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CHOICE AND JUSTIFICATION OF AN INTEREST RATE

IRWIN T. VANDERHOOF

ABSTRACT

This paper, which is based on material presented by the author in a lecture given at an earlier meeting of the Society, develops a new method of determining long-range interest rate assumptions. The importance of interest rate assumptions is discussed first, after which the author describes a new scientific method of predicting interest rates for use in designing premium rates for new business, in dividend scales, and in adjusted earnings. A discussion of techniques for predicting the interest rate environment is followed by development of a life insurance asset model. Finally, examples of the results obtained from a computer model embodying the new techniques that have been described are presented to illustrate the effects of varying the crucial parameters.

INTRODUCTION

ONE of the aims of our society is to replace impressions with demonstrations; perhaps replacing conjecture with calculation might be another appropriate phrase. In the case of the interest rate assumption, something along these lines is overdue.

Actuaries have approached a decision about interest rate assumptions by first conferring with colleagues, reinsurers, investment officers, and economists. After consideration of their advice and following a review of recent trends of interest earnings in his company and in the marketplace, the actuary makes a decision based on his judgment. This method has obvious disadvantages:

1. Its success depends on the judgment or luck of the actuary, and it is not possible to determine in retrospect whether judgment or luck was the essential factor leading to good or bad results.
2. There is no way to teach others this method.
3. Differences of opinion are difficult to reconcile.
4. The advisers, including economists, have a poor record on their predictions, especially over the long periods which actuaries must consider, that is, twenty years or more.
5. No one has experience appropriate for the environment of 7.5 per cent high-grade, long-term rates.

6. The use of unaided judgment is less defensible when possible variations become large. Since yields ten years from now could be argued to range from 3.5 to 10 per cent, this situation now exists.

Therefore, a new approach is desirable, for the following reasons:

1. We have a professional duty to develop and adopt improved techniques in all areas of our profession.
2. The move toward reporting adjusted earnings places a new kind of responsibility on actuaries. The choice of interest rate assumptions has been reserved to actuaries because of their professional expertise, and this implies that we have techniques not available to others. We cannot accept such responsibility if, in fact, we do not have such techniques and a willingness to apply them. Only the application of scientific actuarial techniques can avoid possible professional embarrassment or even actual personal liability in the event that an assumption fails to be realized. Both the profession and the individual would be ill served if management, after ten years of operation of a company, was forced to accept the fact that the adjusted earnings over those years were overstated because of an unmet interest assumption. Earnings for each of those years would have to be restated, which, at the least, would cause stockholder shock. The management and the actuary should have something better to say than that "it was just an error in judgment."
3. One important function of the change to adjusted earnings will be to allow comparability among earnings of different companies. If judgment is the only guide on the interest assumption, such comparisons may not be valid.
4. Insurance departments will probably be more receptive to adjusted earnings reserves based upon scientific assumptions than to those arrived at by judgment.
5. A scientific approach will aid in isolating reasons for variance of actual from predicted results and provides early warning of unfavorable trends. On the other hand, should a variance develop in a noncrucial factor, the remainder of the interest prediction need not be affected.

Throughout this paper the effects of income tax are ignored.

IMPORTANCE OF INTEREST RATE ASSUMPTION

The importance of not only the level of interest rates but also the pattern of interest earnings will be demonstrated by illustrations based on the two main uses of interest assumptions, namely, gross premium rates for nonparticipating insurance and benefit reserves for purposes of adjusted earnings.

Gross Premiums

In Table 1 we detail a series of specimen gross annual premium rates for a nonparticipating endowment at age 95, male aged 40. The calculation at the bottom of the table shows two "needed premiums" and profit margins. The twenty-year needed premium is the gross premium, according to the assumptions, that will exactly accumulate to the statutory reserve at the end of twenty years. The twenty-year profit per cent is [100 − 100 × (needed premium ÷ 19.95)]. This is the per cent of the gross premium that is not needed to accumulate to the reserve and is therefore the per cent of the gross premium that is profit. Correspondingly, the final needed premium is the premium to exactly pay all benefits and expenses when due according to the assumptions and leave exactly $1,000 for each survivor at age 95. The profit per cent is calculated as above. Assumptions are detailed in Appendix II.

TABLE 1

SPECIMEN PREMIUM RATES

(Endowment at Age 95, Nonparticipating—Male Aged 40, Gross Premium $19.95)

Interest rate for:							
1st 5 years. . . .	6.5 %	7.5 %	6.5 %	6.5 %	6.5 %	6.5 %	7.5 %
2d 5 years.	6	6	7	6	6	6	7
3d 5 years. . . .	5	5	5	6	5	5	6
4th 5 years. . . .	4	4	4	4	5	4	5
Thereafter. . . .	3.5	3.5	3.5	3.5	3.5	4.5	4.5
20-year needed premium.	$19.82	$19.75	$19.53	$19.43	$19.41	$19.82	$18.69
20-year profit %.	0.7%	1.0 %	2.1 %	2.6 %	2.7 %	0.7 %	6.3 %
Final needed premium.	$19.91	$19.86	$19.67	$19.59	$19.57	$19.00	$18.12
Final profit %. . .	0.2 %	0.5 %	1.4 %	1.8 %	1.9 %	4.8 %	9.1 %

The upper section of Table 1 details various sets of interest rate assumptions. The first column of interest rates corresponds to a set used several years ago by a large stock company. The next five columns illustrate the effect of changing one of the period interest rates by 1 per cent: the second column differs from the first only by the use of 7.5 per cent for the first five years, the third differs from the first only by the use of 7 per cent for the second five-year period, and so on. The last column has, for each period, the highest value used by any of the alternatives.

Inspection of the results of these calculations leads to several interesting conclusions:

1. Methods of premium calculation which focus on the first twenty years only do not solve the problem of very-long-term interest rates but only hide it. In the first column of figures we see that $19.82 is all that is needed to accumulate to the reserve at the end of twenty years. However, the needed

premium over the whole period is $19.91, so that we can see that the accumulation of the reserve at the end of twenty years does not guarantee adequacy of the premium for the whole term of the policy if the interest earnings after the initial period are as low as the reserve interest rate. The margin in the reserve is only the conservatism inherent in 3.5 per cent interest. Conversely, high earnings after twenty years render the sixth column the next to most favorable.

2. The assumed interest rate during the first five years is relatively unimportant, while the importance of each successive five-year period increases.
3. If we can earn favorable interest rates over a long period, the postulated relatively low rate contains substantial profit margins. An additional 1 per cent per year increases the profit per cent from 0.2 per cent, or $0.04 per year per $1,000, to 9.1 per cent, or $1.83 per year per $1,000.
4. The importance of interest is not limited to stock companies in their gross premiums but is correspondingly reflected in mutual companies in the dividends they pay policyholders. An increase of 1 per cent in the dividend interest rate by a mutual company might raise the twenty-year total dividends by as much as 20 per cent.

Adjusted Earnings

Prior to the American Institute of Certified Public Accountants audit guide for life insurance companies, the most widely accepted adjustments for earnings were those according to the method of the Association of Insurance and Financial Analysts published by *Best's*. The adjustment was developed by adding together several items, one of which was an interest rate adjustment. This came from an application of the rule that a 1 per cent increase in the reserve valuation rate would produce a 10 per cent change in the reserves and their increase. Tables 2 and 3 illustrate the effect of changes in the valuation rate, according to this rule. Table 2 shows the results of this rule applied to the earnings of several companies; in Table 3 the same rule is applied to obtain the corresponding modification in surplus. These tables are intended only to illustrate the possible levels of effect on earnings and surplus of a change in the valuation rate of interest. Such a change is, of course, part of the final generally accepted accounting principles (GAAP) modification in financial statements.

Table 4 explores in somewhat more detail the effects of specific patterns of interest rates. Columns 1–4 of Table 4 indicate the effects of the assumptions displayed in the upper section of the table. The size of the benefit reserves decreases with increasing interest rates as we would expect. However, we should note that the 3.5 per cent statutory reserve at the end of twenty years is $358.00, compared to the 3.5 per cent benefit reserve of $376.03 and the 4.5 per cent benefit reserve of $346.96.

Also, the age 65 statutory reserve is $457.00, compared to the 3.5 per cent benefit reserve of $469.90 and the 4.5 per cent benefit reserve of $438.83. The necessary conclusion is that, although the illustration may not turn out to be appropriate in a particular situation, it is possible that the reserves on a 3.5 per cent basis will exceed statutory and 4.5 per cent will be only slightly below statutory. Benefit reserves and the interest rate used need to be examined carefully before being introduced into financial statements, since they may not have as favorable an effect as one would expect. This problem may be avoided if the benefit reserve is

TABLE 2

EFFECT OF A 10 PER CENT CHANGE IN INCREASE IN LIFE INSURANCE RESERVES ON 1971 EARNINGS OF VARIOUS COMPANIES

(Data from 1972 *Best's;* All Data in Thousands)

Company	Life Reserves as a % of Total Reserves (1)	Increase in All Reserves (2)	Gain from Operations (3)	% Change in Gain from Operations Resulting from 10% Change in Increase in Life Insurance Reserves = 10% × (1) × (2) ÷ (3) (4)
Stock A....	34%	$247,656	$73,890	11%
Mutual A..	82	136,019	4,722	236
Mutual B..	80	40,774	8,288	40
Stock B....	88	6,002	1,046	50
Mutual C..	81	96,705	5,536	141
Stock C....	44	191,518	59,802	14
Mutual D..	61	13,700	2,209	38

TABLE 3

EFFECT OF 10 PER CENT CHANGE IN LIFE RESERVES ON UNASSIGNED SURPLUS OF VARIOUS COMPANIES AS OF DECEMBER 31, 1971

(Data from 1972 *Best's;* All Data in Thousands)

Company	Life Reserves	Unassigned Surplus	10% of Life Reserves as a % of Unassigned Surplus
Stock A........	$1,941,389	$190,188	102%
Mutual A......	1,910,201	121,679	157
Mutual B......	639,342	55,153	116
Stock B........	55,223	2,701	204
Mutual C......	1,856,825	109,891	169
Stock C........	1,550,128	98,266	158
Mutual D......	149,435	10,501	142

set to endow the statutory reserve after twenty or thirty years, but one then has the problem of testing the adequacy of the statutory reserve as discussed above.

The real surprise comes upon examination of columns 5–8. Columns 5 and 6 use moderately increased interest rates for the first twenty years, while column 7 uses the pattern of the first column of figures in Table 1. Column 8 uses the most favorable pattern of the last column of Table 1. The reserve at age 65 in column 7 exceeds every other reserve shown, including the statutory reserve, while the most favorable assumption in column 8 produces long-term reserves that are about the equivalent of

TABLE 4

GAAP RESERVES—AGE 40 NONPARTICIPATING ENDOWMENT AT AGE 95

	1	2	3	4	5	6	7	8	CRVM Statutory Reserve
	Interest Rate Assumptions								
1st 5 years	3.5%	4.5%	5.5%	6.5%	5.5%	6.5%	6.5%	7.5%	....
6th–10th years...	3.5	4.5	5.5	6.5	5.5	6.5	6.0	7.0	....
11th–15th years...	3.5	4.5	5.5	6.5	5.5	5.5	5.0	6.0	....
16th–20th years...	3.5	4.5	5.5	6.5	5.5	5.5	4.0	5.0	....
Thereafter	3.5	4.5	5.5	6.5	4.0	4.0	3.5	4.5	....
	Benefit Reserve								
Year 5....	$ 89.87	$ 79.84	$ 71.24	$ 63.87	$ 77.99	$ 77.44	$ 85.94	$ 75.88	$ 70.00
Year 10...	182.22	163.91	147.92	133.99	164.37	169.53	186.16	166.24	161.00
Year 20...	376.03	346.96	320.64	296.90	377.51	382.16	397.29	364.61	358.00
Age 65....	469.90	438.83	410.18	383.89	468.40	472.48	488.44	454.37	457.00
	Benefit Reserve Increase in Year								
Year 1....	$18.16	$16.05	$14.26	$12.75	$15.42	$15.02	$16.31	$14.36	$ 2.00
Year 2....	18.84	16.63	14.75	13.16	16.08	15.80	17.28	15.20	16.00
Year 3....	17.61	15.63	13.94	12.48	15.26	15.15	16.66	14.70	17.00
Year 4....	17.54	15.64	14.00	12.58	16.41	15.45	17.07	15.12	17.00
Year 5....	17.72	15.89	14.29	12.90	15.82	16.02	18.62	16.50	18.00
Year 10...	19.00	17.47	16.08	14.82	18.35	21.20	23.13	20.88	18.00
Year 20...	19.22	18.56	17.89	17.20	25.61	25.50	21.23	20.39	20.00
Age 65....	18.52	18.29	17.97	17.59	18.01	17.89	17.98	17.83	20.00

level 4 per cent reserves. This result should be disheartening because the normal pattern of reducing interest rates actually penalizes earnings compared to statutory reserves.

Even worse results follow. Comparing the effects of the reducing assumptions on early reserves and reserve increases, one sees that, even though reserves are increased at the higher durations, they are lower at the short durations.

If one compares the situation of a company choosing between column 2 and column 8 as a basis for reserves, the situation is revealed in all its unfavorable aspects. In the first place, if the companies actually have earnings according to the pattern of column 8, the company using column 8 as a basis will have higher earnings in the first four years because the reserve increase will be lower. Thereafter the company using the column 8 basis will have lower earnings, and by the eighth year it will have lost the initial advantage. After that the total earnings will be lower than for the company using a flat earnings assumption. However, if earnings drop quickly to 4.5 per cent the company using the column 8 basis would presumably have to restate its reserves on a more conservative basis (the meaning of "conservative" in this context is uncertain). The reducing rate assumption has the twin disadvantage of reducing the company's earnings over a long period while increasing the possibility of an embarrassing restatement of earnings if the interest assumption does not hold up. This discussion specifically refers to a single year of issue followed through. The results in the aggregate would depend upon the actual mix of business involved.

Reflection indicates that this result is actually in accordance with the audit guide requirement that profits emerge in relation to premium revenues. However, nowhere in the audit guide is there any hint that varying interest rate assumptions were contemplated. There is also no indication that a system of computing reserves that defers income to a greater extent than statutory accounting was expected or intended. Before making a decision to use such an assumption, actuaries should not only be aware of the consequences but also alert their managements to this effect. Experimental use of a model, such as that described later in this paper, under different investment policies should precede such a decision. An investigation of the technique of immunization to justify a flat rate at a high level also should be carried out. In view of the effect on current earnings and earnings over the near term, such co-ordination of the investment and actuarial functions not only should be worthwhile but could be the most important work going on in a company.

PREDICTION OF INTEREST RATES

Actuaries sometimes seem to treat the interest earnings rate of their company as if it were a stochastic variable fluctuating with random variations around some midpoint or trend line. A little consideration indicates that this is not true. If we know what the earnings rate of a company is this year, we know, within reasonable limits, what the rate will be next year. The reason is that next year's earnings will be from next year's assets, and, since most of the assets the company will have next year are unchanged from this year, the yield on them will be known.

If we postulate that 90 per cent of the assets we will have next year are now in our portfolio, then we need only speculate on the return to be achieved on next year's new investments. If we receive 6 per cent on the 90 per cent still invested next year and the possible variation for next year's new investment rate ranges from a low of 6 per cent to a high of 8 per cent, our total earned rate has limited possible variations:

90% × 6% = 5.4%	90% × 6% = 5.4%
10% × 6% = 0.6%	10% × 8% = 0.8%
6.0%	6.2%

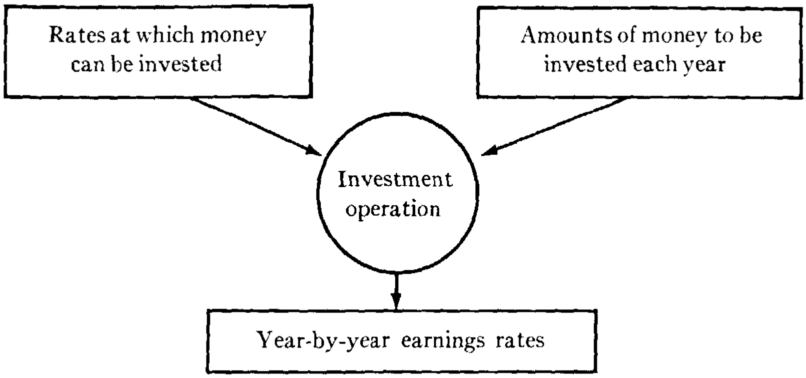
If the variation in the rate on next year's investments is from 5 to 9 per cent, our calculation would be as shown below:

90% × 6% = 5.4%	90% × 6% = 5.4%
10% × 5% = 0.5%	10% × 9% = 0.9%
5.9%	6.3%

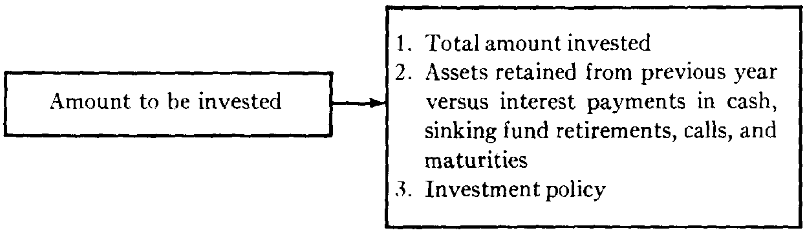
It seems clear, then, that next year's earnings rate is very closely related to that of the current year because of the assets remaining in our portfolio from one year to the next.

If we wish to estimate rates further into the future, we immediately face the problem of the increasing complexity of our calculation. It is easy enough to guess that 90 per cent of next year's assets are in our portfolio now. It is a different kind of problem to try to say what earnings will be in five, ten, or twenty years, using this kind of approach. To do this, we would have to know the various assets then owned and their yields when purchased.

Let us try to analyze the factors involved in such a long-term prediction. In a simple sense, there are two factors—the environment and the amount invested. Actual interest rates will result from the interaction of several factors, shown in the following diagram:



However, this seems incomplete because the amounts to be invested are actually a composite of several factors:



Clearly the total amount invested, if it is known, is one of the factors. If we know the total amount to be invested and then know the actual investments retained from previous years, we know as a difference the amount invested in the current year.

Fortunately we have a reasonable surrogate for the total assets of a company, namely, the reserves of the company. We can use the reserves in place of the total invested assets with reasonable accuracy because companies tend to keep a relatively constant ratio of total assets to reserves—in the area of 105–110 per cent. Thus about 5–10 per cent of the assets are the surplus of the company, and, by their character, companies tend to prefer relatively stable ratios of capital and surplus. In stock companies this can be adjusted by dividends to stockholders. Reserve strengthening can also affect this surplus per cent, but companies cannot allow a substantial drop in surplus, and, to the extent that reserve strengthening can be foreseen, the argument for the use of reserves still holds true. Also, the effect on future assets and earnings is a factor which must be considered when reserve strengthening is under-taken. In mutual companies the surplus is maintained at a reasonably constant ratio by adjusting the dividend scale. Laws in some states—New York for example—forbid surplus for mutual companies to exceed 10 per cent of reserves. A stable ratio in the 105–110 per cent area is then probably an acceptable level for most mutual companies.

There are then two faculties that are needed: (1) a method of predicting the environment—the rate at which new money can be invested—and (2) a method of interrelating the environment with the actual assets a company has and will acquire in each future year according to its investment policy. Will a particular asset mature or be called, or will it stay on the books with its continuing yield adding stability to yields? We need a mathematical model for our assets.

PREDICTING THE INTEREST RATE ENVIRONMENT

Predicting the future level of interest rates is an extremely difficult task, since, until recently, little was known about the variables crucial to the rates and how they interacted. The literature of economics is replete with predictions that were embarrassing to their authors by the time they were published. Naturally, under these circumstances, everyone believes that he is quite competent to make such predictions.

Historically several methods have been used. Perhaps the most fascinating, if not the most successful, is the idea that various economic activities follow a cyclical pattern with fixed periods. In the area of interest rates the Russian economist Kondratieff postulated the existence of a cycle of slightly over fifty years for a variety of economic activities. The worst problem with this long cycle is that there have not been enough repetitions under stable conditions to regard it as proved. An additional problem is that a pure cyclic approach gives no reason or explanation for the theory if it is true. However, one interesting feature is that the series that seem to move together include commodity prices as well as interest rates. This provides some support for the relationship between inflation and long-term interest rates that we will discuss later. Since the cycle was first postulated in the 1920's and predicted a peak about 1970, it has not been disproved by experience.

Perhaps the most frequently used method is that of extrapolation. The most naïve form is to simply presume that current interest rates will continue indefinitely. Use of this technique during any period over the last twenty years would have been wrong. Rates have changed regularly, and even change has not been dependable, since there was one period during the early 1960's when rates were stable.

A more sophisticated version is to extrapolate a trend. This technique would also have proved in error during several periods in the recent past. The period of the highs in 1969 would have been a particularly poor time to use extrapolation.

Among the mathematical techniques for extrapolation, perhaps those of Box and Jenkins [3] are the most sophisticated. David Halmstad, A.S.A., applied these algorithms to a series of rates for call-protected AA utility obligations from 1951 to 1971. We were unhappy but perhaps not surprised when the results clearly established that there was no usable regression.

Using data from earlier periods would probably not correct the problem; the 1940–51 period was one of administered interest rates, while 1919–39 encompassed the greatest abnormalities in the history of our economy when the forces of the federal government and the Federal Reserve were both acting in a way that we know now made the variations more extreme. With our current knowledge, our government would be acting differently, and any conclusion we could draw from that period might not apply. Obviously, data prior to 1913 are useless for this purpose because in this country we did not then have a flexible reserve central bank system and the rates were affected by a money supply that depended on the accidents of gold discoveries.

As a last alternative, econometric models are achieving considerable popularity as a method of summarizing data and providing quantitative as well as qualitative predictions. The most important characteristic of these models may be that, by analysis of the important factors, we determine which are the crucial items. Then, if we cannot necessarily predict parameters such as interest rates, we can at least understand the controlling items which prevent successful predictions. The *Review of the Federal Reserve Bank of St. Louis* contains authoritative economic data and econometric models that work. There also are other competent workers in this field, and their results, while not identical, are certainly consistent.

To provide examples of the material presented, the following is taken from Yohe and Karnosky [9] and Anderson and Carlson [2].

Long-term interest rates are the result of three effects:

$$R_t = R_t^0 + \dot{P}_t^e , \qquad (1)$$

$$R_t^0 = c_0 + c_1\dot{M}_t^0 + c_2\left(\sum_{i=0}^{n} u_i\dot{X}_{t-i}\right) , \qquad (2)$$

$$\dot{P}_t^e = \sum_{i=0}^{n} z_i\dot{P}_{t-i} , \qquad (3)$$

where R_t is the market rate of interest, R_t^0 is the real rate of interest, and $\dot{P}_t^e$ is the expected rate of change in prices. The real rate R_t^0 is composed of a constant factor; a factor $\dot{M}_t^0 = \dot{M}_t/P_t$, or deflated rate of change in money supply; and a series of factors $\dot{X}_{t-1}$ relating to rate of change in gross national product. Finally, in equation (3) inflationary price expectation is, in this model, related only to a series of past actual inflationary price changes.

The effect of $\dot{M}_t^0$ on reduction in interest rates because of increased money supply is the "Wicksell effect." The effect of inflationary expectation on rates is the "Fisher effect." Almost all models must include these factors, although additional factors may be included. The importance of the factors may be varied, and individual economists find it desirable to manage the model to allow for noneconometric considerations.

The combined form of these equations currently applicable would be

$$R_t = 2.82 - 0.06\dot{M}_t^0 + 0.15\sum_{i=0}^{16} u_i \dot{X}_{t-i} + 1.00\sum_{i=0}^{16} d_i \dot{P}_{t-i} ,$$

where

$$\Sigma u_i = 1.0 , \qquad \Sigma d_i = 1.0 .$$

In other words, the current base rate is 2.82 decreased by 0.06 for each 1 per cent increase in the money supply in the current period while it is increased by 0.15 for each 1 per cent increase in the economy and by 1 per cent for each 1 per cent of inflationary expectation based on recent history of price increases. Some independent economists, such as Argus Research, believe that in the foreseeable future the real rate of interest, excluding inflation expectation, will exceed 4 per cent, based upon more rapid growth in the economy and increased need for capital. To this need be added, then, only a perceived rate of price increase to arrive at a market rate.

The various coefficients for this equation and for those following were estimated by using the Almon distribution lag technique [1]. By constraining the distribution of coefficients to fit a polynomial curve of degree n, it is designed to avoid the bias in estimating distributed lag coefficients which may arise from multicollinearity in the lag values of the independent variables. In the case of the above equation the constraints were that $\dot{X}$ and $\dot{P}$ were second-degree polynomials. The period studied extended from the first quarter of 1955 through the second quarter of 1970, but the form of the equation is applicable for the period 1961–70 only.

Perhaps the most valuable use of this type of equation is that it allows us, by considering the ramifications of an action, to decide on its likelihood. From this series of equations we can see that a reduction of interest rates from their current level depends upon a reduction in the rate of inflation. From other analysis most students believe that the economy has adjusted to an inflation rate of 3–3.5 per cent and that further reduction in this rate will require a prolonged period of reduced growth in the economy and higher unemployment, since both of these are related to slowing demand involved in reducing inflation. To the extent to which we believe that our government cannot make this choice, we would expect 7.5 per cent rates to continue, along with increasing expense rates. Reduction in rates to the 4.5 per cent level would have to be gradual indeed to avoid shock effects on the economy.

Besides allowing predictions of results and recognition of the involved factors, this type of equation also indicates why other methods of prediction do not work. If the money supply directly, and money supply and government expenditures indirectly through their effect on the whole economy, are the factors that determine interest rates, then approaches such as extrapolation or the Box-Jenkins techniques or autoregressions cannot work because the crucial factors are exogenous rather than endogenous and depend upon day-by-day and year-by-year decisions on the Federal Reserve and the federal government.

Michael W. Kernan [6] extended the models to include stock market prediction. His model is the most successful that the author of this paper has seen and is available in two versions. The simplest version includes the long-term interest rate specifically:

$$SP_t = 12.33 - 19.3R_t + 3.03R_{t-1} + 4.44\left(\sum_{i=0}^{19} f_i E_{t-i}\right),$$

where SP_t is the Standard and Poor's index at time t, E_t is the corresponding corporate earnings, R_t is the market long-term interest rate, and $\Sigma f_i = 1.0$.

In this version the effect of the long-term interest rate is apparent and, even though the equation is in linear compound form, the coefficients create, within reasonable limits, the same results as the more basic discounting form,

$$SP_t = \sum_i \left[E_{t+i} \Big/ \prod_{j=0}^{i} (1 + R_{t+j})\right].$$

The more complex form, which replaces the interest rate by the explicit terms developed above, is as follows:

$$SP_t = -30.68 + 0.57\dot{M}_t^0 + 0.52\dot{M}_{t-1}^0 + 0.21\dot{M}_{t-2}^0 - 5.37\sum_{i=0}^{7} j_i X_{t-}$$

$$- 11.96\sum_{i=0}^{16} k_i \dot{P}_{t-i} + 4.8\sum_{i=0}^{19} l_i E_{t-i}^0,$$

where $\Sigma j_i = \Sigma k_i = \Sigma l_i = 1.0$. Specific values for these various coefficients are given in Mr. Kernan's paper.

In this form, increases in the money supply and corporate earnings increase the general level of the market, while, surprisingly, growth in the general economy and, as we know, inflation tend to decrease the general market level. This form is particularly valuable because it quantitatively refutes the inflation-hedge argument of common stocks while allowing some confidence in the future of equity growth based upon future earnings increases. The earlier form is valuable because it allows predictions of common stock prices that are consistent with changes in base prices based on interest rate changes. The future level of stock prices in general or even of a specific security can be calculated to be consistent with a given postulated future level of long-term interest rates. This means that empirical justification exists for including appreciation of common stocks as a result of reduced interest rates in an insurance asset model.

Review of the various theories discussed above reveals that only the pure cyclic theory of Kondratieff gives a clear indication of a trend in long-term rates. Other approaches, even though more scientific, leave predictions for a long period in terms of other variables that are subject to political control.

A clear-cut association exists between general economic activity, inflation, and interest rates, and a general decline in interest rates seems to require a lowering of business activity and a lessening of inflationary forces in the economy. Some reputable economists take the position that since inflation can be reduced only with concomitant unemployment, inflation cannot be reduced, and interest rates will not fall from their current levels. The prudent assumption for us, however, is that they will fall eventually. The long-term level of interest rates in this country has seemed to be about 4.5 per cent, and an eventual drop to that area seems to be the only defensible assumption. The best opinion that the author has been able to obtain from the experts in this field is that it might be politically possible to achieve a reduction in interest rates to the 4.5 per cent level over a period of ten years without an impossible amount of unemployment and slowdowns in the economy. The experts also insist that this will not occur. This situation is quite unsatisfactory from our point of view. It is also one we will probably have to live with.

In the illustrations of the use of a computer model later in this paper, the author was forced to make a choice for the new-money rate over the next twenty years and tried to compromise the arguments of the economists with some conservatism. These illustrations have been used for two sets of rates. They are shown in Table 5. The "high" set assumes that 7.5 per cent rates are available through 1977 and that they reduce to 4.5 per cent over a ten-year period. The "low" set assumes that the reduction starts immediately. One advantage of this type of computer model is that it allows the determination of the extent to which this kind of a change is crucial to the outcome and also permits investigation of several different sets of assumptions. Obviously, it is also possible to update the projections as new data become available.

TABLE 5

ASSUMED NEW-MONEY RATES
(Highest-Quality Issues Only)

Year	High	Low	Year	High	Low
1972	7.5%	7.5%	1982	6.0%	4.5%
1973	7.5	7.2	1983	5.7	4.5
1974	7.5	6.9	1984	5.4	4.5
1975	7.5	6.6	1985	5.1	4.5
1976	7.5	6.3	1986	4.8	4.5
1977	7.5	6.0	1987	4.5	4.5
1978	7.2	5.7	1988	4.5	4.5
1979	6.9	5.4	1989	4.5	4.5
1980	6.6	5.1	1990	4.5	4.5
1981	6.3	4.8	1991	4.5	4.5

NOTE.—Hickman study [5] indicates 4.5 per cent as an average rate for the United States.

A LIFE INSURANCE ASSET MODEL TO PREDICT AND CONTROL INTEREST EARNINGS AND NEW BUSINESS

The general purpose of business computer modeling is to allow projections of crucial variables when many possibilities must be analyzed, to test various assumptions about the future, and to explore possible strategies to determine the crucial variables. Insurance companies are ideal business organizations for such simulation, and the model office of the actuary may be the oldest mathematical model for any business.

Our regular model office, however, concentrates on claims, expenses, and income. There is no need for asset analysis, since interest rates are presumed to be stable and/or predictable. Interest rates are the connecting link between the assets and the liabilities, and an asset model is the necessary complement to our regular model office when this connecting link is fluctuating. Interest earnings rates are needed for both existing and new business separately.

Our computer model for assets requires four specific inputs:

1. Reserves for business in force and one year's new business for each future year to be studied. In our examples, twenty years is the period. The starting reserve is normalized to equal the assets, described in item 2 below, and subsequent reserves are automatically similarly adjusted. These reserves are independent inputs, so that, even though we are using ordinary life reserves, we could use group annuity or health reserves if that were our business.
2. Actual existing assets of the company with all necessary data—yield, book value, call year and price, sinking funds, quality according to rating agencies or by the company's own evaluation, and so on—specifically stated. Common stocks are not included in our illustrations, but they can be included, using a variety of models. Real estate is handled as common stocks. Mortgages are grouped by year of inception, yield, term, and quality if that is a factor.
3. Average annual interest rates at which money can be invested in each future year. There should be separate rates for mortgages, short-term investments, long-term bonds, and other forms of investment. The rates may be for prime obligations only or may vary by quality. In the latter case the output yields must be adjusted later to compensate for the risk premiums.
4. A company investment policy. This is a detailing of the proportion of available funds in each future year that will go into each separate type of investment: discount bonds, various quality securities, stocks, mortgages, and so on. It might be desirable to vary this proportion from one year to another to express different investment attitudes depending on money market conditions.

Examination of Figure 1 provides some insight into the characteristics of an asset model (see Appendix III). Model A is from an earlier paper by the author [8] and, as represented in Figure 1, indicates as a lower line the reserves for the business written through 1971, while the upper line indicates the reserves for all business written through 1972. The difference between the two lines represents the reserves on account of the business written during the year 1972. Following these reserves on the business remaining in force through the year 1991, we find that the assets on account of the existing business (1971 and prior) in 1991 are about the same as they were in 1976, and the total assets are about the same as they were in 1978. This means that, depending on how the assets matured, all the assets related to 1972 business could have been purchased prior to 1978. Assets purchased between 1978 and 1984 may have a long-term effect, but they could also be the ones liquidated during the period 1984–91. Because of the dropoff in assets between 1984 and 1991, a possibility exists for capital losses if interest rates are substantially higher during this period. Fortunately, business written in later years obviates any real possibility of having to liquidate any holdings at a loss. The important things to note from Figure 1 are (1) the general pattern of growth and decline in reserves; (2) the fact that the assets many years in the future could be the same assets the company is holding today; and (3) the fact that there may be, over a period of years, a change in attribution of ownership of some assets so that the interest rate attributable to 1972 business after 1991 may be related to investments made no later than 1978.

Examination of Figure 2 (see Appendix IV), which has the same life insurance reserves plus about $50,000 immediate annuity reserves, emphasizes the above conclusions. In this case, because of the rapid decline in annuity reserves, the assets on account of existing (1971 and

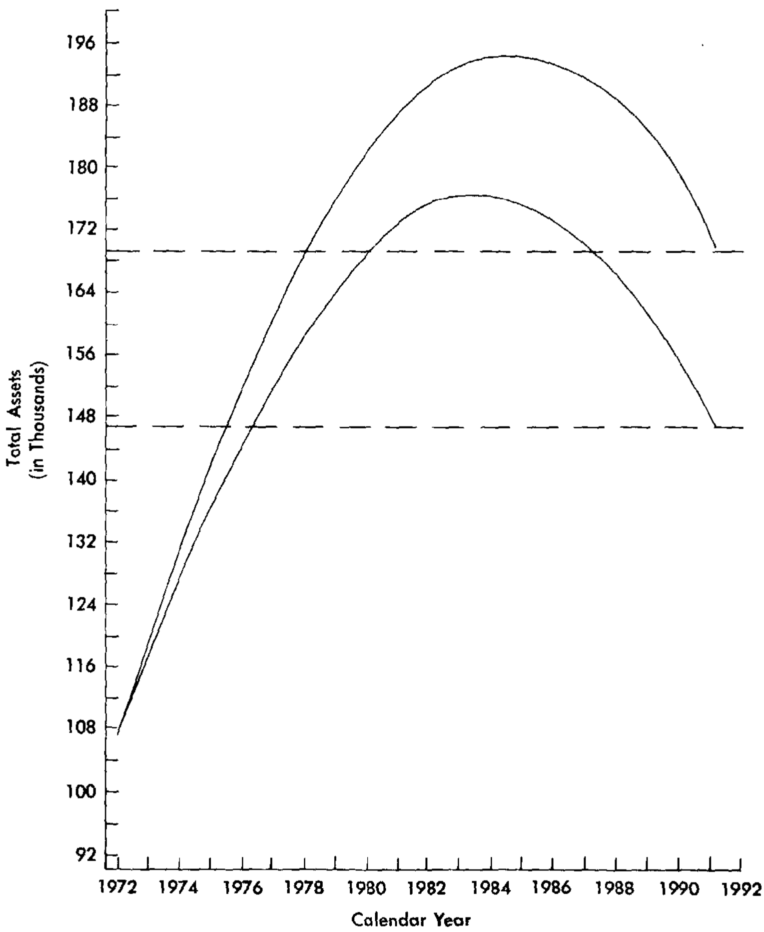


FIG. 1.—Policy reserves in Model A and growth of total assets

prior) business in 1991 are less than they were in 1973, and the total assets are about the same in 1991 as they are in 1975. The required maturity of assets is more important in this case, and the change in attribution of ownership from existing to new (1972) business is more pronounced.

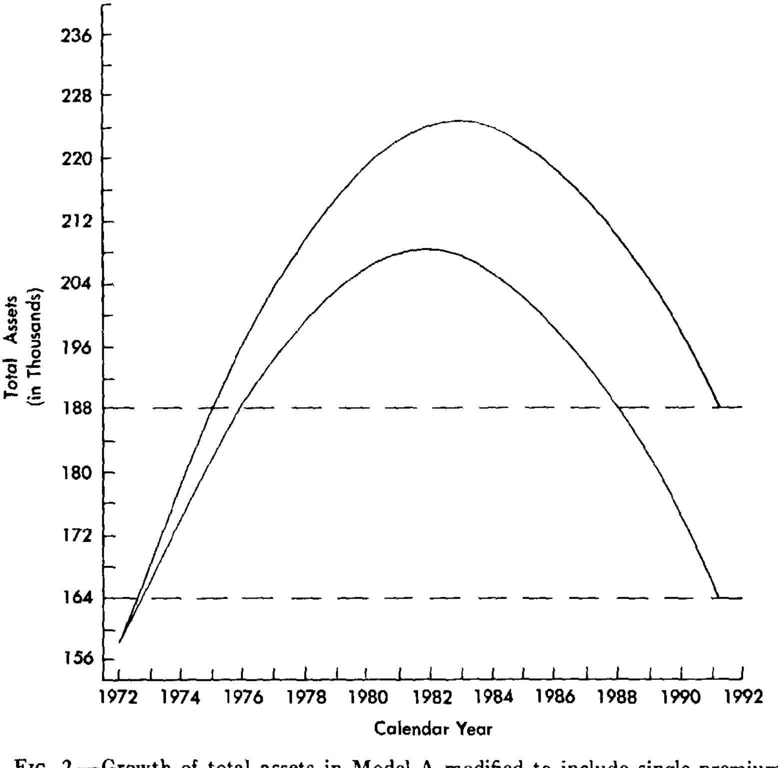


FIG. 2.—Growth of total assets in Model A modified to include single premium annuities.

Finally, Figure 3 (see Appendix V) shows the situation for a company about ten years old writing slowly increasing volumes of insurance. In this case, interest rates earned in the distant future are dependent upon investments made later than in the cases cited previously, although the pattern of increasing and decreasing investments still exists.

To develop the model upon which our projections will be based, first let

R_0^E = Reserves on existing business actually in force at beginning of period;

R_t^E = Reserves on existing business still in force at year t;
R_t^N = Reserves at end of year t on business written in year 1 and still in force in year t;
R_t^T = $R_t^E + R_t^N$ = Total reserves, year t; hence the change in total reserves equals the sum of the changes on account of old and new business, or
$\Delta R_t^T = \Delta R_t^E + \Delta R_t^N$.

All reserves should be increased by the same per cent so that they are equal to the total of the specific assets of the company. This provides

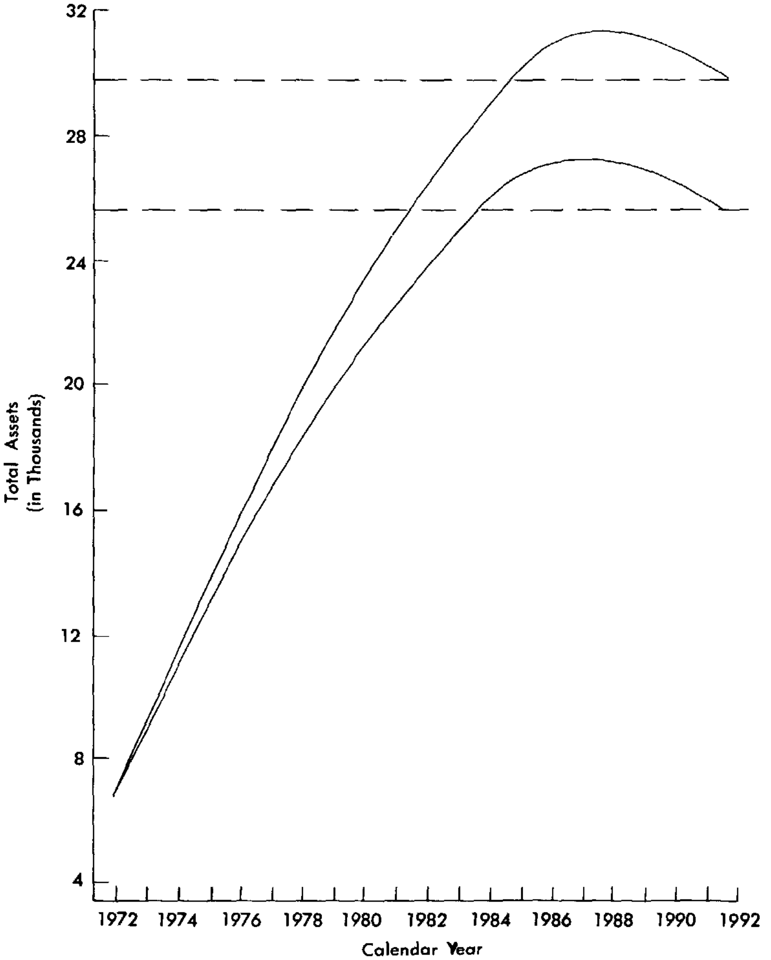


FIG. 3.—Growth of total assets for a newer company

for surplus and is presumed to be done throughout. Let ${}_jA_t$ be the value of the jth asset in year t. In each year,

$$A_t^T = \sum_{j=1}^{\omega} {}_jA_t = R_t^T\,, \quad A_t^E = \sum_{j=1}^{n+kt} {}_jA_t^E = R_t^E\,, \quad A_t^N = \sum_{j=1}^{n+kt} {}_jA_t^N = R_t^N\,,$$

and, if ${}_ji_t$ is the interest rate earned on the jth asset, then the total interest earned on all assets during year t will be

$$I_t^N = \frac{1}{2}\sum_{j=1}^{n+kt}({}_jA_t^N\,{}_ji_t + {}_jA_{t-1}^N\,{}_ji_{t-1})\,; \quad I_t^E = \frac{1}{2}\sum_{j=1}^{n+kt}({}_jA_t^E\,{}_ji_t + {}_jA_{t-1}^E\,{}_ji_{t-1})\,,$$

and the average interest rate earned is

$$\bar{i}_t^N = I_t^N \Big/ \frac{1}{2}\sum_{j=1}^{n+kt}({}_jA_t^N + {}_jA_{t-1}^N)\,; \quad \bar{i}_t^E = I_t^E \Big/ \frac{1}{2}\sum_{j=1}^{n+kt}({}_jA_t^E + {}_jA_{t-1}^E)\,.$$

During each year specific kinds of assets will be purchased and a proportion ${}_kr_t$ of the total change in assets invested in each type. We will assume for simplicity that the number of assets purchased in each future year is the same k. Assets may have a value of zero but are assumed never to be dropped from the array.

If the number of assets at time 0 is n, then the number at time 1 must be $n + k$ and at time 2, $n + 2k$. Each asset has its own characteristics, and these must be considered. Some, like mortgages and sinking fund bonds, have an automatically decreasing asset value, while others actually increase in value as, for example, through the accrual of discount on a bond purchased below par. In addition, some assets will mature in a specific year, while others, if the terms of the asset allow it, will be called if the interest rate falls substantially below the coupon rate. Mortgages are included, using summaries for year and term rate. The set of proportions ${}_kr_t$ constitutes our investment policy.

If in year 0 we have n assets, then, at the end of year 1,

$$\sum_{j=n+1}^{n+k} {}_jA_1^T = \Delta R_t^T - \sum_{j=1}^{n} \Delta_j A_0^T\,,$$

where

$$_{n+m}A_1^T = {}_mr_1 \sum_{j=n+1}^{n+k} {}_jA_1^T \quad \text{and} \quad 1 \le m \le k\,.$$

Since assets are purchased only by the new-business fund,

$$\sum_{j=n+1}^{n+k} {}_jA_0^N = \Delta R_0^N\,, \quad {}_jA_1^N = {}_jr_1 \Delta R_0^N\,.$$

For the analysis of future years, we must subdivide our changes into two types. Type 1, endogenous ${}^1\Delta$, is a change created by the nature of the asset—maturity, accrual, or call. Type 2, exogenous $^2\Delta$, is purchase of a new asset, sale of an asset at the then market value, or transfer of ownership of an asset between existing and new business.

Six situations can occur as far as the reserves are concerned. These are listed in the accompanying tabulation. In situation 1 both segments are

Situation	Existing Business	New Business	Total
1.	Increase	Increase	Increase
2.	Increase	Decrease	Increase
3.	Decrease	Increase	Increase
4.	Increase	Decrease	Decrease
5.	Decrease	Increase	Decrease
6.	Decrease	Decrease	Decrease

purchasing assets. In situation 3 the existing business is reducing its assets. If $^1\Delta A^T < \Delta R^t$, then additional assets will still be purchased. If $^1\Delta A^T = \Delta R$, then no asset transactions take place. If $^1\Delta^E > \Delta R^E$, then some assets owned by the existing business will have to be taken over by the new business. Unless $^1\Delta A^T$ is positive and large, some new assets will be purchased, but if $^1\Delta A^T > \Delta R^T$, some assets will have to be sold according to a predetermined pattern for the sale of such assets.

In situation 5, if $^1\Delta A^T > \Delta R^T$, then assets will have to be sold. If not, the transfer of assets between existing and new business should balance the books. In situation 6 transfers of ownership will probably be taking place, and purchases in total will take place if $^1\Delta A^T < \Delta R^T$, while sales take place if $^1\Delta A^T > \Delta R^T$. Situations 2 and 4 are unlikely but would be handled in a corresponding fashion.

This procedure creates two portfolios of assets flowing into the future. The first, relating to existing business, is composed of assets owned by the company at time 0 plus assets purchased each year according to the company's investment policy adjusted for assets disposed of because of maturities, calls, and the like, or $^1\Delta$'s, or increases of accruals. Assets may also be reduced as $^2\Delta$'s by sale in the open market or to the new-business fund. The fund on account of new business develops correspondingly, except that there are no assets at the beginning of the period. It should be emphasized that this is an analysis of what will happen year by year to each asset, depending on its own characteristics, the environment, and the fact of increase or decrease in the reserve funds.

Obviously, policy loans have a special characteristic as an asset, since they increase automatically. The presumption must be that they increase as reserves are increasing, and more rapidly if the environmental interest rate is above the maximum policy loan interest rate.

As we move from year t to year $t + 1$, we first develop

$$\Delta R_t^T = \Delta R_t^N + \Delta R_t^E .$$

Determine

$$^2\Delta A_t^T = \Delta R_t^T - {}^1\Delta A_t^T ,$$

where

$$^1\Delta A_t^T = \sum_j {}^1\Delta_j A_t^T .$$

Repeating, $^1\Delta$ is developed by actually investigating each separate asset and establishing the amount of accrual of discount, amortization of premium, maturity, sinking fund, or call. $^2\Delta$ represents the adjustments made by the system to the situation revealed by the right-hand side of the equation. $^2\Delta$ is the purchase of additional assets according to investment policy, or the sale of assets at the then market price according to some rule such as the sale that creates the least capital gain or loss. Sales require a re-examination of each of the assets to meet the predetermined condition.

We have then established that

$$A_{t+1}^T = \sum_j {}_j A_{t+1}^T .$$

However, the split between new and existing business remains. Develop

$$^2\Delta A_t^E = \Delta R_t^E - {}^1\Delta A_t^E \quad \text{and} \quad {}^2\Delta A_t^N = \Delta R_t^N - {}^1\Delta A_t^N ,$$

where

$$^2\Delta A_t^N + {}^2\Delta A_t^E = {}^2\Delta A_t^T .$$

If both $^2\Delta$'s are positive, then the ownership of the new assets is split between the new and existing business in proportion to the $^2\Delta$'s although each asset need not be split in the same proportion. If the $^2\Delta$'s are of opposite sign, then the transfer of ownership between new and existing business takes place. If both $^2\Delta$'s are negative, then the sales must be taken from each section in accordance with the total assets in each section. Then each section's ownership of the specific assets sold must be determined and a final transfer of ownership take place to bring each section's ownership of individual assets into correspondence with its total assets.

Examples

Because there is considerable variation in the way all these elements are integrated, some examples of a specific model that has been developed are included. Table 5 shows two sets of assumed new-money rates for the environment. The initial assets were assumed as described in the earlier paper [8] (see Appendix I). The high interest rate assumption presumes continuation of a 7.5 per cent interest rate for five more years and then a ten-year decline to 4.5 per cent, while under the low assumption the decline commences immediately. The ten-year period for the decline was chosen because it is the fastest that could be found in an examination of the history of interest rates. It may be the most rapid decline that would not bring considerable social disturbance.

Table 6 shows a series of rates earned on existing and new business, based on a variety of reserve models, single asset investment policies, and environmental assumptions. The various rates result from the interaction of purchase, calls, transfers of ownership, and changes in the amount of low-rate policy loans. The results in columns II, III, and VI are most interesting because they demonstrate the extent to which portfolio rate can be held above the rates of the environment described in Table 5.

Finally, Table 7 may be compared with Table 1 to show the effect of applying the results of our model to a gross premium calculation. This is especially interesting because the assumption as to the environment does not differ greatly from the assumption of Table 1 but the results are drastically different. Tables 8 and 8A provide a sample of the output of our computer model showing the summaries as well as detailed asset transactions.

Immunization

Immunization is the technique for integrating asset maturity schedules with the operation of a company so that the company's ability to meet future obligations when due and continue with predetermined profit margins is unaffected by future changes in the interest rate environment. This result is achieved by setting equal the first, or first and second, derivatives of the asset and operations income streams. The asset stream is the interest payment, maturities, and so on. The operations stream is premium less claim expenses and the like. In practical terms, if interest rates fall in the future, the company makes enough on capital gains to compensate for the loss of interest on future investments. The author's earlier paper [8] gives a detailed explanation of the practice.

Depending upon the investment policy of the company, immunization will be possible for most companies at some time during the twenty-year period. If the method described as net premium reserve immunization is used, this result may be possible before the end of the first ten years. This should mean that an assumption of over 6 per cent indefinitely can be justified from column II of Table 6 for new business or even for a new

TABLE 6

EARNED PORTFOLIO INTEREST RATES

Company / Investment Policy / New-Money Rate	I Model A Current Coupon 5-Year Non-callable — Low		II Model A Discounts Only — Low		III Model A Discounts Only — High		IV Model A + Ann. Discounts Only — High		V Model A + Ann. Discounts Only — Low		VI New Company Discounts Only — High	
	Old Bus.	New Bus.	Old Bus.	New Bus.	Old Bus.	New Bus.	Old Bus.	New Bus.	Old Bus.	New Bus.	Old Bus.	New Bus.
1972	5.76	....	5.76	....	5.76	....	5.76	....	5.76	....	5.76	....
1973	5.76	7.20	5.76	7.20	5.76	7.30	5.76	7.30	5.76	7.20	5.76	7.30
1974	5.89	7.02	5.89	7.02	5.92	7.30	5.87	7.30	5.85	7.02	6.14	7.30
1975	6.00	6.85	5.98	6.85	6.06	7.30	5.96	7.30	5.91	6.85	6.38	7.30
1976	6.03	6.70	6.03	6.71	6.17	7.30	6.05	7.30	5.95	6.71	6.53	7.30
1977	6.04	6.57	6.05	6.58	6.25	7.30	6.11	7.30	5.97	6.58	6.64	7.30
1978	5.89	6.45	5.89	6.47	6.32	7.30	6.17	7.30	5.79	6.47	6.72	7.30
1979	5.80	6.22	5.89	6.35	6.37	7.26	6.21	7.26	5.80	6.35	6.76	7.25
1980	5.71	5.93	5.87	6.24	6.40	7.20	6.24	7.20	5.79	6.24	6.77	7.19
1981	5.46	5.53	5.71	5.98	6.41	7.14	6.26	7.14	5.63	5.98	6.76	7.10
1982	5.52	5.45	5.83	6.02	6.42	7.06	6.27	7.06	5.76	6.02	6.74	7.01
1983	5.18	5.26	5.81	5.92	6.27	6.98	6.08	6.98	5.76	5.92	6.63	6.91
1984	5.06	5.06	5.81	5.84	6.27	6.90	6.09	6.93	5.76	5.84	6.60	6.80
1985	5.05	5.03	5.81	5.77	6.28	6.83	6.10	6.88	5.77	5.83	6.56	6.72
1986	5.04	5.00	5.82	5.77	6.15	6.65	5.96	6.69	5.77	5.82	6.39	6.48
1987	5.03	4.98	5.82	5.77	6.32	6.73	6.14	6.77	5.77	5.81	6.53	6.54
1988	5.03	4.98	5.83	5.77	6.33	6.71	6.15	6.74	5.78	5.80	6.53	6.54
1989	5.01	4.97	5.86	5.76	6.37	6.70	6.19	6.71	5.82	5.78	6.55	6.54
1990	5.00	4.95	5.91	5.74	6.44	6.68	6.26	6.68	5.87	5.76	6.60	6.53
1991	5.01	4.93	5.96	5.72	6.52	6.67	6.34	6.65	5.93	5.73	6.66	6.52

TABLE 7

EXAMPLE: ENDOWMENT AT AGE 95 PREMIUMS, BASED UPON RESULTS OF TABLE 6

Company Investment Policy New-Money Rate	Model A Discounts Only High 3.5% after 20 Years	Model A Discounts Only Low 3.5% after 20 Years	New Company Discounts Only High 4.5% after 20 Years
20-year needed premium	$17.85	$18.82	$17.83
20-year profit %	10.6 %	5.7 %	10.6 %
Final needed premium	$18.24	$19.08	$17.45
Final profit %	8.6 %	4.4 %	12.5 %

TABLE 8

SOURCES OF FUNDS FOR INVESTMENT

	Summary of Results for the Year					
	1973			1974		
	Amounts on Account of Old Business	Amounts on Account of Current Year's New Business	Total	Amounts on Old Account of Business	Amounts on Account of Current Year's New Business	Total
Total assets	$238,715	$1,678	$240,394	$250,765	$4,640	$255,406
Net accrual	0	0	0	652	68	721
Maturities	0	0	0	0	0	0
Sinking fund maturities	3,718	0	3,718	3,718	0	3,718
Calls	0	0	0	0	0	0
Capital gains	0	0	0	0	0	0
Increase in assets	12,040	1,678	13,719	12,049	2,962	15,011
Net available	14,796	1,544	16,340	14,151	2,656	16,808
Total investment income	13,061	0	13,061	13,965	117	14,082
Earned rate	5.762%	....	....	5.850%	....	....
New-money rate	7.200	....	....	6.900	....	....

TABLE 8A

DETAILS OF INVESTMENTS CONTAINED IN PORTFOLIO

END OF YEAR 1973

Investment	Coupon Rate (1)	Number of Coupons (2)	Month and Year of Maturity (3)	First Year of Call (4)	Face Value of Original Investment (5)	Schedule D Yield (6)	Book Value (7)	Yield in Year (8)	Sinking Fund in Year (9)	Accrual of Discount (10)
	Investments Made Prior to 1973									
Mortgages	7 %	1	July, 1992	...	$24,375	7 %	$23,156	$1,706	$1,218	...
Government bonds	3	2	July, 1992	1992	11,068	3	11,068	332	...	...
Corporate bonds	5	2	July, 1992	1977	50,000	5	50,000	2,500	...	...
Corporate bonds	7	2	July, 1992	1972	73,098	7	73,098	5,116	...	...
Mortgages	5	2	July, 1992	...	50,000	5	47,500	2,500	2,500	...
	New Investments Made in 1973									
Discount bonds*	4.82%	2	July, 1993	1974	$21,787	7.2%	$16,340	...	...	...

Policy loans: amounts prior to 1973, $18,134; new amounts in 1973, $1,097

* Split between assets owned by existing portfolio, 90.5 per cent; by 1973 new business, 9.5 per cent.

TABLE 8A—*Continued*

END OF YEAR 1974

Investment	Coupon Rate (1)	Number of Coupons (2)	Month and Year of Maturity (3)	First Year of Call (4)	Face Value of Original Investment (5)	Schedule D Yield (6)	Book Value (7)	Yield in Year (8)	Sinking Fund in Year (9)	Accrual of Discount (10)
	Investments Made Prior to 1973									
Mortgages	7 %	1	July, 1992	....	$24,375	7 %	$21,937	$1,620	$1,218	....
Government bonds	3	2	July, 1992	1992	11,068	3	11,068	332	....	....
Corporate bonds	5	2	July, 1992	1977	50,000	5	50,000	2,500	....	....
Corporate bonds	7	2	July, 1992	1972	73,098	7	73,098	5,116	....	....
Mortgages	5	2	July, 1992	....	50,000	5	45,000	2,375	2,500	....
	New Investments Made in 1973									
Discount bonds	4.82%	2	July, 1993	1974	$21,787	7.2%	$17,061	$1,176	....	$721
	New Investments Made in 1974									
Discount bonds†	4.57%	2	July, 1994	1975	$22,411	6.9%	$16,808	....	....	....

Policy loans: amounts prior to 1974, $19,231; new amounts in 1974, $1,199

† Split between assets owned by 1972 and prior business, 84.1 per cent; by 1973 and 1974 business, 15.9 per cent.

company in column VI of Table 6. The goal of immunization in the near future tremendously strengthens our ability to control the future operations of our business and decreases our need to rely on long-range environmental projections.

Income Tax

The above discussion has been gross of taxes rather than net. Since the tax law provides a variety of modes of taxation, the company should calculate its income tax liability independent of the investment rate.

Risk Factors

Our examples have assumed that there is only one interest rate and that it is risk-free. This is, of course, not true, but analysis of the level of risk must again depend upon the actual assets of the company. Fraine's study [4] provides an appropriate measure of risk for various categories of corporate bonds, based upon the Hickman study [5] according to the rating of Moody's, Standard and Poor's, and Fitch (see accompanying tabulation).

Agency Category	Loss Rate*	Agency Category	Loss Rate*
I	0.1	IV	0.9
II	0.3	V	2.8
III	0.6	VI–IX	3.9

* Projection less realized yield.

If the above deductions are made from the yield on each asset, a risk-free yield results. This method assumes that the world is not malevolent and that the changes of default experience do not increase when the assets are high. Since these losses include the period from 1930 to 1940 and it may reasonably be argued that we now know enough to avoid a repetition of that experience, we might use significantly lower deductions. One-half of the above might be reasonable.

Loss rates, also available from the same source, relate loss experience to the premium in interest rate available above the best-quality bonds (see the tabulation below). In the event that we have private place-

Market Rating	Loss Rate	Market Rating	Loss Rate
Under 0.5%	0.4	1.5–2.0%	1.2
0.5–1%	0.5	2.0–2.5%	1.0
1.0–1.5%	0.6	2.5% and over	1.4

ments or nonrated obligations, a reasonable approach would probably be to reduce the yield in each case by between one-half and three-fourths of the excess of yield in that asset over the highest-grade obligation available at that time. These deductions would seem to be the appropriate kind of adjustment implied by release from risk theory.

CONCLUSION

The author has tried to detail in this paper the importance of the assumption that actuaries make about future interest earnings. Illustrations have been used to show the effects that different interest rates for various future periods have on profit margins in gross premiums and GAAP reserves, with a hint of the effect on dividend scales. The development shows that there is a rational method of arriving at future earnings rates. This method is more difficult and time-consuming than the "judgment" or "meditative contemplation" approach, but it has certain advantages.

First, since the method is rational, we can see the crucial factors in producing the answer. We can then develop appropriate alternatives if there is adequate reason. Second, the method seems to justify more liberal assumptions than pure judgment would allow.

I do not know how long present interest levels will continue. From this level they may rise but more likely will fall. Actuaries, in setting adjusted earnings standards, are developing patterns that their companies will follow for some time to come. To the extent to which this is a usable method, I hope that it will be applied.

ACKNOWLEDGMENT

The author wishes to acknowledge a most substantial debt of gratitude to Mrs. Anna Rappaport, F.S.A., who not only worked closely with him on all phases of this project but also supervised the calculations and actually developed the computer model which is essential to the successful application of this method.

APPENDIX I[1]

THE ASSETS

Kind of Asset	Face Amount of Asset	Distribution within Category
Government security	$ 11,068	20-year 3% bond
Corporate bonds	73,098	$50,000 20-year, 5%; $23,098 20-year, 7%
Common stock and real estate equity	21,740	This is market value, so assume 4.5% return on market to get present values, half in 3% perpetual growth and half in Williams 10% for 10 years
Mortgages	74,375	$50,000 at 5%; $24,375 at 7%
Policy loans	16,064	All 5%
Total	$196,345	

NOTE.—The earnings rate on this portfolio is 5.3 per cent, the same as the average rate for the industry in 1970.

APPENDIX II

THE LIABILITIES

For purposes of an illustrative model office, only one plan of insurance, a nonparticipating endowment at age 95 issued at age 40 (premium $19.95), and one annuity, immediate at age 65, were used. The same plan of insurance is presumed to have been issued throughout the history of the office since 1918. The sales assumptions are given in Table IIB and are roughly proportional to the sales of the life insurance industry since 1918. Lapses are according to Moorhead S rates, except that a 50 per cent lapse ratio at attained age 65 is assumed. This high lapse at age 65 is intended to balance, somewhat, the lack of any endowments and also to recognize that many individuals purchase life insurance with the expectation of utilizing settlement options at age 65. Mortality is assumed to follow the 1955–60 Select and Ultimate, adjusted for age last birthday. In the case of the annuity, mortality is a-1949 projected for thirty years. Only the net benefit payout is considered (expenses are ignored). The initial reserve is $50,249. Cash values are a current scale equaling the 3.5 per cent CRVM reserve after twenty years. Per cent of premium expenses are assumed to be 98 per cent in the first year, 15.5 per cent in the second, 13 per cent in the third, 10 per cent for years 4–10, and 5 per cent thereafter. Per $1,000 expenses are assumed to be $10.29 per $1,000 for the first year and $0.50 per $1,000 thereafter. The detailed claim and cash-value assumptions for twenty years of the contract are given in Table IIA.

[1] Portions of these appendixes have been drawn in whole, or in part, from material used in development of the model office discussed in the author's paper "The Interest Rate Assumption and the Maturity Structure of the Assets of a Life Insurance Company," *TSA*, XXIV, 157.

TABLE IIA

Mortality and Cash Values

Year	Mortality	Cash Value	Year	Mortality	Cash Value
1	1.33	0	11	6.06	180
2	1.76	10	12	6.76	200
3	2.15	29	13	7.59	219
4	2.55	49	14	8.49	239
5	2.86	69	15	9.33	259
6	3.37	87	16	11.53	279
7	3.76	105	17	12.66	298
8	4.20	124	18	13.93	318
9	4.69	142	19	15.33	338
10	5.33	161	20	16.88	358

TABLE IIB

Sales Assumptions

Year	Sales Assumption	Year	Sales Assumption
1970	$132,980	1943	$ 8,022
1969	123,500	1942	7,041
1968	112,266	1941	7,935
1967	103,545	1940	7,022
1966	95,987	1939	6,886
1965	89,643	1938	6,745
1964	79,430	1937	7,593
1963	68,862	1936	7,314
1962	61,259	1935	7,550
1961	58,888	1934	7,363
1960	56,183	1933	6,786
1959	55,138	1932	7,896
1958	50,839	1931	10,161
1957	48,937	1930	11,905
1956	38,941	1929	12,305
1955	32,207	1928	11,654
1954	26,824	1927	10,777
1953	24,908	1926	10,508
1952	21,579	1925	10,060
1951	19,000	1924	8,764
1950	18,260	1923	8,273
1949	15,848	1922	6,720
1948	15,787	1921	6,248
1947	16,131	1920	7,634
1946	16,244	1919	6,369
1945	10,577	1918	3,520
1944	9,184		

APPENDIX III

RESERVES FOR CURRENT AND ONE YEAR'S NEW BUSINESS: MODEL A

Year	Current	New	Total
1972	$108,694	$ 0	$108,694
1973	118,968	1,177	120,145
1974	129,251	3,254	132,505
1975	138,627	5,292	143,919
1976	146,870	7,208	154,078
1977	154,144	8,810	162,954
1978	160,522	10,316	170,838
1979	165,960	11,829	177,789
1980	170,498	13,152	183,650
1981	173,961	14,477	188,438
1982	176,171	15,708	191,879
1983	176,887*	16,938	193,825
1984	176,692	17,991	194,683*
1985	175,490	19,040	194,530
1986	173,479	20,001	193,480
1987	170,538	20,850	191,388
1988	166,736	21,537	188,273
1989	161,742	22,209	183,951
1990	155,275	22,803	178,078
1991	147,378	23,316	170,694

* Highest value.

APPENDIX IV

RESERVES FOR CURRENT AND ONE YEAR'S BUSINESS: MODEL A + ANNUITIES

Year	Old	New	Total
1972	$158,943	$ 0	$158,943
1973	167,386	1,177	168,563
1974	175,835	3,254	179,089
1975	183,372	5,292	188,664
1976	189,781	7,208	196,989
1977	195,224	8,810	204,034
1978	199,780	10,316	210,096
1979	203,413	11,829	215,242
1980	206,159	13,152	219,311
1981	207,850	14,477	222,327
1982	208,310*	15,708	224,018
1983	207,305	16,938	224,243*
1984	205,423	17,991	223,414
1985	202,563	19,040	221,603
1986	198,936	20,001	218,937
1987	194,421	20,850	215,271
1988	189,088	21,537	210,625
1989	182,612	22,209	204,821
1990	174,715	22,803	197,518
1991	165,438	23,316	188,754

* Highest value.

APPENDIX V

RESERVES FOR CURRENT AND ONE YEAR'S NEW BUSINESS: NEW COMPANY

Year	Current	New	Total
1972	$ 7,098	$ 0	$ 7,098
1973	9,188	1	9,189
1974	11,359	330	11,689
1975	13,384	660	14,044
1976	15,257	1,040	16,297
1977	16,982	1,320	18,302
1978	18,610	1,620	20,230
1979	20,147	1,890	22,037
1980	21,577	2,160	23,737
1981	22,924	2,430	25,354
1982	24,169	2,710	26,879
1983	25,316	3,000	28,316
1984	26,343	3,210	29,553
1985	27,151	3,450	30,601
1986	27,506	3,660	31,166
1987	27,516*	3,900	31,416*
1988	27,332	4,080	31,412
1989	26,904	4,260	31,164
1990	26,388	4,340	30,728
1991	25,693	4,600	30,293

* Highest value.

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DISCUSSION OF PRECEDING PAPER

DALE R. GUSTAFSON:

It is my purpose to comment on only one minor aspect of Mr. Vanderhoof's excellent and timely paper. In the abstract the author refers to the paper as developing "a new scientific method of predicting interest rates for use in . . . dividend scales." At the end of the section entitled "Gross Premiums," conclusion 4 reads in part, "The importance of interest is not limited to stock companies in their gross premiums but is correspondingly reflected in mutual companies in the dividends they pay policyholders."

The purpose of this comment is simply to point out that the interest assumptions used in developing a dividend scale are essentially different from the interest assumptions used in developing nonparticipating gross premiums and in the adjusted earnings situation. Both of these latter situations do, indeed, involve an element of long-range prediction. However, in developing interest assumptions for dividend scales, the longest period of time involved is the number of years that this dividend scale may remain in effect. With normal company practices of changing dividend scales quite frequently, this period of time ordinarily will not exceed two or three years. To emphasize the point in different words: dividend scales are based on current experience and are neither estimates nor projections of the future.

This small point in no way mars a very important paper, but the possibility of misinterpretation seemed to indicate a need for clarification.

CLAUDE Y. PAQUIN:

One who would go along with the proposition that only the actuarial profession is entitled to determine generally accepted actuarial principles (not precluding, of course, prior consultations with all interested parties) also probably would accept willingly the proposition that generally accepted accounting principles are for the accounting profession to determine. One may disagree with the accountants' principles, but one who is not an accountant should learn to live with promulgated accounting principles.

This paper makes statements which are at variance with the current generally accepted accounting principles recently promulgated in the industry audit guide prepared by the Committee on Insurance Accounting and Auditing of the American Institute of Certified Public Accountants and entitled *Audits of Stock Life Insurance Companies*. The reader unfamiliar with the audit guide should be made aware of this.

The paper states that "[o]nly the application of scientific actuarial techniques can avoid possible professional embarrassment or even actual personal liability in the event that an assumption fails to be realized." This statement is open to challenge. An actuary is not an insurer, and he does not guarantee what he assumes. An assumption is a goal, not a prediction, and it can well be argued that the investment department is to blame for not realizing interest assumptions (although the buck can be passed on, almost endlessly, to the Federal Reserve Board, Congress, the President, etc.). An actuary, of course, has the duty to be reasonable, and the investment department should be consulted about the "goals" it can achieve. (The very careful actuary will even assemble written memos from all his advisers to forestall "actual personal liability" and show what a reasonable and prudent man he was.)

The related statement that "[t]he choice of interest rate assumptions has been reserved to actuaries because of their professional expertise" is quite flattering, but nowhere can I find authority for it. The audit guide states (p. 75) that "[t]he selection of such an interest assumption is a subjective judgment which must be made by the company in light of the long term nature of life insurance, the contractual obligations under life insurance policies, and the inherent inability to forecast the future with certainty." The audit guide states further, (p. 96) that the auditor must be satisfied as to "the reasonableness and appropriateness of the basic underlying assumptions." With all due respect, actuaries cannot so easily reserve to themselves the choosing of interest rate assumptions. It is a collective responsibility.

It is difficult to accept the statement that adjusted earnings for each of ten years might have to be restated because unmet interest assumptions had caused an overstatement of earnings in those ten years. Assumptions are seldom met exactly, one way or another. If interest rates actually earned are less than those that were assumed, a loss might have to be reported. But then the audit guide provides (p. 86) that "the original assumptions will continue to be used ("locked-in") during the period in which reserves are accumulated so long as reserves are maintained at a level sufficient to provide for future benefits and expenses." When assumptions are not being met, so that the actual results are financially adverse to an insurer, the actuary must check that, prospectively, "reserves are maintained at a level sufficient to provide for future benefits and expenses." I fail to see how adjusted earnings can be overstated in a year when an interest assumption is not met. Interpretation 1-A(3) of the

Committee on Financial Reporting Principles of the American Academy of Actuaries states clearly that the operating profit or loss reported will include the effect of the difference between actual experience and valuation assumptions. Earnings will be reported as they are, not as they were expected to be.

The paper does not indicate how the gross premium of $19.95 was arrived at or why the author uses two "needed premiums." The conclusion that "the accumulation of the reserve at the end of twenty years does not guarantee adequacy of the premium for the whole term of the policy if the interest earnings after the initial period are as low as the reserve interest rate" needs some explaining. Apparently, by the word "reserve" the author meant "statutory reserve" in that context. Premium adequacy is generally, however, a function of the size of the premium, and of actual mortality, actual persistency, actual interest and actual expenses. Outstandingly good mortality could offset poor interest, for instance, and it is manifestly untrue to state that "the margin in the reserve is only the conservatism inherent in 3.5 per cent interest" unless the statement is properly qualified.

Confusion between reality and the make-believe world of actuarial assumptions is implicit in such statements as the following: "If we can earn favorable interest rates over a long period, the postulated low rate contains substantial profit margins." It is an accepted actuarial dogma that reserves do not affect real earnings and affect only their incidence. So far as reporting earnings is concerned, if you report a little more today you will be reporting a little less tomorrow, and vice versa. It has to come out in the wash. One important thing to keep in mind is that actual earnings are made up of operating earnings and of interest on past earnings. When operating earnings are reported earlier, as where early reserve increases are smaller, there will be smaller operating earnings reported later (when reserve increases will be accelerating) but larger interest earnings on past earnings accumulations. Looking only at operating earnings is like comparing two actuarially equivalent dividend scales by the "traditional" net cost method: the scale which can postpone dividends (operating earnings) the most always seems to produce the best results.

It is apparent from the foregoing remarks that I cannot, then, subscribe to comments that "the normal pattern of reducing interest rates actually penalizes earnings" or to other statements of the same nature. I am not saying that earnings will be *reported* in the same amount each year no matter what the interest assumptions implicit in the reserves; what I am saying is that any method which delays the reporting of operating earnings will produce greater reported *operating* earnings after a while, and

I say further that, when interest earnings on past reported earnings are considered, the actuarial "present value" of all these earnings is the same regardless of the reserving system.

The statement that there is an "audit guide requirement that profits emerge in relation to premium revenues" should not go unchallenged. There is no such requirement. The audit guide states simply (p. 72) that "acquisition expenses should be deferred and charged against income in proportion to premium revenues recognized."

(AUTHOR'S REVIEW OF DISCUSSION)

IRWIN T. VANDERHOOF:

Mr. Gustafson's comments are appreciated and deserve careful consideration. The question of the interest rate used in dividend scales has been discussed in the literature, and there seems to be general agreement with Mr. Gustafson's position. Dividend scales are neither projections nor estimates—they are the results of application of the current scale to the various policies involved. Companies are prohibited by law, at least in some states, from providing estimates or projections. Even if they were not so prohibited, companies certainly would be reluctant to have their estimates or projections represent even a moral commitment to pay dividends in the distant future on the basis of current conditions. The whole point of participating business is that equity will be preserved between classes of policyholders and costs of insurance will be assessed on the basis of actual experience as it develops. But is that all there is to it?

The following is my personal point of view and does not represent the views of any company. I think that the traditional company position is satisfactory for companies but is not good enough for professionals such as ourselves. When an agent makes a presentation, he does not tell the prospect that the dividend illustration is applicable only for the next year or so and may very well never be applied to policies now being sold. He does not tell the prospect that extending the illustration over a period of twenty or thirty years is a purely mathematical exercise and has no exact meaning in connection with the expected costs of the policy. He is more likely to say that the company has always been conservative in its illustrations and that in the past it has paid more than would be implied by the illustrations presented. Not only do we prepare illustrations for plans already in our portfolio of policies—we also prepare them for new policies being introduced, where the existing dividend formula may never be applied because no dividends will be payable for several years. In the case of a new policy form, expense factors may be used that have never been applied to any existing policy. In the case of juvenile forms, we carry out these illustrations for as much as sixty-five years in the future. In addition, we make data readily available to allow the conversion of dividend accumulations and the cash value of paid-up additions to income so that the prospective policyholder may include the monthly payments in his retirement planning.

The agents do not invent these sales techniques. We provide them material to use, we train them to use it, and we encourage them to use it. This kind of estate planning is the "highest" kind of insurance salesmanship. In addition, some states and various consumer groups are circulating cost comparisons based upon dividend illustrations with the intent of making it easier for purchasers to compare costs between companies. The use of these long-term illustrations as a basis for cost comparisons therefore is encouraged by these groups. The pressure upon actuaries to find a way to improve their illustrations will become more intense.

Although companies do not have a clear responsibility to pay dividends according to their illustrations, in my opinion we as professionals do have a clear responsibility to ensure that, according to our judgment, it is reasonable to expect that such dividends will be paid and therefore will provide a reasonable basis for decision by the buyer. The method commonly used in illustrations is to say simply that the most recent years' operations are the best estimate of the future. However, for a company to continue to earn a portfolio rate of 6 per cent, the future market level of interest rates must be 6 per cent. This seems to imply that an inflation rate of 2 or 3 per cent will be with us indefinitely. Continuation of the portfolio rate of interest earnings is then inconsistent with the level expenses implicit in the use of the current formula as the best estimate of the future. Logic then says that the use of illustrations of this kind—projecting a current formula into the future—could easily show dividends that are too high. I believe that the prohibition against estimates is designed to avoid overstating expected future dividends. The reverse effect occurs in this instance.

In providing materials for use by a salesman to facilitate his sales, we should have the same sense of responsibility that would be appropriate if we were doing work for a pension fund, a stock company, or a fraternal. We should treat our work as if we were presenting it personally to the client and suggesting that he make his decision to purchase based on it and our assurance that it was a reasonable basis for such a decision.

This calls for a different kind of responsibility from that which we are accustomed to, but I do not see how we can have less concern for the presumably unknowledgeable individual life insurance purchaser who is encouraged to plan his retirement on the basis of our work than we have for a presumably knowledgeable pension fund or insurance company. I am calling not for a standard more liberal than that previously used but for one which sets more responsibility on the individual actuary and which usually would be less liberal than the simple illustration of the continuation of the current dividend scale.

Mr. Claude Paquin raises a number of points specifically in reference to the audit guide and generally accepted principles. I will comment on his various points without restating them.

I do not agree with his initial proposition that actuaries and accountants each have the right to determine the principles on which their own profession is based. Each of us is supposed to be expert in the fields covered by our profession, and both actuaries and accountants are making serious efforts to solve difficult and ever changing problems. I consider the endeavors of both to be closer to the scientific problem of discovering truth than is the concept of "entitled to determine" expressed by Mr. Paquin. If a nonactuary can show that generally accepted actuarial principles are incorrect, then the actuarial profession must change its principles, and I would guess that accountants also feel that they are engaged in a scientific endeavor and not simply entitled to set up rules because they have passed examinations. The following note from *Accounting Principles Board Opinion No. 11* implies that the accountants recognize that there are possible correct alternatives to accepted practices: "While it is recognized that general rules may be subject to exception, the burden of justifying departures from Board Opinions must be assumed by those who adopt other practices."

Whether statements in the paper or statements in Mr. Paquin's discussion are at variance with generally accepted accounting principles will be left to the reader of these discussions to determine.

The question of possible personal liability of an actuary is more serious than Mr. Paquin indicates. If we are a learned profession, then we can reasonably expect to be held accountable for our work. We can also reasonably expect to be held accountable if the actions taken in our work are not in accordance with the best techniques available within the area of our discipline. In my view an actuarial assumption is not a goal but is a conservative prediction. The term "goal" would be appropriate if there were some action which we should be taking to achieve an objective. In the case of investments, presumably someone other than an actuary will be investing the money, and we actuaries are endeavoring to determine, on a reasonably conservative basis, the results of their activities. Mr. Paquin's comments on a careful actuary assembling written memorandums to forestall "actual personal liability" are clear evidence that he concedes the possibility. The only correct statement would be that actuaries have not in the past been held personally liable for their errors in judgment. As stated on page 64 of the audit guide (Part 2, "Principles of Accounting"), "The choice of actuarial assumption and the disciplining of that choice are the primary responsibilities of the actuarial profession." Providing the actuaries with specific tools to handle the interest rate assumption is of course the point of this paper and of my work over the past few years.

If interest earnings in a specific year are less than those assumed in the benefit reserve calculation and if the pattern of development of such benefit reserves has been upset without any reason to believe that it will come back on track, the original reserves set up could be less than the amount necessary to provide the future benefits and expenses. Mr. Paquin's assumption that earnings can be reported "as they are" misses the fundamental point—we are trying to determine what they are. Such a determination depends upon the development of the benefit reserves, which in turn relates both to the interest assumptions and to such revisions in those assumptions as may be necessary in the light of developing experience.

Table 1 was intended to illustrate the effects of varying interest assumptions on possible premium rates for the specified contract. The rate of \$19.95 per \$1,000 was chosen as being appropriate for illustrative purposes and has no other significance. As stated in the first paragraph of the section on "Gross Premiums," the "first needed premium" is the amount that will accumulate to the underlying statutory reserve at the end of twenty years, after paying expenses, mortality, and so on. The "final needed premium" is the amount necessary to pay all costs according to the assumptions detailed in Appendix II, Tables IIA and IIB. In the first column of Table 1 the premium of \$19.82 is adequate to accumulate to the statutory reserve after paying the costs of having the insurance on the books. Since this is less than the final needed premium of \$19.91, it should be obvious that the accumulation of reserve plus the continuation of the \$19.82 premium for the remainder of the contract would produce a deficit at the end of the entire term. The premium of \$19.82 is, therefore, an adequate premium for the entire contract only if interest earnings exceed the $3\frac{1}{2}$ per cent implicit in the statutory reserve. It has been common to presume that if the premium for a contract paid all expenses and accumulated to the statutory reserve at twenty years, then no further tests of adequacy were necessary. The point of Table 1 is that this assumption is not necessarily true unless we also have assumed that the $3\frac{1}{2}$ per cent earnings rate will be exceeded after the twentieth year. Mr.

Paquin's comments about the adequacy being determined finally by the actual results is correct, of course, but it is of little value to the actuary trying to set rates at the present time without the benefit of absolute knowledge of those future results.

I am disturbed by the negative attitude toward responsibilities of the actuarial profession implicit in Mr. Paquin's reference to the "make-believe world of actuarial assumptions." Some companies and pension funds have actually gone out of business on account of errors in actuarial assumptions, and many companies have been financially embarrassed by such errors. An actuarial assumption determines actions taken in the real world with respect to gross premiums and therefore is a part of this real world. I take such assumptions seriously and believe that most other actuaries do. I would hope that Mr. Paquin will revise his attitude toward their importance. The balance of Mr. Paquin's comment appears to be based on an assumption that the subject of the section entitled "Gross Premiums" is in fact "Benefit Reserves." The subject is, however, gross premiums, and the statement of mine which he quoted is in fact correct.

Since the pattern of reducing interest rates defers substantial amounts of income until the contract is terminated and the benefit reserve released, and since in most cases there will be a substantial number of policies which persist until the far distant future, the statement made in the paper about penalizing earnings is, for practical purposes, correct. One purpose in the adoption of generally accepted accounting principles was to avoid the kind of long-term deferral of earnings which Mr. Paquin feels is of no importance. The basic problem of the accounting profession is to match revenues and expenses so that income is recorded, as far as possible, in the correct year, and this is what GAAP earnings were supposed to do. Page 68 of the audit guide includes the statement: "Any profit in the premium in excess of the provisions for adverse deviation will emerge in relation to premium revenues. Profits emerging as a level percentage of premiums give recognition to the import of the sales effort as a source of profit." The various "releases from risk" were not germane to the subject of that particular sentence, and the statement in the paper is therefore correct as phrased.

In conclusion I would like to thank the many people otherwise unnamed who helped and encouraged me in this endeavor. I wish to restate my hope that other actuaries will share my interest not only in the field of investments but also in the integration of investments and insurance operations of life insurance companies. I believe this to be the next area of a possibly major advance in actuarial science and in the actual operation of insurance companies and pension funds.