan of insurance is limited pay life paid up at age w.

The reserve at age z for a continuous premium endowment or partial endowment maturing at age z may be expressed by formula (10) as

$$I_{x_m}\frac{M_z}{D_z}+F_{x_m}\frac{10^7}{D_z}-I_{x_m}\pi_{x_m}\frac{N_z}{D_z}\geq I_{x_m}k\,,$$

so that

$$\frac{F_{x_m}}{I_{x_m}\pi_{x_m}}\geq\frac{N_z}{10^7}+\frac{I_{x_m}(kD_z-M_z)}{I_{x_m}\pi_{x_m}\times 10^7}\,; \tag{19}$$

z is the lowest age that satisfies formula (19). The plan of insurance is continuous premium endowment of $I_{x_m}k$ maturing at age z.

The reserve at age w for an endowment maturing at age z with premiums payable to age w may be expressed by formula (10) as

$$I_{x_m}\frac{M_w}{D_w}+F_{x_m}\frac{10^7}{D_w}-I_{x_m}\pi_{x_m}\frac{N_w}{D_w}\geq\frac{I_{x_m}M_w+I_{x_m}(kD_z-M_z)}{D_w}\,,$$

so that

$$\frac{F_{x_m}}{I_{x_m}\pi_{x_m}}\geq\frac{N_w}{10^7}+\frac{I_{x_m}(kD_z-M_z)}{I_{x_m}\pi_{x_m}\times 10^7}\,; \tag{20}$$

w is the lowest age that satisfies formula (20). The plan of insurance is endowment of $I_{x_m}k$ maturing at age z with premiums paid to age w.

In each case, when the amount of insurance and the gross premium are selected at random, the reserve will exceed by a small amount the value necessary at the end of the premium-paying period. The excess may be disposed of by recomputing the net and gross premiums so that the actual gross premium payable is slightly less than the amount elected. Alternatively, the policy may provide that the excess is paid in addition to the last dividend on a term or continuous premium endowment policy. On limited pay life or limited pay endowment, the excess may be used to provide a paid-up policy of slightly larger amount than the face amount during the premium-paying period.

Illustration

Table 2 illustrates the results of defining two of the three elements (plan of insurance, gross premium, and amount of insurance) and solving for the third element.

TABLE 2

ILLUSTRATIONS OF INPUT OF TWO ELEMENTS OF THE POLICY AND SOLUTION FOR THE THIRD ELEMENT

(Elements: Plan of Insurance, Gross Premium, and Amount of Insurance)

Category	Age x_m	Plan k	Plan z	Plan w	Amount of Insurance	Gross Premium	Nonrepeating Premium	Issue	Solved Value
Term	35	0	60	60	50,000	Solve	0	$m=1$	506.18
Term	35	0	60	60	Solve	750.00	0	$m=2$*	91,748
Term or life	35	Solve	Solve		50,000	750.00	0	$m=1$	Term to 70; $k=0$
Term or endowment	35	Solve	Solve		50,000	1,000.00	0	$m=2$	Term to 96; $k=0$
Limited pay life	35	0	100	65	50,000	Solve	0	$m=1$	1,292.88
Limited pay life	35	0	100	65	Solve	1,500.00	0	$m=2$	63,542
Limited pay life	35	0	100	Solve	50,000	1,500.00	0	$m=1$	Paid up at age 58
Limited pay life	35	0	100	Solve	50,000	1,500.00	5,000.00	$m=2$	Paid up at age 48
Endowment	35	1	60	60	25,000	Solve	0	$m=1$	968.96
Endowment	35	1	60	60	Solve	1,000.00	0	$m=2$	30,026
Endowment	35	1	Solve		25,000	1,000.00	0	$m=1$	Maturing at age 60
Limited pay endowment	35	1	60	Solve	25,000	1,000.00	5,000.00	$m=2$	Paid up at age 44

* When $m = 2$, the previous status of the policy is assumed to be issue age 27, amount 20,000, net premium 280.30, and reserve at age 35 of 1,893.82. The plan of insurance is life paid up at age 65.

VIII. ADJUSTABLE INDIVIDUAL POLICY PENSION COVERAGE

The adjustable principles may be applied to individual policy pension coverage. When a number of policies are issued on the life of an employee that adjust the prospective pension income to increases in salary, the record-keeping becomes onerous. The adjustable system would require a single policy with reissues producing new schedules of total values. A single dividend would be paid each year, and a single equivalent to the policy fee would apply.

In many situations it is appropriate to specify a maximum initial death benefit. The maximum may be adjusted downward for substandard risks. The adjustable principle will permit the relationship of initial face amount of death benefit to maturity value to be a decreasing ratio as coverage for additional pension is issued through adjustments to the original policy.

If the face amount before the reserve exceeds it is related to current salary, and the maturity value is to be sufficient to pay a pension related, for example, to the average of the five years of consecutive salary giving the highest amount, a single policy will apply with the ratio of maturity value to death benefit changed as required.

If the trustees of a pension plan should decide to increase the percentage of current investments in equities, the increase could be accomplished by reducing the premiums of the individual policies. If the ratio of fixed-dollar investments to equities is to be increased, the result may be accomplished by increasing the pension policy premiums or by paying non-repeating premiums from proceeds of the sale of equities.

Net Premiums

The net premiums for adjustable pension policies involve values of k where $k > 1$ and, for a partial endowment, $0 < k < 1$. Formulas (6)–(8) cover partial endowments by using the relationship $0 < k < 1$.

$k > 1$, $\pi_{x_m} < {}_{19}P_{x_m+1}$, $I_{x_m}\Delta_{x_m}$ is positive, and a is duration where insurance exists:

$$I_{x_m}\pi_{x_m} = \{I_{x_m} k v^{(z-x_m-a)} D_{x_m+a} + I_{x_m}(M_{x_m+1} - M_{x_m+a}) - [I_{x_m-1}(\pi_{x_m-1} - c_{x_m-1}) + {}_{\overline{x_m-x_1}}V_{x_1}]D_{x_m}\} / (N_{x_m+1} - N_{x_m+a} + \ddot{a}_{\overline{z-x_m-a}|}D_{x_m+a}) . \tag{21}$$

$\pi_{x_m} \geq {}_{19}P_{x_m+1}$, and $I_{x_m}\Delta_{x_m}$ is positive:

$$I_{x_m}\pi_{x_m} = \{I_{x_m} k v^{(z-x_m-a)} D_{x_m+a} + I_{x_m}(M_{x_m} - M_{x_m+a}) + [(I_{x_m} - I_{x_m-1})\ {}_{19}P_{x_m+1} - I_{x_m}c_{x_m} + I_{x_m-1}c_{x_m-1} - {}_{\overline{x_m-x_1}}V_{x_1}]D_{x_m}\} / (N_{x_m} - N_{x_m+a} + \ddot{a}_{\overline{z-x_m-a}|}D_{x_m+a}) . \tag{22}$$

$I_{x_m}\Delta_{x_m}$ is negative and taken as zero:

$$I_{x_m}\pi_{x_m} = \frac{I_{x_m} k v^{(z-x_m-a)} D_{x_m+a} + I_{x_m}(M_{x_m} - M_{x_m+a}) - {}_{\overline{x_m-x_1}}V_{x_1}D_{x_m}}{N_{x_m} - N_{x_m+a} + \ddot{a}_{\overline{z-x_m-a}|}D_{x_m+a}} . \tag{23}$$

The formulas require a trial value of a. If the reserve value is assumed to increase by equal increments, the coverage may be represented by a diagram such as that shown in Figure 1 (p. 252), having two similar triangles, ABC and $AB'C'$. The trial value of a is

$$\frac{a}{I_{x_m} - {}_{\overline{x_m-x_1}}V_{x_1}} = \frac{z - x_m}{I_{x_m}k - {}_{\overline{x_m-x_1}}V_{x_1}} ,$$

so that

$$a = \frac{(z - x_m)(I_{x_m} - {}_{\overline{x_m - x_1}}V_{x_1})}{I_{x_m}k - {}_{\overline{x_m - x_1}}V_{x_1}} . \tag{24}$$

The true value of a will normally be greater than the above trial value taken to the next higher integer. Trial values are increased by 1 until the net premium and a meet the test:

$$I_{x_m}kv^{(z-x_m-a)} - I_{x_m}\pi_{x_m}\ddot{a}_{\overline{z-x_m-a}|} \leq I_{x_m}$$

and

$$(I_{x_m}kv^{(z-x_m-a)} - I_{x_m}\pi_{x_m}\ddot{a}_{\overline{z-x_m-a}|} + I_{x_m}\pi_{x_m})(1+i) \geq I_{x_m} .$$

Reserves when $k > 1$

The reserve factor in formula (9) and the attained-age reserve formula (10) produce reserve values of the coverage through the a period. Thereafter the reserve factor and attained-age reserve formula change.

If the functions involving life contingencies in the attained-age reserve formula (10) are taken as analogous to interest functions at attained age y (that is, A_y to $v^{(100-y)}$, D_y to v^y, and $\ddot{a}_y$ to $\ddot{a}_{\overline{100-y}|}$), then the reserve factor F'_{x_m} may be found by equating the adjusted attained-age formula to the prospective reserve formula:

$$I_{x_m}kv^{(100-y)} + \frac{F'_{x_m}}{v^y} - I_{x_m}\pi_{x_m}\ddot{a}_{\overline{100-y}|} = I_{x_m}kv^{(z-y)} - I_{x_m}\pi_{x_m}\ddot{a}_{\overline{z-y}|} ; \tag{25}$$

$$\begin{aligned}
F'_{x_m} &= v^y[I_{x_m}\pi_{x_m}(\ddot{a}_{\overline{100-y}|} - \ddot{a}_{\overline{z-y}|}) + I_{x_m}k(v^{(z-y)} - v^{(100-y)})] \\
&= v^y\Big[I_{x_m}\pi_{x_m}\left(\frac{1 - v^{(100-y)}}{i} - \frac{1 - v^{(z-y)}}{i}\right)(1+i) \\
&\qquad + I_{x_m}kv^{-y}(v^z - v^{100})\Big] \\
&= I_{x_m}\pi_{x_m}\left(\frac{v^z - v^{100}}{i}\right)(1+i) + I_{x_m}k\left(\frac{v^z - v^{100}}{i}\right)(1+i)\left(\frac{i}{1+i}\right) \\
&= I_{x_m}\pi_{x_m}v^z\ddot{a}_{\overline{100-z}|} + I_{x_m}kiv^{(z+1)}\ddot{a}_{\overline{100-z}|} .
\end{aligned}$$

If

$$f_1 = v^z\ddot{a}_{\overline{100-z}|} , \quad f_2 = iv^{z+1}\ddot{a}_{\overline{100-z}|} ,$$

then

$$\begin{aligned}
F'_{x_m} &= I_{x_m}\pi_{x_m}f_1 + I_{x_m}kf_2 \\
&= (\text{Net premium} \times f_1) + (\text{maturity value} \times f_2) .
\end{aligned} \tag{26}$$

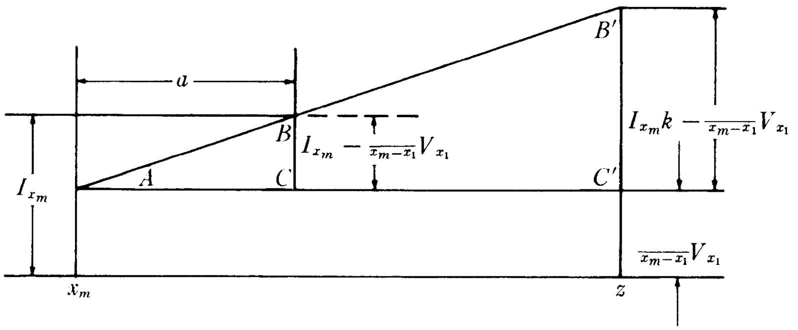

FIG. 1

Tables for calculating reserve factors and terminal reserves after the end of the a period are given in the Appendixes to this paper.

IX. DETERMINATION OF GROSS PREMIUM, AMOUNT OF INSURANCE, AND RATIO OF MATURITY VALUE TO AMOUNT OF INSURANCE WHEN $k > 1$

The methods of determining the gross premium and amount of insurance are similar to the methods described in Section VII. Determination of the ratio of maturity value to amount of insurance is an additional flexibility in a policy used for funding pension plans. All policies are assumed to have continuous premiums to maturity.

Gross Premium

Given the amount of insurance I_{x_m} and the plan of insurance identified by k, z, and a, the net premium is calculated by formulas (21)–(23), stopping at the calculation that satisfies the tests:

$\pi_{x_m} < {}_{19}P_{x_m+1}$	$I_{x_m}\Delta_{x_m}$	Correct Formula
Yes	Positive	(21)
No	Positive	(22)
Yes or No	Negative	(23)

When the correct net premium formula has been determined, the calculation of the gross premium is performed as described under "Gross Premium" in Section VII.

Amount of Insurance

Given the gross premium ($I_{xm}G_{xm}$) and plan of insurance identified by k, z, and a, the value of $I_{xm}\pi_{xm}$ in terms of $I_{xm}G_{xm}$ and loading factors in formulas (1a) and (1b) may be equated to the values of $I_{xm}\pi_{xm}$ in formulas (21)–(23) to give the following formulas for solving for the amount of insurance (I_{xm}).

$I_{xm} \geq J$, $\pi_{xm} < {}_{19}P_{xm+1}$, and $I_{xm}\Delta_{xm}$ is positive:

$$\begin{aligned} I_{xm} = {} & \{(I_{xm}G_{xm} - \lambda_s J)\alpha_s(N_{xm+1} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a}) \\ & + [I_{xm-1}(\pi_{xm-1} - c_{xm-1}) + {}_{\overline{xm-x1}}V_{x1}]D_{xm}\}/ \\ & [kv^{(z-xm-a)}D_{xm+a} + M_{xm+1} - M_{xm+a} \\ & + \alpha_s\beta_s(N_{xm+1} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a})]\,. \end{aligned} \tag{27}$$

$I_{xm} \geq J$, $\pi_{xm} \geq {}_{19}P_{xm+1}$, and $I_{xm}\Delta_{xm}$ is positive:

$$\begin{aligned} I_{xm} = {} & \{(I_{xm}G_{xm} - \lambda_s J)\alpha_s(N_{xm} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a}) \\ & + [I_{xm-1}({}_{19}P_{xm+1} - c_{xm-1}) + {}_{\overline{xm-x1}}V_{x1}]D_{xm}\}/ \\ & [kv^{(z-xm-a)}D_{xm+a} + M_{xm} - M_{xm+a} + ({}_{19}P_{xm+1} - c_{xm})D_{xm} \\ & + \alpha_s\beta_s(N_{xm} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a})]\,. \end{aligned} \tag{28}$$

$I_{xm} \geq J$, and $I_{xm}\Delta_{xm}$ is negative:

$$\begin{aligned} I_{xm} = {} & [(I_{xm}G_{xm} - \lambda_s J)\alpha_s(N_{xm} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a}) \\ & + {}_{\overline{xm-x1}}V_{x1}D_{xm}]/[kv^{(z-xm-a)}D_{xm+a} + M_{xm} - M_{xm+a} \\ & + \alpha_s\beta_s(N_{xm} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a})]\,. \end{aligned} \tag{29}$$

$I_{xm} < J$, $\pi_{xm} < {}_{19}P_{xm+1}$, and $I_{xm}\Delta_{xm}$ is positive:

$$\begin{aligned} I_{xm} = {} & \{I_{xm}G_{xm}\alpha_s(N_{xm+1} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a}) \\ & + [I_{xm-1}(\pi_{xm-1} - c_{xm-1}) + {}_{\overline{xm-x1}}V_{x1}]D_{xm}\}/ \\ & [kv^{(z-xm-a)}D_{xm+a} + M_{xm+1} - M_{xm+a} \\ & + \alpha_s(\beta_s + \lambda_s)(N_{xm+1} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a})]\,. \end{aligned} \tag{30}$$

$I_{xm} < J$, $\pi_{xm} \geq {}_{19}P_{xm+1}$, and $I_{xm}\Delta_{xm}$ is positive:

$$\begin{aligned} I_{xm} = \{ & I_{xm}G_{xm}\alpha_s(N_{xm} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a}) \\ & + [I_{xm-1}({}_{19}P_{xm+1} - c_{xm-1}) + {}_{\overline{xm-x1}}V_{x1}]D_{xm}\}/ \\ & [kv^{(z-xm-a)}D_{xm+a} + M_{xm} - M_{xm+a} + ({}_{19}P_{xm+1} - c_{xm})D_{xm} \\ & + \alpha_s(\beta_s + \lambda_s)(N_{xm} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a})] \,. \end{aligned} \tag{31}$$

$I_{xm} < J$, and $I_{xm}\Delta_{xm}$ is negative:

$$\begin{aligned} I_{xm} = & I_{xm}G_{xm}\alpha_s(N_{xm} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a}) + {}_{\overline{xm-x1}}V_{x1}D_{xm}]/ \\ & [kv^{(z-xm-a)}D_{xm+a} + M_{xm} - M_{xm+a} \\ & + \alpha_s(\beta_s + \lambda_s)(N_{xm} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a})] \,. \end{aligned} \tag{32}$$

Formula (28) is the most likely formula for trial calculations; $(z - x_m)/k$ from formula (24) may be used as a trial value of a. When ${}_{\overline{x_m-x_1}}V_{x_1}$ is significantly large, successive trial values of a may decrease to find the true value that satisfies the tests that follow formula (24).

Ratio of Maturity Value to Amount of Insurance when $k > 1$

Given the gross premium, the amount of insurance, and the maturity age z, the maturity value $I_{xm}k$ is to be determined. $I_{xm}\pi_{xm}$ is obtained from formula (2a) if $I_{xm} < J$ or from formula (2b) if $I_{xm} \geq J$. $I_{xm}\Delta_{xm}$ is obtained from formula (3) or formula (4), and ${}_{\overline{xm-x_1}}V_{x_1}$ is available if $m > 1$. F_{xm} may be calculated from formula (9). By application of formula (19) for an endowment of I_{xm}, a maturity age is found which is equal to $x_m + a + 1$; therefore, a is determined. The formulas for the net premium, (21)-(23), may be rewritten to solve for k:

$\pi_{xm} < {}_{19}P_{xm+1}$, and $I_{xm}\Delta_{xm}$ is positive:

$$\begin{aligned} k = \{ & I_{xm}\pi_{xm}(N_{xm+1} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a}) - I_{xm}(M_{xm+1} - M_{xm+a}) \\ & + [I_{xm-1}(\pi_{xm-1} - c_{xm-1}) + {}_{\overline{xm-x1}}V_{x1}]D_{xm}\}/(I_{xm}v^{(z-xm-a)}D_{xm+a}) \,. \end{aligned} \tag{33}$$

$\pi_{xm} \geq {}_{19}P_{xm+1}$, and $I_{xm}\Delta_{xm}$ is positive:

$$\begin{aligned} k = \{ & I_{xm}\pi_{xm}(N_{xm} - N_{xm+a} + \ddot{a}_{\overline{z-xm-a}|}D_{xm+a}) - I_{xm}(M_{xm} - M_{xm+a}) \\ & - [(I_{xm} - I_{xm-1})\ {}_{19}P_{xm+1} - I_{xm}c_{xm} + I_{xm-1}c_{xm-1} \\ & - {}_{\overline{xm-x1}}V_{x1}]D_{xm}\}/(I_{xm}v^{(z-xm-a)}D_{xm+a}) \,. \end{aligned} \tag{34}$$

$I_{x_m}\Delta_{x_m}$ is negative and taken as zero:

$$k = [I_{x_m}\pi_{x_m}(N_{x_m} - N_{x_m+a} + \ddot{a}_{\overline{z-x_m-a}|}D_{x_m+a}) - I_{x_m}(M_{x_m} - M_{x_m+a}) \\ + {}_{\overline{x_m-x_1}}V_{x_1}D_{x_m}]/(I_{x_m}v^{(z-x_m-a)}D_{x_m+a}) . \tag{35}$$

Retirement Annuity Coverage

When the a period in the net premium formulas equals zero, no life insurance coverage exists. The terms $M_{x_m} - M_{x_m+a}$ and $N_{x_m} - N_{x_m+a}$ become zero. D_{x_m+a} becomes D_{x_m}, and this function cancels out in numerator and denominator. Formulas (22) and (23) automatically apply to retirement annuity coverage if I_{x_m} is treated as the number of units of $I_{x_m}k$ in the maturity value, and the statutory expense involving life insurance functions, ${}_{19}P_{x_m+1}$, c_{x_m}, and $c_{x_{m-1}}$, is retained.

Formula (21) may be modified to apply to retirement annuity coverage when $\pi_{x_m} < {}_{19}P_{x_m+1}$. If the statutory expense is analogous to preliminary term life insurance,

$$I_{x_m}\pi_{x_m}\ddot{a}_{\overline{z-x_m}|} = I_{x_m}kv^{(z-x_m)} + I_{x_m}(\pi_{x_m} - c_{x_m}) \\ - I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}}) - {}_{\overline{x_m-x_1}}V_{x_1} ;$$

$$I_{x_m}\pi_{x_m} = \frac{I_{x_m}kv^{(z-x_m)} - \{I_{x_m}c_{x_m} + I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}}) + {}_{\overline{x_m-x_1}}V_{x_1}\}}{\ddot{a}_{\overline{z-x_m}|} - 1} . \tag{36}$$

Formulas (27)–(35) may be adjusted similarly for use in retirement annuity coverage.

Illustration

Table 3 (p. 256) illustrates the results for pension policies of defining two of the three elements (gross premium, amount of insurance, and ratio of maturity value to amount of insurance) and solving for the third element.

X. PROOF THAT RESERVES SATISFY THE STANDARD VALUATION LAW

As noted above, reserves are by the Commissioners Reserve Valuation Method. If a limited pay life policy in its $(m - 1)$st status with premiums payable to age w passes to the mth status by increase in both amount of insurance and premium, retaining age w as the age to which premiums are payable, the reserves during the mth status may be compared with the sum of reserves if (a) the $(m - 1)$st status of the original policy had been continued and (b) the increase in amount of insurance and premium had been issued in a separate policy.

If π'_{x_m} is the net premium of the separate policy, the difference in net

TABLE 3

ILLUSTRATIONS FOR PENSION POLICIES OF INPUT AND SOLUTIONS FOR GROSS PREMIUM, AMOUNT OF INSURANCE, AND RATIO OF MATURITY VALUE TO AMOUNT OF INSURANCE*

Policy	m	Age	k	z	a	Amount of Insurance	Gross Premium	Nonrepeating Premium	Reserve	Solved Value	a
Income endowment	1	35	1.333	65	Solve	25,000	Solve	0	0	971.59	24
	2	40	1.333	65	Solve	35,000	Solve	0	3,202.48	1,456.60	20
	3	50	1.333	65	Solve	Solve	2,500	0	16,392.03	46,758	10
	4	55	Solve	65	Solve	50,000	4,000	5,000	28,841.28	1.702675	3
Partial endowment	1	35	0.300	65	30	25,000	Solve	0	0	467.32	...
	2	40	0.500	65	25	50,000	Solve	0	1,151.84	1,277.29	...
	3	50	0.500	65	15	Solve	2,000	0	9,833.15	65,550	...
	4	55	Solve	68	13	60,000 (k units)	2,500	2,000	17,046.56	0.868862	...
Retirement annuity. .	1	35	1.333	65	0	25,000	Solve	0	0	838.77	...
	2	40	1.333	65	0	35,000	Solve	0	3,140.12	1,262.07	...
	3	50	1.333	65	0	Solve	2,400	0	16,658.84	48,893	...
	4	55	Solve	65	0	60,000	3,800	5,000	30,302.96	1.463475	...

* In the illustrations an original age of 35 and status changes at ages 40, 50, and 55 are assumed.

premiums between the two policies and the adjustable policy in its mth status is

$$I_{x_{m-1}}\pi_{x_{m-1}} + (I_{x_m} - I_{x_{m-1}})\pi'_{x_m} - I_{x_m}\pi_{x_m} .$$

Evaluation of Expression when $\pi_{x_{m-1}} < {}_{19}P_{x_{m-1}+1}$ *and* $\pi_{x_m} < {}_{19}P_{x_m+1}$:

$$I_{x_{m-1}}\pi_{x_{m-1}} + (I_{x_m} - I_{x_{m-1}})\pi'_{x_m} - I_{x_m}\pi_{x_m} = I_{x_{m-1}}\pi_{x_{m-1}}$$

$$+ \frac{1}{N_{x_m+1} - N_w}[\![(I_{x_m} - I_{x_{m-1}})M_{x_m+1}$$

$$\text{(I)} \qquad - \{I_{x_m}M_{x_m+1} - [I_{x_{m-1}}(\pi_{x_{m-1}} - c_{x_{m-1}}) + {}_{\overline{x_m-x_1}}V_{x_1}]D_{x_m}\}]\!] .$$

Expressing ${}_{\overline{x_m-x_1}}V_{x_1}D_{x_m}$ in terms of ${}_{\overline{x_{m-1}-x_1}}V_{x_1}D_{x_{m-1}}$ and terms which advance the reserve to x_m, we have

$${}_{\overline{x_m-x_1}}V_{x_1}D_{x_m} = {}_{\overline{x_{m-1}-x_1}}V_{x_1}D_{x_{m-1}} + I_{x_{m-1}}\pi_{x_{m-1}}(N_{x_{m-1}} - N_{x_m})$$

$$- I_{x_{m-1}}\Delta_{x_{m-1}}D_{x_{m-1}} - I_{x_{m-1}}(M_{x_{m-1}} - M_{x_m}) .$$

Substituting this expression in the right-hand side in (I) yields

$$
\text{(II)}\quad
\begin{aligned}
&\frac{1}{N_{x_m+1}-N_w}[I_{x_{m-1}}\pi_{x_{m-1}}(N_{x_m+1}-N_w+N_{x_{m-1}}-N_{x_m}+D_{x_m})\\
&\qquad + I_{x_{m-1}}(M_{x_m}-M_{x_m+1}-c_{x_{m-1}}D_{x_m})\\
&\qquad + {}_{\overline{x_{m-1}-x_1}}V_{x_1}D_{x_{m-1}} - I_{x_{m-1}}M_{x_{m-1}} - I_{x_{m-1}}\Delta_{x_{m-1}}D_{x_{m-1}}]\,.
\end{aligned}
$$

In (II),

$$
I_{x_{m-1}}(M_{x_m}-M_{x_m+1}-c_{x_{m-1}}D_{x_m}) = I_{x_{m-1}}\left(C_{x_m}-C_{x_{m-1}}\frac{D_{x_m}}{D_{x_{m-1}}}\right)
$$

and

$$
\begin{aligned}
&-I_{x_{m-1}}M_{x_{m-1}} - I_{x_{m-1}}\Delta_{x_{m-1}}D_{x_{m-1}} + {}_{\overline{x_{m-1}-x_1}}V_{x_1}D_{x_{m-1}}\\
&\qquad\qquad = -I_{x_{m-1}}\pi_{x_{m-1}}(N_{x_{m-1}}-N_w)\,.
\end{aligned}
$$

Substituting the right-hand side of these expressions in (II) results in

$$
\text{(III)}\quad
\begin{aligned}
&\frac{1}{N_{x_m+1}-N_w}\Big[I_{x_{m-1}}\pi_{x_{m-1}}(N_{x_m+1}-N_w+N_{x_{m-1}}-N_{x_m}\\
&\qquad + D_{x_m}-N_{x_{m-1}}+N_w) + \left(C_{x_m}-C_{x_{m-1}}\frac{D_{x_m}}{D_{x_{m-1}}}\right)\Big]\,.
\end{aligned}
$$

Since the coefficient of $I_{x_{m-1}}\pi_{x_{m-1}}$ is zero, the evaluation of the original expression is

$$
\begin{aligned}
&I_{x_{m-1}}\pi_{x_{m-1}} + (I_{x_m}-I_{x_{m-1}})\pi'_{x_m} - I_{x_m}\pi_{x_m}\\
&\qquad\qquad = \frac{C_{x_m}-C_{x_{m-1}}(D_{x_m}/D_{x_{m-1}})}{N_{x_m+1}-N_w}\,. \qquad (37)
\end{aligned}
$$

Evaluation of Expression when $\pi_{x_m} \geq {}_{19}P_{x_m+1}$ *(regardless of whether* $\pi_{x_{m-1}}$ *is less than, equal to, or greater than* ${}_{19}P_{x_{m-1}+1}$*):*

$$
\text{(I)}\quad
\begin{aligned}
&I_{x_{m-1}}\pi_{x_{m-1}} + (I_{x_m}-I_{x_{m-1}})\pi'_{x_m} - I_{x_m}\pi_{x_m} = I_{x_{m-1}}\pi_{x_{m-1}}\\
&\quad + \frac{1}{N_{x_m}-N_w}[\![(I_{x_m}-I_{x_{m-1}})[M_{x_m}+({}_{19}P_{x_m+1}-c_{x_m})D_{x_m}]\\
&\qquad - \{I_{x_m}M_{x_m} + [(I_{x_m}-I_{x_{m-1}})\;{}_{19}P_{x_m+1}\\
&\qquad\quad - I_{x_m}c_{x_m} + I_{x_{m-1}}c_{x_{m-1}} - {}_{\overline{x_m-x_1}}V_{x_1}]D_{x_m}\}]\!]\,.
\end{aligned}
$$

Expressing ${}_{\overline{x_m - x_1}}V_{x_1}D_{x_m}$ in terms of ${}_{\overline{x_{m-1} - x_1}}V_{x_1}D_{x_{m-1}}$ and terms which advance the reserve to $_{x_m}$, we have

$$\begin{aligned}{}_{\overline{x_m - x_1}}V_{x_1}D_{x_m} = {}_{\overline{x_{m-1} - x_1}}V_{x_1}D_{x_{m-1}} &+ I_{x_{m-1}}\pi_{x_{m-1}}(N_{x_{m-1}} - N_{x_m}) \\ &- I_{x_{m-1}}\Delta_{x_{m-1}}D_{x_{m-1}} - I_{x_{m-1}}(M_{x_{m-1}} - M_{x_m})\,.\end{aligned}$$

Substituting this expression in the right-hand side in (I) yields

$$\text{(II)}\quad \begin{aligned}\frac{1}{N_{x_m} - N_w}[&I_{x_{m-1}}\pi_{x_{m-1}}(N_{x_m} - N_w + N_{x_{m-1}} - N_{x_m}) \\ &- I_{x_{m-1}}M_{x_m} - I_{x_m}c_{x_m}D_{x_m} + I_{x_{m-1}}c_{x_m}D_{x_m} \\ &+ I_{x_m}c_{x_m}D_{x_m} - I_{x_{m-1}}c_{x_{m-1}}D_{x_m} + {}_{\overline{x_{m-1} - x_1}}V_{x_1}D_{x_{m-1}} \\ &- I_{x_{m-1}}\Delta_{x_{m-1}}D_{x_{m-1}} - I_{x_{m-1}}(M_{x_{m-1}} - M_{x_m})]\,.\end{aligned}$$

In (II),

$$I_{x_{m-1}}(c_{x_m}D_{x_m} - c_{x_{m-1}}D_{x_m}) = I_{x_{m-1}}\left(C_{x_m} - C_{x_{m-1}}\frac{D_{x_m}}{D_{x_{m-1}}}\right)$$

and

$${}_{\overline{x_m - x_1}}V_{x_1}D_{x_{m-1}} - I_{x_{m-1}}\Delta_{x_{m-1}} - I_{x_{m-1}}M_{x_{m-1}} = -I_{x_{m-1}}\pi_{x_{m-1}}(N_{x_{m-1}} - N_w)\,.$$

Substituting the right-hand side of these expressions in (II) results in

$$\text{(III)}\quad \begin{aligned}\frac{1}{N_{x_m} - N_w}\Big[&I_{x_{m-1}}\pi_{x_{m-1}}(N_{x_m} - N_w + N_{x_{m-1}} - N_{x_m} - N_{x_{m-1}} \\ &+ N_w) + I_{x_{m-1}}\left(C_{x_m} - C_{x_{m-1}}\frac{D_{x_m}}{D_{x_{m-1}}}\right)\Big]\,.\end{aligned}$$

Since the coefficient of $I_{x_{m-1}}\pi_{x_{m-1}}$ is zero, the evaluation of the original expression is

$$\begin{aligned}I_{x_{m-1}}\pi_{x_{m-1}} + (I_{x_m} - I_{x_{m-1}})\pi'_{x_m} - I_{x_m}\pi_{x_m} \\ = \frac{C_{x_m} - C_{x_{m-1}}(D_{x_m}/D_{x_{m-1}})}{N_{x_m} - N_w}\,.\end{aligned} \quad (38)$$

Since the numerator of the right-hand sides of both formula (37) and formula (38), $C_{x_m} - C_{x_{m-1}}(D_{x_m}/D_{x_{m-1}})$, is positive except in possible instances under age 10, the reserve of the adjustable policy in the mth status is generated from a slightly smaller net premium than the sum of the net premiums from continuation of status $(m - 1)$ plus the net premium of a separate policy issued at attained age x_m, payable to age w, for the increased insurance. The reserves of the adjustable policy are, therefore, slightly larger than the reserves of the separate comparable policies.

In most instances, a change producing an adjustable policy with a higher amount of insurance or premium, or both, will have the new premium terminate either after or before the termination date of the premium in the prior status. If the increase in benefits during the period in which the premium in the new status overlaps the premium in the prior status is identified and treated as the benefits of a separate policy, formula (37) or formula (38) will apply if additional statutory expense is involved in the change. This aspect of formulas (37) and (38) indicates that they have general application—that, whenever a change involves additional statutory expense, the reserve of the adjustable life during the overlap period will be greater than the sum of reserves in the prior adjustable life status and the additional benefits treated as in a separate policy.

Illustrations

1. In the sixth illustration in Table 2, 20,000 life paid up at age 65, issued at age 27, is changed at age 35 to 63,542 life paid up at age 65, with appropriate increase in the gross premium. Here the premium period of the policy after the change exactly overlaps the premium period in the prior status. On separate policies:

Net premium on 20,000 adjustable	280.30
Net premium on 43,542 life paid up at age 65 in a separate policy	869.67
A. Total nets	1,149.97
B. Net if adjustable policy changed	1,149.40
Net on adjustable lower by	0.57
Excess of A over B by formula (37)	0.57

2. In the fourth illustration in Table 2, 20,000 life paid up at age 65, issued at age 27, is changed at age 35 to 50,000 term at age 96. The premiums overlap from age 35 to age 65. The reserve on 50,000 term to age 96 is 26,426.32 at age 65; the reserve at 20,000 life paid up at age 65 would be 13,794.50 if there were no change. A separate policy with the term to age 96 premium may be regarded as having 30,000 insurance maturing as a partial endowment at age 65 for the difference in reserve of 12,631.82. Under separate policies:

Net premium on 20,000 life at age 65	280.30
Net premium on 30,000 partial endowment	464.13
A. Total nets	744.43
B. Net if adjustable policy changed	743.85
Net on adjustable lower by	0.58
Difference by formula (37)	0.57

3. In the tenth illustration in Table 2, 20,000 life paid up at age 65, issued at age 27, is changed at age 35 to 30,026 endowment at age 60. The premiums overlap from age 35 to age 60. At age 60 the maturity value of the endowment of 30,026 exceeds the reserve at age 60 of the 20,000 life paid up at age 65, if continued, by 18,652. Assume that a separate policy provides 10,026 insurance and matures at age 60 for 18,652. Under separate policies:

Net premium on 20,000 life paid up at age 65	280.30
Net premium on 10,026 insurance maturing at age 60 for 18,652	507.59
A. Total nets	787.89
B. Net if adjustable policy changed	787.29
Net on adjustable lower by	0.60
Excess of A over B by formula (37)	0.59

4. A 10,000 term to age 65 adjustable policy is issued at age 35. At age 40 the plan of insurance is changed to life paid up at age 65 by increasing the premium, but no change is made in the amount of insurance. The reserve of the life paid up at age 65 policy is 6,897.25 at age 65, and the reserve if the term were continued would be zero. Assume that a separate policy is issued at age 35 for the increase in benefit—a pure endowment for 6,897.25. This would be on the preliminary term basis. Under separate policies:

Net premium on 10,000 term to age 65	84.15
Net premium on 6,897.25 pure endowment	155.09
A. Total nets	239.24
B. Net if adjustable policy changed	238.61
Net on adjustable lower by	0.63
Difference by formula (37)	0.63

5. Assume that the term policy in illustration 4 is changed at age 60 to life paid up at age 65. Since the net premium at age 60 exceeds the nineteen-pay life net premium at age 61, formula (38) would normally apply. However, in formula (4), $I_{xm} = I_{x_{m-1}}$, so that $I_{xm}\Delta_{xm}$ is negative, and the net premium from age 60 to age 65 is the level net. In this case, a separate policy would have zero insurance and 6,897.25 difference in reserves at age 65. Since there was no insurance, there would be no statutory expense; the net premium would be the level net. Under separate policies:

Net premium on 10,000 term to age 65	84.15
Level net premium on 6,897.25 pure endowment	1,163.63
A. Total nets	1,247.78
B. Level net on adjustable policy	1,247.79

There is no difference in net premiums, except for rounding, and no difference in reserves from age 60 to age 65.

XI. RELATION OF ADJUSTABLE POLICY CASH VALUES TO MINIMUM CASH VALUES UNDER THE STANDARD NONFORFEITURE LAW

Adjustable policy cash values, as stated in the Introduction, are equal to the CRVM reserves taken to the nearest dollar. During the initial status of a policy the net premium is a uniform percentage of the gross premium and provides for amortizing, during the entire premium-paying period, an expense equal to

$$I_{x_1}(\pi_{x_1} - c_{x_1}) \quad \text{when} \quad \pi_{x_1} < {}_{19}P_{x_1+1} ;$$

$$I_{x_1}({}_{19}P_{x_1+1} - c_{x_1}) \quad \text{when} \quad \pi_{x_1} \geq {}_{19}P_{x_1+1} .$$

The adjusted net premium under the Standard Nonforfeiture Law (SNFL) is a uniform percentage of the gross premium that provides for amortizing, during the entire premium-paying period, an expense equal to the least of $I_{x_1}(0.02 + 0.40P'_{x_1} + 0.25P_{x_1})$, $I_{x_1}(0.02 + 0.65P'_{x_1})$, and $(0.046I_{x_1})$, where P'_{x_1} is the adjusted net premium for the plan of insurance, and P_{x_1} is the adjusted net premium for a whole life policy with continuous premiums. As long as the SNFL expense exceeds the CRVM statutory expense, the SNFL adjusted net premium will exceed the CRVM net premium. Consequently, in the initial status of an adjustable policy, the present value of the policy benefit over the present value of future net premiums will produce greater cash values in the case of the adjustable policy. The CRVM statutory expense does not exceed the SNFL expense until age 75.

A question to be answered is: Is there any possibility that, after one or more changes of status, the SNFL adjusted net premium will produce cash values greater than the CRVM cash values in the adjustable life policy? Section X develops a proof that the adjustable policy produces reserves by the formulas in this paper that are slightly higher than the sum of reserves when each accretion of benefits is carried in a separate policy with uniform premiums. It may be inferred that, since all cash values in such separate policies would exceed the minimum cash values under the SNFL, it follows that all cash values in the adjustable policy must, in all possible instances, after a change in status, exceed the minimum SNFL cash values.

TABLE 4

ILLUSTRATION OF TEST THAT EXPENSE CHARGES IN SNFL ARE GREATER THAN EXPENSE CHARGES IN ADJUSTABLE POLICY

Illustration	Plan of Insurance (1)	Equivalent Level Amount of Insurance (2)	Value of Expense Charges in Adjustable Policy (3)	Expense Charges in SNFL Adjusted Net (4)	Difference [(4)−(3)] (5)
1.......	Life @ 65	61,303	825.75	1,792.39	966.64
2.......	Term to 96	48,453	536.73	1,385.94	849.21
3.......	Endt. @ 60	29,653	415.57	945.41	529.84
4.......	Life @ 65	10,000	182.72	327.42	144.70
5.......	Life @ 65	10,000	59.78	327.42	267.64

This conclusion may be tested by comparing, as of the original issue age of the adjustable policy, (*a*) the value of the sum of the CRVM statutory expenses charged at the beginning of each of the several statuses and (*b*) the expenses charged in the computation of the SNFL adjusted net premium for the plan of insurance in the last status and for the equivalent level amount of the varying amounts of insurance in the adjustable policy. Table 4 applies this comparison to five adjustable policies illustrated in Section X. Since the SNFL charges are greater, the test is satisfied.

Table 5 compares (*a*) the difference in the charges shown in column 5 of Table 4 amortized over the entire premium-paying period for the plan of insurance in the last status and (*b*) the difference between the level equivalent of the adjustable policy net premium and the SNFL adjusted net premium. The annual amortization of the difference in net premium charges shown in column 2 agrees with the difference in net premiums in column 5, except for illustration 2. In this case the difference is due to the SNFL net premium being exact, while the adjustable policy net premium is partially derived from a net premium which was used in the last status to find the plan of insurance and was slightly overstated.

The test involves the plan of insurance in the last of two or more known or hypothetical statuses and assumes no further change in status. The equivalent level adjustable policy net premium would be a uniform percentage of an equivalent level adjustable policy gross premium.

TABLE 5

ILLUSTRATION THAT DIFFERENCE BETWEEN SNFL NET PREMIUM AND ADJUSTABLE NET PREMIUM EQUATES TO DIFFERENCE IN EXPENSE CHARGES

Illustration	Difference in Charges from Table 4, Col. 5 (1)	Annual Amortization of Col. 1 (2)	Equivalent Level Adjustable Policy Net Premium (3)	SNFL Adjusted Net Premium (4)	Difference in Nets [(4) − (3)] (5)
1	966.64	44.36	863.05	907.41	44.36
2	849.21	35.01	606.66	641.35	34.69
3	529.84	25.97	609.00	634.96	25.96
4	144.70	7.68	200.13	207.80	7.67
5	267.64	14.21	193.60	207.80	14.20

XII. ADAPTATION OF CUSTOMARY PROCEDURES TO AN ADJUSTABLE POLICY

Underwriting

An increase in amount of insurance requested by the policyowner to take effect at the time he elects requires evidence of insurability. The suicide and incontestability provisions should provide the same exempt period with respect to the increase that applied to the original insurance.

If an option to purchase additional insurance without evidence of insurability is included by rider to the policy, subject to an additional premium, such additional insurance for prescribed maximum amounts may be purchased at specified policy anniversaries, subject to a limit on the number of purchases and some age after which purchases cease.

A natural reason for an increase in insurance is to keep the amount of insurance consistent with increasing price levels as recorded by the consumer price index. Insurance increases from this cause may be provided for at regular intervals, such as every three years. Since the date and amount of an increase would be independent of a policyowner's initiative, underwriting at issue should suffice for the benefit. If the option is not accepted at issue, underwriting should be required for later acceptance. Normally, the plan of insurance would not change when an increase is made. The premium would be increased in the same way as in other changes. A limiting age for continuance of the benefit, a maximum percentage of the existing face amount for any one increase, and a maximum dollar amount of increase should be adopted.

A reduction of premium is not associated with a reduction of reserve. Underwriting may not be required in this situation.

If the policy includes a waiver of premium disability benefit, antiselection exists if the policyowner elects a change to a very high premium in anticipation of qualifying for disability once the change is allowed. As most changes will not materially increase the disability risk, limited underwriting should cover the situation.

Substandard Insurance

Substandard insurance may be issued by use of an extra premium, the policy values, dividends, and other figures being those applicable to a standard issue. The extra premium may be the difference in net premiums by formulas (8) and (23) computed by a multiple of the CSO Table q_x. The difference in net premiums would be increased by an appropriate loading.

Dividends

A three-factor dividend formula is well adapted to the adjustable policy. Excess interest at a rate over the guaranteed rate used for reserves is computed on the initial reserve at each annual duration. A mortality gain factor may be some portion of the CSO q_x applied to the net amount at risk at each annual duration. The third factor, a return of some part of the gross premium, requires an adjustment in the actual duration of the insurance to reflect the effect of changes. An "adjusted duration" may be the lesser of (*a*) the sum of the amount of insurance in each year divided by the amount in the current year and (*b*) the sum of the amount of premium in each year divided by the current premium, omitting in each case years where the amount or premium has been reduced and adding 1 to the quotient for each year that the reduction applies. The adjusted duration will determine the appropriate percentage of the gross premium to be returned as the third factor in the dividend.

Guarantees

The basis of gross premiums and nonforfeiture values is guaranteed by the policy even though a very considerable expansion of premium and insurance may be anticipated. Some limitation of ultimate amount of insurance, subject to waiver or extension by the company, may be included in the policy provisions.

A successor mortality table to the 1958 CSO Mortality Table may be adopted at some future date and become mandatory for new insurance after a specified date. An adjustable policy should include a provision permitting past and future reserves to be determined by the new table, adjusted by difference in mortality only, subject to the revised nonforfeiture values and gross premiums after the change being as favorable as if computed by the 1958 CSO Table. Such an adjustment would occur when a policy is changed after the new mortality table becomes mandatory.

Annual Valuation

A seriatim attained-age valuation, where the attained-age multipliers in formulas (10) and (25) are adjusted to yield the mean reserve, is indicated for this type of insurance.The reserve factor is available in the data maintained for each policy.

Agents' Compensation

The objective is for agents' compensation to be comparable to compensation on regular policies. At original issue, a first-year commission based on size of the premium per 1,000 of insurance and typical nine-year renewal commissions may apply. An increase in premium up to a given level per 1,000 of insurance should be entitled to commission at the first-year rate. The unpaid renewal commissions on the previous status plus nine years' renewals on the increase in premium may be added together and paid out at the customary renewal rate on the new premium until the sum is exhausted.

Sales Illustrations

Four types of illustrations have been found useful by agents:

1. A table relating amount of insurance, annual premium, and plan of insurance. One or two panels of a folder will contain columns headed by amount of insurance. Down each column will be annual premiums which increase by increments of 1 or 2 dollars per 1,000. The table is extended to the right by additional panels containing columns headed by each age. These columns contain plan-of-insurance designations in each line. If any selection of amount and premium is located, then, by following the line to the right to the column headed by the insured's age, the plan of insurance is found. The table may be used in reverse to locate an amount or a premium.
2. Ledger sheets showing year-by-year values for typical combinations of

amount, premium, and plan. The illustrations usually include annual dividends and paid-up dividend additions. The paid-up dividend additions may be in terms of paid-up whole life insurance for all plans of insurance.
3. Specific illustrations in the same form as item 2 above, where any two of the three elements of the policy—amount, premium, or plan—are included in an order for the computer run.
4. A specific illustration where the figures assume that changes are made in future years.

XIII. AN EMPHASIS ON SERVICE

The experience developed so far by the adjustable policies described in this paper suggests that this class of individual policy life insurance will operate differently in some respects as compared with typical individual policies. On an initial sale there is little difference. An adjustable policy may be found that will reproduce any level amount and level premium individual policy. It may reproduce closely nonlevel insurance in an illustration where future reductions in insurance are assumed.

At time of an original sale, the agent should emphasize that inevitably future changes will be called for by changes in the insured's life situation. Under present inflationary conditions, practically all purchasers will accept the automatic cost-of-living increase provision. Acceptance of the increase in insurance when it first occurs and payment of the higher premium that goes with it will familiarize the policyowner with the change process.

Whenever a service interview occurs, both agent and policyowner will have a ledger statement which instantly gives the status of premiums, nonforfeiture values, and dividends. The normal increase in a policyowner's responsibilities should lead to buildup of the policy beyond cost-of-living increases.

Repeated experiences of adjusting a single policy without altering the policy provisions will increase the policyowner's understanding of his policy. He will become more confident in his own plans for developing his insurance estate. An important service is the capacity to preserve a policy from lapse by a temporary reduction of premium without a reduction of insurance, to tide the policyowner over a period of financial stress. The flexibility of premiums and use of a nonrepeating premium offer the policyowner better opportunities to coordinate his life insurance with other investments. The single-policy form will apply to any class of insurance buyers. The counsel and assistance of the agent are basic in all aspects of service.

APPENDIX I

TABLE FOR FINDING LIFE VALUATION FACTOR

$$\text{Factor} = F_{x_m} = I_{x_m}\pi_{x_m}\frac{N_{x_m}}{10^7} + \left({}_{x_m-x_1}V_{x_1} + \gamma B_{x_m} - I_{x_m}\Delta_{x_m}\right) \times \frac{D_{x_m}}{10^7} - I_{x_m}\frac{M_{x_m}}{10^7}$$

Age x_m	$N_{x_m}/10^7$	$D_{x_m}/10^7$	$M_{x_m}/10^7$
20....	13.834281	.535127	.132187
21....	13.299154	.518611	.131257
22....	12.780543	.502585	.130336
23....	12.277958	.487039	.129428
24....	11.790919	.471959	.128534
25....	11.318960	.457338	.127659
26....	10.861623	.443160	.126802
27....	10.418462	.429409	.125959
28....	9.989053	.416073	.125129
29....	9.572980	.403134	.124309
30....	9.169846	.390578	.123495
31....	8.779268	.378394	.122688
32....	8.400874	.366569	.121883
33....	8.034305	.355091	.121082
34....	7.679214	.343949	.120282
35....	7.335265	.333130	.119481
36....	7.002135	.322615	.118669
37....	6.679520	.312391	.117842
38....	6.367129	.302443	.116993
39....	6.064685	.292751	.116109
40....	5.771935	.283300	.115186
41....	5.488635	.274078	.114215
42....	5.214557	.265073	.113193
43....	4.949484	.256279	.112120
44....	4.693204	.247688	.110993
45....	4.445516	.239290	.109809
46....	4.206226	.231078	.108567
47....	3.975148	.223040	.107259
48....	3.752108	.215166	.105881
49....	3.536942	.207447	.104430
50....	3.329495	.199874	.102899
51....	3.129621	.192438	.101248
52....	2.937182	.185131	.099582
53....	2.752051	.177949	.097792
54....	2.574102	.170884	.095911
55....	2.403218	.163933	.093936
56....	2.239285	.157089	.091867
57....	2.082196	.150347	.089700
58....	1.931849	.143699	.087432
59....	1.788150	.137142	.085060
60....	1.651008	.130672	.082585
61....	1.520336	.124286	.080004
62....	1.396050	.117982	.077321
63....	1.278067	.111761	.074536
64....	1.166306	.105623	.071653
65....	1.060683	.099569	.068675
66....	0.961114	.093599	.065606
67....	0.867514	.087716	.062449
68....	0.779798	.081922	.059209
69....	0.697876	.076221	.055894
70....	0.621655	.070626	.052519

APPENDIX II

TABLE FOR FINDING ANNUITY VALUATION FACTOR

$$\text{Factor} = F'_{x_m} = I_{x_m}\pi_{x_m}f_1 + I_{x_m}kf_2$$

Maturity Age = z	$f_1/v^2\ddot{a}_{\overline{100-z}\rceil}$	$f_2/iv^{z+1}\ddot{a}_{\overline{100-z}\rceil}$	Maturity Age = z	$f_1/v^2\ddot{a}_{\overline{100-z}\rceil}$	$f_2/iv^{z+1}\ddot{a}_{\overline{100-z}\rceil}$
50.	6.045216	.176074	65.	3.240395	.094380
51.	5.817109	.169430	66.	3.093981	.909116
52.	5.595645	.162980	67.	2.951832	.085976
53.	5.380633	.156717	68.	2.813824	.081956
54.	5.171883	.150637	69.	2.679835	.078053
55.	4.969213	.144734	70.	2.549749	.074265
56.	4.772445	.139003	71.	2.423451	.070586
57.	4.581408	.133439	72.	2.300833	.067015
58.	4.395936	.128037	73.	2.181785	.063547
59.	4.214867	.122792	74.	2.066206	.060181
			75.	1.953991	.056912
60.	4.041042	.117700			
61.	3.871309	.112757			
62.	3.706519	.107957			
63.	3.546529	.103297			
64.	3.391201	.098773			

APPENDIX III

TABLE FOR FINDING LIFE RESERVES

(1958 CSO 3 Per Cent)

$$\text{Reserve} = {}_{\overline{y-x_1}}V_{x_1} = I_{x_m}A_y + F_{x_m}\frac{10^7}{D_y} - I_{x_m}\pi_{x_m}\ddot{a}_y$$

Attained Age y	A_y	$10^7/D_y$	$\ddot{a}_y$	Attained Age y	A_y	$10^7/D_y$	$\ddot{a}_y$
25.	.279136	2.186568	24.749676	50.	.514817	5.003142	16.657937
26.	.286132	2.256520	24.509470	51.	.526321	5.196471	16.262982
27.	.293031	2.328780	24.262308	52.	.537901	5.401573	15.865404
28.	.300739	2.403426	24.007952	53.	.549551	5.619591	15.465402
29.	.308359	2.480565	23.746396	54.	.561260	5.851906	15.063405
30.	.316186	2.560307	23.477619	55.	.573017	6.100054	14.659758
31.	.324232	2.642745	23.201366	56.	.584810	6.365811	14.254864
32.	.332497	2.728002	22.917598	57.	.596622	6.651301	13.849309
33.	.340989	2.816178	22.626035	58.	.608436	6.958982	13.443704
34.	.349710	2.907409	22.326613	59.	.620233	7.291711	13.038673
35.	.358662	3.001835	22.019257	60.	.631999	7.652727	12.634712
36.	.367836	3.099670	21.704311	61.	.643712	8.045964	12.232564
37.	.379227	3.201112	21.381890	62.	.655358	8.475845	11.832700
38.	.386926	3.306403	21.052294	63.	.666922	8.947638	11.435683
39.	.396615	3.415877	20.716217	64.	.678384	9.467621	11.042143
40.	.406585	3.529825	20.373918	65.	.689725	10.043309	10.652764
41.	.416723	3.648599	20.025827	66.	.700921	10.683819	10.268368
42.	.427045	3.772544	19.672144	67.	.711942	11.400382	9.889996
43.	.437490	3.901991	19.312842	68.	.722754	12.206738	9.518791
44.	.448115	4.037340	18.948062	69.	.733321	13.119771	9.155976
				70.	.743628	14.159164	8.802119
45.	.458896	4.179021	18.577908				
46.	.469826	4.327544	18.202628				
47.	.480895	4.483509	17.822612				
48.	.492091	4.647573	17.438197				
49.	.503703	4.820503	17.049840				

APPENDIX IV

TABLE FOR FINDING RESERVES AFTER *a* PERIOD

$$\text{Reserve} = {}_{y-x_1}V_{x_1} = I_{x_m} k v^{(100-y)} + \frac{F'_{x_m}}{v^y} - I_{x_m}\pi_{x_m}\ddot{a}_{\overline{100-y|}}$$

Attained Age y	v^{100-y}	$1/v^y$	$\ddot{a}_{\overline{100-y|}}$	Attained Age y	v^{100-y}	$1/v^y$	$\ddot{a}_{\overline{100-y|}}$
20.....	.093977	1.806111	31.106786	50.....	.228107	4.383906	26.501657
21.....	.096796	1.860295	31.009990	51.....	.234950	4.515423	26.266707
22.....	.099700	1.916103	30.910290	52.....	.241999	4.650886	26.024708
23.....	.102691	1.973587	30.807598	53.....	.249259	4.790412	25.775449
24.....	.105772	2.032794	30.701826	54.....	.256737	4.934125	25.518713
25.....	.108945	2.093778	30.592881	55.....	.264439	5.082149	25.254274
26.....	.112214	2.156591	30.480667	56.....	.272372	5.234613	24.981902
27.....	.115580	2.221289	30.565088	57.....	.280543	5.391651	24.701359
28.....	.119047	2.287928	30.246040	58.....	.288959	5.553401	24.412400
29.....	.122619	2.356566	30.123421	59.....	.297628	5.720003	24.114772
30.....	.126297	2.427262	29.997124	60.....	.306557	5.891603	23.808215
31.....	.130086	2.500080	29.867038	61.....	.315754	6.068351	23.492462
32.....	.133989	2.575083	29.733049	62.....	.325226	6.250402	23.167235
33.....	.138009	2.652335	29.595040	63.....	.334983	6.437914	22.832252
34.....	.142149	2.731905	29.452892	64.....	.345032	6.631051	22.487220
35.....	.146413	2.813862	29.306478	65.....	.355383	6.829983	22.131837
36.....	.150806	2.898278	29.155673	66.....	.366045	7.034882	21.765792
37.....	.155330	2.985227	29.000343	67.....	.377026	7.245929	21.388766
38.....	.159940	3.074783	28.840353	68.....	.388337	7.463307	21.000428
39.....	.164789	3.167027	28.675564	69.....	.399987	7.687206	20.600441
40.....	.169733	3.262038	28.505831	70.....	.411987	7.917822	20.188455
41.....	.174825	3.359899	28.331005	71.....	.424346	8.155357	19.164108
42.....	.180070	3.460696	28.150936	72.....	.437077	8.400017	19.327081
43.....	.185472	3.564517	27.965464	73.....	.450189	8.652018	18.876842
44.....	.191036	3.671452	27.774428	74.....	.463695	8.911578	18.413147
				75.....	.477606	9.178926	17.935542
45.....	.196767	3.781596	27.577660				
46.....	.202670	3.895044	27.374990				
47.....	.207850	4.011895	27.166240				
48.....	.215013	4.132252	26.951227				
49.....	.221463	4.256219	26.739764				

DISCUSSION OF PRECEDING PAPER

ALLAN S. EDWARDS:

I would extend a special commendation to Mr. Chapin for including the sections "Adaptation of Customary Procedures to an Adjustable Policy" and "An Emphasis on Service." He could easily have put down his pen after presenting the concept in algebraic terms; instead he chose to supplement the theory by fitting it into a practical framework. In my opinion, this supplement has enhanced the value of the paper. However, a few points remain that I should like to have clarified.

The first concerns the expense of the changes. Although a policy change has been reduced to the replacement of a single page, the corresponding clerical work and adjustment of computer records still costs something. Would a change fee be employed? If not, do the dividends for adjustable policies reflect this extra expense relative to conventional policies?

If a company were to adopt this type of policy, it would seem that the conventional forms were no longer necessary. What would be included in the rate manual? How would existing policies fit into this scheme?

I read with some alarm that "the basis of gross premiums and nonforfeiture values is guaranteed by the policy even though a very considerable expansion of premium and insurance may be anticipated." With rapid inflation in expenses in the past few years, and the possibility of more to come, this provision makes me uneasy. Although the high interest rates traditionally linked with inflation might provide a sufficient offset to increased expenses on a company-wide basis, this offset approach is unlikely to prove equitable for individual ages and plans. The problems associated with a change in mortality table are discussed briefly, but I am not convinced that they have been adequately resolved. This guarantee of gross premium basis makes the plan unsuitable for nonparticipating use and could be uncomfortable on a participating basis. Is this guarantee necessary?

The underlying concept of three principal elements of a policy, any one of which may be freely adjusted, is impressive in its simplicity and remarkable in its flexibility. Mr. Chapin has done an admirable job of transforming this simple concept into a practical product. I compliment him on an interesting and thorough presentation.

WILFRED A. KRAEGEL:

Walter Chapin really started at the beginning of the adjustable policy concept with Edward A. Rieder's idea in 1947 about using the computer for policies that could expand to meet life insurance needs over a lifetime. To my knowledge, Mr. Rieder's idea was not considered again for many years. Even the extensive set of papers on electronic data processing at the 1957 International Congress of Actuaries reveals no reference to the idea. Life insurance companies quickly learned to use computers to process existing types of policy contracts more efficiently but not to design more efficient types of policies. In part this was the state of the art, but also in part it illustrates the difficulty of introducing major conceptual changes.

The decade of the 1960's saw a gradual awakening of interest in a more flexible policy, with the strongest impetus coming from the 1967 Future Outlook Study sponsored by the Institute of Life Insurance. It is amazing that the elapsed time from the 1947 idea to the current definitive paper has been twenty-nine years. Mr. Chapin deserves our gratitude for finally giving articulate form to a previously vague but intriguing concept. It was a pleasant surprise for me to see how the adjustability idea fits so well into much of the actuarial profession's traditional life contingency structure, and Mr. Chapin has proved to be an excellent interpreter of new ideas using traditional terminology.

Since there may be some difficulty in understanding and accepting this conceptual change, perhaps Mr. Chapin will not object if I try to amplify his remarks about the nature of his approach. As I see it, the adjustable life policy has two primary characteristics that distinguish it from the traditional level amount/level premium policy.

First, at time of issue, the life insurance portfolio has consisted of a number of specific plans out of a continuum—for example, five-year term, whole life, twenty-pay life, and endowment at age 65. The adjustable policy, on the other hand, makes available the entire continuum, including such plans as seventeen-year term, life paid up at age 61, and endowment at age 53. Whether or not a company makes that full range of plans available is a matter of choice.

Second, at time of change in a policyowner's needs and/or ability to pay, the traditional approach for increased coverage has been to issue a new policy for incremental needs, at a premium level reflecting current ability to pay. The adjustable policy, however, can accommodate the change in the original contract, thereby avoiding the proliferation of additional policies on the same life.

Some may ask how useful the adjustability concept is in actual practice. Only time will tell, but there are several intuitive indicators which seem to be of significance. In my opinion:

1. A single up-to-date policy is easier for the policyowner to understand and relate to than multiple policies.
2. The policyowner will sense a greater continuity in relationship with the agent and the company.
3. If the contract is properly constructed, the undesirable and obsolete features of earlier contracts may be rectified in later adjustments.
4. The agent should experience greater productivity for two reasons: the policyowner's sense of continuity in relationship and the reduction in the number of records to be handled. Repeat sales should be easier, thereby making more time available for new clients.
5. The company should also experience greater productivity, for similar reasons.

On the other hand, the policy adjustments may give rise to some complicated problems. For example:

1. How should the lives be treated for mortality study purposes? Is the policy after adjustment entirely select, or entirely nonselect, or must the individual pieces retain their identity?
2. Can amortization of expenses be done on a blended basis (Mr. Chapin proposes one such approach), or must the individual pieces again be retained?
3. Commission problems can be especially difficult. If the policy is originally whole life for $25,000, and five years later, changes to seventeen-pay life for $50,000, which commission rates apply to which parts?
4. How would lapse studies be conducted? Would a reduction in face amount be treated as a lapse for the amount of the decrease? If so, would the same rule apply if the reduction in face amount were accompanied by an increase in premium (e.g., as the policyowner nears retirement)?
5. After six or eight changes, can the nonforfeiture values still be assumed equal to or greater than those initially illustrated? Can differences in interest and/or mortality assumptions be handled?
6. If the policy loan interest rate for new issues is different at the time of a change, which rate applies to the updated policy?
7. How can different owners be accommodated within the same contract?

These are difficult questions, and there are more waiting to be asked. I will not ask Mr. Chapin to answer them, because my intention is to illuminate the concept, not to oppose it. The Chapin approach brings to us not only solutions to old problems but new problems as well.

I will not try to answer these questions specifically either, but I would like to express some thoughts about the ways in which they can be viewed. First, the answer to some of the questions is simply to issue a new policy and retain the old in situations where the old and new circumstances are incompatible. A second answer is to accept the requirement that certain historical pieces of the policy be retained—for example, for mortality studies. A third answer is more subtle. Some of our practices and procedures are the direct result of having multiple policies on a life, and we must consider new practices and procedures that reflect adjustability. Perhaps we should redefine the bases for lapse studies, mortality studies, expense amortization, and so on.

In short, an adjustable policy may be more complicated than a single traditional policy, but that is not a valid comparison. Instead, the adjustable policy must be compared with two or more traditional policies, and then it will probably compare quite favorably.

In closing, I would like to thank Mr. Chapin for widening our horizons significantly in the field of life insurance contract design.

ROBERT E. HUNSTAD:

Mr. Chapin is to be congratulated for the major contribution that his paper makes to the Society's literature. Not only is this an important product innovation; it also may mean a new direction in the marketing and servicing of ordinary life insurance in the future.

This discussion will review the concept behind this product, particularly in reference to traditional products, and also comment on the practical application of the material contained in the paper.

The Concept

There are two important features that could be described as differences between the adjustable life product and traditional products marketed in the past:

1. An in-force adjustable life policy may be changed to provide additional insurance, more premium, reduced insurance, or less premium. Is this really a departure from traditional products? Those familiar with policy change provisions and practices of individual companies will recognize that each of the last three is done with current policy change provisions. Thus, at least with respect to these three, the "flexibility" is not totally new.
2. The policy plan is freed from a limited list of possible plans and allowed to vary over an entire spectrum of term, limited pay life, endowment, and so on. Here is the unique contribution made by this paper. The paper provides a mathematical relationship of reserves, net premiums, and gross premiums that allows the plan of insurance to be removed from the restrictions of traditional products.

The paper also makes important contributions in suggesting techniques for handling small residual amounts of reserve at the end of a term period, premium-paying period, or endowment period. As the paper states, this approach is heavily reliant on the availability of modern computing machinery; it could not have been successful without that important tool.

There are several implications of this latter contribution:

1. Policy flexibility is significantly enhanced. Any combination of premium, face amount, and plan may be allowed, subject to constraints that are desired by the company.
2. Many of the restrictions on policy changes can be eliminated. For example, in a traditional policy that permits a change to a lower-premium plan of insurance, the release of cash value necessitates some underwriting investigation to determine the appropriateness of that release. With the adjustable life concept, there need not be a cash-value release at the time the premium is reduced. Thus there is no underwriting constraint. It is also possible to design a waiver of premium benefit that has limits such that an increase in premium per thousand likewise does not require evidence of insurability. Such a waiver benefit could provide that, upon disability, only a whole life premium is waived rather than a limited pay life premium.
3. The flexibility gives a unique opportunity for companies to market cost-of-living coverage. Previous marketing attempts have generally had limited success. The primary deterrent was the requirement that a new policy be written to provide the additional face amount. With adjustable life, this is no longer necessary. The additional amount can be incorporated into the original policy. Incorporating the provision within each contract and making the provision automatic at each option date has been shown, by experience, to be a highly successful approach.
4. Finally, the flexibility offers immense service opportunities. When the policy is originally sold on the basis of only need and ability to pay (and not plan of insurance), it offers the opportunity for the salesman to return to the client for a current evaluation of need and ability to pay. Certain events (or a lack of events) within the contract itself can also produce a series of "sales leads." For example:
   a) Coverage may be expiring within a few years.
   b) The cash value may have reached a maximum and may soon begin to decline.
   c) Premiums for a supplemental benefit (guaranteed purchase option, for example) may be terminating, thus allowing an additional premium to be applied to the basic coverage.
   d) There may have been no change activity for a few years.

Each of these offers an opportunity for the salesman to discuss with the owner his current insurance needs and ability to pay.

Application of the Concept

The theory presented in Mr. Chapin's paper can be and has been applied to produce a marketable product. The Minnesota Mutual Life has marketed an adjustable life policy since 1971. The *Record* gives sales results (I, No. 1, 56 and II, No. 4, 810). The product being offered encompasses only two of the five possible categories of policies suggested by Mr. Chapin; only term and limited pay whole life are provided.

Previous surveys of actuaries indicated concern about whether this approach would be accepted by the various states. A policy form and an actuarial memorandum consistent with the formulas in Mr. Chapin's paper have been submitted to each of the fifty states, the District of Columbia, and Puerto Rico. Forty-eight of these jurisdictions have approved the policy and the nonforfeiture basis.

Two of the nonapprovals relate to the nonforfeiture treatment. One of the jurisdictions has suggested that all the possible combinations of results (a virtually infinite number) must be submitted before the nonforfeiture basis can be approved. The other disapproving jurisdiction has suggested that it is inappropriate to provide additional expense allowance for any change in status of the policy. Thus, with the exception of these two states, it would seem that the nonforfeiture basis for an adjustable life policy is not a significant deterrent to the marketing of such coverage.

The other two states in which approval has not been secured both feel that the product is too complicated and potentially misleading. In past surveys actuaries have also expressed the opinion that the product may be too complex and difficult to explain to the consumer. Our experience has indicated the opposite. While the mathematical properties may be complex, the operation of the product, in the view of the policyholder, can be quite simple. Our experience has borne out the statement in the concluding paragraph of Mr. Chapin's paper: "Repeated experiences of adjusting a single policy without altering the policy provisions will increase the policyowner's understanding of his policy."

CHARLES E. ROHM:

I want to thank Mr. Chapin for this fine pioneering paper. I believe that the introduction of an adjustable individual life policy is a major event in the evolution of the life insurance business. It is a concept that has long been discussed. Mr. Chapin's ingenuity and skill have now made the dream a reality. His innovation will have a significant and far-reaching effect on the market for individual life insurance.

There are some real advantages that flow from the flexibility inherent in the reserve and cash-value mechanics that Mr. Chapin has introduced into the adjustable life policy. These advantages deserve some discussion because many are not immediately apparent.

For most people an adjustable life policy can be the only life insurance policy they will ever need. Its flexibility permits adjustment of the death benefit, the premium amount, or both to meet the changing needs and the changing premium payment ability of the policyholder. Instead of the typical stack of policies and riders that accumulate through the normal development of a life insurance program, there can be a single contract. The current policy data page, policy values page, and, perhaps, a computer-prepared "ledger statement" would give a coordinated and up-to-date presentation of the individual's insurance program. This should produce a great improvement in policyholders' understanding and appreciation of their programs.

An adjustable life policy facilitates the development of an interrelationship among the policyholder, the agent, and the company that should benefit all parties. As Mr. Chapin noted, there is an inherent emphasis on service. If the agent and the company do their part with good reports, frequent contacts, and so on, the policyholder and the policy will receive good service.

The result should be a long-lasting relationship. The flexibility of the policy should make it lapse-resistant. It cannot really be sold improperly because, if it does not fit, it can be adjusted. This should be true not only for the first few years after issue but throughout the life of the policyholder. For many of the same reasons, it should also be replacement-resistant. An agent who stays in touch and practices professionalism should have excellent client control. The policyholder will have a single contract, and a single agent that can provide a coherent life insurance program and keep it abreast of changing needs. As a result, the insurance company should benefit from better persistency of both policyholders and agents.

An adjustable life policy gives a policyholder more ability to adjust his or her insurance program to reflect decreases in the purchasing power of the dollar. The flexibility of the policy makes possible a cost-of-living option that is the best solution to the problem of inflation that the life insurance industry has yet been able to devise.

Another advantage of adjustable life is that it provides an attractive alternative for a policyholder who is underwritten as substandard. Of course, the traditional method of charging an extra premium that maintains the desired plan and face amount can be used. However, it is also possible to keep the face amount and premium constant but to change the plan. In this way, the anticipated extra mortality affects the plan and cash values instead of the premium. This could improve significantly the acceptance of rated policies and increases in coverage.

A new type of dividend option is possible with an adjustable life policy. This option is an enhancement of the dividend additions option that is customary with a static whole life type of policy. Under the new "policy improvement option," the dividend would go directly into the cash value of the policy rather than into a separate fund. This increment to the cash value would be used either (1) to increase the face amount if the policy provides lifetime protection (similar to the effect of dividend additions) or (2) to change the plan by extending the protection period, with no change in face amount, if the policy provides protection for a limited period. Often, the best disposition of dividends is to use them to increase the basic policy cash value. The second alternative would obviously not be possible with a static policy and can be used only with an adjustable life policy, where the plan can easily be updated.

Additional flexibility is available in the selection of the nonforfeiture option if a zero premium situation becomes necessary. Like any other policy that has cash values, adjustable life would have extended insurance and reduced paid-up options available. Extended insurance can be seen as one extreme of the possible adjustments to a zero premium situation; the face amount is held at the previous level, and the adjustment is made by revising the plan (to paid-up term insurance). Reduced paid-up is at the other extreme of zero premium adjustments; the plan is specified (paid-up whole life insurance), and the adjustment is made by reducing the amount of the death benefit. Under adjustable life it is possible to have either of these extremes or any intermediate form of adjustment. Thus it is possible, if lapse occurs, to tailor-make the combination of death benefit amount and plan.

Reinstatement also has a new look with adjustable life. Because an adjustment can be made in the plan (that is, to the cash values), it is not necessary at reinstatement to require repayment of back premiums with interest. If the policy is in force as paid-up term insurance (extended insurance option), the policyholder could simply begin premium payments in an amount that meets the requirements of the adjustment option without paying past-due premiums. Again, because of the flexibility, the necessary adjustment (this time for reinstatement) can be made in the plan rather than by requiring additional charges.

Finally, a potential advantage would be to supplement or replace the policy loan provision with a partial withdrawal provision. Currently, the only way a policyholder can get money out of a policy without surrendering it is through a policy loan. With an adjustable life policy another method, partial withdrawal, is possible. The policyholder could withdraw funds from the policy in an amount not to exceed the cash value. The policy would remain in force subject only to payment of sufficient future premiums, and the decrease in cash value would be reflected by a change in plan.

There are very many difficult practical problems involved in the design, development, and implementation of an adjustable life policy. Perhaps the most challenging is to design an actuarial basis that fulfills all the requirements of an adjustable changing-needs policy while still satisfying the nonforfeiture and valuation statutes and regulations of the several states. Mr. Chapin's creative approach solves this most difficult of problems, and does it in such a way that cash values can recognize the incidence of expenses and still come within the Standard Nonforfeiture Law.

There are also problems in developing the pricing. The scales of adjustable life gross premiums, cash values, and dividends must be consistent not only internally but also with the scales for the regular individual policy portfolio. It is not easy to devise an adjustable life pricing system that meets these consistency objectives and at the same time is adequate, equitable, and not too complex. The job is made even more difficult by the absence of data regarding the effect of the adjustable life features on basic experience factors. For example, there is no experience base for projecting the persistency of an adjustable life policy either from issue or from subsequent adjustments. In addition, one can only guess at the effect on mortality experience of the use of either the regular adjustment option or a cost-of-living option. Finally, as Mr. Chapin has noted, for most policyholders adjustable life entails an emphasis on service by both the agent and the company. The expense effect of this additional service must be considered in the pricing calculations.

An important practical question that must be decided fairly early in the development of adjustable life is the range of plans that will be available. Mr. Chapin has created a theoretical structure that will accommodate a spectrum of plans from one-year term to whole life insurance, and then down either one branch to paid-up life or a second branch to paid-up endowment. There are some difficult practical problems in extending adjustable life to the farthest ends of the spectrum. One is the complication involved in having both a limited pay life path and an endowment path. Since there currently are few prospects interested in buying short-term endowments, one solution is to simplify the choices by eliminating endowment possibilities. Even then, there are problems with either the very short term plans or the very short premium period whole life plans. These involve mainly the need for price consistency referred to in the preceding paragraph. The difficulties can be dealt with by curtailing further the plan possibilities for adjustable life and by making adjustments in the regular portfolio.

Earlier I mentioned the advantages of the use of a cost-of-living option with an adjustable life policy to meet the inflation problem. There are many variables involved in the design of such a cost-of-living provision. Some of the questions involved are the following.

1. Should it be automatically included at no extra premium, or should it be optionally available at a stated extra cost?
2. How often should adjustments be made?
3. How should the amount be determined?
4. Should there be a limit on the dollar amount of increase?
5. Should the standard adjustment be an increase in face amount with the same plan (and a disproportionate increase in premium), or an increase in premium equal to the percentage increase in death benefit (with a cheapening of the plan)?

These and other questions must be answered in a way that provides a cost-of-living provision that is mutually advantageous to policyholders and company and can be provided at the lowest possible cost. Mr. Chapin suggests good answers to some of these questions.

The waiver of premium benefit when offered with adjustable life also takes on a new appearance and presents new problems. The design of this benefit and the method of pricing are likely to be different for adjustable life. In the first place, a waiver benefit is needed that can continue to function and do a proper job throughout the various changes in amount and plan that can be expected over the lifetime of an adjustable life policy. The determination of this design requires a decision as to the basic function of the waiver of premium benefit. Should the benefit incorporate (1) a pure waiver of premium approach where whatever premium is in force at the time of disability is waived into the future, (2) a true preservation of the death benefit approach that has no concern with the amount of premium waived or the buildup of cash values, or (3) a combination approach that seeks to preserve the death benefit and also provide a reasonable buildup of values within the policy? Whatever design is picked should be one that will be reasonable to the policyholder, clearly understood by all involved, and compatible with the company's administrative systems.

An adjustable life policy also raises interesting questions in regard to price competition and cost comparisons. Most current static plans offered by reputable companies are quite similar. Although there are technical problems in making meaningful cost comparisons of these policies, such comparisons can be made. Adjustable life introduces new considerations into this price competition picture.

First, the only recognized bases for comparing prices among competing life insurance products (interest-adjusted net payment and net cost indexes) require that the two products be similar as to death benefits and premium-payment periods. An adjustable life policy can come in many forms and can be comparable to almost any given standard plan. However, the reverse is not always true. If an individual has determined that his needs are best met by an adjustable life policy that is initially term to age 73, there can be no meaningful price comparison except with another company's adjustable life policy.

There will, of course, be some situations where cost comparisons between adjustable life and traditional plans appear to be appropriate and can be made. There is a logical fallacy, however, in comparing an adjustable life policy on a static basis—that is, on the assumption of no change in plan for as long as twenty years. If the change is indeed unlikely, then adjustable life probably should not be under serious consideration. If change is likely, then static comparisons lose most of their significance.

Adjustable life obscures pure price competition with traditional static policies by adding important quality features. The adjustment features and a cost-of-living option are the most obvious of these. The public can be expected to appreciate these and other quality features of adjustable life and to expect them to add something to the cost. Thus, in the near future adjustable life will shift the competitive emphasis away from cost to other considerations. Eventually, as adjustable life policies are marketed more widely, price competition among different adjustable life policies can be expected.

Of course, this list of practical development problems could go on and on. In addition to those Mr. Chapin has explored, it might include such items as design of agent compensation, preparation of sales promotion material, field training, policy form drafting and submission, electronic data processing systems, routine administrative procedures, and policyholder service requirements. To survey these areas even briefly would require an extensive paper, and, because they are less actuarial than those discussed above, I will make only this reference to them.

It is sufficient to say that the development and introduction of an adjustable life policy are a mammoth job. One must "reinvent the wheel" for a portfolio of individual life insurance policies. It is necessary to reconsider carefully all aspects of plan design and policy provisions in regard to such criteria as the theoretical ideal, consistency with products and practices within your own company and other companies, and anticipated acceptance by the public and the sales force.

It is necessary that those with experience in individual life insurance adjust their thinking to adjustable life. Traditional attitudes and approaches are not always appropriate. This is particularly true in regard to two aspects of designing and analyzing an individual's insurance program.

One is concerned with the order of priority in a life insurance purchase decision. As we view life insurance today, the prospect and the agent tentatively decide upon a plan and amount of insurance and then go to the ratebook to determine the required premium. If the premium fits the prospect's ability to pay, the purchase decision may move ahead; if not, an adjustment in the plan or amount or both may be called for. With adjustable life, plan does not need to come first, and the order of setting specifications can be changed. The initial premium and initial face amount can be specified and the plan solved for. This should appear to the prospect to be a more logical approach. Moreover, it can be used at future dates when adjustments in the insurance program are made. Under the adjustable life mechanism, the individual's current situation is automatically integrated into the newly adjusted program.

The plan is often not of immediate interest to the prospect. Furthermore, if the prospect can be assured that at any future time the plan can be changed by adjusting the amount of insurance or the premium or both, there may be minimal interest in whether the initial arrangement is technically term or permanent.

This leads to the second attitude adjustment that is needed. The old distinction between term and permanent is usually not appropriate for an adjustable life policy. In every sense an adjustable life policy should be a permanent policy regardless of what the current static plan may be. Its flexibility means that it can be the only policy a person ever owns even if the initial version corresponds to a ten-year term plan.

An adjustable life policy can be adjusted upward or downward in amount and/or premium to accommodate the needs and premium capabilities of the insured. It seems better to look at it in terms of what it can adjust to in the future rather than to concentrate on whatever plan today's premium/face amount relationship requires for defining values and dividends. The emphasis should be on the basic permanent result that flows from the adjustability.

In conclusion, I want to point out again my belief that Mr. Chapin's creation will become a major factor in the life insurance business. It takes the strengths of individual life insurance as we know it today and adds the tremendous advantage of true flexibility. He has made a significant contribution to both the actuarial profession and the life insurance industry.

(AUTHOR'S REVIEW OF DISCUSSION)

WALTER L. CHAPIN:

The four discussions of the paper provide an excellent exposition of adjustable life. At this early stage of the product, it is fortunate in having the views of four actuaries with different degrees of contact with adjustable life, ranging from one who was introduced to it by the paper to one who has been responsible for a going operation. My comments will be confined to one or two topics from each discussion.

In his discussion Mr. Edwards asks whether a fee should be charged for an adjustment. In most cases part or all of the cost will be covered by the expense allowance resulting from the change. One decision might be that costs not covered by the expense allowance should be reflected in the premium rates, since all policyholders have an equal right to make adjustments.

Typical agency material probably does not fit into a company rate manual. The material may include tables showing combinations of premium, amount of insurance, and plan at each issue age, and illustrative ledger statements and the procedure for requesting these statements for specific cases. One can imagine a large agency equipped with a computer terminal into which an agent may input age, sex, amount of insurance, and premium, and within five minutes have a complete ledger statement including dividend illustrations. Proposals may include ledger statements illustrating hypothetical future changes.

Mr. Edwards questions the extent to which premiums for future increases to a participating policy may be guaranteed, and points out that a guarantee of the premium basis makes adjustable life unsuitable for a nonparticipating company. Mr. Kraegel touches on the same point in the fifth of his list of problems that need to be solved: "After six or eight changes, can the nonforfeiture values still be assumed equal to or greater than those initially illustrated? Can differences in interest and/or mortality assumptions be handled?"

While participating policies may rely on the dividend cushion to avoid a new gross premium and nonforfeiture value basis where relatively small changes in interest, mortality, or expense levels occur, there will be times when some change of basis for new issues is necessary. A new mortality table mandatory for new issues is one example. The problem is how to make an adjustment when the premium and benefits of a policy on an old basis are increased after a new basis is in effect for new issues.

When a new basis goes into effect, all policies on the old basis will represent one block of business, and a separate block of business will develop for new issues. When the premium and benefits on an old-basis policy are increased, the future gross premium and nonforfeiture values should be on the new basis. The policy may then transfer to the new-business block of policies.

An equitable criterion for a transition from one basis to another is that a continuing adjustable policy have a gross premium and cash values at least as favorable as the sum of (1) the gross premium and cash values in the adjustment policy if continued without change and (2) the gross premium and cash values of a separate policy that provides coverage equal to the increase in benefits on the new basis. The procedure to be adopted for effecting a transition to a new basis is dependent on the factors entering into the change of basis. In an extreme situation, such as a change in the nonforfeiture value interest rate from 4 to 3 per cent, it may not be possible to devise an equitable procedure for transition. In such a case, the increase in benefits should be provided by a separate policy instead of by adjustment to the original policy. However, in the majority of changes in bases, the changes are not extreme and may involve favorable as well as restrictive changes. In these cases it seems possible to devise an equitable transition method for both participating and nonparticipating policies.

Mr. Kraegel raises other questions to be resolved, among which are questions relating to mortality and lapse studies. Adjustable life experience indicates better-than-average persistency by number of policies. Even more favorable has been the persistency experience by amount of premium and amount of insurance. This is the result of increases exceeding decreases on in-force policies, primarily because of a cost-of-living feature in the policy providing for increases in amount and premium at three-year intervals without evidence of insurability. The experience indicates only a very small decrease in premium and amount by duration, and it is quite likely that for a year of issue the total premium and amount will be at least equal to the initial premium and amount for a number of years, since surrenders will decrease with increasing duration. These cost-of-living increases are of course acquired at lower expense than original issues.

Mr. Kraegel suggests that an adjustable life policy may best be compared with two or more traditional policies instead of being dissected into parts comparable to separate policies. This point of view has been followed in explaining the policy in state filings. With approval by 92 per cent of United States jurisdictions, as reported by Mr. Hunstad, there is a sufficiently clear field for adjustable life to operate under present law. The objections in the four states seem of a type that might be overcome if the policy becomes successful in a number of companies.

Mr. Hunstad comments on the treatment of a small surplus reserve that occurs when premium and amount are given and the item solved for is the plan of insurance. One choice is to use the first computer solution of the plan to recompute the premium, treating plan and amount as known. The other choice is to make no second calculation but to dispose of the small surplus amount at expiry of a term policy by adding it to the dividend or providing for a short extension of the period of coverage. In the case of whole life, the amount of paid-up insurance at the end of the premium period may be increased. Mr. Rohm points out the advantages, in a sale, of reaching agreement on amount of insurance and premium and then solving for the plan. Since so many sales occur in this manner, it may be better to issue the policy for the exact premium in the application rather than to correct it downward by a small amount.

Mr. Rohm discusses how far an adjustable policy should go in providing for different plans of insurance. Term with continuous premiums to any age at expiry and whole life with premiums payable to any age offer a wide range of plans. From the computer programming standpoint, continuous or limited pay endowments may be included without complication. One question is whether confusion will develop when a given amount of insurance and premium could produce either a limited pay whole life plan, a continuous premium endowment, or a limited pay endowment. If it is desired to have every possible term, life, or endowment plan available, the agent may choose whole life or continuous premium endowment as plan categories. A choice of limited pay endowment would require indicating it as a separate category and naming maturity age.

Mr. Rohm comments on a temporary reduction of the premium to avoid a lapse. If reduction to zero is employed, the extended term nonforfeiture option is appropriate. Another approach related to saving a policy from lapse is to allow reduction of the premium to some small amount, such as a policy fee of $20. The policy then continues as premium-paying, although the period it will run is limited because of the rapid reduction in the reserve. With regard to reinstatement, the process described by Mr. Rohm of restoring the premium without requiring payment of back premiums, either by amortizing the back premiums over the future premium-paying period or by adjusting the plan, is another method of conserving insurance.

Mr. Rohm also discusses the possibility of withdrawing cash from the policy rather than obtaining it through a loan. The reverse of withdrawal is the payment of a nonrepeating premium to build up the policy value.

The discussions have made a very valuable contribution by expanding on the special elements of adjustable life—a flexible individual life product providing a better vehicle for changing needs; the immediate delivery of ledger statement information; and the possibilities of better rapport between agent and customer, improved income to the agent from buildup sales, better persistency, and potential home office economies. I wish to thank Allan Edwards, Wilfred Kraegel, Robert Hunstad, and Charles Rohm for their contributions.