

A METHOD FOR PRICING RETURN OF PREMIUM
BENEFITS UNDER DISABILITY
INCOME POLICIES

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ABSTRACT

A method is described for pricing return of premium benefits under disability income policies. Discrete bivariate frequency distributions are used to determine gross premiums, surrender percentages, and expected profit levels. Expected profits (\pm) are broken down by termination mode (i.e., death, voluntary lapse, involuntary lapse, and maturity) and policy year of termination. Reserve and surplus levels are also considered.

I. RIDER BENEFIT DEFINITION

IN THIS paper we assume that a return of premium rider is to be attached to a disability income policy at issue. The rider provides a maturity benefit equal to the return of premium under the policy and rider, reduced by the sum of any disability benefits paid under the base policy (including waived policy and rider premiums). Upon surrender or lapse prior to maturity, a benefit similar to the maturity benefit is payable, except that the total of the rider and policy premiums is first reduced by the complement of a so-called surrender percentage (designated by k_t in this paper).

II. DEFINITIONS OF THE OPERATORS

We shall find it convenient to define a few special operators on discrete bivariate frequency distributions. For this purpose, assume that the matrix

$$[x_i, y_i, p_i]$$

represents a discrete bivariate frequency distribution, where i refers to the number of the row (or line); x_i and y_i are real numbers; and p_i is the probability or frequency of occurrence of the particular combination of x_i and y_i .

We shall take the liberty of referring to such a matrix as a frequency distribution even when the sum of the frequencies (i.e., $\sum_{i=1}^n p_i$, where n is the number of rows or lines) is less than unity.

The unary operators are "Split," designated by \Leftarrow , and "Transform," designated by \rightarrow . There are two types of Split operators, namely, "Horizontal Split" and "Vertical Split."

Transform:

$$[x_i, y_i, p_i] \xrightarrow{ax_i \rightarrow x_i} [ax_i, y_i, p_i],$$

where a is a given real number; transformations involving different formulas would be analogously defined.

Horizontal Split:

$$[x_i, y_i, p_i] \begin{cases} \xrightarrow{y_i \geq m} [x_j, y_j, p_j], & \text{where } y_j \geq m \text{ for all } j \\ \searrow_{y_i < m} [x_k, y_k, p_k], & \text{where } y_k < m \text{ for all } k, \end{cases}$$

where m is a given real number.

Vertical Split:

$$[x_i, y_i, p_i] \begin{cases} \xrightarrow{p_i q \rightarrow p_i} [x_i, y_i, p_i q] & \text{for each } i \\ \searrow_{p_i(1-q) \rightarrow p_i} [x_i, y_i, p_i(1-q)] & \text{for each } i, \end{cases}$$

where q is a given real number. (Note: The i subscripts in the above definitions are often omitted when the meaning is obvious; e.g., $ax \rightarrow x$ would mean $ax_i \rightarrow x_i$.)

The binary operators are "Merge," designated by \textcircled{M} , and "Convolute for Sums," designated by \oplus .

Merge:

$$[x_i^{(1)}, y_i^{(1)}, p_i^{(1)}] \textcircled{M} [x_j^{(2)}, y_j^{(2)}, p_j^{(2)}] = \begin{bmatrix} x_i^{(1)}, y_i^{(1)}, p_i^{(1)} \\ x_j^{(2)}, y_j^{(2)}, p_j^{(2)} \end{bmatrix},$$

that is, the resulting matrix is simply the union of the first and second matrices.

Convolute for Sums:

$$[x_i^{(1)}, y_i^{(1)}, p_i^{(1)}] \oplus [x_j^{(2)}, y_j^{(2)}, p_j^{(2)}] = [x_i^{(1)} + x_j^{(2)}, y_i^{(1)} + y_j^{(2)}, p_i^{(1)} p_j^{(2)}],$$

where i assumes each integer value from 1 to the number of lines in the first matrix; and, for each such value of i , j assumes each integer value from 1 to the number of lines in the second matrix; thus the resulting

matrix is obtained by calculating the triplet of values $(x_i^{(1)} + x_j^{(2)}, y_i^{(1)} + y_j^{(2)}, p_i p_j)$ for each combination of i and j . The superscripts (1) and (2) merely identify whether the value originates from the first or second matrix, respectively.

The Convolute for Sums operator \oplus and the Merge operator \otimes are commutative and associative in an algebraic sense. If a "Convolute for Products" operator \odot were defined in an analogous manner, it would not be distributive with respect to the Convolute for Sums operator \oplus . These statements can be verified easily by the reader.

III. DETERMINATION OF EXPECTED PROFIT LEVELS GIVEN RIDER PREMIUM (P^R) AND SURRENDER PERCENTAGES (k_i)

Considering only one insured life, suppose that we had available, for each mode (lapse, death, and maturity) and year of termination, a 3-column matrix $[x, y, p]$ which is a discrete bivariate frequency distribution, where

- x = Accumulation, from issue to the end of the policy year of termination, *at interest only*, of rider premium income less rider expenses, commissions, federal income taxes, premium taxes, etc., under a return of premium rider attached at issue to a disability income policy; any rider premiums waived would not be included in this accumulation;
- y = Sum, *without interest*, of disability benefits paid, from issue to the end of the policy year of termination, under the disability income policy; y would include any premiums waived under the base policy and the rider during the same period; y would not include any return of premium benefit;
- p = Probability at issue of the occurrence of a particular combination of x and y , and the rider's termination by the given mode at the end of the given year of termination.

Taken all together, such frequency distributions would, in effect, enumerate all of the possible outcomes in connection with that single insured life.

The return of premium benefit¹ can be defined in symbols as follows:

$$\max \{0, k_{it}(P^B + P^R) - y\}, \tag{1}$$

where t is the policy year, P^B is the gross annual premium for the base policy, P^R is the gross annual premium for the rider, and k_t is an arbitrary

¹ See Sec. I, "Rider Benefit Definition."

“surrender percentage” specified in the rider for each policy year t of termination.

Then each of the original discrete bivariate frequency distributions can be transformed into a discrete univariate frequency distribution of present value of profit (\pm), as follows:

$$\left[(x - \max \{0, k_t(P^B + P^R) - y\}) \frac{1}{(1+i)^t}, p \right], \quad (2)$$

where i is worth of money (an interest rate). There would be one such distribution for each mode and policy year of termination. Note that the phrase “present value of profit (\pm)” refers here to the present value of profit (\pm) from issue to the end of the policy year of termination for the given mode of termination.

The mean or expected present value of profit (\pm), from issue to the end of the policy year of termination, for a particular mode of termination, is simply the sum of the products of corresponding amounts and probabilities in the appropriate discrete univariate frequency distribution derived according to expression (2). The sum of the corresponding probabilities or frequencies is referred to as the cumulative probability or cumulative frequency (“cum”).

Table 1 shows the cumulative frequency “cum” and the mean present value of profit (\pm) from each such mode and policy year of termination, and is based on the assumptions given in Appendix I. Summing the means of the frequency distributions for all modes and policy years of termination produces the mean or expected present value of profits ($-\$54.48$ in Table 1) from the rider on the single insured life. Naturally, the sum of all the cumulative probabilities equals unity.

In Table 1 the figures in the column labeled “Mean/Cum” should be used with care, because the calculation of the mean profit figures in some cases involves both positive and negative numbers. “Involuntary lapse” refers to lapse of the rider which occurs because total disability benefits paid to date (i.e., y) exceed $k_n n(P^B + P^R)$, where n is the number of years from issue to maturity; that is, the insured lapses because there remains no possibility for him to collect any “return” benefits.

Thus, given appropriate bivariate frequency distributions $[x, y, p]$ for a given insured life, we can calculate the present value of profit (\pm) expected from each source. Of course, the mean or expected present value of profit (\pm) from a portfolio of such riders would be simply the sum of the corresponding mean figures for the individual lives. Let us now turn to the problem of how to determine the appropriate discrete bivariate frequency distributions.

TABLE 1—ONE INSURED LIFE: PRESENT VALUE AT ISSUE OF PROFITS BY SOURCE; DISABILITY INCOME—RETURN OF PREMIUM RIDER (TWO-YEAR ACCIDENT AND SICKNESS, 90-DAY ELIMINATION PERIOD, CLASS 2A) PER \$100 OF MONTHLY INCOME

$P^R = 35.84$, $P^B = 48.88$; $P^R/P^B = 73.32$ Per Cent

POLICY YEAR OF TERMINA- TION	MODE OF RIDER TERMINATION											
	Voluntary Lapse			Death			Involuntary Lapse			Maturity		
	Mean	Cum	Mean/Cum	Mean	Cum	Mean/Cum	Mean	Cum	Mean/Cum	Mean	Cum	Mean/Cum
1.....	1.74	0.1984	9	0.01	0.0033	4						
2.....	2.98	0.0786	38	0.08	0.0039	20	-0.01	0.0044	-1			
3.....	2.76	0.0629	44	0.13	0.0044	29	0.03	0.0040	9			
4.....	2.55	0.0502	51	0.18	0.0046	40	0.15	0.0040	38			
5.....	1.95	0.0398	49	0.20	0.0046	43	0.26	0.0040	65			
6.....	1.30	0.0312	42	0.20	0.0050	40	0.38	0.0041	92			
7.....	0.61	0.0239	25	0.15	0.0053	29	0.50	0.0042	117			
8.....	0.11	0.0222	5	0.08	0.0054	14	0.61	0.0044	140			
9.....	-0.39	0.0206	-19	-0.02	0.0054	-4	0.74	0.0045	163			
10.....	-0.97	0.0191	-51	-0.16	0.0055	-30	0.86	0.0047	184			
11.....	-1.49	0.0176	-84	-0.33	0.0056	-58	0.98	0.0048	205			
12.....	-1.95	0.0162	-120	-0.56	0.0060	-92	1.09	0.0049	224			
13.....	-2.43	0.0148	-164	-0.87	0.0065	-134	1.20	0.0049	244			
14.....	-2.84	0.0135	-211	-1.21	0.0068	-179	1.29	0.0049	262			
15.....				-1.60	0.0069	-231	1.37	0.0049	280	-64.14	0.2491	-257
Total	3.93	0.6090		-3.72	0.0792		9.45	0.0627		-64.14	0.2491	

	Mean	Cum
Voluntary lapse.....	3.93	0.6090
Death.....	-3.72	0.0792
Involuntary lapse.....	9.45	0.0627
Maturity.....	-64.14	0.2491
All modes.....	-54.48	1.0000

NOTE.—Each "Mean" figure shown in the body of this table represents the present value at issue of profits from issue to the end of the indicated policy year, for a life exiting according to the indicated mode and policy year of termination.

IV. CONVOLUTION-STATE DIAGRAM

The "convolution-state diagrams"² shown at the end of this section (Figs. 1-4) document an approach to calculating the required bivariate frequency distributions, moving through one policy year at a time. The process could be refined to move through one policy day or one policy month at a time, but the number of operations would be increased considerably.

Essentially, the process involves moving through the life of one rider, recording (1) rider net cash income (accumulated with interest), (2) disability benefits (not accumulated with interest), and (3) the associated probabilities in the form of a 3-column matrix or bivariate frequency distribution. In the process, bivariate frequency distributions $[x, y, p]$ are captured in each policy year for each mode of termination (voluntary lapse, involuntary lapse, and death) and, finally, at maturity.

The names assigned to output bivariate frequency distributions are indicative of the modes of exit: *A* refers to active, *I* refers to disabled, *D* refers to dead, *W* refers to voluntary lapse, *INV* refers to involuntary lapse, and *MAT* refers to maturity. Table 1A shows the names of these output bivariate frequency distributions which were transformed into univariate frequency distributions according to expression (2) and used to obtain the figures in Table 1 (see Appendix III for more information about the bivariate output frequency distributions).

The names assigned to the input bivariate frequency distributions are indicative of the progression of states involved:

- AAA* refers to a life who is active at the beginning of the policy year and remains active until the end of the policy year;
- AIA* refers to a life who is active at the beginning of the policy year, becomes disabled during the year, recovers, and remains active to the end of the policy year;
- AII* refers to a life who is active at the beginning of the policy year, becomes disabled during the year, and remains disabled at least until the end of the policy year;
- AAD* refers to a life who is active at the beginning of the policy year and dies, while active, by the end of the policy year;
- ID* refers to a life who is in a disabled status at the beginning of the policy year and dies, while disabled, by the end of the policy year;
- IA* refers to a life who is in a disabled status at the beginning of the policy year, recovers, and is active at the end of the policy year;
- II* refers to a life who is in a disabled status at the beginning of the policy year and remains disabled at least until the end of the policy year.

² This term was coined by the author and his associates.

TABLE 1A—RELATIONSHIP OF TABLE 1 AND CONVOLUTION-STATE DIAGRAM: * ONE INSURED LIFE; PRESENT VALUE AT ISSUE OF PROFITS BY SOURCE; DISABILITY INCOME—RETURN OF PREMIUM RIDER (TWO-YEAR ACCIDENT AND SICKNESS, 90-DAY ELIMINATION PERIOD, CLASS 2A) PER \$100 OF MONTHLY INCOME

POLICY YEAR OF TERMINATION	MODE OF RIDER TERMINATION											
	Voluntary Lapse			Death			Involuntary Lapse			Maturity		
	Mean	Cum	Mean/Cum	Mean	Cum	Mean/Cum	Mean	Cum	Mean/Cum	Mean	Cum	Mean/Cum
1.....			W(01)			D(01)			INV(01)		
2.....			W(02)			D(02)			INV(02)		
3.....			W(03)			D(03)			INV(03)		
4.....			W(04)			D(04)			INV(04)		
5.....			W(05)			D(05)			INV(05)		
6.....			W(06)			D(06)			INV(06)		
7.....			W(07)			D(07)			INV(07)		
8.....			W(08)			D(08)			INV(08)		
9.....			W(09)			D(09)			INV(09)		
10.....			W(10)			D(10)			INV(10)		
11.....			W(11)			D(11)			INV(11)		
12.....			W(12)			D(12)			INV(12)		
13.....			W(13)			D(13)			INV(13)		
14.....			W(14)			D(14)			INV(14)		
15.....						D(15)			INV(15)			MAT
Table....	W(01) (M)	W(02) (M)	... (M) W(14)	D(01) (M)	D(02) (M)	... (M) D(15)	INV(01) (M)	INV(02) (M)	... (M) INV(15)			MAT

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	Mean	Cum
Voluntary lapse.....	W(01) (M)	W(02) (M) ... (M) W(14)
Death.....	D(01) (M)	D(02) (M) ... (M) D(15)
Involuntary lapse.....	INV(01) (M)	INV(02) (M) ... (M) INV(15)
Maturity.....	MAT	
All modes.....	The results of performing the Merge operation using all of the above files	

* See also Appendix III, "Output Frequency Distributions."

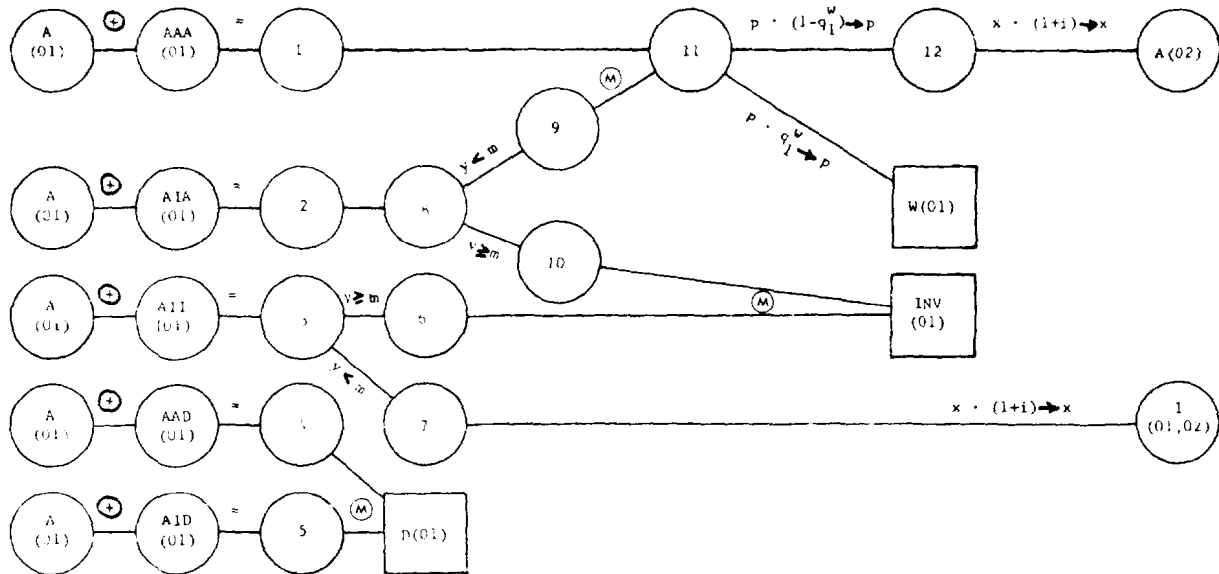


FIG. 1.—Convolution-state diagram: year 1.

In all figures, \oplus means "Convolute for Sums," as defined in the text; \otimes means "Merge," as defined in the text; $m = 15(100\%) (P^B + P^R) =$ Total of rider and policy premiums from issue to maturity; $q_1^w =$ Voluntary lapse rate for the t th policy year; x , y , and p are real numbers defined in the text and constitute lines in bivariate frequency distribution; and $i =$ an interest rate. Capital letters indicate names assigned in the text to bivariate frequency distributions; see text, Appendixes II and III. Circles containing only an integer represent intermediate resulting bivariate frequency distributions.

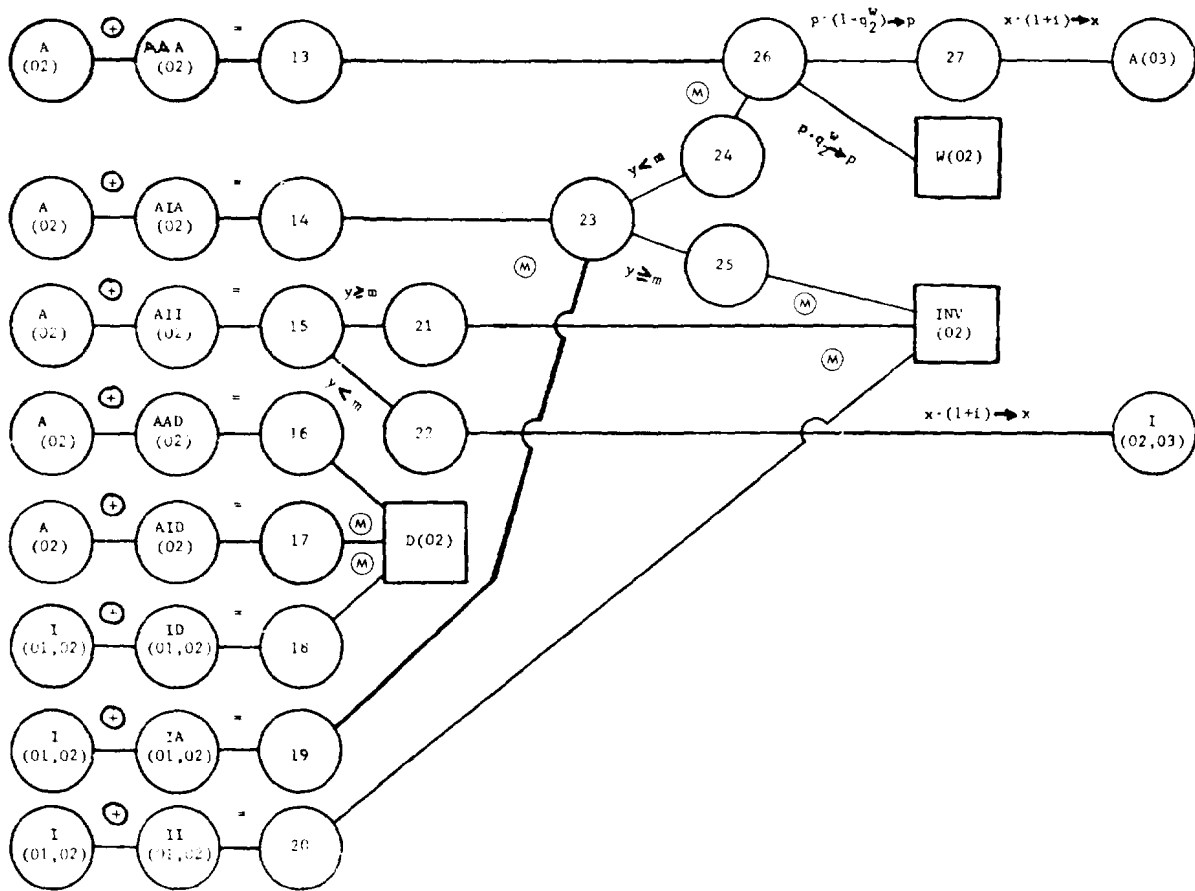


FIG. 2.—Convolution-state diagram: year 2

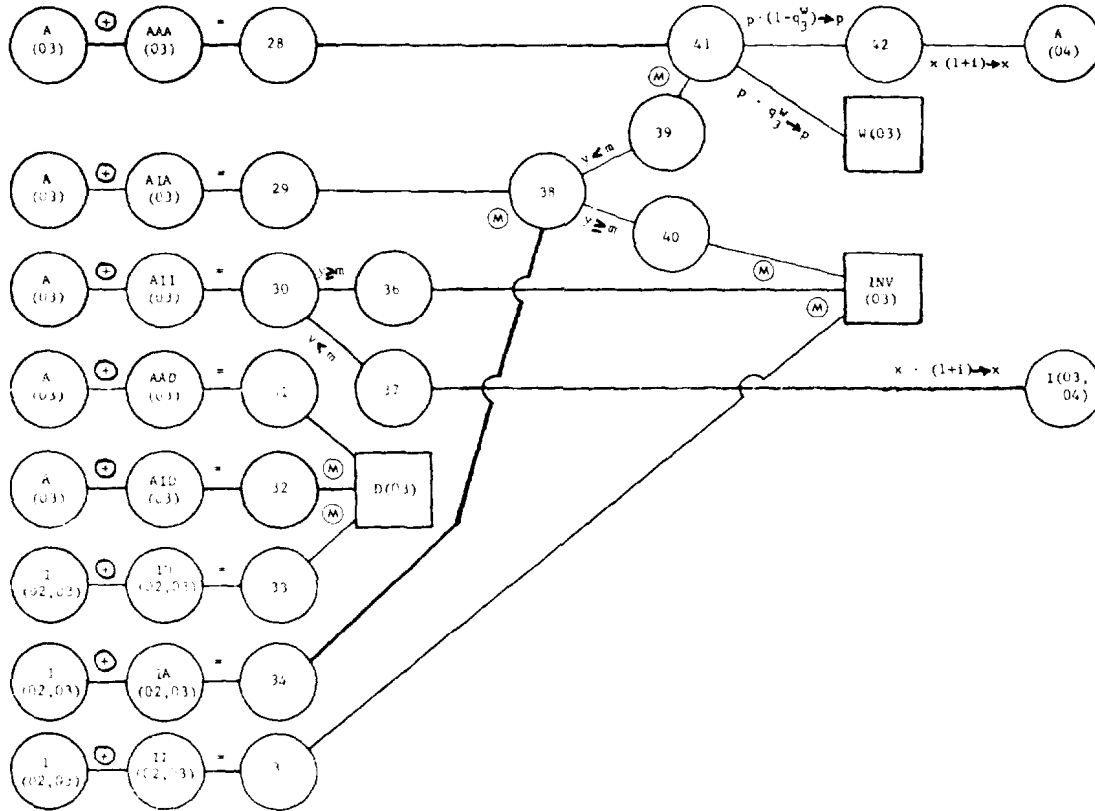


FIG. 3.—Convolution-state diagram: year 3.

In all figures, \oplus means "Convolute for Sums," as defined in the text; \otimes means "Merge," as defined in the text; $m = 15(100\%)$ ($P^B + P^R$) = Total of rider and policy premiums from issue to maturity; q_i^v = Voluntary lapse rate for the i th policy year; x , y , and p are real numbers defined in the text and constitute lines in bivariate frequency distribution; and i = an interest rate. Capital letters indicate names assigned in the text to bivariate frequency distributions; see text, Appendixes II and III. Circles containing only an integer represent intermediate resulting bivariate frequency distributions.

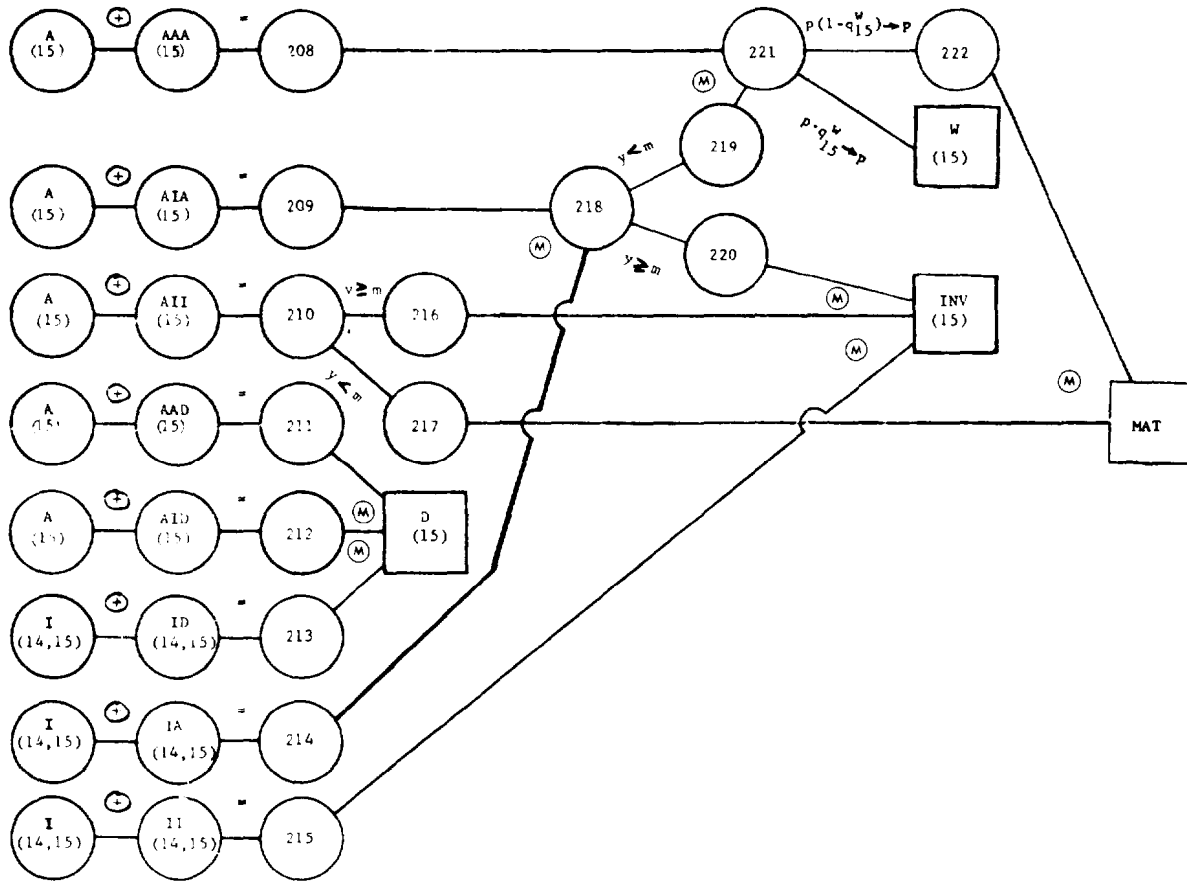


FIG. 4.—Convolution-state diagram: maturity year

Appendix II contains detailed formulas for the input bivariate frequency distributions.

V. SOME POSSIBLE MODIFICATIONS OF THE PROCESS

The process described above can be modified to handle various other situations; some examples are the following:

1. Where the maximum benefit period for accident differs from that for sickness, a life disabled by sickness is followed separately from a life disabled by accident.
2. We have assumed that a life disabled in the previous policy year and remaining disabled for a second year is automatically an involuntary lapse. If more than two years of disability are required before an involuntary lapse would occur, then a disabled life would have to be followed separately for more than two years.
3. If there is a minimum period of disability income guaranteed beyond maturity, then a disabled life at maturity can be followed beyond maturity in order to reflect properly a postmaturity recapture, by the insurer, of return benefits paid out prematurely at maturity.
4. We have assumed that voluntary lapse occurs only where the insured is in an active state and that a voluntary lapse is independent of the disability benefits paid to date. An alternative assumption would be that a voluntary lapse occurs because the y -value is large, even if the y -value does not exceed $nk_n(P^B + P^R)$, where n is the number of years from issue to maturity. This can be handled easily by a Horizontal Split operator specifically designed for the purpose.

VI. DETERMINATION OF SURRENDER PERCENTAGES (k_t), GIVEN RIDER PREMIUM (P^R) AND PROFIT OBJECTIVES

Suppose that gross premium rates (P^R) are given and surrender percentages (k_t) are to be determined; that is, values of k_t are not given. Of course, the precise size of the surrender percentages will depend on the objectives sought. Suppose the objective is as follows: "Deaths and voluntary lapses for a given policy year are to be treated together in such a way that the mean profit (\pm) from deaths and surrenders will be zero (r per cent of cash premiums for profits having already been deducted)."

To accomplish this objective, the bivariate frequency distributions for deaths ($D(t)$) and voluntary lapses ($W(t)$) for the t th policy year would be "merged." Then formula (2) would be applied to the resulting bivariate frequency distribution, with trial values of k_t , in order to produce a univariate frequency distribution of present value of profit (\pm), with a mean of zero. By interpolating linearly on the best two of the previous three trial values of k_t , and continuing in a recursive fashion, we obtain

the value of k , after a few trials; the number of trials depends on how close to zero the mean profit from deaths and voluntary lapses is required to be. A similar procedure would be followed for each policy year.

VII. DETERMINATION OF RIDER PREMIUM (P^R), GIVEN SURRENDER PERCENTAGES (k_i) AND PROFIT OBJECTIVES

Suppose that surrender percentages (k_i) are given for each policy year, and the rider gross premium (P^R) is to be determined. This problem is not quite as simple as the situations discussed in Sections III and VI, because both x and y are dependent to some extent on P^R . If x and y were each linear functions of P^R , the difficulty would disappear.

The total disability benefits paid (y) would be a linear function of P^R if the elimination period for base disability income benefits were the same as the waiting period for a nonretroactive waiver benefit; however, the elimination period for base disability income benefits might be seven days, whereas the waiting period for waiver is likely to be ninety days with waiver retroactive to the beginning of the elimination period. Also, x would be a linear function of P^R if all deductive items (expenses, commissions, premium tax, etc.) were expressed as a percentage of premium; however, some expenses probably are not properly assessed strictly as a percentage of premium; for example, federal income tax (other than on investment income) might best be expressed as a percentage of net cash flow.

It is, of course, possible to redetermine all the bivariate frequency distributions for each trial value of P^R . However, to minimize time and expense, one approach which can be used, but with care, is to assume that x is a linear function of P^R and that y does not vary significantly with a small change in P^R . Note that y includes base disability benefits plus waived premiums, with waived premiums normally a small portion of the total. Occasional redetermination of all the bivariate frequency distributions can be used to confirm that the approximate P^R indeed is close enough to the accurate P^R .

VIII. USE OF UNIVARIATE FREQUENCY DISTRIBUTIONS

Under certain circumstances it may be possible to work exclusively with univariate frequency distributions, namely, $[x, p]$ and $[\max \{0, k_i l(P^B + P^R) - y\}, p]$. If we are interested only in mean values of accumulated profits (\pm), then the mean of $[x, p]$ minus the mean of $[\max \{0, k_i l(P^B + P^R) - y\}, p]$ would give us the mean profit, for each profit source.

Using only univariate frequency distributions, we would be able to determine the variability in x and the variability in $\max \{0, k_i l(P^B +$

$P^R) - y\}$ but not the variability in $x - \max \{0, k_d(P^B + P^R) - y\}$, because x and $\max \{0, k_d(P^B + P^R) - y\}$ are not independent random variables. When a waived premium (or a portion thereof) is included in y , then x reflects the fact that a premium (or a portion thereof) has been waived and hence not collected in cash. (In calculating x it is important to consider how expenses, commissions, profit deductions, and so on, are to be treated where a premium is waived rather than paid.)

On the other hand, given a frequency distribution of present value of profit (z) for each rider in a portfolio of N riders, these frequency distributions could be "convoluted for sums,"

$$[z, p]_{1st\ life} \oplus [z, p]_{2d\ life} \oplus \dots \oplus [z, p]_{Nth\ life}, \quad (3)$$

to obtain a frequency distribution of the present value of profit (\pm) expected from such portfolio of riders. (For some sample distributions of this type, based on the assumptions in Appendix I, see Table 2 in Sec. XII, "Surplus Levels.")

The convolution of two univariate frequency distributions is defined as follows, analogous to the convolution of two bivariate frequency distributions previously described:

$$[x_i^{(1)}, p_i^{(1)}] \oplus [x_j^{(2)}, p_j^{(2)}] = [x_i^{(1)} + x_j^{(2)}, p_i^{(1)} p_j^{(2)}],$$

where i assumes each integer value from 1 to the number of lines in the first matrix; and, for each such value of i , j assumes each integer value from 1 to the number of lines in the second matrix; thus the resulting matrix is obtained by calculating the pair of values $(x_i^{(1)} + x_j^{(2)}, p_i^{(1)} p_j^{(2)})$ for each combination of i and j . The superscripts (1) and (2) merely identify whether the value originates from the first or second matrix, respectively.

Since the \oplus convolution operation is associative, the meaning of expression (3) above is well defined.

IX. EVALUATION OF PROFIT (\pm) EXPECTED FROM RIDER AND POLICY TREATED AS A PACKAGE

The basic differences between this approach and the rider approach are as follows: x would represent the accumulation, *with interest only*, of premium income (*from the rider and policy*) less (*rider and policy*) expenses, commissions, federal income taxes, premium taxes, and so on, and *base disability income benefits paid*; any (rider or policy) premiums waived would not be included in this accumulation; y would still represent the corresponding sum, *without interest*, of disability benefits paid under the disability income policy; y would include any premiums waived under

the base policy and rider; p would present the probability at issue of the occurrence of a particular combination of x and y , and the rider's terminating by the given mode at the end of the given year of termination.

The process, as described by the convolution-state diagram, would be different insofar as involuntary lapse is concerned. An involuntary lapse would be treated not as a termination but rather as discontinuance of the return feature, together with a reduction in premium from $P^B + P^R$ to P^B . Thus an involuntary lapse would be carried forward separately through each policy year (subsequent to the occurrence of the involuntary lapse) and each mode of exit (death, voluntary lapse, active, disabled), similar to the situation where an insured has not become an involuntary lapse. If it were felt that the experience among the class of insureds who become involuntary lapses would be significantly different from that of the other continuing class of insureds, then appropriately different morbidity, lapse, and/or death rates could be applied to the continuing involuntary lapses.

X. SOME MODIFICATIONS IN THE DEFINITION OF THE RETURN OF PREMIUM BENEFIT

There are several types of return of premium benefit being marketed. This section mentions a few of the common variations in definition and indicates briefly how the procedure set forth in this paper can be adapted to handle such definitions.

First, k_t might apply to the excess of total premiums over the total disability benefits. The formula for the return benefit would be

$$\max \{0, k_t(t(P^R + P^B) - y)\},$$

which simply would be used in lieu of the original formula (1). This same return benefit formula would be used whether waived premiums are included or excluded in determining both total premiums and total disability benefits, since the algebraic result would be the same in either case.

Second, there might be a "cutoff" provision, under which the amount of the return benefit would be determined according to formula (1), but no benefit would be paid if total disability benefits exceeded c per cent of total premiums. The formula for the return benefit might be

$$k_t(P^B + P^R) - y, \quad \text{provided } y < ct(P^B + P^R),$$

which logic would be applied in lieu of the original formula (1).

Third, k_t might apply to "total premiums paid by the policyholder, excluding premiums waived by the company." The formula for the return benefit might be

$$\max \{0, k_t z - y\},$$

where y is defined as before but excludes any waived premiums and z is the total premiums paid (not waived). This situation would require a trivariate frequency distribution, recording $[x, y, z, p]$, where x is defined as before. (Presumably the return benefit would not be defined in such a way that y would include waived premiums and x would exclude waived premiums.)

In the three situations described above, the formulas used to determine the timing of involuntary lapse would have to be suitably modified.

In a fourth situation, rollover periods might be involved in the definition of the return benefit. Where this is the case, frequency distributions could be subdivided according to the number of years since a rollover period was restarted. Fortunately, under most such designs the maximum number of years possible from restart to involuntary lapse is fairly small for a disabled life; also, a disabled life may be able to be considered to be "ultimate" with respect to probabilities of death and recovery after a relatively brief "select" period. Thus the number of frequency distributions required is not unmanageable. (See alternative approaches presented in ref. [2].)

XI. RESERVES

Reserves for the rider must be approached seriatim and prospectively, because the reserve at a given duration (t_0) depends upon the amount (y_0) of disability benefits paid under the policy prior to the valuation date. The usual definition applies—namely, the present value of future return benefits minus the present value of future premiums. The calculation involves construction of bivariate frequency distributions, similar to those described in the pricing process above, but starting on the valuation date with $[0, y_0, 1]$. These bivariate frequency distributions (one for each policy year and mode of exit) can be transformed into the univariate frequency distributions

$$\left[(-x + \max \{0, k_{it}(P^B + P^R) - y\}) \frac{1}{(1+i)^{t-t_0}}, p \right],$$

which represent frequency distributions of the excess of the present value³ of future benefits over the present value³ of future premiums. The mean of this frequency distribution is the reserve.

Note that one mode of exit is by involuntary lapse. Thus the reserves so calculated reflect the fact that the rider will terminate as soon as it can be determined that no return benefit will ever be paid.

The specific formulas and actuarial assumptions to be used depend on whether the reserve is to be on a statutory, GAAP, or other basis, as well as on the precise form of the return benefit. In any event, a seriatim

³ Discounted at interest only.

valuation is indicated, requiring at least the following information for each rider:

1. Return of premium benefit definition
2. P^B (premium for the base policy)
3. Age at issue
4. Sex
5. Duration
6. Elimination period
7. Maximum benefit period for accident and sickness
8. Size of policy
9. Waiver of premium provision
10. Term period of coverage
11. Status of insured (i.e., active or disabled, and, if disabled, how long disabled)
12. Total (y_0) of the disability benefits paid under this policy prior to the valuation date
13. The existence of other riders affecting the return benefit which also must be taken into account

Presumably, accurate reserves would be calculated only for a skeleton of such parameters, the final reserves being obtained by interpolating between the skeletal values.

Since the valuation date usually will not be coincident with the end of a policy year, the reserve could be calculated by interpolating between an initial reserve and a terminal reserve, in each case reflecting the (same) amount of disability benefits (y_0) which were paid prior to the valuation date. An alternative would be to refine the reserve calculation process to a monthly rather than an annual basis, at least during the policy year in which the valuation date falls; however, this may not be practical.

XII. SURPLUS LEVELS

Having the reserve frequency distributions determined in the previous section at a given policy duration, we can convolute (\oplus) such constructed univariate frequency distributions for sums to produce a frequency distribution of aggregate natural reserves for a portfolio of riders (see Table 2). These frequency distributions can be helpful in deciding how much surplus is needed in order to provide for adverse random fluctuations. Of course, additional surplus is required to provide for other contingencies. Table 2 is based on the parameters and assumptions set forth in Appendix I.

XIII. CAUTION

The particular figures shown in this paper are for the purpose of illustrating a set of results; different parameters (age at issue, sex, elimination period, maximum benefit period, term period, base policy

premium [P^B], commissions, expense rates, etc.) and different actuarial assumptions (e.g., morbidity rates, voluntary lapse rates, death rates among active lives, death and recovery rates among disabled lives, interest rate[s], etc.) will produce results which may differ widely from those shown.

XIV. CONCLUSION

The pricing (and reserving) of disability income return of premium benefits probably is one of the most complicated technical problems being solved by individual health insurance actuaries today. This paper has attempted to outline a method of attacking these problems which, the author hopes, is theoretically correct and yet practical. The degree of refinement is left up to the individual actuary who is responsible for the final product.

TABLE 2
NATURAL RESERVES PER \$100 OF DISABILITY INCOME
FOR A PORTFOLIO OF RIDERS
(Duration 0)

CUMULATIVE FREQUENCY	NUMBER OF RIDERS			
	10	100	1,000	10,000
0.01.....	- 49.40	+19.75	+43.42	+50.98
0.05.....	- 22.16	+29.43	+46.57	+51.95
0.10.....	- 6.19	+34.97	+48.26	+52.52
0.50.....	+ 53.10	+54.36	+54.55	+54.46
0.90.....	+117.18	+73.75	+60.60	+56.40
0.95.....	+135.87	+79.98	+62.29	+56.97
0.99.....	+170.63	+90.36	+65.67	+58.01

The derivation of the probabilities for use in constructing the input bivariate frequency distributions has not been covered in the paper, but they can be computed by applying standard actuarial techniques to appropriate morbidity and mortality experience. In investigating such experience, disabled life terminations would be split into recoveries and deaths.

This paper describes a method for treating observed probabilities stochastically in order to determine expected profit levels. The determination of appropriate reserves and surplus levels is approached using the same method. Further work is needed to build in the interrelationship between economic conditions and morbidity rates.

Techniques similar to those described in this paper can be used to price almost any type of insurance. Use of such method is more necessary in

the case of benefits with minimum or maximum features, such as minimum death benefit provisions under variable life insurance; (nonnegative) dividends or refunds and forgiveness features under group life or health insurance; stop-loss provisions under reinsurance or other arrangements; and offsets and cutoffs under return of premium contracts (health, disability income, and so on).

XV. ACKNOWLEDGMENTS

The author wishes to express his appreciation to Mr. Thomas M. Chiappetti, who, in addition to contributing the idea of using the convolution-state diagram as a visual aid, developed the formulas shown in Appendix II and the morbidity rates underlying Table 1, and to Mr. Larry Hickey, of M & R Services, Inc., who provided significant computer programming assistance.

APPENDIX I

PARAMETERS AND ASSUMPTIONS USED IN DETERMINING PROFIT MARGINS IN TABLE 1

1. *Issue age*: 50.
2. *Elimination period*: 90 days.
3. *Insurance period*: To age 65.
4. *Per cent of premium expenses*:

Policy Year	Total
1.....	72.0%
2-10.....	9.5
11-15.....	4.5

5. *Per policy expenses*: \$4 first year, increasing 4 per cent per year (geometrically).
6. *Federal income taxes* (other than tax on investment income): 0 per cent. Note: profit (\pm) figures shown might be multiplied by $(1 - 0.24)$ to obtain profit (\pm) after federal income tax.
7. *Average size of policy*: \$400 per month (disability income).
8. *Net investment earnings*: 5 per cent, after deducting federal income tax on investment income.
9. *Voluntary lapse rates*:

Policy Year	Rate	Policy Year	Rate
1.....	20%	5.....	7%
2.....	10	6.....	6
3.....	9	7+.....	5
4.....	8		

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- 10. *Morbidity basis:* 120 per cent of 1969 M & R Disability Income Tables, Group II.
- 11. *Mortality basis:*
 - a) Active lives: 1955-60 Modified Basic Select and Ultimate Table.
 - b) Disabled lives: 1965 Railroad Retirement Board Disabled Annuitants Mortality Table.
- 12. *Gross annual premiums:*

$$\begin{aligned}
 P^B &= \$ 48.88 \\
 P^R &= 35.84 \\
 P^B + P^R &= \$ 84.72
 \end{aligned}$$

$$m = 15(100\%)(P^B + P^R) = \$1,270.80$$

- 13. *Arbitrary surrender percentages (k_t) for male issue age 50:*

Policy Year	k_t	Policy Year	k_t
1.....	0%	9.....	48%
2.....	0	10.....	56
3.....	10	11.....	64
4.....	15	12.....	72
5.....	21	13.....	81
6.....	27	14.....	90
7.....	34	15.....	100
8.....	41		

APPENDIX II

INPUT FREQUENCY DISTRIBUTIONS

The x , y , and p variables in the input frequency distributions [x , y , p] are defined as follows, making the assumptions stated at the end of this appendix.

Name of Input
Bivariate
Frequency
Distribution

Definitions

$AAA(t)$ $x = \{P^R(1 - E_t^{\%}) - E_t^c\} \bar{s}_{\overline{1}|}$;

$y = 0$;

p = Probability of an active life at the beginning of policy year t remaining in an active state throughout the policy year,

where

$E_t^{\%}$ = Commissions and percentage of premium expenses for the t th policy year and

E_t^c = Per policy expenses for the t th policy year.

Name of Input
Bivariate
Frequency
Distribution

Definitions

$AAD(t)$ $x = \{P^R(1 - E_t^{\%}) - E_t^c \bar{s}_{\overline{1}|i}\}(1 + i)^{1/2},$
 $y = 0,$

p = Probability of an active life at the beginning of policy year t dying while in the active state, without having been disabled during policy year t .

$AID(t)$ $x = \{P^R(1 - E_t^{\%}) - E_t^c \bar{s}_{\overline{T}|i}\}(1 + i)^{1-T}$ if $T < WP$
 $= 0 - E_t^c \bar{s}_{\overline{T}|i}(1 + i)^{1-T}$ if $T \geq WP;$

$y = 1,200(T - EP)$ if $T < WP$
 $= 1,200(T - EP) + (P^B + P^R)T$ if $T \geq WP;$

p = Probability of an active life at the beginning of policy year t becoming disabled during policy year t and then dying after having been disabled for T years ($T \leq 1$), given an elimination period EP ,

where

T = Length of disability,

WP = Waiting period (expressed as a fraction of a year) for disability waiver benefit which is retroactive to the beginning of the waiting period, and

EP = Elimination period (expressed as a fraction of a year) for disability income benefit.

$AIA(t)$ $x = \{P^R(1 - E_t^{\%}) - E_t^c \bar{s}_{\overline{1}|i}\}$ if $T < WP$
 $= P^R(1 - E_t^{\%}) \bar{s}_{\overline{1-T}|i} - E_t^c \bar{s}_{\overline{1}|i}$ if $T \geq WP ;$

$y = 1,200(T - EP)$ if $T < WP$
 $= 1,200(T - EP) + (P^B + P^R)T$ if $T \geq WP ;$

p = Probability of an active life at the beginning of policy year t becoming disabled, remaining disabled T years ($T \leq 1$), recovering to the active state, and remaining active to the end of the policy year, given an elimination period of EP .

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Name of Input Bivariate Frequency Distribution	Definitions
<i>AI</i> (<i>t</i>)	$x = 0 - E_t^c \bar{s}_{\overline{1} }$ $y = 1,200(1 - EP) + (P^B + P^R),$ <p><i>p</i> = Probability of an active life at the beginning of policy year <i>t</i> becoming disabled during policy year <i>t</i> and remaining disabled at least to the end of the policy year, given an elimination period of <i>EP</i>.</p>
<i>IA</i> (<i>s</i> , <i>t</i>)	$x = P^R(1 - E_t^{\%}) \bar{s}_{\overline{1/2} } - E_t^c \bar{s}_{\overline{1} }$ $y = 600 + \frac{1}{2}(P^B + P^R);$ <p><i>p</i> = Probability of a life who was initially disabled in policy year <i>s</i>, and who continued to be disabled until the beginning of policy year <i>t</i>, recovering to an active state by the end of the policy year <i>t</i>.</p>
<i>ID</i> (<i>s</i> , <i>t</i>)	$x = 0 - E_t^c \bar{s}_{\overline{1/2} }(1 + i)^{1/2};$ $y = 600 + \frac{1}{2}(P^B + P^R);$ <p><i>p</i> = Probability of a life who was initially disabled in policy year <i>s</i>, and who continued to be disabled until the beginning of policy year <i>t</i>, dying while continuing to be disabled in policy year <i>t</i>.</p>
<i>II</i> (<i>s</i> , <i>t</i>)	$x = 0 - E_t^c \bar{s}_{\overline{1} };$ $y = 1,200 + (P^B + P^R);$ <p><i>p</i> = Probability of a life who was initially disabled in policy year <i>s</i>, and who continued to be disabled at least until the beginning of policy year <i>t</i>, remaining disabled until the end of policy year <i>t</i>.</p>

Some assumptions used in this appendix are the following:

1. Disablement, if any, occurs at the beginning of the policy year.
2. No more than one disability per life occurs during a policy year.
3. Interest, premiums, and expenses accrue continuously during the policy year.
4. Per cent of premium expenses are not incurred while premiums are being waived, whereas per policy expenses are so incurred.

Assumptions differing from these are possible, and may be desirable.

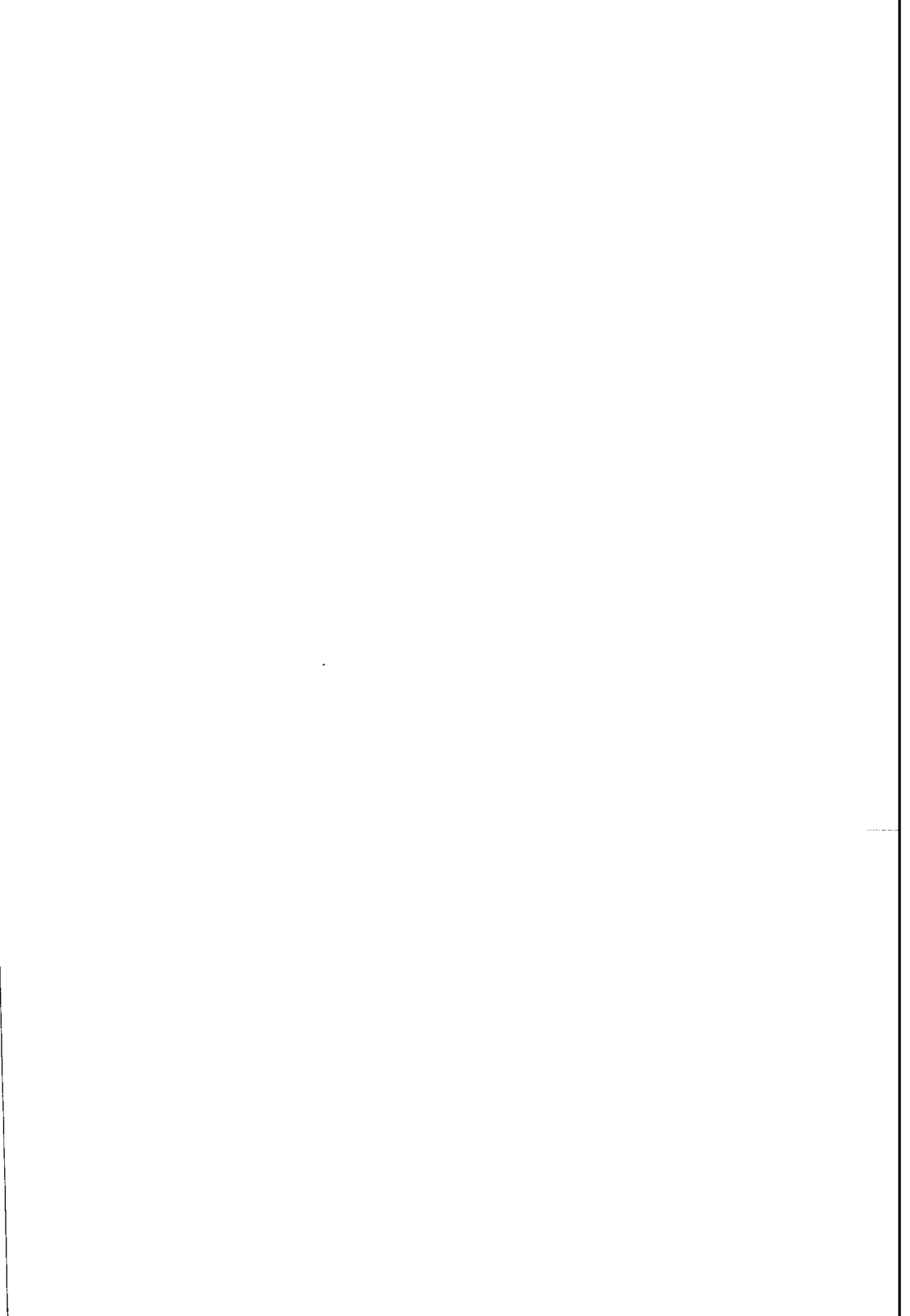
APPENDIX III
 OUTPUT FREQUENCY DISTRIBUTIONS

Name of Output Bivariate Frequency Distribution	Sum of p -Values in Each of These Bivariate Frequency Distributions
$D(01)$ to $D(15)$	Probability (at issue) of the insured's dying in the policy year indicated
$W(01)$ to $W(14)$	Probability (at issue) of the insured's voluntarily lapsing at the end of the policy year indicated
$INV(01)$ to $INV(14)$	Probability (at issue) of the insured's involuntarily lapsing the rider at the end of the policy year indicated, that is, terminating because the total of the disability benefits already paid was large enough to rule out the possibility of any return benefit ever being paid under the rider
$A(01)$ to $A(15)$	Probability (at issue) of the insured's entering the policy year indicated in an active life status
$I(01, 02)$ to $I(14, 15)$	Probability (at issue) of the insured's entering the indicated policy year in a disabled life status, having become disabled during the previous policy year
MAT	Probability (at issue) of the insured's persisting to maturity

Note: Of course, in each bivariate frequency distribution each individual value of p corresponds to a particular combination of x and y .

REFERENCES

1. WILLIAM FELLER. *An Introduction to Probability Theory and Its Applications*, Vols. I and II. 2d ed. New York: John Wiley & Sons, Inc.
2. ERNIE FRANKOVICH. "Health Insurance—Return of Premium Revisited," *TSA*, XXV, 363.
3. E. PAUL BARNHART. "The Return of Premium Benefit in Health Insurance," *TSA*, XXII, 235.



DISCUSSION OF PRECEDING PAPER

PAUL W. ROBBERSON:

The pricing of the return of premium policy as described by Mr. William Bailey is indeed a complicated affair. Mr. Bailey has certainly devised a very interesting and ingenious method for handling the problem. His illustration is also good, in that it shows that just the fact that a profitable disability income product is loaded 73 per cent to provide a return of premium rider does not guarantee profitability of the rider. In choosing to write such business, companies must be very careful in their choice of benefits, premiums, and reserves.

In selecting the type of rider, careful consideration should be given to using the deferred benefit with interim surrender benefits as described in the paper. This particular design does not have some of the questionable features of others which do not provide surrender benefits or tend to negate the basic protection by reducing return benefits because of a claim to a greater extent than the claim amount. This design also fits the model recommended by the National Association of Insurance Commissioners (NAIC *Proceedings*, II (1971), 413) and should, therefore, find better acceptance by state insurance departments. The guidelines of the NAIC give other provisions which may have to be considered.

One advantage to the company choosing this type of return rider is that to a large degree claim payments offset eventual return of premium payments. The effect is not good enough to provide complete immunity to adverse experience, but a relatively large amount of insensitivity to such fluctuations can result.

In order to gain this feature, however, the company must choose carefully the base policy to which the rider is to be attached. Only benefits which will produce substantial offsets should be considered. It can be seen, then, that long maximum periods are to be avoided, since claims running far in excess of the total premium to be paid during the life of the policy will never go toward reducing return payments. Likewise, long elimination periods subvert the offset principle by making the policyholders tend either to have claims or to not have them, thus eliminating the spread of claims necessary for substantial offsets.

A method which incorporates these ideas using the familiar book profit, or asset share, type of calculation can be used to illustrate the working of the return of premium benefit and aid in setting premiums and reserves.

The method, within limits, will give results comparable to that of Mr. Bailey's. Such a calculation can yield a wide variety of information which can be useful in setting premiums and reserves. The calculation assumes a number of issue units (l_0) which progresses according to a total decrement, $(q^T) = q^d + q^w$, of lapses and deaths.

This approach utilizes net annual claim costs and cash, or surrender, values calculated in advance to reflect the partial offset of claims. The book profits are then calculated, year by year, and all discounted to issue. This can be done for the rider only or for the base policy and rider in combination.

Once the original work is done, any change in expenses and interest can be made solely in the asset share calculation, the surrender benefits being unchanged. If any change in premium is required, one need only recalculate the surrender benefits and terminal reserves as described and then recalculate the asset share. This method does not develop the profits which arise from the various modes of terminations, as Mr. Bailey's does, but the approach has several features which make it useful.

1. It is a common tool, well understood by all actuaries and therefore relatively easy to use to achieve proper results.
2. The use of net annual claim costs provides an easy way to account for many variations in benefits without changing the logic of the method. For instance, the disability benefit may provide for yearly increasing benefits during disability, say an extra 5 per cent per year. The benefits may include (or be) a hospital indemnity amount. Others which are more subtle could be benefit provisions for nondisabling injury, partial accident, and the like.
3. The figures from the asset share can be used, for individual ages or model-office combinations, to keep a check on the accuracy of the assumptions. As mentioned, the effect of adding the return of premium may have unpredictable effects on either the incidence or the continuance of claims. The effect on lapses may also be significant. The percentage claims which are effective in offsetting surrender benefits can also be easily reviewed on an aggregate basis.
4. The calculation of the surrender benefits, as explained above, provides an easy way of computing adjusted reserves. They can be treated simply as any cash value would be in such a calculation.

Let the following definitions apply, using Bailey's notation where possible:

$$l_t = l_{t-1}(1 - q_t^T) = \text{Lives at the end of year } t; \quad (1)$$

$$(P^B + P^R) \frac{l_{t-1} + l_t}{2} = \text{Average premium received in year } t; \quad (2)$$

TABLE 1

Policy Year	Lives at Beginning of Year (1)	Premium Income (2)	Expenses (3)	Claims (4)	Surrender Benefit (5)	Reserve at End of Year (6)	Book Profit (7)
Rider Only							
1. . . .	1,000	\$32,184	\$24,172	\$0	\$ 0	\$ 0	\$ 8,012
2. . . .	796	27,005	3,393	0	0	41,067	(-)17,455
3. . . .	711	24,246	3,071	0	1,228	75,461	(-)12,394
4. . . .	642	21,988	2,814	0	2,028	104,961	(-) 8,581
5. . . .	585	20,124	2,596	0	2,927	130,987	(-) 6,177
6. . . .	538	18,583	2,416	0	3,747	154,565	(-) 4,609
7. . . .	499	17,329	2,280	0	4,375	177,072	(-) 4,105
8. . . .	468	16,236	2,160	0	5,839	196,820	(-) 2,657
9. . . .	438	15,160	2,040	0	7,982	213,327	(-) 1,528
10. . . .	408	14,121	1,921	0	9,970	227,590	(-) 1,367
11. . . .	380	13,135	1,153	0	12,353	239,210	(-) 612
12. . . .	353	12,186	1,088	0	14,824	248,236	(-) 791
13. . . .	327	11,272	1,030	0	17,609	254,725	(-) 1,444
14. . . .	302	10,376	971	0	21,263	257,818	(-) 2,215
15. . . .	277	9,928	926	0	0	282,842*	(-) 3,131

Present value of book profit at issue. \$(-)46,097

Rider and Base Policy Combined							
1. . . .	1,000	\$76,079	\$55,777	\$ 8,910	\$ 0	\$ 0	\$11,392
2. . . .	796	63,837	6,892	8,225	0	41,070	7,650
3. . . .	711	57,313	6,213	8,213	1,228	83,204	1,579
4. . . .	642	51,976	5,663	8,328	2,028	118,537	4,784
5. . . .	585	47,570	5,204	8,511	2,927	148,888	6,504
6. . . .	538	43,927	4,824	8,784	3,747	175,542	7,362
7. . . .	499	40,962	4,525	9,180	4,375	200,085	7,116
8. . . .	468	38,378	4,264	9,652	5,839	220,626	8,086
9. . . .	438	35,837	4,005	10,063	7,982	236,706	8,738
10. . . .	408	33,380	3,750	10,391	9,970	249,569	8,241
11. . . .	380	31,050	1,960	10,744	12,353	258,785	9,255
12. . . .	353	28,805	1,836	11,143	14,824	264,402	8,324
13. . . .	327	26,644	1,722	11,645	17,609	266,422	6,868
14. . . .	302	24,526	1,608	12,083	21,263	264,052	5,263
15. . . .	277	23,467	1,535	12,954	0	282,843*	3,390

Present value of book profit at issue. \$74,677

* Total maturity benefits.

$$(2) \times E\% + E^C = \text{Expenses in year } t; \quad (3)$$

$$S_t l_{t-1} = \text{Net claim cost for year } t; \quad (4)$$

$$SV_t = \max \left\{ F^1 K_t l (P^B + P^R), K_t l (P^B + P^R) - F^2 \sum_{z=1}^t S_z \right\} \quad (5)$$

= Expected surrender benefit in year t ;

$$V_t = \text{Terminal reserve at end of year } t. \quad (6)$$

The terminal reserves are the sum of the regular benefit reserve and a reserve for the return benefit which might be defined as the one-year preliminary term reserve for the expected surrender benefit at maturity. Surrender benefits for each year on a block of policies are defined as the maximum benefit, $k_t l (P^B + P^R)$, less the accumulated claims which do not exceed the maximum, although in no case may the benefit be less than $(0 \leq F^1 \leq 1)$ of the maximum. The use of F^1 and F^2 is the key to this method. While theoretically these values vary with plan age, and duration, and are quite complex in their probabilistic determination, some argument can be made for the fact that they should lie in neighborhoods of 0.5 (for F^1) and 0.75 (for F^2) for a rather wide range of benefit descriptions. Mr. Bailey's method very cleverly incorporates automatically the partial claim offset against maximum surrender. However, since the nature of claims may belie the assumed morbidity, the varying of F^1 , F^2 , or S_t allows a simple sensitivity test on the level of profits. In practice, the SV_t 's and V_t 's are calculated with a trial premium.

The book profit for a policy year (t) can be expressed as

$$BP_t = (2) - (3) - (4) - (5)(l_{t-1} - l_t) + {}_{t-1}V(1+i)l_{t-1} - {}_tVl_t.$$

Using assumptions which are substantially the same as Bailey's, we obtain the results shown in Table 1 of this discussion.

Again, Mr. Bailey is to be thanked for his addition to, and prompting of, discussion about the body of information relating to the pricing of this benefit.

(AUTHOR'S REVIEW OF DISCUSSION)

WILLIAM A. BAILEY:

My appreciation to Mr. Robberson for his comments on my paper and the presentation of some of his thoughts on designing and pricing return of premium benefits. Even where refined methods are to be used ultimately in the pricing process, cruder methods can be valuable in making initial estimates of the premiums to be tested for profitability.

The following numbering coincides with Mr. Robberson's numbered remarks on the features of his suggested approach.

1. Mr. Robberson's approach is a straightforward application of the traditional book profit method, assuming that the values of F^1 and F^2 are known or can be assumed. The paper implies possibly different values of F^1 and F^2 for each mode of termination, year of termination, and value of the other variables listed in Section XI, and the voluntary lapse rates, morbidity rates, and mortality rates assumed. It is not obvious to me how Mr. Robberson can be confident of his chosen values of F^1 and F^2 without first making more refined calculations to confirm the level and relative constancy of these parameters. But perhaps he has done so.

In developing the method described in the paper, I was trying to meet several objectives, in addition to the development of a calculation procedure for expected value of profits; the method was required to

- a) Permit the determination of theoretically appropriate cash values as a by-product.
- b) Reflect properly the effect of involuntary lapses; whether the rider is separable or not, there may be pressure to permit discontinuance of the rider premium once it becomes clear that no return benefit will ever be paid.
- c) Produce a frequency distribution for each insured life, which can in turn be used to calculate frequency distributions of aggregate natural reserves, in order to determine appropriate surplus levels.
- d) Not be dependent upon any particular reserve method in the calculation of expected value of profits.

To meet all these objectives, the effect of the waiver-of-premiums benefit had to be built in as an integral part of the method. I am not sure whether such benefits were evaluated in Mr. Robberson's numerical example.

If these requirements are removed, then the whole problem becomes much simpler. I would resort to univariate frequency distributions as defined in Section VIII of the paper. An example of the results of using univariate frequency distributions is given in reference [4]; the return benefit being addressed there is of the rollover variety.

2. Each of the variations in benefits mentioned by Mr. Robberson can be handled under the method described in the paper, by modifying the input frequency distributions. Under his method the effect of different net annual claim costs on the maturity benefit expected may in some cases be approximated closely enough for practical purposes. However, the effect on the incidence of involuntary terminations will depend on the incidence and severity pattern underlying the net annual claim costs; even similar net annual claim costs may produce different levels of profits from involuntary terminations.
3. The traditional book profit approach depends upon reserves which in turn are based on an "expected surrender benefit at maturity." The method of

the paper was designed so as not to rely on a reserve method which disregards the retrospective claim history of the individual in-force policies. However, if such individual claim histories are to be ignored in calculating reserves, then the "Mean/Cum" figures shown in Table 1 of the paper can be distributed by policy year to produce results in the more traditional book profit form.

4. The topic of natural reserves under the rollover type of return benefit is touched on in my discussion of Ernie Frankovich's paper.¹ The method suggested therein could not be applied directly using the figures in Table 1 of the paper, because the waiver of premium benefit has been built into Table 1 and in my discussion I assumed that the waiver of premium benefit is evaluated separately. Otherwise, the natural reserve method suggested in my discussion could be applied.

Thus, most of the features of Mr. Robberson's suggested method can also be considered to be features of the method in the paper, under certain simplifying assumptions. My thanks to Mr. Robberson for his clear and succinct presentation.

The cash-value type of return benefit is a special case of the rollover type of return benefit, where

1. The rollover period is the number of years from issue to maturity, instead of, say, ten years.
2. "Involuntary terminations" replace "restarts."
3. Cash values are payable (there is no theoretical reason to prevent cash values being payable under the rollover type of benefit, but the benefit would be more expensive if cash values were offered).

The remainder of this discussion will focus on the rollover type of return benefit. The same general technique described in the paper can be applied where the return benefit is of the rollover type. The pricing of the rollover type of return benefit is approached by treating each 10-year rollover period as a separate calculation. That is, frequency distributions of present value of profits (\pm), corresponding to the figures in Table 1 of the paper, are calculated assuming a maturity date ten years away. Voluntary lapses involve the payment of no cash values; deaths may or may not involve the payment of death benefits, and involuntary lapses become restarts entering a new rollover period. The profits (\pm) accruing subsequent to restart are handled later in the calculations, but, first, the return benefits payable during or at the end of the particular ten-year rollover period are assumed to be funded from the premiums received prior to restart during that ten-year rollover period.

¹ Ernie Frankovich, "Health Insurance—Return of Premiums Revisited," *TSA*, XXV, 391.

Then, after constructing frequency distributions of present value of profits (\pm) at the beginning of each ten-year rollover period for that ten-year rollover period, a recursive process is applied starting with the rollover period nearest to the expiry date of coverage and ending with the ten-year rollover period beginning at issue.

Under the most common types of rollover provisions, the rollover period grades from ten years down to one year just prior to expiry. In order to describe the backward recursive procedure used, more specific notation will be used as indicated below.

The steps shown must be performed in the order indicated, except insofar as the operations of \otimes and \oplus are each commutative. Once again, capital letters designate frequency distributions; transforms are indicated above the file to which they apply; and z is the age at which coverage ceases under the policy.

Symbols used in the paper for bivariate frequency distributions:

$$W(t), \quad D(t), \quad INV(t), \quad MAT(t).$$

Symbols used in this discussion for the corresponding univariate frequency distributions of present value of profits (\pm):

$$W(x, s, t), \quad D(x, s, t), \quad INV(x, s, t), \quad MAT(x, s, t),$$

where

x = Issue age;

s = Policy duration at beginning of the rollover period;

t = Policy year measured from beginning of the rollover period.

$$B(x, z - x - 1) = D(x, z - x - 1, 1) \otimes W(x, z - x - 1, 1) \\ \otimes INV(x, z - x - 1, 1) \otimes MAT(x, z - x - 1, 1).$$

$$F(01) = INV(x, z - x - 2, 1) \overset{vx \rightarrow x}{+} B(x, z - x - 1);$$

$$B(x, z - x - 2) = D(x, z - x - 2, 1) \otimes W(x, z - x - 2, 1)$$

$$\otimes F(01) \otimes D(x, z - x - 2, 2)$$

$$\otimes W(x, z - x - 2, 2) \otimes INV(x, z - x - 2, 2)$$

$$\otimes MAT(x, z - x - 2, 2).$$

$$Y(01) = INV(x, z - x - 3, 1) \oplus B(x, z - x - 2) ;$$

$$Y(02) = INV(x, z - x - 3, 2) \oplus B(x, z - x - 1) ;$$

$$B(x, z - x - 3) = D(x, z - x - 3, 1) \textcircled{M} W(x, z - x - 3, 1) \\ \textcircled{M} Y(01) \textcircled{M} D(x, z - x - 3, 2) \\ \textcircled{M} W(x, z - x - 3, 2) \textcircled{M} Y(02) \\ \textcircled{M} D(x, z - x - 3, 3) \textcircled{M} W(x, z - x - 3, 3) \\ \textcircled{M} INV(x, z - x - 3, 3) \textcircled{M} MAT(x, z - x - 3, 3).$$

$$Y(01) = INV(x, z - x - 10, 1) \oplus B(x, z - x - 9) ;$$

$$Y(02) = INV(x, z - x - 10, 2) \oplus B(x, z - x - 8) ;$$

$$Y(03) = INV(x, z - x - 10, 3) \oplus B(x, z - x - 7) ;$$

⋮

$$Y(09) = INV(x, z - x - 10, 9) \oplus B(x, z - x - 1) ;$$

$$B(x, z - x - 10) = D(x, z - x - 10, 1) \textcircled{M} W(x, z - x - 10, 1) \\ \textcircled{M} Y(01) \textcircled{M} D(x, z - x - 10, 2) \\ \textcircled{M} W(x, z - x - 10, 2) \textcircled{M} Y(02) \\ \dots \\ \textcircled{M} D(x, z - x - 10, 9) \textcircled{M} W(x, z - x - 10, 9) \\ \textcircled{M} Y(09) \textcircled{M} D(x, z - x - 10, 10) \\ \textcircled{M} W(x, z - x - 10, 10) \textcircled{M} INV(x, z - x - 10, 10) \\ \textcircled{M} MAT(x, z - x - 10, 10) .$$

$$Y(01) = INV(x, z - x - 11, 1) \oplus B(x, z - x - 10) ;$$

$$Y(02) = INV(x, z - x - 11, 2) \oplus B(x, z - x - 9) ;$$

⋮

$$Y(10) = INV(x, z - x - 11, 10) \oplus^{v^{10}x \rightarrow x} B(x, z - x - 1) ;$$

$$Y(11) = MAT(x, z - x - 11, 10) \oplus^{v^{10}x \rightarrow x} B(x, z - x - 1) ;$$

$$B(x, z - x - 11) = D(x, z - x - 11, 1) \textcircled{M} W(x, z - x - 11, 1)$$

$$\textcircled{M} Y(01) \textcircled{M} D(x, z - x - 11, 2)$$

$$\textcircled{M} W(x, z - x - 11, 2) \textcircled{M} Y(02)$$

.....

$$\textcircled{M} D(x, z - x - 11, 10) \textcircled{M} W(x, z - x - 11, 10)$$

$$\textcircled{M} Y(10) \textcircled{M} Y(11) .$$

Etc.

$$Y(01) = INV(x, 0, 1) \oplus^{vx \rightarrow x} B(x, 1) ;$$

$$Y(02) = INV(x, 0, 2) \oplus^{v^2x \rightarrow x} B(x, 2) ;$$

⋮

$$Y(10) = INV(x, 0, 10) \oplus^{v^{10}x \rightarrow x} B(x, 10) ;$$

$$Y(11) = MAT(x, 0, 10) \oplus^{v^{10}x \rightarrow x} B(x, 10) ;$$

$$B(x, 0) = D(x, 0, 1) \textcircled{M} W(x, 0, 2) \textcircled{M} Y(01) \textcircled{M} D(x, 0, 2)$$

$$\textcircled{M} W(x, 0, 2) \textcircled{M} Y(02)$$

.....

$$\textcircled{M} D(x, 0, 10) \textcircled{M} W(x, 0, 10)$$

$$\textcircled{M} Y(10) \textcircled{M} Y(11) .$$

Note that v^t could be redefined to reflect interest rates varying by policy year. $B(x, 0)$ is the frequency distribution of the present value of profits (\pm) at issue for the whole period from issue to expiry. If only the expected value of profits (\pm) is to be used, then the volume of calculations in the backward recursive procedure can be reduced by collapsing each of the intermediate and frequency distributions to "one-liners" of the form [Mean/Cum, Cum].

