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HEALTH INSURANCE—RETURN OF PREMIUM REVISITED

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ABSTRACT

In 1970 and 1971 interest and controversy soared to high levels as more and more companies began issuing a return of premium health insurance rider which provided for payment of a periodic amount if claims did not exceed a certain level prior to its payment. The most popular of these was the rider which paid 80 per cent of premiums less claims during a ten-year cycle if claims did not exceed 20 per cent of premiums during that cycle. The question of how to price this benefit adequately still remains.

Thus the primary purpose of this paper is to present an alternative method of determining the profitability of the premium for a return of premium benefit which will pay X per cent of accumulated premiums less claims if such accumulated claims during the t-year period prior to payment of the benefit do not exceed Y per cent of the accumulated premiums.

The paper then extends the concepts for testing the profitability of the premiums to develop a method for estimating the theoretical level of reserves after claim offset.

ALTERNATIVE METHODS OF TESTING PROFITABILITY FOR THE RETURN OF PREMIUM HEALTH INSURANCE RIDER

The Equation Approach

IN HIS paper "The Return of Premium Benefit in Health Insurance" (*TSA*, XXII, 235), Mr. E. Paul Barnhart used equations to determine the gross premiums and reserves for the return of premium benefit. In Section II of the Appendix to his paper, Mr. Barnhart utilized approximation techniques to calculate the cost of the return of premium benefit in successive ten-year rolling cycles.

Simulation Technique

A second approach which could be utilized for testing the profitability of this product is that of simulation. On page 246 of the same paper, Mr. Barnhart states, "One method of dealing with the problem is by computer simulation, synthesizing the values by means of a mathematical population model." In the discussion of Mr. Barnhart's paper, Mr. Henry K. Knowlton corroborates this: "Since it is not possible to predict accurately the number of individuals receiving the return of premium benefit on a ten-year 80-20 per cent basis without a computer simulation of the expected results. . . ."

From Mr. Barnhart's paper, I assume that the simulation program would utilize assumptions as to mortality, withdrawal, rates of disablement, and rates of termination of disability. The thought of writing a program involving these variables and then running it on the computer could cause a data-processing department to have apoplexy. As Mr. Barnhart goes on to state, "It is desirable, however, to have available alternative approximate methods which can be handled less elaborately."

The Combination Approach

The alternative method is an asset share approach utilizing values obtained from a limited simulation program. With careful definition, this approach will have most of the advantages without the complexities or the larger computer-time requirement of the model-office simulation approach. The following is the five-step procedure used to test the profitability of the premium for a return of premium benefit:

1. Calculate premiums for the basic disability benefit, using traditional methods.
2. Obtain the value of the following by using a simulation program:
   a) The probability that a person aged x at issue who is still in force at the end of the cycle will receive a return of premium (ROP) benefit. (Hereinafter this will be called the "probability of a ROP benefit.")
   b) The expected benefit to be paid to each person aged x at issue receiving the ROP benefit at the end of the cycle. (Hereinafter this will be called the "average ROP claim.")
3. Using the same assumptions which were the basis for the basic disability premiums, generate retrospective asset shares to the date on which the first ROP benefit will be paid. After the asset shares have been calculated, the following is the minimum information which should be available.
   a) Accumulated assets due to the basic policy at the end of each year.
   b) Accumulated assets due to the ROP rider at the end of each year.
   c) The number of people beginning each policy year.
   d) The number of people who die during the tenth policy year.
4. Calculate the expected amount to be paid at the end of the first cycle for the ROP benefit by multiplying the number of people in force at the beginning of the tenth policy year less the number dying during that year by the probability of a ROP benefit and then multiplying the result by the average ROP claim. Compare the result with the accumulated assets due to the ROP rider.

5. Repeat steps 3 and 4 after changing all assumptions except those pertaining to the disability risk to those expected for disability policies having a ROP benefit. Logically we can then attribute the change in accumulated assets due to the basic policy to the ROP benefit and add this change to the accumulated assets due to the ROP rider. The sum, the additional assets due to the ROP benefit, would be available to pay the ROP claims when they mature.

THE KEYS TO THE COMBINATION APPROACH

The Morbidity Tables

Two different, although interrelated, morbidity tables are needed in the combination approach. Asset shares require the traditional morbidity table involving discounted benefit costs for each age at disablement. These discounted benefit costs are derived from a continuance table created from experience on lives who terminate disability by either death or recovery.

In the combination approach the results of step 2 are applied to the insureds who survive to the end of the cycle. This means that the probability of a ROP benefit is a conditional probability dependent on the fact that the insured aged x at issue is alive and in force at the end of the cycle. Consequently, the morbidity table used in the simulation program should be derived from the experience on insureds who are alive and in force at the end of the cycle. Thus we see that the probability of a ROP benefit derived in the simulation program will be independent of the lapse rate and the mortality rate if we assume that there is no antiselection by those people who lapse. This concept will allow the actuary to vary any assumption which does not affect the morbidity risk in the asset shares without rerunning the simulation program.

In the simulation program a person who is disabled and then recovers in the same policy year is re-exposed for the portion of the policy year remaining after recovery. Therefore, the rate of disablement in the second morbidity table should represent the probability that a person will be disabled at least once during the year. This means that both the number of claims and the exposure should be adjusted for multiple claims incurred in the same policy year by each insured. In the belief that the effects were small and not readily derived from published data, these adjustments were not made.

The 1964 Commissioners Disability Table (CDT) was used in this paper, since it is a relatively recent disability table having both the claim costs and the underlying continuance table available in published form. Unfortunately, the 1964 CDT does not indicate the proportion of the disabled lives who recover or the proportion of the disabled lives who die while disabled. We can, however, obtain an approximation to the proportion of disabled lives who terminate disability by recovery from the Report of the Committee on Disability and Double Indemnity published in the 1952 Reports of Mortality and Morbidity Experience (*TSA, 1952 Reports*).

Appendix I discusses the validity of using the experience from the 1952 disability study to adjust the continuance table published in Volume II of the 1964 CDT.

The Simulation Program

Step 2, which depends on a simulation program which does not require large amounts of computer time to generate usable values and which also does not involve lapse, mortality, or interest assumptions, is the main key to the combination approach. A description of the logic in the simulation program is given in Appendix II.

For the following reasons, we believe that processing time was reduced by not utilizing withdrawal or interest assumptions:

1. Routines required to determine whether the insured died or lapsed were eliminated.
2. A smaller number of policies could be "issued," yet the results would be credible at the end of the cycle.

As with any simulation program, the results must be checked for reasonableness and possible bias. However, this testing is difficult because theoretical values are not readily obtainable. Appendix III describes the tests utilized and their results.

ILLUSTRATIVE EXAMPLE

The following example is introduced only to illustrate the combination approach and should not be considered indicative of experience for any particular company. The assumptions used to calculate level, term to age 65 experience premiums for the basic disability policy are shown in Table 1. The experience premiums were then loaded for profit (10 per cent of the gross premium) and for the waiver of premium benefit (variable percentage based on the plan and issue age). The resultant gross annual premiums are shown in Table 2 with the proportion of the gross premium required to fund the expenses and each of the benefits.

For the ROP rider, we chose the "80 per cent return with 20 per cent or less claims" described on page 236 of Mr. Barnhart's paper. Briefly, the rider provides that if claims during a cycle do not exceed 20 per cent of gross premium, then the company will pay 80 per cent of the accumulated gross premium less accumulated accrued claims during a ten-year cycle. The gross premium for this rider will be 30 per cent of the gross premium for the basic policy to which it is attached.

Although it is required in the recommended guidelines of the National Association of Insurance Commissioners Accident and Health Protection Subcommittee (C-1), no death benefit is included in the rider benefits for this example. To include such a death benefit during simulation would needlessly complicate the operation of the simulation program. The simulation program would require two distinct continuance tables (one for those who recover from disability and one for those who die) and increased programming logic. In addition, the number of policies "issued" must be increased to ensure credible results for each possible payment of the ROP benefit. Both the increased programming logic and the increased number of policies "issued" will lengthen the run time of the simulation program.

For this paper, each plan and issue age combination was simulated twice. The first trial used rates of disablement and rates of termination of disability derived from the continuance table in Volume III of the 1964 CDT. This table is based on disabled lives who terminate disability by either death or recovery, and for the remainder of this paper the phrase "based on death and recovery" will indicate results obtained by using this table in the simulation program.

TABLE 1

ASSUMPTIONS FOR GROSS DISABILITY PREMIUMS

Commissions and field and acquisition expenses:

Year	Per Cent
1	70%
2	25
3–10	10
11+	5

Taxes, licenses, and fees: $2\frac{1}{2}$ per cent all years

Per policy expenses: $76.00 first year plus $12.00 renewal years

Claim expense: 5 per cent of claims

Average-size policy: $400 monthly income

Total decrement—death and lapse: 35 per cent first year, 20 per cent second year, 15 per cent third year, 10 per cent fourth year, 7 per cent thereafter

Interest—after federal income tax adjustment: 4 per cent for all years

Underwriting selection factors:

Year	Per Cent
1	60%
2	80
3	100

Claims: 1964 Commissioners Disability Table discounted at 3 per cent interest with claims incurred at the beginning of the policy year

TABLE 2

GROSS ANNUAL PREMIUMS PER $100 MONTHLY INDEMNITY

ISSUE AGE	TOTAL GROSS PREMIUM	PROPORTION OF PREMIUM TO FUND		
		Disability Benefit	Waiver of Premium	Expenses
	2-Year Indemnity Period—7-Day Elimination Period			
25......	$ 26.79	44.68%	0.35%	44.98%
35......	34.34	49.50	0.54	39.95
45......	51.90	55.11	1.08	33.81
55......	80.07	53.44	1.85	34.71
	2-Year Indemnity Period—30-Day Elimination Period			
25......	$ 18.71	34.85%	0.35%	54.78%
35......	24.48	42.32	0.54	47.14
45......	39.54	51.47	1.08	37.46
55......	64.78	51.41	1.85	36.74
	5-Year Indemnity Period—7-Day Elimination Period			
25......	$ 29.20	46.47%	0.45%	43.08%
35......	39.63	51.75	0.73	37.52
45......	65.40	57.16	1.61	31.24
55......	105.09	54.72	2.97	32.32
	5-Year Indemnity Period—30-Day Elimination Period			
25......	$ 21.09	38.45%	0.45%	51.11%
35......	29.72	45.66	0.73	43.61
45......	52.81	54.93	1.61	33.46
55......	89.18	53.55	2.97	33.48
	To Age 65 Indemnity Period—7-Day Elimination Period			
25......	$ 34.89	49.70%	0.68%	39.61%
35......	50.67	54.77	1.17	34.06
45......	81.82	58.38	2.46	29.16
55......	117.61	54.61	4.11	31.27
	To Age 65 Indemnity Period—30-Day Elimination Period			
25......	$ 26.71	44.36%	0.68%	44.96%
35......	40.61	51.79	1.17	37.03
45......	68.91	56.99	2.46	30.56
55......	101.19	53.66	4.11	32.23
	To Age 65 Indemnity Period—90-Day Elimination Period			
25......	$ 21.22	38.45%	0.68%	50.85%
35......	32.80	48.23	1.17	32.43
45......	56.77	55.10	2.46	32.45
55......	83.36	52.19	4.11	33.70

Table 3 compares the two morbidity tables at selected ages at disablement for the rates of disablement and the probability that a person who is disabled on the eighth day is still disabled on the tth day. The rate of disablement is defined as the probability that a person (x) will be disabled at least once in a year for more than seven consecutive days. The rate of termination of disability can easily be derived from the probability that a person disabled on the eighth day is still disabled on the tth day. From Table 3 we find that the rates of disablement based on recovery are fairly level when compared with the rates of disablement based on death or recovery.

TABLE 3

COMPARISON OF DISABILITY TABLES BASED ON DEATH AND RECOVERY (D + R) WITH TABLES BASED ON RECOVERY (REC.)

Duration (t)			Age at Disablement							
			27		37		47		57	
Yr.	Mo.	Day	D+R	Rec.	D+R	Rec.	D+R	Rec.	D+R	Rec.
			Probability of Being Disabled More than 7 Days							
			10.7%	10.2%	12.6%	11.6%	15.0%	12.4%	18.1%	11.5%
			Probability of Being Disabled on the tth Day if Disabled on the 8th Day							
0	0	12	82.3%	82.3%	84.0%	84.0%	86.3%	86.1%	88.9%	88.6%
		15	71.2	71.1	73.8	73.7	77.2	77.0	81.5	80.8
	1	0	36.3	36.2	39.8	39.7	46.2	45.7	54.2	52.6
		15	20.8	20.6	23.8	23.6	29.7	29.0	37.6	35.4
	2	0	13.1	12.9	15.5	15.3	20.4	19.6	27.2	24.7
0	3		6.2	6.0	7.8	7.5	11.2	10.3	17.2	14.3
	4		3.4	3.2	4.3	4.1	6.9	6.0	11.9	8.9
	5		2.1	2.0	2.7	2.5	4.6	3.7	9.0	5.9
	6		1.5	1.3	1.9	1.7	3.4	2.5	7.3	4.1
	9		0.9	0.7	1.1	0.9	2.2	1.3	5.4	2.1
1	0		0.7	0.5	0.9	0.7	1.8	1.0	4.6	1.4
	3		0.6	0.5	0.8	0.6	1.7	0.9	4.4	1.2
	6		0.6	0.4	0.8	0.5	1.6	0.8	4.2	1.1
	9		0.5	0.4	0.7	0.5	1.5	0.7	4.1	1.0
2	0		0.5	0.4	0.7	0.5	1.5	0.6	3.9	0.9
3	0		0.4	0.3	0.6	0.3	1.2	0.5	3.4	0.6
4			0.3	0.2	0.5	0.3	1.1	0.4	3.0	0.4
5			0.3	0.2	0.4	0.3	1.0	0.3	2.8	0.3

TABLE 4

RESULTS OF SIMULATION
PROBABILITY OF ROP BENEFIT—AVERAGE ROP CLAIM

Issue Age	Percentage of Premium* for Benefits	Probability of ROP Benefit Based on D+R	Probability of ROP Benefit Based on Rec.	Average ROP Claim Based on D+R	Average ROP Claim Based on Rec.
	2-Year Indemnity Period—7-Day Elimination Period				
25	45.03%	59.83%	61.37%	$ 262.68	$ 263.29
35	50.04	55.64	59.50	333.87	336.29
45	56.19	52.30	62.67	498.01	504.73
55	55.29	50.17	70.23	759.82	779.89
	2-Year Indemnity Period—30-Day Elimination Period				
25	35.20%	78.67%	79.47%	$ 190.93	$ 191.05
35	42.86	72.63	75.70	248.74	249.14
45	52.55	67.53	75.53	395.11	399.47
55	53.26	62.57	79.17	639.82	650.43
	5-Year Indemnity Period—7-Day Elimination Period				
25	46.92%	60.27%	62.13%	$ 286.78	$ 286.65
35	52.48	58.93	61.00	385.76	386.36
45	58.77	60.07	70.87	628.64	635.79
55	57.69	57.90	76.83	1,000.60	1,023.92
	5-Year Indemnity Period—30-Day Elimination Period				
25	38.90%	77.70%	80.60%	$ 215.26	$ 215.68
35	46.39	74.87	77.87	300.80	301.95
45	56.54	74.77	79.97	525.87	531.02
55	56.52	66.67	82.90	883.43	895.13
	To Age 65 Indemnity Period—7-Day Elimination Period				
25	50.38%	65.50%	66.23%	$ 341.38	$ 342.74
35	55.94	64.63	68.67	492.24	494.89
45	60.84	64.50	75.07	783.98	795.73
55	58.72	61.13	79.83	1,116.40	1,149.31
	To Age 65 Indemnity Period—30-Day Elimination Period				
25	45.04%	81.73%	82.98%	$ 272.01	$ 272.45
35	52.96	78.67	80.60	410.61	412.44
45	59.45	77.03	83.97	685.31	691.76
55	57.77	69.73	84.37	998.55	1,016.25
	To Age 65 Indemnity Period—90-Day Elimination Period				
25	39.13%	95.31%	95.37%	$ 220.26	$ 220.31
35	49.40	93.20	94.40	339.36	339.33
45	57.56	88.50	93.20	585.02	586.30
55	56.30	80.57	91.27	854.10	858.96

In fact, the rates of disablement based on recovery decrease from 95 per cent of the rate of disablement based on death or recovery at age 27 to about 60 per cent of the rate at age 57. We also see that the probability that a person who is disabled on the eighth day is still disabled on the tth day generally is slightly less in the table based on recovery when t equals 12. However, this difference becomes greater as either the age at disablement or the duration from date of disablement increases.

In each simulation trial, we assumed the following:

1. Three hundred policies would be issued in ten different companies for each plan and age combination.
2. Underwriting selection would result in the morbidity in the first and second policy years being 60 and 80 per cent of the ultimate morbidity, respectively.
3. A person will receive the ROP benefit if accumulated claims have not exceeded 20 per cent of accumulated premiums during the paid ten-year cycle.

The results of the simulations are shown in Table 4. The following may be seen from the data:

1. Theoretically, the probability of a ROP benefit's being paid should increase as the percentage of the gross premium required to fund the disability benefit and waiver of premium decreases. However, Table 4 shows that the probability of a ROP benefit increases as the indemnity period increases, even though the percentage of premium required to fund the benefits increases. The conclusion is that the additional premium needed to fund a longer indemnity period has the same effect on the ROP benefit as the additional premium for expenses or profit.
2. When based on death or recovery, the probability of a ROP benefit's being paid decreases as the issue age increases. When based on recovery, the probability of a ROP benefit's being paid is almost level or increases slightly as the issue age increases.
3. If the percentage of premium needed to fund the disability and waiver of premium benefits is held constant, it appears that, when based on recovery, the probability of a ROP benefit's being paid increases as the issue age, the indemnity period, or the elimination period increases.
4. The average ROP claim is slightly larger when based on recovery than when based on death or recovery.

Tables 5–7 contain the following information for each issue age and plan combination:

1. Accumulated assets on the tenth policy anniversary due to the basic policy.
2. Accumulated assets on the tenth policy anniversary due to the ROP rider and before payment of the ROP benefit. These assets equal the "net" premium paid each year accumulated at interest. Theoretically, the fund established by the gross premium for the ROP rider should be decreased by all additional expenses attributed to this rider. These would include com-

missions, premium tax, the premium required for the waiver of premium benefit, and the expenses required to administer this rider. For simplicity we assume that the insurer incurred no additional expenses for administering this rider and paid no commissions on the premium paid for the rider. Therefore, the "net" premium would equal the gross premium for the ROP rider less amounts for the waiver of premium benefit and for premium taxes.

3. The additional assets on the tenth policy anniversary due to the ROP rider (Tables 6 and 7 only). These equal the accumulated assets due to the ROP rider plus the increase (decrease) in the accumulated assets due to the basic policy resulting from the improved persistency. This would then be the amount theoretically available for paying the ROP claims falling due at the end of the tenth policy year.
4. The expected payout for the ROP benefit on the tenth policy anniversary, based on death and recovery ("D + R") and on recovery only ("Rec."). The expected payout for the ROP benefit equals the product of the following:
   a) The probability of the ROP benefit.
   b) The average ROP claim.
   c) The number of people beginning the tenth policy year, less expected deaths from the Commissioners 1958 Standard Ordinary Mortality Table.

Tables 5–7 are concerned with cash flow and therefore do not include an estimate of the reserves required to pay the ROP benefit in the eleventh and subsequent policy years or the statutory guaranteed renewable reserves. Thus these exhibits compare the additional assets due to the ROP benefit at the end of the tenth policy year with the expected amount to be paid as the ROP benefit to those who qualify at that time.

Table 5 presents the financial position if experience actually follows that assumed in Table 1 (step 4 in the combination approach). The following comments pertain to the data given in Table 5.

1. For policies having a seven-day elimination period, the accumulated assets due to the ROP rider are greater than the expected payout based on death or recovery.
2. However, for policies having a thirty-day or a ninety-day elimination period, the accumulated assets due to the ROP rider are less than the expected payout based on death and recovery. The deficit is greatest at the lower issue ages.
3. The expected payout based on recovery is greater than that based on death or recovery. This difference increases more rapidly as the issue age of the policy increases. Consequently, the excess of the expected payout over the accumulated assets due to the ROP rider increases as the issue age increases.

Experience reported to date indicates that disability policies have better persistency if a ROP rider is attached. Therefore, Table 6 seeks to determine the impact of improved persistency on policies having a ROP

TABLE 5—FINANCIAL POSITION ON TENTH POLICY ANNIVERSARY
BASED ON ASSUMPTIONS IN TABLE 1

Issue Age	Accumulated Assets due to		Expected Payout Based on	
	Base Plan	ROP Rider	D+R	Rec.
	2-Year Indemnity Period—7-Day Elimination Period			
25	$ 6,375	$ 47,047	$ 43,393	$ 44,613
35	15,512	60,180	51,155	55,100
45	37,446	90,420	71,223	86,497
55	33,983	138,432	102,431	147,174
	2-Year Indemnity Period—30-Day Elimination Period			
25	$ 2,835	$ 32,831	$ 41,464	$ 41,912
35	10,740	42,891	49,747	51,933
45	30,897	68,915	72,960	82,504
55	26,955	111,926	107,570	138,366
	5-Year Indemnity Period—7-Day Elimination Period			
25	$ 8,557	$ 51,204	$ 47,718	$ 49,169
35	23,104	69,336	62,614	64,923
45	52,748	113,311	103,262	123,213
55	44,630	179,575	155,678	211,391
	5-Year Indemnity Period—30-Day Elimination Period			
25	$ 5,049	$ 36,987	$ 46,181	$ 47,998
35	18,268	51,987	62,023	64,755
45	46,033	91,505	107,515	116,119
55	37,852	152,347	158,261	199,394
	To Age 65 Indemnity Period—7-Day Elimination Period			
25	$13,961	$ 61,083	$ 61,736	$ 62,673
35	33,989	88,191	87,607	93,585
45	50,732	140,540	138,279	163,352
55	49,954	198,491	183,378	246,534
	To Age 65 Indemnity Period—30-Day Elimination Period			
25	$10,356	$ 46,746	$ 61,372	$ 62,404
35	29,046	70,662	88,952	91,540
45	43,771	118,311	144,349	158,835
55	42,969	170,780	187,080	230,370
	To Age 65 Indemnity Period—90-Day Elimination Period			
25	$ 7,215	$ 37,168	$ 57,963	$ 48,012
35	23,653	57,108	87,099	88,213
45	34,530	97,529	141,575	149,420
55	35,340	140,721	184,913	210,662

TABLE 6—FINANCIAL POSITION ON TENTH POLICY ANNIVERSARY
BASED ON ASSUMPTIONS IN TABLE 1 WITH IMPROVED PERSISTENCY

Issue Age	Accumulated Assets		Additional Assets due to ROP Benefit	Expected Payout Based on	
	Base Plan	ROP Rider		D+R	Rec.
	2-Year Indemnity Period—7-Day Elimination Period				
25......	$22,882	$ 63,089	$ 79,596	$ 74,531	$ 76,627
35......	35,338	80,699	100,525	87,863	94,639
45......	65,826	121,251	149,631	122,332	148,567
55......	59,714	185,632	211,363	175,935	252,784
	2-Year Indemnity Period—30-Day Elimination Period				
25......	$16,520	$ 44,025	$ 57,710	$ 71,218	$ 71,988
35......	27,326	57,515	74,101	85,445	89,200
45......	55,280	92,412	116,795	125,315	141,708
55......	48,586	150,089	171,720	184,761	237,656
	5-Year Indemnity Period—7-Day Elimination Period				
25......	$26,200	$ 68,663	$ 86,306	$ 81,960	$ 84,452
35......	46,294	92,978	116,168	107,545	111,511
45......	86,761	151,947	185,960	177,362	211,630
55......	79,064	240,805	275,239	267,391	363,083
	5-Year Indemnity Period—30-Day Elimination Period				
25......	$19,889	$ 49,599	$ 64,439	$ 79,320	$ 82,441
35......	38,017	69,713	89,462	106,530	111,223
45......	75,995	122,705	152,667	184,667	199,445
55......	68,377	204,293	234,818	271,828	342,478
	To Age 65 Indemnity Period—7-Day Elimination Period				
25......	$34,296	$ 81,191	$102,246	$106,037	$107,647
35......	61,753	118,262	146,026	150,473	160,739
45......	84,790	188,460	222,518	237,507	280,572
55......	94,826	266,170	311,042	314,969	423,445
	To Age 65 Indemnity Period—30-Day Elimination Period				
25......	$27,859	$ 62,685	$ 80,188	$105,412	$107,185
35......	53,480	94,755	119,189	152,783	157,228
45......	73,671	158,652	188,552	247,932	272,814
55......	83,711	229,011	269,753	321,327	395,682
	To Age 65 Indemnity Period—90-Day Elimination Period				
25......	$22,606	$ 49,841	$ 65,232	$ 99,557	$ 99,641
35......	45,206	76,579	98,132	149,601	151,514
45......	59,853	130,783	156,106	243,168	256,642
55......	72,383	188,702	225,745	317,605	361,832

TABLE 7—FINANCIAL POSITION ON TENTH POLICY ANNIVERSARY BASED ON 5 PER CENT INTEREST AND IMPROVED PERSISTENCY

Issue Age	Accumulated Assets		Additional Assets due to ROP Benefit	Expected Payout Based on	
	Base Plan	ROP Rider		D+R	Rec.
	2-Year Indemnity Period—7-Day Elimination Period				
25......	$22,919	$ 67,166	$ 84,563	$ 74,531	$ 76,627
35......	36,241	85,914	106,822	87,863	94,639
45......	68,828	129,086	159,108	122,332	148,567
55......	62,798	197,628	225,003	175,935	252,784
	2-Year Indemnity Period—30-Day Elimination Period				
25......	$16,289	$ 46,870	$ 61,285	$ 71,218	$ 71,988
35......	27,856	61,232	78,727	85,445	89,200
45......	57,716	98,384	124,156	125,315	141,708
55......	51,032	159,788	182,812	184,761	237,656
	5-Year Indemnity Period—7-Day Elimination Period				
25......	$26,397	$ 73,100	$ 91,682	$ 81,960	$ 84,452
35......	47,889	98,986	123,465	107,545	111,511
45......	91,330	161,766	197,837	177,362	211,630
55......	82,034	253,366	292,545	267,391	363,083
	5-Year Indemnity Period—30-Day Elimination Period				
25......	$19,851	$ 52,804	$ 68,432	$ 79,320	$ 82,441
35......	39,232	74,218	95,061	106,530	111,223
45......	80,000	130,634	162,403	184,667	199,445
55......	70,723	217,494	249,510	271,828	342,478
	To Age 65 Indemnity Period—7-Day Elimination Period				
25......	$34,997	$ 87,204	$108,652	$106,037	$107,647
35......	64,365	125,904	155,236	150,473	160,739
45......	89,302	200,638	236,796	237,507	280,572
55......	97,347	283,370	330,253	314,969	423,445
	To Age 65 Indemnity Period—30-Day Elimination Period				
25......	$28,268	$ 66,736	$ 85,167	$105,412	$107,185
35......	55,721	100,878	126,730	152,783	157,228
45......	77,612	168,904	200,662	247,932	272,814
55......	85,604	243,810	286,334	321,327	395,682
	To Age 65 Indemnity Period—90-Day Elimination Period				
25......	$22,767	$ 53,062	$ 69,265	$ 99,557	$ 99,641
35......	46,968	81,528	104,302	149,601	151,514
45......	62,952	139,234	166,159	243,168	256,642
55......	73,478	200,896	239,432	317,605	361,832

rider. Asset shares were revised by changing the withdrawal assumptions from all causes to 20, 12, 8, and 6 per cent in the first, second, third, and fourth and subsequent years, respectively. Assuming no morbidity antiselection on withdrawals, the change in persistency assumptions does not require that step 2 in the combination approach be repeated.

Table 6 presents the financial position on the tenth policy anniversary, assuming the revised persistency assumptions. Since a ROP rider improves persistency, the resulting increase in the accumulated assets due to the base plan was added to the accumulated assets due to the ROP rider to determine the total additional assets due to the ROP rider which can then fund the expected ROP claims. Rather surprisingly, the improvement in persistency magnified the excess (or deficit) of the additional assets due to the ROP rider over the expected payout.

A level 4 per cent net interest assumption after federal income tax may be low on the basis of current interest earnings. Consequently, Table 7 presents the financial position on the tenth policy anniversary, assuming 5 per cent net interest earnings and the improved persistency assumption. Obviously the premium for the ROP rider will generate a larger amount of assets with which to pay the ROP benefit. In most instances the increase is not sufficient to offset previously noted deficits.

In Tables 5–7 we compare the expected payout to be paid on the tenth policy anniversary for the ROP rider to the additional assets due to the ROP rider. We did not consider the ROP benefits payable on the eleventh, twelfth, and subsequent policy anniversaries. Theoretically, on the tenth policy anniversary we should establish a reserve which, together with future rider premiums and interest, would be sufficient to pay the ROP benefits as they mature.

A different approach utilizes the following assumptions:

1. Whenever an insured begins a new cycle, he enters a new group comprised of similar insureds in the same rating classification.
2. Premium income from the rider can be allocated to each such group of policyholders. Premium income plus its accumulated interest earnings for each group should be sufficient to fund the ROP benefit for that group.
3. Any additional assets due to the basic policy which are attributable to the addition of the ROP rider will be allocated first to the group whose cycle terminates on the tenth policy anniversary and then to each successive group as its cycle ends.

For this approach we maintained a count in the simulation program of insureds who were still in the initial ten-year cycle at the beginning of each policy year. From this count we derived the probability that a person who paid his premium at the beginning of duration t was still in the initial ten-year cycle. Table 8 presents the resultant probabilities for a basic policy having a two-year indemnity period with a seven-day elimination period.

Using these probabilities, we determined the portion of the premium income for the ROP rider which is to be allocated to the initial group whose cycle ceases on the tenth policy anniversary. The premiums allocated to this group were then accumulated at interest.

TABLE 8

PROBABILITY THAT A PERSON AGED x AT ISSUE WHO IS IN FORCE AT BEGINNING OF DURATION t WILL NOT HAVE HAD CLAIMS IN EXCESS OF 20 PER CENT OF TEN YEARS' ACCUMULATED GROSS PREMIUM

(2-Year Indemnity Period—7-Day Elimination Period)

Duration	Issue Age 25	35	45	55
	Based on Death and Recovery			
1	1.0000	1.0000	1.0000	1.0000
2	0.9647	0.9617	0.9639	0.9667
3	0.9258	0.9183	0.9160	0.9347
4	0.8910	0.8730	0.8707	0.8860
5	0.8510	0.8303	0.8227	0.8363
6	0.8050	0.7857	0.7727	0.7920
7	0.7580	0.7283	0.7170	0.7207
8	0.7127	0.6800	0.6597	0.6593
9	0.6753	0.6393	0.6127	0.6060
10	0.6347	0.5927	0.5677	0.5497
	Based on Recovery Only			
1	1.0000	1.0000	1.0000	1.0000
2	0.9707	0.9687	0.9730	0.9830
3	0.9327	0.9270	0.9427	0.9647
4	0.8907	0.8820	0.9060	0.9383
5	0.8507	0.8373	0.8637	0.9077
6	0.8120	0.7923	0.8233	0.8767
7	0.7760	0.7457	0.7853	0.8380
8	0.7360	0.7067	0.7360	0.8060
9	0.6967	0.6610	0.6990	0.7667
10	0.6543	0.6233	0.6610	0.7313

Table 9 attempts to illustrate the effect of this approach on Tables 5 and 6 by comparing the changes made in the amount of the accumulated assets due to the ROP rider. In this table we note that there is a significant reduction in the accumulated assets due to the ROP rider used to fund the expected payout. This reduction varies from 11 to 15 per cent when the experience is based on death or recovery and from 8 to 12 per cent when it is based on recovery.

TABLE 9

FINANCIAL POSITION OF ROP RIDERS TERMINATING INITIAL CYCLE ON TENTH ANNIVERSARY

(2-Year Indemnity Period—7-Day Elimination Period)

Issue Age	Accumulated Assets due to ROP Rider Based on			Expected Payout	
	Total In-Force	D+R	Rec.	D+R	Rec.
	Based on Assumptions in Table 1				
25	$ 47,047	$ 41,410	$ 41,733	$ 43,393	$ 44,613
35	60,180	52,118	52,685	51,155	55,100
45	90,420	77,731	80,750	71,223	86,497
55	138,432	119,798	127,724	102,431	147,174
	Based on Assumptions in Table 1 with Improved Persistency				
25	$ 63,089	$ 54,441	$ 54,929	$ 74,531	$ 76,627
35	80,699	68,331	69,186	87,863	94,639
45	121,251	101,756	106,369	122,332	148,567
55	185,632	156,905	169,091	175,935	252,784

Issue Age	Total Assets due to ROP Rider Based on			Expected Payout	
	Total In-Force	D+R	Rec.	D+R	Rec.
	Based on Assumptions in Table 1 with Improved Persistency				
25	$ 79,596	$ 70,948	$ 71,436	$ 74,531	$ 76,627
35	100,525	88,157	89,012	87,863	94,639
45	149,631	130,136	134,749	122,332	148,567
55	211,363	182,636	194,822	175,935	252,784

RESERVES

Earlier we stated that we should establish on the tenth policy anniversary a reserve which, together with future rider premiums and interest, would be sufficient to pay the ROP benefits as they matured. The following is a method for determining the relative level of reserve *after* claim offset without running additional simulations. These reserves would be based on the following concepts which were discussed earlier:

1. Through a simulation program, estimate
   a) The probability of a ROP benefit.
   b) The expected ROP claim.
   c) The probability that a person who pays a premium at the beginning of the policy year is still a member of the group.
2. Allocate premium received each year to each group. A group would be defined as those policyholders who have the same issue age, issue date, gross annual premium per unit of insurance, and attained age at the beginning of the current ten-year cycle.
3. For a particular ten-year cycle beginning at age y for individuals aged x at issue, there is a net premium ${}^x\pi_y$ which, when accumulated at interest and "survivorship" for the group, will be sufficient to fund the ROP benefit at the end of the ten-year cycle.

These concepts lead to the following definitions and equations:

${}^x_t p_y$ = Probability that a person aged x at issue who began a new ten-year cycle at age y will still be a member of the group at the beginning of year $(t+1)$ after the group began;

l_{y+t-1} = Expected number of people in force at the beginning of policy year $(y-x+t)$;

${}^xB_y$ = Average ROP claim for a person aged x at issue who began a new ten-year cycle at age y and will receive a ROP benefit at age $y+10$;

$$D_{y+t-1} = l_{y+t-1}\, v^{y+t-1}\,;$$

$${}^xD'_{y+t-1} = D_{y+t-1}\, {}_{t-1}^{\;\;x}p_y\,.$$

The net premium is

$${}^x\pi_y = {}^xD'_{y+10}\, {}^xB_y \Big/ \sum_{t=1}^{10} {}^xD'_{y+t-1}\,.$$

By the prospective approach, the reserve is given by

$${}^x_tV_y = \left({}^xD'_{y+10}\, {}^xB_y - {}^x\pi_y \sum_{r=t+1}^{10} {}^xD'_{y+r-1}\right) \Big/ {}^xD'_{y+t}\,,$$

and by the retrospective approach, the reserve is

$$ {}^{x}_{t}V_{y} = {}^{x}\pi_{y}\sum_{r=1}^{t}{}^{x}D'_{y+r-1}\Big/{}^{x}D'_{y+t} \; . $$

On the basis of the formulas stated above, the reserves after claim offset were calculated for the ROP rider attached to a basic policy having a two-year indemnity period with a seven-day elimination period. The reserves and net premiums for policies in the initial cycle used the values shown in Tables 4 and 8 and the Commissioners 1958 Standard Ordinary Table with $3\frac{1}{2}$ per cent interest and are shown in Table 10. Of interest is

TABLE 10

RESERVES FOR ROP BENEFIT

(Basic Plan: 2-Year Indemnity Period—7-Day Elimination Period)

	ISSUE AGE			
	25	35	45	55
	Based on Death and Recovery			
Duration:				
1	$ 16.89	$ 20.31	$ 27.89	$ 37.42
2	35.14	42.41	58.70	77.89
3	54.61	66.63	92.32	124.24
4	76.07	92.92	129.83	176.40
5	100.22	122.12	172.16	233.90
6	127.17	157.27	221.76	310.29
7	156.95	195.10	279.86	397.23
8	188.19	235.45	342.56	496.03
9	224.05	283.88	414.18	619.05
	Based on Recovery Only			
1	$ 17.06	$ 21.38	$ 31.76	$ 48.27
2	35.56	44.72	65.87	99.80
3	55.77	70.32	103.23	156.26
4	77.61	98.36	144.78	218.50
5	101.28	129.31	190.13	286.83
6	126.72	164.05	239.52	365.50
7	155.38	200.86	298.75	450.12
8	186.95	244.30	359.95	549.99
9	223.16	290.07	429.23	660.56
Gross premium	$ 8.04	$ 10.30	$ 15.57	$ 24.02
Net premium (D+R)	15.79	18.91	25.96	34.66
Net premium (Rec.)	16.05	20.06	29.84	45.47

the fact that the gross annual premium for the ROP rider is approximately one-half the appropriate net premium. However, this is not necessarily alarming, because of the effects of lapses in the early policy years.

The reserves in Table 10 are for policies within the initial ten-year cycle. Theoretically, for each issue age and basic disability plan combination a separate set of reserve factors must be established for each attained age at which a new cycle can begin. Obviously this would require a substantial amount of work. Therefore, we must look toward approximate methods to eliminate the work involved.

The obvious first step would be to reduce the number of simulations required to derive the probability of a ROP benefit, the average ROP claim, and the probability that a person who pays the premium at the beginning of policy year t is still in a particular group. A study of morbidity data reveals that the probability that an insured person will be disabled t or more days increases as the age of the insured increases. This is reflected in the increased probability of disablement and the increased probability of remaining disabled. Thus we would expect the probability that a person will have accumulated claims exceeding a specified amount (the claim cutoff level for a person aged x at issue) in duration t of a particular ten-year cycle to increase as the age at the beginning of the cycle increases. Since ${}^{x}_{t}p_{y}$ is the complement of this probability, the following inequality would be true:

$$ {}^{x}_{t}p_{x} \geq {}^{x}_{t}p_{x+1} \geq {}^{x}_{t}p_{x+2} \geq \ldots $$

for all x and t. Similarly, we would expect the amount to be paid at the end of a ten-year cycle to a person with original issue age x to decrease as the age at the beginning of the cycle increases. Or, expressed symbolically,

$$ {}^{x}_{10}p_{x}\,{}^{x}B_{x} \geq {}^{x}_{10}p_{x+1}\,{}^{x}B_{x+1} \geq {}^{x}_{10}p_{x+2}\,{}^{x}B_{x+2} \geq \ldots $$

for all x.

If ${}^{x}_{t}p_{x}$ and ${}^{x}B_{x}$ are substituted for ${}^{x}_{t}p_{y}$ and ${}^{x}B_{y}$ in all equations, then the resultant reserves will be greater than minimum. Using this concept, reserves for an insured aged 25 at issue were calculated assuming that y, the age at which a new ten-year cycle begins, equals 25, 35, 45, and 55. Table 11 shows the resulting reserves for the ROP rider attached to a policy with a two-year indemnity period and a seven-day elimination period. Note that the net premiums and reserve factors are lowest at age 55 and highest at age 25.

This approach could be used to calculate the reserves for the ROP rider before claim offset. To do this, ${}^{x}B_{y}$ would be defined as the maximum benefit which would be paid to a person receiving the ROP benefit. All other items would retain their original definitions.

SUMMARY

The combination approach is an excellent tool for testing gross premium assumptions and then for establishing reserve factors for the ROP rider. Through careful definition of the simulation program, needless computer time can be eliminated. In fact, the following steps can be

TABLE 11

RESERVES FOR ROP BENEFIT

(Basic Plan: 2-Year Indemnity Period—7-Day Elimination Period; Original Issue Age 25)

	Age at Beginning of Cycle			
	25	35	45	55
	Based on Death and Recovery			
Duration:				
1	$ 16.89	$ 16.72	$ 16.23	$ 15.04
2	35.14	34.81	33.84	31.52
3	54.61	54.13	52.73	49.38
4	76.07	75.45	73.68	69.43
5	100.22	99.49	97.43	92.46
6	127.17	126.36	124.15	118.76
7	156.95	156.14	153.97	148.64
8	188.19	187.50	185.65	181.10
9	224.05	223.61	222.44	219.53
	Based on Recovery Only			
1	$ 17.06	$ 16.90	$ 16.40	$ 15.20
2	35.56	35.23	34.25	31.91
3	55.74	55.28	53.87	50.46
4	77.61	76.98	75.19	70.87
5	101.28	100.54	98.47	93.48
6	126.72	125.92	123.73	118.40
7	155.38	154.59	152.45	147.21
8	186.95	186.27	184.44	179.94
9	223.16	222.72	221.55	218.67
Gross premium	$ 8.04	$ 8.04	$ 8.04	$ 8.04
Net premium (D+R)	15.79	15.62	15.12	13.91
Net premium (Rec.)	16.05	15.88	15.37	14.14

accomplished through one simulation run without increasing computer time:

1. Determine the profitability of the premium for the ROP rider on the basis of the initial assumptions used to calculate gross premiums for the basic policy.
2. Determine the effect on profitability of changes in any assumption except the underlying morbidity and the percentage of accumulated premium which claims must not exceed in order for the insured to be eligible for the ROP benefit.
3. Determine the effect on the probability of a ROP benefit, the average ROP claim, and ultimately profits, of a change in premium for the basic plan or ROP rider. This is done in the simulation program by appropriate use of accumulators.
4. Calculate reserve factors based on plan and issue age where duration t is measured from the date the current ten-year cycle began and not from the issue year.
5. The expected percentage of policies which are in the initial ten-year cycle can be compared easily to the actual percentage on the valuation date. This would be a rough measure of the adequacy of the reserves and the profitability of the ROP rider.

The combination approach looks at the problem of the ROP benefit by viewing the characteristics of the disability claims of a person whom we know to be in force at the end of the cycle. Ignoring the effects of anti-selection, we would expect the claim characteristics for this group to be different from those of all policies issued in the same group, including those who died or lapsed. Table 3 shows that a disability table based on recovery is significantly different from a disability table based on death and recovery, especially at the older ages. The remainder of the tables illustrate the impact that this difference can have on the probability of a ROP benefit and the average ROP claim.

As indicated earlier, the purpose of this paper is to present an approach to testing the profitability of the premium and to establishing a reserve basis for a ROP benefit. It does not answer the questions about the morbidity on the group of policyholders in force at the end of the cycle or about the effect of death benefits on the profitability of the ROP rider.

Hopefully, the companies issuing this rider will publish their experience in the *Transactions*. Although experience on the rider is not old enough to make available morbidity data near the end of the cycle, whatever is available would be helpful. In addition to actual claim experience, one possibility would be to give the proportion to policies which are still in the initial ten-year cycle. These data, by issue age and policy duration, could be obtained readily as a by-product of each year's valuation.

APPENDIX I

DISCUSSION OF DISABILITY TABLES

As stated in the main body of this paper, the tables contained in Volume III of the 1964 CDT were used as the basis for the expected disability costs. However, the continuance table based on only those people who terminate disability by recovery was derived from the 1964 CDT by using the experience contained in the Report of the Committee on Disability and Double Indemnity published in *TSA, 1952 Reports*.

Use of these data from the *1952 Reports* is actually invalid because the data were based on experience derived from life insurance policies which define disability differently. However, in order to simplify work for this paper, the data were used unadjusted and extended in a simple manner to those areas not included in the *1952 Reports*. For example, the probability that a termination in the fifth month was due to recovery was applied to all terminations of disability which occurred prior to the fifth month, and for age at disablement 62 the probability that any termination of disability was due to recovery was considered to be the same as the appropriate one for age at disablement 57.

TABLE IA

RATES OF DISABLEMENT PER 1,000
BASED ON DISABILITIES GREATER THAN 89 DAYS

Age at Disablement	1964 CDT	1930–50 Experience	
		Benefit 2	Benefit 3
22	6.64	2.44	3.71
27	6.57	2.93	4.01
32	7.78	3.49	4.63
37	9.81	4.43	6.34
42	12.57	6.32	8.99
47	16.76	9.34	11.85
52	22.39	13.57	16.87
57	31.10	21.64	28.17

When we compare the morbidity experience on the 1964 CDT to that in the 1952 disability study, we find that (1) the probability that a person is disabled ninety days or more is considerably lower in the 1952 disability study than in the 1964 CDT and (2) the rates of termination of disability for the 1964 CDT are considerably higher at the earlier durations and then gradually grade into the values from the 1952 disability study at the end of the second year of disability.

Table IA compares the probability that a person will become disabled for a continuous period of three or more months, based on the 1964 CDT, the period 2 graduated experience for benefit 2, and the period 2 graduated experience for benefit 3. Table IB compares the monthly termination rates per 1,000 lives disabled at the beginning of the month for the 1964 CDT, the 1930-50 graduated termination rates for benefit 2, and the 1930-50 graduated termination rates for benefit 3. Table IC compares the probability that termination of disability was due to recovery and not to death for benefit 2 with that for benefit 3.

TABLE IB

MONTHLY TERMINATION RATES PER 1,000

Month of Recovery	Age at Disablement			
	25–29	35–39	45–49	55–59
	1964 Commissioners Disability Table			
4	447.5	442.4	391.4	306.1
5	369.1	365.6	321.6	244.7
6	296.9	293.9	255.8	185.9
7	229.8	224.5	190.3	137.2
8	161.3	152.6	124.7	92.6
9	115.4	105.6	84.9	66.4
10	87.0	76.4	71.9	52.6
11	71.4	60.2	48.4	45.7
12	51.3	48.0	40.7	39.9
	1930–50 Graduated Termination Rates—Benefit 2			
4	102.5	95.7	81.7	56.2
5	93.6	88.6	74.7	50.2
6	85.9	82.3	69.9	45.7
7	78.2	75.6	64.5	41.1
8	70.7	68.6	58.6	36.7
9	63.7	61.7	52.6	32.5
10	57.4	55.2	46.9	28.8
11	51.9	49.3	41.6	25.5
12	47.1	44.1	37.1	22.7
	1930–50 Graduated Termination Rates—Benefit 3			
4	156.2	135.8	109.8	83.5
5	129.8	114.1	91.3	63.6
6	109.4	98.4	80.1	54.4
7	92.2	84.7	70.0	46.3
8	78.3	73.0	61.0	39.4
9	67.2	63.4	53.3	33.7
10	58.7	55.6	46.8	29.2
11	52.2	49.2	41.4	25.5
12	47.2	44.1	37.0	22.7

In reviewing these three tables, we find that the probability that termination of disability was due to recovery for a particular age at disablement varies inversely with the magnitude of the termination rates, which, in its turn, increases as the probability of disablement increases. Consequently, we would expect that the probability that the termination of disability was due to recovery would be somewhat higher than that actually used in the paper.

TABLE IC

PROBABILITY THAT TERMINATION OF DISABILITY WAS DUE TO RECOVERY, BASED ON 1930–50 GRADUATED TERMINATION RATES PER 1,000

MONTH OF RECOVERY	AGE AT DISABLEMENT 25–29	35–39	45–49	55–59
	Benefit 2			
4	91.7%	85.1%	72.7%	54.5%
5	92.6	85.7	74.7	57.4
6	92.2	85.3	74.5	58.4
7	91.8	85.1	74.4	58.6
8	91.5	84.8	74.2	58.6
9	91.2	84.8	74.1	57.5
10	91.1	85.0	74.0	56.3
11	91.1	85.0	73.8	54.5
12	91.1	84.8	73.6	52.4
	Benefit 3			
4	95.2%	90.1%	78.5%	55.8%
5	95.7	91.6	82.5	63.1
6	95.2	91.1	82.3	63.4
7	94.5	90.4	81.7	63.3
8	93.7	89.6	80.8	62.2
9	93.0	88.5	79.6	60.5
10	92.3	87.7	77.8	58.6
11	91.8	86.2	76.9	55.7
12	91.3	85.3	74.3	52.9

APPENDIX II

THE SIMULATION PROGRAM

The Monte Carlo method was used for the simulation program because it required the least amount of initial expertise to develop. However, through subsequent work we found that many of the simulation techniques used for life insurance could not be applied to this problem. In the first instance, the probability that a person will receive the ROP benefit at the end of each ten-year cycle and the expected average claim are not readily available. In fact, these are the two items which we are trying to calculate. Also, the technique of exposing a number of lives on one throw of a random number is inappropriate because we are concerned with the accumulation of premiums and claims for each individual. In addition, the technique of exposing one life for a number of policy years is not readily feasible because each individual must be re-exposed to the probability of disability after recovery. Establishing the programming logic to recalculate the required probabilities is difficult.

The following is a brief description of the steps in the simulation program:

1. A policy is issued to an individual aged x.
2. A random number N is cast to determine whether the person is disabled and how long he has been disabled.
3. If the policyholder is not disabled, he is advanced to the next policy anniversary and the program proceeds to step 8.
4. A new number N is cast to determine when during the policy year the policyholder was disabled. (It is possible to assume that all claims occur in the middle of the policy year.)
5. On the basis of the indemnity period, the elimination period, the duration of disablement, and the date of disablement, the company then pays the claim. The claim payment would include both the waiver of premium benefit for that portion of disability in excess of the elimination period for the waiver of premium benefit and the monthly income benefit. The disability and the waiver of premium benefits paid in each policy year would be directly proportional to the amount of the benefits which accrued during the policy year.
6. The date on which the policyholder recovers from disability is determined. If an anniversary date is crossed during a period of disability, the company checks the following on each anniversary date:
   a) If the accumulated paid claims during the current ten-year cycle exceed 20 per cent of ten annual premiums, then the insured begins a new ten-year cycle and consequently enters a new group. Therefore, the policy is removed from the in-force.
   b) If the current ten-year cycle terminates on this policy anniversary, then the ROP benefit is calculated and paid to the policyholder. Again, this policy is removed from the in-force.
7. When the policyholder recovers from disability, we must re-expose him to disability from the date he recovered to the next policy anniversary. Therefore, we go to step 2 if the date on which the policyholder recovers is not a policy anniversary.
8. At this point the individual is not disabled, and the policy is on an anniversary date. The program now does the tests performed in step 6.
9. If the policy is still in force, we return to step 2.
10. Whenever a policy is removed from the in force, the program continues issuing a new policy to an individual aged x by returning to step 1 until a specified number of policies have been issued.

11. During processing of the simulation program, appropriate accumulators maintain a record of claims paid for disability income, number of days for which premiums were waived, number of policies in force at the beginning of each duration, number of individuals to whom the ROP benefit was paid, and the amount paid for the ROP benefit. After the specified number of policies has been issued at age x, the computer prints the values contained in these accumulators.

The random number generator was the same one described by Mr. Robert C. Tookey in the discussion of the paper by Mr. Russell M. Collins, Jr., "Actuarial Application of the Monte Carlo Technique," *TSA*, XIV, 377. Briefly stated, a fixed multiplier, 9,677,214,091, was first squared. Then it was multiplied by the 10 right-hand digits of the resultant number to obtain a 20-digit number. From this 20-digit number, the middle 12 digits were used to obtain two 6-digit random numbers. Successive random numbers were obtained by repeatedly multiplying the 10 right-hand digits of previous 20-digit numbers by the fixed multiplier.

The simulation program was run on a UNIVAC 9200 with 16K storage. Using this computer, the average processing time for one plan having issue ages 25, 35, 45, and 55 was $1\frac{3}{4}$ hours. However, this time would be reduced by assuming that all disabilities occur in the middle of the policy year and by eliminating some extraneous calculations and printing performed in the program. Also, a scientific computer would probably reduce the CPU time considerably.

APPENDIX III

TESTING RESULTS OF SIMULATION PROGRAM

A major difficulty encountered with this simulation program was that of reviewing and checking the results. Obviously the results cannot be verified readily by comparing them with "expected values" for the "no claims" ROP benefit.

The first method tested the simulation program on a series of one-year term contracts having seven-, fourteen-, and thirty-day elimination periods. Using the continuance table in Volume III of the 1964 CDT, the theoretical probability of a person's not having a disability claim exceeding the deductible within one policy year was calculated. Thus the probability of a ROP benefit calculated by the simulation program for a ROP rider which will pay a percentage of gross premium if the individual has not had a claim exceeding the deductible within one policy year could be compared to the expected values. The results of this comparison are shown in Table IIIA.

The second method, which used the same basic concepts, extended the term of the contracts to ten years. As in the first method, the results of the simulation program were compared, in Table IIIB, to the expected number of people not having a continuous period of disability exceeding the elimination period during ten policy years.

When reviewing these comparisons, remember that the expected probability assumes that a person disabled for a period less than the elimination period is not re-exposed for the remainder of the policy year after recovery. Because the program re-exposes a person to disability for the portion of the policy year remaining after recovery, the "actual" results are expected to be slightly lower than the theoretical values. This factor would have larger impact on the ten-year contract than on the one-year contract.

TABLE IIIA

TEST OF SIMULATION PROGRAM
USING ONE POLICY YEAR AS DURATION OF CYCLE

Issue Age	Probability of No Claims Exceeding Deductible (1)	Number of People with No Disability Claims per 5,000 Lives Issued: Expected (2)	Actual (3)	Actual/Expected [(3)/(2)] (4)
	7-Day Elimination Period			
27	89.3%	4,466	4,467	100.0%
32	88.4	4,420	4,444	100.5
37	87.4	4,369	4,367	99.9
42	86.3	4,314	4,315	100.0
47	85.0	4,252	4,247	99.9
52	83.6	4,181	4,182	100.0
57	81.9	4,092	4,137	101.1
62	79.8	3,992	4,008	100.4
	14-Day Elimination Period			
27	92.0%	4,601	4,603	100.0%
32	91.2	4,560	4,562	101.1
37	90.3	4,514	4,516	100.0
42	89.2	4,462	4,450	99.7
47	88.0	4,401	4,376	99.4
52	86.6	4,329	4,276	98.8
57	84.8	4,240	4,253	100.3
62	82.7	4,137	4,146	100.2
	30-Day Elimination Period			
27	96.1%	4,806	4,799	99.9%
32	95.6	4,781	4,789	100.2
37	95.0	4,749	4,788	100.8
42	94.1	4,705	4,711	100.1
47	93.1	4,654	4,626	99.4
52	91.8	4,592	4,619	100.6
57	90.2	4,509	4,529	100.4
62	88.3	4,413	4,422	100.2

TABLE IIIB

TEST OF SIMULATION PROGRAM
USING TEN POLICY YEARS AS DURATION OF CYCLE

Issue Age	Probability of No Claims Exceeding Deductible (1)	Number of People with No Disability Claims per 5,000 Lives Issued: Expected (2)	Number of People with No Disability Claims per 5,000 Lives Issued: Actual (3)	Actual/Expected [(3)/(2)] (4)
	7-Day Elimination Period			
25	30.7%	921	918	99.7%
30	27.5	825	805	97.6
35	24.4	731	687	94.0
40	21.3	638	614	96.2
45	18.2	546	529	96.9
50	15.0	451	480	106.4
55	11.9	358	348	97.2
	14-Day Elimination Period			
25	41.6%	1,249	1,247	100.0%
30	37.8	1,135	1,099	96.8
35	33.9	1,018	970	95.3
40	29.9	897	827	92.2
45	25.7	771	746	96.8
50	21.3	640	644	100.6
55	17.0	510	489	95.9
	30-Day Elimination Period			
25	65.6%	1,969	1,954	99.2%
30	61.8	1,854	1,808	97.5
35	57.0	1,711	1,684	98.4
40	51.6	1,547	1,478	95.5
45	45.7	1,370	1,377	100.5
50	39.0	1,169	1,152	98.5
55	32.0	956	894	93.5

DISCUSSION OF PRECEDING PAPER

WILLIAM A. BAILEY:

Introduction

Mr. Frankovich has presented results of a "combined" approach (based on Monte Carlo sampling) to a disability income return of premium pricing problem. My discussion will present some results based on an alternative method which does not require Monte Carlo sampling. Rather than present a large array of results, I will focus on a single policy issued at age 35 with zero-day accident/seven-day sickness elimination periods. The benefits are of the 80-25 variety, as contrasted with the 80-20 definition used in Mr. Frankovich's paper; that is, 80 per cent of ten years' premiums are returned if total disability benefits received during that ten-year period do not exceed 25 per cent of the total premiums (rider and policy) paid during the period. The results shown are illustrative only and would not apply necessarily to any actual company.

Univariate Frequency Distribution of Claims

Referring to the 1969 Milliman and Robertson Disability Tables and the *1971 Reports* of the Society of Actuaries, we set down the morbidity rates shown in Table 1. Although we could interpolate to obtain corresponding figures for ages 36–44, we interpolated for age 40 and used the results as though they were applicable each year, producing claim costs (for balance of first year of disability) of \$11.95 for accident and \$11.40 for sickness; probabilities (of a claim) of 0.0855 for accident and 0.0720 for sickness; and probabilities (of becoming disabled and remaining in a disabled state at least to the end of the first year of disability) of 0.000798 for accident and 0.001241 for sickness. The next step was the construction of a frequency distribution of claims during the first year of disability. For simplicity we fitted a mathematical curve which would reproduce the age 40 morbidity rates derived above. The rounded numerical results are shown in Table 2. The particular "grid" was selected bearing in mind that the total gross premium for policy and rider was \$111.72 (i.e., 4 times the rider premium of \$27.93), so that no return benefits would be payable at the end of ten years if total disability claims exceeded \$279.30 (25 per cent $\times$ \$111.72 $\times$ 10).

Profit Studies

We adopted the same approach as did Mr. Frankovich, one in which the return benefits payable at the end of a 10-year rollover period are assumed to be funded from the premiums paid prior to rolling over during that ten-year period. However, we used a convolution approach.

The fundamental convolution operation is

$$[x_i^{(1)},\ p_i^{(1)}] \oplus [x_j^{(2)},\ p_j^{(2)}] = [x_i^{(1)} + x_j^{(2)}, p_i^{(1)}\ p_j^{(2)}]\ ,$$

where i assumes each integer value from 1 to the number of lines in the first matrix; and, for each such value of i, j assumes each integer value from 1 to the number of lines in the second matrix; thus the resulting

TABLE 1

	Age 35		Age 45	
	Accident	Sickness	Accident	Sickness
Elimination period	0 day	7 days	0 day	7 days
Claim cost for balance of first year of disability	\$13.00	\$9.20	\$10.90	\$13.60
Probability of a claim	0.096	0.067	0.075	0.077
Probability of having incurred a claim and remaining in a disabled state at least to the end of the first year of disability	0.000561	0.000709	0.001035	0.001773

matrix is obtained by calculating the pair $(x_i^{(1)} + x_j^{(2)},\ p_ip_j)$, for each combination of i and j. The superscript (1) and (2) merely indicate whether the value originates from the first or the second matrix, respectively.

Starting out with a matrix [0 1], the total disability claims for the first policy year are obtained by convoluting the claim frequency distribution (Table 2) with [0 1]. After adjusting the resulting frequency distribution to reflect terminations (i.e., multiplying the frequencies $p_i^{(1)}p_j^{(2)}$ by the complement of the withdrawal rate for that policy year) and restarts (i.e., eliminating lines where $x_i^{(1)} + x_j^{(2)} > \$279.30$), we convolute the remaining frequency distribution with Table 2 again. We perform the same type of convolution for each policy year and store the cumulative frequency for terminations and restarts separately. The frequency distribution of present value of profits (±) from survivors of the ten-year period without restarting can then be calculated: we know that ten

TABLE 2

Grid (Days)	Total Claims (*X*) per $100 of Disability Monthly Income	Frequency (*P*)
	Accident	
0	$ 0.00	0.8425
>0, <7	11.27	0.011458
≥7, <14	34.27	0.010090
≥14, <21	57.28	0.008862
≥21, <28	80.29	0.007763
≥28, <35	103.30.	0.006782
≥35, <42	126.31	0.005908
≥42, <49	149.32	0.005131
≥49, <56	172.32	0.00444
≥56, <63	195.33	0.003834
≥63, <70	218.34	0.003298
≥70, <80	245.96	0.003907
≥80, <90	278.81	0.003110
≥90, <120	339.11	0.005865
≥120, <150	436.96.	0.002656
≥150, <180	534.60	0.001079
≥180, <365	663.29.	0.000516
365 or over*	1,200.00*	0.000798
	Sickness	
>0, <7	See 0.00 above	....
≥7, <14	$ 11.31	0.008284
≥14, <21	34.32	0.007452
≥21, <28	57.33	0.006690
≥28, <35	80.33	0.005992
≥35, <42	103.34	0.005354
≥42, <49	126.35	0.004773
≥49, <56	149.36	0.004243
≥56, <63	172.37	0.003763
≥63, <70	195.38	0.003328
≥70, <80	223.06.	0.004081
≥80, <90	255.92	0.003388
≥90, <120	317.20	0.006938
≥120, <150	415.19	0.003633
≥150, <180	513.00	0.001740
≥180, <365	652.02.	0.001101
365 or over*	1,200.00*	0.001241
Total	....	1.00000

* The distribution can be extended as necessary, to meet the benefit requirements of the specific plan under study.

premiums were paid on this portion of the business; we can calculate the return benefit, take present value at interest only, and subtract the result from the present value of rider premiums after deducting expenses. For each policy year the frequency distributions of profits ($\pm$) from terminations and restarts are simply one-line frequency distributions of the present value at interest only of premiums less expenses, since no return benefit was payable. Finally, the frequency distributions for survivors, terminations, and restarts are merged to give us a complete frequency distribution of the present value of profits ($\pm$).

Rider expenses were assumed to be 57 per cent (of gross rider premiums) in the first policy year and 27 per cent in renewal policy years. Total termination rates (deaths and lapses) were assumed to be as follows:

Policy Year	q^w	Policy Year	q^w
1	50%	5	10%
2	25	6	8
3	18	7 and over	7
4	12		

The probabilities at issue of first rolling over (restarting) or terminating in the indicated policy year turned out to be as shown in Table 3. The first restart probabilities (${}_{t-1|}q_0$) reflect the operation of the total termination rates and are not independent of them.

The probability of surviving (without restarting) to the end of ten years turned out (as shown in Table 3) to be 0.108498, and the expected value of the loss (present value at issue, discounted at 5 per cent interest) for such policies was \$36.25. Offsetting these losses are the expected gains ($\pm$) (present value at issue) shown in Table 4. Thus the expected present value (at issue) of profits is $-$\$8.37 (i.e., $-$\$36.25 + \$27.88). The figures in Table 4 are per \$100 of monthly disability income. Present values were calculated discounting at an interest rate of 5 per cent per annum. These figures (including the expected loss for policies surviving the ten-year period without restarting) were calculated by recording the univariate frequency distributions of total disability claims from year to year during the ten-year period. The rider produces an expected loss (present value at issue) of \$8.37, considering the return benefits payable at the end of the tenth policy year and the premiums received prior to restart during that ten-year period. Calculated separately, the present value of profits from the base policy during the first ten years was \$12.93, reflecting the same total termination rates (death and lapse) together with expense and other assumptions appropriate to the base policy. Thus the net present value of profits for the first ten years (treating the rider as indicated above) was $4.55 under these particular assumptions. The present value at issue of ten years' gross annual premiums (not including rider premiums payable after restart), if desired for measurement purposes, can be determined using the probabilities in Table 3 to construct a survival table.

Natural Reserves

Interestingly enough, natural reserves for the rider can be calculated as a by-product of these and similar calculations for ten-year rollover periods starting at the end of each policy year. By "natural reserve" I

TABLE 3

Policy Year t	First Restart $_{t-1\|}q_0$	Death or Lapse $_{t-1\|}{}_1q_0$	$_{10}P_0$	Total
1	0.012783	0.500000		
2	0.011049	0.121804		
3	0.010024	0.063786		
4	0.009479	0.033667		
5	0.008967	0.023741		
6	0.008522	0.016376		
7	0.008073	0.012586		
8	0.007561	0.011140		
9	0.007012	0.009831		
10	0.006450	0.008652	0.108498	
Total	0.089920	0.801583	0.108498	1.00000

TABLE 4

Policy Year	Present Value of Profits (±) from:		
	Deaths and Lapses	Restarts	Total
1	$ 6.00	$0.16	
2	3.84	0.35	
3	3.18	0.50	
4	2.31	0.64	
5	2.03	0.75	
6	1.62	0.85	
7	1.45	0.93	
8	1.45	0.98	
9	1.47	1.01	
10	−2.66	1.02	
Total	$20.69	$7.19	$27.88

TABLE 5

CONDITIONAL PROBABILITIES OF THE NEXT RESTART OCCURRING IN POLICY YEAR $s+t$ AFTER COMMENCEMENT OF A 10-YEAR RESTART PERIOD AT END OF POLICY YEAR s

DURATION (t)*	$s=0$ †		$s=1$		$s=2$		$s=3$		$s=4$	
	(1) ‡	(2) §	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
1	0.012783	0.500000	0.019175	0.250000	0.020965	0.180000	0.022499	0.120000	0.023010	0.100000
2	0.011049	0.121804	0.018120	0.131549	0.021260	0.095884	0.023334	0.085750	0.024395	0.070159
3	0.010024	0.063786	0.017642	0.069739	0.021170	0.068189	0.023752	0.059873	0.025102	0.054771
4	0.009479	0.033667	0.017061	0.049378	0.020929	0.047403	0.023736	0.046535	0.025085	0.049179
5	0.008967	0.023741	0.016498	0.034187	0.020458	0.036694	0.023203	0.041616	0.024521	0.043981
6	0.008522	0.016376	0.015851	0.026366	0.019655	0.032693	0.022291	0.037079	0.023558	0.039186
7	0.008073	0.012586	0.015017	0.023410	0.018621	0.029029	0.021118	0.032923	0.022318	0.034794
8	0.007561	0.011140	0.014063	0.020721	0.017438	0.025694	0.019778	0.029140	0.020901	0.030796
9	0.007012	0.009831	0.013043	0.018286	0.016173	0.022674	0.018342	0.025716	0.019385	0.027177
10	0.006450	0.008652	0.011997	0.016093	0.014876	0.019955	0.016872	0.022632	0.017830	0.023918
	${}_{10}P_0=0.108498$ $\Sigma=1.00000$		${}_{10}P_1=0.201807$ $\Sigma=1.00000$		${}_{10}P_2=0.250240$ $\Sigma=1.00000$		${}_{10}P_3=0.283809$ $\Sigma=1.00000$		${}_{10}P_4=0.299934$ $\Sigma=1.00000$	

* Measured from beginning of the 10-year restart period indicated, i.e., measured from end of policy year s.

† From Table 3.

‡ (1) $= {}_{t-1|1}q_s^{\text{next restart}}$.

§ (2) $= {}_{t-1|1}q_s^{\text{death or lapse}}$.

TABLE 6

Policy Year (t)*	10-Year Restart Period Begins at End of Policy Year s									
	$s=0$†		$s=1$		$s=2$		$s=3$		$s=4$	
	Deaths + Lapses	Restarts	Deaths + Lapses	Restarts	Deaths + Lapses	Restarts	Deaths + Lapses	Restarts	Deaths + Lapses	Restarts
1	$ 6.00	$0.16	$ 5.09	$ 0.43	$ 3.67	$ 0.43	$ 2.45	$ 0.46	$ 2.04	$ 0.47
2	3.84	0.35	5.24	0.72	3.82	0.85	3.41	0.93	2.79	0.97
3	3.18	0.50	4.07	1.03	3.98	1.23	3.49	1.38	3.19	1.46
4	2.31	0.64	3.75	1.30	3.60	1.59	3.53	1.80	3.73	1.90
5	2.03	0.75	3.17	1.53	3.40	1.90	3.86	2.15	4.08	2.27
6	1.62	0.85	2.86	1.72	3.55	2.14	4.03	2.42	4.26	2.56
7	1.45	0.93	2.90	1.86	3.60	2.31	4.08	2.62	4.31	2.76
8	1.45	0.98	2.87	1.95	3.56	2.41	4.03	2.74	4.26	2.89
9	1.47	1.01	2.78	1.98	3.45	2.46	3.91	2.79	4.14	2.95
10	−2.66	1.02	−4.80	1.98	−5.95	2.46	−6.75	2.79	−7.13	2.95
	$20.69	$7.19	$27.93	$14.46	$26.68	$17.78	$26.04	$20.08	$25.67	$21.18
Total	$27.88		$42.39		$44.46		$46.12		$46.85	
"Maturity"	−36.25		−65.71		−81.47		−92.40		−97.65	
	−$8.37		−$23.32		−$37.01		−$46.28		−$50.80	

* Measured from beginning of 10-year restart period indicated; i.e., measured from end of policy year s.

† From Table 4.

mean the negative of the present value of future profits, given a valuation date. (The calculations for subsequent rollover periods involve the termination rates [death and lapse] assumed for each policy year; for example, the calculations for the ten-year rollover period beginning at the end of the second policy year would involve termination rates of 25 per cent, 18 per cent, etc., rather than 50 per cent, 25 per cent, etc. Thus it is not likely that the calculations for one ten-year period could be modified to produce the figures for a subsequent ten-year rollover pericd, even if the morbidity rates did not differ materially during the two ten-year periods. It is practical to assume termination rates which are a function of years since rollover, or partly a function of policy years since issue and partly a function of policy years since last rollover.)

TABLE 7

t	$s=0$	$s=1$	$s=2$	$s=3$	$s=4$	Total Per Cent Surviving
0	1.000000					1.0000
1	0.487217	0.012783				0.5000
2	0.354364	0.009342	0.011294			0.3750
3	0.280555	0.007429	0.009024	0.010493		0.3075
4	0.237410	0.006312	0.007701	0.008998	0.010181	0.2706

After the calculations (similar to those described above) have been produced for each ten-year rollover period (i.e., beginning at the end of policy years 1, 2, . . .), the natural reserves can then be calculated with the aid of a recursive procedure which determines the probabilities of rolling over in a given year. The recursive procedure (used to obtain the figures shown in Table 7) is required because of the many different combinations of restarts which may have occurred prior to the restart for which we want the probability.

Table 5 presents probabilities similar to those in Table 3, for ten-year rollover periods starting at the end of policy years 0 (from Table 3), 1, 2, 3, and 4. The columns of Table 6 present expected present value (at end of policy year s) of gains (+) or losses (−), for the ten-year rollover period starting at the end of policy year $s = 0$ (from Table 3), 1, 2, 3, and 4.

The natural reserve at the end of the second policy year may be calculated as an example. Using the probabilities in Table 5 and proceeding recursively, calculate the survival probabilities shown in Table 7 as of issue of the policy. Calculate the portion of the retrospective

TABLE 8

RATIO OF NATURAL RESERVES TO RIDER PREMIUM

Duration	Assumption a			Assumption b			Assumption c		
	Base Policy	Rider	Total	Base Policy	Rider	Total	Base Policy	Rider	Total
0	−1.86	0.00	−1.86	−4.86	3.02	−1.84	−16.00	13.12	−2.88
1	−2.05	0.78	−1.27	−5.20	4.02	−1.18	−16.24	14.58	−1.66
2	−1.28	1.22	−0.06	−4.61	4.90	0.29	−14.92	16.11	1.19
3	−0.93	1.58	0.65	−4.07	5.71	1.64	−13.55	17.74	4.19
4	−0.70	1.92	1.22	−3.43	6.53	3.10	−12.11	19.45	7.34

natural reserve for policies which reach the end of the second policy year without restarting,

$$[(8.37 + 0.43 \times 27.93) \times 1.05 + 0.73 \times 27.93 \times 0.487217]$$
$$\times 1.05 = 21.40 + 9.93 \times 1.05 = 32.90\,; \quad (1)$$

for those policies which restarted at the end of the first policy year,

$$(23.32 + 20.39) \times 0.012783 \times 1.05 = 0.59\,; \quad (2)$$

and for those policies which restarted for the first time at the end of the second policy year,

$$37.01 \times 0.011294 = 0.42\,. \quad (3)$$

The sum of (1), (2), and (3) is 33.91, the natural reserve for the rider at the end of the second policy year.

The natural reserve for the base policy at the end of the second policy year was −\$35.75, on the basis of a separate traditional calculation. Thus the natural reserve for the combined policy and rider would be −\$1.84 (i.e., \$33.91 − \$35.75).

The ratios of natural reserves to the rider premium at issue and at the end of each of the first four policy years are shown in Table 8 under three assumptions: (*a*) the same as that used in the example shown above, (*b*) total termination rates reduced by 50 per cent at all durations and the morbidity in the third and later policy years reduced to 60 per cent of that used in assumption *a*, and (*c*) zero lapse, zero morbidity, and deaths according to the 1958 CSO Mortality Table.

E. PAUL BARNHART:

I was extremely pleased to see this valuable additional contribution to the literature on health insurance return of premium from Mr. Frankovich. The subject has needed much more research beyond the "scratching of the surface" I was able to give to it in my own 1970 paper. It remains an extremely important and timely subject, because many companies have entered this field and the potential liabilities involved are enormous. I think that Ernie is particularly to be commended for his work on the simulation technique, on the concept of a recovery-only continuance table, and on his treatment of the concept of current cycle classes or groups of policies. I have a number of observations to offer, some of them merely in the nature of questions seeking clarification of items in the paper.

1. Ernie devotes considerable attention to the alternatives of using disability continuance tables based on death and recovery as opposed to

recovery only. As I understand his reasoning, we should be interested in the probabilities arising from a recovery-only table because we are concerned about the probability of return of premium among those policies in force at the end of the cycle and therefore obviously involving living survivors. Upon termination of a disability brief enough to avoid disqualification, we can return the policy to the "qualified exposure" only if termination was the result of recovery rather than death. I agree with this, and the recovery-only table would appear to be the appropriate basis as long as it reflects properly the relative incidence of recovery by duration of disability. Once a claim has disqualified the policy and the policy drops out of the cycle, it does not matter thereafter (so far as the original cycle is concerned) whether later termination is by death or by recovery.

I feel that the paper, having drawn this important distinction, should be more emphatic in asserting that the recovery-only table is the proper basis. Much of the treatment deals with both bases, as though they were more or less optional alternatives. At the older ages, especially, the results differ dramatically, so it seems quite crucial which form of table is used.

I also hope that, in his reply to this discussion, Ernie will provide more detail as to the construction of his recovery table, perhaps with some quantitative illustration if this could be presented briefly enough.

2. In his discussion following Table 7, Ernie mentions that "Tables 5–7 are concerned with cash flow and therefore do not include . . . reserves." I assume that neither ROP reserves *nor* basic policy reserves are considered. This seems quite significant, because in Table 6, where improved persistency is analyzed, some portion of the increased assets on the basic policy, arising because of improved persistency, would surely have to be allocated to policy reserves, since more basic policies remain in force. I would therefore have to disagree with Ernie's conclusion that the entire "increase in the accumulated assets due to the base plan" can be added to the ROP assets to "fund the expected ROP claims." It would seem that only any excess over the increase necessary to fund the additional policy reserves could be regarded as available to fund ROP claims.

3. I like the author's treatment of reserve classes and reserve funding on the basis of current cycles. This does present some problems, at least in theory, because it amounts to a form of reclassification, which could create difficulties if the original class is regarded as guaranteed. It also creates some difficulty where the level ROP gross premium is graded by issue age; this is the case even under the constant 30 per cent loading used in Ernie's illustrations, because this 30 per cent is applied to a base

rate that increases with issue age. The ROP valuation net premium changes, upon each cycle reclassification, while the original age gross premium remains constant. Hence the margin in the gross premium changes, and the result is an uneven emergence of profit (or deficit).

In spite of such problems, I like the method. In my 1970 paper I dealt with reserves on the basis of original issue date and age, which, while actuarially consistent with original age classification and level premium, leads to its own set of problems, such as negative reserves at certain durations under the "sawtooth" reserve pattern that develops. Mid-terminal reserves also give serious trouble. All things considered, the method of reserving purely in relation to entry age and duration in relation to the current cycle is certainly simpler to work with and much more practical.

4. In his discussion following Table 10, Ernie draws attention to the fact that the ROP gross annual premium is approximately one-half the appropriate net premium. He comments that "this is not necessarily alarming, because of the effect of lapses in the early policy years." I agree with this, but I conclude from his earlier developments of the tenth-year asset funds in relation to the ROP expected payouts that the assumed 30 per cent ROP gross premium is, in most instances, seriously deficient, so that surely we have much cause for alarm, entirely aside from the magnitude of the valuation net premium in relation to the gross premium.

5. Throughout the paper, and as indicated in step 2 of his Summary, Ernie deals with determining the effects of changes in any assumption "except the underlying morbidity." It seems to me that the question of the underlying morbidity is an absolutely crucial one, and I could never feel satisfied with an analysis that did not test the effects of changes in the morbidity. There are at least four basic considerations affecting morbidity, all of which I believe must be tested, for their possible effect:

a) The question of what basic expected standard of morbidity is to be used in the first place. It will not do simply to use a conservative basis of morbidity such as reasonably might be used under the basic policy, because this is anticonservative when it comes to the ROP rider itself.

b) The question of what effect the presence of the ROP rider itself, particularly under assumptions of improved persistency, may have upon the basic morbidity otherwise to be expected.

c) The expectation that the probability of disablement is increased among those lives with a prior instance of disablement, as pointed out by Niels Fischer in his discussion of my 1970 paper. Hence, the probability must be assumed to remain relatively more select among those lives that have not experienced claims. This probability needs to be measured in some way, because the effect will be to increase the proportion of lives qualifying for the ROP benefit, since relatively more of the total volume of disability will occur among lives already partially disqualified or else totally disqualified from the ROP cycle.

d) Finally, the tendency toward some nonreporting or underreporting of smaller disabilities, particularly those that would fall between 20 and 80 per cent of ten years' premiums. It is hard to imagine that policyholders will be so naive as to fail to protect their qualification, particularly if the claim is likely to be worth between 20 per cent and, say, 50 or 60 per cent of the premiums. Lacking demonstrated evidence to the contrary, I would have to assume a significant degree of nonreporting of claims in favor of the policyholders. This, again, clearly seems in need of measurement.

Accordingly, these several effects need to be tested, and it seems to me that it is possible to do so without too much trouble, using either Ernie's simulation approach or another direct approach which I have used for this purpose. The technique I have employed consists of actual enumeration and accumulation of all the possible combinations of claim probabilities, carried through the second claim within the ten-year cycle. There are very few cases of two claims that still leave the policy qualified, so a very slight adjustment of the number of two-claim qualifiers provides for approximation of the very small number of three or more claim qualifiers. For the probability calculations, I employ functionally graduated tables such as those presented in my "1971 Modification" paper earlier this year (*TSA*, XXV, 119), which permit efficient data storage and convenience of computation.

I first ran calculations through the ten-year cycle on the same assumptions as used by Ernie, in order to check the relative results arising from the two methods. The table used here is my functionally graduated version of the 1964 Commissioners Disability Table; slight differences would arise from this, since Ernie used the actual table. Also, my calculations are on the "death and recovery" basis, sufficient at least for comparative calculations. For three or more claim qualifiers, I simply increased the two-claim qualifiers by five per cent. To obtain the average ROP claim, I used 80 per cent return among the no-claim qualifiers (exact) and 70 per cent return among the one- and two-or-more claim qualifiers (approximate, but certainly close—the minimum possible return, of course, is 60 per cent, if the policy qualifies for ROP at all). The results appear in Section I of Table 1 of my discussion.

Next I tested, for one set of assumptions, the possible influence of the "Fischer principle," which is that no-claim lives remain at least partially select, while prior-claim lives become antiselect, with a higher claim probability. My assumptions were intended to maintain the same approximate aggregate frequency of disability among the total group. An outline of the assumptions tested is as follows:

a) For a year following a first claim, the policyholder becomes subject to a 25 per cent increased probability of claim under a seven-day elimination plan and a 50 per cent increased probability under a thirty-day elimination plan. Each subsequent year without further claim, these substandard extras are cut progressively in half. Upon a second claim, the following year becomes subject to 50 per cent extra (seven-day) and 80 per cent extra (thirty-day). Again, for each subsequent year without claim, these extras are also cut progressively in half.

b) The no-claim survivors become subject to a partial select factor, which cuts in as the initial select period wears off, so that this partial select factor becomes a ceiling on the ultimate morbidity of the no-claim survivors. To maintain aggregate total morbidity, this factor has to vary by both age and elimination period. I used the following tabulation:

Elimination Period	Issue Age	
	35	55
7 days	82%	70%
30 days	90	77

The results of this test are shown in Section II of Table 1.

I next tested the further effect of an assumed amount of nonreporting of claims, on the seven-day plan only, where this effect would probably be most significant. I added this adjustment on top of the Section II test, simply by reducing the no-claim select factors by another 10 per cent, to 74 per cent for issue age 35 and 63 per cent for issue age 55. (These Sec. II and Sec. III assumptions are, of course, entirely arbitrary but still give some idea of the possible effects of these influences.) The results are shown in Section III of Table 1.

Finally, I tested the effects of using a net experience table instead of the 1964 CDT, holding all other assumptions the same as in Section I (Ernie's assumptions). Here I simply substituted the 1971 Modification of the 1964 CDT for the 1964 Table and tested this change on the seven-day elimination/twenty-four-month maximum plan. The results are shown in Section IV of Table 1.

In Section I of the table my results are in general quite close to Ernie's, except for the age 55 probability under the two-year/seven-day plan. (I have not been able to determine any particular reason for this one

TABLE 1

ALTERNATE TEST RESULTS OF PROBABILITY OF ROP CLAIM AND AVERAGE ROP CLAIM AT END OF 10-YEAR CYCLE (80% RETURN IF CLAIMS DO NOT EXCEED 20% OF PREMIUMS)

(Death and Recovery Basis)

	PROBABILITY OF BENEFIT				AVERAGE ROP CLAIM	ROP CLAIM COST [(4)×(5)]	% RATIO TO SECTION I VALUE
	0 Claims (1)	1 Claim (2)	2+ Claims (3)	Total Qualifiers (4)	(5)	(6)	(7)
	I. Using Assumptions in Paper						
2-year period/7-day elimination:							
Age 35	26.3%	24.8%	1.8%	52.9%	$334.37	$176.88	
	.....	.....	.....	55.6*	333.87	185.63	
Age 55	13.3	21.9	4.4	39.6	763.93	302.52	
	.....	.....	.....	50.2*	759.82	381.43	
5-year period/30-day elimination:							
Age 35	60.3	15.8	0.3	76.4	300.78	229.80	
	.....	.....	.....	74.9*	300.80	225.30	
Age 55	34.5	26.3	2.2	63.0	874.98	551.24	
	.....	.....	.....	66.7*	883.43	589.25	

* From Table 4 of paper.

TABLE 1—*Continued*

	Probability of Benefit				Average ROP Claim	ROP Claim Cost [(4)×(5)]	% Ratio to Section I Value
	0 Claims (1)	1 Claim (2)	2+ Claims (3)	Total Qualifiers (4)	(5)	(6)	(7)
II. With Assumed Adjustments for Select No-Claim Group and Antiselect Prior-Claim Group							
2-year period/7-day elimination:							
Age 35	32.5%	21.2%	1.5%	55.2%	$338.49	$186.85	105.6
Age 55	23.0	18.0	3.0	44.0	783.36	344.68	113.9
5-year period/30-day elimination:							
Age 35	63.3	19.9	0.3	77.5	301.85	233.94	101.8
Age 55	43.2	20.4	1.9	65.5	887.85	581.54	105.5
III. With Further Assumption as to Nonreporting of Claims							
2-year period/7-day elimination:							
Age 35	35.7%	20.2%	1.4%	57.3%	$340.04	$194.84	110.2
Age 55	26.0	17.4	2.8	46.2	787.53	363.84	120.3
IV. Substituting 1971 Modification Table for 1964 CDT							
2-year period/7-day elimination:							
Age 35	72.5%	14.6%	0.1%	87.2%	$349.41	$304.69	172.3
Age 55	30.7	26.4	1.4	58.5	783.58	458.39	151.5

significant discrepancy.) In Section II the effect of the particular assumptions tested is a moderate increase in the ROP claim cost, although this increase comes out rather lower than I would have expected. In Section III the further assumption as to nonreporting creates a further increase in cost of 5–7 per cent, although the nonreporting would of course produce a partially offsetting reduction in claim losses under the basic contract. In Section IV the effect is a dramatic increase in ROP claim cost. It must also be recognized, however, that if this alternate morbidity table had been used in calculating rates for the basic plan, those rates would have been considerably lower to begin with, so again there is a partially offsetting effect under the basic contract.

As a concluding comment, I might mention that the cumulative generation of these probabilities throughout a ten-year cycle, using actual calculation of all probabilities from a functional table instead of simulation, is a reasonably efficient process. Each full ten-year-cycle calculation (for one issue age and plan and for one set of assumptions) consumed less than 10 minutes of running time on a WANG 720 Programmable Calculator.

(AUTHOR'S REVIEW OF DISCUSSION)

ERNIE FRANKOVICH:

I would like to thank Mr. William A. Bailey and Mr. E. Paul Barnhart for contributing discussions to my paper. I found both discussions thought-provoking and excellent supplements to my paper.

Mr. William Bailey described the convolution approach which he uses to determine premiums and reserves for a return of premium rider. For an example, Mr. Bailey used the 80-25 ROP rider with a rider premium equal to $33\frac{1}{3}$ per cent of the premium for the basic policy. Due to different benefits, premiums, and basic assumptions, the results were not directly comparable to those given in the paper. Therefore, I sought to find a common ground on which to compare the results to mine.

The morbidity assumption, I found, conformed to the experience of Male Occupational Group II in the first year and then somewhat less than Male Group II, thereafter, as published in the *1971 Reports* of the Society of Actuaries (see Table 1 below).

In his Table 3 Mr. Bailey showed the probability *at issue* of the first restart occurring or the policy terminating in a particular policy year. In my paper I assume that the probability *at the beginning of a specified duration* that a person who is then in force is still in the initial cycle (or has not had a restart prior to that point in time) is independent of the total termination rates. To obtain the probability at issue that the person

TABLE 1

Item	Mr. Bailey's Value Age (35/45)	*1971 Reports* Value Age (35/45)	1964 CDT Value Age (37/42)
Probability of a claim:			
Accident	(0.096/0.075)	(0.096/0.075)	(0.045/0.043)*
Sickness	(0.067/0.077)	(0.067/0.077)	(0.081/0.094)
Claim cost for balance of first year of disability:			
Accident	(13.00/10.90)	(13.00/10.90)	(5.90/6.30)†
Sickness	(9.20/13.60)	(9.20/13.60)	(7.70/12.80)†
Probability of having incurred a claim and remaining in a disabled state at least to the end of the first year of disability:			
Accident	(0.000561/0.001035)	(0.00106/0.00096)	(0.00043/0.00054)
Sickness	(0.000709/0.001773)	(0.00087/0.00240)	(0.00077/0.00118)

* Frequency for accident benefit with 7-day elimination period.

† Estimated costs for ages 35 and 45 according to *1971 Reports* of the Society of Actuaries.

is still in the initial cycle (the complement of the sum of Mr. Bailey's probabilities) at the beginning of a particular duration, the appropriate probability that a person who is in force is still in the initial cycle must be multiplied by the probability that the person who was issued the policy is still in force at the beginning of that duration.

Table 2 of my discussion restates Mr. Bailey's Table 3 within the concepts stated in my paper and then compares the results with similar values published for the plan having a two-year indemnity period and a seven-day elimination period in my paper. We find that the probability

TABLE 2

Policy Year	Survivors Beginning of Year	Probability at Issue of Person in Initial Cycle:		Probability of Person Being in Initial Cycle at Beginning of Policy Year	Probability that Person Who Is in Force at Beginning of Policy Year Is Still in Initial 10-Year Cycle	
		Withdrawing	Restarting		Mr. Bailey [(5)÷(2)]	Mr. Frankovich
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	1.0000	0.500000	0.012783	1.00000	1.00000	1.0000
2	0.5000	0.121804	0.011049	0.48722	0.97444	0.9617
3	0.3750	0.063786	0.010024	0.35437	0.94499	0.9183
4	0.3075	0.033667	0.009479	0.28056	0.91239	0.8730
5	0.2706	0.023741	0.008967	0.23741	0.87736	0.8303
6	0.2435	0.016376	0.008522	0.20471	0.84068	0.7857
7	0.2241	0.012586	0.008073	0.17981	0.80236	0.7283
8	0.2084	0.011140	0.007561	0.15914	0.76364	0.6800
9	0.1938	0.009831	0.007012	0.14044	0.72467	0.6393
10	0.1802	0.008652	0.006450	0.12360	0.68590	0.5927
11	0.1676	....	....	0.10850	0.64741	....

that a person who is in force at the beginning of a particular policy year is still in the initial cycle is slightly greater for the plan with the 80-25 ROP rider than for the plan with the 80-20 ROP rider. This is to be expected.

I next tried to determine the average claim which would be paid to the 10.8498 per cent of the people being issued the policy. Since Mr. Bailey stated that \$36.25 was the present value at issue of the loss, I obtained \$799.85 as the average claim (about 71.6 per cent of the total gross premium for the ten-year cycle) by using the following formula:

$$0.108498\{(\text{Av. claim})(1.05)^{-10} - 27.93[(1-0.27)\ddot{a}_{\overline{10}|} - 0.3]\} = 36.25\ .$$

I then tried to estimate the average claim by assuming that the average ROP claim for those people who had one or more claims and were eligible to receive the ROP benefit at the end of the tenth policy year was 67.5 per cent of the total gross premium for the ten years. Those who had no claims obviously would receive 80 per cent of the premium. Since 18.018 per cent of the people surviving to the end of the tenth policy year would have no claims [$(0.8425)^{10}$], the expected average claim would be $729.90, or 70.98 per cent of the total gross premium for ten years. This is very comparable to the 74.78 per cent of the total gross premium which I indicate would be the average ROP benefit for the 80-20 ROP rider on the two-year indemnity period with a seven-day elimination period. The difference of 3.8 per cent of gross premium is due to larger claim offset afforded under the 80-25 ROP rider.

The introduction of natural reserves for the ROP rider is excellent. From the reference to the negative of the present value of future profit I infer that the natural reserve is simply the excess of the present value of future benefits and expenses over the present value of future gross premiums (a gross premium valuation). Obviously, if an individual is calculating natural reserves, the calculations for one ten-year period probably should not be modified to produce the values for a subsequent ten-year cycle, because of the impact of select morbidity and early termination rates.

In my paper I used a statutory mortality table without provision for voluntary withdrawals because I was illustrating a simplified approach to estimating an adequate reserve for statutory purposes. I was attempting to reduce the number of reserve factors which must be stored in a file and yet maintain a reasonable but conservative statutory reserve. Consequently, the calculations for one ten-year period could be modified to produce the figures for subsequent ten-year cycles.

Mr. Bailey indicates that it is practical to assume termination rates which are at least partially a function of years since rollover. This is a valid assumption which emphasizes the care that a company must take to ensure that asset shares on the policy and rider combination, excluding provision for reserves on the basic policy, are at least positive immediately following payment of the ROP benefit at the end of the tenth year or any subsequent year.

Mr. E. Paul Barnhart discusses the actuarial assumptions to be used in calculating or testing the premium for the ROP rider and a calculation technique which he uses to test gross premiums for the ROP rider.

By making his observations, Paul focuses attention on specific actuarial assumptions in my paper and then discusses them. In the first of these observations Paul gives a different and probably better explanation of why the morbidity table used in determining the cost of the ROP benefit should be based on "recovery only." Although I did not place emphasis on it, I also feel that the recovery-only table, or a modification of it, is the proper basis. However, the approach used in my paper to develop the recovery-only table is conservative with respect to the cost of the ROP benefit for the following reasons:

1. A person can be disabled on the tenth policy anniversary and subsequently terminate disability by either death or recovery. This person would receive the ROP benefit if the total disability claims paid and accrued did not exceed the 20 per cent claim cutoff level. Thus the claim frequencies and claim termination rates of the recovery-only table should be modified in the later years of the cycle to reflect the fact that the person is alive on the tenth policy anniversary, is disabled, and dies subsequently before recovering from the disability.
2. The frequency of disability varies depending upon a number of factors, of which occupation and benefit elimination period are examples. If the frequency of disability is increased or decreased, the proportion of disabilities resulting in death will decrease or increase, respectively. This will be especially true with regard to changes in frequencies due to claim antiselection. However, we cannot assume that the number of disabilities resulting in death will remain constant, because a change in frequency due to a higher occupation risk will be accompanied by an increased number of disabilities resulting in death.
3. The actual incidence of the termination of disability by death or recovery and the proportions of such terminations resulting from death also vary depending upon a number of different factors, such as occupation and claim antiselection.

Since the example was meant to illustrate the potential impact of the recovery-only concept, I was not overly concerned with introducing refinements to the following approach to constructing a recovery-only continuance table.

1. Determine the number of people who terminate disability either by death or by recovery in successive intervals since the date of disablement. This table should be consistent with the claim costs used in developing the premiums for the basic policy. For example, 182 people (age at disablement, 57) terminated disability in the seventh month of disability (based on the 1964 CDT).
2. Estimate the proportion of people who recover from disability during each of the intervals. For example, 16 per cent of the individuals (age at disablement, 55–59) recovered from disability in the seventh month (based on the 1952 disability study).
3. Multiply the appropriate values from step 1 by those in step 2. The result is the number of people who recover from disability during each period on a

basis consistent with the morbidity table used in developing the premiums for the basic policy.

4. By appropriate summation of the values obtained in step 3, construct a table showing the number of people disabled t days or more. The result will be a continuance table based on recovery only.
5. Because some individuals who became disabled and then died were probably included in the exposure, such exposure should be adjusted. Since a traditional method of calculating exposure is to use in-force data which include disabled individuals, I reduced the exposure basis for the continuance table in the 1964 CDT by 50 per cent of the number of people who became disabled and did not recover.
6. To obtain the frequency of disability, divide the number of people who are disabled and who recover (step 4) by the adjusted exposure (step 5).

In his second observation Paul is correct when he states that Tables 5–7 are concerned with cash flow and therefore do not include provision for reserves. Company management, when contemplating the addition of the ROP rider to their disability portfolio, should consider at least the following:

1. *Cash position of the rider and of the combination policy and rider at the end of the tenth policy year.*—As Mr. Bailey implied in his discussion of my paper, heavier than normal lapsation of the rider is expected after payment of the ROP benefit. Consequently, the amount of the accumulated assets on the policy and rider immediately following the payment of the ROP benefit should never be negative. If it is, high lapses may prevent the company from ever realizing even a meager profit.

2. *Surplus strain on the company immediately following payment of the ROP benefit.*—The statutory reserves on both the basic policy and the ROP rider should be considered. Currently, disability policies require statutory reserves which are generally greater than the accumulated assets for at least ten years after issue if experience actually were to follow the assumptions included in the gross premiums. Using assets accumulated on the basic policy to fund part of the ROP benefit will result in larger surplus strains even if actual withdrawals follow those assumed. It is within this context that the *excess* of the additional assets over the increase in the statutory reserves due to the more favorable persistency can be used to help fund ROP claims.

In his fifth observation, Paul emphasizes that "the question of the underlying morbidity is an absolutely crucial one," and I agree that its impact should be tested. In the paper, however, I attempted to sidestep actuarial questions which pertain to the morbidity assumptions and which could not be resolved by substituting facts for appearances. Many considerations enter into the selection of morbidity tables for the disability policy and the ROP rider. Briefly, these would include:

1. The morbidity cost of the disability class itself.
2. The underlying continuance table which produces morbidity cost.
3. The impact of the recovery-only concept.
4. The impact of an increased susceptibility to additional disabilities after an individual recovers from disability.
5. The impact of nonreporting or underreporting of claims on the disability costs, the disability continuance table, and the recovery-only continuance table.
6. The impact of insured antiselection at issue and after issue. Only the better risks will want the ROP rider, unless the company establishes too-liberal underwriting rules which allow uninsurables to obtain the disability policy with the ROP rider and subsequently terminate the rider.

Paul then describes the technique that he uses to calculate or test gross premiums for the ROP rider. It is readily apparent that the two techniques produce comparable results. After analyzing the procedures and results obtained by the two approaches, I feel that the following are the probable reasons for any differences in the results:

1. *Assumed incurred date of the disability claim.*—The assumption that claims are incurred in the middle of the policy year will result in fewer ROP claims than the assumption that disability claims occur equally throughout the year. If the ROP rider states that claims paid or claims incurred (probably defined as paid claims plus accrued unpaid liability through the date the ROP benefit becomes due) must be less than a specified amount, a claim incurred near the due date of the ROP benefit may not disqualify the insured even if he remains disabled five or more years after the ROP benefit is paid. Consider, for example, the results for issue age 55 on the two-year indemnity period with a seven-day elimination period. A claim incurred within the last sixty-nine days of the cycle would not disqualify an insured for the ROP benefit if he had no previous claims. On the basis of the 1964 CDT, there is a 5.84 per cent chance that the insured will incur a claim exceeding the 20 per cent claim level if we assume that claims are incurred in the middle of the policy year. However, there is only a 4.76 per cent probability of this occurring if claims are assumed to be equally distributed. I feel that the difference in assumptions will have the largest impact where the frequency of a disqualifying claim is the highest (whether it be the first claim or the fifth claim).

2. *Assumption that the average ROP benefit payable to those people still qualified with exactly one claim within the ten-year cycle is 70 per cent of the total gross premium for the ten-year cycle.*—It seems to me that this probability should be greater than 70 per cent, since the number of people being disabled exactly t days longer than the elimination period is larger than the number disabled t days less than the maximum number of days before disqualification for the ROP benefit.

3. *Assumption that the average ROP claim for those who qualify with two or more claims will be 70 per cent of the total gross premium.*—I feel that this percentage

should be less than 70 per cent. However, this assumption in combination with the prior assumption generally produces reasonable values in relationship to my values. Any differences are probably due to the differing mix of 1, 2, 3, or more claim qualifiers.

4. *Assumption that the number of people who qualify for the ROP benefit with three or more claims is 5 per cent of those who qualify with exactly two claims.*—Without specific proof, I feel that the actual percentage will vary considerably by plan and issue age. As support for this position, one notes readily that the differences in "total qualifiers" vary in direct relationship to the proportion of two-claim qualifiers. These differences also appear to be related to the frequency with which a claim will occur.

5. *Impact of re-exposing individuals to a claim after they recover from the disability in the policy year in which the claim was incurred.*—In my simulations I re-expose the individual to the possibility of a claim if he recovers in the policy year in which the claim is incurred. This has an impact on claim incidence similar to that which Paul attempts to produce by assuming that a person who has been disabled previously is more likely to become disabled subsequently than is a person with no claims. My select period, however, is less than a year. Even in this case, the frequency of disability should be adjusted in the simulation program in order to reproduce the real world as closely as possible.

Mr. Bailey introduced a variation in the ROP rider, a 25 per cent claim rollover provision with a gross premium for the rider equal to $33\frac{1}{3}$ per cent of the premium on the basic policy. An additional variation along these lines would be a 30 per cent claim rollover provision with a 40 per cent gross premium. Other companies have modified the concept of a new ten-year cycle beginning on the policy anniversary following the date when claims exceeded the claim rollover level by stating that they will pay the ROP benefit at the end of *any* ten-year period in which claims do not exceed the claim rollover level. This latter definition is slightly more liberal than the original ten-year cycle definition if the insured does not die. Since generally the ROP benefit includes a death benefit, this modification to the definition of the ten-year cycle becomes more significant.

It appears that the ROP rider remains a controversial subject, although the intensity of the controversy has diminished since Mr. Barnhart wrote his paper in 1970. It had been hoped that companies with experience on this product would present some data as part of the discussion of my paper. Unfortunately, this did not occur. Consequently, insurance management is forced to choose between the advice given by the proponents of this product and that given by its opponents. Neither side in reality, has made available much factual evidence. Although I am against this benefit, because I feel that it will result in losses to the insurance company, with little possibility of producing profit, I can understand the position in which a small insurance company finds itself, with the agents clamoring for the product and other competitors achieving record sales through the use of it.

If management decides to issue the ROP rider, I feel that as soon as possible they should establish mechanisms for determining the profitability of the rider. These mechanisms are readily available as by-products of my simulation technique (the expected values) and as a by-product of the valuation program (the actual values). The actual and expected experience can be compared by means of the following tests.

1. PERSISTENCY OF POLICIES HAVING THE ROP BENEFIT

Substantially improved persistency will result in an inadequate premium for the ROP benefit. It is expected, however, that the improved persistency on the basic policy will result in additional profits to offset the losses experienced on the ROP rider. Asset shares studies should be made to ascertain the expected profits, and the actual persistency should be reviewed in order to verify that, because of drastically improved persistency, the company is not going to sustain a loss on the combination.

2. FREQUENCY OF THE ROP CLAIMS

An extremely vital segment in the profitability of the ROP rider is the ex-expected number of claims which will be paid on the rider. Unfortunately, by the time the first ROP claim is paid, it may be too late for the insurance company to act because the first claim is not payable until ten years after the policy was first issued. For the small, aggressive life insurance company this could be critical financially, especially if the product is as unprofitable as the opponents state. Consequently, the percentage of people in force who are still in the initial cycle should be followed very carefully.

A by-product of the simulation program is the probability that a person who is in force is still eligible to receive the ROP benefit at the end of ten years. These values progress on a pattern which is not linear toward the probability that a person who is in force at the end of the tenth policy year will receive the ROP benefit. Although these values vary by plan, issue age, and size of margin in the gross premium, a small sample set of expected values can be obtained readily. The insurance company maintains in the valuation record the year of issue and the date the person is eligible to receive a ROP benefit. Thus the company, as a by-product of the valuation, can divide the number of people who are still in the initial cycle by the number of policies issued in the appropriate year and still in force.

3. COMPARISON OF ACTUAL WITH EXPECTED RESERVES

Another important factor in the profitability of the ROP rider is the claim offset provision. The premium contemplates that a certain percentage of claims will be used to reduce the ROP benefit on those actually receiving it, and a steady progression of small claims eventually will disqualify a number of people who would otherwise receive the ROP benefit.

Since this test will reveal the profitability or unprofitability of the ROP rider earlier than the second test, the company should make the additional effort to determine whether the actual reserves are greater than, equal to, or less than expected. This can be accomplished readily in the valuation program.

First calculate the reserve factors based on an assumed level of incurred claims by using the formulas (developed in my paper) to produce reserve factors after claim offset. Hereinafter this reserve factor times the number of people still in the initial cycle will be called the "expected reserve."

Next calculate reserve factors before claim offset. This can be done easily by defining ${}^{x}B_y$ as the maximum ROP claim for a person aged x at issue who began a new ten-year cycle at age y and will receive a ROP benefit at age $y + 10$. In reviewing the formulas, we find that the reserve factors before claim offset and the associated net premiums will equal the appropriate reserve after claim offset and the associated net premiums times the ratio of the maximum ROP benefit to the average ROP benefit. The valuation program would store one set of reserve factors and a set of ratios for use in going from one reserve basis to the other. Thus it would be easy for both valuations to be done simultaneously.

In analyzing the results of this test, the results of test 2 above should be considered. Poorer experience on the ROP rider will be revealed when the actual reserve established on people still in the initial cycle becomes greater than the expected reserve established on the same people. After a short period of time, we would expect to find that the actual percentage of people who are still in the initial cycle is greater than the expected percentage.

I would like to thank Mr. William A. Bailey and Mr. E. Paul Barnhart again for contributing discussions of my paper. We hope that companies will publish actual data on the ROP rider as soon as possible.