TRANSACTIONS OF SOCIETY OF ACTUARIES
1973 VOL. 25 PT. 1 NO. 73

THE DEVELOPMENT OF MEAN NATURAL RESERVE FACTORS AND METHODS OF AMORTIZING ACQUISITION EXPENSES IN ADJUSTING LIFE INSURANCE COMPANY EARNINGS

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ABSTRACT

Mean reserve factors should be those factors which, under the assumptions, are best related to the population or in-force to which they will be applied. To compute mean natural reserve factors properly requires close attention to the sequence and timing of the insurance events which take place within each policy year.

There can be as many techniques for the amortization of expenses as there are for the depreciation of assets. The actual pattern of expense amortization should accord with a conscious policy decision rather than the vagaries of any set formula. A mortgage amortization technique can, when used properly, produce the same results as an expense runoff, including an amortization directly proportional to premium income when an interest rate of zero per cent is used. This technique can accommodate actual amortizable expenses arising in uneven amounts over a number of policy years.

Earnings, statutory or adjusted, can be developed through the use of mean reserve factors, although, for adjusted earnings, an expense runoff charge may be used instead of expense reserve factors. The process of adjusting earnings changes their incidence. As earnings are reported, they are transferred to a general corporate account, which thereafter will produce earnings no longer specifically ascribed to the insurance policies which produced the original earnings. By taking this feature into account, one can demonstrate that, no matter how reported, earnings scales are actuarially equivalent.

The main challenge to the actuary is in developing a system of reporting earnings which is workable and fairly fulfills its intended purpose.

I. INTRODUCTION

With the growing recognition that certain acquisition expenses in the sale of life insurance must be amortized so as to produce realistic reports of earnings, it becomes important to consider anew the traditional practical means of computing individual life insurance reserves, and to undertake a fresh consideration of some amortization methods and of their effect on reported earnings. In view of the popularity of the seriatim and group methods of valuation, which consist in applying a mean reserve factor to one policy or a group of similar individual policies in force at the calendar year end, one important purpose of this paper is to examine how mean reserve factors can best be calculated in light of the new accounting reporting requirements for the earnings of stock life insurance companies. At the same time we shall examine some methods of expense amortization, including the reserve factor method, and also attempt to show an informative comparison of how reported calendar-year earnings can be expected to emerge with statutory mean reserve factors, natural mean reserve factors, and assorted expense amortization methods.

II. MEAN RESERVE FACTORS

A. *A Theory of Mean Reserves*

Mean reserve factors are used as a means to an end. The end is to produce the most practical and rational approximation possible to what the true reserve ought to be at the time of valuation, traditionally a calendar year end. By convention, the mean reserve for any given policy year is the average of the initial and the terminal reserves for that policy year. If the initial and terminal reserves are amounts which are quite close to one another, the mean reserve will produce a very satisfactory approximation to the "true" reserve at the time of valuation. If the initial and terminal reserves are far apart, however, the resulting mean reserve may be off substantially, as where the mean reserve factors are applied to a distribution of in-force by policy anniversary which is not uniform throughout the calendar year as of the end of which a reserve valuation is undertaken.

If we let

${}_tV$ = Terminal reserve at the end of policy year t,
${}_tP$ = Net annual premium payable at the beginning of policy year t, and
${}_tM$ = Mean reserve for policy year t,

the tradition has been to define

$$ {}_tM = \tfrac{1}{2}[({}_{t-1}V + {}_tP) + {}_tV] , \qquad (1) $$

where $({}_{t-1}V + {}_tP)$ is regarded as the initial reserve.

But consider the case of a twenty-year endowment insurance policy. What is the terminal reserve for the twentieth year? It is either the face amount, if one looks at the policy one instant before the endowment benefit is paid, or zero, if one looks at the policy one instant after the endowment benefit is paid. If one encounters a whole life policy with a pure endowment of $100 at the end of every fifth policy year, the mean reserve factor for policy year 5 must be

$$_5M = \tfrac{1}{2}[({}_4V + {}_5P) + {}_5V]\,, \tag{2}$$

where symbols are defined as previously and where ${}_5V$ *includes* the $100 pure endowment. But the mean reserve factor for policy year 6 would have to be

$$_6M = \tfrac{1}{2}[({}_5V - {}_5B + {}_6P) + {}_6V]\,, \tag{3}$$

where ${}_5B$ represents the $100 pure endowment benefit.

This illustrates the need to define the terminal reserve as the year-end reserve *before* all policy-year-end transactions. Policy-year-end transactions, in that context, should include not only benefit payments but also population decrements (as will be explained hereafter).

Likewise, the initial reserve which becomes the component for the same year's mean reserve should be that reserve which is applicable *after* all policy-year-beginning transactions (hence, also, after all transactions of the previous policy year end).

Where major transactions are assumed, generally quite properly, to take place at policy year ends and at the beginnings of policy years—as, for example, in the collection of pure endowment benefits or dividends, the surrender of policies for their cash value or equivalent benefit, the payment of annual premiums by the policyowner, and the disbursement of commissions and payment of other expenses by the insurer—the only factors which should normally be assumed to cause the difference between the initial reserve and the terminal reserve are deaths (a source of decrement) and interest (a source of increment). Death claims and interest earnings are in the nature of continuous operations, and their effect can probably best be estimated by calculating mean reserves as the average of the corresponding initial and terminal reserves. Where annual premium payments are the rule, which is presumed here, practically all transactions affecting benefits and expenses, other than death and the slow continuous earning of interest, take place at policy year ends.

Using so-called natural reserves, that is, reserves which take expenses and withdrawals into account, it becomes important to determine what major transactions are to be assumed to take place. This should be done as realistically as possible, if mean reserves approximating the "true" reserves are desired. To be considered are the payment of cash benefits, such as coupons, pure endowments, dividends, and cash values; payments to survivors; the receipt of premiums; and the payment of expenses. Transactions which should logically be bunched up at policy year ends (depending upon the insurer's experience with a particular type of policy) should be taken into account at the proper time in the derivation of the initial and terminal reserves which will serve to make up the mean reserve factors.

We indicated earlier that policy-year-end transactions should include not only benefit payments but also population decrements. The question one may ask about reserves is, "To what group of people do they pertain?" The pre–policy year end transactions terminal reserve is applicable to those insureds who are then members of the insured population. Hence, in a population where deaths and withdrawals can occur, but where withdrawals are presumed to take place only at policy year ends, while deaths are continuous, the terminal reserve will be, in a rough way of speaking, the equity of those who are then around and in a position to make some kind of claim against the insurer. Those who claim their cash value will not only withdraw a part of the reserve fund of the insurer for that group—they will also be withdrawing from the group and hence will cause a reapportionment, among the remaining members, of the remaining fund. This may create an instant survivorship benefit for the nonwithdrawing survivors if cash values are smaller than reserves.

B. *Mean Reserve Calculation Example for Benefits*

Since current adjusted earnings approaches contemplate the calculation of separate reserves for benefits and for expenses eligible for amortization, the transition from initial to terminal reserve, and then from terminal to initial reserve, will be considered separately for each.

Let us define the following terms:

l_{t-1} = Number of insureds who pay the annual premium at the beginning of policy year t;

${}_tI^B$ = Initial reserve for benefits as of the beginning of policy year t (after all beginning-of-year transactions);

i = Interest rate assumed to be earned during policy year t;

d_{t-1} = Number of insureds dying during policy year t $= l_{t-1}q_{t-1}^{(d)}$;

${}_tV^B$ = Terminal reserve for benefits as at the end of policy year t (before all end-of-year transactions).

Then the transition from initial to terminal reserve can be viewed by the following actuarial equation (which assumes that a $1,000 benefit is payable on death):

$$l_{t-1}\,{}_tI^B(1+i) - 1{,}000(1+i)^{1/2}d_{t-1} = (l_{t-1} - d_{t-1})\,{}_tV^B\,. \quad (4)$$

This is a far from startling result unless one realizes that withdrawal benefits are contemplated in the formulas for ${}_tI^B$ and ${}_tV^B$. The terminal reserve ${}_tV^B$ pertains to all survivors, including those about to collect cash benefits and even withdraw.

Let us define a few more terms:

${}_tCV$ = Cash value available at the end of policy year t;

w_{t-1} = Number of insureds withdrawing at the end of policy year t
$= l_{t-1}(1 - q_{t-1}^{(d)})q_{t-1}^{(w)} = (l_{t-1} - d_{t-1})q_{t-1}^{(w)}$;

${}_tY$ = Dividend or coupon payable at the end of policy year t (regardless of whether the next premium is paid);

${}_tP^B$ = Annual premium for benefits payable at the beginning of policy year t.

Note that

$$l_t = l_{t-1} - d_{t-1} - w_{t-1} = l_{t-1}(1 - q_{t-1}^{(d)})(1 - q_{t-1}^{(w)}) . \qquad (5)$$

The transition from one terminal reserve (at the end of policy year t) to the next following initial reserve can be viewed by the following actuarial equation, which takes into account all major policy-year-end transactions:

$$(l_{t-1} - d_{t-1})\ {}_tV^B - {}_tCV\ w_{t-1} - {}_tY(l_{t-1} - d_{t-1}) + l_t\ {}_{t+1}P^B = l_t\ {}_{t+1}I^B . \qquad (6)$$

The mean reserve factor, which one would then expect to apply to a population of $l_{t-1} - \frac{1}{2}d_{t-1}$, the expected in-force at calendar year end midway in policy year t, would quite properly be seen as $\frac{1}{2}({}_tI^B + {}_tV^B)$. In effect, if one should follow the progress of ${}_tI^B$ for a half-year, one would have

$$l_{t-1}\ {}_tI^B(1 + i)^{1/2} - \tfrac{1}{2}[1{,}000(1 + i)^{1/4}]d_{t-1} = (l_{t-1} - \tfrac{1}{2}d_{t-1})\ {}_tM^B , \qquad (7)$$

where ${}_tM^B$ is the tth-year mean reserve for benefits.

From that point on, the mean reserve would progress roughly in this fashion:

$$(l_{t-1} - \tfrac{1}{2}d_{t-1})\ {}_tM^B(1 + i)^{1/2} - \tfrac{1}{2}[1{,}000(1 + i)^{1/4}]d_{t-1} = (l_{t-1} - d_{t-1})\ {}_tV^B . \qquad (8)$$

If one multiplies equation (7) by $(1 + i)^{1/2}$ and substitutes in equation (8) for the term involving the mean reserve factor ${}_tM^B$, one will reproduce, after taking the practical liberty of setting $\frac{1}{2}(1 + i)^{3/4} + \frac{1}{2}(1 + i)^{1/4} = (1 + i)^{1/2}$, equation (4), namely,

$$l_{t-1}\ {}_tI^B(1 + i) - 1{,}000(1 + i)^{1/2}d_{t-1} = (l_{t-1} - d_{t-1})\ {}_tV^B .$$

It is conceded that to calculate the mean reserve as the arithmetic average of the initial and terminal reserves is a *practical* step. The main point is that, if initial and terminal reserves are calculated properly, the practical mean reserve will be much closer, in most situations, to the correct or true mean reserve, while improper initial and terminal reserves may cause greatly distorted earning patterns and inadequate or redundant natural reserves.

C. *Mean Reserve Calculation Example for Expenses*

Let us again define a few additional terms:

${}_tI^E$ = Initial reserve for expenses eligible for amortization as of the beginning of policy year t (after all beginning-of-year transactions);

${}_tV^E$ = Terminal reserve for expenses as of the end of policy year t (before all end-of-year transactions);

${}_tP^E$ = Annual premium for expenses payable at the beginning of policy year t, or, viewed differently, level amount available from each policyowner paying the tth premium for the amortization of eligible expenses;

${}_tE^E$ = Actual amortizable expenses paid at the beginning of policy year t.

The transition from initial to terminal reserve can be viewed through the equation

$$l_{t-1}\ {}_tI^E(1+i) = (l_{t-1} - d_{t-1})\ {}_tV^E\ , \tag{9}$$

while the transition from each terminal reserve to the next following initial reserve can be seen as

$$(l_{t-1} - d_{t-1})\ {}_tV^E + l_t({}_{t+1}P^E - {}_{t+1}E^E) = l_t\ {}_{t+1}I^E\ . \tag{10}$$

Again mean reserve factors will be taken as $\frac{1}{2}({}_tI^E + {}_tV^E)$ and will be expected to apply to an in-force of $l_{t-1} - \frac{1}{2}d_{t-1}$.

The progress of the reserve in the first half of the policy year will be expected to conform to the equation

$$l_{t-1}\ {}_tI^E(1+i)^{1/2} = (l_{t-1} - \tfrac{1}{2}d_{t-1})\ {}_tM^E\ , \tag{11}$$

and in the second half to

$$(l_{t-1} - \tfrac{1}{2}d_{t-1})\ {}_tM^E(1+i)^{1/2} = (l_{t-1} - d_{t-1})\ {}_tV^E\ . \tag{12}$$

III. AMORTIZATION OF EXPENSES

A. *Theoretical Considerations*

A cash outlay is a fact, while depreciation and its twin, amortization of expenses, are fictional, although they may seek to reflect reality. (Likewise, it may be philosophically observed that while earnings are ultimately a fact, *reported* earnings have as such a more fictional or intellectual reality.) Depreciation and amortization are essentially allocation devices between defined time periods, and this allocation generally reflects the allocator's objectives, subject, of course, to such external constraints as may exist on his discretion to allocate. Depreciation and amortization are natural corollaries of accrual basis accounting and have no place in a strict cash basis of accounting. The amortization of expenses could be viewed essentially as a depreciation of the asset which the expense to be amortized secured.

Depreciation can be accelerated, constant, or decelerated. For instance, a cash expenditure of $1,000 could be reported according to the patterns shown in Table 1, over a ten-year period.

TABLE 1

EXAMPLES OF ASSORTED PATTERNS OF DEPRECIATION

YEAR	CASH BASIS	ACCRUAL BASIS			
		Double-declining Balance Method*	Sum-of-Years Digits Method	Straight-Line Method	Sinking Fund Method (8%)
1	$1,000.00	$ 200.00	$ 181.82	$ 100.00	$ 69.03
2	0	160.00	163.64	100.00	74.55
3	0	128.00	145.45	100.00	80.52
4	0	102.40	127.27	100.00	86.96
5	0	81.92	109.09	100.00	93.91
6	0	65.54	90.91	100.00	101.43
7	0	65.54	72.73	100.00	109.54
8	0	65.54	54.55	100.00	118.30
9	0	65.53	36 36	100.00	127.77
10	0	65.53	18.18	100.00	137.99
Total	$1,000.00	$1,000.00	$1,000.00	$1,000.00	$1,000.00

* With conversion to the straight-line method in the last few years, as commonly used.

Actuarially, under normal theories of compound interest, these schedules for reporting a $1,000 one-time cash expenditure are not equivalent (unless a 0 per cent interest rate is chosen), but the point to remember is that they are fictional, that is, tools of the mind, and fictional amounts do not earn actual interest, or cost any. If the expenditure caused an interest-earning asset such as cash to be exchanged for another asset (such as life insurance in force) which contributes to earning profits, it is difficult to see what would be gained by treating the cash expenditure as an interest-bearing loan where the interest is both charged and paid to oneself: the net effect on total actual or reported earnings is bound to be zero. Lending money to oneself accomplishes nothing, as an over-all result. Hence the postulate is stated here that, no matter what amortization method is chosen, and no matter what the period over which the amortization takes place, neither more nor less than the actual acquisition expense may be charged to income.

B. *Expense Runoffs*

The privilege of amortizing a cash expenditure over a number of fiscal periods for accounting purposes is, in fact, recognition that the expenditure secured some form of depreciable capital asset (even if intangible). It is difficult to think of any reason why the amortization of expenses could not be handled with as much intelligence, imagination, and flexibility as depreciation currently is. Thus it can be argued that as much flexibility should be allowed in the choice of an amortization method as in the choice of depreciation methods.

One more point should be noted: the value placed on the depreciable asset which the amortizable acquisition expense is deemed to have secured is conservatively assumed not to exceed the expense of acquiring it. This process, in effect, establishes a book value for the in-force asset, and this book value should cast no reflection upon the in-force's market value.

Before the amortization process can take place, it is important to first characterize expenses so that only those expenses eligible for amortization, under prevailing standards, are in fact subjected to the amortization process. The determination of what expenses are eligible for amortization is beyond the scope of this paper, but the constant references to expenses eligible for amortization or to amortizable expenses should serve here as a sufficient reminder of the importance of characterizing expenses properly before proceeding to amortize.

The proposition has been advanced that the amortization of acquisition expenses eligible for amortization should be in direct proportion to premium income. Since premium payers have the privilege of ceasing to pay premiums when they see fit, a perfect amortization would require a flawless prediction of future premium income (or else a long, long wait before earnings could be reported). Since this is asking too much, reasonable actuarial assumptions must be used, subject to appropriate mid-course corrections, as in lunar voyages.

One amortization method is to have a direct proportional amortization, as illustrated in column 3 of Table 2. This method is conceptually equivalent to the straight-line method of depreciation. (In examining Table 2 and subsequent tables, note that all examples in this paper assume, for practical purposes, that all policies terminate after twenty-five years.)

Another method is to discount expected premium income, at interest, and to amortize expenses eligible for amortization in the proportion of each year's premium income discounted to the time of issue to all years' premium income discounted to the time of issue. As the illustration of column 4 of Table 2 shows, this method accelerates the amortization of expenses. No particular logic seems to warrant this approach.

A third method is the sinking fund approach illustrated in column 5 of Table 2. The sinking fund method also incorporates an interest rate. As a depreciation method, it calls for charging as depreciation each year a level amount plus interest on past depreciation charges, in such a way that a replacement fund will be available to replace the depreciated asset, at its original cost, at the expiration of its useful life. While this rationale may be totally unrealistic in most situations, the existence and mechanics

TABLE 2

EXPENSE RUNOFFS UNDER ASSORTED AMORTIZATION METHODS

Policy Year (1)	Proportion of Premium Paid (2)	Amortization Schedule for $1,000 Acquisition Expense: Straight Runoff (3)	With Premiums Discounted to Issue* (4)	Under Sinking Fund Approach* (5)
1	1.00000	$ 98.22	$ 151.53	$ 37.42
2	0.79938	78.51	114.27	32.16
3	0.70279	69.03	94.78	30.48
4	0.63179	62.05	80.38	29.65
5	0.57542	56.52	69.06	29.32
6	0.52857	51.92	59.85	29.32
7	0.48965	48.09	52.30	29.63
8	0.45737	44.92	46.09	30.19
9	0.42982	42.22	40.86	30.98
10	0.40550	39.83	36.37	31.92
11	0.38404	37.72	32.49	33.04
12	0.36433	35.78	29.08	34.28
13	0.34619	34.00	26.07	35.66
14	0.32947	32.36	23.41	37.17
15	0.31402	30.84	21.04	38.82
16	0.29972	29.44	18.95	40.62
17	0.28583	28.07	17.05	42.54
18	0.27240	26.75	15.33	44.59
19	0.25940	25.48	13.77	46.77
20	0.24683	24.24	12.36	49.11
21	0.23466	23.05	11.09	51.60
22	0.22287	21.89	9.93	54.26
23	0.21145	20.77	8.89	57.08
24	0.20037	19.68	7.95	60.09
25	0.18962	18.62	7.10	63.30
Total	10.18149	$1,000.00	$1,000.00	$1,000.00

* Using 6 per cent annual interest.

of this method show that the use of interest may serve to slow up depreciation or the amortization of expenses. Hence one may use interest to accelerate amortization, as in column 4, or to postpone it, as in column 5. (The sinking fund depreciation method is a recognized, but little used, depreciation method.)

What the figures of Table 2 perhaps illustrate best is the variety of results which the use of interest can produce in assorted amortization formulas.

A special comment should be made about an actuarial technique which, for expense amortization purposes, should be handled with care. One may be tempted to find a level percentage of premiums, expressed as an amount, such that the value of all such amounts (using customary factors of life contingencies and interest), at any time, is equal to the value, at the same time, of the amortizable expenses. This technique creates a special amortizable expense premium akin to the net level premium used in statutory life reserve calculations. Referring to the basic data from Table 2, one will find that the annual amortizable expense premium, at 6 per cent, comes to $151.525 and that $1,542.74 ($151.525 times 10.18149) is expected to be charged over the twenty-five-year useful life assumed for each item of business produced by the $1,000 amortizable expense. As will be demonstrated later, the use of reserve factors will, even under this actuarial technique, cause no more than $1,000 to be amortized for each $1,000 expended. But there is some danger, when this technique is used under the runoff method, that the interest charged to oneself will not be counted also as interest paid to oneself. If interest is counted in both places, or in neither place, then no more or less than the actual acquisition expense is charged to income, and the system of amortization contains no inherent flaw. Table 2A illustrates results under this technique, with the same assumptions on premiums paid and interest (6 per cent) as in Table 2. One will see, from a comparison of the straight runoff amortization of Table 2 and the "actuarial" runoff of Table 2A, that the latter is not proportionate to premiums paid and is less conservative in the long run (after year 7 a greater amount has been amortized under the straight runoff method). Pharr has demonstrated (*TSA*, XXIV, 25) that, under this approach, when assumptions are realized, the runoff method and the reserve factor method produce the same amortization. The caveat here is that, when the actuarial runoff method is used, the presence of interest requires proper and careful handling. More specifically, the net amount to be amortized each year is that shown in column 4 of Table 2A, not that in column 2, which is the "net level premium for amortizable expenses" multiplied by the in-force volume. (For greater ease of description, this approach might best be referred to as the "mortgage amortization approach," since the word "actuarial" does not provide as graphic a description.)

C. *Use of Mean Reserve Factors*

The mean reserve factor method of amortization normally contemplates the use of an interest rate (which, however, could be 0 per cent), and the effect of using such an interest rate is difficult to isolate. The requirement of an actuarial equivalence between retrospective and prospective calculations, not present with the runoff methods (except indirectly, with the mortgage amortization approach), has a substantial smoothing effect.

TABLE 2A

EXPENSE RUNOFF BY "ACTUARIAL" OR "MORTGAGE" AMORTIZATION TECHNIQUE*

Policy Year (1)	Amortizable Expense Premium Charge (2)	Interest Charge (3)	Net Amortization [(2) − (3)] (4)
1	$ 151.53	$ 0.00	$ 151.53
2	121.13	50.91	70.22
3	106.49	46.70	59.79
4	95.73	43.11	52.62
5	87.19	39.95	47.24
6	80.09	37.11	42.98
7	74.19	34.53	39.66
8	69.30	32.15	37.15
9	65.13	29.93	35.20
10	61.44	27.81	33.63
11	58.19	25.80	32.39
12	55.20	23.85	31.35
13	52.46	21.98	30.48
14	49.92	20.14	29.78
15	47.58	18.36	29.22
16	45.42	16.61	28.81
17	43.31	14.88	28.43
18	41.27	13.17	28.10
19	39.31	11.49	27.82
20	37.40	9.82	27.58
21	35.56	8.16	27.40
22	33.77	6.52	27.25
23	32.04	4.88	27.16
24	30.36	3.25	27.11
25	28.73	1.63	27.10
Total	$1,542.74	$542.74	$1,000.00

* Col. 2: amortizable expense premium times projected in-force (${}_tP^E$), or "mortgage loan payment"; col. 3: interest portion contained in ${}_tP^E$ charged on unamortized expense balance or on "loan principal outstanding"; col. 4: reduction in expenses remaining to be amortized or in "loan principal."

Under this amortization method, a fictitious net premium is first created, from which reserve factors are then developed in accordance with normal actuarial formulas. These reserve factors are negative, since the expenses are paid first and the premiums collected later. These reserve factors are then applied to the actual in-force at each valuation. Table 3 illustrates the amortization results this approach can create, and the comparison with the results of Table 2, where the same assumptions are used, is also informative. For the sake of comparing the results under two different interest rates, column 5 of Table 3 shows the amortization pattern to be expected where 3 per cent interest is used in the formulas rather than the 6 per cent used elsewhere. Column 6 shows the corresponding results when the same approach is used, but with a 0 per cent interest rate: it

TABLE 3

AMORTIZATION PATTERNS WITH RESERVE FACTOR METHOD

Policy Year	Proportion of Premium Paid	Amortization of $1,000 Acquisition Expense			
		Initial Reserve Factor, Using 6% Interest	Amount Amortized at 6%	Amount Amortized at 3%	Amount Amortized at 0%
(1)	(2)	(3)	(4)	(5)	(6)
1	1.00000	−$ 848.47	$ 151.53	$ 124.64	$ 98.22
2	0.79938	− 973.57	70.22	73.38	78.51
3	0.70279	− 1,022.30	59.79	63.54	69.03
4	0.63179	− 1,053.89	52.62	56.59	62.05
5	0.57542	− 1,075.04	47.24	51.27	56.52
6	0.52857	− 1,089.01	42.98	46.96	51.92
7	0.48965	− 1,094.60	39.66	43.52	48.09
8	0.45737	− 1,090.62	37.15	40.80	44.92
9	0.42982	− 1,078.63	35.20	38.59	42.22
10	0.40550	− 1,060.40	33.63	36.72	39.83
11	0.38404	− 1,035.31	32.39	35.15	37.72
12	0.36433	− 1,005.28	31.35	33.75	35.78
13	0.34619	− 969.89	30.48	32.50	34.00
14	0.32947	− 928.74	29.78	31.39	32.36
15	0.31402	− 881.38	29.22	30.40	30.84
16	0.29972	− 827.31	28.81	29.53	29.44
17	0.28583	− 768.04	28.43	28.69	28.07
18	0.27240	− 702.75	28.10	27.87	26.75
19	0.25940	− 630.70	27.82	27.09	25.48
20	0.24683	− 551.07	27.58	26.34	24.24
21	0.23466	− 462.90	27.40	25.61	23.05
22	0.22287	− 365.10	27.25	24.91	21.89
23	0.21145	− 256.39	27.16	24.23	20.77
24	0.20037	− 135.28	27.11	23.58	19.68
25	0.18962	0	27.10	22.95	18.62
Total	10.18149	....	$1,000.00	$1,000.00	$1,000.00

will be observed that the results there are absolutely identical with the straight runoff schedule results of column 3 in Table 2. This result indicates that absolute faithfulness in matching amortized expenses with premium income is possible with the mean reserve factor approach, provided that, as in the straight runoff schedule method, assumptions are exactly realized. One may also note that the results of column 4 in Table 3 are identical with those of column 4 in Table 2A, a situation which illustrates again the correspondence possible between the runoff and the reserve factor methods.

The use of mean reserve factors applied to the actual in-force has the main advantage of producing an automatic adjustment for deaths and withdrawals which do not conform to original assumptions. Where terminations are heavier than was originally assumed, the amortization is accelerated. But, as the initial reserve factors shown in column 3 indicate, there can be some durations where the factors increase, and, if the actual in-force does not decrease enough to offset the increase in reserve factors from one year to the next, a negative amortization will take place, that is, the deferred acquisition expense asset will be increased when, in fact, no additional expense has been made. This is a rather limited danger, but it is present. When an insurer's whole business is aggregated, net overstatements seem very unlikely. Where an insurer has good reason to be fearful of this phenomenon, it can modify its reserve factors so that they are never permitted to increase in size beyond any additional amortizable acquisition expense (in a manner analogous to the usual treatment of negative statutory reserves for decreasing term insurance, which are taken as zero). In normal cases this special adjustment does not seem to be favored by the accounting profession.

The danger inherent in the use of mean reserve factors should be compared with the opposite danger in the use of amortization schedules. In the case of the latter, the amount amortized will be too small if terminations exceed original expectations.

The possibility exists, of course, of using the reserve factor method to derive an expected amortization schedule (such as the ones shown in cols. 4 and 5 of Table 3), and of using the latter as the actual amortization runoff schedule without relating it directly to the in-force at each valuation date.

Where the acquisition expenses to be amortized can include amounts in early renewal years, such as the excess of heaped renewal commissions over long-term commissions or service fees, the pattern of reserve factors and of amortized amounts will be different. One may consider the results shown in Table 4 for an example involving expenses of \$600, \$200, \$100,

\$75, and \$25 for policy years 1–5, respectively. Because of terminations in the course of these five years, in fact only \$891.93 is expected to have to be amortized. Table 4 illustrates the pattern of the initial reserve factors for amortizable expenses as well as that of the amount effectively amortized from year to year.

As can be seen by examining column 4, if there were considerably fewer terminations than are built into the expense reserve calculations, the amount of expense to be amortized could be increased by more than the current year's amortizable expense. For instance, if no one terminated in the first policy year, the deferred acquisition expense asset would be increased from \$512.50 to \$753.39, which is \$40.99 more than the \$200 additional acquisition expense added in that year to the amortization

TABLE 4

RESERVE FACTOR METHOD: AMORTIZATION PATTERN WITH SUCCESSIVE AMORTIZABLE EXPENSES

Policy Year (1)	Proportion of Premium Paid (2)	Amortizable Expense per Policy in Force (3)	Expense Initial Reserve Factor (4)	Initial Amount Amortized at 0% (5)	Additional Actual Charge [(2)×(3)] (6)	Net Acquisition Expense Charge [(5)+(6)] (7)
1	1.00000	\$ 600.00	−\$512.40	\$ 87.60	\$ 0	\$ 87.60
2	0.79938	200.00	− 753.39	− 89.85	159.88	70.03
3	0.70279	100.00	− 869.33	− 8.71	70.28	61.57
4	0.63179	75.00	− 954.43	7.96	47.38	55.34
5	0.57542	25.00	− 985.32	36.02	14.39	50.41
6	0.52857	0	− 985.05	46.31	0	46.31
7	0.48965	0	− 975.76	42.89	0	42.89
8	0.45737	0	− 957.00	40.07	0	40.07
9	0.42982	0	− 930.75	37.65	0	37.65
10	0.40550	0	− 898.97	35.52	0	35.52
11	0.38404	0	− 861.60	33.64	0	33.64
12	0.36433	0	− 820.61	31.92	0	31.92
13	0.34619	0	− 776.00	30.33	0	30.33
14	0.32947	0	− 727.78	28.86	0	28.86
15	0.31402	0	− 675.99	27.51	0	27.51
16	0.29972	0	− 620.64	26.26	0	26.26
17	0.28583	0	− 563.20	25.04	0	25.04
18	0.27240	0	− 503.37	23.86	0	23.86
19	0.25940	0	− 440.98	22.73	0	22.73
20	0.24683	0	− 375.84	21.62	0	21.62
21	0.23466	0	− 307.73	20.56	0	20.56
22	0.22287	0	− 236.40	19.53	0	19.53
23	0.21145	0	− 161.57	18.52	0	18.52
24	0.20037	0	− 82.90	17.55	0	17.55
25	0.18962	0	0	16.61	0	16.61
Total		\$1,000.00		\$600.00	\$291.93	\$891.93

schedule. Again, in practice this phenomenon is unlikely to distort results appreciably, and it is confined to a few years following the issue of a policy; it can be prevented entirely and automatically by limiting the increase in the reserve to the maximum new amortizable expense for that year, as we discussed previously. This latter approach would change the initial reserve factors and expected net acquisition charges in the manner illustrated in Table 5.

TABLE 5

AMORTIZATION PATTERN OF SUCCESSIVE AMORTIZABLE EXPENSES WITH ADJUSTED RESERVE FACTORS

(Same Assumptions as in Table 4)

Policy Year	Proportion of Premium Paid	Expense Initial Reserve Factor		Net Acquisition Expense Charge	
		Before Adjustment	After Adjustment	Before Adjustment	After Adjustment
(1)	(2)	(3)	(4)	(5)	(6)
1	1.00000	−$512.40	−$512.40	$87.60	$ 87.60
2	0.79938	− 753.39	− 712.40	70.03	102.80
3	0.70279	− 869.33	− 812.40	61.57	68.81
4	0.63179	− 954.43	− 887.40	55.34	57.68
5	0.57542	− 985.32	− 912.40	50.41	50.03
6	0.52857	− 985.05	− 912.40	46.31	42.74
7	0.48965	− 975.76	− 912.40	42.89	35.51
8	0.45737	− 957.00	− 912.40	40.07	29.46
9	0.42982	− 930.75	− 912.40	37.65	25.13
10	0.40550	− 898.97	− 898.97	35.52	27.63

(Remainder as in Table 4)

One may note from Table 5 that expense reserve factor adjustments will change only the incidence of the net acquisition expenses amortized (in a conservative direction) and that the total amount of acquisition expenses amortized over the amortization period will remain the same.

IV. THE DEVELOPMENT OF EXPECTED EARNINGS

A. *Theoretical Considerations*

When considering the seriatim and group methods of individual life insurance reserve valuation, it is good to keep in mind the fact that while reserve *factors* may depend on assumptions, the reserves, to the extent to which they are dependent upon the actual in-force, do not. Whether they are redundant, adequate, or inadequate, for the purpose of reporting each year's earnings they should be taken as a fact. Everything that goes toward making reported earnings, all items of income and outgo, and the changes in the insured population as of each year end are all, for this purpose, facts. Only assumptions produce expected earnings, and it may be argued that one test of a good system in the development of reserve factors is that, when assumptions are realized, the pattern of earnings has a certain consistent smoothness about it.

We shall, in this section, consider a small, fairly realistic model, which will test the formulas and thinking so far developed. Expected statutory earnings will be developed by using statutory mean reserve factors and applying them to our assumptions as if our assumptions were facts. Expected adjusted earnings will be developed in exactly the same way, except that natural reserve factors will be used, in lieu of the statutory ones. With respect to amortizable acquisition expenses, we shall illustrate the difference between the adjusted earnings formula resulting from the use of expense amortization runoffs and that resulting from the use of expense amortization reserve factors, although the two formulas produce identical results when assumptions are realized.

In determining statutory earnings, it is important that the actuary identify all elements of statutory reserves. For instance, the statutory mean reserve factor for a life insurance policy with a coupon will normally provide for the coupon. Where a dividend is present, however, the normal statutory mean reserve factor includes no provision for it; provision for the coming dividend nonetheless is made in item 7 of the Liabilities page in the United States Annual Statement form. Therefore, insurers with participating policies must be careful to add the dividend component to their main statutory reserve factor when computing after-dividends statutory earnings, before comparing them with the corresponding adjusted earnings.

Another point to keep in mind is that earnings are normally reported as of the end of a calendar year. Earnings of \$10 arising on January 1 will, in the normal case where invested assets produce an annual interest return of 6 per cent, promote themselves to \$10.60 by calendar year end. Conversely, losses will pull earnings down with some additional interest burden. The whole subject has a nice philosophical nebulosity about it, but it must be remembered that the fiscal year end is the normal point in time from which we will view earnings.

A few additional symbols need to be defined:

${}_tG$ = Annual gross premium per \$1,000 of insurance, paid at the beginning of policy year t;

${}_tE^T$ = Total actual expenses per \$1,000 of insurance, paid at the beginning of policy year t;

${}_tM^S$ = Statutory mean reserve factor for all benefits, applicable to policy year t;

${}_tEE^S$ = Expected statutory earnings in the calendar year ending during policy year t;

${}_tEE^A$ = Expected adjusted earnings in the calendar year ending during policy year t.

Then, for statutory earnings,

$$\begin{aligned}{}_tEE^S = {} & (l_{t-2} - \tfrac{1}{2}d_{t-2})\ {}_{t-1}M^S(1+i) - \tfrac{1}{2}(1{,}000)d_{t-2}(1+i)^{3/4} \\ & - \tfrac{1}{2}(1{,}000)d_{t-1}(1+i)^{1/4} - w_{t-2}\ {}_{t-1}CV(1+i)^{1/2} \\ & - (l_{t-2} - d_{t-2})\ {}_{t-1}Y(1+i)^{1/2} + l_{t-1}({}_tG - {}_tE^T)(1+i)^{1/2} \\ & - (l_{t-1} - \tfrac{1}{2}d_{t-1})\ {}_tM^S\,. \end{aligned} \tag{13}$$

For adjusted earnings, the formula is identical, except that $(M^B + M^E)$ must be substituted for the corresponding M^S symbols, where the superscripts B and E pertain to all policy benefits and to amortizable expenses, respectively, if the reserve factor method is to be used in the treatment of amortizable acquisition expenses. If the expense runoff method is used for amortizable acquisition expenses, then

$$\begin{aligned}{}_tEE^A = {} & (l_{t-2} - \tfrac{1}{2}d_{t-2})\ {}_{t-1}M^B(1+i) - \tfrac{1}{2}(1{,}000)d_{t-2}(1+i)^{3/4} \\ & - \tfrac{1}{2}(1{,}000)d_{t-1}(1+i)^{1/4} - w_{t-2}\ {}_{t-1}CV(1+i)^{1/2} \\ & - (l_{t-2} - d_{t-2})\ {}_{t-1}Y(1+i)^{1/2} \\ & + [l_{t-1}({}_tG - {}_tE^T) - {}^{PY}_{\ t}E^{RO}](1+i)^{1/2} - (l_{t-1} - \tfrac{1}{2}d_{t-1})\ {}_tM^B\,, \end{aligned} \tag{14}$$

where ${}^{PY}_{\ t}E^{RO}$ stands for the net amount of amortizable acquisition expenses due to be charged at the beginning of policy year t under the runoff method. In a year when an acquisition expense is in fact paid, this amount could be negative and serve as an offset to the actual ${}_tE^T$ caused by the survivors. (A technical adjustment for ${}^{PY}_{\ t}E^{RO}$ is necessary in the first calendar year.)

These formulas can apply for all calendar years, including that of issue, for which all functions pertaining to a time before issue are taken as zero.

B. *A Concrete Example*

Table 6 presents an example of assumptions which may be made concerning a plan for which natural reserves are to be calculated into the two components of benefit reserves and amortizable expense reserves. The plan is an endowment at age 85 especially created to serve as an example, with no special features. The premium is the average actual

TABLE 6

MODEL PLAN ASSUMPTIONS

Issue Age, 35
Annual Premium, $19.79
Average Policy Size, $9,200
Interest from Money Invested, 6 Per Cent

Year	Total Expenses			Amortizable Expenses		
	Per Policy	Per $1,000	%	Per Policy	Per $1,000	%
1	$55.00	$2.80	0.9312	$37.00	$1.50	0.9062
2	8.00	0.80	0.1600	0.00	0.00	0.1350
3	8.00	0.80	0.0800	0.00	0.00	0.0550
4	8.00	0.80	0.0800	0.00	0.00	0.0550
5	8.00	0.80	0.0800	0.00	0.00	0.0550
6-10	8.00	0.80	0.0800	0.00	0.00	0.0550
11 and over	8.00	0.80	0.0250	0.00	0.00	0.0000

Year	Deaths	Lapses	CSV	Dividend	Death Benefit	Mean Reserve
1	0.00077	0.200	$ 0.00	$0.00	$1,000.00	$ 1.21
2	0.00095	0.120	0.00	1.28	1,000.00	14.94
3	0.00114	0.100	11.60	1.56	1,000.00	29.04
4	0.00134	0.088	26.47	1.86	1,000.00	43.50
5	0.00154	0.080	41.69	2.16	1,000.00	58.29
6	0.00177	0.072	57.22	2.46	1,000.00	73.41
7	0.00204	0.064	73.06	2.77	1,000.00	88.83
8	0.00238	0.058	89.22	3.09	1,000.00	104.57
9	0.00273	0.054	105.69	3.41	1,000.00	120.61
10	0.00308	0.050	122.48	3.74	1,000.00	136.95
11	0.00349	0.048	139.56	4.08	1,000.00	153.60
12	0.00396	0.046	156.93	4.42	1,000.00	170.54
13	0.00450	0.044	174.58	4.77	1,000.00	187.76
14	0.00511	0.042	192.48	5.12	1,000.00	205.23
15	0.00577	0.040	210.62	5.48	1,000.00	222.95
16	0.00661	0.040	228.97	5.84	1,000.00	240.89
17	0.00729	0.040	247.53	6.20	1,000.00	259.04
18	0.00802	0.040	266.29	6.57	1,000.00	277.38
19	0.00882	0.040	285.23	6.94	1,000.00	295.91
20	0.00969	0.040	304.33	7.32	1,000.00	314.61
21	0.01066	0.040	323.59	7.70	1,000.00	333.47
22	0.01173	0.040	342.98	8.08	1,000.00	352.47
23	0.01291	0.040	362.48	8.46	1,000.00	371.59
24	0.01424	0.040	382.06	8.85	1,000.00	390.81
25	0.01571	1.000	401.72	9.24	1,000.00	410.09

gross premium per $1,000 of insurance. The net expected interest return on the insurer's invested assets is shown as interest from money invested. Mortality rates and lapse rates are shown on a policy-year basis, with deaths presumed to be taking place uniformly during the year while lapses occur only at policy year end. Cash values and dividends are payable at year end, and no part of the dividends is paid unless the insured survives at year end (whether he then lapses or not). (Dividends are illustrated because there is a great deal of similarity in the treatment of dividends and coupons, and because stock insurers will fairly often issue participating policies.) Note that all policies lapse after twenty-five years.

Using the formulas presented earlier in this paper, the figures and reserve factors of Table 7 can be derived. The "valuation premium" shown in that table should be understood to be the accountants' valuation premium, which is the actuaries' natural premium, that is, that gross premium which, when all assumptions are realized, would exactly support all benefits and expenses, with no added margin for profit. The "benefit premium" is that portion of the gross premium needed to support the plan benefits, while the "amortizable expense premium" would support amortizable expenses. Note that in Table 7 the amortizable expense premium was calculated by a formula following the mortgage amortization technique and using 6 per cent interest. The statutory mean reserve figures of Table 7 include provision for the dividend payable at the end of the current policy year. With the payment of the twenty-fifth gross premium at the beginning of policy year 25 (assumed to be the last one), the initial reserve for amortizable expenses becomes zero, and hence the mean and terminal reserve is zero.

Using the figures of Tables 6 and 7, Table 8 shows the resulting "expected earnings," both adjusted and statutory. These earnings are shown as of the end of the calendar year ending within the policy year shown in the leftmost column. The net expense runoff is also shown, the leftmost column under that caption showing the runoff for first-year expenses alone (renewal acquisition expenses being absorbed directly by the insurer and amortized indirectly through the craggy pattern of the first-year amortizable expense runoff) and the rightmost column that for all amortizable expenses. These figures are shown with three decimals so that the results might more conveniently be read for $1,000,000 of business issued rather than only $1,000. All the results in Table 8 are expressed on the basis of each $1,000 of life insurance *issued.* (Total reported earnings may not add up precisely to the total shown, because of computer printout rounding.)

One may wonder why statutory expected earnings are shown for year

26 in Table 8. These earnings are really earnings from the last half of policy year 25. The formulas for adjusted earnings anticipated full withdrawal at the end of policy year 25 and recognized profits, by design, when premiums were paid. With no premium paid in the last half of policy year 25, and all assumptions realized as made, no adjusted earnings developed. But matters are different when the statutory mean reserve held at the beginning of the second half of policy year 25 turns out to be more than adequate to cover the benefits and withdrawals at the end. The remainder ends up as earnings. It will be of interest to note, from the

TABLE 7

MODEL PLAN RESERVES FOR ADJUSTED EARNINGS

(Expenses Amortized with 6 Per Cent Interest)

Valuation Premium, $18.65
Ratio of Valuation Premium to Gross Premium, 0.94249
Benefit Premium, $11.87
Amortizable Expense Premium, $4.38

Year	Benefit Reserves			Amortizable Expense Reserves			Statutory Mean Reserve
	Initial	Terminal	Mean	Initial	Terminal	Mean	
1	$ 11.87	$ 11.79	$ 11.83	−$19.07	−$20.23	−$19.65	$ 1.21
2	26.61	27.25	26.93	− 23.58	− 25.02	− 24.30	16.22
3	41.38	42.74	42.06	− 25.14	− 26.68	− 25.91	30.60
4	56.33	58.41	57.37	− 26.35	− 27.97	− 27.16	45.36
5	71.32	74.12	72.72	− 27.37	− 29.06	− 28.22	60.45
6	86.46	89.99	88.22	− 28.30	− 30.05	− 29.17	75.87
7	101.74	105.96	103.85	− 29.08	− 30.89	− 29.99	91.60
8	117.12	121.99	119.55	− 29.71	− 31.57	− 30.64	107.66
9	132.59	138.11	135.35	− 30.22	− 32.12	− 31.17	124.02
10	148.22	154.42	151.32	− 30.66	− 32.60	− 31.63	140.68
11	164.03	170.88	167.45	− 29.94	− 31.84	− 30.89	157.67
12	180.03	187.50	183.77	− 29.07	− 30.94	− 30.00	174.95
13	196.21	204.27	200.24	− 28.05	− 29.86	− 28.95	192.52
14	212.51	221.13	216.82	− 26.86	− 28.61	− 27.73	210.34
15	228.91	238.07	233.49	− 25.49	− 27.17	− 26.33	228.41
16	245.37	254.98	250.18	− 23.92	− 25.53	− 24.72	246.71
17	261.84	272.03	266.94	− 22.21	− 23.71	− 22.96	265.22
18	278.46	289.23	283.84	− 20.32	− 21.71	− 21.02	283.92
19	295.21	306.54	300.87	− 18.24	− 19.50	− 18.87	302.82
20	312.07	323.95	318.01	− 15.93	− 17.06	− 16.50	321.89
21	329.01	341.42	335.21	− 13.39	− 14.34	− 13.86	341.13
22	346.00	358.90	352.45	− 10.56	− 11.32	− 10.94	360.50
23	363.01	376.36	369.68	− 7.41	− 7.96	− 7.69	380.00
24	379.99	393.73	386.86	− 3.91	− 4.21	− 4.06	399.60
25	396.87	410.96	403.91	0.00	0.00	0.00	419.26

TABLE 8

EXPENSE RUNOFFS AND EXPECTED EARNINGS FOR MODEL PLAN

(Expenses Amortized with 6 Per Cent Interest)

YEAR	PORTION OF ISSUES		EXPENSE RUNOFF		EXPECTED EARNINGS	
	Paying Premium	At Calendar Year End	First Year	All Years	Adjusted	Statutory
1	1.00000	0.99962	$3.809	$3.809	−$0.205	−$9.236
2	0.79938	0.79900	0.229	2.365	1.130	− 0.157
3	0.70279	0.70239	1.219	1.984	0.993	2.355
4	0.63179	0.63137	1.051	1.739	0.893	2.087
5	0.57542	0.57498	0.923	1.549	0.814	1.784
6	0.52857	0.52810	0.819	1.395	0.748	1.587
7	0.48965	0.48915	0.736	1.269	0.693	1.433
8	0.45737	0.45683	0.671	1.169	0.648	1.279
9	0.42982	0.42923	0.618	1.086	0.609	1.172
10	0.40550	0.40488	0.573	1.014	0.575	1.100
11	0.38404	0.38337	0.965	0.965	0.544	1.462
12	0.36433	0.36361	0.934	0.934	0.517	1.392
13	0.34619	0.34541	0.908	0.908	0.491	1.321
14	0.32947	0.32863	0.887	0.887	0.468	1.259
15	0.31402	0.31311	0.870	0.870	0.446	1.194
16	0.29972	0.29873	0.858	0.858	0.426	1.129
17	0.28583	0.28479	0.847	0.847	0.406	1.088
18	0.27240	0.27130	0.837	0.837	0.387	1.076
19	0.25940	0.25826	0.829	0.829	0.369	1.060
20	0.24683	0.24563	0.822	0.822	0.351	1.043
21	0.23466	0.23341	0.816	0.816	0.334	1.024
22	0.22287	0.22157	0.812	0.812	0.317	1.002
23	0.21145	0.21008	0.809	0.809	0.301	0.980
24	0.20037	0.19894	0.808	0.808	0.285	0.952
25	0.18962	0.18813	0.808	0.808	0.270	0.925
26	....	....	....	....	....	3.082

	EARNINGS	
	Adjusted	Statutory
Total reported	$12.810	$23.393
Value at issue*	7.504	7.509

* Discounted at 6 per cent.

figures at the bottom of Table 8, that the discounted value of the earnings by one method, at the chosen interest rate of 6 per cent, is equal to the discounted value of the earnings produced by the other method. Timing differences cause total earnings of $12.81 to be reported over twenty-five calendar years by the adjusted earnings method, while total earnings of $23.39 can be reported over twenty-six years by the statutory method. It should be realized, naturally, that earnings, as developed here, do not include interest on previous years' accumulated earnings, but, if they did, the equivalence of the two scales of earnings would be demonstrated (an asset share of $33.19 per $1,000 issued would be accumulated by the calendar year end within policy year 26, at 6 per cent). The philosophical problem presented here is: "How long does one compute interest earnings on past earnings as continuing to be earned by a given block of issued policies?" The practical answer implied by the results presented here is, "Not beyond the fiscal year end when the profit (or loss) is produced and reportable as such." Hence, after one year's earnings on a block of business have been reported, they are transferred to general corporate funds, which, in turn, will generate earnings from investments (either in new life insurance business or in securities and the like) which will no longer be counted as earnings from this particular block of business. But these earnings, no matter what origin is assigned to them, will still be there.

From column 2 of Table 2 and column 6 of Table 3, the reader will have observed that the amortization of acquisition expenses with a 0 per cent interest rate, by either the runoff method or the reserve factor method, produces an amortization directly proportional to expected premium income (compare, for instance, cols. 2 and 6 of Table 3).

Tables 9 and 10 duplicate Tables 7 and 8 in all respects but one: amortizable expenses are amortized at 0 per cent interest in Tables 9 and 10 instead of at 6 per cent.

It will be observed from Tables 7 and 9 that the "amortizable expense premium" reduces from $4.38 to $2.96 when interest is not used, quite the same way that one's monthly payments on the mortgage on one's home would be decreased if the lender made the loan without interest; the amount of "principal" would of course remain the same, but it would be amortized somewhat differently. In both Tables 7 and 9, the amortizable principal starts at $23.45 ($19.07 initial reserve plus $4.38 "premium" in Table 7, and $20.49 plus $2.96 in Table 9) and ends at zero. Benefit reserves and statutory reserves are unaffected. The $2.96 amortizable expense premium of Table 9 has no meaning and is merely a tool for amortization. The corresponding premium of $4.38 in Table 7 has more meaning, inasmuch as it represents the portion of the gross premium which the insurer would have to charge *to the policyowner* for amortizable expenses and 6 per cent interest on unamortized balances. But, since an insurer cannot charge interest to itself in determining its earnings, the $4.38 has no other special meaning and is simply a tool in a mortgage amortization formula.

A comparison of the amortizable expense items in Tables 8 and 10 is also informative. The expense runoff for all years in Table 10 shows a much smoother progression, and the amortization expense charges for all years ($30.19, including $6.74 in renewal amortizable expenses) are

TABLE 9

MODEL PLAN RESERVES FOR ADJUSTED EARNINGS

(Expenses Amortized with No Interest)

Valuation Premium, $18.65
Ratio of Valuation Premium to Gross Premium, 0.94249
Benefit Premium, $11.87
Amortizable Expense Premium, $2.96

Year	Benefit Reserves			Amortizable Expense Reserves			Statutory Mean Reserve
	Initial	Terminal	Mean	Initial	Terminal	Mean	
1	$ 11.87	$ 11.79	$ 11.83	−$20.49	−$20.51	−$20.50	$ 1.21
2	26.61	27.25	26.93	− 25.34	− 25.36	− 25.35	16.22
3	41.38	42.74	42.06	− 26.95	− 26.98	− 26.96	30.60
4	56.33	58.41	57.37	− 28.10	− 28.14	− 28.12	45.36
5	71.32	74.12	72.72	− 28.97	− 29.02	− 29.00	60.45
6	86.46	89.99	88.22	− 29.67	− 29.72	− 29.69	75.87
7	101.74	105.96	103.85	− 30.15	− 30.21	− 30.18	91.60
8	117.12	121.99	119.55	− 30.40	− 30.47	− 30.44	107.66
9	132.59	138.11	135.35	− 30.47	− 30.56	− 30.51	124.02
10	148.22	154.42	151.32	− 30.42	− 30.52	− 30.47	140.68
11	164.03	170.88	167.45	− 29.16	− 29.26	− 29.21	157.67
12	180.03	187.50	183.77	− 27.77	− 27.88	− 27.83	174.95
13	196.21	204.27	200.24	− 26.26	− 26.38	− 26.32	192.52
14	212.51	221.13	216.82	− 24.63	− 24.76	− 24.69	210.34
15	228.91	238.07	233.49	− 22.88	− 23.01	− 22.94	228.41
16	245.37	254.98	250.18	− 21.00	− 21.14	− 21.07	246.71
17	261.84	272.03	266.94	− 19.06	− 19.20	− 19.13	265.22
18	278.46	289.23	283.84	− 17.04	− 17.17	− 17.10	283.92
19	295.21	306.54	300.87	− 14.92	− 15.06	− 14.99	302.82
20	312.07	323.95	318.01	− 12.72	− 12.84	− 12.78	321.89
21	329.01	341.42	335.21	− 10.41	− 10.53	− 10.47	341.13
22	346.00	358.90	352.45	− 8.00	− 8.10	− 8.05	360.50
23	363.01	376.36	369.68	− 5.47	− 5.54	− 5.50	380.00
24	379.99	393.73	386.86	− 2.81	− 2.85	− 2.83	399.60
25	396.87	410.96	403.91	0.00	0.00	0.00	419.26

TABLE 10

EXPENSE RUNOFFS AND EXPECTED EARNINGS FOR MODEL PLAN
(Expenses Amortized with No Interest)

Year	Portion of Issues		Expense Runoff		Expected Earnings	
	Paying Premium	At Calendar Year End	First Year	All Years	Adjusted	Statutory
1	1.00000	0.99962	$2.965	$2.965	$0.639	$ −9.236
2	0.79938	0.79900	0.234	2.370	1.074	−0.157
3	0.70279	0.70239	1.319	2.084	0.843	2.355
4	0.63179	0.63137	1.185	1.873	0.715	2.087
5	0.57542	0.57498	1.080	1.706	0.621	1.784
6	0.52857	0.52810	0.992	1.567	0.549	1.587
7	0.48965	0.48915	0.919	1.452	0.495	1.433
8	0.45737	0.45683	0.858	1.356	0.456	1.279
9	0.42982	0.42923	0.806	1.274	0.426	1.172
10	0.40550	0.40488	0.761	1.202	0.403	1.100
11	0.38404	0.38337	1.139	1.139	0.399	1.462
12	0.36433	0.36361	1.080	1.080	0.409	1.392
13	0.34619	0.34541	1.026	1.026	0.420	1.321
14	0.32947	0.32863	0.977	0.977	0.432	1.259
15	0.31402	0.31311	0.931	0.931	0.445	1.194
16	0.29972	0.29873	0.889	0.889	0.459	1.129
17	0.28583	0.28479	0.847	0.847	0.471	1.088
18	0.27240	0.27130	0.808	0.808	0.482	1.076
19	0.25940	0.25826	0.769	0.769	0.492	1.060
20	0.24683	0.24563	0.732	0.732	0.501	1.043
21	0.23466	0.23341	0.696	0.696	0.509	1.024
22	0.22287	0.22157	0.661	0.661	0.516	1.002
23	0.21145	0.21008	0.627	0.627	0.521	0.980
24	0.20037	0.19894	0.594	0.594	0.526	0.952
25	0.18962	0.18813	0.562	0.562	0.530	0.925
26	....	....	....	....	....	3.082

	Earnings	
	Adjusted	Statutory
Total reported	$ 13.332	$ 23.393
Value at issue*	7.504	7.509

* Discounted at 6 per cent.

amortized in direct proportion to expected premium income ("portion of issues paying premium"), which is not the case in Table 8. Greater amounts remain to be amortized in the later years in Table 8 than in Table 10, although both tables eventually amortize $23.45 in first-year expenses and $30.19 in total expenses. The pattern of expected adjusted earnings is also smoother in Table 10, and the effect of excess interest earnings on increasing amounts of natural reserves (reference here is being made to natural reserves which would take into account all expenses and which are neither shown nor discussed in this paper) is more apparent there, since earnings curve gently upward after the acquisition expense incurral period. As expected, statutory earnings and the discounted value of earnings remain unchanged. Also of substantial interest might be the notable improvement in first-year adjusted earnings produced by the "no interest" amortization method.

The reader who has observed that the author shows some partiality toward amortizing expenses with 0 per cent interest may also have noted that the use of interest in calculating benefit reserves has pretty much been taken for granted. There is a reason for this. To the extent to which policy benefits have a built-in promise of interest to the policyowner (such as cash values and excess interest through policy dividends), the natural reserve formulas for benefits must normally include a suitable interest rate, to allow the interest portion of these benefits to spring up from a less limited source than gross premiums, namely, from invested past asset accumulations. The case is different for amortizable expenses: they must normally come out of gross premiums, since expenses are largely independent of any policy asset accumulation.

One more important comment might be appropriate. Reported adjusted earnings are just that: reported adjusted earnings. They may help in judging management performance, but they will normally buy no groceries (unless they have induced the management to declare and pay a bigger cash dividend to the stockholders). Grim reality, insofar as an insurer's management is concerned, will certainly include statutory earnings. As long as our statutory insurance solvency standards remain as they are, life insurer managements would be wise to keep an eye on both types of earnings.

V. CONCLUSION

For the American actuary, influences outside the profession have caused the amortization of life insurance company expenses to become less a question of "whether" than of "how." The life insurance business is unique and complex enough to require its own imaginative solutions. Inasmuch as earnings adjustments are sought for the purpose of stating life insurance company earnings on a basis comparable to that obtaining for other kinds of business, it seems proper to proceed by analogy with the not very inflexible treatment of expenses in other businesses. This is why it may be quite relevant to consider various flexible approaches to asset depreciation and expense amortization in industry in general, before setting out to consider specific life insurance company problems.

When facing the practical task of making the earnings adjustment system work, the actuary may find it useful to consider the two main tools at his disposal: the expense runoff and the reserve factor approach. Each has respective advantages and pitfalls, the expense runoff schedule offering the stability and inflexibility characteristic of tangible assets, and the reserve factor approach the more erratic and flexible characteristics of living organisms. The use of the reserve factor method, optional in the handling of amortizable expenses, is well-nigh inescapable in the handling of most common individual life insurance benefits and requires theoretical development beyond that customary in the handling of statutory reserves. The proper use of that method represents the actuary's principal professional challenge and contribution.

Finally, earnings adjustment methods can best be utilized when one understands the results that they produce and appreciates the elusiveness of the concept of "earnings," as well as their genealogical characteristics—earnings producing earnings, which together produce more earnings, which together produce even more earnings, and so on. There must be finality in a practical system of reporting earnings, and at the same time a full realization of the fictional aspects of the system. The system is essentially a tool for the measurement of a life insurance company management's performance; the actuary's role is to make the system a workable one, as well as to make it fair for both stockholders and management. This paper aims to assist the actuary in meeting better or more easily this truly professional challenge.

DISCUSSION OF PRECEDING PAPER

ROBERT H. DREYER AND STEVEN D. SOMMER:

Mr. Paquin is to be commended for the effort put into this paper; its substance is as useful as its title is long. In particular, we would like to call attention to Section II and its formulation of the "mean" reserve where lapses must be taken into account, because the traditional interpolation formula gives no clue as to the proper handling of lapses.

On several occasions we have seen formulas where year-end lapses were removed from the terminal reserve before interpolation to obtain the mean reserve. It is not readily apparent that such an approach is improper; trying to explain it to an auditing accountant, untrained in actuarial science, can make it even more confusing. The clear development and description of the proper approach set forth by Mr. Paquin should help anyone faced with such a problem.

Having recognized that different approaches exist, the question arises whether or not those differences are material. For distinction, we will refer to reserves calculated by the approach described in the paper as "intermediate reserves" and those calculated by the alternate approach as "modified mean reserves"—"modified" in the sense that expenses are removed at the start of the year and lapses at the end (before interpolating). The "true" midyear reserve is shown for comparison. Table 1 on page 486 takes two very general cases and sets forth the three types of reserves and the magnitude of differences arising at selected durations. It is left to the reader to judge whether or not these differences could be material in any specific instance, noting that the lapse rate, duration, and plan appear to be critical factors.

PAUL R. MILGROM:

I would like to thank Mr. Paquin for his daring sally into the theory of mean reserves. His comments on the nature of a terminal reserve contribute greatly to an intuitive understanding of the problem. Mr. Paquin's suggested redefinition of the terminal reserve symbol, however, leaves something to be desired for at least three reasons: (1) it is unnecessary, (2) the terminal reserves so defined are inappropriate for gain and loss analysis, and (3) the resulting mean reserves are proper only for annual premium policies.

TABLE 1

COMPARISON OF NATURAL RESERVE FACTOR METHODS

DURATION (AGE 35)	WHOLE LIFE PLAN			10-YEAR TERM PLAN		
	Benefit	Expense	Net	Benefit	Expense	Net
	"True" Reserve at Midyear					
1	$ 11.45	−$24.28	−$ 12.83	$1.07	−$6.77	−$5.70
2	26.04	− 29.49	− 3.45	1.91	− 7.34	− 5.43
3	41.38	− 32.06	9.32	2.61	− 7.19	− 4.58
4	56.11	− 33.11	23.00	3.12	− 6.61	− 3.49
5	71.25	− 33.91	37.34	3.52	− 5.93	− 2.41
10	153.16	− 37.59	115.57	1.55	0.00	1.55
20	335.32	− 36.14	299.18	....	....	....
30	510.02	− 28.42	481.60	....	....	....
	Intermediate Reserve					
1	$ 11.46	−$24.29	−$ 12.83	$1.06	−$6.76	−$5.70
2	26.05	− 29.50	− 3.45	1.90	− 7.35	− 5.45
3	41.39	− 32.07	9.32	2.60	− 7.20	− 4.60
4	56.13	− 33.12	23.01	3.11	− 6.62	− 3.51
5	71.27	− 33.93	37.34	3.52	− 5.94	− 2.42
10	153.20	− 37.61	115.59	1.53	0.00	1.53
20	335.38	− 36.16	299.22	....	....	....
30	510.05	− 28.43	481.62	....	....	....
	Modified Mean Reserve					
1	$ 12.88	−$27.41	−$ 14.53	$1.17	−$7.76	−$6.59
2	27.52	− 31.18	− 3.66	2.03	− 7.96	− 5.93
3	42.21	− 32.93	9.28	2.72	− 7.61	− 4.89
4	56.79	− 33.82	22.97	3.26	− 6.99	− 3.72
5	71.95	− 34.63	37.32	3.68	− 6.27	− 2.59
10	153.96	− 38.35	115.61	1.53	0.00	1.53
20	336.04	− 36.55	299.49	....	....	....
30	510.36	− 28.36	482.00	....	....	....
	Excess of Modified Mean over Intermediate					
1	12.4%	−12.8%	−13.3%	10.4%	−14.8%	−15.6%
2	5.6	− 5.7	− 6.1	6.8	− 8.3	− 8.8
3	2.0	− 2.7	− 0.4	4.6	− 5.7	− 6.3
4	1.2	− 2.1	− 0.2	4.8	− 5.6	− 6.3
5	1.0	− 2.1	− 0.1	4.5	− 5.6	− 7.0
10	0.5	− 2.0	0.0	0.0	0.0	0.0
20	0.2	− 1.1	0.1	....	....	....
30	0.1	0.2	0.1	....	....	....

Consider first gain and loss analysis. Let ${}_tV_x^B$ be the traditional benefit reserve factor. Then

$$ {}_tV_x^B = [({}_{t-1}V_x^B + P_{[x]}^B)(1+i_t) - q_{[x]+t-1}^{(d)}\ {}_tDB(1+\tfrac{1}{2}j_t) - q_{[x]+t-1}^{(w)}\ {}_tCV_x]/(1 - q_{[x]+t-1}^{(d)} - q_{[x]+t-1}^{(w)}) \,, \quad (1) $$

$$ {}_0V_x^B = 0 \,. $$

Here the $q^{(w)}$'s are probabilities rather than rates. The short-term interest rate symbol j_t is used to calculate interest lost by immediate payment of death claims. Its use simplifies the ensuing development. Then

$$ {}_tV_x^B = ({}_{t-1}V_x^B + P_{[x]}^B)(1+i_t) - q_{[x]+t-1}^{(d)}[{}_tDB(1+\tfrac{1}{2}j_t) - {}_tV_x^B] - q_{[x]+t-1}^{(w)}({}_tCV_x - {}_tV_x^B) \,. \quad (2) $$

Now an analysis of gain and loss for a policy year reveals that the net GAAP gain is (assuming that $j_t' = j_t$)

$$ \begin{aligned} \mathrm{Gain}_{[x]+t-1} = {} & (GP_{[x]} - P_{[x]})(1+i_t') + (i_t' - i_t)({}_{t-1}V_x^B + P_{[x]}^B) \\ & + (q_{[x]+t-1}^{(d)} - q_{[x]+t-1}^{(d)\prime})[{}_tDB(1+\tfrac{1}{2}j_t) - {}_tV_x^B] \\ & + (q_{[x]+t-1}^{(w)} - q_{[x]+t-1}^{(w)\prime})({}_tCV_x - {}_tV_x^B) \,, \end{aligned} \quad (3) $$

where the prime indicates actual experience and $GP_{[x]+t-1}$ is the gross premium. "Gain" is expressed on a per-beginner basis at the end of the year. The point is that, in gain and loss analysis, both the tabular costs—that is, $q_{[x]+t-1}^{(w)}({}_tCV_x - {}_tV_x^B)$ and $q_{[x]+t-1}^{(d)}[{}_tDB(1+\tfrac{1}{2}j_t) - {}_tV_x^B]$—and the reserves released are based properly on the more usual style of terminal reserves.

Let ${}_tV_x^{AE}$ be the terminal unamortized acquisition expense factor for a company using a factor approach to amortizing acquisition expenses. Then, to illustrate the use of terminal reserves in gain and loss analysis, consider the "gain from withdrawals" per unit.

Tabular cost of withdrawals (sometimes negative),

$$ q_{[x]+t-1}^{(w)}({}_tCV_x + {}_tV_x^{AE} - {}_tV_x^B) \,, $$

plus reserves released by withdrawal,

$$ q_{[x]+t-1}^{(w)\prime}\ {}_tV_x^B \,, $$

less unrecovered acquisition expense,

$$ q_{[x]+t-1}^{(w)\prime}\ {}_tV_x^{AE} \,, $$

less cash values paid,

$$q^{(w)\prime}_{[x]+t-1}\ {}_tCV_x\,,$$

equals gain from withdrawals:

$$(q^{(w)}_{[x]+t-1} - q^{(w)\prime}_{[x]+t-1})({}_tCV_x + {}_tV^{AE}_x - {}_tV^B_x)\,.$$

For companies writing off acquisition expense according to a fixed schedule, the best method of computing gains or losses by source will depend on the type of schedule used.

The problem in the mean reserve calculation for annual premium policies is that the terminal reserve contains no provision for the tabular cost of lapse and is, therefore, an inappropriate interpolation point. Replacing ${}_tV^B_x$ with ${}_tV^B_x + q^{(w)}_{[x]+t-1}({}_tCV_x - {}_tV^B_x)$ in the interpolation formula gives

$${}_t\text{Mean reserve}^B_x = \tfrac{1}{2}[{}_{t-1}V^B_x + P^B_{[x]} + {}_tV^B_x + q^{(w)}_{[x]+t-1}({}_tCV_x - {}_tV^B_x)]\,. \quad (4)$$

A further argument favoring this method is that it also works for modes of premium payment other than annual. Assume for the moment that appropriate terminal reserves, ${}^M_tV^B_x$, for all modes combined have been developed. For all modes combined, experience may show that a fraction $\bar{\alpha}_t$ of the cost of lapse of the policy year occurs before the valuation date. Then an appropriate mean reserve is

$$\begin{aligned}\text{Mean reserve}^B_x = {} & \tfrac{1}{2}({}^M_{t-1}V^B_x + P^B_{[x]} + {}^M_tV^B_x) \\ & + (\tfrac{1}{2} - \bar{\alpha}_t)q^{(w)}_{[x]+t-1}({}_tCV_x - {}^M_tV^B_x)\,.\end{aligned} \quad (5)$$

A rather detailed theory in support of this approach is contained in a paper which I am preparing. For the moment, it can be said that appropriate mixed-mode terminal reserves are developed by using mixed-mode withdrawal rates and making small modifications in the death benefit and cash value.

The modification in the death benefit is a fraction of the valuation premium depending on the mix of modes. The modification raises the reserve by an amount equal to the theoretical reserve for nondeduction of deferred premiums at death.

Limits on the modified cash value may be developed by considering the two boundary cases. The first is the usual annual premium case, with the full premium for the year paid by all starters and with lapses occurring at the year's end. The second boundary case assumes that no premium for the year is paid by lapsing policies and that all lapses occur at the beginning of the year. In this second case, the correct terminal reserves must obey the formula

$$\begin{aligned} {}_tV_x^B = \{[{}_{t-1}V_x^B &+ P_{[x]}(1 - q_{[x]+t-1}^{(w)})](1 + i_t) \\ &- q_{[x]+t-1}^{(w)}\ {}_{t-1}CV_x(1 + i_t) \\ &- (1 - q_{[x]+t-1}^{(w)})q_{[x]+t-1}^{(d)}\ {}_tDB(1 + \tfrac{1}{2}j_t)\}/ \\ &\qquad [1 - q_{[x]+t-1}^{(w)} - (1 - q_{[x]+t-1}^{(w)})q_{[x]+t-1}^{(d)}] . \end{aligned} \tag{6}$$

Rearranging terms,

$$\begin{aligned} {}_tV_x^B = \{[{}_{t-1}V_x^B &+ P_{[x]}(1 - q_{[x]+t-1}^{(w)})](1 + i_t) \\ &- q_{[x]+t-1}^{(w)}\ {}_{t-1}CV_x(1 + i_t) - q_{[x]+t-1}^{(d)}\ {}_tDB(1 + \tfrac{1}{2}j_t) \\ &+ q_{[x]+t-1}^{(w)}\ q_{[x]+t-1}^{(d)}[{}_tDB(1 + \tfrac{1}{2}j_t) - {}_tV_x^B]\}/ \\ &\qquad (1 - q_{[x]+t-1}^{[w]} - q_{[x]+t-1}^{[d]}) . \end{aligned} \tag{7}$$

If the same valuation premium and reserve are to be generated using the traditional annual premium formula and the modified cash value ${}_tCV_x^M$, then we must have

$$\begin{aligned} {}_tV_x^B = [({}_{t-1}V_x &+ P_{[x]})(1 + i_t) - q_{[x]+t-1}^{(w)}\ {}_tCV_x^M \\ &- q_{[x]+t-1}^{(d)}\ {}_tDB(1 + \tfrac{1}{2}j_t)]/(1 - q_{[x]+t-1}^{(w)} - q_{[x]+t-1}^{(d)}) . \end{aligned} \tag{8}$$

Equating the right-hand sides of equations (7) and (8) and solving for ${}_tCV_x^M$ gives

$${}_tCV_x^M = ({}_{t-1}CV_x + P_{[x]})(1 + i_t) - q_{[x]+t-1}^{(d)}[{}_tDB(1 + \tfrac{1}{2}j_t) - {}_tV_x^B] . \tag{9}$$

The modified cash value of the first boundary case is, of course, ${}_tCV_x$. When ${}_{t-1}CV_x > 0$, equation (9) suggests that the second boundary cash value will be close to ${}_tCV_x$ and no adjustment may be necessary. For term plans, however, and in early years on permanent plans when ${}_{t-1}CV_x = 0$, some adjustment seems to be called for.

LEONARD H. MC VITY:

My discussion of this paper refers primarily to Section III(B), "Expense Runoffs," and to Section III(C), "Use of Mean Reserve Factors." Although I was not able to reproduce exactly the factors appearing in columns 2 of Tables 2 and 3, they appear to approximate closely the probability of survivorship of a male life aged 32 for periods of from one to twenty-five years, subject to 1955–60 Select Mortality and Linton B withdrawal rates.

In Table 2 of the paper the figures in column 3 were obtained by applying the probabilities of survivorship in column 2 to the twenty-five-year annuity obtained by dividing $1,000 by $10.18149. The resulting annuity, $98.217, was reduced progressively by the probability of survivorship to obtain the runoff. However, I am puzzled by the results shown in Mr. Paquin's Table 3. Using completely parallel methods in Table 1 on page 491, I believe I prove that the yearly amortization using no interest differs little from the corresponding amortization using 6 per cent interest, bearing in mind that the yearly amortization is the difference between the remainders at the end of the current and previous years.

The figures in my column 6 can be proved correct easily by the following demonstration. Dividing $1,000.00 by the annuity value 6.59956 as shown in the first line of column 5 gives an initial payment of $151.525. Deducting this from $1,000.00 and adding 6 per cent interest to the remainder gives $899.384 at the end of the first year. The deduction at the beginning of the second year is $151.525 × 0.79938 = $121.126. Again adding 6 per cent interest to the remainder, the value at the end of the second year is $824.953. This value agrees with the second remainder factor in column 6 of my table, and subsequent values can be checked similarly. Column 7 of my table is directly comparable to column 10, which, in turn, agrees, except for two different roundings, with the figures in column 6 of Mr. Paquin's Table 3.

I am of the opinion that the use of an interest assumption in amortizing deferred acquisition expenses is perfectly proper and is completely analogous to expense reserve computations made by true actuarial procedures. The expense portion of the actuarial reserve should, however, be computed in two parts, reflecting deferrable acquisition expenses and all other expenses separately. The negative reserve representing the deferrable acquisition expenses will be the unamortized expenses asset, provided that the actual expenses, mortality rates, withdrawal rates, and interest rate assumed are in accordance with the GAAP assumptions.

ANTHONY C. SHARP:

I note that Mr. Paquin's paper quite properly makes no mention of the audit guide. Perhaps a few words about accounting for unamortized acquisition expenses as set out in the guide are in order.

Appendix B of the audit guide states in the first paragraph that "ac-

TABLE 1

COMPARISON OF YEARLY AMORTIZATION—6 PER CENT INTEREST AND NO INTEREST

n	$s_{[x]+n-1}$ (1)	$[(1)\times v^{n-1}$ $(i=0.06)]$ (2)	$\uparrow\Sigma(2)$ (3)	$\ddot{a}_{[x]+n-1:\overline{25-n+1}\rvert}$ $[(3)\div(2)]$ (4)	$[(4)\times(1)]$ (5)	Remainder Factor $[(5)\div 6.59956]$ (6)	Expenses Amortized from (6) (7)	$\uparrow\Sigma(1)$ (8)	Remainder Factor $[(8)\div 10.18149]$ (9)	Expenses Amortized from (9) (10)
1.	1.00000	1.00000	6.59956	6.59956	6.59956	. . . .	. . . .	10.18149	. . . .	. . . .
2.	0.79938	0.75413	5.59956	7.42519	5.93555	0.89939	$ 100.61	9.18149	0.90178	$ 98.22
3.	0.70279	0.62548	4.84543	7.74674	5.44433	0.82495	74.44	8.38211	0.82327	78.51
4.	0.63179	0.53046	4.21995	7.95527	5.02606	0.76158	63.37	7.67932	0.75424	69.03
5.	0.57542	0.45579	3.68949	8.09472	4.65786	0.70578	55.80	7.04753	0.69219	62.05
6.	0.52857	0.39498	3.23370	8.18700	4.32740	0.65571	50.07	6.47211	0.63567	56.52
7.	0.48965	0.34518	2.83872	8.22388	4.02682	0.61016	45.55	5.94354	0.58376	51.91
8.	0.45737	0.30418	2.49354	8.19758	3.74933	0.56812	42.04	5.45389	0.53567	48.09
9.	0.42982	0.26967	2.18936	8.11866	3.48956	0.52876	39.36	4.99652	0.49075	44.92
10.	0.40550	0.24001	1.91969	7.99838	3.24334	0.49145	37.31	4.56670	0.44853	42.22
11.	0.38404	0.21445	1.67968	7.83250	3.00799	0.45579	35.66	4.16120	0.40870	39.83
12.	0.36433	0.19192	1.46523	7.63459	2.78151	0.42147	34.32	3.77716	0.37098	37.72
13.	0.34619	0.17205	1.27331	7.40081	2.56209	0.38822	33.25	3.41283	0.33520	35.78
14.	0.32947	0.15447	1.10126	7.12928	2.34888	0.35591	32.31	3.06664	0.30120	34.00
15.	0.31402	0.13889	0.94679	6.81683	2.14062	0.32436	31.55	2.73717	0.26884	32.36
16.	0.29972	0.12506	0.80790	6.46010	1.93622	0.29339	30.97	2.42315	0.23800	30.84
17.	0.28583	0.11252	0.68284	6.06861	1.73459	0.26283	30.56	2.12343	0.20856	29.44
18.	0.27240	0.10116	0.57032	5.63780	1.53574	0.23270	30.13	1.83760	0.18048	28.08
19.	0.25940	0.09088	0.46916	5.16241	1.33913	0.20291	29.79	1.56520	0.15373	26.75
20.	0.24683	0.08158	0.37828	4.63692	1.14453	0.17343	29.48	1.30580	0.12825	25.48
21.	0.23466	0.07317	0.29670	4.05494	0.95153	0.14418	29.25	1.05897	0.10401	24.24
22.	0.22287	0.06556	0.22353	3.40955	0.75989	0.11514	29.04	0.82431	0.08096	23.05
23.	0.21145	0.05868	0.15797	2.69206	0.56924	0.08625	28.89	0.60144	0.05907	21.89
24.	0.20037	0.05246	0.09929	1.89268	0.37924	0.05746	28.79	0.38999	0.03830	20.77
25.	0.18962	0.04683	0.04683	1.00000	0.18962	0.02873	28.73	0.18962	0.01862	19.68
							28.73			18.62
	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	$1,000.00	. . . .	. . . .	$1,000.00

quisition expenses should be charged against income in proportion to premium recognized." It then states that "some techniques will tend to produce unacceptable results" and gives an example (straight-line charge-off per in-force policy per year plus immediate charge-off of the unamortized amounts on terminated policies) which "will not result in a reasonable association of expenses with related revenues" and also "is not consistent with the concept that aggregate acquisition costs . . . are expected to be recovered from aggregate premium revenue over the life of each block." Only two other examples are given. One is an amortization for each major block of business in proportion to expected premium revenue ("sum of the premiums" method). The next and last example given is a method which approximates the "expense factor" method used by actuaries, and the statement is made that "to be fully consistent with actuarial concepts, the rate of amortization should give effect to interest assumed in benefit reserve calculations." The statement also is made that "any 'worksheet' approach to amortization should be based on the fact that interest affects the rate of recovery of costs." These last two statements about interest are consistent with the stated objective of the method, which "approximates the techniques used by actuaries in the determination of reserve valuation factors under the single valuation reserve method" because this technique evidently was presumed to involve the use of interest (other than zero).

Whether this or any other method is consistent with the quotation "reasonable association of expenses with related revenues" in the above paragraph will depend on the specific circumstances in which it is applied. For example, to obtain amortization in proportion to actual premiums, it is necessary to make adjustments for the difference between actual and expected terminations. The audit guide does not specify any method for amortization but, in addition to the fact that expenses must be recoverable to be amortizable, does provide a guide in which the following principles are set forth:

1. There should be a reasonable association of expenses with related revenues.
2. Aggregate acquisition costs for each year's block of business are expected to be recovered from aggregate premium revenue over the life of each block.

An approach suggested by Mr. Paquin under which a zero interest rate is used for the expense premium and expense amortization could satisfy the above criteria. The fact that interest is used for benefit premiums but not for expense premiums seems to me to have little bearing on the acceptability of this approach if, when applied, it meets the above-mentioned requirements. This is confirmed by my discussion with accountants. The whole procedure of anticipating future gains by deferral of expenses is artificial, as is the interest charged on unamortized expenses. In my opinion this "zero interest" approach on amortization of expenses should be regarded as one of many acceptable approaches to the deferral of expenses, if applied consistently. It is certainly more defensible than some other methods used for similar purposes in accounting. I think that Mr. Paquin has made a useful contribution to the literature on adjusted earnings by suggesting the need for flexibility in the choice of amortization methods.

ROBERT L. COLLETT:

Mr. Paquin's article makes an interesting and useful contribution to the theory and practice of adjusted reserves. Some companies have approached the problem of computing GAAP reserves, taking it for granted that the traditional mean reserve formula, using successive terminal reserves, is adequate for all years after the first. Mr. Paquin's Section II is a very clear and concise explanation of why further analysis by companies is required. Valid alternatives to his adjusted mean reserves would include "intermediate" reserves of the type described in the volume *Natural Reserves and Life Insurance Accounting*, published by Ernst and Ernst.

Section III raises a couple of questions for me. The use of a zero rate of interest is, as he says, a method which is more conservative in the long run than amortization by what he calls the "actuarial technique" (that is, amortization with a positive interest charge each year on the unamortized balance). In this context it is understood that "conservative" means a more rapid write-off of the capitalized asset. The AICPA audit guide for life companies does call for the introduction of conservatism, but it calls for it through the careful selection of assumptions (in this case, those expenses deemed eligible for capitalization) and through meaningful tests for recoverability of any capitalized expense. An accelerated amortization schedule is not cited as a recommended means of achieving conservatism. Indeed, replacement of the most conservative version of accelerated amortization (that is, statutory accounting) would seem to be a key objective of the audit guide.

It also seems to me that the zero interest rate amortization approach can be theoretically confusing. The author apparently recognizes it, but I believe that further emphasis needs to be placed on the fact that the use of a zero rate in no way alters the cost to the company to "finance the sale," the premium needed to repay that cost, or the existence of a premium deficiency, if present when interest is considered. If a company is using a zero-rate approach to achieve a desired amortization pattern, that is one thing. If, however, they entertain a notion of using such a method in order to avoid or reduce gross premium deficiencies, then I believe that such use would constitute a significant misapplication of the concept discussed in the paper.

My final comment has to do with whether use of the zero-rate approach results in proper matching, as contemplated under generally accepted accounting principals. The use of a zero rate of interest does result in a write-down of the capitalized expense in a pattern which is in proportion to premium revenues. However, the exclusion of interest from the calculation results in a lack of matching of net income to premium revenues. Consider the adjusted earnings exhibited in Mr. Paquin's Tables 8 and 10. The premium income in any year could be considered to be his annual premium ($19.79) times the proportion of issues paying premiums, as shown in both tables.

It can be determined from Table 8 that, where interest is included, adjusted earnings (excluding year 1) are a nearly constant 7 per cent of premium revenues over the life of the policy. On the other hand, in Table 10, which involves amortization without interest, profits vary from 7 per cent of premiums in year 2 down to 5 per cent in year 10 and up to more than 14 per cent in year 25. The only obvious improvement in the reported pattern derived from the omission of interest would seem to be in the first year. I suspect that improvements in the first-year figure could be obtained through alternative alterations, which would retain the use of interest and the matching of the earnings and premiums in renewal years.

It is not my intention here to take any particular position as to whether or not flexibility such as amortization without interest should be permitted. The practical convenience to management of being able to choose alternatives is clear. However, I believe that the theoretical bases for such alternatives should be subjected to further scrutiny.

HENRY KUNKEMUELLER:

I believe that the separation of the policy reserve into a benefit reserve and an expense asset is somethat arbitrary, and, if the assumptions used for the expense asset are different from those used for the benefit reserve, one may encounter difficulties. I would suggest that, if one decides to use different assumptions for the expense asset as opposed to the benefit reserve, one tests the over-all results against a common-sense standard, such as eighteen-month preliminary term reserves, to make certain that any anomalies will be avoided.

(AUTHOR'S REVIEW OF DISCUSSION)

CLAUDE Y. PAQUIN:

I am grateful to the discussants for their interest in the subject and their contribution to the body of thought which is rapidly emerging on the subject of adjusted earnings.

Messrs. Dreyer and Sommer make reference to three types of mean reserves, namely "intermediate," "modified mean," and "true midyear." They define as intermediate reserves the mean reserves used in the paper, namely, the arithmetic mean of the post-beginning of year transactions initial reserve and of the pre-year end transactions terminal reserve. They define as modified mean reserves what I with less charity would call the "thoughtless" mean reserves, that is, the average of a post-year end transactions terminal reserve and an improperly defined initial reserve (previous terminal reserve plus premium). They do not define "true midyear reserves."

The true midyear approach to which they refer seems to involve moving the initial reserve forward by half a year or bringing the terminal reserve back half a year, using in each case the expression $(1 - \frac{1}{2}q_x^{(d)})$ to take survival into account. Such a formula is based upon the premise that $(1 - \frac{1}{2}q_x^{(d)})^2 = (1 - q_x^{(d)})$, which simply is not "true." The assumption that deaths are uniformly distributed through the policy year is better reflected, it would seem, in a simple averaging of the initial and terminal reserve factors (properly defined) to produce the mean reserve. The latter method is admittedly also based upon the premise that $(1 + i/2)^2 = (1 + i)$, which also is not true. However, the distortion implicit in that traditional method is at least uniform and guaranteed to be small. Moving a year-end reserve forward or back by half a year really is not a bad method, but to qualify the resulting reserve as "true" and those used in this paper as merely "intermediate" is subtly misleading and perhaps mildly presumptuous. (The truest mean reserve factor of all would probably be of the form $\frac{1}{2}[{}_tI(1 + i)^{1/2} + v^{1/2}\ {}_tV]$, where the terms ${}_tI$ and ${}_tV$ are as defined in the paper.)

Messrs. Dreyer and Sommer, after comparing so-called modified mean reserves with correctly calculated reserves, leave it to the reader to decide whether the differences are material. Of course, the responsibility for the decision is well placed, but what is most important to note is not the materiality of the difference in the reserves but the materiality of the difference in the resulting earnings. One may note, for instance, that where a stock price is determined by a price-earnings ratio of, say, 15, the difference in reported earnings can be magnified 15 times. It is true that in the long run reserves do not affect the amount of earnings and affect only their incidence. However, in practice this simply means that a distortion in reported earnings eventually will be followed by an actuarially equivalent distortion in the opposite direction.

It is, in a way, regrettable that Mr. Milgrom should, in his discussion, adopt the formula $l_t = l_{t-1}(1 - q_{t-1}^{(d)} - q_{t-1}^{(w)})$, whereas the paper uses $l_t = l_{t-1}(1 - q_{t-1}^{(d)})(1 - q_{t-1}^{(w)})$. His use of probabilities of decrement, predicated upon the existence of multiple-decrement tables for ordinary life insurance (for which data are usually analyzed and available only in terms of single decrements), is difficult to reconcile with the assumption that withdrawals take place only when scheduled annual premium payments are not made and complicates the comparing of his approach with mine. In other words, the paper shows successive single decrements, deaths being uniformly distributed over each policy year and withdrawals being instantaneous at policy year ends; Mr. Milgrom uses multiple decrements operating jointly and uniformly over the policy year. His discussion is clever and informative, and one can but look forward to a paper which would advance the actuarial art in the handling of nonannual premium payments as well as in meaningful gain and loss analysis.

One point my paper attempted to make was that a reserve cannot be used properly (except by chance) unless it is defined properly (in relation to the timing of insurance events, including population decrements). There is really no standard definition of "terminal reserves" which is complete and satisfactory (Jordan's *Life Contingencies* states simply that a terminal reserve relates to the end of a policy year). After all these years, the term still must be defined, and one will find that good definitions make good formulas. It is important, and perhaps not emphasized enough in the paper, that the actuarial profession defines all its terms with meticulousness. Like the jet age, GAAP is thrusting many new concepts upon us, and unless we begin to agree on a set vocabulary, our discussions will be but a Babelized conundrum of garbled thoughts. Until we can settle our differences in terms, let us say that Mr. Milgrom prefers to work from a post-year end transactions terminal reserve and to convert it, for the purpose of calculating mean reserves, into a pre-year end transactions terminal reserve of the form ${}_tV_x^B + q_{[x]+t-1}^{(w)}\,({}_tCV_x - {}_tV_x^B)$, where ${}_tV_x^B$ is the post-year end transactions terminal reserve and where there are no coupons or dividends paid. Contrary to Mr. Milgrom's assertion, I am not suggesting a redefinition of the terminal reserve symbol; what I am suggesting very strongly is that there is more than one kind of terminal reserve factor, namely, a pre-year end transactions kind and a post-year end transactions kind, and as soon as the Committee on Standard Notation and Nomenclature tells me which symbol to use for each kind I will comply gladly. Meanwhile, I ask a question. What is the amount represented by ${}_{20}V_x$ for $1,000 twenty-year endowment insurance policy? Those who answer $1,000 are thinking of this symbol, as I did, as representing a pre-year end transactions reserve.

I would suggest that all pre-year end transactions terminal reserves be called "pre-terminal reserves," and that post-year end transactions terminal reserves can be called "post-terminal reserves." Likewise, pre-beginning of year transactions initial reserves could be called "pre-initial reserves," and post-beginning of year transactions initial reserves could be called "post-initial reserves." Normally, the pre-initial reserve and the post-terminal reserve of the next preceding policy year will be identical. This convention may prove helpful and will be used in the remainder of this author's review of discussion. It will be apparent that this paper defines the mean reserve as the average of the post-initial reserve and the pre-terminal reserve.

The definition of gain which Mr. Milgrom presents in his formula (3) is clearly flawed by the absence of provisions relating to expenses. Yet, on the whole, there is good food for thought in his discussion. Mr. Milgrom obviously would like a more complete and more advanced treatment of the subject of reserve valuation, and I am in sympathy with him. I might explain, however, that it seemed preferable to me to keep this paper reasonably short and simple, in order not to deter anyone who might benefit from it from reading it, to keep my work within reasonable bounds and to present my paper to the profession at a time when it could be of most practical value.

In his discussion Mr. McVity, apparently puzzled by the figures of column 4 of Table 3, failed to observe that they could be extracted directly from column 4 of Table 2A. I would not dare challenge Mr. McVity's figures, but I would dare say that he missed a point which, if discovered, would have shown the reason for our difference in results. The point is that costs should be recognized at the same time as premium revenues are recognized. Mr. McVity looked at costs from the end of the first policy year, gave himself 6 per cent interest on his beginning-of-year premium of $151.525, charged himself 6 per cent on his $1,000 initial expenditure, and so found himself with $899.384 at the *end* of the policy year, from which he concluded that he should deduct $100.61 as of the *beginning* of the first policy year. What, in effect, he does is to calculate the amortized cost as of the end of each policy year, but then, inexplicably, he charges it off at the beginning of that same policy year. Were he to look at expenses from the end of each calendar year, as might be the normal practice at the end of a fiscal year, presumably he would accrue 6 per cent interest all around for six months, so that he would have $873.93 unamortized after six months, which means that he amortized $126.07 the first year (rather than $100.61).

One important point of this paper is that interest can play tricks in the amortization of expenses and that it might be wise for an actuary to stay away from the resulting complications. The following paragraphs will attempt to explain how the rabbit comes out of the hat when one insists on using interest.

The paper illustrates how the use of interest in amortizing expenses can be likened to a home mortgage amortization. A home mortgage amortization is characterized by decreasing interest payments and increasing principal reductions as part of each level installment. There are, however, two main differences peculiar to a block of life insurance business. First, the installments (premium payments) decrease from year to year as a result of the decrease in in-force brought on by mortality and withdrawals. While each installment may contain a proportionately greater principal reduction (expense amortized or, in the accountants' terminology, "cost recovered"), the reduction in the size of the installments still may show a deceptively diminishing *amount* of principal reduction. *In relation to the premium revenue,* this principal reduction is, in fact, increasing from year to year, and there is a deferment of the amortization of the principal. A comparison of columns 4 and 6 in Table 3 of the paper (comparing policy years 16, 17, and 18, for example) would show this clearly. Second, the first installment is paid concurrently with the "borrowing" of the "principal," while a home mortgage normally calls for the first installment to be paid one period after the lending of the principal. Hence, with a block of life insurance business, there is no interest payment in the first installment, and all of it can be devoted to "principal reduction" (expense amortized or cost recovered). This is illustrated in column 4 of Table 3 of the paper.

One possible way (which, as may be seen, still presents problems) of remedying the problem of the "balloon" first-year principal reduction inherent in the mortgage amortization approach is to use a modification which one might call "the mortgage amortization with prepaid interest" approach. Under this approach, the first year's interest would be forced out of the first installment by being discounted. The same process would apply to all other installments as well. The amount of principal upon which interest is discounted is, of course, the balance of the principal after the current year's principal repayment. This can be expressed by the following formula:

$$ {}_tP^E = {}_tA^E + \frac{{}_ti}{1 + {}_ti} ({}_tL^E - {}_tA^E) , \tag{1} $$

where

${}_tP^E$ = tth amortizable expense premium (payable at the beginning of each period);

${}_tA^E$ = Amount of amortizable expenses amortized upon the payment of ${}_tP^E$;

${}_tL^E$ = Amount of amortizable expenses yet unamortized immediately before ${}_tP^E$ is paid; and

${}_ti$ = tth period's interest rate used in the determination of ${}_tP^E$ and for the amortization process.

In this equation, all factors are known quantities except ${}_tA^E$. Rearranging the equation above produces

$$ {}_tA^E = {}_tP^E - {}_ti({}_tL^E - {}_tP^E) . \tag{2} $$

Formula (2) above insidiously found its way into Appendix B of the AICPA audit guide (as released in "final" form in the spring of 1973). This formula can be extracted from a tabulation in the appendix which purports to illustrate an amortization method "with interest." Since the audit guide mandates (p. 72) that "acquisition expenses . . . be deferred and charged against income *in proportion* to premium revenues recognized" (emphasis mine) it might be appropriate to examine briefly the nature and characteristics of the mortgage amortization approach with prepaid interest, which, incidentally, is that which Mr. McVity illustrates in the table of his discussion and which Joe B. Pharr also illustrated in his paper (*TSA*, XXIV, 25). Referring to the "Examples of Assorted Patterns of Depreciation" in Table 1 of the paper, consider a ten-year mortgage with 8 per cent interest and level installments payable at the beginning of each year (tantamount to considering a ten-year insurance policy with no benefits, no mortality, no withdrawals, and a first-year amortizable expense of \$1,000).

The value of P, which may be derived by use of the formula $P\ddot{a}_{\overline{10}|} =$ 1,000, is found to be 137.99. Application of formula (1) or formula (2) above will produce the values shown in Table R1 on page 500.

The format of Table R1 is analogous to that of Table 2A of the paper so that the reader may observe more easily the similarities and differences between the two mortgage amortization techniques, that illustrated in Table 2A being for interest charged "when earned" rather than prepaid at the beginning of each year.

An interesting observation may be made by comparing the amortized amounts of Table R1 (col. 5) below, with the amounts of depreciation taken under the sinking fund method as shown in Table 1 of the paper. The amounts are identical, which shows that the "mortgage amortization with prepaid interest" approach is akin to a sinking fund method depreciation. From Table R1 one may observe that the amounts amortized each year (col. 5) are far from being in proportion to the annual payments (premium revenues), and the amortization is the opposite of conservative.

For life insurance the annual payments (premium revenues) normally are not level, as are those of Table R1, but are in proportion to a decreasing in-force. Were the normal sinking fund depreciation approach

TABLE R1

MORTGAGE AMORTIZATION WITH PREPAID INTEREST

Year (1)	Principal before Current Amortization (2)	Annual Payment (3)	Interest Portion of Payment (4)	Current Amortization (5)
1	$1,000.00	$ 137.99	$ 68.96	$ 69.03
2	930.97	137.99	63.44	74.55
3	856.42	137.99	57.47	80.52
4	775.90	137.99	51.03	86.96
5	688.94	137.99	44.08	93.91
6	595.03	137.99	36.56	101.43
7	493.60	137.99	28.45	109.54
8	384.06	137.99	19.69	118.30
9	265.76	137.99	10.22	127.77
10	137.99	137.99	0	137.99
Total		$1,379.90	$379.90	$1,000.00

to be followed in such a case, the amount amortized each year would represent a combination of increasing interest (on the fictitious accumulated fund for replacement) and of decreasing "premium income," producing the results illustrated in column 5 of Table 2 of the paper. The mortgage amortization with prepaid interest approach for life insurance is a special version of the sinking fund depreciation approach. The values it produces are illustrated in Table R2 on page 501, which should be compared with Table 2A of the paper as well as with Mr. McVity's table. As one can observe from column 4, the balloon first-year principal reduction problem essentially has been cured. However, the nonproportionality problem remains (in year 25, where 1.86 per cent of the revenues are expected, 2.87 per cent of the expenses are amortized).

Normally it is not the function of an author's review of discussion to dwell upon actuarial techniques and approaches which the author deems questionable. But this departure from custom seems appropriate here because it has been reported that the mortgage amortization with prepaid interest approach was introduced into Appendix B of the AICPA audit guide at the last minute and without being shown in any of the prior drafts upon which comments were sought. This approach, like other

TABLE R2

EXPENSE RUNOFF BY MORTGAGE AMORTIZATION WITH PREPAID INTEREST TECHNIQUE

Policy Year (1)	Amortizable Expense Premium Charge (2)	Discount Charge (3)	Net Amortization [(2)−(3)] (4)	Proportion of Premiums Paid (In-Force) (5)
1	$ 151.53	$ 50.91	$ 100.62	1.00000
2	121.13	46.70	74.43	0.79938
3	106.49	43.11	63.38	0.70279
4	95.73	39.95	55.78	0.63179
5	87.19	37.11	50.08	0.57542
6	80.09	34.54	45.55	0.52857
7	74.19	32.15	42.04	0.48965
8	69.30	29.93	39.37	0.45737
9	65.13	27.82	37.31	0.42982
10	61.44	25.80	35.64	0.40550
11	58.19	23.85	34.34	0.38404
12	55.20	21.97	33.23	0.36433
13	52.46	20.15	32.31	0.34619
14	49.92	18.36	31.56	0.32947
15	47.58	16.60	30.98	0.31402
16	45.42	14.88	30.54	0.29972
17	43.31	13.17	30.14	0.28583
18	41.27	11.48	29.79	0.27240
19	39.31	9.82	29.49	0.25940
20	37.40	8.16	29.24	0.24683
21	35.56	6.52	29.04	0.23466
22	33.77	4.88	28.89	0.22287
23	32.04	3.25	28.79	0.21145
24	30.36	1.63	28.73	0.20037
25	28.73	0	28.73	0.18962
Total	$1,542.74	$542.74	$1,000.00	10.18149

approaches "with interest," fails to produce an amortization which, in the words of the audit guide, is "in proportion to premium revenues recognized" (as mandated on p. 72 of the guide). Moreover, it produces, in relation to premium income, a deferment in the recognition of expenses. As Mr. Sharp's discussion points out, the audit guide really contradicts itself when it says, on the one hand, that expenses are to be amortized in proportion to premium revenues recognized and, on the other, that the use of interest is expected "for full consistency with actuarial concepts." From a legalistic standpoint, an appendix should have a lesser standing than the main body of an audit guide requiring proportionality in amortization. From an actuarial standpoint, this paper should demonstrate that the use of interest in the treatment of expenses is not necessary to ensure consistency with actuarial concepts. But we are there entering the realm of opinion. Assume that interest is used. The paper makes clear, in the presentation of Table 2A, that amounts amortized under the runoff approach must be net of interest. Where these amounts are calculated through reserve factors, this "netting" process is automatic, although it is not easy to see how it is taking place. Therefore, let us examine what happens when "true actuarial concepts" or "true actuarial formulas" are applied.

The paper points out that earnings cannot be determined or reported without reference to a point in time. Two points in time usually are used by actuaries: for theoretical work, the policy year end is often used, while for practical work (such as preparing an annual statement), the calendar year end is used.

Proceeding from the more familiar to the less familiar, note how statutory earnings are determined in the annual statement. The profit is determined essentially as being made up of gross premiums plus interest income, less the following items: (*a*) benefits (of all kinds), (*b*) expenses (of all kinds, including commissions and taxes), and (*c*) the increase in reserves. How is the increase in reserves determined traditionally? Let us look at the increase in reserves symbolically.

Let

${}_tM$ = Mean reserve factor applicable for a policy in its tth policy year;

l'_{t-1} = In-force by which the mean reserve factor for the tth policy year is multiplied

$= l_{t-1} - \frac{1}{2}d_{t-1}$; and

${}_t^cIR$ = Increase in reserves at the calendar year end with respect to the beginning-of-year in-force represented by l'_{t-2}.

Then

$$ {}_t^cIR = l'_{t-1}\ {}_tM - l'_{t-2}\ {}_{t-1}M\ . \qquad (3)$$

If the reader is willing and able to accept the foregoing equation as representative of what every life insurance company does at calendar year end, let us see what happens to amortizable expense reserves on a policy-year basis. As the paper explains, a reserve factor cannot be described properly and used without being associated with a population (or in-force), by reference either to that population or to a specific point in time which indicates what the population will be. In the case of expenses, where only withdrawals are expected at year end, the policy-year-end reserve (not the reserve factor, but the total reserve) may be described as either $(l_{t-1} - d_{t-1})_t V^E$, where ${}_t V^E$ represents the pre-terminal amortizable expense reserve factor previously used in the paper, or as $l_t \, {}^{\text{post}}_{\;\;t}V^E$, where ${}^{\text{post}}_{\;\;t}V^E$ would represent the corresponding post-terminal reserve factor. In effect,

$$(l_{t-1} - d_{t-1})_t V^E = l_t \; {}^{\text{post}}_{\;\;t}V^E \,. \tag{4}$$

Let ${}^p_t IR^E$ represent the increase in expense reserves at the tth policy year end with respect to the previous policy year end's in-force represented by l_{t-1} (this last symbol being defined in the paper). Then

$$ {}^p_t IR^E = l_t \; {}^{\text{post}}_{\;\;t}V^E - l_{t-1} \; {}^{\text{post}}_{\;t-1}V^E \,. \tag{5}$$

From manipulation of formulas (10)–(12) of the paper and of formulas (4) and (5) above, one can derive

$$ {}^p_t IR^E = l_{t-1}({}_t P^E - {}_t E^E)(1 + i) + l_{t-1} \; {}^{\text{post}}_{\;t-1}V^E i \,. \tag{6a}$$

It might be simpler to consider this last formula in a year when no new amortizable expense has been incurred, that is, when ${}_{t-1}E^E$ equals zero. Then

$$ {}^p_t IR^E = l_{t-1} \; {}_t P^E(1 + i) + l_{t-1} \; {}^{\text{post}}_{\;t-1}V^E i \,. \tag{6b}$$

This formula expresses symbolically the point I made earlier in this reply with respect to Mr. McVity's discussion. The expense premium, ${}_t P^E$, paid by l_{t-1} persons, is accrued with interest to the end of the year, and interest on the previous reserve, $l_{t-1} \; {}^{\text{post}}_{\;t-1}V^E$, is deducted (it is a deduction, not an addition, because the expense reserve is negative). Hence, using the traditional "increase in reserves" deduction approach is equivalent to paying interest to oneself on premiums received and charging interest to oneself on the yet unamortized amount, and charging off the remainder (the amortizable expense effectively amortized) as of the *end* of the policy year.

However, in practice earnings are calculated not through the use of policy-year-end factors but through the use of mean reserve factors. Hence my paper attempted to show what it is that an actuary does to himself (or his employer or client) when he uses mean reserve factors.

Formula (3) of this reply is a generalized expression which shows what the actuary does in practice at each calendar year end. The following formula for the increase in reserves charged at year end applies more specifically for amortizable expenses:

$$ {}^{c}_{t}IR^{E} = l'_{t-1}\ {}_{t}M^{E} - l'_{t-2}\ {}_{t-1}M^{E} \,. \tag{7} $$

A transformation similar to that employed on formula (5) of this reply (using again formulas [10]–[12] of the paper) results in:

$$ {}^{c}_{t}IR^{E} = l_{t-1}({}_{t}P^{E} - {}_{t}E^{E})(1+i)^{1/2} + l'_{t-2}\ {}_{t-1}M^{E}i \,. \tag{8a} $$

Again, consider formula (8a) in a year when no new amortizable expenses has been incurred, that is, when ${}_{t}E^{E}$ equals zero. Then

$$ {}^{c}_{t}IR^{E} = l_{t-1}\ {}_{t}P^{E}(1+i)^{1/2} + l'_{t-2}\ {}_{t-1}M^{E}i \,. \tag{8b} $$

This latter formula shows how half a year's interest is accrued on premiums received and a whole year's interest is deducted ("charged to oneself") on the previous reserve. For the first year of a policy's life, only half a year's interest is charged upon the whole amortizable expense incurred at issue.

It might be informative to refer to Tables 7 and 8 of the paper and to derive the expense runoff amounts by both formulas (7) and (8a) of this reply. For instance, the effective expense runoff for the calendar year ending within policy year 12 can be expressed as

$$ l'_{11}\ {}_{12}M^{E} - l'_{10}\ {}_{11}M^{E} \,, $$

or $(0.36361)(-30.00) - (0.38337)(-30.89) = 0.934$, or as

$$ l_{11}({}_{12}P^{E} - {}_{12}E^{E})(1+i)^{1/2} + l'_{10}\ {}_{11}M^{E}i \,, $$

or $(0.36433)(4.3815 - 0)(1.0296) + (0.38337)(-30.89)(0.06) = 0.934$. From this demonstration we may conclude that the actuary who develops mean reserve factors for amortizable expenses with interest is, in effect, contemplating amortizing expenses in accordance with the expense runoff listed in Table 8.

The expense runoff of Table 8 really is a hybrid of the two mortgage amortization techniques described earlier, namely, (1) that with interest when earned and (2) that with prepaid interest. It shows how, in practice, with the expense reserve factor approach, one does not amortize, in the first year, the full expense premium of \$4.38 (more accurately \$4.3815), as contemplated by the normal mortgage amortization technique, nor does one amortize the \$3.24 which the policy-year-end approach or mortgage amortization technique with prepaid interest would have produced. Pharr, in his paper, limits his showing of equivalence between the reserve factor approach and the amortization schedule to the situation where earnings are looked at from the end of a policy year. In my opinion, it is more practical to look at mean reserve factors, because they are what actuaries use in practice. This discussion demonstrates that the mean reserve factor approach with interest does, for expenses, create a balloon first-year amortization. The normal mortgage amortization technique would amortize \$4.38 the first year, the mortgage amortization with prepaid interest technique would amortize \$3.24, the mean reserve factor approach would amortize \$3.81 (from Table 8), while the mean reserve factor approach with a 0 per cent interest rate would amortize \$2.97 (from Table 10).

Once one understands what the traditional "increase in mean reserves method" of charging off amortizable expenses does, should one be willing to use it? The audit guide contains two important criteria: (1) amortizable expenses must be associated with the corresponding premium revenues, and presumably must be recognized at the same time, and (2) amortizable expenses should be charged against income in proportion to premium revenues recognized. The paper has sought to show, *inter alia*, that using a 0 per cent interest rate in preparing mean reserve factors for amortizable expenses satisfies these two criteria admirably well.

It is possible, for one who wishes to use the mortgage amortization with prepaid interest technique, to derive mean reserve factors in such a way that the amount effectively amortized each year is that which a policy-year-end valuation would have amortized. The solution is to force the mean reserve factor in a manner such that, if the mean reserve factor is multiplied by the projected in-force at calendar year end, it will produce the net amortization contemplated by the mortgage amortization with prepaid interest technique. For instance, if one wants to amortize \$1,000 by charging off \$100.62 in year 1, \$74.43 in year 2, and so on,

$$ {}_1^cIR^E = l_0' \, {}_1M^E - {}_1^{\text{pre}}I^E , $$

or $100.62 = 0.99962\,{}_1M^E - (-1{,}000)$, from which the value of ${}_1M^E$ is -899.72. For year 2, and through the general recursion formula thereafter,

$$ {}_2^cIR^E = l_1' \, {}_2M^E - l_0' \, {}_1M^E , $$

or $74.43 = 0.79900\,{}_2M^E - 0.99962(-899.72)$, from which ${}_2M^E$ is $-1{,}032.48$; and so on. This approach reflects more ingenuity than accord with established precedents of actuarial practice representing adherence to true actuarial concepts and formulas. However, a clever actuary can contrive formulas which do precisely what he wants them to do. In fact, one important point of this paper was that a formula exists which amortizes expenses precisely as the audit guide demands, and that formula is, for amortizable expenses, the normal formula with a 0 per cent interest rate.

Mr. Collett's discussion would lead me to re-emphasize two points. The first is that the audit guide calls for a system which produces a recognition of costs in proportion to revenues being recognized, and that profits (earnings) are expected to fall where they may. The objective is not specifically that of producing earnings which are proportional to revenues. The primary objective really is to adjust expenses, although that process ends in an adjustment of earnings. If earnings end by being disproportionate to premium revenues, that normally should not disturb anyone. The second point is that there is more to earnings than those shown in Tables 8 and 10 of the paper. These earnings possibly could be described as "operating earnings." Looking at two scales of operating earnings is a bit like looking at two different but actuarially equivalent dividend scales. Some adjustment for interest on prior earnings needs to be made (at least mentally). The sum of the adjusted earnings comes out to \$12.81 in Table 8 and to \$13.33 in Table 10, while statutory earnings total \$23.39. All of these earnings scales have a discounted value at issue of \$7.51, which proves that they are actuarially equivalent. As the paper states, if they were accumulated at interest, as insurance policy dividends can be, they would all accumulate to \$33.19 per \$1,000 issued, $25\frac{1}{2}$ years after issue. This implies that, looking at the over-all financial operation of a life insurance company, there is an accretion of some earnings which are earnings resulting from prior earnings. This is a nebulous and theoretical area because we are dealing with a process of allocation where two sources may compete for recognition as the producer of the earnings. (If a thousand policies issued in 1971 produced enough profits in 1972 to make it possible to issue ten new policies in 1972, and if the 1972 policies produced \$100 profit in 1973, should this \$100 profit be counted as earnings produced by the 1971 policies? What if the 1972 profits from the 1971 issues are invested at 8 per cent interest instead?)

Writing a paper on the various subjects I have covered may have been a mistake, for a book would really appear to be needed. I trust, nevertheless, that my remarks have not been unduly burdensome to read and comprehend. I thank the discussants for challenging me to clarify or expand on some of the points covered in the paper. My main aim was to provoke fresh thinking on subjects which surround the adoption of generally accepted accounting principles. Any computer can grind out reams of figures, and producing reserve factors, however burdensome it may be, is not our real professional challenge. The actuary's challenge is to think through his formulas before he uses them, to define his objectives clearly, and to devise new tools and new approaches as circumstances demand. This paper may represent a modest contribution to meeting the challenge presented by GAAP.

I would be remiss if I did not acknowledge the important contribution to the development of my thinking by my colleague John L. Street, a member of the American Academy of Actuaries, whose oral discussions with me on the subjects covered in the paper and the review of discussions have added to my perception of problem areas. To him, as well as to all those who contributed directly or indirectly to the advancement of our understanding of concepts relating to GAAP, I express my sincere appreciation.