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INTEREST AND INFLATION ASSUMPTIONS IN PENSION PLAN VALUATIONS

RICHARD K. KISCHUK

ABSTRACT

This paper deals with the selection of appropriate interest and inflation assumptions, which is one of the most important considerations in the valuation of a defined benefit pension plan. The paper begins by describing a simplified model in which the only important consideration is the selection of an appropriate differential between interest and inflation assumptions. The discussion then considers some situations that might lead the actuary to attempt to select a "realistic" level of interest and inflation assumptions, in addition to an appropriate differential.

The author's conclusion is that in some situations the absolute level of the interest and inflation assumptions can be all but meaningless, and the differential between the two assumptions is the important factor. In other situations the use of this technique will produce serious inaccuracies in the valuation results. As always, the actuary must analyze each situation in order to determine which of the available techniques will produce the best results with the least effort, given the degree of accuracy required.

INTRODUCTION

THE relationship between interest and inflation assumptions used in pension plan valuations has been the center of a great deal of debate over recent months. In the past, the offsetting of a liberal salary-scale assumption with a conservative interest assumption and turnover assumption was a popular practice in the valuation of pension plans. Recently this technique has been refined by using the argument that the absolute levels of the interest and inflation assumptions are unimportant; it is the differential between them that is important. Most recently, the technique of offsetting one assumption with another has come under fire. The opponents of this technique contend that a reasonable differential between given interest and inflation assumptions is not a sufficient excuse for offsetting one against the other; in all cases realistic assumptions should be used.

It is the author's contention that the truth lies somewhere in between the two opposing viewpoints. In some situations the absolute levels of the interest and inflation assumptions are all but meaningless and the differential between the two assumptions is the only important factor. In other situations the use of offsetting assumptions will produce serious inaccuracies in the valuation results.

The following discussion will be centered in the development of a simplified model in which the differential between interest and inflation assumptions is the determining factor. The remainder of the discussion will deal with the application of the results of the simplified model to the selection of interest and inflation assumptions in realistic situations.

SIMPLIFIED MODEL

Consider a pension plan with a unit benefit formula, where the benefit earned each year is based upon the final salary earned at age 65. The plan provides f per cent of final salary for each year of service.

Each year, on the valuation date of the plan, the entire pension fund is fully reinvested in one-year certificates of deposit. This process has been continuing since plan inception. The interest rate earned on the one-year certificates includes an allowance for inflationary price increases during the succeeding one-year period. Further, the allowance for inflation which is included in this rate of interest is exactly realized in all wages and prices during the one-year period. Inflation is the only cause of changes in wages and prices. Thus the interest rate r_t obtainable in one-year time certificates at the beginning of year $t + 1$ may be expressed as follows:

$$(1 + r_t) = (1 + i)(1 + i'_t) ,$$

where i is the base rate of interest, assumed to be constant in all years, and i'_t is the actual rate of inflation applicable to wages and prices during year $t + 1$.

In valuing the plan liabilities and costs, the actuary has at his disposal a record of all investment yields obtained in past years, from which he can also deduce the corresponding rates of inflation. Having considerable background in economics and having developed an infallible computer model of the economy, the actuary is also able to predict without error the investment yields, r_t, to be obtained on one-year certificates of deposit in all future years. Knowing the base rate of interest i, he is able to predict the precise rate of inflation, i'_t, which will prevail over each year in the future.

Retirement benefits are payable at the beginning of each year and are subject to cost-of-living increases based upon the rate of inflation to be experienced during the coming year.

CALCULATION OF THE NORMAL COST FACTOR

Utilizing the entry age normal cost method, the normal cost factor may be computed as follows:

$$(NCF)_{[x]+t}(AS)_{[x]+t}\,\frac{{}^{S}N_{[x]}}{{}^{S}D_{[x]+t}} = (AS)_{[x]+t}(65-x)f\,\frac{{}^{S}D_{[x]+(65-x)}}{{}^{S}D_{[x]+t}}\,{}^{S}\ddot{a}_{[x]+(65-x)}\,, \tag{1}$$

where

$[x]$ = Entry age;

$[x]+t$ = Attained age;

$(NCF)_{[x]+t}$ = Normal cost factor at attained age $[x]+t$;

f = Percentage credit given for each year of service;

D and N = Commutation functions based on service table and varying interest rates, r_t, where

$$N_{[x]} = \sum_{t=0}^{65-x-1} D_{[x]+t}\,;$$

${}^{S}D$ and ${}^{S}N$ = Commutation functions based on service table, varying rates of interest, i_t, and salary scale, $s_{[x]}$, based on varying rates of inflation, i'_t, where

$${}^{S}D_{[x]} = s_{[x]}D_{[x]}\,,$$

$${}^{S}N_{[x]} = \sum_{t=0}^{65-x-1} {}^{S}D_{[x]+t}\,;$$

$(AS)_{[x]+t}$ = Annual salary earned from age $[x]+t$ to age $[x]+t+1$;

${}^{S}\ddot{a}_{[x]+(65-x)}$ = Present value of increasing annuity with payments due at the beginning of each year, starting with \$1 at age $[x]+(65-x)$ and increasing annually depending on rates of inflation i'_t.

Solving equation (1) for the normal cost factor,

$$(NCF)_{[x]+t} = \frac{(65-x)f\;{}^{S}D_{[x]+(65-x)}\;{}^{S}\ddot{a}_{[x]+(65-x)}}{{}^{S}N_{[x]}}\,. \tag{2}$$

Equation (2) can be simplified further by considering the implications of the definitions of commutation functions ${}^{S}D$ and ${}^{S}N$:

$$\begin{aligned}
{}^{S}D_{[x]+t} &= s_{[x]+t}D_{[x]+t} \\
&= s_{[x]+t}v^{[x]+t}l_{x+t} \\
&= \frac{\prod_{t=0}^{x+t-1}(1+i'_t)}{\prod_{t=0}^{x+t-1}(1+r_t)}\, l_{x+t} \\
&= \frac{1}{(1+i)^{x+t}}\, l_{x+t} \\
&= D^{i}_{x+t}
\end{aligned}$$

and

$$\begin{aligned}
{}^{S}N_{[x]} &= \sum_{t=0}^{65-x-1} D_{[x]+t} \\
&= \sum_{t=0}^{65-x-1} D^{i}_{x+t} \\
&= N^{i}_{x}\,.
\end{aligned}$$

Similarly,

$$\begin{aligned}
{}^{S}\ddot{a}_{[x]+(65-x)} &= \frac{{}^{S}N_{[x]+(65-x)}}{{}^{S}D_{[x]+(65-x)}} \\
&= N^{i}_{65}/D^{i}_{65} \\
&= \ddot{a}^{i}_{65}\,.
\end{aligned}$$

Substituting these results in equation (2),

$$(NCF)_{[x]+t} = (NCF)_x = \frac{(65-x)fD^{i}_{65}\ddot{a}^{i}_{65}}{N^{i}_{65}}\,. \tag{3}$$

The equation for the normal cost factor, in terms of varying rates of interest and inflation, thus has been reduced to an equation in terms of the base rate of interest i, with no inflation assumption. The right-hand side of the equation for the normal cost factor is also independent of duration, t.

Stated another way, the equation for the normal cost factor, $(NCF)_x$, is independent of the absolute level of the interest and inflation rates; rather, it depends solely upon their differential.

From the simplified model we now proceed to explore the effects of relaxing some of the assumptions which were made earlier. In the process some of the considerations which would be used in the selection of interest and inflation assumptions will become apparent.

LONG-TERM INVESTMENTS

Earlier we assumed that the pension fund was reinvested annually in short-term securities that always provide a net rate of return i after allowance is made for inflation. Ignoring for the time being the question of what is meant by the term "inflation," we may examine some of the consequences of relaxing this assumption.

First, it is clear that most investments of a pension fund are not risk-free. The market value of nearly any investment fluctuates with prevailing market conditions. A pension fund with a net cash inflow over a substantial period of time is in a position to accept a sizable degree of investment risk. On the other hand, increased risk normally will be reflected in a higher investment yield after allowance for inflation. Thus the inflation-adjusted investment yield will tend to be higher as more risk is assumed but will also tend to fluctuate more. This fluctuation can be smoothed out by use of an appropriate asset valuation method, with any remaining fluctuation being reflected in the form of actuarial gains and losses to the plan.

Second, the fact that most pension funds are heavily invested in long-term rather than short-term securities does not necessarily affect the overall results of the simplified model to any great extent. Long-term, like short-term, investments include an allowance for inflation expected over the period of investment. Thus the concept of using an assumed base rate of interest, after allowance for inflation, would seem to be as applicable to long-term investments as to short-term investments. To the extent that the allowance for inflation built into the long-term yields is imperfect, there will be gains or losses to the pension plan similar to the short-term investment situation.

Third, the rate of return after inflation will tend to fluctuate with general economic conditions even in the absence of investment risk. Two important factors in this regard are the overall growth rate of the economy and the growth rate of the money supply. The importance and impact of these factors will vary by type of security. Thus, general economic considerations will affect the choice of a "base rate of interest" for use in valuing plan liabilities and costs.

It would appear that the overall results of the simplified model are usable if sufficient care is taken in selecting an appropriate differential between interest and inflation assumptions. In making this decision, the actuary should take into consideration the degree of investment risk, the distribution of asset maturities, and the overall economic conditions. To absorb fluctuations from the assumed base rate of interest, the actuary can fall back on a prudent selection of asset valuation method and the offsetting of actuarial gains and losses from year to year.

INFLATIONARY AND NONINFLATIONARY WAGE INCREASES

The simplified model assumes that all wage increases are inflationary and correspond exactly to increases in prices. In relating this assumption to the real world, we immediately come up against the ambiguous nature of the term "rate of inflation."

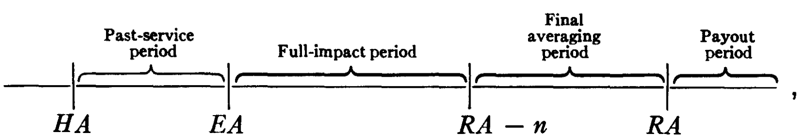
How fast wages and prices are increasing depends upon one's vantage point within the economy. Depending upon one's point of view, wages and prices may seem to be rising more or less rapidly than popular indexes of "inflation." Thus each individual's measure of inflation will tend to be highly subjective. This applies to employers as well as individuals. It is highly unlikely that, from any given vantage point within the economy, the rates of inflation reflected in wages, prices, and interest rates will exactly coincide.

Furthermore, there are many factors besides inflation that determine the rate at which wages increase. For any one individual, increases in salary due to merit may be large or small. Increases and decreases in productivity can have large effects on wages paid in specific industries as well as in the economy as a whole. Supply-and-demand forces in the labor market can drive wages up or down.

In selecting the desired differential between interest and inflation assumptions, the actuary should consider all these factors. Some of these factors will be reflected in terms of a salary scale other than the compound interest variety.

EFFECT OF PLAN PROVISIONS

The underlying mechanism by which plan provisions may affect the overall problem may be seen by diagraming the progressive stages through which a given participant progresses during his lifetime:



where HA is age at hire, EA is age at entry into the plan, n is the number of years in the final averaging period, and RA is age at retirement.

In the simplified model, the "payout" period was characterized by annual cost-of-living increases in retirement benefits. Removing the cost-of-living provisions from the pension plan also removes the inflation assumption from the valuation of benefits after retirement. The differential between interest and inflation assumptions thus becomes irrelevant, and the absolute level of the interest assumption is of paramount importance.

The assumptions of the simplified model implied a nonexistent "final averaging period." Where benefits at retirement are based upon the average salary over n years prior to retirement, there will be a decelerating effect upon benefit increases due to inflation during these final n years. If, prior to the beginning of the final averaging period, interest earnings in excess of the base rate of interest i exactly offset benefit increases due to inflation, then excess interest earnings will more than offset these inflationary increases in benefits following the commencement of the final averaging period. Therefore, whenever there is some type of final averaging period (including career average), then the absolute levels of the interest and inflation assumptions become important.

The simplified model made use of a benefit formula which produced benefit increases in the same proportion as salary increases during the "full-impact period." Other types of benefit formulas include "dollars times years of service" formulas, where the retirement benefit does not depend upon salary at all, and step-rate integrated formulas, where the retirement benefit may increase at a higher rate than the rate of increase in salary. In these latter types of formulas, the absolute levels of the interest and inflation assumptions are of critical importance.

It is important to note that the entry age normal cost method, used in the simplified model, makes retroactive use of the valuation assumptions. These retroactively applied assumptions may or may not coincide with the actual type of benefit provided for the "past-service period." Thus, in setting the assumptions for a given pension plan valuation, the actuary should first compare the provisions of the plan with those of the simplified model. If they are the same, then the absolute levels of the interest and inflation assumptions are of no consequence and the actuary need only select an appropriate differential between them. Needless to say, this greatly simplifies the process of selecting assumptions.

On the other hand, where the formula for retirement benefits is, say, $\$X$ times years of service, with no cost-of-living increases before or after retirement, then the absolute level of the interest assumption becomes very important.

In between these two extremes, one can imagine a plan which provides, say, f per cent of final n-year average salary for each year of credited service, with no cost-of-living increases in benefit after retirement. Here the absolute levels of the interest and inflation assumptions are unimportant prior to the attainment of the final averaging period; the differential between these assumptions is the crucial factor. However, during the final averaging and payout periods, the absolute levels of these assumptions are of paramount importance. The longer these two periods are, the more important are the absolute levels of the assumptions. Of the two periods, the payout period usually will be the most influential in selecting valuation assumptions.

For those participants who are many years from retirement, the absolute levels of the assumptions relating to interest and inflation are irrelevant during the full-impact period so long as the differential is reasonably correct. The final averaging and payout periods will be reached so far into the future that a reasonable set of ultimate assumptions will suffice for these periods.

For those participants who are already within the final averaging or payout period, or for those who will reach one of these periods in just a few years, the absolute levels of the interest and inflation assumptions are of more immediate importance. Here the actuary must attempt to set realistic assumptions, possibly varying them according to groupings of participants by age. However, in this case we are dealing with events occurring during the more immediate future, where the actuary may have more confidence in predicting interest rates and rates of inflation.

Analyzing this specific situation, the appropriate technique appears to be to use ultimate assumptions, with an appropriate differential between them, for those who are many years from retirement, phasing into realistic assumptions for those participants who have reached the final averaging and payout periods. A similar analysis applied to other combinations of plan provisions would lead to similar types of conclusions with regard to the levels of assumptions and the differential between them. In some cases, realistic assumptions would be necessary; in others, merely selecting an appropriate differential together with a good set of ultimate assumptions would prove to be satisfactory.

CONCLUSION

The selection of appropriate assumptions for anticipated rates of interest and inflation is more difficult than many people might imagine. It is not a question of merely selecting any set of assumptions so long as the differential between them meets certain criteria. Even if this were true, the selection of the appropriate differential is often a complex and difficult process.

On the other hand, the selection of these assumptions can be less difficult than others might imagine. After analyzing a given set of plan provisions, the actuary often will conclude that a good set of ultimate assumptions, together with an appropriate differential between them, will be sufficient for the task at hand. Any effort directed toward obtaining a realistic level of interest and inflation assumptions will, in these cases, be of limited value.

The indiscriminate use of rules of thumb is as inappropriate to the valuation of pension plans as is the unyielding insistence upon the use of realistic assumptions. As always, the actuary must analyze each situation in order to determine which of the available techniques will produce the best results with the least effort, given the degree of accuracy required.

DISCUSSION OF PRECEDING PAPER

BARNET N. BERIN:

I suggest that the author supplement his paper by making some cost calculations involving a career average pension plan, an updated career average pension plan, and a final pay pension plan, including a salary scale in these cost calculations.

He should choose a recognized funding method under ERISA and an amortization period. Then he should construct a summary cost table for each of the three pension plans. The following tabulation presents an example, in this case for a career average pension plan.

Valuation Interest Rate	Salary-Scale Rate*						
	2%	3%	4%	5%	6%	7%	8%
5%......	100%†	103%	107%	110%	114%	117%	121%
6%......	....	89	93	96	100	103	107
7%......	....	....	80	83	87	90	94
8%......	....	....	....	72	74	78	81

* For convenience assume a compound interest scale.

† Basic cost index.

The pattern of cost indexes for the other two tables will be similar.

Examine the *constant* positive or negative differences of interest rate less salary-scale rate, by moving down a diagonal. For any constant difference, positive or negative, the costs decrease as the interest and salary-scale rates increase.

Examine the situation where the valuation rate equals the salary-scale rate. The costs decrease as the interest and salary-scale rates increase.

The same increase added to both the interest rate and the salary-scale rate lowers costs. (This would be an increase, for example, of 1 per cent to each rate, say from a 5 per cent interest rate and a 2 per cent salary-scale rate to a 6 per cent interest rate and a 3 per cent salary-scale rate.)

We can consider the cost significance of a postretirement benefit increase where the increase is equal to the salary-scale rate. For example, in simplest terms, it is possible to conclude that a 5 per cent valuation interest rate and a 2 per cent salary-scale rate with a postretirement benefit increase of 2 per cent is roughly equivalent to a 3 per cent valuation interest rate. It is possible to consider a series of such combinations (see the accompanying tabulation).

Valuation Interest Rate (1)	Salary-Scale Rate (2)	Postretirement Benefit Increase (3)	Revised Valuation Interest Rate (4)
5%.....	2%	2%	2.94%
6%.....	3	3	2.91
7%.....	4	4	2.88
8%.....	5	5	2.86

Even if you are impressed with the right-hand column and are ready to value at 2.9 per cent, there should be a restraining influence. Columns 1 and 2 are *estimates* that are subject to the testing of emerging experience by means of the actuarial gain and loss analysis. The revised valuation interest rate depends upon the interaction of two estimates, each subject to its own experience gains and losses. Furthermore, in the real world, it is most unlikely that postretirement increases would be constant. The indexing of retirement benefits, and attention to the differences between the valuation interest rate and the salary-scale rate, do not simplify the problems of choosing valuation assumptions.

I have trouble understanding the conclusion of this paper: "The selection of appropriate assumptions for anticipated rates of interest and inflation is more difficult than many people might imagine. . . . On the other hand, the selection of these assumptions can be less difficult than others might imagine. After analyzing a given set of plan provisions, the actuary often will conclude that a good set of ultimate assumptions, together with an appropriate differential between them, will be sufficient for the task at hand." How do we recognize a good set of ultimate assumptions and an appropriate differential?

In addition to questioning the basic proposition of this paper, I find it disappointing that the author does not mention that the actuary's assumptions, whatever they may be, are susceptible to testing on a regular annual basis, by means of the actuarial gain and loss analysis. This testing and occasional changing of assumptions are the essence of pension mathematics.

THOMAS P. BLEAKNEY:

Mr. Kischuk's paper, it is to be hoped, opens the door for a wider discussion of the effects of inflation on the paired actuarial assumptions of investment return and salary growth. My contribution to that discussion was induced by the statement in the introduction to the paper that "the truth lies somewhere in between the two opposing viewpoints." The viewpoints referred to here might be capsulized as the "explicit" and "implicit" methods of recognizing the effect of inflation, or as the "absolute" and "offset" assumptions.

I feel that the casual reader of the paper might get the wrong impression from the quoted statement. Later in the paper Mr. Kischuk clarifies when implicit assumptions are theoretically defensible and when they are inappropriate. What might need more emphasis, in my opinion, is the fact that a proper choice of explicit assumptions will fit his models in *all* cases, while no set of implicit assumptions will be equally acceptable.

This leads to the obvious advantage of proper explicit assumptions. Their use gives universally correct actuarial results, while implicit assumptions are equally accurate only under limited circumstances. Perfect prescience would give rise to an array of explicit assumptions that would be usable for all plans. Given the same prescience, a different set of implicit assumptions would have to be derived actuarially from the explicit assumptions for each situation, depending on the plan provisions.

If the focus is narrowed to a single plan, there is presumably one set of explicit assumptions that is correct. Corresponding to that set is a family of implicit assumptions that will give the same actuarial results. A change in plan benefits will not change the explicit assumptions (except in such areas as retirement rates, which may be directly affected by the benefit change). However, the same change will give rise to a whole new family of implicit assumptions, because of the changed timing of the benefit accrual and cash flow. This phenomenon is a fundamental disadvantage of implicit assumptions.

From a practical standpoint, it is important to note how very limited is the situation where implicit assumptions involving a differential rate may be chosen by a simple division involving the rate of interest and the rate of inflation assumed in the salary scale. As Mr. Kischuk demonstrates, precisely this situation occurs only in a final salary (not final *average* salary) program with a salary-related postretirement increase provision.

This leads to another practical thought for the actuary who strives for conservative errors in the era of ERISA. For the majority of plans using a final average salary formula, the blind use of the differential approach results in an overstatement of costs, due to the use of a conservative annuity "purchase rate" at retirement. That conservatism may be warranted if, for example, a long-term downward trend in interest rates is expected. Such an expectation causes some actuaries to use lower interest assumptions after retirement than before. The differential approach works well in such circumstances. For all years, the interest rate can be set equal to the expected ultimate (postretirement) rate, and the salary inflation factor can be set equal to that interest rate less the expected differential.

The use of the preretirement differential rate for all years is much less conservative in a program involving a postretirement increase tied to the consumer price index. Nevertheless, the results are still conservative if interest rates and salary scales are accurate and the CPI grows at a lower rate than the salary growth rate used in determining the differential. The major threat to conservatism arises from a recognition of all or most of the effect of inflation on investment return, without a corresponding recognition of the inflation factor in salaries or postretirement increases.

Although the implicit approaches just described may have some mechanical advantages in actually carrying out a valuation, it would seem that a forthright use of the explicit rates actually expected would be preferable. Nevertheless, there appears to be a substantial amount of delay within the actuarial profession in adopting such assumptions. Perhaps this is because of the belief that implicit assumptions are inherently conservative in most situations. Even when that conservatism may not be understood, there is also the understandable reluctance to adopt interest assumptions approaching today's record high levels, both because of their unprecedented nature and because of the disasters recently experienced in most stock portfolios. A minor contributing factor to this reluctance might even be semantic; *explicit* tends to connote precision, and what actuary takes any comfort in an extended prediction regarding the effect of inflation on any assumption?

Despite all these objections, I fear the credibility gap between the actuary and his various publics will grow if we persist in using assumptions that are not readily defensible, on their face, in those areas where the public has some commonsense comprehension.

HOWARD J. BOLNICK:

The effect of inflation on the cost of a pension plan varies significantly depending on plan design. Unless the actuary is aware of the effect of inflation on a particular plan, it is easy to misstate the pension cost. Misstatements are particularly easy to make where short cuts have been taken by offsetting various assumptions to calculate pension costs. Mr. Kischuk has done a valuable job of uncovering some implicit assumptions and demonstrating the consequences of offsetting actuarial assumptions.

With a few minor changes in Mr. Kischuk's model, a more generalized statement of inflation's effect on funding can be demonstrated. His equation (2) can be reformulated from

$$(NCF)_{[x]+t} = \frac{(65 - x)f \; {}^{S}D_{[x]+(65-x)} \; {}^{S}\ddot{a}_{[x]+(65-x)}}{{}^{S}N_{[x]}}$$

to

$$(NCF)_{[x]+t} = (65 - x)f \; {}^{S}\ddot{a}_{[x]+(65-x)} \frac{1}{{}^{S}\ddot{s}_{[x]:(65-x)}} . \tag{1}$$

This equation may be interpreted as follows: the normal cost in a given year is a function of three items: (1) the benefit amount (first term of right-hand side of equation); (2) the benefit form (second term); and (3) the funding pattern (third term).

Note that by using life annuities, as Mr. Kischuk does, we take into account forfeitures resulting from participant deaths. Interest-only annuities would include no benefit forfeitures; death and withdrawal annuities include all forfeitures.

Now let us make some basic economic assumptions:

1. Nominal wages equal real wages plus inflation.
2. Nominal interest rates equal real interest rates plus inflation.
3. Real wages are constant.
4. Real interest rates are constant.

Let

r_t = Nominal interest rate in year t ;

w_t = Nominal wages in year t ;

i_t = Real interest rate in year t ;

s_t = Real wages in year t ;

i'_t = Inflation rate in year t .

Then the economic assumptions can be expressed as

$$w_t = s_t + i'_t , \quad r_t = i_t + i'_t ,$$
$$s_t = s \quad \text{for all } t , \quad i_t = i \quad \text{for all } t .$$

These mathematical terms correspond to Mr. Kischuk's assumptions, with the addition of a term for real wages.

The effect of inflation on pension costs can be calculated using Mr. Kischuck's method of analyzing annuity commutation functions. For a given pension plan, the funding and benefit annuities may or may not be a function of wages. If wage increases are germane,

$$\begin{aligned} {}^{S}D_{[x]+t} &= s_{[x]+t}D_{[x]+t} \\ &= s_{[x]+t}v^{x+t}l_{[x]+t} . \end{aligned} \tag{2}$$

Now

$$s_{[x]+t} = \prod_{t=0}^{x+t-1} (1 + w_t) \,,$$

$$(1 + w_t) = 1 + (s + i'_t) \cong (1 + s)(1 + i'_t) \,,$$

and

$$v^{x+t} = \prod_{t=0}^{x+t-1} (1 + r_t)^{-1} \,,$$

$$(1 + r_t) = 1 + (i_t + i'_t) \cong (1 + i)(1 + i'_t) \,,$$

so

$$\begin{aligned} {}^{S}D_{[x]+t} &= \frac{\prod_{t=0}^{x+t-1} (1 + s)(1 + i'_t)}{\prod_{t=0}^{x+t-1} (1 + i)(1 + i'_t)} \, l_{[x]+t} \\ &= \frac{(1 + s)^{x+t}}{(1 + i)^{x+t}} \, l_{[x]+t} \,. \end{aligned} \tag{3}$$

On the other hand, if wage increases are not germane,

$$\begin{aligned} D_{[x]+t} &= v^{x+t} l_{[x]+t} \\ &= l_{[x]+t}\left[(1 + i)^{x+t} \prod_{t=0}^{x+t-1} (1 + i'_t)\right]^{-1} . \end{aligned} \tag{4}$$

A close look at the above commutation functions reveals the basic economic and demographic variables that affect the cost of any pension plan: (1) real wages (s); (2) real interest rates (i); (3) forfeitures for death and withdrawal (l); and (4) inflation (i'_t). These variables affect pension costs through their direct effect on the benefit form and the funding pattern.

Refer back to the components of equation (1) and equations (3) and (4). We see that, for the benefit form,

$$\begin{aligned} {}^{S}\ddot{a}_{[x]+(65-x)} &= \left(\sum_{t=[x]+(65-x)}^{\infty} {}^{S}D_t\right) \Big/ {}^{S}D_{[x]+(65-x)} \\ &= \left[\sum_{t=[x]+(65-x)}^{\infty} \frac{(1 + s)^{x+t}}{(1 + i)^{x+t}} \, l_{[x]+t}\right] \Big/ {}^{S}D_{[x]+(65-x)} \\ &= f(s, 1/i, l) \end{aligned} \tag{5}$$

if wage increases affect the benefit form. That is, the benefit form is directly related to real wages, indirectly related to real interest rates, and directly related to forfeitures. Likewise, for the funding pattern affected by wages,

$$\frac{1}{{}^{S}\ddot{s}_{[x]:(65-x)}} = {}^{S}D_{[x]+(65-x)}\left(\sum_{t=[x]}^{[x]+65-x} {}^{S}D_t\right)^{-1}$$
$$= f(1/s, i, 1/l) \,. \tag{6}$$

(Note: ${}^{S}D_{[x]+(65-x)}$ cancels out of eqs. [5] and [6] when substituted in eq. [1].)

If either the funding pattern or the benefit form is unaffected by wages, the corresponding annuities are related to commutation functions without wage scales (i.e., substitute in eq. [4]):

$$\ddot{a}_{[x]+(65-x)} = f(1/i, 1/i'_t, l) \tag{7}$$

and

$$\frac{1}{\ddot{s}_{[x]:(65-x)}} = f(i, i'_t, 1/l) \,. \tag{8}$$

By simply substituting the appropriate functional form of the funding and benefit annuities (eqs. [5]–[8]) and recognizing the impact of inflation on the benefit amount, the actuary can determine, among other things, how inflation affects the cost of a particular pension plan.

This type of analysis, however, is dependent on the economic assumptions made to simplify the mathematics. In particular, the assumption that real interest rates are constant over time is at best an unlikely assumption. When an actual pension portfolio is analyzed to determine the pension cost, the assumption that real interest rates are constant means that the pension fund earns a constant real fund yield. Even if real new-money interest rates are, in fact, constant, it is highly unlikely that a pension fund, having been invested over a period of years, will maintain a constant real fund yield.

A simple set of examples should suffice to demonstrate this point. Three pension funds have different cash flows over a ten-year period:

A \$1,000 first-year cash flow decreasing by \$100 per year;
A level \$1,000 cash flow per year;
A \$1,000 first-year cash flow increasing by \$100 per year.

Two different sets of nominal new-money interest rates are assumed over the ten-year period: the first increasing and then decreasing, the second decreasing and then increasing. The nominal fund yields and real fund yields for each year are presented in Tables 1 and 2. Real fund yields are simply the nominal fund yields less inflation. Even if the nominal new-money interest rate each year is equal to, say, a constant 3 per cent real new-money interest rate plus inflation, the tables show that the real fund yield will not be level.

If real new-money interest rates decline with increasing inflation and rise with decreasing inflation, as is more likely the case, the pattern of real fund yields will be even more distorted. As inflation accelerates, the real fund yield drops; as inflation decelerates, the real fund yield increases. Over the ten-year inflation cycle, none of the funds can avoid the ravages of inflation and earn a constant real fund yield.

Refer back to equation (3). If the assumption of a constant real interest rate (i.e., a constant real fund yield) is not true, the equation cannot be simplified. Inflation will, then, actually affect the pension costs of the

TABLE 1

Year	Nominal New-Money Interest Rate	Nominal Fund Yield		
		Decreasing Cash Flow	Level Cash Flow	Increasing Cash Flow
1	4%	4.00%	4.00%	4.00%
2	5	4.48	4.51	4.53
3	6	4.96	5.03	5.09
4	7	5.43	5.56	5.66
5	8	5.89	6.11	6.26
6	7	6.05	6.28	6.42
7	6	6.04	6.23	6.34
8	5	5.94	6.04	6.09
9	4	5.78	5.76	5.75
10	3	5.59	5.41	5.33

Year	Real New-Money Interest Rate	Inflation	Real Fund Yield*	Year	Real New-Money Interest Rate	Inflation	Real Fund Yield*
1	3%	1%	3.00%	1	3.00%	1.00%	3.00%
2	3	2	2.51	2	2.50	2.50	2.01
3	3	3	2.03	3	2.00	4.00	1.03
4	3	4	1.56	4	1.50	5.50	0.06
5	3	5	1.11	5	1.00	7.00	(0.89)
6	3	4	2.28	6	1.50	5.50	0.78
7	3	3	3.23	7	2.00	4.00	2.23
8	3	2	4.04	8	2.50	2.50	3.54
9	3	1	4.76	9	3.00	1.00	4.76
10	3	0	5.41	10	3.00	0	5.41

* Level-cash-flow portfolio.

TABLE 2

Year	Nominal New-Money Interest Rate	Nominal Fund Yield: Decreasing Cash Flow	Level Cash Flow	Increasing Cash Flow
1.......	8%	8.00%	8.00%	8.00%
2.......	7	7.51	7.48	7.46
3.......	6	7.02	6.95	6.90
4.......	5	6.45	6.42	6.32
5.......	4	6.08	5.88	5.73
6.......	3	5.64	5.33	5.15
7.......	4	5.45	5.12	4.92
8.......	5	5.41	5.10	4.94
9.......	6	5.45	5.22	5.10
10......	7	5.55	5.44	5.39

Year	Real New-Money Interest Rate	Inflation	Real Fund Yield*	Year	Real New-Money Interest Rate	Inflation	Real Fund Yield*
1....	3%	5%	3.00%	1....	1.00%	7.00%	1.00%
2....	3	4	3.48	2....	1.50	5.50	1.98
3....	3	3	3.95	3....	2.00	4.00	2.95
4....	3	2	4.42	4....	2.50	2.50	3.92
5....	3	1	4.88	5....	3.00	1.00	4.88
6....	3	0	5.33	6....	3.00	0	5.33
7....	3	1	4.12	7....	3.00	1.00	4.12
8....	3	2	3.10	8....	2.50	2.50	2.60
9....	3	3	2.22	9....	2.00	4.00	1.22
10...	3	4	1.44	10...	1.50	5.50	(0.06)

* Level-cash-flow portfolio.

three pension funds. The simplified model may be helpful to the actuary projecting pension costs, but he should not forget the pitfalls inherent in using economic simplifications to obtain mathematical simplifications.

HERBERT L. FEAY

My discussion will be limited primarily to the basic problem expressed in the first sentence of Mr. Kischuk's introduction, the relationship between interest and inflation assumptions to be used for pension plan valuations. The basic issue has been whether or not an interest rate assumption can be combined with an inflation salary index assumption to provide one net rate for valuation purposes.

The usual measure of inflation is the consumer price index ratio. As shown in studies by economists, there is, in the long run, a substantial correlation between consumer prices, interest rates, and salary or wage scales. This is discussed, for example, in a paper published December 14, 1970, by the Congressional Research Service of the Library of Congress. The paper is entitled "Inflation and Interest Rates—Analysis of the 1960's and Some Interpretations for the 1970's" and was written by Asher Achinstein, senior specialist on price economics for the Library.

Mr. Kischuk assumes inflationary increases in pension benefits after retirement and in salaries before retirement. This is an assumption that is not applicable to most plans today. Very few plans provide for automatic increases in pension benefits related to the consumer price index, and the inflationary effects on pension payments probably differ from the effects on salaries. If pensions of retired lives are indexed, the effects of inflation on valuation assumptions should be treated separately for active lives and retired lives. As for salaries, Mr. Kischuk points out that these increase for reasons other than inflation. Also, the inflationary effect on expenses is not included in the problem covered by the paper.

Inflation affects the cost of pension plans before retirement because of its effect on salaries. Obviously, salaries affect costs only if annual normal costs and payments, and the unfunded liabilities, are expressed as percentages of future annual compensation, and if pension benefits are based on average annual compensation before retirement. If the costs and benefits are not related to salaries, salary rates will not be needed for valuation purposes.

Mr. Kischuk's formula for relating the assumed rate of interest for investments, the net annual rate of interest for the valuation, and the salary index rate can be changed to the form I used in *The Actuary*. Using his defined symbols, the net valuation interest rate can be expressed as follows:

$$(1 + i) = (1 + r_t)/(1 + i_t) .$$

Mr. Kischuk assumes that the net valuation rate (which he calls the base rate of interest) remains constant. The annual values of r_t and i_t may vary, but only as permitted by this formula. In other words, if both $(1 + r_t)$ and $(1 + i_t)$ are increased by the factor $(1 + n)$, there is no change in $(1 + i)$. The corollary of this is that, since salary and interest are both increased by approximately equivalent inflationary index rates, the value of $(1 + i)$ is more stable than either $(1 + r_t)$ or $(1 + i_t)$. I again emphasize that this applies only to the determination of present values of future salaries, and of present values of future pensions granted at retirement where pension payments are based on average salaries received before retirement.

The changed expression for the relationship of i, i_t, and r_t simplifies the actuarial formulas and functions. The calculations with net valuation rate i are the same as if there were no inflation factors, except for the changes needed because the same net rates cannot be continued for valuation of pension benefits after termination of employment. I use the expression "termination of employment" advisedly because termination can occur on dates other than normal retirement dates, with pension benefits granted as either immediate annuities or deferred annuities.

As stated in the paper, the development assumes that pension payments are based on the final salary for the year commencing at the date of retirement. I find that the easiest way to change to an average salary is to multiply the pension based on the final salary by a ratio. For example, if the pension is based on the average annual salary for the last five years, the factor is

$$(s_{y-5} + s_{y-4} + s_{y-3} + s_{y-2} + s_{y-1})/5s_y ,$$

where y is retirement age.

As pointed out by Mr. Kischuk, the past salary history is not likely to agree with the backward extension of the current salary rate. The same condition can exist for other items, such as mortality, disability, termination (other than mortality and disability), retirement, and expenses. I see no essential difference in handling experience gains and losses and changes in standards for future valuation for any of the basic factors, including salary scales. The net valuation interest rates probably are more representative of experience than are the termination rates for causes other than death or disability. For example, I wonder about the accuracy of the liability for vested benefits calculated with aggregate attained-age factors when the benefits are determined by years of service. The error involved in this approximation must exceed any error involved in combining salary increase rates with investment income rates for the period of active employment.

I wonder what those who object to combining interest and inflation propose to do regarding inflation. Ignoring inflation is to make the assumption that inflation will be at a zero rate. Under present conditions, this can be more dangerous to the solvency of a pension plan fund with salary-related contributions and benefits than using combined salary inflation rates and interest rates before retirement of currently active employees.

It is not wise to ignore inflation and to use the excuse that any error in past assumptions can be corrected by changes in future normal costs. Why should an actuary expect that an employer would be willing to pay a constantly increasing percentage of payroll into a pension plan fund because the actuary did not allow for future inflation? Will the actuary's report provide an accurate and understandable explanation of the facts regarding inflation and pension costs?

In the paper, Mr. Kischuk has considered only pensions that are determined by one percentage of final salary. The problem is more complicated if the pension is related to final salary by two factors, as in the case of integrated pensions. My solution is to make two valuations. One is based on a percentage of final salary, and the other is based on a percentage of the final salary up to the present value of the integrated amount. For example, if the pension amount for one year of service is 1 per cent of the first \$600 of monthly salary plus 2 per cent of the monthly salary in excess of \$600, I make a valuation assuming 2 per cent of the total monthly salary, and a second valuation assuming 1 per cent of the present equivalent value of the \$600 or of the current monthly salary, whichever is less. The results of the second valuation are subtracted from the results of the first valuation. The present equivalent value of \$600 at attained age $x + n$ is $\$600/(1 + i_t)^n$, or the equivalent multiplications of $(1 + i_t)$ if i_t is a variable.

(AUTHOR'S REVIEW OF DISCUSSION)

RICHARD K. KISCHUK:

I would like to thank Messrs. Berin, Bleakney, Bolnick, and Feay for their very able assistance in exploring further the topic of interest and inflation assumptions as used in pension plan valuations. Their discussions have raised a number of interesting points and questions for further discussion.

Explicit versus Implicit Assumptions

There has been a tendency for some actuaries to insist upon using explicit assumptions in all cases. Other actuaries tend to rely upon an artificially low interest rate assumption to compensate for an inadequate provision for inflation in the assumptions used to project costs.

As Mr. Bleakney points out, a proper choice of explicit assumptions should provide a reasonable estimate of costs in all cases. On the other hand, use of implicit assumptions will not produce a reasonable projection of costs in every case.

It does seem significant, however, that there are situations where the use of implicit assumptions *will* produce good results. Accordingly, in order to evaluate the use of implicit assumptions in any given instance, it is important to know when implicit assumptions will produce good results and when they will not.

The "simplified model" as defined in the paper provides the perfect opportunity for the use of the implicit approach to inflation and interest assumptions. In actual practice, few, if any, plans would be expected to match the specifications of the simplified model exactly. Also, the discussion of the simplified model employs certain assumptions about the economy. These assumptions would probably never be met exactly in the real world.

Mr. Berin recognizes this fact by pointing out several instances of types of plans where the implicit approach does not produce exact results. However, there are a number of situations that come close enough to the specifications of the simplified model that the implicit approach can be used with only minor modifications. Both Mr. Bleakney and Mr. Feay have pointed out some useful techniques in this regard. In general, a modification of the implicit approach is most useful when one is considering nonintegrated final average salary plans. For other types of plans, use of the implicit approach or modifications of this approach will usually lead to significant distortions in projected plan costs.

As Mr. Bleakney has indicated, this leads to the conclusion that the implicit approach is useful only in a fairly limited number of situations. However, in those plans for which this approach is usable, it may provide significant mechanical advantages in carrying out a valuation. And, because of the popularity of final average salary plans, the approach is probably usable for a larger percentage of plans than one might at first expect.

The main disadvantage to the use of the implicit approach, even where it does apply, is that a small change in plan provisions will often make it unusable. For example, if one is valuing a plan that is based upon a benefit of 50 per cent of final five-year average salary, the implicit approach can generally be applied. But if, next year, a minimum benefit of $500 per month is added to the plan, the implicit method probably will no longer be usable. The actuary is then forced into either making an elaborate modification of the implicit method or simply using the explicit approach.

The implicit approach to interest and inflation assumptions can be a very useful technique, but in using it the actuary must be keenly aware of the pitfalls and limitations.

Differential versus Absolute Level

It is very important for the pension actuary to have a good understanding of the relative importance of the absolute levels of the interest and inflation assumptions, and the differential between them, when making pension cost projections. The "simplified model" developed in the paper defines a situation in which the differential between interest and inflation assumptions is all-important; the absolute level of these two assumptions is totally unimportant. If we consider a plan that provides a flat pension of $500 per month, with no cost-of-living increases, we find that the absolute level of the interest assumption becomes all-important. If we consider a step-rate plan integrated at the $6,600 level, the absolute levels of both interest and inflation assumptions become all-important.

An understanding of this aspect is important in discussing the implications of various sets of interest and inflation assumptions with a client. In some situations a 1 per cent increase in both interest and inflation assumptions will be approximately offsetting. In other cases, a 1 per cent increase in both assumptions may increase projected costs or decrease them. Given a good understanding of the interrelationship of interest and inflation assumptions, it is usually possible for the actuary to predict in which direction costs will change, given the basic plan provisions. Given a few general rules of thumb, it is often possible to give a good estimate of the percentage change in projected costs that such a revision in assumptions might create.

Such techniques can obviously be great time savers. The basic approach underlying these techniques is to analyze a given plan in terms of the simplified model. The more closely a given plan resembles the simplified model, the less effect the absolute values of the interest and inflation assumptions will have on projected plan cost once the differential between them is established.

Restating Implicit Assumptions

Mr. Bleakney objects to the use of implicit assumptions on the grounds that "the credibility gap between the actuary and his various publics will grow if we persist in using assumptions that are not readily defensible, on their face, in those areas where the public has some commonsense comprehension." On this point, I agree wholeheartedly. It is very difficult to explain to a client why a 3 per cent interest assumption was used, with no salary scale, when interest rates are in excess of 9 per cent and inflation rates are in excess of 6 per cent.

The implicit approach to valuation of a pension plan is often very advantageous from a mechanical point of view. But for purposes of communication to the client, it is probably preferable to restate the implicit assumptions in terms of some reasonably realistic explicit assumptions, even though the projected costs themselves are changed little if at all.

There is an additional reason for restating assumptions besides communication to the client. This lies in the widespread use of the valuation rate of interest in actuarial gain and loss analysis, and in calculating such items as the maximum tax deduction, the funding standard account minimum contribution, and the pension expense for accounting purposes. There would seem to be little justification for the use of a 3 per cent net valuation rate in these calculations. Instead, a 3 per cent net valuation rate should be restated in terms of realistic interest and inflation rates, such as 7 and 3.88 per cent, respectively.

As Mr. Bleakney has mentioned, failure to restate implicit assumptions in terms of explicit assumptions can lead to serious credibility problems for the actuary. As pointed out in the preceding paragraph, it can lead to serious technical problems as well.

Effect of Investment Rates

Mr. Bolnick points out quite accurately that many of the conclusions that could be drawn from the simplified model may not be valid if the real rate of return on pension fund investments is not considered to be constant.

As demonstrated in Tables 1 and 2 of Mr. Bolnick's discussion, the cash flow of the pension fund can have much to do with the average yield of the fund. Decreasing, level, and increasing cash flows will each have a different impact, given the same available rates of investment return. If the actuary has good reason to expect a certain kind of cash flow that would have a long-term effect on the average fund yield, this should be taken into account in setting the interest assumption for the valuation.

However, there may be good reason to assume that the effects would balance out over long periods. For example, in Mr. Bolnick's Table 1, the three funds all start out at a 4.00 per cent average yield. After five years, with interest rates at 8 per cent, the fund with increasing cash flow enjoys a higher yield (6.26 per cent) than either of the other two funds. After nine years, with interest rates back at 4 per cent, the funds are all substantially even again. After ten years, with interest rates at the 3 per cent level, the fund with the decreasing cash flow has gained the advantage and enjoys a higher average yield (5.59 per cent) than either of the other two funds.

Table 2 shows similar results. By the end of the sixth year, interest rates have declined from 8 to 3 per cent, and the fund with declining cash flow enjoys a substantial advantage over the funds with level and increasing cash flow. Interest rates rise again to 7 per cent by the end of the tenth year, and the fund with increasing cash flow is on the verge of overtaking the fund with decreasing cash flow in terms of average investment yield.

If the plan experiences a prolonged period of increasing or decreasing interest rates, which lasts for a substantial portion of the average remaining working lifetime of current plan participants, it becomes fairly unlikely that the effects of increasing, level, or decreasing cash flow will cancel out. It is significant that the difference between the highest and the lowest funds in Mr. Bolnick's examples is never very much greater than $\frac{1}{2}$ per cent, in spite of some large fluctuations in interest rates over a ten-year period.

In addition to cash flow, it is important to consider whether interest rates and inflation rates move up and down together. In considering this point, it is assumed for the moment that bonds are all valued at amortized cost and that stocks are valued using one of the methods that smooth out fluctuations in market value. On this basis, over the long run, investment yields have shown a tendency to increase as inflation has increased, which is consistent with the assumptions of the simplified model.

There are additional aspects of the problem. In times of rising inflation, interest rates may show a lag in recognizing increased expectations of future inflation. Similarly, in times of declining inflation, interest rates may show a lag in recognizing a much lower expectation of future rate of inflation. At other times, interest rates may overreact, up or down, to short-run changes in inflation rates.

All of the above are valid points to consider. However, it is difficult to build them into the valuation of a pension plan. Unless the actuary has good reason to expect a substantial impact from one or more of these factors, it may be best to assume that the effects will cancel out over the long run.

Actuary's Role in Disclosing the Effects of Inflation on Plan Costs and Benefits

In his discussion Mr. Feay has raised a very interesting question: "Will the actuary's report provide an accurate and understandable explanation of the facts regarding inflation and pension costs?" A related question is, "How far should the actuary go in this regard?"

For example, the simplified model produced a "normal cost factor," which was then applied to the current salary to calculate the current year's normal cost. In computing the normal cost factor, we assumed that salaries would rise each year consistent with an inflation assumption.

Since, in the absence of actuarial gains or losses, the normal cost factor remains constant, we are really projecting that the normal cost expressed in current dollar terms will rise each year at the assumed rate of inflation.

But suppose that projected costs are converted to "real dollars." If this is done, one finds that the normal cost is actually expected to remain *constant* when expressed in real-dollar terms. A statement to this effect included in the actuary's report may be of vital interest to many clients.

As the only person who fully understands the calculations underlying a given valuation, the actuary is in a unique position to restate projected future costs in real-dollar terms consistent with the inflation assumptions that are built into the calculations. This can be very important in helping the plan sponsor to understand the real-dollar cost impact of a given pension plan.

Similarly, the actuary should take a more active role in helping plan sponsors to understand the real value of benefits that will be provided by the plan. For example, career average and fixed-dollar benefits have to be updated continually in order to prevent inflation from eroding benefits. The same type of guidance can be given by the actuary regarding the impact of inflation on benefits to retirees where there are no postretirement cost-of-living adjustments. Even a modest 3 per cent rate of inflation has the effect of a 50 per cent reduction in benefits after twenty-four years have gone by. It is essential that actuaries provide the type of guidance that will help plan sponsors to adapt plan provisions and funding policies to the very difficult problems created by inflation.

Analysis of Gains and Losses

In his discussion Mr. Berin states that "the actuary's assumptions, whatever they may be, are susceptible to testing on a regular annual basis, by means of the actuarial gain and loss analysis." He further states that "this testing and occasional changing of assumptions is the essence of pension mathematics."

This reasoning is applicable to some types of actuarial assumptions. However, it would seem to be somewhat hazardous when applied to interest and inflation assumptions. Interest and inflation assumptions are economic assumptions and should be revised according to changes in the long-term economic outlook. The setting of interest and inflation assumptions is, thus, essentially a *prospective* process.

Actuarial gain and loss analysis is essentially a *retrospective* process. We look at the past history of the plan to see how actual experience has compared with the assumptions used to project costs. Actuarial gain and loss analysis may be a perfectly valid approach to setting such assumptions as mortality, turnover, and noninflationary salary scale. But in economics past history is not a good guide to the future. For example, an average of interest rates over the last ten years probably would not be a good estimate of the average interest rate over the next ten years.

The selection of interest and inflation assumptions is essentially a *prospective* process, which is not particularly related to the *retrospective* process of actuarial gain and loss analysis. For this reason, analysis of actuarial gains and losses was not considered in the context of this paper.