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A MATHEMATICAL MODEL OF THE INCIDENCE OF DISABILITY

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ABSTRACT

Because the disability risk is so volatile, an all-purpose table reflecting age, duration, and the different types of selection is generally thought not to be a realistic goal. Yet, despite all the variables, there are some rather consistent relationships.

The authors have undertaken to create a mathematical model which reflects typical relativities by age, duration of disablement, and deferment period. This model is not a representation of any single body of experience. It is rather a family of equations intended to portray the general shape or pattern of a three-dimensional configuration in which the existence of disability at any point in time is a function of age at disability, duration of disability, and deferment period.

First, some of the factors influencing disability experience and how they are evaluated in the model are discussed. Then the actuarial notation used and the mathematical structure adopted for the model are outlined, and examples of the resulting values are given and compared with relevant experience data. Finally, several applications of the model are described.

I. FACTORS INFLUENCING DISABILITY EXPERIENCE

Deferment Period Selection

DISABILITY rates of occurrence and termination, the contingencies which must be evaluated for premium and reserve computations, have been expressed and presented in various ways. With the exception of the Group Weekly Indemnity Continuation Table [1] and Mr. Barnhart's recently published 1971 Experience Modification of the

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1964 Commissioners Disability Table [2], none of the tables based on North American experience recognizes the phenomenon of elimination or deferment period selection. By this we mean the fact that the probability of becoming disabled at age x and still being disabled at duration t tends to increase as the deferment period e decreases. To avoid notional numbers of disabled lives, it shall always be assumed that t is not smaller than e.

It may be noted that in Sweden an actuarial treatment of disability insurance has been adopted under which a table for a three-month deferment period, but including pro forma values for disablement within the first three months, is modified for shorter deferments by a multiplier which depends on the length of the deferment period [3].

TABLE 1

DISTRIBUTION OF RECOVERIES FROM ACCIDENTAL DISABILITY BY DAY OF RETURN TO WORK

Day of Week when Work Is Resumed	Number per 1,000 Claims	Day of Week when Work Is Resumed	Number per 1,000 Claims
Saturday–Monday	572*	Thursday	105
Tuesday	169	Friday	34
Wednesday	120		
		Total	1,000

* Daily average = 191.

Part of the higher incidence of disability under the shorter deferment periods may be ascribed to a lag in the termination of claims. This lag is due, in part, to the tendency to postpone "recovery" until Monday, as illustrated by the distribution shown in Table 1 of 1,000 claimants under the SUVA (the Swiss occupational accident program) experience of 1965–67 [4].

However, with respect to individual insurance, where the healthier or more responsible applicants seem to choose the longer deferment of benefits, this element of lag or malingering appears to be of relatively small consequence in comparison with deferment period antiselection at issue. When this tendency is recognized in the rate structure, the antiselection could well be progressively intensified.

The existence of this form of selection at issue has been well documented. While first noted in connection with group weekly indemnity insurance by Gilbert W. Fitzhugh [5] and Morton D. Miller [1], its even stronger influence in individual insurance has been discussed recently [2, 6].

A first attempt to create a family of continuance tables with different disability rates for each deferment, but merging at the end of two years of disablement, revealed that such convergence could be achieved only at the expense of severe distortion of the observed data. It was concluded, therefore, that the model should provide for levels of disability differing by deferment period throughout the benefit period. However, in the absence of any data on which to base a differentiation, the termination rates were assumed to be independent of the deferment period after the first two years of disablement. Of course, this results in a uniform relationship among the respective continuance series for durations of two years and longer.

Underwriting Selection

The initial selection of applicants, whether with or without medical examination, does not appear to have an effect on disability experience similar to that generally observed in life insurance. In fact, for the younger ages, there is some evidence that the experience in the early policy years may actually be higher than the ultimate at the same attained age. Only above issue age 50 is there any clear indication that the amount of disability claimed in the early policy durations is significantly lower than the corresponding ultimate experience. However, since the quality of the underwriting of an office is probably of even greater importance for disability than for life insurance, experience under similar policy conditions may differ greatly from one insurer to another in this as well as other respects.

While no published sources of North American experience by policy year are available except with respect to waiver benefits [7], observation of aggregate data compiled for successive years strengthens the conclusion that for some years following issue the experience at the younger ages may improve, while at the higher ages it worsens progressively. Some evidence of these tendencies will be seen in Tables 1, 3, 4, and 5 of an earlier paper in a discussion of initial selection in disability insurance [6].

The authors have been informed that a recent investigation of Swedish experience under waiver of premium benefits showed that claim rates, based on numbers of policies, were higher for the first five policy years than afterward for all age groups under 65.

Although tables of disability claim rates on a select basis have not been published, some insurers use aggregate or ultimate tables and allow discounts for selection in arriving at gross premiums for the higher issue ages only. Lacking a valid statistical basis for reflecting initial selection in the model, we adopted the concept of an "aggregate-ultimate" table. By this we mean an aggregate table with an increasing addition to the observed experience after age 40. Such a basis can be expected to produce adequate active life reserves. For gross premium computations the resulting rates may be redundant, unless discounts are made at the older issue ages based on assumed coefficients of selection.

II. DESCRIPTION OF THE MODEL

Common Characteristics of the Incidence and Persistency of Disability

Although disability experience is marked by tremendous variation from one portfolio to another and, within the same portfolio, over a period of time, there are internal relationships which seem to exist universally. For example, the rate of disability appears to be essentially an exponential function and can be represented by an expression similar to the Makeham law of mortality. The probability of continuance of disability follows another mathematical pattern. There is also a consistency in the relation of incidence and termination rates according to the length of deferment.

The net annual cost of a disability income benefit can be expressed as the discounted value of the product of the claim rate and average claim amount, the latter being the value of a disabled life annuity, which, in turn, depends on the maximum benefit period and on the rates of termination of disability. Although it is convenient, in the analysis of experience and compilation of tables, to deal with the two series of rates separately, a single function will suffice for all calculations of premiums and reserves, namely, the product of (1) the rate of claim and (2) the rate of survival. This single determinant of the cost of a disability benefit is also very useful for experience analysis, since it embraces the contingencies both of incidence and of continuance of claim. Because of the fundamental nature and usefulness of this compound probability, it seems worthy of a name and a symbol. The authors suggest that it be called "disability prevalence probability" and be represented by the term ${}^{e}r_{[x]+t}$. If

$$ {}_{t-e}^{\;\;e}p^{i}_{[x]+e} \tag{1} $$

designates the probability that a person who became disabled at age x and was still disabled at the end of the deferment period e will remain disabled at least until duration t, we have

$$ {}^{e}r_{[x]+t} = {}^{e}r_{[x]+e}\ {}_{t-e}^{\;\;e}p^{i}_{[x]+e}\,. \tag{2} $$

Thus the general term "disability prevalence probability" includes the special situation when $t = e$, the latter being the traditional rate of disability subject to a given deferment period.

In the expression ${}^{e}r_{[x]+t}$ the brackets around x signify selection with respect to the ensuing rates of termination, not with respect to the incidence of disability.

For the construction of tables of the expected number of disabled lives and the definition of commutation functions, the actuarial notation outlined on pages 364 ff. of our earlier paper [6] is used. In order to deal with integral ages at time of disability, it is assumed that the insurance takes effect midway between birthdays. The probability[1] of claim is therefore applied to the number of active lives according to an appropriate mortality table at age $x - \frac{1}{2}$ to produce the number of lives becoming disabled at age x. These values, in turn, are multiplied by the continuance rates—complements of the termination rates—to determine the number remaining disabled for at least duration t; that is,

$$ {}^{e}l^{i}_{[x]+e} = l^{aa}_{x-1/2}\ {}^{e}r_{[x]+e}\,, \tag{3}$$

$$ {}^{e}l^{i}_{[x]+t} = {}^{e}l^{i}_{[x]+e}\ {}_{t-e}{}^{e}p^{i}_{[x]+e} = l^{aa}_{x-1/2}\ {}^{e}r_{[x]+t}\,. \tag{4}$$

If we visualize a curve of disability claims by age with each point the beginning of a continuance curve by duration, we can see that the resulting surface is a complete representation of the incidence and continuance of disability among a stationary population of active lives and that every point on the surface can be defined as a value of the function ${}^{e}l^{i}_{[x]+t}$.

There will be one such surface for each deferment period. From these expressions formulas for commutation functions, net premiums, claim reserves, and active life reserves are easily obtained.

Basic Functions

Using this actuarial notation, a model incorporating the relationships discussed in the first sections has been constructed, based on formulas which will now be presented. The probabilities of becoming disabled at age x and remaining disabled for at least the length of the deferment period e were assumed to be functions of the following type:

$$ {}^{e}r_{[x]+e} = A + Bc^{x}\,. \tag{5}$$

Here e indicates the deferment period; A, B, and c depend on e but not on x. It should be noted, however, that this expression is not a precise analogue of Makeham's law of mortality, because ${}^{e}r_{[x]+e}$ is the probability of becoming disabled, not the force of disability.

[1] The concept may be expressed in terms of either absolute rates or probabilities. The use of the latter simplifies the exposition and the computations.

The basic pattern of the disability prevalence probabilities is expressed by the formula

$$ {}^{e}r_{[x]+t} = {}^{e}r_{[x]+e}\ \alpha^{k_t} , \tag{6} $$

where for each deferment period α is a function of age and k is a function of the duration of disablement.

A modification is required at the higher attained ages where the rising rate of mortality among disabled lives causes the termination rates to increase with duration. For ages over 55 disability prevalence probabilities therefore were developed recursively by the formula

$$ {}^{e}r_{[x]+t} = {}^{e}r_{[x]+t-1/12}(\alpha^{k_t - k_{t-1/12}} - g_{x+t-1/12}) , \tag{7} $$

where g is zero for durations of less than two years and for attained ages under 55. For the shorter and longer durations, intervals other than monthly were chosen.

Evaluation of the Mathematical Model

Numerical values have been derived for five common deferment periods: seven and fourteen days and one, three, and six months. Values used for the various parameters were determined by the following procedure. Target values of ${}^{e}r_{[x]+t}$ were chosen for each deferment period for specimen ages and for up to nine selected durations. The principal sources of experience data used in arriving at those target values were as follows:

1930–50 Experience of eleven United States and Canadian companies under disability benefits supplementary to life policies [7].

1954–63 Disability rates according to Danish bases of calculation in life and pension insurance [8].

1966–69 Experience on men in Occupational Group I of twelve United States companies under individual loss-of-time policies [9].

1965–70 Experience of thirteen United States and Canadian companies under group long-term disability insurance [10].

1955–64 Group waiver of premium termination experience [11].

Swedish experience reflecting deferment period selection.

Since the group long-term disability business is largely on administrative and supervisory personnel and only Group I experience under individual policies was used, the resulting model is heavily influenced by the experience of managers and workers in nonhazardous occupations as well as that of professionals.

The parameters given in Table 2 for the calculation of the probability of disability according to formula (5) were developed to produce reasonable approximations of the chosen target values and a family of curves with internal consistency. The resulting values are illustrated by the figures shown in Table 4.

Provisional values of α and k were determined empirically to approximate the chosen target values of the disability prevalence probabilities at select ages and durations. In order to achieve a high degree of consistency, it was found helpful to have also disability prevalence probabilities for all deferment periods after a common duration. Therefore, for

TABLE 2

CONSTANTS FOR THE PROBABILITY OF DISABILITY WITH INDICATED DEFERMENT

Deferment Period (Months)	A	B	$\log c$
6..........	.0006461	.0000023805	.064650
3..........	.0015491	.000009703	.056775
1..........	.0082131	.0001189104	.044976
14/30......	.033817	.0002548	.042363
7/30.......	.0731	.0004202	.040000

TABLE 3

CONSTANTS FOR THE DISABILITY PREVALENCE PROBABILITIES AT DURATION 2 YEARS

Deferment Period (Months)	A^*	B^*	$\log c^*$
6............	.000308001	.00000116145	.0677944
3............	.000366494	.00000126313	.0672093
1............	.000492777	.00000147223	.0668700
14/30........	.000593505	.00000163205	.0664107
7/30.........	.000668316	.00000184868	.0658192

duration two years a further set of formulas similar to equation (5) has been chosen:

$$^{e}r_{[x]+2} = A^* + B^* c^{*\,x} . \qquad (8)$$

The adopted constants are shown in Table 3 and the resulting disability prevalence probabilities in Table 5.

The function k is basically a polynomial in t, and α is calculated by the formula

$$\alpha_x = \left(\frac{^{e}r_{[x]+2}}{^{e}r_{[x]+e}}\right)^{1/k_2} . \qquad (9)$$

As discussed in Section I, uniform continuance rates were assumed for disablements of more than two years. For all deferments the α_x's calculated with formula (9) for the six-month deferment were used, and a uniform root function in t was adopted for k. This relationship was based on the observation that in the basic underlying continuance table described below the probabilities of remaining disabled at duration 1, 2, 4, 7, 11, 16, . . . years approximate a geometric progression.

When this project was initiated, the most recent termination experience, except for some scant data from group long-term disability, was the 1955–64 experience on group waiver of premium benefits [11]. This experience on group waiver benefits indicated somewhat lower termination rates than the Benefit 2 and Benefit 3 rates [7], despite the fact that the latter, based on income benefits, were distinctly lighter than the corresponding rates under the waiver clause, Benefit 5. To reflect an apparent trend, and to introduce some measure of conservatism, provisional continuance tables based on 75 per cent of the group waiver termination rates were adopted.

The developing termination experience under group long-term disability lends credence to the belief that earlier tables are no longer appropriate, if one is willing to assume that group data on termination experience are relevant to individually underwritten policies. Table D-1B in the latest report on group long-term disability [10] indicates generally lower termination rates as compared with the Commissioners Disability Table rates, which, after the first year of disablement, are the same as those of Benefits 2 and 3 combined [7].

Although our target values for the disability claim rates did not contain any specific contingency margins but only an allowance at the higher ages for the eventual wearing off of selection at issue, the treatment of termination rates was designed to provide reasonable margins for adverse deviations. The rationale for introducing contingency margins in the termination rates and not in the incidence rates is based upon evidence in the Society's 1952 report on disability experience [7], wherein it can be seen that claim rates in Period 1 (1930–35) were as much as 116 per cent higher for all ages combined than those in Period 4 (1946–50), while the accompanying decrease in termination rates was only 17 per cent. While the developing experience as to claim rates can be measured readily by the current reports on new claims, the emergence of credible experience with respect to termination rates obviously takes much longer. Consequently, changes in the incidence of disability can be detected much earlier than changes in claim durations. Of course, the pattern of termination rates can be altered drastically when overinsurance develops through expansion of social security benefits or otherwise, or when administrative policies have the effect, in some situations, of converting disability benefits into unemployment benefits, as has occurred in at least two European countries.

The active life mortality rates have been computed according to the following formula:

$$q_x^{aa} = \begin{cases} 0.025(35 - x) + 1 - 10^{c^{(x-\beta)}} & (x < 35) \\ 1 - 10^{c^{(x-\beta)}} & (x \geq 35)\,, \end{cases} \tag{10}$$

where $\log c = 0.041$ and $\beta = 63.3415$.

Specimen Tables of 1973 Disability Model

The entire set of figures produced, which includes disability prevalence probabilities, commutation functions, one-year term premiums, level premiums, and claim values, is too voluminous for inclusion in this paper, but an extensive presentation of disability probabilities is given in Tables 4–6.

III. COMPARISON OF EXPECTED VALUES ACCORDING TO 1973 DISABILITY MODEL WITH OTHER BASES

One-Year Benefit Period with Short Deferments

For a benefit period of one year and deferments of one month or less the various constants in the model were chosen to produce results which were both internally consistent and reasonably in line with the North American experience of 1966–69, with due allowance for indicated selection at issue operating at the higher ages. Comparisons of claim rates and net annual costs with 5 per cent interest are shown in Table 7.

It seems reasonable, from available data, to assume that the ultimate experience for ages 50–59 will exceed the published experience to date by about one-sixth and for ages 60–64 by one-third. The "ultimate" rates according to this assumption are the basis for section 2 of the comparison shown in Table 7. To make the comparisons at specific ages, the 1971 Experience Table has been used to represent the actual experience.

Second Benefit Year

It was possible to construct the model so as to reflect in a general way both (1) the recent experience under individual loss-of-time policies, which is believed to be heavily weighted by policies with benefit periods not exceeding two years, and (2) experience under long-term disability policies, including both Period 4 experience under benefits attached to

TABLE 4

DISABILITY PREVALENCE PROBABILITIES AT END OF DEFERMENT PERIOD

(Per 1,000)

Age at Disablement	Deferment Period in Months				
	7/30	14/30	1	3	6
20	75.751	35.610	9.157	1.682	0.693
21	76.007	35.793	9.260	1.700	0.700
22	76.288	35.996	9.374	1.721	0.709
23	76.595	36.219	9.500	1.745	0.719
24	76.932	36.465	9.641	1.773	0.731
25	77.302	36.736	9.797	1.804	0.744
26	77.707	37.035	9.969	1.840	0.760
27	78.152	37.365	10.161	1.880	0.779
28	78.639	37.729	10.374	1.926	0.800
29	79.174	38.129	10.609	1.979	0.825
30	79.760	38.571	10.871	2.039	0.853
31	80.402	39.058	11.161	2.107	0.886
32	81.107	39.596	11.482	2.185	0.925
33	81.879	40.188	11.839	2.274	0.970
34	82.726	40.840	12.235	2.376	1.022
35	83.655	41.560	12.674	2.491	1.082
36	84.673	42.353	13.160	2.623	1.152
37	85.790	43.228	13.700	2.773	1.233
38	87.014	44.192	14.299	2.943	1.327
39	88.357	45.255	14.963	3.138	1.437
40	89.828	46.427	15.699	3.360	1.564
41	91.442	47.719	16.516	3.613	1.711
42	93.212	49.144	17.422	3.901	1.882
43	95.152	50.714	18.427	4.230	2.080
44	97.280	52.445	19.541	4.604	2.311
45	99.613	54.354	20.777	5.031	2.578
46	102.171	56.458	22.148	5.517	2.888
47	104.975	58.778	23.669	6.071	3.248
48	108.051	61.335	25.355	6.703	3.665
49	111.423	64.155	27.226	7.422	4.150
50	115.120	67.263	29.300	8.243	4.712
51	119.174	70.690	31.601	9.177	5.365
52	123.619	74.468	34.153	10.243	6.122
53	128.493	78.634	36.984	11.457	7.001
54	133.837	83.225	40.123	12.841	8.021
55	139.697	88.288	43.605	14.418	9.205
56	146.122	93.869	47.466	16.215	10.579
57	153.167	100.022	51.749	18.263	12.173
58	160.892	106.805	56.500	20.597	14.023
59	169.362	114.284	61.769	23.258	16.171
60	178.649	122.528	67.612	26.289	18.663
61	188.833	131.618	74.093	29.745	21.554
62	199.998	141.639	81.282	33.682	24.910
63	212.241	152.686	89.255	38.170	28.805
64	225.665	164.865	98.097	43.285	33.325

TABLE 5

DISABILITY PREVALENCE PROBABILITIES AT DURATION 2 YEARS
(Per 1,000)

Age at Disablement	Deferment Period in Months				
	7/30	14/30	1	3	6
20	0.707	0.628	0.525	0.394	0.334
21	0.713	0.634	0.530	0.399	0.339
22	0.720	0.641	0.536	0.405	0.344
23	0.729	0.648	0.544	0.411	0.350
24	0.739	0.658	0.552	0.418	0.357
25	0.750	0.668	0.562	0.427	0.366
26	0.763	0.680	0.573	0.437	0.375
27	0.779	0.695	0.587	0.449	0.387
28	0.797	0.712	0.603	0.463	0.400
29	0.818	0.731	0.621	0.479	0.415
30	0.843	0.754	0.642	0.498	0.434
31	0.871	0.780	0.667	0.520	0.455
32	0.904	0.811	0.696	0.545	0.480
33	0.943	0.847	0.730	0.575	0.509
34	0.988	0.889	0.769	0.610	0.542
35	1.040	0.938	0.815	0.651	0.582
36	1.101	0.995	0.869	0.698	0.628
37	1.172	1.061	0.931	0.754	0.682
38	1.254	1.138	1.005	0.819	0.746
39	1.350	1.228	1.090	0.894	0.820
40	1.462	1.333	1.189	0.983	0.906
41	1.592	1.456	1.305	1.086	1.007
42	1.743	1.598	1.440	1.206	1.125
43	1.919	1.764	1.598	1.347	1.263
44	2.123	1.957	1.782	1.511	1.425
45	2.362	2.183	1.996	1.703	1.613
46	2.639	2.445	2.247	1.926	1.834
47	2.961	2.751	2.539	2.187	2.092
48	3.336	3.108	2.879	2.492	2.393
49	3.773	3.523	3.276	2.848	2.745
50	4.281	4.008	3.740	3.263	3.157
51	4.872	4.572	4.280	3.748	3.639
52	5.560	5.229	4.911	4.314	4.201
53	6.361	5.995	5.646	4.975	4.859
54	7.292	6.887	6.504	5.746	5.628
55	8.376	7.927	7.505	6.646	6.527
56	9.637	9.139	8.672	7.698	7.577
57	11.105	10.551	10.033	8.925	8.805
58	12.813	12.196	11.621	10.357	10.241
59	14.800	14.113	13.474	12.029	11.919
60	17.113	16.347	15.635	13.981	13.881
61	19.804	18.950	18.155	16.260	16.174
62	22.935	21.983	21.095	18.920	18.854
63	26.579	25.517	24.525	22.025	21.987

TABLE 6

Disability Prevalence Probabilities per 1,000 for Indicated Duration

Deferment Period (e)*	Duration of Disablement												
	8 Days	14 Days	1 Month	3 Months	6 Months	1 Year	2 Years	3 Years	4 Years	5 Years	10 Years	20 Years	30 Years
	Age 27												
7/30	78.152	56.539	30.836	6.357	2.210	1.111	0.779	0.659	0.578	0.517	0.341	0.192	0.125
14/30		37.365	21.065	4.853	1.854	0.983	0.695	0.588	0.516	0.461	0.304	0.172	0.111
1			10.161	3.075	1.413	0.818	0.587	0.496	0.435	0.390	0.257	0.145	0.094
3				1.880	0.983	0.622	0.449	0.380	0.333	0.298	0.196	0.111	0.072
6					0.779	0.527	0.387	0.327	0.287	0.257	0.169	0.096	0.062
	Age 52												
7/30	123.619	99.418	66.109	22.840	11.216	7.062	5.560	5.080	4.727	4.432	3.442		
14/30		74.468	50.822	19.101	10.057	6.591	5.229	4.777	4.446	4.168	3.237		
1			34.153	15.148	8.927	6.155	4.911	4.487	4.175	3.915	3.040		
3				10.243	6.925	5.251	4.314	3.942	3.668	3.439	2.670		
6					6.122	4.965	4.201	3.838	3.572	3.349	2.601		

* Deferment period in months.

individual life policies and the more recent group long-term disability experience.

When deferment period selection is taken into account, the two categories of experience can be accommodated within the same model with relative facility, except for an apparent anomaly with respect to the second year of disablement. In Table 8 the disability cost in the second year of disablement under the individual loss-of-time policy

TABLE 7

RATIOS OF EXPERIENCE ON INDIVIDUAL POLICIES TO MODEL 1966–69 EXPERIENCE—MEN—GROUP I

AGE AT DISABLEMENT	PROBABILITY OF DISABILITY			NET ANNUAL COST FOR FIRST BENEFIT YEAR		
	$e = 7/30$*	$e = 14/30$	$e = 1$	$e = 7/30$	$e = 14/30$	$e = 1$
	1. With No Adjustment for Selection at Issue					
27	96%	91%	94%	106%	94%	80%
37	96	95	98	112	114	86
47	94	104	100	102	106	83
57	86	92	93	74	77	67
62	75	76	82	60	64	64
	2. With Actual Experience Increased 22% for Age 57 and 33% for Age 62 for Assumed Selection					
57	105%	112%	113%	90%	94%	82%
62	100	101	110	80	85	85

* Deferment period in months.

experience is compared with an approximation to the corresponding annual claim cost derived from experience in the second category. It will be noted that, compared with long-term disability income experience, the second-year costs for policies in the first category are (1) considerably higher at ages under 40; (2) somewhat lower, on the average, for ages 40–49; and (3) as much as 44 per cent lower for ages 50 and over.

Similar comparisons with Benefits 2 and 4 in Period 4 showed, for ages 40 and over, second-year costs even higher than the group experience. It will be noted that only the waiver experience, Benefit 5, is consistently lower than the individual income experience, although it is only slightly so for ages 50–59.

The comparisons shown in Table 8 suggest that, analogous to the costs under medical care insurance occasioned by "elective surgery," there are substantial disability costs resulting from what might be termed "discretionary disability." It is fairly obvious that an individual who has a demonstrable bodily or mental impairment will be motivated to seek rehabilitation services or to return to his customary work despite a continuing impairment if his disability benefits definitely terminate at

TABLE 8

PREVALENCE OF DISABILITY OF 12 MONTHS' DURATION
NET ANNUAL COST PER 1 A MONTH OF SECOND YEAR OF DISABLEMENT

Experience	Prevalence per 1,000 at 12 Months' Duration				Net Annual Cost per 1 a Month			
	Under Age 40	40–49	50–59	60–64	Under Age 40	40–49	50–59	60–64
Group LTD—1966–70 experience on men, nonjumbo groups:*								
3-month deferment....	0.84	2.14	6.95	12.62	.0082	.0225	.0771	.1401
6-month deferment....	0.48	1.75	6.03	13.33	.0049	.0188	.0666	.1502
Waiver of premium, benefit 5, period 4........	0.53	1.22	3.73	†	.0050	.0121	.0399	†
Individual loss-of-time‡—men, Group I:								
1965–66 experience....	1.37	1.58	3.79	6.96	.0137	.0139	.0373	.0741
1967–68 experience....	0.81	2.05	4.89	10.28	.0071	.0186	.0475	.1030
1965–68 experience....	1.09	1.83	4.39	8.77	.0105	.0164	.0429	.0897

* Jumbo groups were excluded from claim rates but not from termination rates.

† Not available above age 59.

‡ For individual loss-of-time policies, data based on sickness claims are for the 12-month period beginning 12 months and 7 days after date of disablement.

some point within one or two years following the acute phase of disability. On the other hand, if the benefit is continuable to age 65 or for life, he may opt to enjoy the security and convenience of an insured income rather than to make the effort to obtain gainful employment, with the inherent risk of subsequent unemployment or exacerbation of his impairment.

In Mr. Morton Miller's analysis of group weekly indemnity claims in 1949 [1], evidence was found of selection with respect not only to the elimination period but also to the benefit period. It was concluded, however: "Whereas the data show that it is not possible to represent the 4 day and 8 day plans by the same continuation table satisfactorily throughout, it is possible to combine the data for the same waiting period by duration."

From Table XIV in the paper referred to, it can be determined that the claim frequency under plans with a twenty-six-week benefit period exceeded that for a thirteen-week benefit period by 10 per cent, weighted, or 13 per cent unweighted.

Referring to the more recent group weekly indemnity experience through 1971, we quote from the committee's report [12]: "Perhaps the most interesting trend . . . is the increasing difference between ratios of actual to tabular for thirteen-week plans as compared to twenty-six-week plans. [For 1971 the former] are generally lower than the corresponding ratios for 1969 and 1970, while the 1971 ratios for the twenty-six-week plans are generally higher than at any time in the past."

Although we have this evidence of benefit period selection, no clear pattern has emerged. Therefore, rather than further complicating the model by introducing a variation according to benefit period, we have followed the precedent established by the committees dealing with group weekly indemnity. As a result, the model shows costs generally much higher for the second benefit year than the individual loss-of-time policy experience presented in Table 8. Until more extensive experience is available, it seems preferable to apply special discounts for benefit period selection rather than to introduce this additional variable into basic experience tables or models.

Long-Term Benefits

Table 9 shows comparisons of the 1973 model claim rates with experience on (1) group long-term disability and (2) total and permanent disability in individual life insurance policies for Period 4, 1946–50. Also shown are comparisons of graduated rates, based on experience of 1954–63, which were adopted in 1966 for valuation of the Danish pension system.

Claim Reserves

The use of a disability table which does not allow for deferment period selection is unlikely to produce inadequate premiums or active life reserves, but inequities in the premium structure may result. However, claim reserves can be seriously understated, as explained by Mr. Barnhart [2] and further demonstrated in the latest report on group long-term disability [10]. The reason for this may be seen in the comparisons presented as Figure 1. As any such empirical table must do, the Commissioners Disability Table continuance series starts at a high level and cuts through the corresponding curves for the shorter deferments in progressing to the level of the Benefit 2 and Benefit 3 termination experience. For a long-term benefit this does not greatly increase the present value of future benefits. However, at durations of less than one year the numbers remaining disabled are materially overstated. Thus the resulting claim reserves are understated. This understatement will, in general, increase as the deferment increases.

Since, on a relatively new portfolio, the increase in claim reserves is often the major element in the incurred losses, an understatement of

TABLE 9

LONG-TERM DISABILITY BENEFITS
COMPARISONS OF CLAIM RATES* PER 1,000—MEN

TABLE OR EXPERIENCE	CENTRAL AGES									
	†	22	27	32	37	42	47	52	57	62
	3-Month Deferment Period									
1973 model	2.04	1.72	1.88	2.19	2.77	3.90	6.07	10.24	18.26	33.68
Group LTD, 1966–70	2.13	....	....	....	....	3.51	5.60	9.12	14.21	18.28
Individual, Benefit 2, Period 4	....	....	....	1.35	1.96	3.12	4.81	8.89	16.03	.....
	6-Month Deferment Period									
1973 model	0.85	0.71	0.78	0.93	1.23	1.88	3.25	6.12	12.17	24.91
Danish pension plan‡	....	0.63	....	0.74	....	1.37	2.36	4.64	9.85	21.80
Group LTD, 1966–70:										
Nonjumbo cases	0.71	....	....	....	....	1.72	2.81	5.56	8.60	14.80
All experience	0.70	....	....	....	....	1.78	3.12	6.09	9.87	17.27
Individual:										
Benefit 2, Period 4§ (adjusted)	....	....	....	0.80	1.19	1.97	3.23	6.43	12.64	.....
Benefit 5, Period 4‖ (waiver only)	....	0.61	0.69	0.63	0.82	1.29	2.14	3.84	6.78	.....

* Danish figures are for force of mortality; model figures are probabilities of disablement; other data are rates of disablement.

† All ages under 40, compared with age 30, from the model.

‡ The Danish plan does not incorporate a fixed elimination period but requires a two-thirds reduction in earning power, which reduction "must be considered to have a certain permanency."

§ The comparison of the Period 4, 1946–50, disability income experience with the model was made by first adjusting the Benefit 2 (3-month deferment) to a 6-month deferment on the basis of published termination rates representing the experience of 1930–50. These pro forma rates were then further adjusted to allow for deferment period selection. This was done by reducing the computed values in the ratio of $^{6}r_{[x]+6/12}$ to $^{3}r_{[x]+6/12}$. Similar modifications of Benefit 4 data produced, for ages over 45, somewhat higher rates than those shown for Benefit 2.

‖ The comparison of Benefit 5 with Benefit 2 (adjusted) indicates the substantially higher claim rates when a cash benefit is paid as compared with premium waiver only. The actual cost difference is even greater, owing to the higher termination rates under the waiver benefit. Of course, some of the reduction in claim rates under the waiver benefit results from terminations of disability between the expiration of the deferment period and the due date of the next premium.

In 1970 the group long-term disability experience with six-month deferment was, according to the committee's report [10], "about 25 per cent higher than the years immediately preceding." The rates shown in Table 9 for Benefits 2 and 5 are observed, not graduated, rates.

these reserves is very serious. This is particularly so with respect to the higher ages, not only because of the greater frequency of claims but also because of the tendency, noted earlier, for these frequencies to increase as the business matures. Incidentally, in Table D-1C of reference [10] the ratio of 114 per cent in the penultimate line of the table should be 144 per cent.

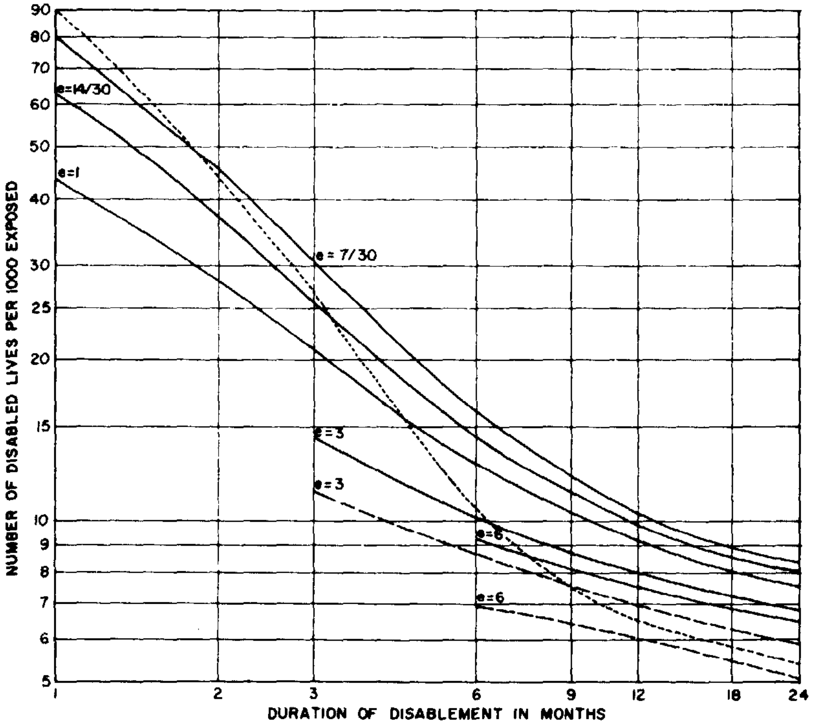

FIG. 1.—Disability prevalence probabilities, for indicated ages at disablement: ——— = 1973 disability model—age 55; — — — = group long-term disability (1966–70 incidence rates, 1962–70 termination rates)—ages 50–59; - - - - - - = Commissioners Disability Table—average of ages 52 and 57.

IV. THE COMPUTER PROGRAM

The parameters for the model were developed largely by manual methods, with some use of charts and computer calculations. For the application of the mathematical model, with either the standard parameters or optional alternate parameters, a comprehensive computer program was developed, capable of producing all the basic probabilities and commutation functions, as well as net premiums and claim values.

The computer program allows the following modifications of underlying mortality and morbidity assumptions: (1) use of any mortality table; (2) modification of the standard active life mortality by a percentage rating, a flat loading, an age adjustment, or a combination of these variations; (3) similar but independent variations in the disablement probabilities; and (4) modification of the termination rates.

Although only the five deferment periods mentioned were provided for in the computer program, other specifications which may be varied are the rate of interest; the benefit period, subject to an age 65 limit; and the maximum covered age, subject to an age 65 limit. In any future revision of the computer program it is intended to remove the age 65 limit and to provide for evaluation of benefits continuing to any desired terminal age.

No provision for lapse rates was included, but a level lapse rate could be reflected by a constant addition to the active life mortality rates. Alternatively, the model program can be used to produce net annual costs which, in turn, could be used in any standard life or disability premium computation program along with the assumed lapse rates.

V. APPLICATION OF THE MODEL

Use of the Model as a "Translator"

Because the model is based on the fundamental function, disability prevalence at each point in time, the computer program can readily translate these values into equivalent values according to any other form or system of expressing the incidence and extent of disability. For example, the first expression on page 192 of the Society's text *Health Insurance Provided through Individual Policies* [13] can be written in terms of the notation of the model as

$$ {}^{d}S_x^{1/12} = \frac{1}{D_x^{aa}} \left({}^{1}H_{[x]+1/12} - {}^{1}H_{[x]+1+1/12} \right) . \qquad (11) $$

Perhaps the earliest actuarial treatment of the disability risk was Richard Price's concept of an assumption as to the portion of the population disabled at any given time, as cited in reference [14]. This concept is currently used in Great Britain in connection with friendly society statistics and also in actuarial practice in Norway. It is frequently adopted for government or medical research studies. By use of the model, the "portion disabled" at age x can be developed from the underlying probabilities by means of the formula

$$ {}^{e}j_x = \frac{1}{l_{x-1/2}^{aa}} \int_{x_0}^{x-e} {}^{e}r_{[z]+x-z}\, l_{z-1/2}^{aa}\, dz , \qquad (12) $$

where x_0 is the lowest age in the relevant table or population. By an approximation employing a formula similar to equation (12) above, the comparison shown in Table 10 has been made between Manchester Unity rates of sickness and corresponding amounts developed from the model. This comparison should be assessed with some caution because (1) the Manchester Unity rates do not reflect the seven-day deferment period, the minimum covered by the model; (2) approximations were used in applying the model; and (3) the experience upon which the standard parameters of the model were based may have treated certain cases of recurring illness on a basis different from that implicit in the Manchester Unity experience.

TABLE 10

COMPARISON BETWEEN MANCHESTER UNITY TABLE (AHJ) AND DISABILITY MODEL (DM)

(Rates of Sickness in Weeks per Annum)

Age	First 3 Months	Second 3 Months	Second 6 Months	First 12 Months	Second 12 Months
27:					
AHJ	0.635	0.085	0.060	0.780	0.038
DM*	0.376	0.048	0.039	0.463	0.047
42:					
AHJ	0.803	0.159	0.124	1.086	0.098
DM*	0.495	0.087	0.079	0.685	0.103
57:					
AHJ	1.262	0.405	0.407	2.074	0.400
DM*	1.044	0.347	0.417	1.808	0.629

* The disablement rates of the model with deferment period 7 days were used.

The Model as a Tool for Graduation, Simulation, or Other Analysis

The model can also be helpful in a variety of calculations, for example, for the following:

1. To project known experience into new areas; an insurer with a large volume of eighth day/one year policies might compare this experience with the model and then deduce a consistent basis for a fifteenth day/two year policy.
2. To facilitate the graduation of insurance experience data and the development of premiums reflecting such experience.
3. To calculate rates, premiums, and reserves on a consistent basis where available experience provides a credible basis for modification of the standard parameters, as for groups for which lower termination rates are likely as a result of particular economic conditions or for insurance in countries where local statistics are lacking but where there is an acceptable basis for modifying the standard parameters.

The model provides a convenient basis for simulating disability rates for valuation or other purposes, in the absence of adequate data derived from actual experience. For example, it has often been observed that disability rates in pension funds or retirement plans, especially public plans, exhibit an age incidence very different from that found in disability insurance. This is quite evident in the experience of public funds in Switzerland [6] and in comparisons of the disability rates in the Railroad Retirement Plan with those under OASDI. Such rates are often very low at the younger ages, reflecting careful selection of new employees, effective rehabilitation measures, or both. Also, a minimum period of service as a condition of eligibility may depress these rates. At the older ages, the normal increase may be sharply accelerated, perhaps reflecting a tendency on the part of employers to retire for disability the redundant employees with long service or those who have become ineffective. By reason of the underlying Makeham-type formula, the disability rates can be steepened readily to reflect such influences. Appropriate modification of the termination rates also can be achieved easily.

Evaluation of Changes in Individual Assumptions

The model also lends itself to evaluation of the financial effect of changes in any of the assumptions underlying the premium structure or reserve basis. As an example, Table 11 illustrates the effect on premiums of a number of modifications. With the high rates prevalent today, interest has become a more important factor, especially in plans which result in substantial claim reserves.

The indicated modifications in the frequency of disability were also applied to the active life mortality. Predictably, the rating in age corresponds closely to a percentage rating, except for one-year term rates at the younger ages, where the constant element of the Makeham formula dominates.

The mortality assumption has the least impact of all. Its effect is parallel to that of the lapse rate but is generally at a much lower level. The interest assumption exerts a similar effect on active life reserves, but this is compounded by the influence of interest on the claim reserves.

The disability termination rate operates from the inception to the termination of the claim and therefore any change in these rates has increasing effect as the age at issue or the deferment period is decreased or as the benefit period is increased. The fact that the impact of a change in termination experience is greatest at the younger ages, where the low incidence rates result in relatively greater variability in the frequency of claims, is an added reason for the use of conservative assumptions as to the rates of termination of disability.

TABLE 11

EFFECT OF VARIOUS MODIFICATIONS OF STANDARD ASSUMPTIONS
ANNUAL PREMIUMS PER 1,000 OF YEARLY BENEFIT PAYABLE MONTHLY*

Modifications	Net Level Premiums Ages			Yearly Renewable Term Net Premiums Ages		
	27	42	57	27	42	57
	I. Benefit Period of 2 Years; Deferment 1 Month					
Standard assumptions†	8.489	17.378	41.125	2.571	5.425	27.917
Interest, $3\frac{1}{2}\%$	10.080	18.958	42.195	2.613	5.519	28.464
	(119)	(109)	(103)	(102)	(102)	(102)
Morbidity:						
150% r_x	12.375	25.530	61.466	3.856	8.137	41.876
	(146)	(147)	(149)	(150)	(150)	(150)
$r_x + 0.001$	8.725	17.723	41.690	2.824	5.736	28.456
	(103)	(102)	(101)	(110)	(106)	(102)
r_{x+3}	10.396	22.113	54.229	2.750	6.469	36.474
	(122)	(127)	(132)	(107)	(119)	(131)
Mortality, 200% q_x^{aa}	8.023	16.673	40.829	2.571	5.425	27.917
	(95)	(96)	(99)	(100)	(100)	(100)
Terminations:						
75% q_x^i‡	12.351	24.250	54.107	4.277	8.581	38.370
	(145)	(140)	(132)	(166)	(158)	(137)
125% q_x^i	5.864	12.451	31.142	1.552	3.423	20.197
	(69)	(71)	(76)	(60)	(63)	(72)
	II. Benefit to Age 65; Deferment 6 Months					
Standard assumptions†	12.349	25.586	42.824	3.101	9.433	45.503
Interest, $3\frac{1}{2}\%$	15.356	28.655	44.464	3.537	10.604	48.283
	(124)	(112)	(104)	(114)	(112)	(106)
Morbidity:						
150% r_x	18.134	37.979	64.457	4.651	14.149	68.254
	(147)	(148)	(151)	(150)	(150)	(150)
$r_x + 0.001$	16.613	29.966	45.310	7.083	14.445	49.241
	(135)	(117)	(106)	(228)	(153)	(108)
r_{x+3}	17.443	38.093	66.180	3.398	12.920	69.760
	(141)	(149)	(155)	(110)	(137)	(153)
Mortality 200% q_x^{aa}	11.838	25.054	43.116	3.101	9.433	45.503
	(96)	(98)	(101)	(100)	(100)	(100)
Terminations:						
75% q_x^i‡	14.684	29.344	46.713	4.274	11.836	51.037
	(119)	(115)	(109)	(138)	(125)	(112)
125% q_x^i	10.503	22.423	39.335	2.300	7.580	40.649
	(85)	(88)	(92)	(74)	(80)	(89)

* Figures in parentheses are percentages of modified to standard premiums.

† Standard assumptions refer to the unmodified model and an interest rate of 5 per cent.

‡ The symbol q_x^i is used here to represent combined rate of termination by death and recovery.

Uses of Model for Problem-solving

Many academic as well as practical questions are solved easily by use of the model. As an example, take the sales promotion misstatement discussed in reference [15] by Mr. Robert L. Whitney: "For a man aged 35 there is a 50-50 chance that he will be disabled for at least 90 days continuously before he reaches age 60." On the basis of the standard disability model assumptions, and disregarding multiple occurrence of disabilities, a probability of 19.3 per cent has been calculated for a man aged 35 to become disabled before reaching age 60, and for terminal age 65, 33.1 per cent, compared with Mr. Whitney's 18.3 per cent and 27.3 per cent calculated on his "low" basis. To take multiple occurrences into account, the population was divided into two groups, those who already had a disability of at least three months, and the others. Counting only the number of people of the second group who became disabled reduces the probabilities to 17.3 and 27.4 per cent, respectively. Those figures should, however, be considered as upper limits, since undoubtedly those who already were disabled have a higher probability of dying or, if they have recovered, of becoming disabled again, which would further reduce the probabilities. Assuming that the people already disabled have mortality and disablement rates 50 per cent above those for the entire population results in probabilities of 16.6 and 25.6 per cent, respectively. If it is assumed that the extra mortality and disablement probabilities are 200 per cent, the figures are further reduced to 14.9 and 21.8 per cent. It is felt that those figures should indicate about the range of the two probabilities. Calculations made with different combinations of extra mortality and extra disablement again showed that the assumption as to active life mortality is of minor importance.

Use of the Model as a Bench Mark for Comparisons

The comparison of one disability experience with another, unless there is reasonable assurance of homogeneity, is fraught with uncertainty. It is believed that, to some extent, such comparisons will have more validity if related to a single standard which reflects typical internal relationships by age, duration, and deferment period. Thus the model might be useful in comparing trends in short-benefit-period individual policy experience with those of long-term group policies or in comparing trends in two portfolios with markedly different distributions by age or deferment period.

Comparisons between different countries may be facilitated by use of the model as a common yardstick, but of course any indications from such comparisons should be assessed with great caution.

Concluding Remarks

We should like to emphasize that the model is conceived not as an experience table but as an instrument for calculations and comparisons. Appropriate use with or without modifications in the standard parameters requires the most careful actuarial judgment. Opportunities for improvement in the structure of the model will doubtless be found as it is used, and as new and continuing investigations of morbidity experience become available. It is our hope that, whatever shortcomings may exist in the results of our efforts, we will have encouraged other actuaries to contribute to the greatly needed extension of knowledge concerning the disability risk.

ACKNOWLEDGMENTS

The authors wish to express their indebtedness to their actuarial colleagues and others in the Swiss Reinsurance Group who have assisted in this effort, especially to Gottlieb Schmidli for his skillful programming and help in selecting the parameters giving the best fit to the target values, as well as to Baruch Berliner, Anni Binder, and Karl Küstahler for their respective contributions.

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DISCUSSION OF PRECEDING PAPER

JOHN C. ANGLE:

John Miller and Simon Courant thoughtfully share with us the method of construction of a computer program that will project the incidence of disability for any desired combination of age, duration of disability, and deferment period. The resulting model promises to be of great value as a tool of analysis. As we share with the authors our pride in receiving this creative work, it seems important to suggest limitations to the underlying theory as an explanation of all conditions of disability.

Thus I shall probe the assertion that the same sort of deferment period selection occurs under both individual and group insurance. I will also consider the notion that discretionary disability exists as an observable phenomenon and is related to the length of the benefit period. Finally, I shall comment on the model's disregard of variations in the definitions by which insurer and insured will attempt to verify "disability."

In their 1972 paper the authors suggested that over four-fifths of the cyclical variation in disability experience was attributable to variations in the frequency of disability. This led them to suggest that an ongoing analysis of claim frequency alone could provide a sensitive gauge of current disability experience. In seeking a basis for monitoring the rate of disability claim, the authors found the 1964 Commissioners Disability Table (CDT) unsuitable for some deferment periods and proceeded to develop the model here presented, which describes a bench mark for the incidence of disability.

While the complete Miller-Courant model is defined by equations, parameters, and instructions that form a part of a computer program, this paper explains at least the basic equations and criteria of the model. The most significant of these criteria, in the eyes of the authors, is one explicitly recognizing deferment period antiselection. Thus the authors present separate parameters for each of five deferment periods to permit projection of incidence of claims reflecting the past behavior of purchasers of disability insurance. Healthier, morally upright men and women apparently select longer deferment periods.

To complete the foundation of the model, the authors are forced to incorporate a variety of found materials. Health insurance actuaries, as a matter of practical necessity, must scour the countryside for material and produce collages rather than single-medium paintings. Such an eclectic approach underlies the Conference Modification of Class (3) Disability Table and the 1964 CDT.

Parameters in the 1973 disability model reproduce target values based upon disability claim frequencies and claim termination rates drawn from such seemingly disparate types of insurance as ordinary waiver of premium benefits, group life waiver of premium benefits, individual accident and sickness short-term loss-of-time benefits, and group long-term disability benefits. Significantly, the authors employ 75 per cent of the disability termination rates of those awarded group life insurance waiver of premium benefits, thereby reflecting both the limited suitability of the experience and an apparent decline in disability termination rates observed in other investigations.

I must ask about the authors' assertion that "there are internal relationships which seem to exist universally" (as to patterns of disability). If the authors mean that the rate of disability can always be expressed as an exponential function of age, or continuance of disability by a polynomial function of duration of disability, I can readily agree. But, if the phase carries the implication that the authors have uncovered universally applicable parameters, then I must object that the case has not been proved.

For one thing, the data are largely limited to less hazardous, white-collar occupations. In examining the fit of the model to group data, one scarcely finds proof of universal applicability of the model. There are, the authors point out, substantial differences between second-year disability benefit costs for group and individual disability benefits. Nor are variations in short-term weekly indemnity experience necessarily extensible to longer deferment periods. Thus the group experience seems not easily accommodated by a model with variables limited to deferment period, age at disability, and duration of disablement. The authors explain the variance of group results from the predicted level of disability as an anomaly caused by "benefit period selection," which gives rise to "discretionary disability."

However, the differences between individual and group insurance entail such significant differences in the insured person's responsibility for adverse deferment period selection and in the sources of moral hazard that I am reluctant to dismiss the differences as mere anomaly. How can one explain imputed behavior that requires hundreds of employees insured under a single group long-term disability contract to exercise collectively the same deferment period antiselection as a single buyer?

If the explanation of the second-year difference in group long-term disability claim costs stems from human behavior that is characterized as discretionary disability, we are left with a serious dilemma. Discretionary disability, if it exists, has ominous portents for individual disability insurers. This is because long, and supposedly benign, deferment periods are commonly paired with long-term benefits. Thus, even moderate amounts of discretionary disability elicited by to age 65 benefit periods will counteract the reductions in disability supposedly achieved by long deferment periods. Additionally, the possibility of discretionary disability may dash our confidence that variations in the rate of disability alone can serve to monitor trends in disability claim costs.

Actually, I believe that we lack sufficient assurance of the homogeneity of group and individual claim experiences to permit sweeping generalizations about possible conceptualizations of universal relationships. As to moral hazard, individual experiences are immune from one influence affecting group experience. This is the inclination of some employers to regard group long-term disability benefits as early retirement benefits.

Finally, there is the vital matter of the definition of disability, which the courts, legislatures, and insurance departments are continually modifying. I believe that any characterization of the disability risk, mathematical or otherwise, must take this into account. Target data incorporated in the model presumably reflect experience under clauses that seem conservative by today's standards. The ordinary waiver of premium clauses, for example, spoke of total and permanent disabilities. One must ask to what extent this experience is transferable to contracts promising disability benefits to age 65 while a disabled insured is unable to engage in his regular occupation or profession, or to the income loss and residual disability contracts that have recently been introduced in the United States. How clearly do these contracts define disability? How will they be viewed in the social milieu of A.D. 2000?

Perhaps these questions arise because actuaries share with economists and other social scientists a discipline in which human behavior, often unpredictable, is an important determinant of outcome. While the Miller-Courant model ascribes values to the behavior of applicants based upon their selection of deferment periods, it cannot hope to represent other behavioral changes acknowledged by the authors as regulating the disability risk. Perhaps one day we shall join economists in including an exogenous variable in our models to measure the influences of forces not explained by a model.

I conclude after reading this paper that it is important to build models but that we should maintain some skepticism about the projections of any model. The art of model-building forces a beneficial analysis of trends in the incidence of disability. Indeed, it seems to give a better basis of expected claims than the 1964 CDT. However, neither mathematics nor a set of computer programs can move us much closer to certainty about future disability claim costs.

Perhaps it is unfortunate that there is so much criticism of the slope of claim costs in the 1964 CDT. In all fairness to Eduard H. Minor and those who shared in the effort, the 1964 CDT was conceived as a valuation table that could accommodate all occupations and ages. Minor noted the curve of claim costs to be less steep for blue-collar than for white-collar risks and suggested that outdoor workers have more robust health after age 50 than their indoor counterparts. He also pointed to "significant differences in the termination rates of the second 12 months, depending on whether payments cease at or continue beyond the end of the second year."[1] If we are to draw a lesson from some of this disenchantment with the 1964 CDT, it may be only that tables universally applicable for valuation, pricing, and analytical purposes are not to be found on this earth.

In a panel discussion several years ago, Dr. Denis F. Johnson, a scholar with long experience in the difficulties of model-building, remarked that "our ability to predict economic and demographic phenomena presupposes our ability to explain these phenomena; given the highly contingent and partial nature of our explanations of these phenomena, it is hardly surprising that our predictions may be characterized in the same manner."[2]

A similar theme was sounded by economist Wassily Leontief in his 1970 presidential address to the American Economic Association. Out of concern for a spreading predilection for mathematics and model-building, Leontief entitled his address "Theoretical Assumption and Nonobserved Facts." He cautioned economists to restrain their enthusiasm for mathematics, which so often concealed what he called the ephemeral arguments hidden behind algebra. What is more important, Leontief concluded, is to find the facts by which we can judge the assumptions contained in our models.[3]

Perhaps the Miller-Courant model can be put to use to find the facts that will help us better understand the incidence of disability. It does seem advisable to caution those who might mistakenly expect infallible predictions of future incidence of disability from a "mathematical model" that claimed to have established "universal relationships" or Newtonian laws of disability. The reliability of any man-made model will be limited by the unpredictable nature of human behavior.

[1] *TSA*, XVII (1965), 370–72.

[2] *TSA*, XXIII (1971), D561.

[3] *American Economic Review*, LXI, No. 1 (March, 1971), 1–2.

E. PAUL BARNHART:

First of all, I want to express my own distinct appreciation for this new contribution to our literature and available tools for working with the disability risk. We really have not had very much available to us in the nature of mathematical techniques for analysis and evaluation of this risk. My own paper on "Continuance Functions," published fifteen years ago, was intended as one comprehensive effort toward this end, and this valuable additional work presented by Mr. Miller and Dr. Courant should provide a major alternative type of mathematical model. Further, while my continuance function technique obviously can be used as a basis for construction and comparison in terms of mathematical models, it was developed and has been used with emphasis primarily on graduation of data and power and convenience in calculation of values. The authors have expanded further the scope and value of the general concept of mathematical representation of the disability risk by placing primary emphasis on the underlying mathematical model itself and its general applications. It would seem useful to draw some comparisons between the two techniques to obtain some feeling for their relative advantages.

Conciseness of Definition

The authors have defined the rate of disability in terms of a Makeham type of function in which age at disablement becomes an intrinsic parameter, and this makes it possible to express the other functional parameters in an extremely concise manner. I have never introduced age directly as an intrinsic functional parameter in working with continuance functions but instead have only kept note as to whether the parameters of a specific graduation seem to maintain a reasonable relationship by advancing ages (usually quinquennial). In general, they have not performed too well in this respect; generally, I have had to sacrifice this characteristic in favor of maintaining a more faithful fit to the underlying data, which in turn suggests that any attempt to define the various parameters directly in terms of age would not prove very satisfactory. In this respect, I wonder how well the Makeham function used by the authors maintains fit at specific ages, and I would value whatever further comment they might offer on this point. If a satisfactory fit at specific ages does seem to be realizable, then the type of function employed by the authors definitely would appear to be more concise in definition and consequently mathematically more elegant than the type of function I have generally used, which as a practical matter usually means working specifically at quinquennial ages and then interpolating final values.

Fit

To consider further the general criterion of fit, I have found almost invariably that a single function, or "element," as I have termed it, will not produce satisfactory fit over all durations (except when applied to some of the hospital confinement continuance data I have worked with). Where disability continuance is concerned, I have always had to employ at least two elements, and occasionally even three, to define the entire range of continuance, even when these elements are adjustable by deferment period. In general, I have favored expressing the continuance as the sum of the values produced by each element, so that each element contributes somewhat to the "disability prevalence probability" at all durations, although each single element normally will predominate over a certain portion of the total range. This proves convenient for calculation of values from the resulting functionalized table, but it also considerably complicates the initial graduation. The alternative is to segmentize the total continuance range and fit each successive segment with its own single graduating element. This simplifies the initial graduation but complicates both the definition of the functions and later calculation of values. I have used this approach rather seldom, although it would appear to be necessary to use the segmentation method if one is to apply the criterion of rates of termination, beyond a certain duration such as two years, which no longer vary by deferment period, as is done by the authors.

As I understand the technique employed by the authors, it is also essentially a segmentation approach, with the dividing point set specifically at two years of disability duration, so that essentially a "two-element" graduation is used. Here again, I would find it very valuable if the authors could give some general comment as to how well this particular approach maintains fit by duration of disablement. Possibly a somewhat more faithful fit can be maintained under the authors' approach, using only two segments connecting at two years' duration, than I usually have been able to maintain under the technique I have been using.

One point at which the authors' technique appears to become relatively awkward is at the higher ages, where it becomes necessary to deal with an increasing rate of termination because of mortality among disabled lives. I have found it possible to handle this problem quite neatly by employing a "lambda" function as my second continuance element at the higher ages. This function has an increasing rate of termination and tends naturally to predominate over the continuance at the longer durations, so that both decreasing and increasing termination rates along the same continuance curve can be readily dealt with.

Computational Convenience and Efficiency

With sufficiently fast and powerful computer equipment, the question of computational convenience and efficiency is undoubtedly a secondary criterion, but I think it remains an important one. Calculation of continuance values, using the type of integrable continuance function I have been working with, is extremely efficient and can be accomplished rapidly and conveniently on a programmable calculator such as the WANG 720 or 2200. Interest discount has to be incorporated in an approximate manner (to achieve maximum efficiency in the integration calculation), but usually I have found the approximation to be sufficiently accurate to cause little trouble. It would not appear that calculation of values such as disabled life annuities and reserves is as efficient a process using the authors' technique, although I may well be misjudging this, and I would invite any additional comment that the authors care to offer on this point.

Adaptability and Ease of Modification

The criterion of adaptability is, of course, a critical one in relation to any technique to be employed in a general mathematical model. With age incorporated as an intrinsic parameter, I think that the technique employed by the authors appears superior where modification by advancing age is concerned, as in introducing progressive conservatism into the values with increasing age. My parameters, while they can be adjusted readily enough on the basis of some set rule or formula, nevertheless have to be modified age by age, and, in the basic modification, it becomes cumbersome to deal with age intervals any smaller than quinquennial.

Modification by deferment period or by duration of disability, on the other hand, seems somewhat more convenient and flexible under my functions, although either technique seems to be readily adaptable to such modification.

Miscellaneous Comments

By way of offering a few minor miscellaneous comments, it seems to me that the authors' notation for the "disability prevalence probability," ${}^{e}r_{[x]+t}$, could be expressed somewhat more logically as ${}^{[e]}r_{x+t}$ (or perhaps ${}^{e}r_{x+[t]}$), since the selection relates to the deferment period or duration of disability rather than to age itself.

One comment relating to mortality tables. Unless a disability table is being built for valuation or other use which requires some specific mortality table, I have found it generally both adequate and convenient to dispense entirely with the use of any specific mortality table in calculating disability experience net or gross premiums. The calculations can be based entirely on a total persistency decrement—particularly if this is graded by some convenient formula both as to entry age and policy duration—as long as the ages do not extend beyond 65, when increasing mortality begins to become too significant to ignore or to assume to be inherent in the total persistency assumption. The authors make the observation that "the mortality assumption has the least impact of all," and I have found that usually it can be ignored entirely as a specific decrement and instead simply be regarded as being provided for in a rough way as inherent in the total persistency decrements employed in the calculations. Experience lapse data are usually available, in any case, in the form of total terminations, not broken down between mortality and other causes.

Again, I greatly appreciate this valuable additional tool developed and presented by Mr. Miller and Dr. Courant, and I anticipate that it will find many practical applications.

OVE LUNDBERG:*

I would like to submit the following comments.

1. An obvious change in the claim losses of Swedish disability income insurance took place in the middle of the 1960's. Having been satisfactory until that time, the risk results began to deteriorate in an accelerated way. A Swedish actuarial report of 1972 for individual insurance threw light on this change.

2. On the basis of the aforementioned experience, a Swedish model of 1973 has been applied as a basic assumption for long-term disability insurance (LTD). I refer to C. G. Dillner's paper in the *Skandinavisk Aktuarietidskrift.* (1974). It is interesting to compare the Swedish model with the 1973 model presented by Miller and Courant. The following differences are deserving of notice.

a) While the model presented defines the probability of becoming disabled and remaining disabled at least the length of the deferment period e, the Swedish model defines in a formal way for each deferment period e the force of becoming disabled at age x, by the product

$$\nu_x^e = r_e\left(\frac{0.4}{l_x}\right),$$

where r_e is the factor of the deferment period e and

$$-\ln l_x = \int_0^x \mu_t dt = \alpha_x + \beta\left(\frac{e^{\nu x} - 1}{\gamma}\right),$$

* Dr. Lundberg, not a member of the Society, is editor of the *Skandinavisk Aktuarietidskrift.*

α, β, and γ being the Makeham parameters of the mortality table M64 of Swedish life insurance with $\alpha = 0.0006$, $\beta = 0.000034$, and $\gamma = 0.042/\log e$.

b) The logarithm of the survival function is not a polynomial $k(t)$ in t as in the model presented, but the survival function itself is a weighted sum of exponentials $e^{-a_\nu t}$ ($\nu = 1, 2, \ldots, 5$), where t is measured from the beginning of the disability period and the weights are dependent on the age x.

c) The dependence of the deferment period e is, according to the Swedish model E72, limited to periods e shorter than three months. The survival function is assumed to be independent of the deferment period e. The deferment factor r_e is assumed to be independent of age. It is set equal to 1 for $e \geq \frac{3}{12}$ and for $e = \frac{1}{12}$ is equal to 1.40. This means that the rates of becoming disabled at all ages and for all durations $t \geq \frac{3}{12}$ year are assumed to be 40 per cent higher for a deferment period of one month than for a deferment period of three months or longer.

TABLE 1

RATES PER 1,000 OF BECOMING DISABLED AND REMAINING DISABLED FOR 6 MONTHS

Age	LTD Males		Age Group	Group		
				Males		Females G74
	I64	E72		G64	G74	
			25–29	1.9	1.1	3.4
30	4.0	4.5	30–34	2.5	1.5	4.4
			35–39	3.2	2.0	5.6
40	5.7	7.1	40–44	4.3	3.0	7.1
47	7.6	10	45–49	6.1	4.8	9.2
52	9.6	14	50–54	9.3	8.9	12.1
57	13	21	55–59	15.0	16.5	16.7
62	17	33	60–64	25.0	32.6	24.1

3. As Simon Courant has said in his comments, the level of disability experience in one country gives practically no indication of what is to be experienced in another. The social security schemes are different, and the policy conditions are also different.[1] It can, however, be of interest to notice that the rates of becoming disabled at age x (<60) and remaining disabled for at least t months are, for $t < 2$ years, essentially higher according to the new Swedish model (E72) than according to the model presented by Miller and Courant. The differences are, however, partly compensated in the net premiums by higher termination rates (cf. item 5).

Table 1 of this discussion shows, for $t = \frac{1}{2}$ year, rates according to the new Swedish model E72 of LTD in comparison with the former model I64.

[1] The Swedish LTD conditions include partial disablement of at least 50 per cent.

For comparison the rates according to models G64 and G74 of Swedish group pension disability also are given. The model G74 (which commencing in 1974 is to be applied as the basic assumption for group pension disability) has not yet been presented for publication.

4. The differences in the rates between males and females can be studied in Table 1 for group pension disability. The curve by age is not at all as steep for females as it is for males. The disability rates have, according to the new experience, been reduced for females of all ages up to 60 years, beyond which age there does not exist any experience before 1970.

Statistics of 1971 from the Swedish National Insurance show that the rates of being disabled for ages above 50 years are higher for males than for females. Also, grouping according to income will show the comparison to be still more favorable for females.

5. The effects on survival functions of changes in termination rates are illustrated in Figure 1 of the paper. The survival functions of Swedish individual disability insurance (LTD) give for two and ten years the following probabilities per thousand of remaining disabled beyond six months' duration:

Duration in Years	Males and Females Age at Onset of Disablement					
	40		50		60	
	I64	E72	I64	E72	I64	E72
0.50	1,000	1,000	1,000	1,000	1,000	1,000
2	240	239	287	324	349	472
10	89	102	109	165	160	277

It has been considered justified to use the same assumption for females as for males, but it must be said that the statistical experience for females is poor.

For group pension disability insurance the "lingering" of the disability periods is still more obvious for disabled men. For females the "lingering" is less pronounced, but the termination rates for females are still assumed to be a little lower than those for males.

6. The claim reserves are calculated for the outstanding claims on the basis of the disabled life annuities. For individual disability insurance (LTD) the disabled life annuities are calculated up to an ultimate age 67 (Table 2 of this discussion). The basic force of interest corresponds

TABLE 2

DISABLED LIFE ANNUITIES TO AGE 67

DURATION IN YEARS	INDIVIDUAL LTD—MALES AND FEMALES AGE AT ONSET OF DISABLEMENT								
	40			50			60		
	I64	E72	P	I64	E72	P	I64	E72	P
0.25..	1.1	1.6	10.9	1.2	2.2	8.9	1.2	2.2	5.0
0.50..	2.2	2.4	11.0	2.3	2.9	8.9	1.9	2.6	4.8
2.....	6.7	7.3	11.0	5.7	6.8	8.7	3.2	3.6	4.1
10....	9.5	9.9	9.9	5.4	5.5	5.5			

to an interest rate of 3 per cent yearly. "P" means capital values according to E72 for "permanent" disability. The basic assumptions for females are the same as those for males as a consequence of the same survival functions (i.e., continuance rates).

Special valuation tables according to E72 (column "P" of Table 2) are used for disability pensions of disabled persons who have been entitled to national disability pensions ("permanent" disability). This valuation principle is considered to be a safeguard against the effects of extended entitlement to "permanent" pensions due to the national law.

7. The effect of the liberalized rules for entitlement to disability pension—even temporary disability pension (for one, two, or three years)—according to the Swedish National Insurance Act can be studied for the insured members of the group pension plan as illustrated below for men in the age group 60–64 years.

In dividing the number of men disabled at this age who have become entitled to a national disability pension (NDP) by the number of disabled men "under risk" of becoming entitled, we obtain an estimate of the rate of entitlement to NDP for a disabled man. Thus, for males aged 60–64, we obtain the rates shown in the accompanying tabulation. The rates of

YEAR OF ENTITLEMENT	AVERAGE YEARLY NUMBER OF ENTITLEMENTS	YEARLY RATES OF BECOMING ENTITLED TO NDP WHEN THE DISABLEMENT HAS LASTED		
		0–1 Year	1–2 Years	2–4 Years
1966–68......	148	15%	47%	18%
1971–72......	388	30	65	42

becoming entitled to NDP have increased significantly for all durations. The increased rates for 2–4 years are, however, of less interest for the age group 60–64, since the rates correspond here to small numbers of entitlements (only 5–10 per cent of the whole number).

The increase of the rates according to the table could, if studied separately, be regarded as solely the effect of a quicker transfer from one state of disablement to another. The dominant element behind the increase in the rates is, however, the appearance of new disablements, initiated by the difficulties encountered by older people in retaining employment and stimulated by the liberalized medical requirements for entitlement to NDP by people above age 60.

JAMES J. OLSEN:

The authors of this paper have done a very fine job of developing a method which should help to solve some of the elusive problems of obtaining claim rates and net annual claim costs for the disability income risk.

Unfortunately, since the network of the premium structure is so large, consisting of age, sex, occupational class, deferment period (or elimination period), and maximum duration of benefits, it becomes very difficult, even on an intercompany basis, to collect a significant amount of experience. Therefore, it becomes essential for the actuary to use a substantial amount of judgment in order to derive a reasonable representation of net annual claim costs which will result in a consistent set of premium rates. The authors' discussion of the various problems and their suggested model should be helpful to the actuary in devising techniques to obtain premium rates.

The authors have emphasized that the values shown in the paper should be used with utmost caution. I believe that the primary purpose of the paper was to explain how, and the reasons why, the model was developed. The values shown are of secondary importance.

The derivation of the formulas used is quite clear and easy to follow. The most difficult part is the determination of the objectives or the parameters at the later durations of disability.

I would like to add some of my thoughts to those of the authors. Annual studies of my company's morbidity experience agree with the authors' statement that "the probability of becoming disabled at age x and still being disabled at duration t tends to increase as the deferment period e decreases." A few years ago my company discontinued issuing policies with deferment periods of less than two weeks, and now our shortest deferment period is two weeks. I am inclined to believe that persons who would normally buy a policy with a deferment period of one week will now be encouraged to buy a policy with a two-week deferment period. Therefore, it is reasonable to believe that our claim costs for a two-week deferment period will be higher than we experienced previously. Since other companies also have discontinued the sale of policies with short deferment periods, this probably will result in an increase in the claim frequencies and the morbidity costs for a two-week deferment period in the intercompany morbidity studies. We are also emphasizing a disability plan with a four-week deferment period for mortgage protection purposes, and probably this will tend to increase the normal claim frequencies and net annual claim costs for a four-week deferment period.

The authors state: "In fact, for the younger ages, there is some evidence that the experience in the early policy years may actually be higher than the ultimate at the same attained age." I believe the reason for this is that for ages below 30 a substantial proportion of the claims arise from accidents. A study we made many years ago of accident-only policies showed that for attained ages below 30 the net annual claim costs in the first policy year were about twice as large as they were in the sixth policy year. The net annual claim costs decreased from the first to the sixth policy year, where they seemed to level off. The experience on the accident portion of sickness and accident policies was similar, although not as pronounced as for accident-only experience. My explanation for this antiselection is that those insureds who tend to be poor accident risks also tend to be haphazard in the way they live and, as a result, have high lapse rates. Thus, as the policies become older, we end up with a better class of risks.

Another factor to consider for the age group below 30 is that, because of the accident hazard, the over-all cost of disability benefits for sickness and accident combined is probably somewhat higher at age 20, and it gradually decreases until about age 30 and then increases. In regard to attained age 20, the experience can include only the first few policy durations; but with each passing year another policy duration is added, so that at age 29 the experience will consist of up to about the first ten policy durations, the later policy durations having fewer exposures because of lapsation. Back in the 1950's, when a number of the life insurance companies entered the disability income business, their contributions initially consisted only of the early policy durations. However, as time went by, more and more of their contributions reflected experience for the later policy durations. Thus, from the earlier studies to the more current studies, there has been a shift in the age and the policy-year duration.

The authors state that "only above issue age 50 is there a clear indication that the amount of disability claimed in the early policy durations is significantly lower than the corresponding ultimate experience." I believe that this statement is true, but it may be possible that the experience is not as low in the early policy durations as some people think it is. I am assuming that the authors may have made their statement to some extent on the basis of the published reports of the Society's intercompany studies.

In 1952, when we entered the disability income business, our highest issue age was 54. Therefore, when we first contributed our experience, our average age for attained-age group 50–59 was low, and the average policy duration was low. As the years went by and we increased our maximum issue-age limits, our data gradually reflected the experience at the higher issue ages and the higher policy durations. As a result, our average age and average policy duration for age group 50–59 gradually increased. On the assumption that many of the other contributors to the intercompany morbidity studies were in the same position as we were, I believe it is entirely possible that each new intercompany study will show higher results, particularly for ages 50 and above, than the previous studies, to a large extent because of the increasing average age and average policy duration.

The above remarks, which were applicable to males, are even more applicable to females. Originally in 1955 we issued coverage to females on a noncancelable to age 60 basis, with a maximum issue age of 50. In 1968 we issued to age 52 on a noncancelable to age 62 basis, and in 1972 we issued to age 59 on a noncancelable to age 65 basis.

The previous remarks pertaining to ages 50–59 would be even more applicable to age group 60–69. In addition, I have always assumed that the amount of experience above age 64 is quite small, and the experience for age group 60–69 really reflects the experience for age group 60–64.

There is another point that the authors have indicated clearly but that I would like to reiterate. One of the main points made by the authors is that a different table is needed for each deferment period, but there is also the question whether a particular table for a given deferment period is applicable regardless of the maximum duration of benefits. For example, if the experience for a two-year maximum benefit period is compared with the experience for a maximum benefit period to age 65, with respect to a four-week deferment period, is it reasonable to expect that the experience for the first two years beyond the deferment period is the same? I believe that the experience for the maximum duration to age 65 plan will be better. The type of risk applying for the longer duration is probably inherently a better risk, and I would also expect that there would be a higher degree of underwriting for the longer-duration plan, so that a company not willing to issue a longer-duration plan to some applicants would be willing to issue a shorter-duration plan.

I like the term "deferment period" which is used instead of the term "elimination period." It seems to have a much more positive meaning, and perhaps someday it will be used by the industry.

(AUTHORS' REVIEW OF DISCUSSION)

JOHN H. MILLER AND SIMON COURANT:

In our "Concluding Remarks" we intended to warn against the indiscriminate use of the model as a table "for all seasons," but we are glad that Mr. Angle has added emphasis to our caveat. The "internal relationships" which we discerned refer to broad patterns, whereas the parameters used were termed "standard" with the intent that they should be modified to conform to observed experience or, in the absence of relevant experience, modified by judgment.

We agree that one would not expect deferment period selection to be the same for individual as for group experience, and yet the evidence of Figure 1 indicates that, for the group LTD, the disparity between the three-month and six-month deferment experience is even greater than that indicated by the model. The quotation on page 355 of reference [6] may also be cited in this connection.

Although Table 9 lists the Benefit 5 experience, this was only for comparison and to add further evidence of discretionary disability. The ordinary waiver experience was not relied upon in fixing the claim rate target values.

Mr. Angle refers to our comments on "benefit period selection" as an explanation of "the variance of group results from the predicted level." This was not the intent. Although, in Table 8, we did compare individual and group experience in the second year, we added the observation that similar comparisons with Benefits 2 and 4 showed even higher costs for ages 40 and over. Can we not draw the conclusion that, rather than being left with a dilemma, we see evidence that the extension of disability coverage in either direction, that is, by a shorter deferment or by a longer benefit period, tends to increase the prevalence of disability at every point in time in relation to that for the benefit specifications before the extension? Of course, as Mr. Olsen points out, there may be countervailing forces, such as different standards of acceptance or selection procedures.

The definition of disability is certainly the heart of the policy contract, and any significant change from the definition prevailing in the 1960's would require a change in the parameters of equation (5).

It was not our intent to deprecate the excellent craftsmanship which entered into the construction of the 1964 CDT. This table was intended, we understand, to be used solely for valuation purposes, particularly for active life reserves. It is unfortunate that this statement of purpose, included in the early report drafts, was omitted from the final publication.

Mr. Angle has echoed our best hopes for our effort in stating that perhaps the model "can be put to use to find the facts that will help us better understand the incidence of disability."

Mr. Barnhart's comparison of techniques that he has employed with those underlying the model will be most useful to others who pursue mathematical approaches in dealing with the disability risk. He discusses our use of a Makeham-type formula as it affects the resulting continuance functions or disability prevalence values. We feel that we have reasonably established the validity of this approach both by producing, for different deferment periods, satisfactory incidence rates and also, as illustrated in Table 3, prevalence rates at a specified duration. In fact, Table 3 suggests the possibility that all prevalence values could be expressed by a family of equations similar to equation (8), every value of A^*, B^*, and log c^* being a function of the deferment period and of the duration of disablement.

Mr. Barnhart's use of a "lambda" function to reverse the slope of the disability termination rates at the higher ages might well be explored in any future development of the techniques underlying our model. Mr. Barnhart's techniques may offer greater convenience, as he suggests, but we have found our approach to be quite practical. If a comprehensive computer program is employed, percentage or additive changes in termination rates or alternate interest rates are readily introduced. If manual calculations are to be accommodated, suitable commutation functions can be created initially to facilitate these. For any deferment period between one and twelve months, inclusive, the parameters in the formula for the rate of disability can be developed by a rather simple mathematical expression. Thus, within this range, a computer program could be devised to reproduce values for any desired deferment. For the seven- and fourteen-day deferments, however, we found it necessary to resort to an empirical approach.

We agree that, from the expression ${}^{e}r_{[x]+t}$, one might infer that selection by age at issue is indicated, but we were following the precedent of the expression $q^{i}_{[x]+t}$. A resolution by the Committee on Standard Notation and Nomenclature would be desirable in this instance.

Dr. Lundberg has provided some interesting notes on the Swedish actuarial techniques, which include an explicit recognition of deferment period selection for periods of less than three months. The startling increase in the rates of becoming entitled to national disability pensions and the concurrent acceleration in long-term disability rates offer a timely reminder to us that this is a potentially explosive line of insurance and that the insurance companies are not insulated from the effects of changes in social benefits. To what extent the inclusion of rather liberal benefits for certain levels of partial benefits has affected the Swedish disability incidence and termination rates is not revealed, but these data from overseas should give pause to anyone contemplating more liberal policy conditions or underwriting standards.

Mr. Olsen's observations and comments on selection at issue are of great interest. His explanation of some of the circumstances affecting the published rates from the Society's studies of individual loss-of-time policies is quite helpful in the interpretation of these results. However, our observations relative to selection at the higher issue ages were based largely on the waiver of premium study (ref. [7]) and on the comparisons cited in our earlier paper [6].

We agree with Mr. Olsen that there are positive factors which might be expected to result in better experience under a benefit to age 65 as compared with a two-year benefit. Nevertheless, the evidence of Table 8 indicates that, at the higher ages, the attraction of a long-term benefit may inhibit the motivation toward rehabilitation.

It is gratifying that Mr. Olsen has commented favorably on our use of the expression "deferment period." His examples of how circumstances not explicitly disclosed in published reports, such as age-at-issue limits and changes therein, can affect the indicated experience provide a valuable warning to anyone who uses experience data without full knowledge of the underwriting rules and practices underlying the source material.

The authors wish to express their thanks to all who have discussed their paper. Each discussion constitutes a valuable addition and should be helpful to anyone seeking a better understanding of a very complex subject.