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INTEREST ALLOCATION USING A COMPUTER MODEL

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ABSTRACT

This paper describes a method of interest allocation which relies on a computerized model of invested assets. Characteristics of this method, known as the "investment generation model" method, are discussed in relation to other allocation methods in current use and in relation to desirable criteria of interest allocation given in this paper. The paper concludes by identifying important management benefits associated with this method of allocation. The Appendix gives a simple illustration of the application of this method to a hypothetical company.

INTRODUCTION

THE primary objective of any interest allocation process is to achieve equity among classes of policyholders, that is, to credit each major class of policyholders with its fair share of the investment return from a common pool of assets, recognizing the significant characteristics of the contributions of each class which affect the actual investment return. Equity is a relative concept and is achieved only in degree, not absolutely. Hence its achievement is often as much an art as it is a science. Interest allocation, with its desire to achieve equity and with its many practical constraints, requires that the actuary subdue his natural punctilious inclinations, substituting satisfaction for certainties and practicalities for precision.

ALLOCATION CRITERIA

For an allocation method to be both satisfying and practical, it should fulfill certain desirable allocation criteria. The following should be basic to any method:

1. Ensure that each class of participating policyholders receives the benefit of its actual contributions to the company's investment return, so that it is possible to provide each class with its insurance coverage at cost.
2. Ensure that there is consistency between the interest assumptions in premium rates and the interest credited to any line of business, so that products can be marketed competitively and management reports on the earnings of each line of business will be meaningful.

In addition, a good allocation method should satisfy a number of practical criteria. It should do as many as possible of the following:

1. Recognize, for each class of policyholder, the characteristics of investment contributions which significantly affect the company's over-all rate of return. These characteristics include the amount of contributions, the time of receipt, and any condition imposed on investment, such as policy loans or marginal rate considerations.
2. Achieve the stability and diversity of pooled investments which result from all lines sharing in the return of a common investment pool. A direct implication of this criterion is that particular lines of business should not be directly affected by investment activities or changes in portfolio resulting from the exercise of managerial judgment, under particular market conditions, except as such activity affects the over-all investment return on the pool.
3. Satisfy regulatory authorities.
4. Exhibit simplicity in concept in order that the implications of the method may be understood and accepted. The method should act as an aid, rather than a hindrance, to communication among those involved in investment, actuarial, and marketing activities.
5. Exhibit simplicity of application so that solutions to the problems associated with record-keeping, computer usage, use of obsolete historical records, availability of input data, and alternative methods of introduction do not become so detailed and complex that the costs of implementation and operation outweigh the benefits.
6. Demonstrate consistency between the bases used to value the assets underlying the allocation process and the bases used to measure actual investment income. Other objectives of the allocation process might be thwarted if, for example, the asset allocation base were valued at market and the actual investment income valued on the basis of amortized cost.
7. Allow flexibility and independence, within over-all guidelines, of the investment operation. Investment strategy should be free to pursue over-all portfolio objectives and should not be constrained artificially by the requirements of an allocation process.
8. Accommodate all types of investment situations found in a life company, including policy loans, equities, reinvested assets, head office buildings, and cash.
9. Be suitable for the allocation of both realized and unrealized capital gains and losses. If possible, the method should have the capacity to be extended to the allocation of other investment-related items, such as the mandatory securities valuation reserve and federal income tax.
10. Result in an interest allocation which facilitates product pricing and dividends and is so structured that investment antiselection on the part of policyholders will be discouraged.
11. Have the capacity to be directly extended to sublines or to policyholder experience funds within lines of business, if this is desirable.

METHODS IN CURRENT USE

It has been almost twelve years since Edward A. Green, in his paper "The Case for Refinement in Methods of Allocating Investment Income" (*TSA*, XIII, 308), described the investment generation method as an alternative to the mean fund method then in general use. Since that time there has been both widespread adoption of the investment generation approach and also experimentation with, and adoption of, other methods. Since all the methods which have evolved have certain similarities, it is not possible to identify a pre-emptive list of allocation methods. There are, however, four general approaches to allocation. Each of the four approaches, with either the characteristics described below or some modification of them, is in use today by North American companies. The following descriptions highlight the main features of each approach.

Mean Fund Method

Each class of policyholders shares in the investment income of all the company's assets in proportion to the net contribution of the class to the aggregate book value of the company's assets (funds). The result of this method is that each dollar of net investment, regardless of time or conditions of receipt, earns the same rate of interest—the average rate earned on invested assets. Money invested by any class, regardless of whether market yields are high or low at the time of investment, will be credited the average rate of return earned, from time to time, on the company's entire portfolio.

Segregated Portfolio Method

Each class of policyholders receive the investment income of a specially earmarked group of assets. This earmarking of assets is for the purpose of allocating investment income only, and the capital values of all assets are available for the protection of all policyholders. The result of this method is that each dollar of net investment earns the average rate of interest experienced by the "segregated fund" in which it is invested. Money invested by a particular class is credited with the actual investment return of the assets specially earmarked for that class. This return will reflect both the interest rates available at the time money was invested and the type of assets included in the "segregated" portfolio.

Investment Generation Method

Each class of policyholders shares in the investment income of separate "generations" of assets in proportion to the net contribution of the class

to the investable funds making up each generation. An investment generation is defined as a given period of time, often a calendar year. The result of this method is that each dollar of net investment in a given investment generation will be credited with the investment and reinvestment experience of the assets acquired for that generation. Money invested by a class when market yields are high will achieve a better return than money invested when yields are low. A corollary is that money invested in a particular investment generation will achieve the portfolio mix of that generation only (and its subsequent reinvestments) and, if the method is extended to capital gains and losses, will experience the full effects of capital gains or losses of its own investment generation.

Accumulated Unit Method

Each class of policyholders shares in the investment income of all the company's assets in proportion to the net contribution of the class to the aggregate market value of the assets. The portion of the market value applicable to any class of policyholders is determined in the same way as the number of units in a mutual investment fund. The result of this method is that each dollar of net investment will be credited with a share of the company's total investment return in proportion to its share of the total number of units. Because unit values vary directly with market values, money invested by a class when market yields are high will achieve a better return than money invested when market yields are low.

EVALUATION OF METHODS

Experience has indicated that each of the four general approaches described above fails in some important way to satisfy the allocation criteria given. Rather than attempting a complete evaluation, the following points highlight the major deficiencies of each method which tend to limit its usefulness.

MEAN FUND

1. This method, because it fails to recognize the time of receipt of different investment contributions, often gives inequitable results, especially in periods of changing interest rates. As a consequence, this method has fallen from its place of pre-eminence among allocation processes to one of relative disfavor among most insurance companies.
2. It is inconsistent with the assumptions built into many premium rates. The result is that financial operating gains become unintelligible and investment antiselection on the part of policyholders is encouraged.

SEGREGATED PORTFOLIO

1. This method, by definition, negates the stability and diversity of return associated with a single, common investment pool.

2. It may result in a less autonomous investment operation, since each "segregated" portfolio will have different constraints on its investment strategy dictated by the different product requirements represented in each portfolio. There is the additional difficulty of deciding which portfolio gets which investment, how to split up large investments, and what to do with uninvested funds.

INVESTMENT GENERATION

1. This method, even in its simplest application, requires very detailed and complex record-keeping. As a result, it is difficult to implement and impractical without computerized investment records.
2. It is often deemed unsuitable for handling capital gains and losses and usually requires special treatment for reinvested assets, cash, and equities.
3. The method fails to accomplish a complete pooling of investment risk, and the return on particular lines of business may be influenced directly by specific investment decisions. The reason for this is that, if the investment results of the assets acquired in a particular generation have deviated significantly from the expected, this experience will be reflected in the lines of business that contributed investable funds to that particular generation of assets.

UNIT ACCUMULATION

1. This method has theoretical advantages, but it is inconsistent with the book value approach to the measurement of investment income which is integral to both life insurance accounting and premium setting.
2. It requires frequent market valuations of all assets.

INVESTMENT GENERATION MODEL ALLOCATION METHOD

The observed deficiencies of methods in current use prompted the search for a method of allocation which would provide greater satisfaction in terms of the criteria given. Most of the deficiencies observed in more refined existing methods are directly attributable to the variety and complexity of a company's actual invested assets. An improved method would be one which would eliminate most of the detail and complexity associated with a company's actual asset portfolio, substituting for it a much more simplified model of invested assets. The interest allocation method which takes this approach, which incorporates the best features of the four methods in common use and also produces the most satisfying results in terms of the given criteria, can be described as the "investment generation model" (IGM) allocation method.

The IGM method is similar to the conventional investment generation method in that it identifies generations of assets, where each generation has different investment characteristics and where the funds making up each generation are identified as having been originally contributed by specific lines of business. As in the investment generation method, total

assets under the IGM method are equal to the company's actual invested assets. The IGM method is, however, different in a fundamental way from the investment generation method. Instead of earmarking the actual assets purchased for each generation, the IGM method develops a model of assets which, under the then current investment conditions, might reasonably have been purchased for each generation. This requires that the model assets be given investment characteristics, such as rate of return and term of investment, which are representative of assets which could have been acquired, under actual market conditions, for each generation. The model assets are kept as simple as possible and are given only those characteristics which are essential to accomplish an equitable allocation of investment results in accordance with the desirable criteria. Hence the assets in the model will not necessarily have the same characteristics as those found in the company's actual portfolio. The combination of all asset generations in the model then becomes a simplified representation of what the company's invested assets might reasonably be like if investment expectations, current at the time each new generation of assets was acquired, had actually been realized.

Operation of this asset model generates what may be termed "expected investment income" for each asset generation and each line of business. Actual investment income is then allocated by line of business in proportion to "expected." Since expected investment return depends on the definition given to it by the user, expected return can encompass all facets of investment performance, including net capital gains. Hence the IGM method can be extended easily to allocate capital gains and losses.

With the IGM method, each class of policyholders shares in the total investment return of all the company's assets in proportion to its share of the expected investment income generated by the asset model. Since the asset model is built on the investment generation principle, the effect of the IGM method is to recognize the potential impact of the time of receipt of money on actual investment return. This approach allows the IGM method to adopt the best characteristics of the other allocation methods.

INVESTMENT GENERATION MODEL METHOD

ILLUSTRATIVE CHARACTERISTICS

Because the IGM method is a generalized method, it can take on different forms to suit particular circumstances. For example, the investment characteristics assigned to one company's model of assets might legitimately be quite inappropriate for another company's model. Con-

sequently, the following characteristics, based on a particular application, are given for the purpose of illustration only and should be considered in no way completely definitive of the method.

1. The IGM method takes into account the original source of investable funds, including lines of business, profit, and country of origin giving rise to the investable cash flow, and attributes to such funds an expected investment return based on the prevailing investment conditions at the time of receipt. Investment income actually earned during any period is then allocated to the sources of investable funds in proportion to their contribution to the total expected return over the period.
2. Each year's investable funds are used in a computer model to develop invested assets analyzed by investment generation. Each generation is originally made up of
   a) The net cash flow from insurance operations of the current generation.
   b) The reinvestment or rollover of previous generations of assets according to the assumption in the asset model.
   c) Actual net investment income and capital gains and losses earned in the current generation.
3. Each generation of model assets is made up of three categories of a generalized form of asset:
   a) Bonds and preferred stocks.
   b) Mortgages.
   c) Equities, including common stock and real estate.

   The funds assigned to the current generation are divided among the three categories in the proportions dictated by the company's long-run investment mix objective.

   The form of the generalized asset may be any representation deemed to be suitable. The most straightforward choice is a form which repays the original principal in equal amounts over a fixed term and earns a uniform effective rate of interest.
4. Investable funds included in any generation of model assets are assigned investment characteristics such that the model assets can generate "expected" interest for any time period. The ability of assets to generate expected interest is hereafter referred to as "investment potential." The investment potential attributed to any generation is a function of the combination of
   a) The new-money rate of return expected for each category of generalized asset at the time funds are received.
   b) The term of investment over which this rate is expected to be earned.
   c) The proportion of each category of generalized asset included in the generation.

   The new-money rate of return would normally be the over-all expected return on investment, including that portion of the return expected to accrue through net capital gains.

5. To ensure equity between generations of policyholders, the investment potential has to be determined for each investment generation according to some objective, standard approach. Greater equity is achieved by choosing the characteristics of the model of invested assets in such a way that the investment potential attributed to each generation of funds is objective, and consistent, from one year to the next, in relation to market conditions and to the company's long-range investment policy. While every company may have such a policy for its general portfolio, the investments actually acquired in any time period often do not fit this policy because of current market conditions or investment strategies. The IGM method tends to mirror the company's long-range investment policy rather than following short-term swings in investment practice. Hence, for purposes of the allocation model, the rollover period and the new-money rate for each category of assets are determined in light of prevailing investment opportunities in the marketplace at the time each investment generation is set up and do not necessarily agree with the actual investment result of the year. Objectivity and consistency can be achieved in the IGM method by, for example, relating new-money interest rates to Moody's corporate bond rate or the LIAA experience mortgage rate; by having the expected term of the investment a function of the level of the new-money rate and the category of investment; by having the mix of the current year's investments agree with a target portfolio mix the company is trying to achieve. The use of such standard points of reference results in the investment characteristics attributed to the asset model reflecting what could actually have been achieved under reasonable current investment conditions irrespective of the strategies currently being employed by the investment operation.
6. All funds included in a particular investment generation, within a given country of origin, are assigned the same investment potential regardless of the line of business contributing the funds. However, it is possible under the IGM method to assign different investment potential to different lines of business within the same country. For example, a higher new-money rate might be assigned to single premium immediate annuity premiums than would be assigned to other new money in the current generation, on the premise that single premium immediate annuity premiums represent "marginal" new money. With this approach the IGM method would take on the dominant characteristic of the segregated portfolio method. The approach taken here will depend upon a company's philosophy on the pricing of so-called marginal products.
7. Annual increases in policy loan balances are a first charge against the investable funds received from the lines of business giving rise to such increases in policy loans. These amounts are deemed to be invested in policy loans and do not have attributed to them the investment potential of other funds. Actual policy loan interest is credited to the lines of business from which the policy loans arose.
8. Having assigned investment potential to all investable funds, the operation

of the model generates an "expected" investment return for any period of time. The expected return so determined would result from the aggregate of the investment potential still remaining after rollover in each investment generation. In order to have the total value of the model assets contributing to expected interest at any time equal to the actual amounts of a company's invested assets available to generate investment income, the amount of investment income added to the current year's generation of model assets is the actual investment income and not the "expected" investment income generated by the model. Although this approach ensures that the total amount of model assets agrees with the total of actual assets, the potential attributed to model assets would, however, reflect the investment conditions at the time at which the assets were initially set up or at the time which they were assumed to have been reinvested.

9. Capital gains, realized and/or unrealized, can be allocated by the IGM method in proportion to the expected investment return developed in the asset model. The investment potential assigned to each generation of assets reflects the total expected investment return on those assets, whether that return is realized in the form of dividends, rents, or capital gains. A fundamental premise of the IGM method is that actual results will be allocated in proportion to expected, where expected, through the assignment of investment potential, reflects the fairly assessed contribution of each generation of funds to the over-all investment result. Each company will have to determine, however, whether the most reasonable expectation from its common stock component, for example, would be a new-money rate of the order of 3 per cent, reflecting only expected dividends, or perhaps 10 per cent, reflecting both expected dividends and net capital gains.
10. The method can be introduced on a retroactive, retrospective, or prospective level as desired. The reader may refer to Mr. Green's paper (p. 314) for a more complete discussion of levels of introduction.
11. For companies which operate in more than one country but do not necessarily hold the assets in the country from which investable funds originated, the method can easily be adapted to allocate the interest from a common asset pool back to the countries of origin.
12. Because the method relies on a model of invested assets, there is a high degree of stability introduced into the rollover from old investment generations into the current generation. As a result, even when current cash flow in any line of business is substantially negative, there is usually enough rollover to ensure that all generations of funds are positive.

MANAGEMENT BENEFITS

The IGM allocation method not only satisfies the criteria of a good allocation method but also can provide important additional management benefits. Two examples follow. In each case, use of the IGM method provides an explicit, quantified basis for communication with

the investment operation in terms of both investment objectives and results.

In the first example, an objective is established for the investment results of each generation. The IGM method of allocation uses standard, independent criteria to determine the investment potential of each generation. Since the investment potential is expressed in terms of rate, term of investment, and mix of assets, it can be used as a generalized investment objective to which the marketing, actuarial, and investment operations can relate. This generalized investment objective can be used as an effective basis of communication without requiring that the objective be interpreted or achieved in its exact form. For example, the investment program may call for all of the current generation funds to be invested in real estate with an expected return of 10 per cent over a term of forty years. The generalized objective, however, may be stated, for example, as in the accompanying tabulation. In this illustration

Asset Mix Proportion	Effective Rate	Term
40%	8%	20 years
40	9	15 years
20	11	Perpetuity

the investment in real estate would be expected to generate investment income which would equal or exceed the requirements imposed by the stated objective. At the same time, actuaries can use the objective as a point of reference and are free to interpret the implications of the objective into rate-setting processes in any convenient manner. The only constraint is that the objective, however stated, represents the maximum "guarantee" of investment division performance for funds invested in the current generation.

The second example is concerned with the desire of management to identify some standard against which actual investment results can be measured. An integral part of the IGM method is "expected interest," which is generated by the asset model and is the basis for allocation of actual interest. If the nature of the investment potential used in the asset model is understood by the investment division and if the assumptions inherent in it are agreed upon and realistic, then the resulting expected interest can act as a standard of investment performance for the company's portfolio. This approach to determining a standard of investment performance is appealing from several points of view. First, the fact that expected interest is developed from a model means that the

impact of various possible investment objectives on dividend scales and earnings can be tested in advance. Second, the expected interest or standard of performance can be said to be impartial if it results from investment potential determined in relation to prevailing market conditions. Finally, since any of the factors contributing to expected interest may be negotiated from time to time by the investment, marketing, and actuarial divisions, the process should result in consistent, competitive, and realistic interest assumptions in the company's various products.

CONCLUSION

While there have been many refinements in interest allocation in the last decade, most of the processes adopted have achieved improvements in equity at the expense of technical complexity and cost. It is hoped that the investment generation model method, as outlined here, will permit not only a simplified and equitable allocation process but also the development of investment objectives and standards of performance which will contribute toward more effective, co-ordinated management of insurance company funds.

APPENDIX

INTRODUCTION

This Appendix presents a simple illustration of the operation of the investment generation model (IGM) method for a hypothetical insurance company.

The XYZ Insurance Company has been operating for some time and will begin allocating interest by the IGM method. The method will be introduced prospectively with the first year of allocation on the IGM method designated as year 1. The illustration covers the first three years of operation under the new method. The company has three lines of business: participating ordinary insurance, nonparticipating ordinary insurance, and group insurance. In the illustration a fourth "line of business" called "unallocated" is introduced. The purpose of this fourth category is to recognize the existence of invested assets which contribute to investment income but which cannot be allocated directly to a particular line of business. The tables of figures, in most cases, are given only for participating ordinary insurance and for total company, since figures for the other lines add nothing to the illustration.

The assumptions, definitions, and formulas used are for purposes of the illustration only, and, while they are intended to be representative of a real situation, they should not be construed as limiting the definition or scope of the IGM method.

PROCESS

The following basic steps are performed in the allocation process. Step 1 is an initialization procedure; steps 2–5 must be performed for each generation.

1. Set up starting assets in model form, assign investment characteristics and allocate to lines of business. Table A, I, gives the balance sheet of XYZ Company at time 0, showing that most liability items are capable of being analyzed by line of business with most assets being common to all lines. Table A, II, shows the situation after (*a*) bonds, mortgages, common stocks, cash, and accrued interest have been allocated to the lines of business in proportion to allocated liabilities less policy loans and (*b*) cash has been allocated to the three major asset categories that will be used in the model in proportion to existing assets in these categories. In this example, then, the three categories of assets which are recognized in the model are bonds (called Fund A), mortgages (Fund B), and common stocks (Fund C). The rollover period and interest rates associated with these beginning model assets are given in Table F. Note that these initial investment characteristics relate to assets which are in the existing portfolio, whereas investment characteristics assigned in subsequent generations relate to newly acquired assets. Once the initial asset model has been established, the allocation process can proceed by introducing the insurance cash flow and investment results for each succeeding generation. This process is described in steps 2–5, which are repeated for each generation.

2. Determine the percentage distribution of each new generation and model assets between Funds A, B, and C. In this illustration the percentage distribution of the total model assets at the end of the generation is specified, and the model assets set up in the current generation are then distributed by fund in such a way that the specified over-all distribution is accomplished. All lines of business are given the same fund distribution in any generation.

3. Calculate expected interest from the asset model for each line of business. Since the assets are analyzed by investment generation and fund as well as by line of business, the expected interest for any line will be the aggregate of expected interest for that line by generation and fund.

4. Allocate all other than policy loan actual investment results by line of business in proportion to the model expected interest (except that the unallocated line is not allocated any actual investment return, even though it generates expected interest). Since actual cash investment results form part of the current generation of model assets, there is a further allocation within line of business to the three funds on the basis identified in item 2 above.

5. Determine the current investment generation assets, analyzed by line of business and fund, and assign rollover periods and new-money rates to this generation.

ASSUMPTIONS USED IN THIS ILLUSTRATION

1. Investment income and realized capital gains received in the current generation are received at the end of the period and do not contribute to the exposure for expected interest in the current generation. Insurance cash flow contributes to exposure for expected interest for one-half of the current generation.

2. Rollover of prior generations occurs immediately at the beginning of the current generation.

3. Assets associated with the unallocated line generate expected interest but this line is not allocated any actual interest.

4. The distribution of assets by fund within line of business will be the same, in any given investment generation, for all lines.

5. Expected rates of interest assigned to model assets are net of expected investment expense, and the resulting net expected interest is appropriate to allocate net investment income.

6. Realized capital gains and the change in excess of statement values over cost are allocated in proportion to expected interest.

7. Interest on policy loans is recorded in the company ledger by line of business and consequently needs no allocation.

DEFINITION OF TERMS

The following items are used in the formulas given below and in the tables. They represent all the items defined for a small computer system which produced the illustration. The columns may be interpreted as follows:

Source:

I	The item was originally input to the system either as part of the preliminary asset model or as input to a subsequent generation.
C	The item is calculated internally in the system.

Allocation:

L	The item is allocated by line of business exactly in the company ledger, for example, cash flow by line of business.
S	The item is allocated by line of business within the system in proportion to expected interest.

Type:

A	The item is an amount.
R	The item is a rate (e.g., interest rate) used in the system.

Mnemonics are given in the table on page 344 to facilitate the reading of formulas and column headings.

Modifiers are used in the formulas below to denote different dimensions of a particular item. When used, modifiers are of the form XXXX(F, G, L), where F refers to fund, G refers to generation, and L refers to line of business. When an item is given in its unmodified form, it is taken as the total of the item. Hence XXXX = XXXX(F, G, L).

FORMULAS

The following is not an exhaustive list of steps performed in the allocation process, but is should be sufficient, along with the tables, to allow the reader to follow the illustration.

$$\text{IVAS}(1) = \text{IVAS}(0) + \text{CINO} + \text{IPOL} + \text{NCIV} + \text{RCAP};$$

$$\text{OTAS}(1) = \text{OTAS}(0) + \text{CPOL} + \text{ACDI} + \text{ACPL} + \text{STEX};$$

$$\text{PUDF(F)} = \frac{\text{IVAS(1)}*\text{ADFZ(F)} - [\text{IVAS(0)(F)} - \text{ROLL(F)}]}{\text{IVAS(1)} - \text{IVAS(0)} + \text{ROLL}};$$

ROLL(F, G, L) = IVAS(1)(F, G, L)/ROLP(F, G);

Current income, midyear (Table C) = CINO;

Current income, end of year (Table C) = NCIV + IPOL + RCAP;

XPOS(F, current, L) = ROLL(F, all prior, L) + $\frac{1}{2}$CINO(F, current, L);

XPOS(F, prior, L) = IVAS(0)(F, G, L) − K * ROLL(F, G, L),

where K = number of years since generation established;

XPIN(F, G, L) = XPOS(F, G, L) * RATE(F, G).

Mnemonic	Item	Source	Allocation	Type
ACDI	Increase in accrued interest (other than policy loans)	I	S	A
ACPL	Increase in accrued policy loan interest	I	L	A
ADFZ	Desired distribution of total model assets by fund at the end of each generation	I	.....	R
CINO	Net cash flow from all sources except investment items otherwise defined	I	L	A
CPOL	Net change in policy loan balances	I	L	A
IPOL	Net cash interest on policy loans	I	L	A
IVAS	Cost value of model assets analyzed by investment generation at the beginning (0) or end (1) of the period	I	L	A
NCIV	Net cash investment income (other than policy loan)	I	S	A
OTAS	Value of model assets not analyzed by investment generation at the beginning (0) or end (1) of the period	I	L	A
PUDF	Factor for distribution of current generation model assets by fund	C	.....	R
RATE	Interest rate applied to declining balance of model assets by generation and fund to derive expected interest	I	.....	R
RCAP	Realized net capital gain	I	S	A
ROLL	The rollover, or amount by which model assets decline each year during the lifetime of a generation	C	S	A
ROLP	Rollover period—the term over which model assets decline on a straight-line basis	I	.....	R
STEX	Increase in the excess of statement value over cost value of assets	I	S	A
XPIN	Expected net investment income	C	S	A
XPOS	Exposure for the purpose of calculating XPIN	C	S	A

TABLE A

BEGINNING VALUES FOR MODEL ILLUSTRATION

I. Balance Sheet, XYZ Insurance Company at Time 0

ASSETS

Bonds		$ 40,000
Mortgages		35,000
Common stock		10,000
Cash		2,000
Policy loans:		
Ordinary insurance, par	$4,500	
Ordinary insurance, nonpar	3,500	8,000
Accrued interest		1,500
Total assets in allocation model		$ 96,500
Other assets		3,500
Total assets		$100,000

LIABILITIES

	Ordinary Insurance, Par	Ordinary Insurance, Nonpar	Group Insurance	Total
Policy reserves	$44,500	$29,000	$6,500	$ 80,000
Deposits	3,000	2,000	0	5,000
Dividend provisions	2,000	0	0	2,000
Surplus	3,000	3,000	1,000	7,000
Amounts owing	1,500	1,000	1,500	4,000
Total amounts allocated by line	$54,000	$35,000	$9,000	$ 98,000
Suspense				2,000
Total liabilities				$100,000

II. Assets Used in Allocation Model Allocated to Funds and Lines of Business

ASSETS AT TIME 0

	Ordinary Insurance, Par	Ordinary Insurance, Nonpar	Group Insurance	Total
Fund A	$22,517	$14,329	$4,094	$40,940
Fund B	19,701	12,537	3,582	35,820
Fund C	5,632	3,584	1,024	10,240
Total assets to be analyzed by investment generation	$47,850	$30,480	$8,700	$87,000
Policy loans	4,500	3,500	0	8,000
Accrued interest	825	525	150	1,500
Excess statement value over cost	0	0	0	0
Total assets used in allocation model	$53,175	$34,475	$8,850	$96,500

TABLE B1

RECONCILIATION BY LINE OF BUSINESS OF ASSETS USED IN ALLOCATION MODEL

(Year 1)

	Ordinary Insurance, Par	Ordinary Insurance, Nonpar	Group Insurance, Nonpar	Unallocated	Total, All Lines
	I. Assets Analyzed by Investment Generation				
1. IVAS(0)	$47,850	$30,450	$ 8,700	$ 0	$87,000
2. CINO	200	−800	3,000	50	2,450
3. NCIV	3,042	1,905	653	0	5,600
4. IPOL	250	180	.....	.....	430
5. RCAP	0	0	0	0	0
6. IVAS(1)	$51,342	$31,735	$12,353	$50	$95,480
	II. Assets Not Analyzed by Investment Generation				
1. OTAS(0)	$ 5,325	$ 4,025	$ 150	$ 0	$ 9,500
2. CPOL	800	300	.....	.....	1,100
3. ACPL	6	4	.....	.....	10
4. ACDI	33	20	7	0	60
5. STEX	0	0	0	0	0
6. OTAS(1)	$ 6,164	$ 4,349	$ 157	$ 0	$10,670

TABLE C1

RECONCILIATION BY FUND AND INVESTMENT GENERATION OF ASSETS ANALYZED BY INVESTMENT GENERATION

(Year 1)

	Investment Generation					PUDF
	0	1	2	3	Total	
	I. Ordinary Insurance, Participating					
Total funds:						
1. Value, beginning of year	$47,850	$ 0			$47,850	
2. Rollover	−2,451	2,451			0	
3. Current income, midyear		200			200	1.0000
4. Current income, end of year		3,292			3,292	
5. Value, end of year	$45,399	$ 5,943			$51,342	
Fund A:						
1. Value, beginning of year	$22,517	$ 0			$22,517	
2. Rollover	−901	1,066			165	
3. Current income, midyear		87			87	0.4345
4. Current income, end of year		1,430			1,430	
5. Value, end of year	$21,616	$ 2,583			$24,199	
Fund B:						
1. Value, beginning of year	$19,701	$ 0			$19,701	
2. Rollover	−986	1,159			173	
3. Current income, midyear		95			95	0.4732
4. Current income, end of year		1,558			1,558	
5. Value, end of year	$18,715	$ 2,812			$21,527	
Fund C:						
1. Value, beginning of year	$ 5,632	$ 0			$ 5,632	
2. Rollover	−564	226			−338	
3. Current income, midyear		18			18	0.0923
4. Current income, end of year		304			304	
5. Value, end of year	$ 5,068	$ 548			$ 5,616	

TABLE C1—*Continued*

	Investment Generation					PUDF
	0	1	2	3	Total	
	II. Total, All Lines					
Total funds:						
1. Value, beginning of year	$87,000	$ 0			$87,000	
2. Rollover	−4,458	4,458			0	
3. Current income, midyear		2,450			2,450	1.0000
4. Current income, end of year		6,030			6,030	
5. Value, end of year	$82,542	$12,938			$95,480	
Fund A:						
1. Value, beginning of year	$40,940	0			$40,940	
2. Rollover	−1,639	$ 1,938			299	
3. Current income, midyear		1,064			1,064	0.4345
4. Current income, end of year		2,620			2,620	
5. Value, end of year	$39,301	$ 5,622			$44,923	
Fund B:						
1. Value, beginning of year	$35,820	0			$35,820	
2. Rollover	−1,793	$ 2,110			317	
3. Current income, midyear		1,159			1,159	0.4732
4. Current income, end of year		2,853			2,853	
5. Value, end of year	$34,027	$ 6,122			$40,149	
Fund C:						
1. Value, beginning of year	$10,240	0			$10,240	
2. Rollover	−1,026	$ 410			−616	
3. Current income, midyear		227			227	0.0923
4. Current income, end of year		557			557	
5. Value, end of year	$ 9,214	$ 1,194			$10,408	

TABLE D1

DERIVATION OF EXPECTED INTEREST FROM ASSETS
ANALYZED BY INVESTMENT GENERATION

(Year 1)

	Investment Generation				
	0	1	2	3	Total
	I. Ordinary Insurance, Participating				
Fund A:					
1. Exposure	$21,616	$1,100			.....
2. Interest rate	0.0650	0.0725			.....
3. Expected interest	$ 1,405	$ 80			$1,485
Fund B:					
1. Exposure	$18,715	$1,206			.....
2. Interest rate	0.0700	0.0750			.....
3. Expected interest	$ 1,310	$ 91			$1,401
Fund C:					
1. Exposure	$ 5,068	$ 235			.....
2. Interest rate	0.1000	0.1000			.....
3. Expected interest	$ 507	$ 24			$ 531
Total funds:					
3. Total expected interest	$ 3,222	$ 195			$3,417
	II. Unallocated				
Total funds:					
3. Total expected interest	$ 0	$ 2			$ 2
	III. Total, All Lines				
Total funds:					
3. Total expected interest	$ 5,858	$ 434			$6,292

TABLE E1

ALLOCATED INVESTMENT INCOME

(Year 1)

LINE OF BUSINESS	OTHER THAN POLICY LOANS		POLICY LOAN INTEREST	TOTAL REVENUE INTEREST
	Expected	Actual		
Ordinary insurance, par	$3,417	$3,075	$256	$3,331
Ordinary insurance, nonpar	2,140	1,925	184	2,109
Group insurance, nonpar	733	660	.....	660
Unallocated	2	.....	.....	.....
Total	$6,292	$5,660	$440	$6,100

TABLE B2

RECONCILIATION BY LINE OF BUSINESS OF ASSETS USED IN ALLOCATION MODEL

(Year 2)

	Ordinary Insurance, Par	Ordinary Insurance, Nonpar	Group Insurance, Nonpar	Unallocated	Total, All Lines
	I. Assets Analyzed by Investment Generation				
1. IVAS(0)	$51,342	$31,735	$12,353	$ 50	$95,480
2. CINO	800	−600	800	100	1,100
3. NCIV	3,339	2,024	837	0	6,200
4. IPOL	300	225	.....	.....	525
5. RCAP	108	65	27	0	200
6. IVAS(1)	$55,889	$33,449	$14,017	$150	$103,505
	II. Assets Not Analyzed by Investment Generation				
1. OTAS(0)	$ 6,164	$ 4,349	$ 157	$ 0	$ 10,670
2. CPOL	500	400	.....	.....	900
3. ACPL	15	10	.....	.....	25
4. ACDI	41	24	10	0	75
5. STEX	−242	−147	−61	0	−450
6. OTAS(1)	$ 6,478	$ 4,636	$ 106	$ 0	$ 11,220

TABLE C2

RECONCILIATION BY FUND AND INVESTMENT GENERATION OF ASSETS ANALYZED BY INVESTMENT GENERATION

(Year 2)

	Investment Generation					PUDF
	0	1	2	3	Total	
	I. Ordinary Insurance, Participating					
Total funds:						
1. Value, beginning of year	$45,399	$ 5,943	$ 0		$ 51,342	
2. Rollover	−2,449	−281	2,730		0	
3. Current income, midyear			800		800	1.0000
4. Current income, end of year			3,747		3,747	
5. Value, end of year	$42,950	$ 5,662	$ 7,277		$ 55,889	
Fund A:						
1. Value, beginning of year	$21,616	$ 2,583	$ 0		$ 24,199	
2. Rollover	−901	−108	1,168		159	
3. Current income, midyear			342		342	0.4276
4. Current income, end of year			1,602		1,602	
5. Value, end of year	$20,715	$ 2,475	$ 3,112		$ 26,302	
Fund B:						
1. Value, beginning of year	$18,715	$ 2,812	$ 0		$ 21,527	
2. Rollover	−985	−118	1,337		234	
3. Current income, midyear			391		391	0.4893
4. Current income, end of year			1,833		1,833	
5. Value, end of year	$17,730	$ 2,694	$ 3,561		$ 23,985	
Fund C:						
1. Value, beginning of year	$ 5,068	$ 548	$ 0		$ 5,616	
2. Rollover	−563	−55	225		−393	
3. Current income, midyear			67		67	0.0831
4. Current income, end of year			312		312	
5. Value, end of year	$ 4,505	$ 493	$ 604		$ 5,602	

TABLE C2—*Continued*

	Investment Generation					PUDF
	0	1	2	3	Total	
	II. Total, All Lines					
Total funds:						
1. Value, beginning of year	$82,542	$12,938	$ 0		$ 95,480	
2. Rollover	−4,452	−614	5,066		0	
3. Current income, midyear	.....	.....	1,100		1,100	1.0000
4. Current income, end of year	.....	.....	6,925		6,925	
5. Value, end of year	$78,090	$12,324	$13,091		$103,505	
Fund A:						
1. Value, beginning of year	$39,301	$ 5,622	$ 0		$ 44,923	
2. Rollover	−1,638	−236	2,167		293	
3. Current income, midyear	.....	.....	470		470	0.4276
4. Current income, end of year	.....	.....	2,961		2,961	
5. Value, end of year	$37,663	$ 5,386	$ 5,598		$ 48,647	
Fund B:						
1. Value, beginning of year	$34,027	$ 6,122	$ 0		$ 40,149	
2. Rollover	−1,791	−257	2,480		432	
3. Current income, midyear	.....	.....	538		538	0.4893
4. Current income, end of year	.....	.....	3,388		3,388	
5. Value, end of year	$32,236	$ 5,865	$ 6,406		$ 44,507	
Fund C:						
1. Value, beginning of year	$ 9,214	$ 1,194	$ 0		$ 10,408	
2. Rollover	−1,023	−121	419		−725	
3. Current income, midyear	.....	.....	92		92	0.0831
4. Current income, end of year	.....	.....	576		576	
5. Value, end of year	$ 8,191	$ 1,073	$ 1,087		$ 10,351	

TABLE D2

DERIVATION OF EXPECTED INTEREST FROM ASSETS
ANALYZED BY INVESTMENT GENERATION

(Year 2)

	Investment Generation				
	0	1	2	3	Total
	I. Ordinary Insurance, Participating				
Fund A:					
1. Exposure	$20,715	$2,475	$1,339		.....
2. Interest rate	0.0650	0.0725	0.0730		.....
3. Expected interest	$ 1,346	$ 179	$ 98		$1,623
Fund B:					
1. Exposure	$17,730	$2,694	$1,533		.....
2. Interest rate	0.0700	0.0750	0.0755		.....
3. Expected interest	$ 1,241	$ 202	$ 116		$1,559
Fund C:					
1. Exposure	$ 4,505	$ 493	$ 259		.....
2. Interest rate	0.1000	0.1000	0.1000		.....
3. Expected interest	$ 451	$ 49	$ 26		$ 526
Total funds:					
3. Total expected interest	$ 3,038	$ 430	$ 240		$3,708
	II. Unallocated				
Total funds:					
3. Total expected interest	$ 0	$ 3	$ 4		$ 7
	III. Total, All Lines				
Total funds:					
3. Total expected interest	$ 5,525	$ 937	$ 430		$6,892

TABLE E2

ALLOCATED INVESTMENT INCOME

(Year 2)

LINE OF BUSINESS	OTHER THAN POLICY LOANS		POLICY LOAN INTEREST	TOTAL REVENUE INTEREST
	Expected	Actual		
Ordinary insurance, par	$3,708	$3,380	$315	$3,695
Ordinary insurance, nonpar	2,248	2,048	235	2,283
Group insurance, nonpar	929	847	.....	847
Unallocated	7	.....	.....	.....
Total	$6,892	$6,275	$550	$6,825

TABLE B3

RECONCILIATION BY LINE OF BUSINESS OF ASSETS USED IN ALLOCATION MODEL

(Year 3)

	Ordinary Insurance, Par	Ordinary Insurance, Nonpar	Group Insurance, Nonpar	Unallocated	Total, All Lines
	I. Assets Analyzed by Investment Generation				
1. IVAS(0)	$55,889	$33,449	$14,017	$150	$103,505
2. CINO	1,000	−850	1,400	75	1,625
3. NCIV	3,491	2,033	926	0	6,450
4. IPOL	310	250	.....	.....	560
5. RCAP	−54	−32	−14	0	−100
6. IVAS(1)	$60,636	$34,850	$16,329	$225	$112,040
	II. Assets Not Analyzed by Investment Generation				
1. OTAS(0)	$ 6,478	$ 4,636	$ 106	$ 0	$ 11,220
2. CPOL	−100	50	.....	.....	−50
3. ACPL	20	20	.....	.....	40
4. ACDI	59	35	16	0	110
5. STEX	−108	−63	−29	0	−200
6. OTAS(1)	$ 6,349	$ 4,678	$ 93	$ 0	$ 11,120

TABLE C3

RECONCILIATION BY FUND AND INVESTMENT GENERATION OF ASSETS ANALYZED BY INVESTMENT GENERATION

(Year 3)

	Investment Generation					PUDF
	0	1	2	3	Total	
	I. Ordinary Insurance, Participating					
Total funds:						
1. Value, beginning of year	$42,950	$ 5,662	$ 7,277	$ 0	$ 55,889	
2. Rollover	−2,449	−280	−365	3,094	0	
3. Current income, midyear	.....	.....	.....	1,000	1,000	1.0000
4. Current income, end of year	.....	.....	.....	3,747	3,747	
5. Value, end of year	$40,501	$ 5,382	$ 6,912	$ 7,841	$ 60,636	
Fund A:						
1. Value, beginning of year	$20,715	$ 2,475	$ 3,112	$ 0	$ 26,302	
2. Rollover	−901	−108	−142	1,332	181	
3. Current income, midyear	.....	.....	.....	431	431	0.4307
4. Current income, end of year	.....	.....	.....	1,614	1,614	
5. Value, end of year	$19,814	$ 2,367	$ 2,970	$ 3,377	$ 28,528	
Fund B:						
1. Value, beginning of year	$17,730	$ 2,694	$ 3,561	$ 0	$ 23,985	
2. Rollover	−985	−117	−162	1,547	283	
3. Current income, midyear	.....	.....	.....	500	500	0.5001
4. Current income, end of year	.....	.....	.....	1,874	1,874	
5. Value, end of year	$16,745	$ 2,577	$ 3,399	$ 3,921	$ 26,642	
Fund C:						
1. Value, beginning of year	$ 4,505	$ 493	$ 604	$ 0	$ 5,602	
2. Rollover	−563	−55	−61	215	−464	
3. Current income, midyear	.....	.....	.....	69	69	0.0692
4. Current income, end of year	.....	.....	.....	259	259	
5. Value, end of year	$ 3,942	$ 438	$ 543	$ 543	$ 5,466	

TABLE C3—*Continued*

	Investment Generation					PUDF
	0	1	2	3	Total	
	II. Total, All Lines					
Total funds:						
1. Value, beginning of year	$78,090	$12,324	$13,091	$ 0	$103,505	
2. Rollover	−4,454	−608	−660	5,722	0	
3. Current income, midyear	.....	.....	.....	1,625	1,625	1.0000
4. Current income, end of year	.....	.....	.....	6,910	6,910	
5. Value, end of year	$73,636	$11,716	$12,431	$14,257	$112,040	
Fund A:						
1. Value, beginning of year	$37,663	$ 5,386	$ 5,598	$ 0	$ 48,647	
2. Rollover	−1,638	−234	−256	2,464	336	
3. Current income, midyear	.....	.....	.....	700	700	0.4307
4. Current income, end of year	.....	.....	.....	2,976	2,976	
5. Value, end of year	$36,025	$ 5,152	$ 5,342	$ 6,140	$ 52,659	
Fund B:						
1. Value, beginning of year	$32,236	$ 5,865	$ 6,406	$ 0	$ 44,507	
2. Rollover	−1,791	−255	−293	2,861	522	
3. Current income, midyear	.....	.....	.....	813	813	0.5001
4. Current income, end of year	.....	.....	.....	3,456	3,456	
5. Value, end of year	$30,445	$ 5,610	$ 6,113	$ 7,130	$ 49,298	
Fund C:						
1. Value, beginning of year	$ 8,191	$ 1,073	$ 1,087	$ 0	$ 10,408	
2. Rollover	−1,025	−119	−111	397	−858	
3. Current income, midyear	.....	.....	.....	112	112	0.0692
4. Current income, end of year	.....	.....	.....	478	478	
5. Value, end of year	$ 7,166	$ 954	$ 976	$ 987	$ 10,083	

TABLE D3

DERIVATION OF EXPECTED INTEREST FROM ASSETS
ANALYZED BY INVESTMENT GENERATION

(Year 3)

	Investment Generation				
	0	1	2	3	Total
	I. Ordinary Insurance, Participating				
Fund A:					
1. Exposure	$19,814	$2,367	$2,577	$1,548	.....
2. Interest rate	0.0650	0.0725	0.0730	0.0740	.....
3. Expected interest	$ 1,288	$ 172	$ 217	$ 115	$1,792
Fund B:					
1. Exposure	$16,745	$2,577	$3,399	$1,797	.....
2. Interest rate	0.0700	0.0750	0.0755	0.0770	.....
3. Expected interest	$ 1,172	$ 193	$ 257	$ 138	$1,760
Fund C:					
1. Exposure	$ 3,942	$ 438	$ 543	$ 250	.....
2. Interest rate	0.1000	0.1000	0.1000	0.1000	.....
3. Expected interest	$ 394	$ 44	$ 54	$ 25	$ 517
Total funds:					
3. Total expected interest	$ 2,854	$ 409	$ 528	$ 278	$4,069
	II. Unallocated				
Total funds:					
3. Total expected interest	$ 0	$ 3	$ 8	$ 3	$ 14
	III. Total, All Lines				
Total funds:					
3. Total expected interest	$ 5,190	$ 888	$ 949	$ 504	$7,531

TABLE E3

ALLOCATED INVESTMENT INCOME

(Year 3)

LINE OF BUSINESS	OTHER THAN POLICY LOANS		POLICY LOAN INTEREST	TOTAL REVENUE INTEREST
	Expected	Actual		
Ordinary insurance, par	$4,069	$3,550	$330	$3,880
Ordinary insurance, nonpar	2,369	2,068	270	2,338
Group insurance, nonpar	1,079	942		942
Unallocated	14			
Total	$7,531	$6,560	$600	$7,160

TABLE F

VALUES ASSUMED BY INVESTMENT GENERATION AND TYPE OF FUND

GENERATION YEAR	FUND A			FUND B			FUND C		
	Rollover Period	Interest Rate	Distribution Factor	Rollover Period	Interest Rate	Distribution Factor	Rollover Period	Interest Rate	Distribution Factor
0	25	0.0650	0.47057	20	0.0700	0.41172	10	0.1000	0.11771
1	24	0.0725	0.4705	24	0.0750	0.4205	10	0.1000	0.1090
2	22	0.0730	0.4700	22	0.0755	0.4300	10	0.1000	0.1000
3	22	0.0740	0.4700	22	0.0770	0.4400	10	0.1000	0.0900

DISCUSSION OF PRECEDING PAPER

ANNA M. RAPPAPORT:

Mr. Chapman is to be commended for a fine and original piece of work. It is most gratifying to see a fresh approach which simplifies things. In this era of computers, there seems to be an irresistible urge to complicate everything.

The success of any model in operation must be measured in terms of the purposes for which it was designed. The use of an investment generation allocation model should reasonably approximate the results which would be obtained by an investment generation allocation. The actuary using such a model must be prepared to validate the model on a continuous basis and to adjust it when the validation shows that it is not working out well. The model should be constructed and the validation schedule designed so that such adjustments do not produce discontinuities in operating results. Unless adjustments are possible, the actual results can diverge from the model results and the allocation can become increasingly distorted.

I am disturbed by the suggestion that long-term policy rather than actual investments be used to create the model pattern. It is quite possible that in *every* year actual investments will differ significantly from long-term policy. Using long-term policy may then produce distorted results and may also complicate the validation process. If the model is to mirror policy rather than its implementation, how does one measure its functioning in practice?

The concept of rollover should be analyzed further. Rollover consists of a number of types of transactions such as maturities, repayments of mortgage principal, calls, and refinancing. These can be divided into rollover which can be predicted at time of investment and rollover which is dependent on future conditions. It seems to me that the model must be able to accommodate the actual results with respect to calls and other transactions. A large number of calls could change very substantially both the amount and the character of assets in a given generation. Since such an event takes place if there is a substantial change in interest rates, it seems desirable that the model should reflect this type of event.

In item 8 on page 338 the author writes about conforming model assets to actual assets. This seems to apply only in the year of investment. If future years' assets are not conformed to remaining assets, then at least they should be tested.

More analysis of the problems of handling equities is needed. The model tends to obscure the real-life problems of equity investments by assigning to them a smooth and predictable result.

ASUTOSH CHAKRABARTI:

1. Mr. Chapman's paper describes a model for an equitable departmental allocation of investment income before deduction of federal income tax. Since the model does not take into account the federal income tax status of the departments, the suggested method may not result in a fair distribution of after-tax investment income. The investments of some of the departments may not be compatible with the over-all company's investment portfolio from a federal tax point of view.

2. Determination of rollover rates to be attributed to the different investment types would be quite subjective.

3. Total funds in a closed generation of assets may be quite different from what is actually held in that generation. For this reason, allocation of investment income and capital gain or loss in proportion to expected investment incomes of the departments may not be equitable.

AUTHOR'S REVIEW OF DISCUSSION

CHRISTOPHER D. CHAPMAN:

Most of the points which have been raised in connection with the investment generation model method relate to differences which may occur, over time, between the structure and characteristics of the model assets and those of the company's actual portfolio. There are many complex factors at work in the real world which make the tracking of actual assets and their allocation by generation and line of business an impossibly frustrating task. In fact, when such things as unusual rollover or large capital gains or losses occur, arbitrary decisions regarding their allocation are often required. It is precisely the desire of the author to "share" such unpredictable events among all generations and lines that results in the model's being built on reasonable expectations, current at the time when money is received, rather than on the details of actual performance. Hence, when deviations of investment performance from "expected" do occur, they are shared over all generations in proportion to each one's contribution to the total "expected" result.

This approach does, however, result in a need for a continuous evaluation of the model in its aggregate terms, as pointed out by Mrs. Rappaport. Anyone using such a model would have to determine his own constraints on the extent of allowable deviation of the model from actual assets. As a minimum, the total assets should be the same and the ex-

pected and actual investment incomes, in total, should be reconcilable and should not diverge over time. If the company's investment policy is never achieved in practice, then the asset structure of the model should be forced to fit the policy actually emerging, rather than the stated policy. However, attempts to validate the model in too much detail would tend to defeat some of the cited benefits to be gained from the IGM method. For example, although initial rollover may be somewhat arbitrary, it can be reasonable and consistent from one generation to the next, and emerging rollover should not be validated strictly against actual results.

The model consequently is bound to develop characteristics different from those found in the actual asset portfolio and different from those which would have accrued to a particular generation under some other approach. However, if the model is constructed with equity as an objective and is being validated in the aggregate under conditions satisfying to its user, then it can be taken to be equitable for allocation purposes.

In closing, the author extends his thanks to Mrs. Rappaport and Mr. Chakrabarti for their constructive comments and to Mr. Peter Wijtkamp, who wrote the program and produced the illustrations.