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INVESTMENT GENERATIONS REVISITED

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ABSTRACT

The principal purpose of this paper is to examine some of the factors that must be considered if the concept of investment generations is applied to the distribution of investment income within individual life insurance lines of business. The investigation is carried out by means of a simple model office designed to simulate the operations of mutual life companies using, under similar circumstances, the portfolio method and, alternately, the investment-generation method. The results obtained by the two approaches prove to be strikingly different.

I. INTRODUCTION

THERE has been considerable discussion in the past few years of the concept of applying investment generations to the distribution of investment income within individual life insurance lines of business. Model-office simulations of life company operations are used here in an attempt to seek answers to some of the fundamental questions surrounding such application. Evaluation of the results can be done only in terms of the somewhat philosophical or speculative considerations underlying investment-generation theory. There is set out first, therefore, one version of the rationale for this theory and its associated history.

II. GENERAL THEORY AND HISTORY

To the life insurance lawyer or the life insurance actuary, the concept of "class of insureds" is an elusive one. The intent of a mutual company is to furnish coverage to the buyer at cost, but not by charging each buyer the costs flowing from his own contract or by averaging the costs to produce a uniform charge for every buyer. What is sought is a middle ground providing for both a pooling of experience, with its associated benefits, and "equitable distinctions" in the costs developed for different classes of insureds. The cost distinctions may be reflected in the premium structure (including the underwriting classifications) and benefit structure of the contracts, or in the apportionment of surplus.

In some respects, the criterion described as "equitable distinctions" is a comfortable one. Everyone agrees that it is proper for each major line of business within a company to bear its own mortality or morbidity experience and its own direct or allocated expenses. Mortality distinctions by age are universal. Individual policies usually are charged with their own specific field sales and service costs, although there may be differences with respect to what costs, beyond pure commissions, are included in this category. By contrast, although differences in female mortality and morbidity were recognized in the cost structures at an early stage in group insurance and in both group and individual annuities, until recently they have been largely disregarded in individual life insurance, on the grounds that they were not very important or were offset by similarly unrecognized differences in expenses. Population statistics still reflect higher mortality rates for nonwhites, but the practice of rating for color has long since bowed to public opinion.

A substantial list could be made of experience distinctions about which actuaries might differ, on either philosophical or practical grounds. To cite just a few examples, should cost structures recognize higher mortality on term business, higher mortality on small-sized policies, higher lapse rates when a modified premium policy reaches the increase-in-premium anniversary, lower investment return on high early-cash-value policies, expense differentials on pension trust business, or exceptional average size on some plans of insurance?

If one were the actuary for the first mutual life insurance company founded in Ruritania, a mythical country with insurance laws roughly similar to those of the United States, one might feel free to define a variety of classes in allocating investment income. For example, there might be a class whose risks were supported only by investment in real estate equities (with suitable restrictions on the eligibility of risks, of course), or a class participating in all investment pools except common stocks, and so on.

As the life insurance business actually evolved in the United States, however, a century of tradition and business practice developed under which every contract shared equally, in proportion to its own invested assets, in the investment yield from all invested assets other than policy loans. Whether this was required by the law of the land may have been debatable. The long-standing practice, in any event, seemed to sanctify the approach as the right and proper one. Accordingly, when it became imperative that life insurance companies be able to offer products whose benefits were to be supported primarily by equity investments, the companies took the precaution of seeking enabling legislation to authorize the creation of separate pools of assets for such business.

The adoption at about that same time of investment-year approaches for pension business was handled differently. Essentially the two cases were similar. They both involved a new definition of classes and the pools of experience they would share. Intuitively, however, the creation of separate accounts seemed to be a departure from pooling, while the investment-year approach was regarded merely as a different method of allocation of the pooled investment income. Consequently, legislation was not sought. Insurance regulators were consulted, however, and procedures for implementing the investment-year method were subjected to regulation.

Table 1 and Figure 1 show the average portfolio rate earned by United States life insurance companies on general account assets over the period 1930–75. For comparison there is also shown a series of corporate bond rates. The latter are gross rates. Year-by-year comparison of these bond rates with new investment yields of one life company shows a fairly close correspondence. For our purposes here, the figures may be taken as reasonably representative of yields available to life companies on new investments.

TABLE 1

INVESTMENT YIELD RATE COMPARISON, 1930–75

Year	Net Rate of Interest Earned, U.S. Life Companies*	Moody's Corporate Bond Yields†	Year	Net Rate of Interest Earned, U.S. Life Companies*	Moody's Corporate Bond Yields†
1930	5.05%	4.77%	1953	3.36%	4.33%
1931	4.93	5.05	1954	3.46	3.58
1932	4.65	5.98	1955	3.51	3.91
1933	4.25	5.23	1956	3.63	4.58
1934	3.92	4.44	1957	3.75	5.43
1935	3.70	3.95	1958	3.85	4.92
1936	3.71	3.46	1959	3.96	5.16
1937	3.69	3.46	1960	4.11	5.34
1938	3.59	3.56	1961	4.22	5.14
1939	3.54	3.22	1962	4.34	4.83
1940	3.45	3.02	1963	4.45	4.69
1941	3.42	2.94	1964	4.53	4.74
1942	3.44	2.98	1965	4.61	4.95
1943	3.33	2.86	1966	4.73	6.02
1944	3.23	2.81	1967	4.83	6.29
1945	3.11	2.71	1968	4.97	7.13
1946	2.93	2.62	1969	5.15	8.44
1947	2.88	2.70	1970	5.34	9.69
1948	2.96	2.90	1971	5.52	8.38
1949	3.06	3.42	1972	5.69	8.01
1950	3.13	3.24	1973	6.00	8.15
1951	3.18	3.41	1974	6.31	9.14
1952	3.28	3.85	1975	6.44	11.29

* Excludes separate accounts.

† Moody's corporate bond yields (average monthly figures): (*a*) 1930–48 = Aa seasoned; (*b*) 1949–51 = Baa seasoned; (*c*) 1952–75 = Baa new.

As we might have guessed, new-money rates were typically below portfolio rates during the latter part of the decade of the 1930's, and were heading lower. It would have been legitimate then to ask whether it was equitable for new money to receive the benefit of the higher rates on investments made earlier. However, that question received little attention until a later date.

In the late 1940's life insurance investment managers were struggling with the difficult task of reorienting their portfolios after the heavy reliance on government bonds during World War II. The supply of private

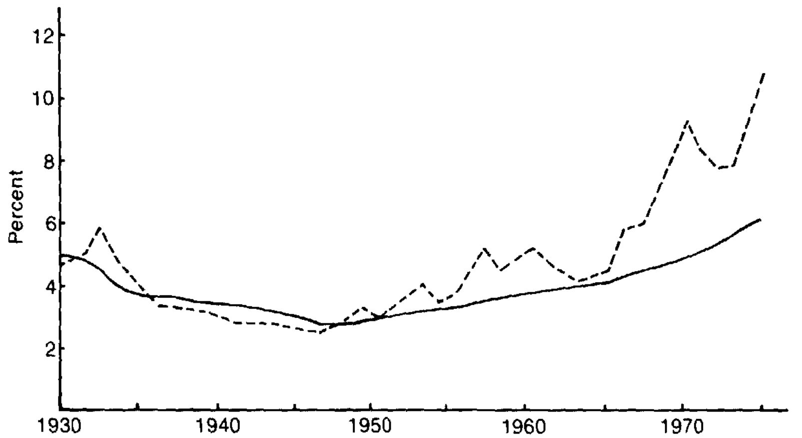

FIG. 1.—Comparison of portfolio and new-money rates of return, 1930–75. *Dashed line*: representative new-money rate (Moody's corporate bond yields: [*a*] 1930–48 = Aa seasoned; [*b*] 1949–51 = Baa seasoned; [*c*] 1952–75 = Baa new). *Solid line:* Portfolio average rate, United States life companies (*source:* Institute of Life Insurance).

investments was inadequate, and the yields were low. Portfolio rates had been dragged down to levels at which they could barely cover reserve interest requirements. Vast sums were being devoted to strengthening reserves by lowering the interest assumptions. At the same time, pension funds were just beginning the enormous growth that has characterized the postwar years. Pension managers, finding life company portfolio rates the most attractive returns available, pushed large sums at the already harassed life companies.

Some years later the situation reversed. New-money rates became attractively higher than portfolio rates. Pension money was welcomed by life companies, but increasingly it was going to other trust funds, where it could get the better new-money yields. The fact that the trust funds did not offer some of the guarantees and administrative services offered by life companies was not a critical deterrent to this trend.

The new factor in the situation was the appearance in the life insurance marketplace of a body of sophisticated buyers who had the power to place money in the companies' hands in large amounts, at irregular intervals, at the buyers' discretion. The question this situation raised was whether it was equitable to allow these buyers to water down portfolio rates when new-money rates were low, to the detriment of other contract holders, and then go elsewhere when new-money rates were high. In response to this question, companies writing group pensions generally adopted the investment-year method of allocating investment income from general account assets to lines of business and, within lines of business, to each group pension contract and, in some instances, to individual annuity contract classes.

By this action, companies redefined the class subdivisions for purposes of distributing investment experience, going from one all-inclusive class to a number of smaller classes defined on the basis of the *time* at which the cash inflow becomes available for investment. The "time" does not necessarily mean the "year," of course; it could be a shorter interval if that seems necessary, or a group of years if that seems sufficient. However, the division into individual years is satisfactory for most purposes and has the advantage of fitting accounting cycles and so on for both life companies and their clients.

Parenthetically, it should be reiterated that, at about this same time, life companies created separate accounts to make available other investment experience classes for buyers who wanted different distributions of investments than were available in companies' general accounts. In view of the success, in those days, of the concept of "buy term and invest the difference," some consideration certainly was given to actions broader than those that were actually taken.

These reactions of life insurance companies in the pension field aroused only a mild interest in applying similar techniques to individual life insurance lines generally. It was noted properly that individual life contract holders generally are not sophisticated investors and that they do not have the option of making large, irregular payments; that the differences between new-money rates and portfolio rates would be small; and that the composite rate on a typical policy would approach the company portfolio rate over the life of the policy. These were sufficient reasons to avoid the complications of investment-year methods until the recent dramatic expansion in the differences between the rates (see Fig. 1).

One additional reason for not embarking on investment-year approaches in the individual life insurance lines is the existence there of guaranteed cash values. This factor usually was described in terms of the potential replacement problem. What could prevent a policyholder from cashing in an old policy to get the benefit of higher, new-money rates on a new policy? In fact, the potential problem is more fundamental than this replacement question seems to imply and is worth exploring more thoroughly. The simple principles involved can be illustrated by examining the operation of a hypothetical fund. The "Nonesuch General Fund" outlined in Table 2 is created by a financial intermediary on December 31, year 0. Initial deposits of $10,000 each of 1,000 individual investors are placed in 5 percent bonds at par. For a period of five years, interest coupons are received and distributed to the investors. Precisely at midnight at the end of year 5, the going rate of interest in the marketplace shifts to 7 percent. Maturities of the bonds are such that the market value

TABLE 2

HISTORY OF "NONESUCH GENERAL FUND"

	December 31, Year 0	December 31, Year 5	January 1, Year 6
Going rate of interest	5%	5%	7%
Book value of fund	$10,000,000	$10,000,000	$10,000,000
Market value of fund	10,000,000	10,000,000	9,000,000

at 7 percent is $9,000,000. Also, a moment after midnight, 500 of the original subscribers withdraw, and 500 new subscribers assume ownership of shares. The question is: at what price?

The investing intermediary deals in the marketplace, and its prices and yields are determined there. However, whether the original subscribers withdraw at book value of $10,000 each or at market value of $9,000 each is determined by the contract with them. Similarly, whether the new subscribers pay $10,000 and receive the portfolio rate of 5 percent or pay $9,000 for a new-money yield of 7 percent is a condition of sale. Literally, however, no securities are sold. The original subscribers transfer their ownership to the new subscribers, and the price had better be the same on both sides of that transaction.

It is worthwhile considering these relationships in terms of the common instruments offered to the public by financial intermediaries. Table 3 contains a partial listing of the customary practices. Group annuity contracts are described as "mixed" partly because of the diverse kinds of contracts currently in force. However, the word also relates to the fact that, in the operation of a total group annuity line of business, there is a large cash outflow each year for benefit payments (including possibly withdrawals) on old contracts and for expenses and taxes. Most of these payments are in stated dollar amounts and therefore are equivalent to book-value withdrawals from the invested funds.

Consider for a moment the operation of an investment-year method of allocation of investment income to lines of business. The investment income arising each year from investments made in year k is available directly from the books, and is distributed to each line in proportion to its original share in the money invested in that year. Cash from maturities and sales of investments is also distributed in this manner. The distribution ratios that will be used in future years to apportion the income from

TABLE 3

COMMON INSTRUMENTS OF FINANCIAL INTERMEDIARIES: BASES OF PURCHASE AND WITHDRAWAL, AND OF INTEREST YIELD

Instrument	Purchases and Withdrawals at	Interest Yield Offered
Corporate bond	Market	New money
Mutual fund	Market	New money
Trust fund	Market	New money
Savings bank account	Book	Portfolio
Insurance company separate account	Market	New money
Ordinary insurance policy	Book	Portfolio
Group annuity contract	Mixed	Mixed

investments made this year can be determined by adding the items contributing to the investable cash this year: (1) the net income from premiums less benefit payments and expenses and taxes; (2) the net investment income from previous years' investments; and (3) the proceeds from sale or maturity of previous investments. Each of these items is available by line. Appropriate modifications are made to accommodate items excluded from the investment-year process, such as fixed and frozen assets, cash float, and possibly some additional asset classes such as common stocks. Modifications also may be necessary for bond rollovers and the like.

The process that has been described actually distributes to lines of business the investment income recorded on the company's books for each investment year. It might be thought of as the "natural" way to administer investment-generation theory among lines of business, for it produces the results that are theoretically desired, and it fits accounting methods well. When other mechanics are used, they customarily are intended to produce results equivalent to those that would be produced by the natural process.

The point at issue here is that not all new money acquires new assets in the marketplace; part of it goes to redeem assets from existing contracts because of obligations to pay fixed dollars to the contract holders or on their behalf. From the seller's viewpoint, this transfer of assets seemingly takes place at book values and is inconsistent with new-money theory.

A partial resolution of this inconsistency would be possible by establishing the rule that proceeds from the maturity or sale of investments go first to classes which, in the current year, have a negative cash flow—up to the amounts that those classes contributed to the generations that acquired the investments originally. This differs from the normal procedure of allocating such proceeds to the classes in proportion to their cash flow in the years in which the assets were acquired. For instance, looking again at the Nonesuch General Fund, suppose that, at the point where half the original holders were about to be paid off, half the original bonds were sold or redeemed a moment before midnight. The proceeds then would be applied to pay off the discontinuing participants. The money paid in by the 500 new participants would then be invested in new securities at 7 percent. The fund would continue with two classes of participants, one earning 5 percent and the other earning 7 percent.

While some may balk at this modification, it does seem to be defensible from the point of view of equity. However, the matching of maturities and sales with the needs of classes that develop net negative cash flow, besides its practical difficulties, inevitably would be quite imperfect. Surely, much of the original inconsistency would remain, and companies might be embarrassed to be in a position to affect the rights of policyholders relative to one another by their decisions to dispose of investments.

Within lines of business, it is not convenient to use directly the natural process described above for distributions to individual contracts or classes of contracts. For later reference, it is worth considering one practical approach for getting equivalent results. First the method determines, for each contract or class, the net cash income or outgo of each historic year (from all sources, including allocated expenses and so on). A current earned rate is also derived for each year, representing the original earned rate for that year updated for the reinvestment process. These rates are then applied to the corresponding historical cash-flow amounts for a given class, whether positive or negative, and the results are summed to obtain the total investment income for that class. The total investment income so distributed for all such classes is then reconciled to the total available to the line.

When group annuity contracts that contemplated possible withdrawal by the buyer were first written, the common practice was to hedge that withdrawal with penalties or restrictions sufficient to yield a result equivalent to withdrawal at market values. More recently, companies have been issuing large volumes of contracts guaranteeing high, new-money rates and certain rights to withdraw principal. Aside from the risk involved in rate guarantees that are possibly too liberal, the fixed-dollar withdrawal privilege conflicts with new-money theory. As a hedge against loss, withdrawal privileges are commonly restricted to stated points in time, and companies are collecting small risk charges. These could be viewed, of course, as premiums to cover the risk of loss due to the difference between market and book values.

In view of the very large sums involved, many companies also are carefully matching investment maturities with withdrawal rights. This is helpful in avoiding the calamity of sacrifice selling, but it may not provide complete comfort with regard to possible loss and the question of who should bear that loss. Under the usual investment-year theory, the benefits arising from the maturity of a security at a time of higher yields belong proportionately to all contributors to the original class of investments and cannot be reserved for the contract taking advantage of favorable withdrawal privileges.

The same question is posed in a somewhat larger context in connection with the normal cycle followed by contracts or classes of contracts, from an initial pay-in period to an ultimate pay-out period. With good fortune, interest rates may be such as to resolve any problems. More likely, a company would be faced with a choice between two actions: (1) transferring assets at book value, which means restricting the application of new-money rates to an amount less than the gross amounts paid in by the contract holders, or (2) transferring assets at market value, and maintaining sufficient surplus on old contracts to cover differences between dollar payouts and the market values of the contract holders' interests in the assets transferred from the payees. If the company follows the method outlined earlier for distributing investment income to contracts and classes within a line of business (including applying current new-money rates to negative cash flow on old business), and if interest rates are high, the surplus of the old business will be diminished because the underlying assets effectively have been transferred at market values to current positive cash income.

A question of equity may be raised here. If a contract holder is promised the right to receive certain payments described in fixed-dollar terms, has that guarantee been fulfilled if the insurance company simply accumulates sufficient surplus on that contract (or class) to cover the difference between book and market values of the underlying assets? If this is an equitable procedure for contracts currently being issued, is it fair to apply it to old contracts that were issued when the rules of the game were different? There will not be universal agreement on the answers to these questions.

III. APPLICATION TO INDIVIDUAL LINES OF BUSINESS

When it comes to the individual life insurance lines, the questions that must be examined are essentially the same ones. The factual situations that may exist, however, are much more difficult to postulate. The typical life insurance contract class will contribute positive cash flow in varying amounts over a long period of time and wind down over a long period of negative cash flow. When account is taken of that fact, and of the reinvestment process, it is not easy to guess at how different the current year's average investment yield rate might be for, say, policies issued in 1951 as compared with policies issued in 1966. If investment-generation theory were applied over a period when new-money rates varied as widely as they did in the 1930–75 period, how much surplus would have had to be accumulated on old policies to allow them to sustain the drain of negative cash flow at high interest rates?

To provide some answers to these questions, two simple model companies were postulated and operated over a hypothetical period described as years K30–K75. In this period, new-money rates were experienced as shown in Table 1 for the corresponding years 1930–75. Both companies issued all business on the whole life form at age 35. Each began with a block of business in force, and amounts of issue increased 6 percent per year after K30. More detailed descriptions of the model parameters are given in the Appendix.

The operation of Company A was simulated by computer through the years K30–K75 to provide a base case, called Company A(PF) because investment income was distributed to years of issue on the portfolio method. Successive portfolio rates (which follow the general pattern of those in Table 1 but are necessarily different) were generated year by year, assuming the investment each year of a portfolio turnover of 5.5 percent plus the net cash flow.

The overriding parameter was the ratio of surplus to reserves. This began at 8 percent on the beginning block of business. As Company A(PF) progressed, the dividend scale was modified each year to maintain reasonable patterns of surplus ratios.

For the Company A(PF) base case, federal income tax was charged to each issue year of business by means of a uniform reduction in the earned rate of investment income. This may be a common method for companies in a Phase 1 situation (tax based on net investment income). An alternate approach to this process, which involves computing the tax on investment income on a gross basis and then subtracting the reserve exemption separately for each duration, can give effect to the varying surplus levels of different years of issue. Such an approach could enter a company's dividend formulas explicitly or could be used in asset shares underlying the formulas. A second base case, called Company B(PF), was constructed using this alternate method. Naturally the resulting dividend scales were somewhat different.

Once the two base case companies had been constructed, the dividend scales were frozen and became part of the fixed input in connection with the operation of the companies on an investment-year method. Company A(IY) operated as did Company A(PF), except that investment income was allocated to each issue year on the investment-year method. Company B(IY-1) and Company B(IY-2) corresponded similarly to base case Company B(PF). For Company B(IY-1) the allocation of federal income tax to years of issue reflected only the variation in surplus levels. For Company B(IY-2) the allocation reflected the variation in both surplus levels and earned rates.

Before moving to the significant findings, several observations may be mentioned in passing. Cash flow from insurance operations typically turned negative at durations 16–21 for the test companies; for net combined insurance and investment cash flow the turning point was at durations 25–28. These points were not significantly affected by the method of allocating investment income or federal income tax.

The beginning block of business exhibited a rising surplus ratio as it declined in size. This seems to indicate that, as an old, closed block of business runs off the books, it may be impossible to prevent some tontine effects in its dividend treatment. This tendency was accentuated in the investment-year cases. The beginning block benefited from the investment-year approach during the years following K35, when new-money rates were lower than portfolio rates. It appears that, once surplus reaches a sufficiently high level, the momentum of surplus building may override some otherwise adverse factors.

Something similar to a tontine tendency seems to be a natural part of the picture when earned rates reach levels well above reserve assumptions. Even with a rather steep annual dividend scale and an average surplus of 8.0 percent of liabilities, the surplus ratios that develop for Company A(PF) and Company B(PF) for policies issued as far back as forty or forty-five years ago become very high. This seems to be natural and would probably be viewed by many as an argument for the use of terminal dividends considerably higher than those of Company A or Company B.

Most of the results can be found by inspection of the operations of each company for the final calendar year K75. For background, Table 4 sets out selected data for that year for the two base cases. The purpose of the exercise, of course, was to examine the differences that would develop in surplus levels and in earned rates, by year of issue. Table 5 displays those results, again for the calendar year K75.

It is readily apparent that the differences in earned rate that develop over time are larger than might be expected intuitively. The same is true of the accumulated differences in surplus. For Company A(IY) the results are extravagant and even include negative earned rates. Analysis of the reasons for the differences is likely to lead to some thoughts about equity. When separate years of issue are treated as separate classes for distribution of investment income, it may be thought unfair not to charge or credit each year with the same resulting rate of return in distributing the company's federal income tax. In that case, only Company B(IY-2) would be a proper alternative to the portfolio approach. However, even for Company B(IY-2)the deficit levels that develop for older years of issue are startling. They also grow rapidly; for instance, for year of issue K30 a surplus ratio of −11.0 percent for calendar year K73 becomes −22.5 percent two years later.

These results cannot represent the real world, even in a model-office sense. The actuaries of Company A and Company B would observe the situation developing as the years pass and would make modifications to dividends as required, to prevent the undesirable events. If the impact of accumulating those changes is as large as the surplus-building momentum of the beginning block that we have noted earlier, the modified dividend scales might not be implausible.

To test these questions, the investment-year versions of Company A and Company B were run again. In this second trial the surplus ratios of the investment-year versions were required to be the same as those of the corresponding portfolio cases. The result was a new set of dividends.

While this procedure was performed for all the investment-year models, only Company B(IY-2) is discussed here. Table 6 compares some of the dividends. One set of modified dividends shows the picture as it would appear to a policyholder who became insured in K30. The second set is what the actuary setting scales would see in K75.

From either point of view the modified dividend scales are much flatter than the original scales. Whether they are too flat is a judgment that

TABLE 4

DATA FROM CALENDAR YEAR K75

(In $1,000's)

ISSUE YEAR	INSURANCE IN FORCE	ASSETS		NET CASH FLOW		NET OPERATING GAIN		ANNUAL DIVIDENDS	
		Company A(PF)	Company B(PF)	Company A(PF)	Company B(PF)	Company A(PF)	Company B(PF)	Company A(PF)	Company B(PF)
Beginning block	$ 644	$ 1,095	$ 1,051	−$ 79	−$ 84	$ 14.9	$ 10.3	$ 30	$ 24
K30	129	138	135	− 11	− 11	0.1	0.1	5	4
K35	286	260	256	− 10	− 11	1.5	1.2	8	8
K40	541	416	414	− 10	− 10	2.3	2.1	14	13
K45	967	618	618	− 12	− 12	2.0	1.9	21	21
K50	1,654	853	853	7	6	3.1	2.9	31	30
K55	2,607	1,043	1,043	33	33	5.7	5.6	38	38
K60	3,914	1,128	1,131	61	61	8.7	8.7	47	47
K65	5,788	1,066	1,070	92	92	10.6	10.8	55	56
K70	8,612	745	751	159	160	34.6	35.2	39	41
K75	13,356	− 90	− 84	− 90	− 84	− 285.3	− 279.6	0	0
Total	$160,111	$32,376	$32,377	$1,800	$1,801	$133.1	$134.0	$1,329	$1,328

TABLE 5

RATIOS OF SURPLUS TO LIABILITIES, AND NET INVESTMENT INCOME RATES, CALENDAR YEAR K75

ISSUE YEAR	RATIO, SURPLUS TO LIABILITIES					NET INVESTMENT INCOME RATE				
	Company A(PF)	Company A(IY)	Company B(PF)	Company B(IY-1)	Company B(IY-2)	Company A(PF)	Company A(IY)	Company B(PF)	Company B(IY-1)	Company B(IY-2)
Beginning block	105.5%	191.5%	97.2%	115.9%	149.3%	6.8%	3.2%	6.8%	2.4%	2.8%
K30	37.7	−69.9	35.2	−7.5	−22.5	6.8	−2.4	6.8	2.6	1.9
K35	28.6	−45.7	26.9	−2.3	−13.9	6.8	2.2	6.8	4.1	3.8
K40	21.9	−15.8	21.4	6.0	0.0	6.8	4.4	6.8	4.9	4.8
K45	17.0	4.9	17.1	11.9	10.1	6.8	5.6	6.8	5.7	5.7
K50	13.4	16.1	13.4	14.3	15.0	6.8	6.3	6.8	6.3	6.3
K55	10.9	19.1	10.9	14.3	15.8	6.8	6.9	6.8	6.8	6.8
K60	7.8	13.9	8.0	10.8	11.7	6.8	7.3	6.8	7.2	7.2
K65	3.8	15.5	4.2	9.8	10.3	6.8	8.1	6.8	8.1	8.1
K70	−6.6	−2.2	−5.9	−3.6	−3.9	6.8	8.6	6.8	8.6	8.6
K75	−145.9	−147.5	−143.0	−143.8	−145.2	6.8	10.8	6.8	10.8	10.8
All years	8.0%	8.0%	8.0%	8.0%	8.0%	6.8%	6.8%	6.8%	6.8%	6.8%

would have to be made bearing in mind that over the years policyholders would be comparing the flattened scales with improving returns offered by competition such as savings accounts or other life insurance policies. The flattening is a consequence of negative cash flow for classes of insureds at a time when new-money rates are rising. While this might be impossible to explain to the layman, it may be that the resulting dividends are not impossible to live with. A company with high terminal dividends might be able to soften the impact by using such dividends to absorb some of the effect. It should be noted that these dividend modifications, which correct the surplus-ratio problem completely, have only a moderate effect in leveling the rates of return, as may be seen from Table 7.

Of course, Company A and Company B must be regarded as particular

TABLE 6

DIVIDENDS FOR COMPANY B(IY-2)

Year of Payment	For Issue Year K30			Year of Issue	For Payment in K75		
	Original Dividend	Modified Dividend	Difference		Original Dividend	Modified Dividend	Difference
K75.....	$34.10	$14.35	−$19.75	K30....	$34.10	$14.35	−$19.75
K70.....	21.23	12.50	− 8.73	K35....	27.00	15.95	− 11.05
K65.....	13.40	10.32	− 3.08	K40....	24.08	17.26	− 6.82
K60.....	10.73	8.47	− 2.26	K45....	21.32	18.00	− 3.32
K55.....	6.73	5.46	− 1.27	K50....	18.18	17.34	− 0.84
K50.....	5.32	3.96	− 1.36	K55....	14.58	14.98	+ 0.40
K45.....	5.85	4.49	− 1.36	K60....	12.02	12.88	+ 0.86
K40.....	6.14	5.30	− 0.84	K65....	9.64	10.92	+ 1.28
K35.....	3.58	3.50	− 0.08	K70....	4.73	5.56	+ 0.83
K30.....	0	0	0	K75....	0	0	0

TABLE 7

NET INVESTMENT INCOME RATES, COMPANY B(IY-2)

Year of Issue	Before Dividend Modification	After Dividend Modification	Year of Issue	Before Dividend Modification	After Dividend Modification
Beginning block.......	2.8%	2.4%	K55........	6.8%	6.8%
K30..........	1.9	3.7	K60........	7.2	7.2
K35..........	3.8	4.6	K65........	8.1	8.1
K40..........	4.8	5.1	K70........	8.6	8.6
K45..........	5.7	5.7	K75........	10.8	10.8
K50..........	6.3	6.3			

creatures in a particular time. For term plans the impact of investment-year treatment would be minimal; for high-premium plans it might provoke severe difficulties. Results would certainly be different over other time periods, too. A comparable study assuming a trend of declining new-money rates might illustrate how difficult the decision might be to pay out large dividends on old business while illustrating much lower dividends on new business.

It may be worthwhile to take a quick look at the K30–K50 period. Company B(IY-2) experienced a slow but inexorable decline in portfolio rates over this period, as shown in Table 8. In K50 the management of Company B(IY-2) would find itself looking at the earned rates shown in Table 9. Those of us who lived through the difficulties of the 1940's can

TABLE 8

PORTFOLIO RATES, COMPANY B(IY-2)

Calendar Year	Rate	Calendar Year	Rate
K31	4.53%	K41	3.95%
K32	4.59	K42	3.86
K33	4.59	K43	3.76
K34	4.56	K44	3.66
K35	4.49	K45	3.56
K36	4.41	K46	3.46
K37	4.32	K47	3.36
K38	4.25	K48	3.29
K39	4.15	K49	3.26
K40	4.05	K50	3.22

TABLE 9

INVESTMENT-YEAR RATES IN CALENDAR K50, COMPANY B(IY-2)

Year of Issue	Rate	Year of Issue	Rate
Beginning block	3.71%	K40	2.44%
K30	2.76	K41	2.44
K31	2.71	K42	2.44
K32	2.63	K43	2.46
K33	2.58	K44	2.48
K34	2.54	K45	2.53
K35	2.52	K46	2.63
K36	2.50	K47	2.75
K37	2.48	K48	2.90
K38	2.45	K49	2.65
K39	2.44	K50	2.74

remember the hard decisions made in those days and the importance attributed to as little as ten basis points in the rate of investment return. However, it is impossible to recapture fully the mood of those days, so we cannot guess whether the Table 9 picture would have led to management decisions different from those that would have been precipitated by the world of Table 8.

IV. SOME CONCLUSIONS

Finally, there remains the question, where does truth lie? Which policyholders are being treated equitably? A century of tradition prejudices many actuaries in favor of Company A(PF) or Company B(PF). However, some actuaries who have become accustomed to the investment-year approach through their companies' use of it over the recent decades may feel that the logic of its application *within* the individual lines is already implicit in the methods they are using to allocate investment income *to* the individual lines. Even actuaries who have not had that conditioning recognize the attraction of the new-money concept when setting rates for products that must compete against those of other financial institutions.

The thought of competing products raises other questions. Company A and Company B live in a world where cash flow follows current experience. Might not the adoption of investment-year methods precipitate massive surrenders by old policyholders to buy new products? If such surrenders were not large enough to provoke chaos, would we not at least be allowing some policyholders to take advantage of the rest by selling their assets at book and buying back in at market?

Perhaps the answers are yes, but some of these questions also arise in the case of the portfolio approach. There, too, in a period like the present, the old departing policyholders sell their assets to the continuing policyholders at a book price well above current market, and thus prevent them from investing all of their cash flow at attractive rates.

The practical problem of potential disorder may override the theoretical considerations of equity. In this connection it is worth reflecting on the underlying fundamentals. All financial intermediaries do their investing under the conditions of the marketplace: market values and new-money rates. Some of them, notably life insurance companies and savings banks (the same is true of savings accounts in any institution), deal with their clients on the completely different basis of book values and portfolio rates. They are constrained by practical problems from mixing the two bases of operation. Thus a savings bank that decided to offer new-money rates to new deposits would see its depositors withdraw and redeposit. They must demand some unattractive sacrifice, such as a restriction on withdrawal, to balance the attractive opportunity.

A financial intermediary that operates on a basis different from the law of the marketplace always faces the possibility of disintermediation. The savings banks could only watch in dismay when depositors withdrew massive amounts several years ago to buy government bonds directly. The fact that life insurance companies and savings banks do not experience such adversities more frequently is traceable to the fact that they offer advantages not readily available in alternative investment vehicles, namely, guaranteed safety of principal, availability in small amounts, and packaging with desirable insurance protection.

People with small savings are loath to expose themselves to a serious risk of loss of principal. They generally seem willing, however, to chance the modest fluctuations in market value that result from changes in the value of money in the marketplace. One could speculate that, if there were available widely, either for direct purchase or as a product of a financial intermediary, a vehicle that offered new-money rates of return on small amounts of capital, with minimal hazard to principal and with a protection benefit added, the threat to the book-value basis of operation of life companies and savings banks would be material. A whole life insurance policy offering new-money rates does not quite meet those specifications, for two reasons. First, considering existing legal requirements, it might have to be offered under conditions accommodating the traditional use of book values on policyholder benefits; second, it does not offer a place for transferred capital accounts to be absorbed immediately. However, a single premium policy based on new-money rates, with benefits somewhat variable to reflect market values, comes very close to being a recipe for disorder.

Since different policyholder dollars really do earn different rates of return, some might feel that we have an obligation to recognize that fact, at least to a limited degree. Can this be done in a way that will avoid the practical problems? No answer to that question is attempted here. This paper is intended only to analyze what, fundamentally, goes on in the application of investment-year theory to individual life insurance; an extremely large field is left open for exploration.

The authors wish to acknowledge the excellent work done by Ms. Pamela Shapiro Baer in preparing the computer programs that were used to simulate the operations of the model companies referred to in the paper.

APPENDIX

DESCRIPTION OF THE CALCULATION PROCESS USED IN THE INVESTMENT-YEAR MODEL OFFICE

I. ASSUMPTIONS AS TO TIMING

All business is issued on January 1. Premiums are received and expenses (including investment expenses) are incurred on January 1. Rollover transactions take place on January 1. Death and withdrawal benefits are paid on December 31. Investment income is received and federal income tax is paid on December 31. Dividends are paid on December 31. (Note that this means there is no dividend liability; thus policy reserves are the only liability.)

Beginning assets are defined to be assets from December 31 of the previous year plus premiums for the current year less expenses for the current year. In other words, beginning assets are assets on January 1 of the current year.

The following equations express the relationships among the various items of income and outgo:

Beginning assets = Assets of previous year + premiums − expenses;

Assets = Beginning assets − claims − surrenders + investment income − federal income tax − dividends;

Insurance operations cash flow = Premiums − insurance expenses − claims − surrenders − dividends;

Investment operations cash flow = Investment income − federal income tax − investment expense;

Total cash flow = Insurance operations cash flow + investment operations cash flow
= Assets of current year − assets of previous year.

The above equations hold true for each generation (year of issue) separately and for the whole company.

II. OTHER SIGNIFICANT ASSUMPTIONS

A. All assets are invested in fixed-income securities—no common stocks or real estate is purchased.

B. There is no tax-exempt investment income.

C. There are no qualified pension reserves.

D. There is no mandatory securities valuation reserve.

E. When existing assets are rolled over, the assets sold are assumed to have the same yield rate as the block of assets whence they came; that is, the remaining assets in that block are assumed to have the same yield rate as that block had before any of its assets were rolled over.

F. The *beginning block* consists of the persisting policies from the sixty-four years of issue immediately prior to K30. It is assumed that $1 billion of business was issued in each of those years, 50,000 policies of $20,000 each. The surplus ratio of the beginning block is 8 percent on December 31, K29. (For a further explanation of the determination of the initial values for the beginning block see Sec. VII of this Appendix.)

G. In K30 $1 billion of business is issued (50,000 policies of $20,000 each). In each subsequent year the number of policies issued grows by 6 percent; the policy size remains constant at $20,000. (Note: An extremely large company resulted from those issue amounts, so after the model was run all aggregate amounts were reduced by a factor of 1,000.)

H. All policies are whole life policies issued to males aged 35.

I. Net level premium reserves are used, based upon the 1958 CSO Mortality Table, continuous functions, and 3 percent interest.

J. Cash values are computed from the above reserves using an excess initial expense of $34.00, and become equal to the full reserve at the end of the tenth year. Cash values are rounded to the next higher dollar.

K. The gross premium per thousand is $22.83; the net premium per thousand is $16.92.

L. Mortality, withdrawal, and expense assumptions are taken from one company's recent asset share experience factors for a whole life plan issued at age 35. These factors remain constant for all calendar years.

M. The same rollover rate applies to the assets of all generations in all calendar years. The assumed rollover rate is 5.5 percent.

N. Investment expenses are assumed always to be 0.5 percent of invested assets, that is, net yield rate equals gross yield rate minus 0.5 percent.

O. The current federal income tax formula applies in all years.

P. The scale of terminal dividends remains constant for all calendar years.

III. CALCULATION OF INVESTMENT INCOME

Investment income is calculated for each generation (year of issue) separately. Beginning assets are the assets invested as of January 1, so the investment income calculation is based on them.

Beginning assets are subdivided into two parts: (1) the portion that earns interest at the yield rate applicable to this generation in the previous calendar year and (2) the portion invested at the current new-money rate. The first part is equal to (*a*) the beginning assets from the previous year times (*b*) one minus the rollover rate; the second part is equal to the beginning assets for the current year less the part 1 amount.

This calculation is expressed by the following equations:

$$\text{Part 1} = (\text{Beginning assets of previous year}) \times (1 - \text{rollover rate});$$

$$\text{Part 2} = (\text{Beginning assets of current year}) - \text{part 1}.$$

Note that the effect of this is to treat the net cash flow from all December 31 and January 1 transactions (including rollover) as being money available for investment at the current new-money rate; the remainder is considered to be invested at the yield rate applicable to it in the previous calendar year.

Investment income is then calculated by applying the yield rate from the previous year to part 1 and the new-money rate to part 2.

For Companies A(PF) and B(PF) only (the portfolio average method companies) the investment income thus obtained is summed for all generations to arrive at the total company investment income. This in turn is divided by the total company beginning assets to obtain the total company yield rate for the current year. Then the investment income is recalculated for each generation separately by setting each generation's current yield rate equal to the company yield rate and applying it to each generation's beginning assets.

For Companies A(IY), B(IY-1), and B(IY-2) (the new-money method companies) the current yield rate for each generation is obtained by dividing the investment income for that generation by the beginning assets for that generation. These yield rates are then carried over to the next calendar year's calculations to be applied to the part 1 assets of each generation.

IV. CALCULATION OF FEDERAL INCOME TAX

The total company federal income tax (FIT) is calculated for Companies A(PF) and B(PF) by the current FIT formula. For Companies A(IY), B(IY-1), and B(IY-2) the total FIT is adjusted to the total FIT obtained for Companies A(PF) and B(PF).

There are several methods of allocating FIT to the generations. For Companies B(PF), B(IY-1), and B(IY-2) the investment income and reserves of each generation are used separately in the FIT formula (the small-business deduction is omitted from the formula here). For Companies B(PF) and B(IY-1) the portfolio average yield rate is used in the FIT formula and is applied separately to each generation. This is done even though the yield rates for Company B(IY-1) are based on the investment-year method. For Company B(IY-2) each generation's actual current yield rate is used in the FIT formula and is applied separately to each generation. The FIT thus obtained is adjusted so that it adds up to the total company FIT.

For Companies A(PF) and A(IY) a rate, called the *FIT rate*, is calculated by dividing the total company FIT by the total company beginning assets. This rate is then applied to each generation's beginning assets to obtain that generation's FIT. Note that this FIT rate may be thought of as a deduction from the gross yield rate to obtain an after-tax yield rate.

The differences among the five companies are summarized in the chart at the top of the following page.

V. CALCULATION OF DIVIDEND FACTORS FOR FIRST TRIAL (CONSTANT DIVIDEND SCALES)

For each company there is a separate dividend scale for each calendar year. A "set" of dividend scales refers to all the dividend scales used in one case. There are only two sets of dividend scales: one set for Companies B(PF),

	FIT Formula Applied to Each Generation Separately	FIT Treated as a Flat Deduction from Yield Rate
Investment income and FIT based on portfolio average	Company B(PF)	Company A(PF)
Investment income based on investment-year method; FIT based on portfolio average yield rate	Company B(IY-1)	Company A(IY)
Investment income and FIT based on investment-year method	Company B(IY-2)	

B(IY-1), and B(IY-2) and another set for Companies A(PF) and A(IY). The set for Companies B(PF), B(IY-1), and B(IY-2) is established in the course of running Company B(PF); the set for Companies A(PF) and A(IY) is established in the course of running Company A(PF).

In order to determine the dividend scales, the mortality and loading elements of the dividends for durations 2–64 are read into the program. (The first dividend is always payable in the second year.) These mortality and loading elements remain constant for all calendar years. Then the dividend excess interest rate must be determined in order to calculate the interest element of the dividend. The dividend excess interest rate is recomputed in each calendar year by the following formula:

$$\text{Dividend excess interest rate} = \text{Net yield rate} - \text{reserve interest rate} - \text{FIT rate.}$$

The FIT rate is simply the total company FIT rate for Company A(PF). For Company B(PF) it must be calculated for each generation separately; it is equal to the FIT divided by the beginning assets for the specific generation. If the dividend excess interest rate turns out to be negative, it is set equal to 0. The dividend excess interest rate is then applied to the sum, for each duration, of the initial cash value plus the net premium, thus obtaining the interest element of the dividends.

The dividend factor for each duration is simply the sum of the interest, mortality, and loading elements for that duration.

The aggregate dividends for the current calendar year are calculated, and then the surplus and net gain are calculated by the following formulas:

$$\text{Surplus} = \text{Assets} - \text{reserves};$$

$$\text{Net gain} = \text{Surplus of current year} - \text{surplus of previous year.}$$

The adjustment to surplus necessary to make the total company surplus ratio equal to 8 percent is now computed. Then a factor called *delta dividend* is computed such that, when applied in the manner described below, it results in a change in the aggregate dividends equal to the desired change in surplus, but with the opposite sign. Delta dividend is applied in the following manner: the full amount is added algebraically to the dividend factors determined previously for durations 10 and later, and a fraction of delta dividend equal to 0.125 × (duration − 2) is added algebraically to the dividend factors determined previously for durations 3–9. Then the aggregate dividends are recomputed for each generation using the adjusted dividend factors.

Another test is then performed. If the surplus ratio for the generation being tested is not higher than the surplus ratio for the same generation in the previous calendar year, and/or is not higher than the surplus ratio of the next younger generation in the same calendar year by at least a predetermined amount (0.1 percent), the aggregate dividend reduction is computed that would make the above inequalities true. This is converted to an adjustment of the dividend factor for that generation. The sum of the aggregate amount of all such adjustments is taken for the K30 and later generations in the current calendar year. Then a level dividend increase for all issue years in the beginning block is calculated such that it offsets the sum of the dividend reductions for issue years K30 and later. Thus, after all these dividend adjustments have been made, the company surplus ratio is still 8 percent for the current calendar year.

After a dividend scale has been determined for each calendar year from K30 to K75 for Companies A(PF) and B(PF), the scales are stored for use with Companies A(IY), B(IY-1), and B(IY-2).

Note that the same set of loading elements is not used as input for both Companies A(PF) and B(PF). The scales of loading dividends are adjusted separately to obtain the desired pattern of surplus ratios for the beginning block.

VI. CALCULATION OF DIVIDEND FACTORS FOR SECOND TRIAL (CONSTANT SURPLUS RATIOS)

The same sets of dividend scales that are used for Companies A(PF) and B(PF) in the first trial are used again for Companies A(PF) and B(PF) in the second trial. For Companies A(IY), B(IY-1), and B(IY-2), however, those dividends are adjusted as necessary so that the resulting surplus ratio for each generation in each calendar year is the same as the corresponding surplus ratio in the portfolio method company. Note that in those cases where this adjustment process produces negative dividend factors, the factors are set equal to zero.

VII. DESCRIPTION OF BEGINNING BLOCK INITIAL VALUES MODEL

The purpose of this model is to compute all the initial values for the beginning block that are needed for the main model (investment-year model office).

The assumption is made that $1 billion of insurance (50,000 policies of $20,000 each) is issued to males aged 35 each year for sixty-four years. The same assumed rates of death and withdrawal that are used in the main program are used here to calculate the amounts in force at each duration on December 31, K29. Reserves are calculated, and assets are taken to be 1.08 times reserves.

Dividend factors are calculated for each duration in the same manner as they are calculated initially in the main program (i.e., before the adjustments to control the surplus ratio).

In determining the dividend excess interest rate, the yield rate is taken to be the K30 portfolio average yield rate that is used as input to the main program. An estimate is used for the FIT rate. After aggregate dividends, adjus'ed reserves for FIT, and other necessary values have been calculated, the K30 investment income is computed using the following formula, which was derived from an implicit equation:

$$\begin{aligned}\text{Investment income} = \{&(\text{Yield rate})\,[\text{assets} + \text{claims} + \text{surrenders} \\ &+ \text{dividends} - 0.48(i' \times \text{adjusted reserves} \\ &+ \text{small-business deduction})]\} \\ &\qquad\qquad \div [1 + (\text{yield rate})(1 - 0.48)].\end{aligned}$$

Now federal income tax may be computed:

$$\begin{aligned}\text{Taxable investment income} = \text{Investment income} &- i' \times \text{adjusted} \\ \text{reserves} &- \text{small-business deduction};\end{aligned}$$

$$\text{FIT} = 0.48(\text{taxable investment income}).$$

Now K29 beginning assets, which is the only other initial value needed, may be calculated as follows:

$$\begin{aligned}\text{Beginning assets} = \text{Assets} + \text{claims} + \text{surrenders} + \text{dividends} \\ - \text{investment income} + \text{FIT}.\end{aligned}$$

The actual FIT rate is calculated by dividing FIT by beginning assets. It is compared with the estimated FIT rate used in determining the dividend factors. If it is different, the actual FIT rate is used to derive new dividend factors. The whole process is repeated as many times as necessary until the actual FIT rate agrees with the FIT rate used in calculating dividends.

Finally, investment income is checked by applying the yield rate to beginning assets to see that it agrees with the investment income figure produced by the earlier formula.

DISCUSSION OF PRECEDING PAPER

CHARLES G. FISHER AND DONALD B. MAIER:

This paper provides us with an excellent analysis of what might have happened to a company's dividend distribution system had they adopted an investment-year method beginning in 1930. It demonstrates clearly many of the problems that would have been encountered, particularly those involving transfers between lines of business with negative and positive cash flows and, in certain instances, the need to cash out at depressed book values.

This paper is presented by the chairman of the Society of Actuaries Committee on Dividend Philosophy and the committee's acting secretary. One of the major reasons for the formation of the committee was to "study in depth the underlying actuarial principles and practical problems relating to the calculation and illustration of dividends, including related matters of philosophy." The paper examines the effect of the investment-year method on dividend distribution but does not explore the related question of the effect on the comparability of dividend illustrations. The purpose of this discussion is to take advantage of the data presented in the paper to see how cost comparisons would be affected by the investment-year method used in the paper. We believe our results indicate clearly that comparisons of cost indexes for policies with illustrated dividends on a new-money basis with those for policies on a portfolio basis are not meaningful.

Background

There has been a growing movement to provide the insurance-buying public with the ability to make intelligent cost comparisons at the time of purchase. This has been encouraged by insurance regulators, professors of insurance, and other consumer advocates who have felt that cost disclosure generally has been neglected by the insurance industry. The NAIC Model Life Insurance Solicitation Regulation requires companies to provide cost data based on the interest-adjusted method, using illustrated dividends.

We have been concerned that this increased emphasis on cost comparisons would create pressures on actuaries to devise methods of dividend distribution that would result in a competitive advantage in the comparison of illustrated dividends without affecting significantly the amount of dividends to be paid. With today's new-money rates, one possibility would be the adoption of an investment-generation method. At the present time, this would improve dividend illustrations while requiring no increase in dividends on those older policies that comprise the bulk of the business. This paper gives us an opportunity to study the comparability of dividend illustrations between companies using an investment-generation approach and companies using the traditional portfolio approach.

Effect of Using an Investment-Generation System for Cost Comparisons

We have made some studies to explore the effect of the adoption of an investment-period system on illustrated and payable dividends. We divided our individual life business into two classes for investment income allocation purposes: business issued January 1, 1974, and later, and business issued prior to January 1, 1974.

We feel that a certain amount of inflation is implied as long as new-money interest rates remain at today's levels. We assumed that expenses would increase 4 percent per year with no change in current new-money rates and somewhat less rapidly if new-money rates were to decrease. We tried several different scenarios for the progression of new-money rates over the next twenty years. (We did not include the possibility of significantly increasing new-money rates.) The studies indicated the following: First, while the illustrated cost position was distinctly superior under the new-money than under the portfolio method, the actual results based on payable dividends over the next twenty years were likely to be only marginally better for the new-money class. Second, under the new-money method dividends actually paid were consistently worse than illustrated, while under the portfolio method there was an improvement in dividends paid over those illustrated in two of three cases.

While it is debatable whether a new-money system is more or less equitable, we concluded that any improvement in equity would result in only moderately greater payable dividends for recently issued business under what we considered to be reasonable projections of interest. This did not seem worth the complexities and future problems that could arise from an investment-period system. We also felt that the dramatic improvement in illustrative dividends likely would be misleading without making some corresponding expense projections. However, the concept of using projected expenses for dividend illustrations would be inconsistent with the relationship between factors for illustrated and payable dividends that we felt were traditional and actually required by some state laws and regulations.

It should be noted that a similar conclusion was reached in a model-office presentation made by Dale Gustafson at a concurrent session on "Philosophy and Practice of Investment Income Allocation" at the June, 1976, Chicago meeting (*Record*, II [No. 3], 547–66). In the summary of that presentation Mr. Gustafson stated, "It seems apparent from this simplified model office that the introduction of an investment year method of allocating investment income and the incorporation of the higher rate in all durations of dividend illustrations introduces a substantive difference in the character of that dividend illustration as compared to a portfolio dividend illustration. Secondly, the model seems to indicate that there is a serious question as to Company B's ability to maintain its current dividend scales, even under the assumption of an indefinite continuation of the current high new money rates."

This paper by Messrs. Matz and Peters presents us with an opportunity to make additional comparisons of cost illustrations under an investment-year method with those under a portfolio method.

Cost Comparisons Using Model Companies from the Paper

We decided to compare illustrated and payable dividends, based on dividend interest rates, for Company A(PF), which used the portfolio method, and Company A(IY), which used an investment-year approach. We used policies issued in 1930, 1935, 1940, 1945, 1950, 1955, and 1960 to see how the difference in dividends actually paid would compare with the difference in illustrated dividends on a twenty-year interest-adjusted basis (fifteen-year interest-adjusted basis for 1960).

Appendix I of this discussion gives the dividend interest rates that would be used for each of these companies, assuming that dividend scales were changed each year to reflect allocated investment income. These rates are based on the actual Moody's corporate bond yields given in Table 1 of the paper for the period 1930–74. Illustrated dividends would assume that the interest rate and expense factors applicable to the current year of issue continue indefinitely. Appendix II gives details of the dividend factor assumptions.

Appendix III compares the illustrated and payable dividends for a policy issued at age 35 by each company during each of the years being tested. The following table summarizes the resulting twenty-year interest-adjusted surrender cost indexes on an illustrated and payable basis:

Year of Issue	Illustrated Twenty-Year Interest-Adjusted Surrender Cost Index			Actual Twenty-Year Interest-Adjusted Surrender Cost Index		
	Portfolio Method (1)	Investment-Year Method (2)	Difference, [(1) − (2)] (3)	Portfolio Method (1)	Investment-Year Method (2)	Difference, [(1) − (2)] (3)
1930...	$ 7.10	$ 7.53	−$0.43	$8.15	$ 8.97	−$0.82
1935...	6.83	8.56	− 1.73	9.03	10.51	− 1.48
1940...	7.60	10.14	− 2.54	9.62	10.72	− 1.10
1945...	8.86	11.10	− 2.24	9.77	10.15	− 0.38
1950...	10.09	10.88	− 0.79	9.47	9.12	0.35
1955...	10.33	10.08	0.25	8.53	7.77	0.76
1960...	9.65*	8.33*	1.32	8.62*	7.97*	0.65

* Fifteen-year figures.

The table shows that the difference between the illustrated indexes is not a very reliable indicator of the difference between the actual results. In addition, it is very difficult to predict whether illustrated differences are likely to overstate or understate actual differences.

In the case of policies issued in 1930 and 1935 the illustrated cost comparisons give a somewhat competitive edge to the portfolio method. The comparison period was characterized by generally decreasing new-money rates such that the new-money rate was lower than the portfolio rate by about the same amount throughout the period. The actual results give pretty much the same magnitude of advantage to the portfolio method.

For policies issued in 1940 and 1945 the illustrated dividends strongly favor the portfolio method; however, during the twenty-year period over which dividends were paid, the trend of new-money rates changed and the period was characterized by rising new-money rates that approached the portfolio rates by the end of the period. The result is that the actual costs under the two methods were much closer together than the illustrated costs.

For issues of 1950 the trend that emerged in the 1940 and 1945 issue comparison continues; the illustrated comparison favors the portfolio company, but the actual results favor the investment-year company.

For 1955 issues a small advantage is illustrated for the investment-year method. However, the period from 1955 through 1975 was characterized by monotonically increasing new-money rates, with the result that the dividends paid by the investment-year company for policies issued in 1955 turn out to be significantly better than those paid by the portfolio company. This is in spite of the fact that the increasing new-money rates enable the portfolio company to pay considerably better dividends than originally illustrated.

For issues of 1960 illustrated dividends overstate the actual advantage of the investment-year method. The fifteen-year interest-adjusted index based on illustrated dividends is dramatically better under the investment-year method. While both companies produce a lower index based on actual dividends, the improvement for the portfolio company is much greater than for the investment-year company.

These results indicate that it is very difficult to predict whether at any point in time an interest-adjusted index under a new-money method is more or less conservative than an interest-adjusted index under a portfolio method. It is possible to develop relationships between the trend of new-money rates and the present difference between new-money and portfolio rates that could be used to predict when for one method an interest-adjusted comparison will be relatively more favorable under a payable dividend basis or an illustrated dividend basis. However, this likely would be too complex to be used by prospects, would require predicting the trend in future new-money rates, and would require an understanding of each company's investment-year method. We believe the basic reason for the incomparability of the investment-year method and the portfolio method for illustrating dividends is that one is dealing with fundamentally different systems of determining dividend interest rates.

Conclusion

The concept of having the option of using an investment-period system or a portfolio system of investment income allocation within a company's individual life insurance line has introduced an entirely new and unanticipated dimension into the area of cost comparisons based on illustrated dividends. If all companies were on a portfolio basis, cost comparisons based on illustrated dividends would be considered reasonable for two basic reasons: discipline and consistency.

Clearly, under a strict portfolio-interest system a company that wants to reflect a higher rate of interest in illustrated dividends is constrained by the requirement that the same rate would need to be used for determining payable dividends on in-force business. Thus, an automatic discipline is imposed by the amount of divisible surplus that can be distributed. An investment-period system presents the opportunity to introduce a new period when it would do the most good for competitive dividend illustrations, without requiring any significant increase in the amount of surplus that would have to be released as payable dividends on older business.

As long as all companies were using the same system for investment income allocation within the individual insurance line, a certain degree of reliance could be placed on competitive rankings based on interest-adjusted indexes. This assumes a consistent interpretation of the requirement, imposed by many states, that illustrated dividends should be based on the same scales as are presently used for corresponding payable dividends. Where some companies are using a new-money method in one form or another, it becomes virtually impossible to make a meaningful comparison of interest-adjusted indexes at any particular point in time. We believe the results described in this discussion show this inconsistency clearly. Even where all other factors in the dividend formula are the same, an apparent cost advantage can diminish or even be reversed depending on the trend of new-money and portfolio rates. While we have not explored the results that would arise from different approaches to the recognition of new money, it is clear that variations would compound the inconsistency.

The use of investment-period systems within individual lines of business introduces a new factor into the cost disclosure area. It is clear to us that the nature of a company's system for determining dividend interest rates should be disclosed along with any illustrated dividend material, especially in connection with interest-adjusted indexes for comparison purposes. Even if this were done, we are doubtful that anyone would be able to make meaningful comparisons among companies using different systems of investment income allocation.

APPENDIX I

DIVIDEND INTEREST RATES
INVESTMENT-YEAR VS. PORTFOLIO METHOD

Dividend Year	New-Money Rate* Less 0.50 Percent	Year of Issue													
		1930		1935		1940		1945		1950		1955		1960	
		Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year
1930	4.27%	4.53%	4.27%												
1931	4.55	4.53	4.69												
1932	5.48	4.59	5.18												
1933	4.73	4.59	5.00												
1934	3.94	4.56	4.67												
1935	3.45	4.49	4.36	4.49%	3.45%										
1936	2.96	4.41	4.06	4.41	2.72										
1937	2.96	4.32	3.86	4.32	2.87										
1938	3.06	4.25	3.72	4.25	2.95										
1939	2.72	4.15	3.58	4.15	2.88										
1940	2.52	4.05	3.43	4.05	2.79	4.05%	2.52%								
1941	2.44	3.95	3.30	3.95	2.71	3.95	2.40								
1942	2.48	3.86	3.20	3.86	2.67	3.86	2.45								
1943	2.36	3.76	3.10	3.76	2.62	3.76	2.41								
1944	2.31	3.66	3.02	3.66	2.57	3.66	2.38								
1945	2.21	3.56	2.93	3.56	2.52	3.56	2.34	3.56%	2.21%						
1946	2.12	3.46	2.85	3.46	2.47	3.46	2.29	3.46	2.08						
1947	2.20	3.36	2.78	3.36	2.44	3.36	2.27	3.36	2.15						
1948	2.40	3.29	2.75	3.29	2.43	3.29	2.29	3.29	2.26						
1949	2.92	3.26	2.76	3.26	2.49	3.26	2.39	3.26	2.46						
1950	2.74			3.22	2.51	3.22	2.44	3.22	2.53	3.22%	2.74%				

* From Table 1 of the paper (Moody's corporate bond yields).

APPENDIX I—*Continued*

Dividend Year	New-Money Rate* Less 0.50 Percent	Year of Issue: 1930		1935		1940		1945		1950		1955		1960	
		Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year
1951	2.91%			3.20%	2.55%	3.20%	2.50%	3.20%	2.61%	3.20%	2.99%				
1952	3.35			3.21	2.63	3.21	2.60	3.21	2.75	3.21	3.22				
1953	3.83			3.26	2.75	3.26	2.75	3.26	2.93	3.26	3.47				
1954	3.08			3.25	2.78	3.25	2.78	3.25	2.95	3.25	3.35				
1955	3.41					3.26	2.85	3.26	3.02	3.26	3.36	3.26%	3.41%		
1956	4.08					3.34	2.98	3.34	3.15	3.34	3.52	3.34	4.42		
1957	4.93					3.48	3.17	3.48	3.37	3.48	3.78	3.48	4.74		
1958	4.42					3.57	3.29	3.57	3.49	3.57	3.89	3.57	4.61		
1959	4.66					3.67	3.41	3.67	3.62	3.67	4.00	3.67	4.62		
1960	4.84							3.79	3.75	3.79	4.12	3.79	4.68	3.79%	4.84%
1961	4.64							3.87	3.85	3.87	4.19	3.87	4.67	3.87	4.54
1962	4.33							3.91	3.89	3.91	4.20	3.91	4.61	3.91	4.41
1963	4.19							3.94	3.92	3.94	4.20	3.94	4.54	3.94	4.32
1964	4.24							3.97	3.95	3.97	4.21	3.97	4.49	3.97	4.29
1965	4.45									4.02	4.23	4.02	4.49	4.02	4.33
1966	5.52									4.18	4.36	4.18	4.62	4.18	4.59
1967	5.79									4.34	4.50	4.34	4.76	4.34	4.81
1968	6.63									4.58	4.70	4.58	4.98	4.58	5.11
1969	7.94									4.93	4.99	4.93	5.31	4.93	5.53
1970	9.19											5.38	5.72	5.38	6.03
1971	7.88											5.65	5.94	5.65	6.27
1972	7.51											5.85	6.09	5.85	6.42
1973	7.65											6.04	6.24	6.04	6.56
1974	8.64											6.32	6.45	6.32	6.79

* From Table 1 of the paper (Moody's corporate bond yields).

APPENDIX II

DIVIDEND FORMULA

The dividend formula used in this study is

$$D_t = \{[CV_{t-1} + G(1 - e_t) - E_t](1 + i_t) - CV_t\}$$
$$-\left[1{,}000\left(1 + \frac{i_t}{2}\right) - CV_t\right] q_{x+t-1} ,$$

where

CV_t = Cash value at duration t; values are shown in Table 1 of this discussion.

G = Gross premium; value for age 35 is \$28.

e_t = Per dollar of premium expense in duration t; values are shown in Table 1.

E_t = Maintenance expense (per \$1,000 of insurance) for year t. Starting with \$2 per \$1,000 in 1930, this factor is assumed to vary with the consumer price index thereafter. Values are shown in Table 2.

q_{x+t-1} = Mortality for issue age x, duration t, based on 1965–70 Ultimate Basic Table, Males and Females Combined. Values are shown in Table 1.

i_t = Dividend interest rate for duration t. The dividend interest rates used for the calculation of payable dividends are those shown in Appendix I. For illustrated dividends, it is assumed that the dividend interest rate shown in Appendix I for the year of issue continues unchanged.

TABLE 1

CASH VALUES, PER DOLLAR OF PREMIUM EXPENSES, AND MORTALITY RATES

DURATION (t)	ISSUE AGE (x) = 35;	GROSS PREMIUM = $28	
	CV_t	e_t	q_{x+t-1}
1	$ 0	*	*
2	21	.10	.00142
3	42	.10	.00154
4	63	.10	.00168
5	84	.08	.00185
6	105	.05	.00207
7	126	.05	.00231
8	147	.05	.00257
9	168	.05	.00286
10	189	.05	.00319
11	209	.05	.00354
12	229	.05	.00391
13	249	.05	.00431
14	270	.05	.00477
15	291	.05	.00531
16	311	.05	.00591
17	332	.05	.00655
18	353	.05	.00722
19	374	.05	.00796
20	395	.05	.00876

* Not shown since no dividend is paid in first year.

TABLE 2

MAINTENANCE EXPENSES PER $1,000 OF INSURANCE

Year	E_t	Year	E_t	Year	E_t
1930	$2.00	1945	$2.16	1960	$3.55
1931	1.82	1946	2.34	1961	3.58
1932	1.64	1947	2.68	1962	3.62
1933	1.55	1948	2.88	1963	3.67
1934	1.60	1949	2.86	1964	3.72
1935	1.64	1950	2.88	1965	3.78
1936	1.66	1951	3.11	1966	3.89
1937	1.72	1952	3.18	1967	4.00
1938	1.69	1953	3.20	1968	4.17
1939	1.66	1954	3.22	1969	4.39
1940	1.68	1955	3.21	1970	4.65
1941	1.76	1956	3.26	1971	4.85
1942	1.95	1957	3.37	1972	5.21
1943	2.07	1958	3.46	1973	5.32
1944	2.11	1959	3.49	1974	5.91

NOTE.—Maintenance expenses were assumed to be $2 per $1,000 in 1930 and to vary with the consumer price index thereafter.

APPENDIX III

DIVIDENDS ILLUSTRATED VERSUS DIVIDENDS ACTUALLY PAID INVESTMENT-YEAR VERSUS PORTFOLIO METHOD

Duration	Year of Issue: 1930				Year of Issue: 1935			
	Illustrated		Actually Paid		Illustrated		Actually Paid	
	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year
1	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0
2	1.83	1.77	2.02	2.06	2.20	1.96	2.16	1.77
3	2.69	2.58	3.10	3.35	3.05	2.60	2.89	2.26
4	3.54	3.37	4.05	4.31	3.89	3.22	3.68	2.71
5	4.95	4.73	5.39	5.49	5.29	4.40	4.98	3.89
6	6.62	6.34	6.95	6.81	6.95	5.83	6.44	5.08
7	7.40	7.06	7.60	7.15	7.73	6.38	6.91	5.31
8	8.17	7.78	8.15	7.46	8.49	6.93	7.23	5.44
9	8.93	8.49	8.78	7.88	9.24	7.46	7.55	5.60
10	9.66	9.16	9.30	8.21	9.96	7.98	7.89	5.81
11	11.40	10.84	10.71	9.40	11.69	9.48	9.18	6.98
12	12.08	11.47	10.99	9.48	12.36	9.95	9.25	6.96
13	12.76	12.10	11.12	9.46	13.03	10.41	9.11	6.80
14	12.40	11.70	10.24	8.45	12.67	9.85	8.13	5.81
15	13.06	12.30	10.41	8.54	13.32	10.28	8.47	6.22
16	14.69	13.88	11.49	9.52	14.95	11.69	9.68	7.47
17	14.28	13.41	10.37	8.34	14.52	11.06	8.71	6.56
18	14.91	14.00	10.09	8.04	15.16	11.48	9.04	7.00
19	15.55	14.57	10.00	7.99	15.77	11.88	9.56	7.66
20	16.16	15.13	10.27	8.30	16.37	12.27	9.85	8.01
20-year interest-adjusted cost	$ 7.10	$ 7.53	$ 8.15	$8.97	$ 6.83	$ 8.56	$9.03	$10.51

APPENDIX III—*Continued*

Duration	Year of Issue: 1940				Year of Issue: 1945			
	Illustrated		Actually Paid		Illustrated		Actually Paid	
	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year
1	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0
2	2.05	1.70	1.95	1.59	1.44	1.14	1.24	0.94
3	2.81	2.15	2.45	1.84	2.11	1.52	1.48	0.97
4	3.56	2.57	2.97	2.11	2.76	1.89	1.84	1.18
5	4.88	3.55	4.09	2.99	3.95	2.79	2.98	2.29
6	6.44	4.78	5.41	4.10	5.41	3.96	4.30	3.57
7	7.11	5.14	5.67	4.17	5.99	4.26	4.54	3.79
8	7.79	5.50	5.72	4.10	6.56	4.54	4.99	4.30
9	8.44	5.83	5.91	4.22	7.11	4.82	5.53	4.97
10	9.08	6.15	6.35	4.69	7.65	5.07	5.96	5.40
11	10.71	7.47	7.71	6.07	9.18	6.32	7.45	6.95
12	11.30	7.75	7.85	6.24	9.67	6.54	8.02	7.58
13	11.88	8.03	8.21	6.69	10.15	6.76	8.70	8.52
14	11.43	7.28	7.72	6.34	9.60	5.94	8.29	8.07
15	11.99	7.52	8.06	6.70	10.06	6.13	9.01	8.86
16	13.52	8.73	9.47	8.19	11.49	7.27	10.77	10.65
17	13.01	7.93	9.03	7.83	10.89	6.40	10.45	10.38
18	13.56	8.15	9.79	8.70	11.33	6.57	11.05	10.98
19	14.09	8.36	10.44	9.41	11.76	6.71	11.60	11.53
20	14.60	8.56	11.22	10.20	12.16	6.85	12.17	12.09
20-year interest-adjusted cost	$ 7.60	$10.14	$ 9.62	$10.72	$ 8.86	$11.10	$ 9.77	$10.15

Duration	Year of Issue: 1950				Year of Issue: 1955			
	Illustrated		Actually Paid		Illustrated		Actually Paid	
	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year	Portfolio	Investment-Year
1	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0
2	0.63	0.52	0.39	0.34	0.30	0.33	0.26	0.49
3	1.21	1.01	0.90	0.91	0.89	0.96	0.82	1.35
4	1.79	1.48	1.49	1.62	1.48	1.57	1.42	2.07
5	2.93	2.51	2.60	2.68	2.62	2.74	2.67	3.47
6	4.30	3.79	4.00	4.11	4.00	4.16	4.22	5.16
7	4.80	4.20	4.57	4.80	4.52	4.71	4.91	5.93
8	5.31	4.59	5.18	5.63	5.03	5.24	5.57	6.60
9	5.79	4.98	5.78	6.32	5.51	5.77	6.19	7.20
10	6.25	5.34	6.47	7.10	5.99	6.28	6.81	7.79
11	7.71	6.70	8.22	8.92	7.45	7.77	8.47	9.45
12	8.13	7.03	8.91	9.64	7.89	8.23	9.30	10.32
13	8.55	7.34	9.51	10.23	8.31	8.69	10.19	11.24
14	7.94	6.64	9.06	9.77	7.71	8.12	10.27	11.35
15	8.33	6.93	9.64	10.33	8.10	8.53	11.72	12.82
16	9.68	8.19	11.25	11.89	9.47	9.94	14.56	15.61
17	9.02	7.42	11.15	11.74	8.81	9.30	14.99	15.95
18	9.38	7.70	12.16	12.73	9.19	9.72	16,18	17.02
19	9.74	7.95	13.45	13.91	9.55	10.10	17.67	18.41
20	10.09	8.20	15.22	15.46	9.90	10.49	19.05	19.57
20-year interest-adjusted cost	$10.09	$10.88	$ 9.47	$ 9.12	$10.33	$10.08	$ 8.53	$ 7.77

Duration	Year of Issue: 1960			
	Illustrated		Actually Paid	
	Portfolio	Investment-Year	Portfolio	Investment-Year
1	$0	$ 0	$ 0	$ 0
2	0.05	0.28	0.04	0.18
3	0.77	1.20	0.73	0.95
4	1.45	2.12	1.42	1.66
5	2.71	3.59	2.69	2.96
6	4.22	5.33	4.22	4.55
7	4.84	6.18	4.98	5.50
8	5.46	7.01	5.80	6.50
9	6.06	7.83	6.74	7.64
10	6.64	8.64	7.92	9.05
11	8.22	10.42	10.40	11.76
12	8.75	11.17	11.66	13.08
13	9.28	11.91	12.68	14.09
14	8.79	11.62	12.97	14.37
15	9.29	12.34	14.13	15.48
15-year interest-adjusted cost	$9.65	$ 8.33	$ 8.62	$ 7.97

PETER F. CHAPMAN:

Messrs. Matz and Peters have served the actuarial profession and the insurance industry well by considering, dispassionately and factually, a subject that up to now has generated more emotion than analysis. Continuing in what I hope will be a similar vein, I would like to suggest three significant problems facing the actuary who might be considering the application of investment-year techniques. Each problem represents an area where substantial thought and research are required.

Dividends in Sales Illustrations

As we are reminded from time to time, the dividends shown in rate-books and sales illustrations are not guarantees, estimates, or projections. They are representations of the current scale. As long as the current scale remains in effect, there is little need for the actuary to be concerned with future trends in long-term interest rates when establishing or illustrating a dividend scale. The portfolio aggregate rate currently credited presumably will be credited in all future calendar years to all funds received. We accept this as a conservative assumption, since even a sharp decrease in bond and mortgage yields would be unlikely to depress current aggregate interest rates. The yields on the older debt instruments that mature or are redeemed typically will continue to be lower than the new instruments that are acquired. In short, the present yield rate is sheltered by the averages.

If the investment-year method (IYM) is used, however, it becomes necessary to "estimate" or "project" the yield to be allocated in each year to funds received in that year. That, in turn, requires estimates or projections of the rate of return on new investments for that year, an adventure that few of us would care to undertake.

Modified Forms of IYM

Most literature on the use of IYM in individual insurance appears to present polarized alternatives. One either ignores entirely the effect of investment timing, or recognizes it by individual year or at least by groups of years. However, there are circumstances when application of the portfolio aggregate method without modification may do a disservice to the actuary's ideal of equity.

For example, the amount of Phase 1 income tax incurred depends heavily on the tabular interest rate used to determine reserves. While this was not the legislative intent, the present level of current earnings rates causes serious questions to be raised about the appropriateness at this time of the adjustment vehicle, the ten-for-one rule. Several recent actuarial papers have demonstrated the inaccuracy of the approximation when current earnings rates exceed tabular interest rates by 250 or 300 basis points.

The federal income tax paid by the typical mutual company on investment income generated by assets underlying non-tax-qualified policy reserves may be approximated with substantial accuracy by

$$0.48(\overline{MMR})_t[Y - CER_t(1 - 10CER_t) - 10i_t^{\mathrm{TAB}}CER_t],$$

where $(\overline{MMR})_t$ is the mean of the mean reserves of a block of business during taxable year t, $\overline{Y}_t$ is the net earned yield on a portfolio aggregate basis, CER_t is the current earnings rate (five-year current average) on the federal income tax yield basis, and i_t^{TAB} is the tabular interest rate used in determining $(\overline{MMR})_t$.

Within the realistic range of CER_t (between 5 and 6 percent), it can be demonstrated easily by the formula that an increase of 50 basis points in i_t^{TAB} is worth between 10 and 15 additional basis points of post-tax net yield. A glance at Exhibit 8 of most mutual companies confirms the belief that the tabular interest rate has increased with the introduction of each new policy series. Should not the lower tax rate for the more recent series be passed on to the policyowners of that series? If this is done, however, would it not represent at least a symbolic departure from the use of the same portfolio aggregate rate for all policies?

Another example concerns policy loans. We have found that our policy loan utilization ratio (outstanding accrued indebtedness to available loan values) tends to increase with the age of the policy. A peak ratio is achieved usually between the seventh and tenth policy years. When new investments are available at a higher net yield than the policy loan rate, does not equity dictate the incorporation of this loss from reduced investment opportunity into the dividend interest term? Would such an introduction represent dividend formula recognition of investment timing?

Cash Flow and IYM

Our asset share analyses indicate invariably that, after the first policy year, the cash flow for policies issued in any particular calendar year becomes positive and remains so for a considerable time. Somewhere about the fifteenth year, depending on experience assumptions and plan and age distributions, a reversal takes place and benefit payments, expenses, taxes, and dividends exceed premium and investment income.

In the absence of extreme policy loan demands or an abnormal economic environment, the aggregate cash flow generally remains positive since at any time the typical company will have an excess of business in the positive-cash-flow phase. If a pure portfolio aggregate dividend interest rate is used, the actuary need not be unduly concerned about the equity of the implicit assumption that all debt instruments are held to maturity.

This picture changes as soon as we move into an IYM environment. We no longer can feel that we are being equitable to all generations of policyowners when we use premiums collected in 1978 for a policy issued in 1972, and investable at 9 percent before taxes, to pay a death claim in the same year on a policy issued in 1955. Equity suggests that we maintain separate fund accounting records for each policy year or group of policy years that are differentiated in our dividend formula.

When the cash flow becomes negative for any year of issue, we must capture in our dividend formula the reality that we are liquidating assets underlying the policy reserves for *that* year of issue. Since we are completing three decades of increasing interest rates, it is obvious that any funds not available from maturities and redemptions will come from assets sold at less than their book value. These capital losses, of course, will have to be offset marginally against federal income taxes and against changes in the mandatory securities valuation reserve.

The two net results are an increasingly burdensome IYM implementation procedure and further exacerbation of the differences in the interest credited through the dividend formula to newer as compared with older generations of insureds.

ALVIN B. NELSEN:

Any paper dealing with the investment-year method is most timely, especially one that deals with the individual lines of business. The authors obviously have given the subject much thought and have come up with many interesting observations.

That the authors have a marked preference for the portfolio average method is evidenced by the statements made early in the paper to the effect that this method has a century of tradition and is sanctified by long-standing practice. References in actuarial literature indicate that the practice was continued simply because of the impracticability of doing otherwise, even though recognition of interest returns by generations would have improved equity. The modern computer changed all this. It became practicable to recognize interest returns by generation. The result has been a continuing trend toward the adoption of investment-year methods in the last two decades.

The recognition of investment generations is certainly common in group lines and as a basis for the allocation of investment income by major lines of business. The approach has been used for some time for individual immediate annuities and more recently for individual deferred annuities, both participating and nonparticipating. Current new-money rates are taken into account in establishing nonparticipating premium rates for life insurance. There has been a somewhat greater lag in utilizing generational interest rates for individual participating life insurance, at least in an explicit form that can be identified readily. The seemingly greater utilization of investment-year methods for individual deferred annuities than for individual life insurance is, in some ways, an anomaly. The considerations and problems are much the same for life insurance and deferred annuities, except that for deferred annuities there has to be much greater concern for the replacement problems, which could serve to defeat the effective operation of the investment-year method.

The paper describes a study based on a model life insurance company assumed to be in operation for sixty-five years prior to 1930. Simulation is made of emerging average portfolio rates to produce given financial objectives, and then of investment-year rates in the period 1930–75 to produce similar financial results. The investment rollover rates are assumed to be a level 5.5 percent of outstanding investments each year. This may be unrealistic since rollovers tend to vary greatly by duration as well as with the trend of interest rates. Furthermore, the rollover rate used corresponds to an average investment lifetime of more than seventeen years. This is too long an average period for maturities under current conditions, and even more so under the conditions that prevailed in the 1930's and 1940's, the period highlighted in the study.

In a declining interest rate market, the return on the total average portfolio can go down very sharply because of repayments and renegotiations of investments that do not have a repayment penalty, to say nothing of defaults. This is supported by Table 1 of the paper, which demonstrates that the actual average portfolio interest rates of companies declined almost on a one-to-one basis with new-money interest rates in the 1930's and 1940's. However, in the model the average portfolio rate does not decline as much as in Table 1. Furthermore, a factor in the amount of decline that does appear is the assumption of an increasing volume of new business issued each year during the period, which serves to disguise the understatement of the rollover rates. Here again, the scenario departs from the actual conditions in the 1930's, when the annual volume of new individual life insurance issued by companies actually declined. The apparent understatement of the rollover rates produces an understatement of the cash flow and tends to cause the emergence of generations with negative cash flow. This is significant since investment immunization results if the benefit payments made for a generation correspond to investment income and rollover.

While this is an interesting study, it should be recognized that it does not come to grips with the realities and modifications that must be made in adapting the investment-year method to dividend scales for individual life insurance and deferred annuities. An example of such a modification is the use of a limited average portfolio for issues of recent years in order to have a basis for dividend illustrations.

JOHN C. ANGLE:

The Securities and Exchange Commission (SEC) will be interested in the arguments in this paper to the effect that individual life insurance is a security. In drafting the Securities Act of 1933, Congress exempted insurance under section 3(a)(8) of the act. The exemption indicated Congress's general satisfaction with state regulation but concern that elements of a security were inherent in life insurance. In June, 1977, the SEC issued Release 5838, which advanced the view that flexible individual annuity contracts were in all respects securities requiring regulation by the SEC. One point made by the SEC staff was that under flexible premium annuities the owner bears the entire risk of interest rate fluctuation. This was said to be the difference between the rate guaranteed in the annuity contract and the higher rate, including excess interest declarations, projected by the insurer at the time of sale.

Does the SEC's position relate to investment-generation theory? I believe it does because of the authors' demonstration that dividends based on new-money interest rates will be more volatile and less predictable than those calculated by the portfolio method. Since "risk" is to a degree synonymous with uncertainty, one could contend that new-money dividend interest rates increase the risk borne by the insured and hence come closer to pushing life insurance across the dividing line between insurance and securities. The line is important because greater disclosure and different regulation will be required if a policy subject to investment-generation dividends is held to be a security.

Let me turn to the related proposition that a mutual life insurer is essentially a financial intermediary analogous to a mutual fund or a savings bank. One argument for this proposition is that insurers cannot ignore the law of the marketplace in an era of interest rates driven up by inflation. The authors say that individual life insurers face real possibilities of disintermediation because of their status as intermediaries. However, the authors' investment-year model produces higher dividends for new issues. The investment-year scale would encourage new sales, which would generate a negative cash flow. Meanwhile the lower dividends paid to older policyholders (as compared with portfolio dividends) would encourage those policyholders to cease premium payments.

While group annuities may be close to securities, I would submit that there are very real differences between individual life insurance and group annuities. The individual purchaser has made an estate gift to his beneficiaries for which he pays a substantial entry fee in order to establish and maintain the gift regardless of future insurability or time of death. An individual insured also has chosen guaranteed cash values and a guaranteed premium in preference to investments that might earn more return but that present a correspondingly greater risk of loss of principal and provide no guaranteed death benefit. On the other hand, the group annuity is a funding vehicle purchased by a corporation that retains the risk of underfunding and that relies on its own actuaries.

It also is to be noted that as an individual insured grows older he becomes a substantial part owner of a mutual insurer. Unlike the group policyholder, the individual becomes the source of capital appropriated for the writing of new business, pays for the maintenance of the insurer's agency force, and helps secure the goodwill of the institution. Eventually, he even may contribute substantially to the surplus of the company. I would submit that a long-term individual policyholder has a more intimate relationship with his company than does an investor, bank depositor, or mutual fund purchaser. However, insureds may leave companies at which they no longer are honored as close members of the family.

The investment-generation theory implies that the present owners of a mutual insurer be put in an inferior position compared with their status under portfolio theory, in order to speed the transfer of the business to new owners. Is this a creditable assumption?

It seems to me that those espousing the investment-generation theory for individual life insurance assume implicitly either indefinite inflation at 6 percent per annum or its actual acceleration to levels above 8 percent per annum. I wonder whether this may not be the latest example of the penchant of life insurers for taking action just as an era draws to a close. We entered the equities field in the late 1960's near the peak of a forty-year bull market. We strengthened policy reserves in the late 1940's just before interest rates bottomed out in 1950. Now we seem willing to assume that interest rates will remain at levels not seen since the Civil War.

Does the investment-generation method protect individual life policyholders better against inflation? The Matz-Peters model tells us that those who bought twenty years ago are protected far better against inflation by a portfolio insurer than by an investment-year insurer. For the older policyholder, investment-year theory makes life insurance a weaker bulwark against inflation.

Let me turn now to some of the assumptions that underlie the Matz-Peters model. My first reaction, which I related to a colleague, was that we were being baited to attack investment-generation theory. Perhaps the authors, adopting a mathematician's practice of testing theories with extreme cases, sought to make such cases. One of those relates to the 80-year-old policyholder who reaches his forty-fifth policy anniversary in model year K75. We are told that under equally plausible assumptions this insured should be paid a per-thousand dividend of either $14.35 or $34.10. Either he pays the insurer a net of $8.48 or he cashes a check for $11.27. Furthermore, he can expect that his widow will receive a terminal dividend under portfolio theory but no terminal dividend under investment-generation theory.

I devote the remainder of my comments to a discussion of the assumptions that cause overstatement of the difference between the dividends under the two approaches.

1. The model ignores the severe policy loan disintermediation seen in 1969–70 and 1973–74. New-money yields were high in 1969–70 but there was very little net cash flow available to purchase securities at those attractive yields. The disintermediation penalized portfolio yields but also would reduce new-money investment income significantly.

2. The model does not allocate investment expenses consistently. The portfolio model uses industry-wide portfolio yields that are net of investment expenses. However, the investment-year model distributes earnings from gross new-money interest rates without deduction for investment expenses.

3. The portfolio and investment-year investments are not comparable. The model compares yields from funds invested 100 percent in newly issued bonds with yields from diversified portfolios that include bonds, mortgages, stocks, and policy loans. A better contrast might result if we used composite new-money interest rates from bonds and mortgages. Alternatively, an investigator could employ the yield obtained from bond portfolios held by life insurers as the portfolio yield. A. M. Best and Company, for example, publishes bond portfolio yields before expenses as part of its annual reports for all United States and Canadian life insurers. While I have no statistics for the entire industry, I can relay the median yields from bond portfolios held by thirty major life companies as tabulated by Mr. A. G. Kearney of Paine, Webber, Jackson and Curtis. Mr. Kearney found that between 1968 and 1976 the median bond portfolio return of the thirty companies moved from 5.08 to

7.60 percent. In 1976 this portfolio yield approached the yield on new bond issues much more closely than did the net rate of interest earned by all United States life companies.

	Net Rate of Interest Earned, U.S. Life Companies	Median Bond Portfolio Yield of 30 Major Life Insurance Companies	Moody's Yields on Newly Issued Baa Bonds
1968	4.97%	5.08%	7.13%
1969	5.15	5.18	8.44
1970	5.34	5.41	9.69
1971	5.52	5.82	8.38
1972	5.69	6.12	8.01
1973	6.00	6.62	8.15
1974	6.31	6.83	9.14
1975	6.44	7.08	11.29
1976	6.68	7.60	9.51
Increase in yield from 1968 to 1976	1.71%	2.52%	2.38%

4. The model fails to adjust the internal rate of return on capital invested in new business when moving from portfolio to investment-year distributions. The internal rate of return seems to be taken as the portfolio rate of return for investment-year dividends and to be constant for all years of issue. In the model, old policyholders provide surplus equal to 145 percent of policy liabilities to finance policies issued in year K75. It seems that the older policyholders should be entitled to at least as high a return on that investment as would be realized if the capital so advanced were invested in new securities. Such new investments, of course, would generate additional earnings and thus higher dividends for older policyholders under investment-generation distributions.

5. An investment rollover rate implies little investment management other than a process analogous to the establishment of an index common stock fund. An assumed investment rollover rate of $5\frac{1}{2}$ percent seems low in light of the investment management experience of some companies.

Messrs. Matz and Peters deserve warm thanks for their paper. This is the first paper to illustrate individual life insurance dividends that vary by issue (or investment) year. I have inspected the pattern of dividends produced and the authors' assumptions. The inspection leads me to conclude that the results overstate the magnitude of the practical difference between dividends resulting under portfolio as compared with new-money interest rates. I also believe the model suggests new reasons for caution in recommending new-money allocations for individual life insurance dividends.

THOMAS G. KABELE:

The authors are to be congratulated for their fine paper on the investment-year method for calculating individual life insurance dividends. They certainly have demolished one myth, which is that "over the long run, the use of a portfolio rate may be as equitable since the results are similar" (see 1974 Part 9I examination, question 8). The authors have shown that the results are certainly *not* similar. In the following paragraphs I will explore certain other myths concerning the investment-year method.

Myth 1: Under the Portfolio Method "Surplus Investment Income Is Distributed Using the Same Interest Rate for All Blocks of Policyholders"

The above quotation comes from a November, 1976, article in *Consumer Reports.* However, the interest rate is the same only if one values all the assets on the unrealistic book-value basis. On that basis the same security can be carried at many different values. In fact, there is a different book value for each purchase lot. For one security owned by my company I found that the book value varied from $1,010 for a lot purchased in 1954 to $660 for a lot purchased in 1974. Under the portfolio method a new policyholder "buys" a pro rata portion—on a book-value basis—of all purchase lots.

If we restated all assets to market value and made some allowance for the tax effects of "deep discounts," we would find that under the *investment-year method* all policyholders receive approximately the same rate of interest.

Myth 2: The Investment-Year Method Cannot Be Used because of the Presence of Guaranteed Cash Values

Clearly a mutual savings bank cannot credit different rates of interest on its various passbook accounts, even though the moneys invested may earn different rates of interest. This is because there are no withdrawal penalties. I contend that from the policyholder's viewpoint there are substantial penalties for surrendering a life insurance policy. These include the following:

1. The old policy offers guaranteed insurability and the protection of the suicide and incontestable clauses.
2. The old policy actually may be cheaper than a similar one purchased today. That is, the effect of new money may be offset completely by the rise in initial expenses that has accompanied the increase in new-money interest rates.
3. The surrendering policyholder must repay all the initial costs if he buys a new policy. Thus, he starts out at the bottom of the cash-value and dividend ladders.

4. The surrendering policyholder loses any interest on surplus funds (above the cash value) that is used in determining his annual dividend.

From the company's viewpoint, after the first few durations the asset share may be greater than the cash value plus any terminal dividend, and even in the first year the gross premium usually is greater than the cash value plus the commission.

It is because of these withdrawal penalties that companies can and must credit the policyholder with the current new-money rate. A life insurance policy is more like a certificate of deposit than a passbook account.

Myth 3: Companies on New Money Will Be at a Competitive Disadvantage when the Portfolio Rate Climbs above the New-Money Rate

If new-money rates drop significantly below the portfolio rate and stay below, I doubt that even portfolio companies will credit new policyholders with the excess interest. Many of these new policyholders would be large pension funds and sophisticated individuals who might withdraw their funds if the situation reversed. In fact, it is doubtful whether insurance departments would allow such a dilution in policyholder surplus.

Also, if new-money rates drop, companies will have a large amount of unrealized capital gains. These gains will tend to be larger for new-money companies, since they gave policyholders more favorable treatment when interest rates were rising. The capital gains may help finance agency expansion or could be used to pay extra dividends to old policyholders. These dividends may reduce lapse rates and hence unit expenses for both old and new policyholders.

The real risk for a life insurance company is not that interest rates will fall but that they will rise to still higher levels. The recent experience of the United Kingdom shows that interest rates of even 16 percent are possible. If interest rates rise, the market values of fixed-income securities fall and companies are subject to potential losses from disintermediation (that is, surrenders exceeding sales). The disintermediation probably will be greater for portfolio companies than for new-money companies.

Myth 4: The Investment-Year Method Will Cause Replacements for Companies Adopting It

I do not believe that investment-year companies will suffer from replacements, for the reasons given earlier. If a replacement problem arises, it will affect all companies, not just those adopting new money. Policyholders of portfolio companies could buy new policies from investment-year companies, from stock companies, or from new subsidiaries of existing companies (for which the portfolio rate would be close to the new-money rate). Also, they may replace a whole life policy with term insurance and a deferred annuity that offers new money. (Even companies that use the portfolio method for whole life and endowment contracts often use new money for individual and group deferred annuities.) Finally, other savings institutions provide replacement vehicles for insurance company policyholders.

Myth 5: In Times of Rising Interest Rates, Old Policyholders Must Lose since New Policyholders Gain

This myth is predicated on the assumption that going to a new-money method will have no effect on sales. Actually, I believe that the new-money method will increase sales of permanent life insurance policies and enable the industry to recapture some of its lost market share. According to the *Life Insurance Fact Book, 1977* (p. 56), the ratio of life insurance premiums to disposable family income has declined from 3.34 percent in 1940 to only 1.90 percent in 1976. With increased sales will come lower unit expenses and higher dividends.

Myth 6: There Is Less Financial Discipline in Investment-Year Dividend Illustrations than in Portfolio Dividend Illustrations

Those who believe in this myth cite examples of what may happen if a company has, say, $1 million extra to distribute as dividends. The portfolio company can use this extra money to pay *all* policyholders, say, an extra $\frac{1}{4}$ percent of excess interest, even if they did not earn it. On the other hand, an investment-year company might spend only one-tenth of the extra money to give policyholders in the current ratebook era the extra $\frac{1}{4}$ percent, with nothing to older policyholders. Thus, the investment-year company gets the same dividend illustrations for new business by spending only a fraction of the amount the portfolio company spends.

To answer this, let me say first that the fraction one-tenth is much too small. Because of increased sales caused by the same inflation that has raised interest rates, the insurance in force is concentrated in the most recent policy years. Furthermore, some of the older blocks also will have large amounts of new money, and under a properly designed investment-year method they also will partake of the extra surplus.

Also, because the portfolio company is not giving new policyholders as favorable a treatment as they can obtain elsewhere, it probably will not have as much money to give policyholders. Perhaps instead of having $1 million to give policyholders, it may have only $100,000 to spread among everybody.

In the following paragraphs I will explore certain other aspects of the paper.

Tax Formulas

As I understand it, the authors considered the following tax formulas (neglecting the small-business allowance):

$$\text{Tax}_b = [r(I_c - i_c^x f_c V_c) A_b]/A_c \; ; \tag{1}$$

$$\text{Tax}_b = r(i_c^c A_b - i_c^x f_c V_b) \; ; \tag{2}$$

$$\text{Tax}_b = r(I_b - i_b^x f_b V_b) \, . \tag{3}$$

The subscript b is used for data pertaining to a particular block (or generation), and the subscript c is used for data pertaining to the entire company. Symbols are defined as follows:

I = Investment income;
A = Assets for tax purposes;
V = Reserves for tax purposes;
i^c = Current earnings rate;
i^a = Five-year average earnings rate;
i^x = Lower of i^a and i^c;
i^{NP} = Nonpension assumed interest rate used to compute V;
f = Ten-for-one rule adjustment factor
$= 1 - 10(i^x - i^{NP})$;
r = Effective tax rate.

I believe that formula (1) is used for companies A(PF) and A(IY), formula (2) is used for companies B(PF) and B(IY-1), and formula (3) is used for company B(IY-2). For companies using the portfolio method of computing interest, formulas (2) and (3) are equivalent, since for these companies $I_b = (I_c/A_c)A_b = i^c A_b$ and $f_b = f_c$. For companies using the investment-year method, however, formulas (2) and (3) can give very different results. In fact, the "required interest" term ($i^x fV$) is not affected greatly by the level of interest rates, whereas the investment income (I) increases linearly with the interest rate. This is illustrated in the accompanying tabulation, where we assume that i^{NP} = 3 percent

	i^x	f	$i^c A$	$i^x f V$
Case 1.....	0.08	0.5	$0.08A$	$0.040V$
Case 2.....	0.07	0.6	$0.07A$	$0.042V$
Case 3.....	0.06	0.7	$0.06A$	$0.042V$
Case 4.....	0.05	0.8	$0.05A$	$0.040V$

and $i^x = i^c$. I believe that if investment income reflects new money then logically the federal income tax formula also should do so, and I agree with the authors that only method IY-2 "would be a proper alternative to the portfolio approach."

To a limited degree, companies can reduce the impact of ridiculously high taxes on new issues by raising the valuation rate (i^{NP}). Unfortunately, the present law seems to limit the value of i^{NP} to $4\frac{1}{2}$ percent for policies with nonforfeiture benefits. Shockingly, some industry groups have tried to reduce the $5\frac{1}{2}$ percent limit that was proposed for single premium life insurance policies.

At the time the current federal income tax law for life insurance companies was enacted, only small differences were contemplated between the valuation rate (i^{NP}) and the average earnings rate (i^a). With today's high level of interest rates, we need either a change in the tax law to an exact revaluation or a change in the Standard Valuation and Nonforfeiture Laws to allow higher valuation rates.

Tax Rate

In their paper the authors use a tax rate (r) of 48 percent. This is perfectly satisfactory if the tax formula recognizes for each block or policy generation a pro rata portion of the small-business and the $250,000 statutory deductions; the tax discounts for preferred stocks, deep discount bonds, and home office real estate; and any tax credits. Instead of recognizing these factors in the formula, one can lower the tax rate. For my company, I have found that using a tax rate of 40 percent takes account of the effect of these factors.

Credibility Formula for the Average Earnings Rate

For the investment-year company IY-2 the authors computed the current earnings rate i^c by the formula $i^c = I/A$ even if both I and A were negative (which they were for the first duration). For an illustrative calculation I can find no fault with this. But in the real world this may lead to difficulty. For example, it is possible for I to be positive and A to be negative (due to an abrupt change in interest rates in the early durations). In this case i^c would be negative. It also is conceivable that i^c could have a ridiculously large absolute value with either a positive or negative sign.

To remedy the above problems I propose that we calculate a "credibility factor" $Z = A/V$ (but not less than zero or greater than 1), and use the following definition of i_b^c for a block b:

$$i_b^c = Z(I/A) + (1 - Z)i_e^c .$$

The five-year average earnings rate i_b^a is calculated as the appropriate average of the i_b^c, except that for the first four years we substitute values of i_c^c for the missing values. (The authors calculated i_b^a by averaging i_b^c over the available years, which means that $i_b^a = i_b^c$ for the first duration.)

The use of the credibility factor prevents a block from receiving a large tax credit ($i^x fV$) on its reserves, even though its investment income (I) is relatively small or even negative. In such cases the block is really borrowing from other blocks, and its tax credits should reflect the earnings on the borrowed money.

Expenses

In order to focus on the method of allocation of investment income, the authors quite rightly have assumed in their illustrative calculations that all other factors, particularly expenses and taxes, remain constant. However, in the real world expenses play an important part in determining the assets of each block or generation of policies. Although expenses and taxes (excluding commissions, premium taxes, and federal income taxes) may be one-third to one-half as large as investment income, there seems to be less standardization in allocation methods. Thus, decisions regarding the allocation of expenses and the bases for unit expense rates can have a greater effect on dividends and surplus than the investment allocation procedures used.

(AUTHORS' REVIEW OF DISCUSSION)

J. EDWIN MATZ AND FRANKLIN E. PETERS:

Messrs. Fisher and Maier have explored the area of illustrated dividends based upon the investment-year method and the portfolio method, and the relationship of such illustrations to the corresponding paid dividends based on each method. This research makes a worthwhile addition to the paper, which dealt only with paid dividends and did not touch upon illustrations. Their work shows clearly the practical difficulties created by the coexistence of illustrated dividends based on the portfolio method and on the investment-year method.

In the first of several points he makes, Mr. Chapman says that under the investment-year method "it becomes necessary to 'estimate' or 'project' the yield to be allocated in each year to funds received in that year." It may be possible to reduce this practical problem by such means as grouping several years into a single generation, as Mr. Chapman himself suggests in his next comment. However, we agree with Mr. Chapman that the new-money rate to be used for the most recent generation of policies is in any case less durable than the portfolio rate.

Mr. Chapman asks whether the tax advantage of the higher valuation rates used for recent blocks of business should be credited to the benefit of those blocks and, if so, whether it would not "represent at least a symbolic departure from the use of the same portfolio aggregate rate for all policies?" Variation of the dividend interest rate in this manner clearly does not fall within classical investment-generation theory; on the other hand, there no doubt are many actuaries who would consider this practice to be a deviation from the strict portfolio method. Perhaps it could be labeled the "tax generation method." In the policy loan example set forth by Mr. Chapman, it appears that the investment-year method would be applied to only one category of assets, while the portfolio method would be used for other assets. Some actuaries have raised essentially the same question about the practice, which has become common recently, of varying the dividend scale with the policy loan rate, particularly under current circumstances where the illustrated scale in many states is based upon an 8 percent policy loan rate while the scale for old business usually is based on a 5 or 6 percent loan rate or a combination of the two.

The final point made by Mr. Chapman regarding the difficulties created under the investment-year method by the existence of generations with negative cash flow is one that was brought out in the paper. In our model those generations with negative cash flow borrowed at the new-money rate from the generations with positive cash flow. This was equivalent to an intergenerational transfer of assets at market value. The loss to the generations with negative cash flow came out of their accumulated surplus. This difficulty led us to ask the questions contained in the final paragraph of Section II of the paper.

We agree with Mr. Nelsen that actual investment rollover rates tend to vary with the age of the asset. However, a single rollover rate for assets at all durations was one of a number of simplifying assumptions that was necessary to make the model manageable. Mr. Nelsen points out that prevailing rollover rates in the 1930's and 1940's were higher than the 5.5 percent assumed in the model. We began with a rate much higher than 5.5 percent but found that it resulted in a 1975 portfolio rate that was unrealistically high. After some experimentation we found that the 5.5 percent assumption produced a fairly high, but believable, yield of 6.8 percent for 1975.

Mr. Nelsen also states that our study "does not come to grips with the realities and modifications that must be made in adapting the investment-year method to dividend scales for individual life insurance and deferred annuities. We acknowledged this in the closing paragraph of the paper, and would welcome an exposition by those who have come to grips with these realities and modifications.

Regarding Mr. Angle's comments, the paper does not argue that life insurance is a security. Rather, Mr. Angle himself makes this argument using certain information from the paper.

Mr. Angle lists five assumptions used in the model that he believes cause it to overstate the difference between the results under the portfolio method as compared with the investment-year method. Two of the items, the assumption of no policy loans and the 5.5 percent rollover rate that also was referred to by Mr. Nelsen, probably do tend to overstate the difference. With regard to the other three items, Mr. Angle apparently has misunderstood our assumptions. First, the model does allocate investment expenses consistently—investment expenses are deducted from the gross yield rates in both the portfolio and the investment-year companies (see Sec. II, Assumption N, in the Appendix). Second, the portfolio and the investment-year investments are identical. The average portfolio rate of United States life companies from Table 1 is used as input to the model only to establish a portfolio rate for the *beginning block* as of K30. The portfolio rate in K31 and all subsequent years is a composite of that K30 portfolio rate and the subsequent new-money rates. This is true of both the portfolio and the investment-year companies. Third, the model does use the correct internal rate of return on capital invested in new business in the investment-year companies. Each year the current generation borrows funds at the new-money rate from the old generations. Thus the old policyholders do, in fact, receive the full new-money rate on funds invested in new business.

There is another erroneous statement about the model in Mr. Angle's discussion. He says that our portfolio companies pay a terminal dividend but that our investment-year companies do not. This is not accurate. Identical terminal dividends were used for the portfolio and investment-year companies.

Mr. Angle suggests that we are baiting readers to attack the investment-year method and that we sought to make extreme cases. This simply is not true. In designing the model we necessarily made a number of simplifying assumptions, some of which, as acknowledged earlier, may have tended to overstate the differences in the results. However, our intent was to make the model as unbiased as reasonably possible within the constraints of the necessary simplifying assumptions. We do not believe that the biases that may have been introduced inadvertently produced any significant effect on the results.

Mr. Kabele sets forth his opinions on a wide range of topics, some of which do not relate directly to this paper. We do not necessarily agree with his opinions, but will limit our comments here to the one part of his discussion that bears directly upon the paper. Specifically, he has interpreted correctly our treatment of federal income tax for the various model companies.

We wish to express our thanks to all those who were kind enough to submit their reactions to the paper.