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# CHARACTERISTICS AND OPERATION OF <br> PROJECTION VALUATION METHODS FOR PENSION PLAN FUNDING 

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#### Abstract

A new approach to computing annual pension costs, based on an open-group projection of the participant population for a number of years, was described in detail in a paper published in the 1975 Transactions, entitled "The Forecast Valuation Method for Pension Plans." The paper described the operation of such a cost method at the initial valuation date.

The following paper discusses several alternative approaches to projection valuation methods and also describes how costs should be computed at valuation dates after the first one. In addition, the author illustrates projected contributions and asset buildups under both traditional (closed-group) cost methods and projected (open-group) cost methods, for a number of years under a variety of assumptions as to the growth rate of the participant group.

Projection valuation methods have not been described heretofore in sufficient detail to make it possible for actuaries to use them on a recurring basis or for the Internal Revenue Service to make a determination as to the acceptability of such approaches for computing tax-deductible pension plan contributions. It is the hope of the author that this paper will fill both of those gaps.


## 1. INTRODUCTION

OPEN-GROUP projection valuation methods, as described in this paper, are actuarial techniques for determining annual contributions to defined benefit pension programs. Unlike conventional actuarial cost methods, projection methods permit the use of explicit assumptions as to future additions to the group of participants. These open-group techniques take into consideration both the number and the age, sex, and salary characteristics of future new entrants for a number of years, and thereby allow the actuary to develop a funding approach that is more likely to follow a predictable pattern than if future entrants are ignored.

While traditional funding methods produce a snapshot view of cost, that is, present value and asset amounts for only a single year, projection (or forecast) methods develop a series of snapshots for a number of years into the future. This series of projected values provides a wealth of data to the actuary, the plan sponsor, and the fund's investment manager; the data are useful in the following ways:
The estimates of future annual costs give the plan sponsor a better understanding of long-range cost amounts and patterns.
The projection of contributions and benefit payments allows the investment manager to plan for expected liquidity requirements, to establish proper asset mixes, and to determine an investment policy that complies with the sponsor's level of risk acceptance.
The projection of assets and present values of accrued and vested benefits provides the actuary with a clear picture of the year-to-year trends in benefitsecurity ratios for a given funding pattern.
Projection methods are quite flexible with respect to (1) varying actuarial assumptions from time to time, (2) incorporating an expected benefit formula change or one-time benefit improvement at some future date, or (3) anticipating the effects of acquisitions or divestitures involving large groups of employees.

Over the last three decades, actuarial funding techniques for pension plans have progressed significantly, from annual premiums based only on mortality and interest assumptions and on the continuation of current salary levels, to annual costs based on additional assumptions as to termination probabilities, ages at retirement, salary increases, social security increases, and in certain cases rates of disability and rates of remarriage. The increased sophistication of actuarial methods has coincided with a rapid increase in the complexity of plan provisions and in the importance of pension costs to plan sponsors.

Retirement plans continue to become more complicated and more costly, and the investment of plan assets continues to grow in importance both to plan sponsors and to the nation's economy in general. In order to meet the requirements for more information and better predictions of costs and cash flow, actuaries need to advance the state of their art beyond the static, closed-group techniques of the past and present. Projection valuation methods represent such an advancement. Furthermore, the following two segments of the Employee Retirement Income Security Act of 1974 (ERISA) appear virtually to require the use of projection methods:

1. The enrolled actuary is required to use his "best estimate of anticipated experience $\{s e c .103(\mathrm{a})(4)(\mathrm{B})$ of the act\}'; this presumably includes his
estimate of future changes in the participant group, both deletions and additions.
2. The committee reports that are part of ERISA contain the following statement regarding section 402 of the act: "Under the Labor provisions of the substitute, each plan is to provide a procedure for establishing a funding policy and method to carry out the plan objectives. This procedure is to enable the plan fiduciaries to determine the plan's short- and long-run financial needs and communicate these requirements to the appropriate persons. For example, with a retirement plan it is expected that under this procedure the persons who manage the plan will determine whether the plan has a short-run need for liquidity (e.g., to pay benefits) or whether liquidity is a long-run goal and investment growth is a more current need. This in turn is to be communicated to the persons responsible for investments, so that investment policy can be appropriately coordinated with plan needs." If this comment is taken literally, it seems to require cash-flow forecasts such as are provided by projection valuation methods.

The purposes of this paper are (1) to define and describe the operation of projection valuation methods (PVM's) in sufficient detail so that actuaries can utilize them; (2) to compare conventional actuarial cost methods with projection techniques; and (3) to discuss prevalent concerns about the use of PVM's and rebut such concerns. This is the first paper that describes how a PVM should operate at the second and subsequent valuation dates, and that compares the cost and asset accumulations over a period of years under projected and conventional cost methods.

The American Academy of Actuaries has already recognized opengroup valuation techniques as acceptable actuarial practices in the Recommendations of the Committee on Actuarial Principles and Practices in Connection with Pension Plans, published June, 1976. It is the hope of the author that the following presentation, in conjunction with articles and papers previously written on this subject, will persuade the Internal Revenue Service to determine that such techniques (1) are acceptable actuarial funding methods for computing minimum and maximum contributions under the Internal Revenue Code for tax-qualified pension plans and (2) in many instances are more appropriate than traditional closed-group funding techniques.

The actuarial theories and basic formulas for projecting benefits, salaries, and present values for an open group of pension plan participants are not covered in this paper; these matters are explained fully in a paper by Donald R. Fleischer entitled "The Forecast Valuation Method for Pension Plans" (TSA, XXVII, 93). Mr. Fleischer's paper is vital to
a full understanding of the rationale for PVM's and the mechanics of their implementation.

It is the author's desire to provoke discussion and stimulate thought on all aspects of the techniques and theories underlying his approach to PVM's. Such discussions can advance significantly the skills an actuary can bring to bear on the tasks he is called upon to perform, to the ultimate benefit of the public he serves.

## II. OPERATION OF A PVM AT THF FIRST VALUATION

## Setting Objectives

The operation of a PVM for a particular pension plan depends on the funding objective selected by the actuary and plan sponsor at the time the PVM is first applied to the plan. The funding objective, in turn, is based on two elements: the type of present value of benefits toward which the funding scheme is aimed and the time period in which full funding of the chosen present value objective ( PVO ) is to be achieved.

Any one of the following types of present values may be useful as a funding objective in a particular situation:

1. Present value of accrued benefits, determined on a continuing plan basis.
2. Present value of accrued benefits, determined on a plan termination basis (100 percent vesting for all participants).
3. Supplemental present value determined under a conventional entry age normal method.
4 Present value of vested accrued benefits.
The author believes that only the first three types are appropriate for use in computing contributions for a tax-qualified plan, since they are most similar to the funding objectives of actuarial cost methods that are currently used.

The time period in which the funding objective is to be achieved should be selected on the same basis as would be used in choosing an amortization period for the unfunded present value under a conventional cost method. One should consider statutory requirements, age of the plan, average age of the participant group, financial condition of the plan sponsor, and type of benefit formula.

The most common use of a PVM will be to determine the level percentage of covered payroll or the level amount per capita that must be contributed annually over the entire selected time period in order to (1) pay current benefits or purchase annuities as they become due and (2) build up a fund that, at the end of the period, will equal the PVO. It is possible, however, to use the information provided in the year-by-year forecast of the plan population to develop nonlevel funding patterns.

The remainder of this paper will deal with patterns of cost that are intended to be level percentages of covered payroll throughout the objective period. The appendixes at the end of the paper contain descriptions of the plan provisions and the actuarial assumptions that were used in developing the tables presented in the paper.

## Initial Valuation Procedures

For purposes of illustration, a funding objective period of twenty years has been chosen. Costs are determined as the level percentages of payroll required to fund fully the three PVO's described in items $1-3$ above, for populations projected on the basis of (1) no new entrants (closed group), (2) a constant number of active participants, and (3) an active participant group that increases at the rate of 5 percent per year.

Tables 1A-1C show, for each of the twenty-five years following the assumed initial valuation date of January 1, 1976, the numbers of active participants, retirees, and vested terminees, the covered payroll, and the amount of annual retirement benefits, for each of the new-entrant assumptions. Although the funding objective period is twenty years, the population has been projected for twenty-five years in order to determine the expected cost levels after the initial objective is achieved. Tables $2 \mathrm{~A}-2 \mathrm{C}$ show the various present values (in thousands of dollars) that are ingredients in the determination of costs under the PVM, for each of the new-entrant assumptions.

The level contribution rate required to accomplish the full funding of a given PVO at the end of an $n$-year period may be expressed as (a) plus (b) minus ( $c$ ), all divided by ( $d$ ), where
(a) is the present value at the initial valuation date of the benefits expected to be paid during the next $n$ years;
(b) is the present value at the initial valuation date of the expected PVO as of the end of the $n$th year;
(c) is the amount of valuation assets at the initial valuation date; and
(d) is the present value at the initial valuation date of the covered payroll expected in the next $n$ years.

For example, if $n$ equals twenty years and the plan assets on the initial valuation date of January 1,1976 , amount to $\$ 2,400,000$, then the level funding rates would be determined as indicated in Table 3 for the three populations and three PVO's described above.

It is evident from Table 3 that, for a given PVO and time period, the funding rate decreases as the assumed number of new entrants increases.

TABLE 1A
Number of Active Participants and Covered annual Payroll

| Year | Closed Group |  | Constant Grotr |  | 5 Percent <br> Increasing Group |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Number | $\begin{gathered} \text { Covered } \\ \text { Payroll } \\ \text { (in } \$ 1,000 \text { 's) } \end{gathered}$ | Number | $\begin{gathered} \text { Covered } \\ \text { Payroll } \\ \text { (in } \$ 1,000 \text { 's) } \end{gathered}$ | Number | Covered <br> Payroll <br> (in $\$ 1,000$ 's) |
| 1976 | 1,209 | \$15,681 | 1,209 | \$15,681 | 1,209 | \$15,681 |
| 1977 | 1,024 | 14,075 | 1,209 | 15,881 | 1,269 | 16,472 |
| 1978. | 878 | 12,689 | 1,209 | 16,121 | 1,333 | 17,406 |
| 1979. | 759 | 11,638 | 1,209 | 16,377 | 1,400 | 18,367 |
| 1980. | 658 | 10,693 | 1,209 | 16,697 | 1,469 | 19,500 |
| 1981 | 579 | 9,930 | 1,209 | 16,943 | 1,543 | 20,576 |
| 1982 | 509 | 9,232 | 1,209 | 17,218 | 1,620 | 21,777 |
| 1983 | 448 | 8,567 | 1,209 | 17,554 | 1,701 | 23,185 |
| 1984 | 395 | 7,937 | 1,209 | 17,816 | 1,786 | 24,563 |
| 1985 | 350 | 7,394 | 1,209 | 18,165 | 1,876 | 26,185 |
| 1986. | 311 | 6,898 | 1,209 | 18,453 | 1,969 | 27,772 |
| 1987. | 277 | 6,437 | 1,209 | 18,754 | 2,068 | 29,498 |
| 1988 | 246 | 5,978 | 1,209 | 19,121 | 2,171 | 31,511 |
| 1989. | 217 | 5,549 | 1,209 | 19,501 | 2,280 | 33,674 |
| 1990 | 193 | 5,147 | 1,209 | 19,926 | 2,393 | 36,101 |
| 1991. | 169 | 4,705 | 1,209 | 20,301 | 2,514 | 38,621 |
| 1992. | 150 | 4,373 | 1,209 | 20,785 | 2,639 | 41,490 |
| 1993. | 133 | 4,059 | 1,209 | 21,324 | 2,771 | 44,710 |
| 1994. | 117 | 3,707 | 1,209 | 21,844 | 2,909 | 48,163 |
| 1995. | 103 | 3,392 | 1,209 | 22,389 | 3,055 | 51,976 |
| 1996. | 91 | 3,098 | 1,209 | 22,920 | 3,207 | 56,010 |
| 1997 | 80 | 2,848 | 1,209 | 23,561 | 3,368 | 60,536 |
| 1998. | 70 | 2,588 | 1,209 | 24, 237 | 3,537 | 65,548 |
| 1999. | 61 | 2,345 | 1,209 | 24,945 | 3,714 | 71,004 |
| 2000. | 52 | 2,079 | 1,209 | 25,645 | 3,899 | 76,956 |

TABLE 1B
Number of Retired Lives and Annual Benefit Payments

| Year | Closed Group |  | Constant Group |  | 5 Percent <br> Increasing Group |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Number | $\begin{gathered} \text { Annual } \\ \text { Benefits } \\ \text { (in } \$ 1,000 \text { 's) } \end{gathered}$ | Number | $\begin{gathered} \text { Annual } \\ \text { Benefits } \\ \text { (in } \$ 1,000 \text { 's) } \end{gathered}$ | Number | Annual Benetits (in $\$ 1,000$ 's) |
| 1976 | 79 | \$ 66 | 79 | \$ 66 | 79 | \$ 66 |
| 1977. | 110 | 109 | 110 | 109 | 110 | 109 |
| 1978. | 129 | 159 | 129 | 159 | 129 | 159 |
| 1979. | 153 | 191 | 153 | 191 | 153 | 191 |
| 1980. | 174 | 227 | 174 | 227 | 174 | 227 |
| 1981. | 189 | 255 | 189 | 255 | 189 | 255 |
| 1982. | 209 | 289 | 209 | 289 | 209 | 289 |
| 1983. | 229 | 328 | 229 | 328 | 229 | 328 |
| 1984. | 243 | 372 | 243 | 372 | 243 | 372 |
| 1985 | 254 | 410 | 254 | 410 | 255 | 410 |
| 1986. | 267 | 448 | 268 | 450 | 269 | 450 |
| 1987. | 277 | 487 | 280 | 491 | 281 | 492 |
| 1988. | 286 | 528 | 289 | 534 | 291 | 536 |
| 1989. | 294 | 565 | 300 | 575 | 302 | 580 |
| 1990. | 306 | 606 | 315 | 623 | 319 | 630 |
| 1991. | 316 | 659 | 329 | 684 | 335 | 696 |
| 1992. | 326 | 694 | 343 | 728 | 351 | 745 |
| 1993. | 331 | 728 | 352 | 773 | 363 | 796 |
| 1994. | 340 | 771 | 367 | 829 | 381 | 861 |
| 1995. | 348 | 817 | 381 | 898 | 401 | 943 |
| 1996. | 353 | 859 | 393 | 967 | 418 | 1,031 |
| 1997. | 360 | 882 | 407 | 1,010 | 437 | 1,090 |
| 1998. | 369 | 911 | 424 | 1,062 | 461 | 1,160 |
| 1999 | 378 | 932 | 440 | 1,108 | 485 | 1,229 |
| 2000. | 389 | 962 | 460 | 1,173 | 514 | 1,324 |

TABLE 1C
Number of Vested Terminees and Total Number of Participants

| Year | Vested Terminees |  |  | Participants |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Closed <br> Group | Constant Group | 5 Percent Increasing Group | Closed <br> Group | Constant <br> Group | 5 Percent Increasing Group |
| 1976 | 317 | 317 | 317 | 1,605 | 1,605 | 1,605 |
| 1977 | 351 | 351 | 351 | 1,485 | 1,670 | 1,730 |
| 1978. | 386 | 386 | 386 | 1,393 | 1,724 | 1,848 |
| 1979. | 414 | 414 | 414 | 1,326 | 1,776 | 1,967 |
| 1980. | 444 | 444 | 444 | 1,276 | 1,826 | 2,087 |
| 1981 | 475 | 475 | 475 | 1,243 | 1,873 | 2,207 |
| 1982 | 499 | 499 | 499 | 1,217 | 1,917 | 2,328 |
| 1983 | 526 | 526 | 526 | 1,203 | 1,964 | 2,456 |
| 1984 | 554 | 554 | 554 | 1,192 | 2,006 | 2,583 |
| 1985 | 576 | 576 | 576 | 1,180 | 2,039 | 2,707 |
| 1986 | 589 | 589 | 589 | 1,167 | 2,066 | 2,827 |
| 1987. | 599 | 603 | 605 | 1,153 | 2,092 | 2,954 |
| 1988. | 608 | 620 | 624 | 1,140 | 2,118 | 3,086 |
| 1989. | 613 | 634 | 643 | 1,124 | 2,143 | 3,225 |
| 1990. | 610 | 643 | 657 | 1,109 | 2,167 | 3,369 |
| 1991 | 551 | 653 | 674 | 1,091 | 2,191 | 3,523 |
| 1992 | 598 | 660 | 690 | 1,074 | 2,212 | 3,680 |
| 1993 | 592 | 670 | 711 | 1,056 | 2,231 | 3,845 |
| 1994 | 579 | 675 | 728 | 1,036 | 2,251 | 4,018 |
| 1995 | 565 | 678 | 745 | 1,016 | 2,268 | 4,201 |
| 1996. | 551 | 682 | 765 | 995 | 2,284 | 4,390 |
| 1997 | 533 | 684 | 785 | 973 | 2,300 | 4,590 |
| 1998. | 512 | 681 | 802 | 951 | 2,314 | 4,800 |
| 1999. | 490 | 678 | 821 | 929 | 2,327 | 5,020 |
| 2000. | 464 | 672 | 839 | 905 | 2,341 | 5,252 |

TABLE 2A
Present Values Needed for Cost Calculations
(Thousands of Dollars)
CLOSED GROUP

| Year $n$ | Present Value at 1/1/76 of Salaries terough Year $n$ | Present Value Objective at End of Year n |  |  | Present <br> Value of Benefit <br> Payments through Year $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Accrued <br> Benelits, Continuing Plan Assumption* | Accrued <br> Benefits, Terminating Plan <br> Assumption* | Entry Age <br> Normal <br> Supplemental <br> Present <br> Value |  |
| 1976. | \$15,681 | \$4,221 | \$4,200 | \$7,622 | \$ 64 |
| 1977. | 28,959 | 4,832 | 4,788 | 8,142 | 163 |
| 1978. | 40,253 | 5,398 | 5,326 | 8,629 | 301 |
| 1979. | 50,024 | 5,968 | 5,866 | 9,102 | 456 |
| 1980. | 58,494 | 6,531 | 6,403 | 9,558 | 631 |
| 1981 | 65,914 | 7,100 | 6,954 | 10,005 | 816 |
| 1982 | 72,422 | 7,662 | 7,511 | 10,436 | 1,014 |
| 1983 | 78,119 | 8,208 | 8,053 | 10,845 | 1,226 |
| 1984 | 83,099 | 8,725 | 8,568 | 11,226 | 1,452 |
| 1985. | 87,476 | 9,222 | 9,065 | 11,585 | 1,688 |
| 1986. | 91,327 | 9,696 | 9,542 | 11,920 | 1,930 |
| 1987. | 94,718 | 10,145 | 9,993 | 12,226 | 2,180 |
| 1988. | 97,689 | 10,563 | 10,416 | 12,503 | 2,435 |
| 1989 | 100,290 | 10,951 | 10,809 | 12,751 | 2,692 |
| 1990 | 102,567 | 11,306 | 11,168 | 12,967 | 2,953 |
| 1991. | 104,530 | 11,602 | 11,469 | 13,131 | 3,220 |
| 1992. | 106,251 | 11,867 | 11,740 | 13,266 | 3,486 |
| 1993. | 107,758 | 12,100 | 11,980 | 13,370 | 3,748 |
| 1994. | 109,057 | 12,285 | 12,172 | 13,431 | 4,011 |
| 1995. | 110,178 | 12,414 | 12,308 | 13,446 | 4,273 |
| 1996. | 111,144 | 12,488 | 12,389 | 13,414 | 4,534 |
| 1997 | 111,982 | 12,531 | 12,440 | 13,354 | 4,786 |
| 1998 | 112,700 | 12,533 | 12,451 | 13,257 | 5,031 |
| 1999. | 113,314 | 12,502 | 12,430 | 13,131 | 5,208 |
| 2000. | 113,828 | 12,407 | 12,361 | 12,946 | 5,499 |

[^0]TABLE 2B
Present Values Needed for Cost Calculations
(Thousands of Dollars)
CONSTANT GROUP

| Year $n$ | Present Value AT $1 / 1 / 76$ of Salaries through Year $n$ | Present Valce Objective at End of Year $n$ |  |  | Present <br> Value of Benetit <br> Payments througe Yeaz $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Accrued <br> Benefits, Continuing Plan Assumption | Accrued <br> Benefits, Terminating Plan Assumption | Entry Age <br> Normal <br> Supplemental <br> Present <br> Value |  |
| 1976 | \$15,681 | \$4,221 | \$4,200 | \$7,622 | \$ 64 |
| 1977 | 30,663 | 4,841 | 4,811 | 8,177 | 163 |
| 1978 | 45,011 | 5,430 | 5,395 | 8,734 | 301 |
| 1979 | 58,762 | 6,036 | 5,998 | 9,308 | 456 |
| 1980. | 71,987 | 6,650 | 6,616 | 9,898 | 631 |
| 1981 | 84,648 | 7,287 | 7,265 | 10,510 | 816 |
| 1982 | 96,786 | 7,940 | 7,937 | 11,140 | 1,014 |
| 1983 | 108,460 | 8,602 | 8,613 | 11,786 | 1,226 |
| 1984. | 119,638 | 9,265 | 9,283 | 12,440 | 1,452 |
| 1985 | 130,390 | 9,938 | 9,952 | 13,109 | 1,688 |
| 1986 | 140,694 | 10,623 | 10,633 | 13,792 | 1,932 |
| 1987 | 150,573 | 11,321 | 11,330 | 14,485 | 2,183 |
| 1988. | 160,075 | 12,028 | 12,036 | 15,192 | 2,441 |
| 1989 | 169,218 | 12,746 | 12,753 | 15,911 | 2,703 |
| 1990. | 178,032 | 13,471 | 13,476 | 16,638 | 2,971 |
| 1991. | 186,503 | 14,178 | 14,181 | 17,356 | 3,248 |
| 1992 | 194,685 | 14,898 | 14,903 | 18,087 | 3,527 |
| 1993. | 202,603 | 15,634 | 15,639 | 18,831 | 3,806 |
| 1994. | 210,256 | 16,365 | 16,371 | 19,575 | 4,088 |
| 1995. | 217,656 | 17,074 | 17,083 | 20,307 | 4,376 |
| 1996 | 224, 803 | 17,757 | 17,769 | 21,021 | 4,669 |
| 1997. | 231,733 | 18,453 | 18,469 | 21,748 | 4,958 |
| 1998. | 238,459 | 19,155 | 19,176 | 22,480 | 5,244 |
| 1999. | 244,990 | 19,869 | 19,897 | 23,223 | 5,526 |
| 2000. | 251,324 | 20,552 | 20,607 | 23,940 | 5,807 |

TABLE 2C
Present Values Needed for Cost Calculations
(Thousands of Dollars)
5 PERCENT increasing group

| Year $n$ | Present Value at $1 / 1 / 76$ of Salaries through Year $n$ | Present Value Objective at End of Year $n$ |  |  | Present <br> Value of Benefit Payments through Year $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Accrued <br> Benefits, Continuing Plan Assumption | Accrued <br> Benefits, Terminating Plan Assumption | Entry Age <br> Norma! <br> Supplemental <br> Present <br> Value |  |
| 1976. | \$15,681 | \$4,221 | \$4,200 | \$7,622 | \$ 64 |
| 1977 | 31,221 | 4,844 | 4,819 | 8,188 | 163 |
| 1978. | 46,712 | 5,442 | 5,420 | 8,771 | 301 |
| 1979 | 62,134 | 6,061 | 6,050 | 9,388 | 456 |
| 1980 | 77,580 | 6,698 | 6,705 | 10,038 | 631 |
| 1981 | 92,956 | 7,368 | 7,403 | 10,732 | 816 |
| 1982 | 108,308 | 8,066 | 8,140 | 11,471 | 1,014 |
| 1983 | 123,727 | 8,791 | 8,898 | 12,257 | 1,226 |
| 1984 | 139,138 | 9,536 | 9,669 | 13,084 | 1,452 |
| 1985 | 154,637 | 10,315 | 10,459 | 13,964 | 1,688 |
| 1986. | 170,145 | 11,136 | 11,290 | 14,899 | 1,933 |
| 1987 | 185,684 | 12,000 | 12,170 | 15,892 | 2,185 |
| 1988 | 201,344 | 12,912 | 13,099 | 16,953 | 2,444 |
| 1989. | 217,131 | 13,878 | 14,081 | 18,086 | 2,708 |
| 1990. | 233,099 | 14,898 | 15,117 | 19,292 | 2,979 |
| 1991 | 249,214 | 15,952 | 16,188 | 20,559 | 3,261 |
| 1992 | 265,546 | 17,080 | 17,339 | 21,918 | 3,546 |
| 1993 | 282,150 | 18,289 | 18,572 | 23,379 | 3,833 |
| 1994 | 299,023 | 19,566 | 19,874 | 24,935 | 4,126 |
| 1995. | 316,202 | 20,897 | 21,236 | 26,580 | 4,429 |
| 1996. | 333,667 | 22,276 | 22,652 | 28,312 | 4,742 |
| 1997 | 351,474 | 23,764 | 24,179 | 30,178 | 5,053 |
| 1998. | 369,663 | 25,361 | 25,820 | 32,184 | 5,366 |
| 1999 | 388,252 | 27,082 | 27,592 | 34,346 | 5,679 |
| 2000. | 407,259 | 28,887 | 29,475 | 36,634 | 5,996 |

TABLE 3
Calculation of Level Twenty-Year Cost Rates under pVM
(Dollar Amounts in Thousands)


[^1]The reason for this is that PVM's effectively spread the initial unfunded supplemental or accrued benefit present value proportionately over the payroll base or the number of participants in the objective period; the more payroll or people expected in that period, the lower will be the cost per dollar of payroll or per capita to amortize the initial unfunded present value. In addition, PVM's smooth the normal costs for the combination of initial and new participants in the objective period to a constant percentage of payroll or amount per capita. As illustrated later in this paper, the PVM's generally produce a more rational and predictable cost pattern than the traditional closed-group actuarial cost methods.

## Additional Information Available

Once the actuary has determined the level funding rate or amount, he may then project the plan assets on a year-by-year basis and compare them at each point with the value of accrued and vested benefits. He may project the fund using the level funding rate and the valuation interest rate to check the cash-flow requirements and benefit-security ratios expected if all actuarial assumptions are realized, or he may project the fund using a higher, a lower, or a nonlevel funding rate, or a higher, a lower, or a fluctuating investment return rate. He thereby can determine the funding pattern needed to meet certain objectives in terms of benefit security, or he can test the effect of asset value fluctuations on such security. The information in Tables 2A-2C can be used to compute level funding rates to meet a given PVO at any intermediate point during the twenty-five-year period, and to determine the level funding rate for the remaining years to reach full funding of the PVO again at the end of twenty-five years. Examples of the rates that can be computed for various objective periods are shown in Table 4.

From the tables presented, the following general conclusions may be drawn regarding the effect of a given funding objective on future contributions and assets:

1. The shorter the original objective period, the higher will be the contribution level during that period and the faster will be the buildup of assets.
2. A PVO based on the entry age normal supplemental present value will develop higher contribution rates than a PVO based on the present value of accrued benefits on either a continuing plan assumption or a terminating plan assumption.
3. The greater the assumed rate of growth of the participant group, the lower will be the annual contribution as a percentage of covered payroll.

PVM's provide the actuary with a much clearer picture of the effect of a particular funding scheme than do conventional actuarial cost

TABLE 4

## Illustration of Funding Rates Derivable from Values in Tables 2A-2C


methods, and the process and rationale of cost determination are much easier to communicate to plan sponsors. The data provided by the projection of costs, assets, and present values give the actuary and plan sponsor the opportunity to make informed decisions about the funding policy for the plan.

## III. COMPARISONS WITH CONVENTIONAL COST METHODS

In order to compare PVM results with those obtained through closedgroup techniques, three conventional cost methods (unit credit, aggregate entry age normal, and aggregate method without supplemental present value) were applied to the population at the beginning of each of the twenty-five years commencing January 1, 1976, as projected under each of the three new-entrant assumptions. For the first two methods, an amortization period of twenty years was used for the funding of the initial unfunded supplemental present value, corresponding to the objective period used for PVM funding rate calculations. As with the PVM cost determinations, all contributions were assumed to be made at the beginning of the year and all actuarial assumptions were assumed to be matched by actual experience in all years.

## Conclusions on Method Comparisons

Although the figures presented in this section are based on only one hypothetical plan, population, and set of actuarial assumptions, the results of the method comparisons, shown in Tables 5A-5C, disclose the following important patterns:

1. Even if all actuarial assumptions are exactly met, conventional methods tend to produce nonlevel patterns of annual costs, primarily because they ignore the effects of future new entrants.
2. Under conventional methods, cost rates in later years tend to increase for the declining population and to decrease for the increasing population. These cost patterns are precisely the opposite of those desirable.
3. For stable populations the conventional cost techniques can produce nearly level contribution rates, but only a projection of the participant population can verify whether such a condition may be expected.
4. Projection methods for determining annual costs can permit the attainment of a variety of funding objectives while maintaining a stable pattern of costs, and the benefit security of the participants generally does not diminish as a result of their use.

## Costs for Closed Group

Of the conventional cost methods, only the aggregate method produces a stable pattern of costs over a long period for a closed group of partici-
pants. The other two methods develop patterns of increasing costs as a percentage of covered payroll, as a result of the amortization in level dollar amounts of the initial unfunded supplemental present value (USPV). After the original amortization period, costs under the entry age normal and unit credit methods drop substantially, dramatically so under the entry age normal method. In Table 5A the results of the conventional methods are shown in conjunction with the funding rates determined under two of the PVM's, each with an initial objective period of twenty years.

It is clear that, if a company is winding down its operations, it should
TABLE 5A
Annual Costs as a Percentage of Covered Payroll
Closed Grour

| Year | Unit Crinit Method |  | Entry Agf Normal Method |  | Asgregate <br> Method |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Conventional | PVM* | Conventional | PVM $\dagger$ |  |
| 1976 | $3.50 \%$ | $5.21 \%$ | $3.67 \%$ | $5.51 \%$ | $5.24 \%$ |
| 1977 | 3.84 | 5.21 | 3.94 | 5.51 | 5.24 |
| 1978. | 4.04 | 5.21 | 4.24 | 5.51 | 5.24 |
| 1979 | 4.42 | 5.21 | 4.51 | 5.51 | 5.24 |
| 1980 | 4.78 | 5.21 | 4.81 | 5.51 | 5.25 |
| 1981. | 5.15 | 5.21 | 5.08 | 5.51 | 5.25 |
| 1982. | 5.48 | 5.21 | 5.37 | 5.51 | 5.25 |
| 1983. | 5.79 | 5.21 | 5.70 | 5.51 | 5.26 |
| 1984 | 6.06 | 5.21 | 6.06 | 5.51 | 5.26 |
| 1985 | 6.34 | 5.21 | 6.42 | 5.51 | 5.27 |
| 1986. | 6.62 | 5.21 | 6.81 | 5.51 | 5.28 |
| 1987 | 6.90 | 5.21 | 7.22 | 5.51 | 5.29 |
| 1988 | 7.19 | 5.21 | 7.71 | 5.51 | 5.30 |
| 1989. | 7.48 | 5.21 | 8.24 | 5.51 | 5.32 |
| 1990. | 7.80 | 5.21 | 8.84 | 5.51 | 5.34 |
| 1991. | 8.04 | 5.21 | 9.63 | 5.51 | 5.36 |
| 1992 | 8.42 | 5.21 | 10.36 | 5.51 | 5.39 |
| 1993 | 8.85 | 5.21 | 11.20 | 5.51 | 5.43 |
| 1994 | 9.33 | 5.21 | 12.38 | 5.51 | 5.48 |
| 1995. | 9.96 | 5.21 | 13.93 | 5.51 | 5.53 |
| 1996. | 6.50 | 6.74 | 1.45 | 1.37 | 5.60 |
| 1997 | 6.71 | 6.74 | 1.46 | 1.37 | 5.68 |
| 1998 | 6.87 | 6.74 | 1.51 | 1.37 | 5.79 |
| 1999 | 7.11 | 6.74 | 1.55 | 1.37 | 5.91 |
| 2000. | 7.35 | 6.74 | 1.64 | 1.37 | 6.07 |

[^2]not be using a closed-group funding method (other than the aggregate method) unless the company's management is fully cognizant of how costs might escalate at a time when the plan sponsor could probably least afford them. Use of a PVM would produce a much more stable cost pattern in a circumstance when such stability would be most desirable.

## Costs for Constant Group

Actuaries encounter stable or increasing participant populations much more frequently than declining ones, and in such circumstances conventional cost methods are less likely to produce unpleasant surprises. Nevertheless, these methods can provide only a single year's cost data and therefore give no insight as to the direction of future costs. Projected costs for twenty-five years under the three conventional methods and two PVM's are presented in Table 5B for the populations derived from the constant-active-group assumption.
After the first two years, costs under the unit credit method demonstrate a remarkable stability, despite the common fear among actuaries that costs under this method will escalate. As the cost figures for 19962000 show, there can be a fairly constant underlying normal cost rate for a mature population, despite the fact that costs per dollar of benefit rise as individuals get older. Nevertheless, it cannot be assumed that such stability will prevail from year to year for most plans or for most populations under the unit credit method; only by making a projection of the population can the method's results be tested.
Annual costs under the aggregate entry age normal method follow a somewhat erratic pattern during the twenty years when the initial USPV is being amortized, despite the stability of the participant group. Costs for 1996-2000 indicate an underlying normal cost rate of about 2 percent of payroll, after the group reaches maturity; but the level amount to amortize the initial USPV represents a varying percentage of covered payroll, as that payroll base fluctuates on account of terminations, salary increases, and new entrants.
The two cost methods that fund toward the entry age normal supplemental present value produce higher contribution requirements in the initial period than the methods that fund the present value of accrued benefits, but the reverse is true after the end of that period. This is because of the faster accumulation of assets under approaches of the entry age normal type than under techniques of the unit credit type.

The aggregate cost method produces a series of consistently declining funding rates over the twenty-five years illustrated in Table 5B. This
cost method can be thought of as a modification of the entry age normal method whereunder the initial USPV is amortized as a level percentage of future payroll for the current participant group, rather than as a level dollar amount. The aggregate method cost inherently consists of two elements: the underlying entry age normal cost, and an amount to fund the USPV, both computed as level percentages of future payroll. However, as the plan adds new members and thereby maintains or expands the future payroll base, the second cost element becomes a steadily decreasing percentage of current compensation, while the normal cost element may remain a fairly constant function of payroll in a mature

TABLE 5B
Annual Costs as a Percentage of Covered Payroll CONSTANT GROUP

| Year | Unit Credit Method |  | Entry Age Normar Method |  | Aggregate Method |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Conventional | PVM* | Conventional | PVM $\dagger$ |  |
| 1976 | $3.50 \%$ | $3.35 \%$ | 3.67\% | $3.82 \%$ | 5.24\% |
| 1977 | 3.46 | 3.35 | 3.72 | 3.82 | 4.91 |
| 1978. | 3.31 | 3.35 | 3.78 | 3.82 | 4.63 |
| 1979. | 3.34 | 3.35 | 3.81 | 3.82 | 4.38 |
| 1980 | 3.33 | 3.35 | 3.82 | 3.82 | 4.15 |
| 1981 | 3.36 | 3.35 | 3.85 | 3.82 | 3.96 |
| 1982 | 3.37 | 3.35 | 3.87 | 3.82 | 3.79 |
| 1983 | 3.36 | 3.35 | 3.88 | 3.82 | 3.64 |
| 1984 | 3.34 | 3.35 | 3.90 | 3.82 | 3.50 |
| 1985 | 3.33 | 3.35 | 3.89 | 3.82 | 3.37 |
| 1986. | 3.35 | 3.35 | 3.89 | 3.82 | 3.26 |
| 1987. | 3.36 | 3.35 | 3.89 | 3.82 | 3.16 |
| 1988 | 3.36 | 3.35 | 3.88 | 3.82 | 3.07 |
| 1989. | 3.35 | 3.35 | 3.87 | 3.82 | 2.98 |
| 1990. | 3.34 | 3.35 | 3.85 | 3.82 | 2.90 |
| 1991 | 3.29 | 3.35 | 3.83 | 3.82 | 2.83 |
| 1992 | 3.30 | 3.35 | 3.80 | 3.82 | 2.76 |
| 1993 | 3.30 | 3.35 | 3.77 | 3.82 | 2.70 |
| 1994. | 3.29 | 3.35 | 3.74 | 3.82 | 2.63 |
| 1995. | 3.27 | 3.35 | 3.71 | 3.82 | 2.58 |
| 1996 | 2.69 | 2.66 | 2.03 | 2.01 | 2.52 |
| 1997 | 2.69 | 2.66 | 2.02 | 2.01 | 2.47 |
| 1998. | 2.68 | 2.66 | 2.02 | 2.01 | 2.42 |
| 1999 | 2.67 | 2.66 | 2.02 | 2.01 | 2.37 |
| 2000. | 2.64 | 2.66 | 2.02 | 2.01 | 2.32 |

[^3]group. In total, costs under the aggregate method thus decline as a percentage of payroll in the circumstance of a stable or expanding participant population. Costs start out at a higher level than under any of the other approaches illustrated, but eventually drop below costs derived on the other bases. Despite the rapid accumulation of assets in the early years, at the end of twenty years the aggregate method assets are less than those produced by the entry age normal-type methods, simply because the "hidden" initial USPV has not been completely funded by that time-and, theoretically, never will be! This characteristic of the aggregate method will prevail under any new-entrant assumption, including a closed group, as long as there are active participants over whose employment years the remaining USPV can be spread.

## Costs for Expanding Group

Although it is unlikely that a company will expand indefinitely at a constant rate, projections were made for twenty-five years on such a basis in order to examine and compare the results under conventional and projection cost methods. Table 5C presents the contribution rates that would be expected in each of the twenty-five years under the three traditional cost methods and the two PVM's previously described, if active participants were to increase by 5 percent per year.

Under all three conventional cost methods, the annual contribution rates decline steadily throughout the first twenty years, while cost rates under the PVM's are constant. The aggregate method produces the steepest rate of decline, while the unit credit and entry age normal follow less severe slopes. In all cases the pattern of decreasing costs results from the spreading of the initial USPV over a steadily increasing payroll base.

As in the case of the closed group and the constant group, the contribution rates for the increasing group under the unit credit method and the PVM that funds the present value of accrued benefits are lower than those produced by the entry age normal and the PVM that funds the entry age normal supplemental present value. Costs under the aggregate method start at a much higher level than under any of the other methods, but soon drop below those of the entry age normal-type methods. After the initial unfunded present values have been amortized, the costs under all methods except the aggregate are nearly the same (about 2 percent of payroll), and this cost level is nearly equal to the contribution rates under the entry age normal-type methods for the constant group in the same years (1996-2000).

TABLE 5C
Annual Costs as a Percentage of Covered Payroll
5 PERCENT INCREASING GROUP

| Year | Unit Credit Method |  | Entry Age Normal Method |  | Aggregate Method |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Conventional | PVM* | Conventional | PVM + |  |
| 1976 | $3.50 \%$ | $2.70 \%$ | $3.67 \%$ | $3.26 \%$ | $5.24 \%$ |
| 1977. | 3.36 | 2.70 | 3.66 | 3.26 | 4.82 |
| 1978 | 3.11 | 2.70 | 3.66 | 3.26 | 4.46 |
| 1979 | 3.04 | 2.70 | 3.62 | 3.26 | 4.14 |
| 1980. | 2.95 | 2.70 | 3.57 | 3.26 | 3.86 |
| 1981. | 2.90 | 2.70 | 3.54 | 3.26 | 3.63 |
| 1982 | 2.84 | 2.70 | 3.50 | 3.26 | 3.44 |
| 1983 | 2.77 | 2.70 | 3.46 | 3.26 | 3.27 |
| 1984 | 2.70 | 2.70 | 3.42 | 3.26 | 3.12 |
| 1985 | 2.64 | 2.70 | 3.36 | 3.26 | 2.99 |
| 1986 | 2.61 | 2.70 | 3.31 | 3.26 | 2.88 |
| 1987 | 2.58 | 2.70 | 3.25 | 3.26 | 2.78 |
| 1988 | 2.54 | 2.70 | 3.20 | 3.26 | 2.70 |
| 1989 | 2.50 | 2.70 | 3.13 | 3.26 | 2.63 |
| 1990. | 2.45 | 2.70 | 3.06 | 3.26 | 2.55 |
| 1991 | 2.40 | 2.70 | 3.00 | 3.26 | 2.49 |
| 1992 | 2.37 | 2.70 | 2.92 | 3.26 | 2.43 |
| 1993 | 2.35 | 2.70 | 2.84 | 3.26 | 2.38 |
| 1994. | 2.31 | 2.70 | 2.75 | 3.26 | 2.33 |
| 1995. | 2.27 | 2.70 | 2.63 | 3.26 | 2.29 |
| 1996 | 2.00 | 1.96 | 2.02 | 1.99 | 2.24 |
| 1997 | 1.99 | 1.96 | 2.01 | 1.99 | 2.20 |
| 1998 | 1.97 | 1.96 | 2.00 | 1.99 | 2.16 |
| 1999 | 1.95 | 1.96 | 1.98 | 1.99 | 2.12 |
| 2000 | 1.92 | 1.96 | 1.97 | 1.99 | 2.08 |

* Based on PVO equal to present value of accrued benefits, computed on a continuing plan basis.
- Based on PVO equal to individual entry age normal supplemental present value.


## Benefit Security under Various Cost Methods

As a means of comparing further the PVM and conventional cost methods, the plan assets that would be developed over twenty-five years by each of the funding techniques have been projected and related to the present value of accrued benefits (on a plan termination basis) at each year-end. Tables $6 \mathrm{~A}-6 \mathrm{C}$ show the assets as percentages of plan termination liabilities, for each of the populations described previously: They demonstrate that projection methods do not impair the plan's funded status significantly, as compared with the results obtained by

TABLE 6A
Benefit-Security Ratios on a Plan Termination basis Closed group

| Year | PresentValue ofAccruedBenefitsat Year-End(in $\$ 1,000$ 's) | Year-End Assets as Percentage of Present Value of Accrued Benefits |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Unit Credit Method | PVM* | Entry Age Normal Method | PVM $\dagger$ | Aggregate Method |
| 1976 | \$4,200 | 72.7\% | 79.6\% | 73.4\% | 80.7\% | 79.6\% |
| 1977 | 4,788 | 77.3 | 87.9 | 78.2 | 89.9 | 88.0 |
| 1978 | 5,326 | 80.7 | 93.9 | 82.1 | 96.5 | 94.0 |
| 1979 | 5,866 | 83.7 | 98.0 | 85.2 | 101.1 | 98.2 |
| 1980. | 6,403 | 86.1 | 100.7 | 87.6 | 104.3 | 101.0 |
| 1981 | 6,954 | 88.0 | 102.4 | 89.4 | 106.3 | 102.7 |
| 1982. | 7,511 | 89.6 | 103.3 | 90.8 | 107.6 | 103.7 |
| 1983. | 8,053 | 90.9 | 103.8 | 92.0 | 108.4 | 104.2 |
| 1984 | 8,568 | 92.0 | 104.1 | 93.1 | 108.9 | 104.6 |
| 1985. | 9,065 | 93.0 | 104.2 | 94.2 | 109.2 | 104.7 |
| 1986 | 9,542 | 93.9 | 104.1 | 95.2 | 109.4 | 104.6 |
| 1987 | 9,993 | 94.8 | 103.9 | 96.3 | 109.4 | 104.5 |
| 1988. | 10,416 | 95.5 | 103.6 | 97.4 | 109.4 | 104.3 |
| 1989. | 10.809 | 96.3 | 103.3 | 98.6 | 109.4 | 104.0 |
| 1990. | 11,168 | 97.0 | 102.9 | 99.9 | 109.3 | 103.7 |
| 1991. | 11,469 | 97.7 | 102.5 | 101.4 | 109.3 | 103.5 |
| 1992 | 11,740 | 98.4 | 102.2 | 103.0 | 109.3 | 103.2 |
| 1993 | 11,980 | 99.1 | 101.7 | 104.7 | 109.2 | 102.9 |
| 1994. | 12,172 | 99.9 | 101.3 | 106.7 | 109.2 | 102.6 |
| 1995. | 12,308 | 100.9 | 100.9 | 109.1 | 109.2 | 102.3 |
| 1996 | 12,389 | 100.7 | 100.9 | 108.1 | 108.3 | 102.1 |
| 1997 | 12,440 | 100.6 | 100.8 | 107.2 | 107.3 | 101.8 |
| 1998 | 12,451 | 100.5 | 100.7 | 106.3 | 106.4 | 101.6 |
| 1999. | 12,430 | 100.5 | 100.6 | 105.5 | 105.6 | 101.3 |
| 2000. | 12,361 | 100.4 | 100.4 | 104.7 | 104.7 | 101.1 |

[^4]TABLE 6B
Benefit-Security Ratios on a plan Termination basis constant group

| Year | Present <br> Vazue of Accrued Benefits at Year-End (in $\$ 1,000$ 's) | Year-End Assets as Percentage of Present Value of Accrued Benepits |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Unit <br> Credit <br> Method | PVM* | Entry Age <br> Normal <br> Method | PVM $\dagger$ | Aggregate Method |
| 1976 | \$4, 200 | 72.7\% | $72.2 \%$ | $73.4 \%$ | $74.1 \%$ | $79.6 \%$ |
| 1977 | 4,811 | 77.1 | 76.2 | 78.6 | 79.6 | 88.5 |
| 1978 | 5,395 | 80.3 | 79.7 | 83.2 | 84.3 | 95.3 |
| 1979 | 5,998 | 83.0 | 82.4 | 87.1 | 88.1 | 100.3 |
| 1980. | 6,616 | 85.1 | 84.6 | 90.4 | 91.4 | 103.9 |
| 1981 | 7,265 | 86.8 | 86.4 | 93.2 | 94.0 | 106.5 |
| 1982 | 7,937 | 88.3 | 87.7 | 95.5 | 96.2 | 108.3 |
| 1983 | 8,613 | 89.6 | 89.0 | 97.8 | 98.3 | 109.7 |
| 1984 | 9,283 | 90.8 | 90.3 | 100.0 | 100.4 | 110.9 |
| 1985 | 9,952 | 91.9 | 91.5 | 102.1 | 102.4 | 111.9 |
| 1986 | 10,633 | 93.0 | 92.6 | 104.1 | 104.2 | 112.7 |
| 1987. | 11,330 | 94.0 | 93.5 | 106.0 | 105.9 | 113.2 |
| 1988. | 12,036 | 94.8 | 94.4 | 107.7 | 107.6 | 113.5 |
| 1989 | 12,753 | 95.6 | 95.2 | 109.4 | 109.1 | 113.7 |
| 1990. | 13,476 | 96.4 | 96.0 | 111.0 | 110.7 | 113.9 |
| 1991. | 14,181 | 97.1 | 96.8 | 112.7 | 112.3 | 114.0 |
| 1992 | 14,903 | 97.8 | 97.6 | 114.2 | 113.9 | 114.1 |
| 1993 | 15,639 | 98.5 | 98.3 | 115.7 | 115.5 | 114.0 |
| 1994 | 16,371 | 99.2 | 99.1 | 117.3 | 117.1 | 114.0 |
| 1995. | 17,083 | 99.9 | 99.9 | 118.9 | 118.9 | 114.0 |
| 1996 | 17,769 | 99.9 | 99.9 | 118.3 | 118.3 | 114.0 |
| 1997. | 18,469 | 99.8 | 99.8 | 117.7 | 117.7 | 113.9 |
| 1998 | 19,176 | 99.8 | 99.8 | 117.2 | 117.2 | 113.8 |
| 1999. | 19,897 | 99.8 | 99.7 | 116.7 | 116.6 | 113.7 |
| 2000 | 10,607 | 99.7 | 99.7 | 116.2 | 116.2 | 113.6 |

[^5]TABLE 6C

## Benefit-Security Ratios on a Play Termination Basis

5 PERCENT INCREASING GROUP

| Year | Present <br> Value of Accrued Benefits at Year-Exd (in \$1,000's) | Year-End Assets as Percentage of Present Value of Accrued Benefits |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { Unit } \\ & \text { Credit } \\ & \text { Method } \end{aligned}$ | PVM* | Entry Age <br> Normal <br> Method | PVM $\dagger$ | Aggregate Method |
| 1976 | \$4,200 | 72.7\% | $69.6 \%$ | 73.4\% | 71.9\% | $79.6 \%$ |
| 1977 | 4,819 | 77.0 | 71.8 | 78.7 | 75.9 | 88.7 |
| 1978 | 5,420 | 80.2 | 73.9 | 83.6 | 79.6 | 95.7 |
| 1979 | 6,050 | 82.7 | 75.6 | 87.8 | 82.9 | 101.0 |
| 1980 | 6,705 | 84.7 | 77.1 | 91.5 | 85.8 | 105.0 |
| 1981 | 7,403 | 86.3 | 78.5 | 94.8 | 88.5 | 108.0 |
| 1982 | 8,140 | 87.6 | 79.7 | 97.6 | 90.9 | 110.2 |
| 1983 | 8,898 | 88.8 | 80.9 | 100.4 | 93.3 | 112.1 |
| 1984 | 9,669 | 89.9 | 82.3 | 103.2 | 95.9 | 113.8 |
| 1985 | 10,459 | 91.1 | 83.7 | 106.0 | 98.6 | 115.4 |
| 1986 | 11,290 | 92.1 | 85.2 | 108.6 | 101.2 | 116.7 |
| 1987 | 12,170 | 93.1 | 86.5 | 111.0 | 103.7 | 117.8 |
| 1988 | 13,099 | 93.9 | 87.9 | 113.2 | 106.3 | 118.7 |
| 1989 | 14,081 | 94.7 | 89.3 | 115.4 | 108.8 | 119.4 |
| 1990 | 15,117 | 95.4 | 90.7 | 117.4 | 111.4 | 120.1 |
| 1991 | 16,188 | 96.1 | 92.2 | 119.3 | 114.1 | 120.7 |
| 1992 | 17,339 | 96.7 | 93.7 | 121.1 | 116.8 | 121.2 |
| 1993 | 18,572 | 97.2 | 95.2 | 122.7 | 119.5 | 121.6 |
| 1994 | 19,874 | 97.8 | 96.7 | 124.1 | 122.3 | 122.0 |
| 1995. | 21,236 | 98.4 | 98.4 | 125.3 | 125.3 | 122.4 |
| 1996 | 22,652 | 98.3 | 98.2 | 125.2 | 124.9 | 122.8 |
| 1997 | 24,179 | 98.2 | 98.1 | 125.0 | 124.7 | 123.2 |
| 1998 | 25,820 | 98.2 | 98.0 | 124.8 | 124.5 | 123.4 |
| 1999. | 27,592 | 98.1 | 98.0 | 124.7 | 124,4 | 123.6 |
| 2000. | 29,475 | 98.0 | 98.0 | 124.3 | 124.3 | 123.7 |

[^6]application of traditional cost methods, and often produce a higher level of security.

In the closed-group situation, the PVM's produce higher ratios than the conventional methods to which they correspond, and the PVM that funds toward the entry age normal supplemental present value produces higher ratios than the aggregate cost method. This results from the fact that the costs under the conventional methods increase from year to year, while PVM costs are level percentages of payroll (see Table 5A). Under both PVM's illustrated here, the accrued benefits are funded fully within five years of the initial valuation date, while it takes fifteen years under the entry age normal and twenty years under the unit credit method to achieve that position. Full funding would be accomplished in five years under the aggregate method.

When the various cost methods are applied to the population developed under the constant-active-group assumption, the PVM's produce benefitsecurity ratios that are very similar to those produced by the corresponding conventional methods with the same funding objective. The aggregate method accumulates assets faster than any other method through sixteen years, but then the entry age normal-type methods develop higher ratios. Full funding of the termination present values is achieved in four years under the aggregate method, in nine years under the entry age normal-type methods, and in twenty years under the unit credit-type methods.

The conventional cost methods, when applied to the increasing population, produce patterns of decreasing contribution rates (see Table 5C), while PVM rates are constant. Therefore, the former methods result in a faster buildup of assets and an earlier fully funded status than the corresponding PVM's. As with the other projections, assets accumulate most rapidly under the aggregate method for a number of years, but ultimately the entry age normal-type methods develop higher levels.

## IV. OPERATION OF A PVM AT THE SECOND AND SUBSEQUENT VALUATIONS

The material presented so far relates to the information available at the first valuation, based on the premise that all actuarial assumptions will be matched by experience throughout the projection period. This section deals with the technique of redetermining costs under a PVM at the second and subsequent valuation dates. The discussion and formulas that follow apply whether valuations are performed annually or are less frequent.

In developing an approach to these valuations, the following two
criteria were established. First, the method should operate so that, if all actuarial assumptions are matched precisely by experience, the original PVO is achieved at the end of the original period. Second, actuarial gains and losses and the effects of plan amendments and changes in assumptions should be spread over a fixed period of years that can extend beyond the end of the original objective period. The latter criterion implies that the full funding of the PVO would not be accomplished as originally scheduled and that, as a result of actuarial gains and losses, a surplus or deficit would be expected at the end of the original period.

The number of years for spreading gains and losses can be defined either as a constant number or as the greater of a constant number and the number of years remaining in the original objective period. For example, if the objective period is chosen as twenty years, the period for amortizing gains and losses may be either twenty years (measured from the current valuation date) or the greater of fifteen years and the number of years left in the original twenty-year period.

Actuarial gains and losses in the context of a PVM must be defined in essentially the same way as for the conventional aggregate cost methodin terms of the effect on the funding rate rather than the effect on the unfunded present value. The PVM's are further analogous to the aggregate method in that the funding of gains and losses is in proportion to covered payroll (or number of participants) expected in the amortization period.

## Second Valuation Procedure

To facilitate the explanation of renewal valuation procedures under a PVM, a number of symbols are defined below:
$i=$ Assumed annual rate of return on plan assets; this need not be a constant for all years;
$v=1 /(1+i) ;$
$P V O_{j}=$ Present value objective at end of year $j$;
$P V B_{a}^{b}=$ Present value at beginning of year $a$ of the benefits expected to be paid in years $a$ through $b$;
$P V S_{a}^{b}=$ Present value at beginning of year $a$ of the covered payroll (or number of participants) expected in years $a$ through $b$;
$A_{j}=$ Plan assets at the beginning of year $j$;
$F D_{a}^{b}=$ Funding deficiency at beginning of year $a$, relative to an objective period ending at year $b$;
$=P V B_{a}^{b}+v^{(b-a+1)} P V O_{b}-A_{a} ;$
$C R_{a}^{b}=$ Level contribution rate for years $a$ through $b$ needed to fund fully the PVO by the end of year $b$;
$=F D_{a}^{b} / P V S_{a}^{b}$.

The steps to follow to compute the contribution rate at the second and subsequent valuations are described below:

1. Project the population, benefits, salaries, and present values for the number of years over which gains and losses will be spread.
2. Using the contribution rate from the previous valuation, project the fund to the end of the original objective period.
3. Determine the "actual" deficit (or surplus) at the end of the original period by subtracting the projected assets from the projected PVO at that point. Then compute the difference between this deficit and the expected deficit determined at the time of the previous valuation. The expected deficit at the second valuation will be zero if the original contribution rate was computed to fund the PVO exactly in full by the end of the objective period.
4. Discount the algebraic difference between actual and expected deficit (or surplus) to the current valuation date, using the valuation interest rate or rates.
5. Divide the amount from step 4 by the present value of salaries (or number of participants) for the period over which gains and losses are to be spread.
6. Add the fraction from step 5 to the contribution rate previously in effect.
7. Again project the fund, using the adjusted contribution rate, to the end of the original objective period. This will provide the expected surplus or deficit at the end of the objective period for use in the next valuation.

To illustrate these procedures, assume that the objective period is twenty years; the PVO is the present value of accrued benefits on a continuing plan basis; the first valuation is on January 1, 1976, and the second valuation is three years later; and gains and losses are spread over twenty years. Using the values for a constant-group assumption from Tables 2B and 3, one finds the initial contribution rate as follows:

1. $P V B_{76}^{95}=4,376,000$.
2. $P V O_{95}=17,074,000$,
3. $A_{78}=2,400,000$.
4. $F D_{75}^{95}=4,376,000+(0.31180)(17,074,000)-2,400,000=7,300,000(i=6$ percent).
5. $P V S_{76}^{95}=217,656,000$.
6. $C R_{86}^{95}=7,300,000 / 217,656,000=0.0335$.

The expected deficit (surplus) at the end of 1995 is zero.
To determine the cost rate effective for 1979 and later years:

1. At January 1, 1979, project the population for twenty years.
2. On the basis of the new participant group and current actuarial assumptions and plan provisions, project the fund through the end of 1995 on the basis of a 3.35 percent of payroll contribution rate.
3. Compute the difference at December 31, 1995, between the projected PVO
and the projected assets; assume there is a deficit of $\$ 1,000,000$. Subtract from this amount the expected deficit, which at this point is zero.
4. The difference between actual and expected deficits, $\$ 1,000,000$, is discounted at the valuation interest rate or rates to January 1, 1979; assume that this amounts to $\$ 600,000$.
5. If $P V S_{79}^{98}=120,000,000$, the additional contribution rate is $600,000 /$ $120,000,000=0.0050$.
6. Add the adjustment to the previous rate of 0.0335 , to derive an adjusted rate of 0.0385 .
7. Project the fund, using a contribution rate of 3.85 percent, to the end of 1995 and redetermine the expected deficit or surplus at that date. If at that point the expected PVO exceeds expected assets by $\$ 200,000$, this figure will be used in step 3 at the next valuation date as the expected deficit.

It is apparent from these procedures (1) that if all assumptions are always realized, the actual and expected surplus or deficit at the end of the objective period will be identical and no adjustment will be necessary, and (2) that if the assumptions are not matched by experience, the effects of the deviations will be spread over a period that does not diminish below a certain length and will be spread in proportion to covered payroll or number of participants. A cumulative surplus or deficit will be built up over the objective period as a result of the spreading of gains and losses over periods that extend beyond the end of the original objective period. At the end of the original period, a new funding objective will be established, and the future cost rates will include an amortization of the residual surplus or deficit that exists at the end of the original period.

If the period for amortization of gains and losses is less than the remaining years in the objective period, the adjustment to the contribution rate will be effective for the smaller number of years, and the contribution rate will revert to the original rate for the years after the gain or loss has been fully amortized.

## Alternative Approaches to Renewal Valuations

If other criteria are chosen for describing renewal valuations under PVM, other procedures will result. The most obvious alternative is to fund all gains and losses by the end of the original objective period, so that full funding is achieved as originally intended. This approach has the drawback of spreading gains and losses over a continually decreasing period. Nevertheless, in most situations this approach will not produce wide swings in contribution rates, even toward the end of the objective period, if the actuary uses a smoothing technique for valuation of assets. However, the author believes that the use of a minimum amortization period, such as fifteen years, is more consistent with conventional
actuarial cost methods and with requirements of the Internal Revenue Code.

An alternative that represents a compromise between the two approaches discussed above would be to retain the original objective and time period except in the event of (1) changes in the funding rate that exceed predetermined bounds or (2) changes in the funding rate that result from certain types of events, such as plan amendments or changes in actuarial assumptions. In those circumstances, a new objective period could be established for the spreading of those extraordinary gains or losses, while retaining the original period for the funding of all other values.

## v. CONCERNS ABOUT PVM'S

Although the actuarial community has largely accepted projection techniques as tools in studies of expected future costs and in tests of various investment strategies, many actuaries have the following important concerns about the use of PVM's for purposes of determining annual contributions:

1. Assumptions as to future new entrants can be abused so that current costs are understated. While this is certainly true, the same can be said about interest rate and termination rate assumptions. It is up to the actuary to be ethical and competent, and no valid technique should be discarded because it might be used unscrupulously.
2. Projecting a population for a long period cannot be done accurately, and the apparent accuracy of the technique is specious. Unquestionably, the longer the projection period, the less accurate will be the assumptions. However, current cost methods are composites of projections that, for the youngest participants, can extend for sixty or more years. Age, sex, and salary patterns of new entrants can be predicted in the same manner as termination, retirement, and salary-increase rates, and total group growth can be predicted on the basis of the plan sponsor's history and forecasts and the forecasts for the particular industry and geographical region. Ignoring future entrants implies a closed-group assumption, and this is probably the least valid assumption in most cases.
3. PVM's are too costly and too complicated. Only with sophisticated computer programs and high-speed, large-capacity hardware can PVM's be utilized currently. Certainly, such efforts and costs are not warranted or feasible for small plans, at least not until the projection techniques become part of master valuation systems. In most cases small plans currently are valued on less sophisticated bases than large plans, and the acceptability of PVM's for large groups is not likely to change this.
4. PVM's are not acceptable under the Internal Revenue Code for determining minimum and maximum tax-deductible contributions. This fact, of course, has
limited the use of PVM's to studies of anticipated costs under varying circumstances. ERISA permits the secretary of the Treasury to issue regulations defining acceptable actuarial cost methods in addition to those listed in section 3(31); it is the author's contention that PVM's, as described herein, should be included in those regulations.

PVM's are not a panacea, but they advance substantially the actuary's ability to provide accurate forecasts of pension costs, liquidity requirements, and benefit-security ratios under a limitless variety of actuarial assumptions and expected plan changes. They permit the development of rational cost patterns that will tend to accomplish a selected funding objective in a specific time frame. PVM's also make it easier to involve a plan sponsor in the establishment of funding goals, since they generally are easier to explain to laymen than conventional cost methods. Data produced by the projections are vital to the fund's investment managers, and to the actuary and plan sponsor as well.

## APPENDIX I

## SUMMARY OF PLAN PROVISIONS

1. Eligibility: Immediate at hire.
2. Normal retirement date: Age 65.
3. Normal retirement benefits: For each year of service, 1 percent of annual salary plus 1.4 percent of the excess of annual salary over the current social security taxable wage base.
4. Vesting upon termination of employment: 100 percent after ten years of service.
5. Early retirement eligibility: Age 55 with ten years of service.
6. Early retirement benefits: Accrued benefit, reduced by one-fifteenth for each of the first five years and by one-thirtieth for each of the next five years by which early retirement precedes normal retirement date.
7. Death benefits: None.
8. Employee contributions: None.
9. Form of annuity: Single life annuity.

## APPENDIX II <br> ACTUARIAL ASSUMPTIONS

1. Interest rate: 6 percent per year.
2. Salary increase rate: 5 percent per year.
3. Taxable wage base increase rate: 5 percent per year after 1976.
4. Mortality rates: 1951 Group Annuity Table for Males, set back two years for males and seven years for females.
5. Termination and retirement rates (decrements are assumed to occur at the end of the year):

| Age | Rate | Age | Rate | Age | Rate |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 20. | 258 | 36 | . 146 | 52 | 034 |
| 21. | . 251 | 37 | . 139 | 53 | 027 |
| 22. | . 244 | 38. | . 132 | 54. | 020 |
| 23. | . 237 | 39. | . 125 | 55 | . 050 |
| 24. | 230 | 40. | . 118 | 56 | . 060 |
| 25. | 223 | 41. | . 111 | 57 | . 070 |
| 26. | 216 | 42. | . 104 | 58 | 080 |
| 27. | . 209 | 43. | . 097 | 59 | 090 |
| 28. | . 202 | 44. | . 090 | 60 | . 100 |
| 29. | . 195 | 45. | . 083 | 61 | 125 |
| 30. | . 188 | 46. | . 076 | 62 | . 150 |
| 31. | . 181 | 47. | . 069 | 63. | . 250 |
| 32. | . 174 | 48. | . 062 | 64. | 1-mortality |
| 33. | . 167 | 49. | . 055 |  | rate |
| 34. | . 160 | 50. | . 048 |  |  |
| 35. | . 153 | 51. | . 041 |  |  |

6. New-entrant pattern (distribution of new entrants by age):

| Age | Males | Females | Total |
| :---: | :---: | :---: | :---: |
| 22 | 9.6\% | $12.4 \%$ | 22.0\% |
| 27. | 20.4 | 15.2 | 35.6 |
| 32. | 16.2 | 6.8 | 23.0 |
| 37 | 6.0 | 2.4 | 8.4 |
| 42. | 3.6 | 1.2 | 4.8 |
| 47. | 2.4 | 1.2 | 3.6 |
| 52. | 1.2 | 0.4 | 1.6 |
| 57. | 0.6 | 0.4 | 1.0 |
| Total | 60.0\% | 40.0\% | 100.0\% |

New entrants are assigned a beginning salary equal to the average salary of continuing participants of the same age and sex.

## DISCUSSION OF PRECEDING PAPER

## CLYDE D. BEERS:

Mr. Schnitzer's paper presents a welcome follow-up to Don Fleischer's original work on forecast methods. Projection valuation methods represent an increased level of sophistication that is extremely useful in many situations.

I was excited to see results that present a useful analysis of the conservatism of the entry age normal cost method compared with projections of the value of accrued benefits. Table 2B, for example, shows the following data in millions of dollars:

| Year | Accrued <br> Benefits | Entry Age <br> Normal <br> Supplemental <br> Present Value | Difference | Ratio |
| :---: | :---: | :---: | :---: | :---: |
| $1976 \ldots .$. | $\$ 4.2$ | $\$ 7.6$ | $\$ 3.4$ | $181 \%$ |
| $1981 \ldots$. | 7.3 | 10.5 | 3.2 | 144 |
| $1986 \ldots$. | 10.6 | 13.8 | 3.2 | 130 |
| $1991 \ldots$. | 14.2 | 17.4 | 3.2 | 123 |
| $1996 \ldots$. | 17.8 | 21.0 | 3.2 | 118 |
| $2000 \ldots$. | 20.6 | 23.9 | 3.3 | 116 |

As time progresses, the traditional, expected conservatism of entry age normal liabilities virtually disappears. Even with an increasing population (shown in Table 2C), the "redundant" liabilities under the entry age normal approach drop to less than 30 percent of the value of the accrued benefits. In the case studied, the conservatism of the entry age normal method is minimized as vested and retired liabilities grow. Tables 6B and 6 C support this conclusion.

Further review of the paper shows that this relationship results from a benefit formula ( 1 percent of salary up to the social security wage base, and 1.4 percent of the excess) and a set of actuarial assumptions that produce rapidly declining benefits compared with final pay. Such a set of circumstances appears likely in low-paying industries, but only if our social security problems are not solved by decoupling or other legislative action. The benefits are based on earnings in each year, rather than, for example, final average earnings. In addition, salaries and the taxable wage base are both assumed to increase 5 percent per year. As a result, the following relationships can be developed from Table 1A:

| Year | Average Pay | Social Security <br> Wage Base | Ratio |
| :---: | :---: | :---: | :---: |
| $1976 \ldots .$. | $\$ 12,970$ | $\$ 15,300$ | $85 \%$ |
| $2000 \ldots$ | 21,212 | 51,811 | 41 |

Therefore, accrual of benefits drops from a level that includes substantial accrual at the 1.4 percent rate to accrual completely at the 1 percent level.
I wish the paper had shown the effect of a more typical situation in which promotional increases result in a salary assumption that is 1-2 $\frac{1}{2}$ percent over the average wage increase inherent in the movement of the social security wage base. Otherwise, we end up in the year 2000 with "all Indians and no chiefs."
In using a forecast method, if an actuary picks a funding target equal to a future entry age normal supplemental present value, the forecast approach differs from a closed group valuation in two respects. First, the normal cost rate for new entrants may differ somewhat from the existing group's normal cost as a percent of pay. Second, amortization of unfunded liabilities is spread over future payroll rather than as a level dollar amount. I have no problem with the first difference being part of an actuary's "best estimate," but I fail to see why the Internal Revenue Service or the Department of Labor should allow the unfunded liabilities to be spread other than in "equal annual amounts (until fully amortized)." I would welcome this flexibility, but it does seem to be trifling with the funding standard account.

Because a forecasting approach provides greater opportunity for careful analysis of pension funding, Mr. Schnitzer's paper is a welcome addition to our literature.

## KEATH P. GIBSON:

Mr. Schnitzer's paper is timely in that it helps clarify what the proponents of projection valuation methods are really proposing.

The second sentence of his introduction says, "Unlike conventional actuarial cost methods, projection methods permit the use of explicit assumptions as to future additions to the group of participants." If this sentence means that the use of conventional methods prohibits the use of explicit assumptions as to future hiring patterns, it is simply untrue. However, it is true that the use of such explicit assumptions will not reduce currently calculated costs significantly under conventional methods other than the aggregate cost method.

The Bell System has operated under regulatory supervision for many
years. The Federal Communications Commission closely scrutinizes all costs including pension costs and decides which ones will be considered allowable as operating costs of doing business. A few years ago the Bell System was under attack for allegedly overstating its pension costs by roughly 50 percent. One of the arguments was that it was remiss in not establishing some funding goal other than the accumulation of the funds needed to pay the prospective benefits. In our successful rebuttal, my testimony stated:

Each year the Bell System determines three separate items, each of which has somewhat the same characteristics as Mr. - --'s "funding goal." These items are:

1. Full service actuarial reserve requirement (as defined in Bell Exhibit 60 ); ${ }^{\text {a }}$
2. Matured liability (the funds necessary to pay pensions to all retired employees and to all employees eligible to retire);
3. Value of vested benefits (calculated under Accounting Principles Board Opinion No. 8, November 1966).
The Bell System has been calculating items 1 and 2 for many years and files the results of the calculations annually with the FCC. .. . The third item, the value of vested benefits, has been calculated since 1969 when the Bell System first liberalized its plan to provide nonforfeitable benefits for employees aged 40 or more with fifteen years or more of service. Each year, the Bell System compares the assets in the funds with each of the above three measures of funding progress. It does not consider any one of these measures as a "funding goal" in and of itself. Mr. ---_'s "funding goal" elevates an artificial standard to a status of meaningfulness which in fact it does not have.

Two cost disallowances in connection with government contracting currently are being appealed. The first was based upon disallowance of any cost that causes the accumulated funds to exceed the proper funding goal for death benefits to retired employees as determined by the terminal funding method. The second disallowance also was based upon the accumulation of funds in excess of a proper funding goal, but in this instance it was not claimed that the proper goal should be determined by the terminal funding method. It was not mentioned why a different goal was adopted in this second instance, but the reason may have been that no one reached retirement age in the first ten years of operation of the plan and thus terminal funding would have resulted in zero costs. It would have seemed ridiculous to claim that a pension plan could be in operation for ten years without costing anything.

[^7]It is not clear whether the government is claiming that the proper funding goal is the funding objective Mr. Schnitzer enumerates as No. 1 or No. 2. However, it is clear that the government is utilizing a funding goal that it now feels should have been adopted fifteen or twenty years ago, and is claiming that any accumulation of funds that exceeds the amounts determined by that goal represents expenses that are not acceptable as costs of performing government contracts. Through this device almost half the costs determined over the years under conventional actuarial cost methods have been disallowed.

Recently a letter was received proposing to disallow 1975 and 1976 costs for government contracting purposes. Again it is claimed that funds have been accumulated in excess of proper funding goals. However, a new reason (at least to me) for disallowing costs has been added: "One of the reasons for the higher cost appears to be the nonuse by the company actuaries of the generally followed actuarial concept of using a 1 percent to 2 percent spread between the assumptions used for salary increases and interest."

I speculate as to how long it will be before the first major cost disallowance based upon failure to adopt modern actuarial techniques as expressed in Mr. Schnitzer's and similar papers will have to be appealed.

Mr. Schnitzer proclaims several desirable characteristics of projection methods. At best these desirable characteristics are arguments for making projections, not for adopting projection methods. Bell System actuaries first started making projections in the early 1920's. These projections formed the basic evidence that successively forced the companies to adopt more adequate funding goals until they finally adopted the only tenable goal of funding the estimated future contractual obligations.

Projection techniques currently in use by the Bell System, although not as flexible as might be desired, can handle readily changes in assumptions as to the rate of investment return and the rate of change in general pay levels. One of the standard projections that is made is for a twentyyear period and involves the following assumptions:

1. There will be a 4 percent decline in employees for each of the next four years and a level work force thereafter.
2. The investment return will average $2 \frac{1}{2}$ percent per annum.
3. General pay levels will increase at a rate of $7 \frac{1}{2}$ percent per annum.
4. Current actuarial assumptions can be maintained in spite of such experience.

If assumptions 1,2 , and 3 turn out to be true, the current actuarial assumptions obviously will have to be strengthened.

Mr. Schnitzer's projections appear to be based on the theory that the mere good faith of an actuary in making an assumption will alter the
course of future events in such a manner as to make the assumption come true. At the very least, Mr. Schnitzer should have made a projection of the assumptions used for a 5 percent increasing group based upon actual experience emerging in accordance with his closed-group assumptions. An even more instructive projection also would include twenty years of actual experience where investment earnings were only 2 or 3 percent per year and salary increase rates were 9 or 10 percent per year, as has occurred in the recent past.

Although such truncated projections as Mr. Schnitzer proposes provide interesting supplementary information, they are not useful for the determination of appropriate rates of contribution. In terms of the Bell System, the pertinent items of information are the following:

1. It is estimated that the Bell System has contracted to pay $\$ 200$ billion in future benefits with respect to people who already have been hired by the operating telephone companies. The purpose of the funding program is to accumulate that $\$ 200$ billion so that the Bell System will not have to default on its contract. Attempts to divert attention from that basic objective must be avoided, whether they are masqueraded as simple statements that a contingency reserve of two or three years of benefit payments is adequate or as projection valuation methods with a specified funding goal at the end of a specified number of years.
2. Each new employee being hired currently increases that $\$ 200$ billion contractual obligation by about $\$ 40,000$.
3. The present value of that $\$ 200$ billion is about $\$ 46$ billion.
4. The operating telephone companies now have on hand $\$ 13$ billion, and thus $\$ 33$ billion of present value remains to be accumulated.
5. If the remaining $\$ 33$ billion of present value of contractual obligations is not accumulated by the time the last of the currently active employees retires, the Bell System probably will never be allowed to charge the unaccumulated costs as an expense of conducting the telephone business.

The problem has been stated, and the question now is how it should be solved. First, the $\$ 33$ billion of remaining cost will grow by the assumed investment earnings each year. Thus, with an assumed 5 percent investment return, at least $\$ 1.65$ billion must be paid each year in order not to lose ground. To support the contention that the problem is being handled prudently, the $\$ 33$ billion probably has to be amortized by at least 1 percent each year, thus effectively establishing that approximately $\$ 2$ billion is the current minimum acceptable rate of annual contribution. If the current rate of annual contribution is such that the $\$ 33$ billion remaining cost will be reduced by more than 4 percent a year, the Bell System probably cannot defend itself against the accusation that it is
overcharging current customers. Thus, $\$ 3$ billion is effectively the maximum acceptable current rate of annual contribution.

Now that the parameters have been stated, we are in a position to start making decisions about the actuarial methods to be used. First, it must be decided whether costs should be expressed, for example, as $\$ X$ per year for $n$ years, or as an amount per person in the United States, or as an amount per telephone in the United States. This is an easy decision. Since the most likely source of error in the estimated $\$ 200$ billion of future contractual obligations is in the estimate of future payrolls, the problem can be minimized by stating costs as a percentage of future payrolls.

Second, it must be decided whether we should look at the past on the basis of what actually occurred or on the basis of using assumptions and conditions appropriate only for current and future periods and assuming that these applied in the past. The adoption of the latter device is very appealing. It enables the actuary to say that the cost of the plan eventually will reduce to the percentage of payroll that currently is being computed as current or normal cost. This is made possible by using the explicit assumption that the distribution of new entrants will follow the actual historical pattern. Other explicit assumptions regarding new entrants might be used if the actuary has adequate evidence that the historical pattern is not suitable and the assumed one is, but this requires departure from the traditional methods of determining current normal cost.

This device also simplifies the making of many projections to the extent that the actuary hardly even needs a pocket calculator to make them. However, it is fraught with danger. It leads naturally to the next actuarial sophistry of assuming that self-acknowledged past deficiencies should be corrected by an amortization program expressed as a percentage of future payrolls, including the assumed future payrolls of employees who are yet to be hired. The result of the two sophistries will be an actuarially determined rate of contribution substantially less than the $\$ 2$ billion minimum acceptable annual rate.

It is apparent that were it not for the clear parameters that have been established, any competent actuary would have an unlimited choice of methods for accumulating the remaining $\$ 33$ billion of cost. Methods could be selected that would result in more than dissipating the $\$ 13$ billion already accumulated or in accumulating more than the total present value of the benefits, at some interim stage prior to the payment of the contractual obligations.

In the absence of a demonstration to the contrary, the adoption of any
open-group method in regard to the amortization of supplemental liability or in connection with the remaining cost (aggregate) method almost certainly will result in failure to comply with the minimum contribution parameter and the intent of the minimum funding standards of ERISA.

The minimum contributions required to maintain qualification of a pension plan in accordance with the 1942 amendments to the 1939 edition of the Internal Revenue Code never were defined clearly by the courts. However, there seemed to exist a general consensus as to what the requirements were. Expressed in terms of our remaining $\$ 33$ billion of present value, perhaps in an oversimplified fashion, this consensus appeared to be as follows: By the use of not unreasonable methods the $\$ 33$ billion could be split into two segments. Segment 1 , plus the $\$ 13$ billion of assets already accumulated, might be considered to represent something other than the regularly recurring costs of maintaining the plan. Segment 2 must be considered to be the regularly recurring costs of maintaining the plan. The required minimum contribution was the sum of the full costs under segment 2 plus a payment in lieu of investment earnings on segment 1 in an amount sufficient to prevent segment 1 from growing.

With regard to ERISA, it appears that one of the primary intentions of Congress in passing this law was to provide that the mere assignment of some part of the $\$ 33$ billion to segment 1 would not relieve us of the requirement to fund it. ERISA requires that segment 1 costs be amortized as a level amount over a period not normally exceeding forty years, and that segment 2 costs be amortized at an even more rapid rate. Therefore, the $\$ 1.923$ billion that represents the amortization of both segment 1 and segment 2 costs over a forty-year period clearly does not meet the minimum requirements of ERISA.

Mr. Schnitzer refers to changes over the last three decades in actuarial techniques for determining pension costs. In addition to being entirely irrelevant, the implications of this paragraph are simply untrue. As one example, 1927 contributions to the Bell System trust funds were determined using assumptions as to interest, wage scales, withdrawal rates, retirement rates, and mortality rates both before and after retirement that were based primarily on actual Bell System experience. Included also was an implicit assumption that no law such as social security ever would be enacted. The assumptions regarding wage scales, withdrawal rates, and retirement rates were all on a select basis by age at entry and were all applied in accordance with proper multiple-decrement theory.

As to Mr. Schnitzer's remark about the increase in importance of pension costs to plan sponsors, I find it impossible to justify a statement that the Bell System's plan costs, although now several times as high a per-
centage of payroll as they were in 1927, are currently as important to the Bell System as they were then, when they forced serious consideration of the termination of the plan. Nor can I support a statement that pension costs are more important to plan sponsors now than they were to the many plan sponsors who were forced to terminate their plans in the 1930's because they could no longer pay the benefits.

Mr. Schnitzer provides a list of funding objectives that may be useful in a particular situation. It is clear that the Department of Defense believes that such objectives are useful devices to disallow legitimately determined pension costs. In fact the Department of Defense also thinks that a terminal funding objective will prove to be a useful device for disallowing proper costs for death benefits. As indicated in an exposure draft issued April 14, 1977, the Financial Accounting Standards Board (FASB) proposes to adopt Mr. Schnitzer's funding objective No. 2, "present value of accrued benefits, determined on a plan termination basis ( 100 percent vesting for all participants)." The FASB has added the obvious requirement that uniformity be accomplished by requiring that current rates as quoted by the Pension Benefit Guaranty Corporation be used as the actuarial standard. The actuarial profession should recognize that the only tenable funding goal is to accumulate funds to meet the estimated future legal obligations. Failure to do so would mean abandonment of any rational basis for opposing inappropriate standards that might be proposed by nonactuaries.

Within the last five years Mr. Paul H. Jackson, a Fellow of the Society of Actuaries, has testified as follows concerning the adoption of funding goals that would result in the accumulation of amounts less than those necessary to pay the prospective benefits:

This is not accepted actuarial procedure. From the time Henry William Manly read his paper "On Staff Pension Funds" to the Institute of Actuaries on January 30, 1911 (Journal of the Institute of Actuaries, XLV, 149), all practicing actuaries have been aware of the dangers of looking at fund levels in determining the appropriate level of pension cost. Briefly, Mr. Manly set forth the facts for one pension program in perfect balance and another one which provided benefits at 50 percent more than could be supported in the long run. The amount in the inadequate fund was indistinguishable from that of the balanced fund for the first forty years. Indeed, a mere five years before the amount of the second fund leveled off and started decreasing, and a mere twenty-seven years before its total bankruptcy, the inadequate fund stood at $96_{2}^{\frac{1}{2}}$ percent of the balanced fund. The measurement of contribution requirements on the basis of fund levels has, from that time forward, been considered too risky a process for a qualified actuary to undertake. While certain short-term cash flows and longterm total fund levels have been estimated as an aid to those responsible for
the investment of pension funds, or their financial support, to the best of my knowledge it has not been customary for the actuary to use them as the basis for his pension cost calculations. Thus, I disagree with Mr. -_-_'s untested concept that a funding goal is a necessary part of an actuary's determination of contribution requirements.

It may be of interest to quote the statement of the minimum allowable funding goal as it was expressed in the Bell System plan as amended through July 1, 1928.

In order to meet its obligations to pay service pensions . . . granted to take effect under the Plan, the Company, effective January 1, 1927, established a trust fund to be known as the "Pension Fund." It undertakes to maintain this Fund by periodic charges to operating expenses and payments to the Fund in such amounts that when an employee becomes eligible under the Plan to receive a service pension, there will be available in the Pension Fund an amount sufficient to provide for him a pension in the amount stated in the Plan.

Another statement of fact concerns an early encounter in 1949 or 1950 with the development of one of the pattern plans. The union used projection methods to demonstrate that a contribution of 3.5 cents per hour would provide adequate financing for the plan. The projection was only for the totally inadequate period of twenty-five years. A simple extension of the projection showed that bankruptcy would occur in the twentyeighth year.

Mr. Schnitzer's concluding section, "Concerns about PVM's," makes it clear that he has failed completely to comprehend the nature of the objections to the use of projection valuation methods. I concur with Mr. Schnitzer that the first three concerns enumerated at the end of his paper are trivial, but they do not portray the legitimate objections to his proposal. The main objections were discussed in my letter printed in the March, 1977, issue of The Actuary and are restated here.

Traditionally the normal cost under a pension plan has been determined from the equation

$$
f(N C)_{n} P V P_{n}=P V B_{n}-A L_{n},
$$

where $f(N C)_{n}$ has generally been the percentage that should be applied to current payroll to determine current normal cost, $P V P_{n}$ is the value at time $n$ of future payrolls to be paid to active participants at time $n, P V B_{n}$ is the value at time $n$ of future benefits to be paid to all participants at time $n$, and $A L_{n}$ is the value at time $n$ of future benefits to all participants at time $n$ less the future normal costs for active participants at time $n$.

During the years there have been various suggestions for modifying the traditional actuarial equation. One common suggestion has been to replace $P V B_{n}$ in the equation by a present value of a "funding goal." I) uring the early 1950 's
it was commonly argued that the union-negotiated plans would last only for the duration of the labor contract. Therefore, it was argued that $P V B_{n}$ should be replaced by (1) the present value of benefits that would be paid before the expiration of the union contract or (2) the present value of benefits to be paid to people who would retire before the expiration of the union contract. Although these two early proposals were discredited long ago, the basic concept of a funding goal persists to this day and may have been influential in the adoption of an "alternative minimum funding standard" in ERISA.

Another common proposal to modify the traditional actuarial equation has been to include terms for both the costs to be paid for prospective new participants and the benefits to be paid to prospective new participants. As long as this is done on the basis that the present value of the costs equals the present value of the benefits, no harm is done to the basic actuarial concepts. Under the proposals that have come to my attention, however, the present value assigned to costs greatly exceeds the present value assigned to benefits. Frequently, this excess is so great that the costs assigned to current periods are less than under the traditional concepts of minimum current cost, that is, normal cost plus interest on the unfunded liability. Under social security the assignment of excess costs to future generations has been defended on the basis that the government has unlimited taxing power and will have no problem collecting the excess costs when they are needed. Increasingly, this argument is becoming suspect. Certainly no corporation or its actuary can argue with validity that it can collect these prospective excess costs from future generations of customers.

Recently, there have been articles written about projection methods for actuarial valuations. All the traditional actuarial methods involve the projection of both benefit payments and costs until the death of the last participant, and the discounting of all these benefits and costs to a common point in time, ordinarily to time $n$, the date of the current valuation. If these values are projected to time $n+t$, the only change is in each discount factor, caused by multiplying by $(1+i)^{\iota}$. There is no change in the computed values of $f(N C)_{n}, P V B_{n}$, or $A L_{n}$. Any apparent change in these values can result only from a calculating error or a modification of the traditional actuarial equation. In my opinion any proposals for changes in the emerging incidence of actuarial cost that are supported by projection methods need further analysis as to the causal agent of the changes, to determine whether the changes are consistent with the basic actuarial equation.

If the use of properly made projections can assist the actuary, his client, or the public to understand pension costs properly, their use can only be applauded.

Mr. Schnitzer contends that his proposed methods should be permitted by regulations that will be issued by the secretary of the Treasury. Mr. Schnitzer's proposals so violate the accepted minimum standards of funding that existed even before the passage of ERISA that they should be rejected out of hand by both the secretary of the Treasury and the actuarial profession.

## CHARLES L. WALLS:

In a more halcyon day the deferred group annuity policyholder, who might question what benefit he was getting for his permanent contribution to surplus, would sometimes get to talk to an actuary. Carefully removing his green eyeshade and wiggling his ears a bit to straighten them out, the actuary would say, "The cost of a pension plan is equal to the benefits and expenses paid less the investment income credited." This left the policyholder a trifle bewildered, but it was something he could repeat to other members of his business circle. When he finally gave up trying to find out about his surplus contribution and "went trusteed," he was pleased to hear the new actuary repeat those very words.

Papers such as Mr. Schnitzer's and some others that have appeared recently leave me wondering whether perhaps the old verities have been repealed, but after a bit of consideration I do not think so. If we put the previous saying into what might be thought of as its most general form, we would have $B \mid a=C a_{-}$, where $\mid a$ and $a_{\mp}$ should be thought of as very general annuity functions. From this equation it is immediately apparent that anything but an open-group valuation necessarily leads to a restriction on the $a$ functions.

If you are looking for a level cost, the $a$ - becomes a perpetuity and the cost is given by $C=i(B \mid a)$, where $i$ is a general interest function. In actual practice $i$ might be the level equivalent of a series of rates. The second and subsequent contributions in this level scheme would be $C=$ $i(B \quad \mid a-A)$, where $A$, the assets, must be valued by some type of in-come-stream method. If you close the group, you attach conditions to both $B$ and $\mid a$ and lose the ability to have an indefinitely level contribution.

Of more interest, however, is the fact that the fundamental equation shows all valuation methods to be actually the same for the same benefits. Note that the benefits definition is a bit tricky on occasion, since it would properly include benefits that have to be paid because the plan was terminated in an overfunded position. Open group, closed group, unit credit, entry age normal, and so on, are all the same thing as a theoretical matter. The question of which to use is a practical problem and simply involves the establishment of an objective, which frequently is a function of the desired incidence of contributions and the funded benefit position.

Others besides the plan sponsor have objectives, and it might be well to close with a look at the practical aspects of taxation. Taxation, including deductibility of expense, is based hopefully on actual events. Our representatives have carved out an exception for insurance in that charges can be deducted currently for events that may take place in the future,
and have left the revenue collectors to determine the rules of the game. In both the United States and Canada these rules (e.g., regarding the use of a salary scale) are difficult to defend as logically consistent, but they always have required that a taxpayer determine who is actually covered by his insurance and pension programs and have permitted tax deductions only for these countable persons. Anyone who wants to do the accounting on an open-group principle is, of course, free to do so. However, the calculation of any required minimum contribution or maximum deductible contribution will remain a necessity.

GERALD B. ANGER:
Mr. Schnitzer's paper is a very succinct description of a form of actuarial methodology that can produce actuarial funding costs incorporating dynamic assumptions about the future-notably assumptions as to new entrants, but also dynamic future assumptions as to investment yield, benefit changes, and so on. As such, funding in accordance with the methodology described in the paper can be every bit as rigorous as, and potentially far more valid than, traditional methods conducted on a closed-group (i.e., no new-entrants) basis. There is, however, a danger that the methodology described in the paper may be misconstrued as being the only set of dynamic funding methods, and this is the area I wish to address.

Basically, given virtually any one of the traditional valuation methods, dynamic projections of their results can be made for any number of years into the future with practically any amount of sophistication as to future happenings. Then, in order to achieve a certain funding objective and having regard for the range of possibilities (i.e., a poor future scenario versus a best future scenario), an accrual cost pattern might be adopted that deviates from the ERISA minimum funding standard account requirement dictated by the nominal traditional method being used (or accrual cost might even be outside the usual maximum tax limits). It should be remembered that the accrual cost for pension purposes that a corporation reports to shareholders is not necessarily the same as its actual contribution or the tax deduction it claims. The adoption of a given accrual cost pattern can be considered almost separately from the formal underlying actuarial method being used and thus becomes a financial planning tool of the plan sponsor to be sold as reasonable to the auditors and shareholders. Of course, the disadvantage of this approach is that instead of being able to rely slavishly upon a given acceptable actuarial method, with the actuary essentially in full control, the forecast
accrual cost pattern becomes an item to be agreed to among the plan sponsor, the actuary, the auditor, and perhaps the fund manager(s).
Thus, while I would agree that there should be room to embrace Mr. Schnitzer's family of projection valuation methods as acceptable actuarial methods, especially for IRS and ERISA purposes, the use of traditional methods in conjunction with dynamic forecasting should not be overlooked as a means of funding properly for the future.

## MICHAEL PIKELNY:

I would like to congratulate Mr. Schnitzer for his fine description of the forecast valuation method, and to make a forecast of my own by predicting that this method will become the primary valuation method of the future. By funding a pension plan in such a manner that, after a fixed number of years, level contributions will bring the assets up to the present value of accrued benefits on a continuing plan basis or on a plan termination basis, we obtain a funding method in which it is no longer necessary to use such terms as unfunded frozen initial liability or present value of future normal costs. No longer need we present to a client one figure called unfunded past-service cost and another figure called unfunded vested liability and later have to explain why the two numbers differ. No longer need we prepare actuarial balance sheets with an unfunded accrued liability figure listed under assets. While the above terms and concepts are based on sound actuarial principles, they often have led to misunderstandings, misinterpretations, and confusion. Now we find they no longer are necessary.

Under the forecast valuation method the consulting actuary, using an agreed set of actuarial assumptions and a funding objective, can determine the required level contribution rate and present the client with a very meaningful exhibit showing year-by-year active and retired population statistics, benefit payments plus expenses, contributions, investment return, and the resulting orderly progress of the assets toward the funding objective. By modifying a given assumption, the actuary can determine the effect of the change not only on the contribution requirement but also on the year-by-year population figures, benefit payments, investment return, and so on.

The forecast valuation method also has the advantage of taking into account new-entrant rates. While many actuaries have expressed concern about the abuse of new-entrant assumptions in order to control current costs, Mr. Schnitzer points out that the same can be said about interest rate and termination assumptions, both of which are an integral part of
the conventional cost methods. It also should be mentioned that the conventional cost methods do not solve the problem of new-entrant rates; they merely avoid it.

Finally, I would like to comment on the terminology used in describing the forecast valuation method. It is a well-known fact that there already is much confusion regarding conventional cost method terminology and that labels such as entry age normal, aggregate method, and attained age normal have different meanings to different actuaries. Now we have two papers on forecast valuation methods and two sets of terminology. Mr. Fleischer refers to forecast valuation methods and Mr. Schnitzer to projection valuation methods. I would like to suggest that the term "projection" be reserved solely for those projected benefit cost methods (e.g., entry age normal and attained age normal) in which the contribution requirements are determined by the liabilities for the projected benefits. The term "forecast" should be applied exclusively to those methods in which not only are benefit amounts projected but the entire scheme of population, benefit payments, liabilities, and assets is forecast.

## (AUTHOR'S REVIEW OF DISCUSSION)

ROBERT J. SCHNITZER:
The foregoing discussions provide the kernels for numerous future research projects and papers on the subject of projection valuation methods and on all valuation methods in general. I would like to thank all the discussants for their time and effort in responding to this paper.

In pointing out the inconsistency of results deriving from the use of equal salary and taxable wage base (TWB) increase rates (i.e., the decrease in average pay as a percentage of TWB), Mr. Beers demonstrates the value of projection techniques in general. Without such a forecast, the actuary would not be able to foresee the possible effect of his assumptions.

However, the problem appears to lie more in the treatment of new entrants than in these two assumptions, since the average pay for the closed group in the year 2000 is 81 percent of the TWB in that year. New entrants are assumed to enter the plan at the pay level of the shortestservice participants of the same age, and this technique appears to understate future payrolls. Mr. Beers has alerted us to a useful measuring device for analyzing the reasonableness of assumptions and techniques employed in a PVM.

Mr. Beers states correctly that PVM's spread unfunded present values over future payroll rather than in level dollar amounts, and he expresses concern that this violates some requirements of the minimum funding
provisions of ERISA. I believe that the effect of PVM's in this regard should be acceptable to the Internal Revenue Service and the Department of Labor because (1) the aggregate method is an acceptable method and spreads unfunded present values over future payroll and (2) PVM's amortize the unfunded present values over a fixed period of years whereas the aggregate method spreads them over an indefinite, unlimited period of time. Another acceptable method, the entry age normal method with frozen initial supplemental present value, has the same characteristics with respect to actuarial gains and losses as the aggregate method. There seems to be adequate precedent for nonlevel amortization of unfunded supplemental present values.

It appears that the term "funding goal" has been used by the government when attempting to disallow pension contributions developed by Mr. Gibson. As a result, he seemingly cannot look objectively at a paper dealing with funding goals. In his closing comment, Mr. Gibson demands that the proposed methods be rejected by the actuarial profession; he apparently is not aware that they already have been accepted by the American Academy of Actuaries and have been recognized by the Society's Committee on Pensions and by the Inter-Professional Actuarial Advisory Group.

As Mr. Walls mentions, the PVM's do not attempt to repeal the fundamental formula relating contributions, interest, and benefits. Plan costs do not depend on the actuarial valuation method employed; the latter affects only the timing and amount of periodic contributions.

I infer from the last part of Mr. Walls's discussion that he believes that under an open-group funding method the plan sponsor is funding in advance for future participants. This would occur only in the rare situation where the smoothed normal cost rate developed for the objective period is higher as a result of assumptions about new entrants (i.e., they have a greater attained or entry age than continuing participants) than would have resulted from a closed-group assumption. Otherwise, a PVM does not cause advance funding for future participants.

Mr. Anger reminds us that funding and expensing of pension costs need not proceed at the same pace, and that a dynamic projection of costs under a traditional valuation method using various sets of assumptions can provide valuable information for the establishment of both funding and expensing patterns. Nevertheless, it seems easier for all parties involved-sponsor, actuary, accountant, participants, and stock-holders-if contributions and cost accruals are identical, and if those amounts are within the minimum and maximum tax-deductible limits.

Internal Revenue Service approval of PVM's would give the involved parties an additional, easily understandable alternative to traditional valuations applied to projected populations.

I wish to thank Mr. Pikelny for his kind comments and his reminder of the simplified client-actuary communications that can be achieved through the use of a PVM. He states correctly that a new name for a previously defined cost method is the last thing the actuarial profession needs, and I apologize for any confusion my nomenclature may have caused. It is my understanding that the Inter-Professional Actuarial Advisory Group is currently proposing that the techniques that Mr . Fleischer calls forecast valuation methods and that I call projection valuation methods will be named "group target methods." Adoption of this recommendation should eliminate any future confusion in this area.


[^0]:    * The present values under the continuing plan assumption are greater than those under the terminating plan assumption due to the inclusion in the former of the value of future early retirement subsidies for current active employees under age 55 , while the present values on the plan termination basis assume that such participants are entitled only to a deferred annuity commencing at age 65.

[^1]:    * These amounts are computed by discounting, for twenty years at the valuation interest rate, the PVO amounts shown as of the end of 1995 in Tables $2 \mathrm{~A}-2 \mathrm{C}$. At a 6 percent annual interest rate, the twenty-year discount factor equals 0.31180 .

[^2]:    * Based on PVO equal to present value of accrued benefits, computed on a continuing plan basis.
    $\dagger$ Based on PVO equal to individual entry age normal supplemental present value.

[^3]:    * Based on PVO equal to present value of accrued benefits, computed on a continuing plan basis.
    $\dagger$ Based on PVO equal to individual entry age normal supplemental present value.

[^4]:    * Based on PVO equal to present value of accrued benefits, computed on a continuing plan basis.
    $\dagger$ Based on PVO equal to individual entry age normal supplemental present value.

[^5]:    * Based on PVO equal to present value of accrued benefits, computed on a continuing plan basis.
    t Based on PVO equal to individual entry age normal supplemental present value.

[^6]:    * Based on PVO equal to present value of accrued benefits, computed on a continuing plan basis.
    $\dagger$ Based on PVO equal to individual entry age normal supplemental present value.

[^7]:    ${ }^{1}$ This is actuarial terminology used by the Bell System since the 1920's for what presently is more commonly termed the actuarial liability on an entry age normal method. (Footnote not in testimony.)

