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AN ANALYSIS OF THE "RULE OF 78"—ACTUARIAL NOTE

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ABSTRACT

The "rule of 78" is used throughout the United States in consumer credit financial calculations. It is an approximate technique whose accuracy usually is determined by reference to the "actuarial method." To the author's knowledge, no systematic analysis of the rule of 78 has been published.

The paper describes a typical consumer credit transaction, shows how the rule of 78 works, identifies its degree of accuracy, and discusses issues raised by its use, including federal truth-in-lending implications.

The author concludes that the rule of 78 is increasingly unfair as the term of an indebtedness increases, especially when combined with higher interest or finance charge rates.

INTRODUCTION

THE "rule of 78" is used widely, if not universally, throughout the United States in determining rebates for prepayment of closed-end[1] consumer installment credit transactions. (The term "rebate" here has none of the pejorative qualities found in insurance usage.) The rule of 78 is also known as the "sum of the digits" or "balance of the digits" method. It is a technique for determining how much of a precomputed[2] finance charge is earned by a creditor from time to time, in particular, at the end of each installment period, during the term of a closed-end credit transaction which is repayable in equal periodic installments, usually monthly. (For purposes of this actuarial note it is

[1] A "closed-end" consumer credit transaction is one which is not subject to alteration without rewriting of the contract and simply defines a class of credit transactions which is not "open ended." A revolving charge account is the best example of an open-ended contract.

[2] "Precomputed," or "precomputation," used in connection with finance charges, refers to the practice of summing all finance charges scheduled to be paid, if payments are made as scheduled, for the term of the indebtedness. Generally, this amount is added to the amount financed, and the sum is divided by the number of monthly payments to obtain the amount of each monthly payment. Interest charges on first mortgage loans have not been precomputed historically and are not now, except for truth-in-lending disclosure purposes.

assumed that all credit transactions to which reference is made are repayable in equal *monthly* installments.)

The rule of 78 also has applications in the conduct of the credit life insurance business, but it can be said that its insurance applications result not from the useful properties of the method itself but rather from the necessity of meshing credit insurance into the credit transaction.

If a closed-end credit transaction includes credit life insurance—almost all do—which will retire the indebtedness at the death of the insured debtor, and the debt is prepaid other than by death (usually as a result of refinancing, for example, trading a two-year-old car for a new one), the debtor is entitled to a refund of the credit life premium which was included in the credit transaction, such refund being computed by the rule of 78.

Further, some insurers calculate earned premiums in credit insurance by the rule of 78. For example, an insurer may receive a single premium from the creditor at the inception of the installment contract and then "earn" it over the duration of the indebtedness in proportion to the rule of 78.

The rule of 78 usually is described as a method which produces a refund sufficiently close to the refund that would be calculated by an "actuarial" calculation. For example, Kripke[3] says: "The [rule of 78] is only an approximation of a true actuarial calculation, but it is a remarkably close approximation."

It is the purpose of this paper to determine the degree of precision in the rule of 78 for a wide range of typical consumer credit transactions. Reference will be made to the "error" in the rule of 78, with due regard for the fact that this rule is specified in the state laws which govern consumer credit transactions and that such rebates, or refunds, are lawful and, thereby, contain no "error."

Of particular interest to actuaries is the common reference in consumer credit literature to the "actuarial method" as the standard against which the rule of 78 ought to be measured. The actuarial method is defined in this paper and used as the basis for determining the error in the rule of 78.

HOW THE RULE OF 78 WORKS

It is helpful in illustrating the rule of 78 to make reference to a typical consumer credit transaction. There are legal distinctions between a loan of money and the installment purchase of goods. The credit transactions illustrated in this paper use terms associated with the installment pur-

[3] Homer Kripke, "Consumer Credit Regulation: A Creditor-oriented Viewpoint," *Columbia Law Review*, LXVIII (1968), 445, 454–55.

chase of goods. It should be understood that the same principles apply to loans of money.[4]

Consider the following transaction, which is designed to be repayable in twelve monthly installments of $100:

1. Unpaid balance of cash price	$1,064.23
2. Credit life insurance single premium	7.20
3. Amount financed (sum of items 1 and 2)	$1,071.43
4. Financed charge	128.57
5. Total amount repayable (sum of items 3 and 4)	$1,200.00
6. Annual percentage rate	21.46%

Line 1: The amount of cash due the seller, or the loan proceeds.
Line 2: The credit life insurance premium calculated at a rate of 60 cents per $100 initial indebtedness, that is, 60 cents × 1,200/100 = $7.20.
Line 3: The amount upon which the finance charge is calculated.
Line 4: This is the precomputed finance charge for the whole term of the indebtedness, calculated at a rate of $12 per $100 per year, a rate typically permitted for financing older cars: 0.12 × $1,071.43 = $128.57.
Line 5: The amount to be repaid by the debtor to the creditor. It is also known as the "initial indebtedness."
Line 6: This is calculated by the actuarial method, defined later.

Upon completion of the credit transaction, the creditor might set up his books as shown in the accompanying tabulation.

Month	Outstanding Indebtedness*	Monthly Payment	Finance Charge Earned	Credit Life Premium Earned
1	$1,200	$100	$ 19.78	$1.11
2	1,100	100	18.13	1.02
3	1,000	100	16.48	0.92
4	900	100	14.83	0.83
5	800	100	13.19	0.74
6	700	100	11.54	0.65
7	600	100	9.89	0.55
8	500	100	8.24	0.46
9	400	100	6.59	0.37
10	300	100	4.95	0.28
11	200	100	3.30	0.18
12	100	100	1.65	0.09
Total	$7,800	....	$128.57	$7.20

* This column is also the scheduled amount of credit life insurance, which is single premium, uniformly declining, nonconvertible term insurance.

[4] Further, the credit transactions illustrated are simplified to include only credit life insurance in addition to the price of the goods financed. In practice, credit accident and health insurance, physical damage insurance, other property insurance, and/or recording fees might be included in the "amount financed."

The rule of 78 is a method for apportioning the total gross income from a precomputed finance charge, or from a credit life premium, to each installment period, in a way that recognizes the declining nature of the indebtedness. In the illustration above, the finance charge earned in month 1 is (1,200/7,800) times the total finance charge, or $\frac{12}{78}$ times \$128.57, which is \$19.78. Similarly, in the second month, $\frac{11}{78}$ times the total charge is earned, and so on.

The number 78 is, of course, the sum of the first twelve digits—hence the name "rule of 78." Some prefer the term "sum of the digits" to describe the method, because the number 78 is applicable only on a twelve-month indebtedness. Nevertheless, it seems that "rule of 78" is the generic term used by finance people, regardless of term of indebtedness.

At about this point in the description of the rule of 78, actuaries, who deal with interest theory as a child with building blocks, intuitively will feel the approximate nature of the method. Why is the method only an approximation? Aside from the easy arithmetical demonstration that it is one, perhaps the best way to state the error in the rule of 78 is to say that it apportions finance charge earnings in the same proportions as would an actuarial calculation using 0 per cent interest, overlooking the fact that, in this case, there would be no finance charge to apportion.

How a creditor apportions his earnings is only of passing interest, except to accountants and stockholders. The debtor's interest is made real if he decides to prepay his indebtedness. Some consumer debts are prepaid in full in cash, but most result from refinancing—trading in the old car, borrowing a little more money, and the like. In such cases a new indebtedness is created, the proceeds of which pay off the old one. The critical calculation, in such event, is the amount required to pay off the old indebtedness. Ignoring late charges, penalties, overdue payments, and so on, that amount is defined in state laws to be equal to the outstanding indebtedness at date of prepayment (generally assumed to be a monthly anniversary) less the refund of that portion of the precomputed finance charge which has not yet been earned, that is, the "unearned finance charge."[5]

[5] Laws governing consumer credit transactions produce some interesting effects. For example, in order to qualify legally for a refund of an unearned finance charge, the debtor first must come up with a sum equal to the unpaid indebtedness (installments remaining unpaid) prior to deduction of the unearned finance charge. Then the debtor may receive his refund. In practice, of course, the net amount is sufficient.

However, the finance laws generally use such language as the following: "If the debtor prepays the unpaid indebtedness in full prior to its maturity date, he is then entitled to a refund of the unearned finance charge, computed according to the [rule of 78]." The reader may have noted earlier that the amount of life insurance corresponds

The unearned finance charge is generally defined by law in language similar to the following:

Section 622. Refund for Prepayment of Contract.

A. The buyer, notwithstanding the provisions of any installment sale contract, shall have the privilege of prepaying at any time all or any part of the unpaid time balance under an installment sale contract.

B. Whenever all of the time balance is liquidated prior to maturity by prepayment, refinancing or termination by surrender or repossession and re-sale of the motor vehicle, the holder of the installment sale contract shall rebate to the buyer immediately the unearned portion of the finance charge. Rebate may be made in cash or credited to the amount due on the obligation of the buyer.

C. The unearned finance charge to be rebated to the buyer shall represent at least as great a proportion of the total finance charge as the sum of the periodical time balances after the date of prepayment bears to the sum of all the periodical time balances under the schedule of payments in the original agreement: Provided, however, the holder shall not be required to rebate any portion of such unearned finance charge which results in a net minimum finance charge on the contract less than ten dollars ($10.00); And provided further, the holder shall not be required to rebate any unearned finance charge when the amount due, computed as herein set forth, is less than one dollar.[6]

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to the unpaid indebtedness. Thus, if the debtor dies, the proceeds payable to the creditor, who is the policyholder/beneficiary, are *inclusive* of the unearned finance charge. It would seem that the debtor's estate should receive the refund, but the question arises: Did the *debtor prepay* the indebtedness prior to its *maturity date?* If he did not, is he entitled to a refund? This question often is resolved in favor of the creditor.

[6] Pennsylvania Statutes Annotated, Title 69, Section 622.

Critics frequently have attacked the rule of 78 for the wrong reasons. Note the $10 "minimum finance charge" in Pennsylvania. Clearly it will have a more significant effect on the amount of the refund in certain cases than would the rule of 78 error.

A more typical approach to compensating the creditor for his paperwork upon prepayment or refinancing is the use of an "acquisition charge," often $25 for sales of motor vehicles and $10 for sales of other goods. For example, New Hampshire Rev. Stat. Ann. 361-A:9 reads in part as follows: "The amount of such refund shall represent at least as great a proportion of the finance charge *after first deducting from such finance charge an acquisition charge of twenty-five dollars*, as the sum of the monthly time balances *after the month in which prepayment is made*, bears to the sum of all the monthly time balances under the schedule of payments in the contract" (italics added).

The acquisition charge provision is less favorable to the debtor than the minimum finance charge provision because a penalty (in addition to the rule of 78 error) is exacted throughout the term of indebtedness. (Note also the extra half-month's earned finance charge, on the average, given the creditor in the New Hampshire statute.)

Minimum finance charges, acquisition charges, and other "breakage" authorized by legislative action clearly affect significantly the refunds debtors receive. This paper deals only with the rule of 78 per se.

The above definition reveals why the rule of 78 is sometimes called the "balance of the digits" method when it is used to calculate refunds of unearned finance charges. For example, in a twelve-month transaction prepaid at the end of the fourth month, the refund factor is the sum of the monthly balances which remain outstanding divided by the sum of all monthly balances scheduled to be outstanding at the inception of the contract. Thus, referring to the example given earlier, the refund factor is

$$(800 + 700 + 600 + 500 + 400 + 300 + 200 + 100)/(1{,}200 + 1{,}100 + 1{,}000 + 900 + 800 + 700 + 600 + 500 + 400 + 300 + 200 + 100) = 0.46154\ ,$$

which would be multiplied by the finance charge, \$128.57, to obtain the unearned finance charge (refund or rebate) of \$59.34. Generalizing,

$$ {}_tRF_n^{78} = \frac{\sum_{s=0}^{n-t-1} (n - t - s)}{\sum_{s=0}^{n-1} (n - s)} = \frac{(n - t)(n - t + 1)}{n(n + 1)} $$

where ${}_tRF_n^{78}$ is the rule of 78 refund factor, n is the term of indebtedness in months, and t is the number of months elapsed at date of prepayment.

ACTUARIAL METHOD

It is the actuarial method which is prescribed for use in determining the annual percentage rate for truth-in-lending purposes. This method is not defined explicitly in the federal law. However, it is defined implicitly as follows:

Section 1606. Determination of annual percentage rate—Definition

(*a*) The annual percentage rate applicable to any extension of consumer credit shall be determined, in accordance with the regulations of the Board

- (1) In the case of any extension of credit other than under an open end credit plan, as
  - (A) that nominal annual percentage rate which will yield a sum equal to the amount of the finance charge when it is applied to the unpaid balances of the amount financed, calculated according to the actuarial method of allocating payments made on a debt between the amount financed and the amount of the finance charge, pursuant to which a payment is applied first to the accumulated finance charge and the balance is applied to the unpaid amount financed.[7]

[7] United States Code Annotated, Title 15. Regulation Z of the Federal Reserve Board, setting forth truth-in-lending regulations, does not define "actuarial method" other than implicitly.

The Uniform Consumer Credit Code, adopted only in the states of Oklahoma and

In the truth-in-lending context, the actuarial method would be described to an actuary as the process by which one finds the effective monthly interest rate and translates it to a nominal annual interest rate, given the amount of a loan or debt and the monthly payment. To a layman it would be described as the method the bank uses in figuring mortgage loans.

The writer has the impression that "actuarial method" is a kind of successor term to the "United States rule," which is also permitted under truth-in-lending. This rule is described as follows: "Under the United States Rule, each installment is first applied against the interest due at the date the partial payment is made, and the balance of the installment is then applied to reduce the principal. Interest is always computed on the reduced 'principal.' "[8]

ERROR IN THE RULE OF 78

This paper was undertaken mainly because of the author's impression that the rule of 78 has been deemed acceptable for use in determining refunds of unearned finance charges and credit life insurance premiums on consumer credit transactions mainly on the basis of inadequate demonstrations that it is a reasonable approximation to a refund calculated more precisely.

The best and most recent example of the way in which the rule of 78 is assumed to be appropriate is found in the report of the National Commission on Consumer Finance entitled *Consumer Credit in the United States.*[9] This report was released in January, 1973, by the commission, which was created by the Federal Truth-in-Lending Act to make findings and recommendations in the field of consumer credit.

After explaining the rule of 78, the commission gives an example based on a $1,200 debt repayable in twelve months, to which is added

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Utah, contains this definition of the actuarial method: " 'Actuarial method' means the method, defined by rules adopted by the Administrator, of allocating payments made on a debt between principal or amount financed and loan finance charge or credit service charge pursuant to which a payment is applied first to the accumulated loan finance charge or credit service charge and the balance is applied to the unpaid principal or unpaid amount financed."

[8] Jules I. Bogen (ed.), *Financial Handbook* (New York: Ronald Press, Co. 1964). This source says that the rule is followed by courts if the contract does not specify otherwise and that it is used in relations with government.

The United States rule would seem to be identical with the actuarial method. Some descriptions of the United States rule indicate that, if a payment is made at an installment due date in an amount less than the interest due, the deficiency is *not* carried forward at interest. This distinguishing feature of the United States rule has no relevance for purposes of this paper.

[9] See pp. 40–41 of the report, under "Rebates for Prepayment."

a finance charge of \$6 per \$100 (10.90 per cent annual percentage rate [APR]), or \$72. The example indicates that, if prepayment is made after four months, the refund under the rule of 78 would be \$33.23, whereas under the actuarial method it would be \$33.63, "a difference of only 40 cents."

The commission's conclusion about the rule of 78 is: "In view of the negligible difference between results of the two methods, and in view of the existing extensive use of balance of the digits refund tables, the Commission recommends the use of either method."

This paper will show that the "negligible difference" can easily reach more than 1,000 times the 40-cent error shown by the commission's example.

Formula for Error in the Rule of 78

For an initial indebtedness of n, repayable in monthly installments of 1 for n months,

$$\text{Amount financed} = a_{\overline{n}|}^{i/12} ,$$

where i is the APR. We then have, for a prepayment at the end of t months,

$$\text{Refund, rule of 78} = {}_tRF_n^{78}(n - a_{\overline{n}|}^{i/12}) ;$$

$$\text{Refund, actuarial} = (n - t) - a_{\overline{n-t}|}^{i/12} ;$$

$$\text{Error} = (n - t) - a_{\overline{n-t}|}^{i/12} - {}_tRF_n^{78}(n - a_{\overline{n}|}^{i/12}) ;$$

$$\begin{array}{l}\text{Error as per cent of}\\ \quad\text{amount financed}\end{array} = 100\left[\frac{(n - t) - a_{\overline{n-t}|}^{i/12} - {}_tRF_n^{78}(n - a_{\overline{n}|}^{i/12})}{a_{\overline{n}|}^{i/12}}\right] .$$

Of major interest is the maximum error generated by the rule of 78. The expression could be differentiated and its derivative set equal to zero, and the maximum error found. However, an expansion of terms is necessary, to about the fourth power of i for reasonable accuracy, so the technique is not particularly satisfactory.

Instead, the expression of first differences for duration of t was found and the errors generated by an iterative process on a Monroe 1266 desk-top, programmable calculator.

We define the symbol ${}_tPE_n$, the percentage error in the rule of 78 as a per cent of the amount financed, for a term of indebtedness of n months, prepaid at the end of t months. Then

$${}_{t+1}PE_n - {}_tPE_n = A\left[B(n - t) - 1 + \frac{(1 + i/12)^t}{(1 + i/12)^n}\right] .$$

Therefore,

$$ {}_tPE_n = \sum_{s=0}^{t-1} A\left[B(n-s) - 1 + \frac{(1+i/12)^s}{(1+i/12)^n}\right], $$

or, for ease of programming,

$$ {}_tPE_n = \sum_{s=1}^{t} A[B(n-s+1) - 1 + C(1+i/12)^s], $$

where

$$ A = 100/a_{\overline{n|}}^{i/12}, \quad B = 2(n - a_{\overline{n|}}^{i/12})/n(n+1), \quad C = 1/(1+i/12)^{n+1}. $$

Illustrations Showing Error in Rule of 78

As indicated earlier, the rule of 78 apportions finance charge earnings in the same proportion that a 0 per cent loan would. This suggests that the more important interest is to a transaction, the greater the error. This is shown in Table 1; the error increases as the interest rate or term of indebtedness, or both, increase.

The selection of interest rates, or finance charges, and terms of indebtedness shown in Table 1 is designed to provide an array of examples representative of actual marketplace transactions. In particular, \$7 per \$100 per year seems to be a typical rate for financing new mobile homes. It is purchases in this category of consumer credit—mobile homes—which are financed over the longest terms—ten years typically, twelve years occasionally, and possibly even fifteen years[10] and which generate the largest percentage and dollar errors.

TABLE 1

ERROR IN RULE OF 78 AS A PER CENT OF AMOUNT FINANCED OR BORROWED

A. 24-MONTH CONTRACTS

Finance Charge/\$100/Year	...	...	...
Annual Percentage Rate	12%	24%	36%
End of Month			
1	0.038%	0.151%	0.337%
6	0.143	0.575	1.298
Max	0.153	0.616	1.395
12	0.133	0.541	1.235
18	0.054	0.225	0.521
Month of maximum error	8	8	8

[10] Rule of 78 refund tables are routinely provided for indebtednesses up to and including fifteen years by companies specializing in financial calculations.

TABLE 1—*Continued*

B. 36-MONTH CONTRACTS

Finance Charge/$100/Year	$5.00	$7.00	$9.00	$11.00	$13.00
Annual Percentage Rate	9.31%	12.83%	16.24%	19.57%	22.81%
End of Month					
1	0.035%	0.066%	0.106%	0.153%	0.207%
6	0.156	0.297	0.476	0.691	0.938
12	0.204	0.389	0.626	0.912	1.244
Max	0.204	0.389	0.626	0.913	1.245
18	0.176	0.338	0.546	0.799	1.093
24	0.108	0.209	0.339	0.498	0.684
30	0.037	0.071	0.116	0.172	0.237
Month of maximum error	12	12	12	13	13

C. 60-MONTH CONTRACTS

Finance Charge/$100/Year	$5.00	$7.00	$9.00	$11.00
Annual Percentage Rate	9.15%	12.50%	15.71%	18.80%
End of Month				
1	0.057%	0.105%	0.166%	0.236%
6	0.288	0.537	0.847	1.210
12	0.462	0.864	1.366	1.958
18	0.538	1.010	1.603	2.305
Max	0.545	1.025	1.631	2.348
24	0.535	1.008	1.606	2.317
30	0.472	0.893	1.429	2.069
36	0.369	0.702	1.128	1.638
42	0.249	0.474	0.765	1.115
48	0.131	0.251	0.406	0.595
54	0.040	0.077	0.126	0.185
Month of maximum error	21	21	21	21

TABLE 1—*Continued*

D. 84-MONTH CONTRACTS

Finance Charge/$100/Year Annual Percentage Rate End of Month	$5.00 8.97%	$7.00 12.16%	$9.00 15.19%	$11.00 18.09%
1	0.077%	0.140%	0.216%	0.304%
6	0.409	0.749	1.163	1.639
12	0.706	1.296	2.019	2.853
18	0.900	1.659	2.592	3.675
24	1.004	1.857	2.911	4.140
Max	1.030	1.912	3.006	4.288
30	1.029	1.911	3.006	4.288
36	0.989	1.843	2.908	4.161
42	0.896	1.675	2.653	3.809
48	0.763	1.433	2.278	3.281
54	0.606	1.143	1.823	2.634
60	0.439	0.832	1.331	1.931
66	0.279	0.529	0.851	1.238
72	0.140	0.268	0.432	0.630
78	0.041	0.079	0.128	0.188
Month of maximum error	29	29	30	30

E. 120-MONTH CONTRACTS

Finance Charge/$100/Year Annual Percentage Rate End of Month	$5.00 8.69%	$7.00 11.69%	$9.00 14.51%
1	0.102%	0.183%	0.279%
6	0.569	1.020	1.555
12	1.031	1.856	2.837
18	1.394	2.518	3.859
24	1.665	3.015	4.634
30	1.849	3.359	5.178
36	1.954	3.562	5.507
42	1.988	3.635	5.637
Max	1.988	3.636	5.641
48	1.958	3.593	5.588
54	1.873	3.448	5.379
60	1.741	3.216	5.032
66	1.570	2.911	4.570
72	1.371	2.551	4.018
78	1.153	2.152	3.402
84	0.926	1.735	2.751
90	0.700	1.317	2.095
96	0.487	0.919	1.468
102	0.298	0.565	0.906
108	0.146	0.277	0.446
114	0.042	0.080	0.129
Month of maximum error	42	43	43

TABLE 1—*Continued*

F. 144-MONTH CONTRACTS

Finance Charge/$100/Year	$5.00	$7.00	$9.00
Annual Percentage Rate	8.52%	11.41%	14.12%
End of Month			
1	0.118%	0.208%	0.314%
12	1.226	2.178	3.296
24	2.067	3.694	5.619
36	2.564	4.609	7.047
48	2.758	4.990	7.670
Max	2.765	5.011	7.716
60	2.698	4.911	7.591
72	2.432	4.456	6.929
84	2.017	3.721	5.821
96	1.513	2.810	4.424
108	0.985	1.842	2.919
120	0.503	0.949	1.514
132	0.147	0.279	0.449
Month of maximum error	51	52	52

G. 180-MONTH CONTRACTS

Finance Charge/$100/Year	$7.00
Annual Percentage Rate	11.04%
End of Month	
1	0.241%
12	2.600
24	4.592
36	6.015
48	6.909
60	7.322
Max	7.364
72	7.305
84	6.916
96	6.220
108	5.289
120	4.203
132	3.054
144	1.940
156	0.974
168	0.281
Month of maximum error	65

Dollar Error for Typical Transactions

By applying the preceding errors (expressed as a per cent of amount financed) to the amount financed, the dollar errors can be obtained readily for given contracts at representative durations of prepayment, including that duration—roughly one-third or slightly more into the full term—which gives the maximum error. The following are examples.

1. \$1,000 small loan, two years, 24 per cent APR:
   Maximum dollar error = \$1,000 × 0.00616
   = \$6.16 (8th month).
2. \$3,000 auto loan, thirty-six months, \$7 add-on (12.83 per cent APR):
   Maximum dollar error = \$3,000 × 0.00389
   = \$11.67 (12th month).
3. \$6,000 farm equipment, sixty months, \$7 add-on (12.50 per cent APR):
   Maximum dollar error = \$6,000 × 0.01025
   = \$61.50 (21st month).
4. \$10,000 mobile home, seven years, \$7 add-on (12.16 per cent APR):
   Maximum dollar error = 0.01912 × \$10,000
   = \$191.20 (29th month).
5. \$12,000 mobile home, ten years, \$7 add-on (11.69 per cent APR):
   Maximum dollar error = 0.03636 × \$12,000
   = \$436.32 (43d month).
6. \$15,000 mobile home, twelve years, \$7 add-on (11.41 per cent APR):
   Maximum dollar error = 0.05011 × \$15,000
   = \$751.65 (52d month).

Refund of Unearned Credit Life Insurance Premiums

The foregoing examples illustrate the errors which occur as a result of using the rule of 78 to compute the refund of the unearned finance charge. However, the rule also is used to compute the refund of unearned credit life insurance premium.

An attempt to work out a refund technique which might be accurate for credit life insurance would undoubtedly become bogged down in the very real controversy over the rating techniques used. First, premium rates are usually the same for all debtors, regardless of age. Second, the rate often is that which the creditor judges to be just below the level the courts would find to be undue enrichment, because creditors recoup for themselves the difference between the premium charged the debtor and insurer retention. In short, there is no prevailing actuarial method in setting credit insurance rates. Often a single premium will be calculated for up to fifteen years without discount for mortality or interest. Unless one defines a system of rate regulation and rationalizes every step of the system, demonstrations of actuarially sound credit life refunds are elusive.

Suffice it to say that the author, at this point in his credit life experience, is willing to argue that, upon prepayment, the appropriate refund method for credit life insurance coverage which declines with the outstanding indebtedness is that which is appropriate for calculating the unearned finance charge.

Thus, in any credit transaction, the error produced by the rule of 78 may be said to be the sum of the error in the unearned finance charge plus that in the unearned credit life insurance premium.

In the example used by the National Commission on Consumer Finance, inclusion of credit life insurance in the example would have increased the "negligible difference" from 40 cents to less than 43 cents, clearly a change of no moment.

However, consider a $12,000 mobile home financed over twelve years at $7 per $100 per year finance charge (11.41 per cent APR). Using a single premium credit life insurance rate of 60 cents per $100 of initial indebtedness per year—a rate recommended as not unreasonable by the NAIC[11]—we have the following elements of the credit transaction:

1. Cash price	$12,000
2. Credit life single premium	1,832[12]
3. Amount financed	$13,832
4. Finance charge	11,620
5. Initial indebtedness	$25,452[13]

The maximum error in the rule of 78 occurs at the end of month 52 in a twelve-year transaction and is 5.011 per cent of the amount financed. Thus

Maximum dollar error

$$= \text{Unearned finance charge plus unearned single premium}$$

$$= \$582 + \$92 = \$674.$$

Refunds of unearned finance charges are not, of course, within the purview of insurance commissioners, but refunds of unearned premiums

[11] The NAIC is currently looking into long-term credit insurance and may revise this recommendation. The author considers it exorbitant.

[12] Note that an algebraic equation is needed to determine the initial indebtedness and, thereby, the credit life single premium.

[13] Note that the amount of insurance, which corresponds to the outstanding indebtedness, is far in excess of the cash price of the mobile home, giving rise to very large refunds of unearned finance charges upon death, if such refunds are made by the creditor.

are. However, regulation of the rate charged is obviously much more critical than refinements of the refund technique.

Maximum Dollar Error as a Per Cent of Actuarial Refund

The tables presented show the error in the rule of 78 as a per cent of amount financed because the appropriateness of approximate methods in consumer finance needs to be measured by the dollar impact on the consumer, and this percentage is easily applied to the sales price, or amount borrowed.

Of course, the error in the rule of 78 as a per cent of the actuarial refund would be a better measure of the rule of 78 per se. The accompanying tabulation shows some results of this test at that duration where the dollar error for the transaction illustrated is at a maximum.

Term of Indebtedness in Months	Finance Charge per $100 per Year	Annual Percentage Rate	% Error in Rule of 78 as % of Actuarial Refund
24.........	....	24.00%	4.81%
36.........	$ 7.00	12.83	3.95
36.........	11.00	19.57	6.26
60.........	7.00	12.50	6.43
60.........	9.00	15.71	7.84
84.........	7.00	12.16	8.29
84.........	9.00	15.19	10.29
120........	7.00	11.69	11.15
120........	9.00	14.51	13.16
144........	7.00	11.41	12.70
144........	9.00	14.12	14.67
180........	7.00	11.04	14.62

TRUTH-IN-LENDING IMPLICATIONS

The APR disclosed to the debtor at the inception of a closed-end, consumer installment credit transaction is the rate the debtor actually pays only if he does not prepay the contract.

For example, on a 120-month debt at $7 per $100 per year finance charge rate, the APR is 11.69 per cent. However, if the debt is prepaid at the end of 43 months, one can solve for the APR on the basis of the actual payments made. It turns out to be 12.59 per cent, or 0.90 per cent more than disclosed.

On a 144-month transaction, prepaid at the end of 52 months, the APR's ($7 per $100 per year) are 11.41 per cent disclosed and 12.37 per cent actual, a discrepancy of 0.96 per cent.

It is true that APR's are designed more for comparison shopping than for their abstract value. Since all creditors use the rule of 78, one could argue that there are no truth-in-lending implications necessarily to be identified. Nevertheless, it seems that the heavy penalties imposed by the rule of 78 in a credit transaction represent an important item of disclosure not now available to the buyer.

REFUNDS OF CREDIT ACCIDENT AND HEALTH PREMIUMS

The rule of 78 is also used in all but three states (Connecticut, New Hampshire, and Tennessee) to determine the refund due when an indebtedness which includes credit accident and health insurance is prepaid.

The rule of 78 has no theoretical or approximate application to credit accident and health insurance due to recoveries from disability. The curve of costs per claim is more nearly flat than uniformly decreasing.

To the extent that the rule of 78 is permitted on credit accident and health, it has to be rationalized as an administrative convenience for the creditor; in fact, its use should be viewed as an important part of the permitted maximum rate structure.

SUMMARY

The rule of 78 is seen to be an increasingly inaccurate method for use in long-term credit transactions, or at very high interest rates, or especially when both of these combine.

In effect, the rule of 78 exacts a very heavy penalty for prepayment in many instances. In this connection, it is interesting that the sales finance laws of most states permit the creditor to deduct from the finance charge, prior to application of the rule of 78 factor, an amount ranging from $10 to $35. This additional penalty, known usually as an acquisition charge, may be appropriate as a protection of the creditor if the debtor returns very soon after the sale and prepays the debt. But, as the months pass, it is hardly needed; the rule of 78 does the job.

At what interest rate and/or term of indebtedness the rule of 78 is no longer appropriate is a question that could occupy the lifetime of a consumer advocate. The only comment the author has on this point is to recall that phrase in every sales finance law and every credit insurance law—"If the refund so calculated is less than $1, it need not be made"—or words similar to these. If this measure of "negligible difference" were applied to the question whether the rule of 78 is appropriate in consumer finance, the rule of 78 would go the route of the comptometer.

DISCUSSION OF PRECEDING PAPER

HOWARD H. KAYTON:

Mr. Hunt has presented a paper which seeks to explore the validity of the "rule of 78" for refunds of credit life insurance. A more correct title for the paper might have been "The Inappropriateness of the 'Rule of 78' for Refunds of Consumer Credit Finance Charges."

The rule of 78 is encountered in credit life insurance in three separate areas: (1) in the calculation of reserves as a measure of solvency; (2) in the calculation of earned premiums for measuring experience; and (3) in the calculation of amounts to be refunded.

Mr. Hunt has focused only on item 3, and then only as an extension of an analysis of refunds of the finance charge. However, the finance charges refunded do not follow the same pattern as the insurance premium that is unearned. This discussion will present a justification for the rule of 78 for refunds of life insurance premiums and then explore the two areas not considered by Mr. Hunt.

Error in Rule of 78

The formulas given on page 232 of the paper are applicable only to the refund of the *finance* charge. For life insurance the amount being repaid to the lending institution is not the present value of future payments but is in fact the sum of future payments without discount. The lending institution, on receiving this amount from the insurance company, then credits the beneficiary with a rule of 78 unearned interest charge (even where the amount insured is the present value of future finance payments, the formulas given are not applicable to refunds of insurance premiums).

The formula for the refund would therefore be

$$\frac{\sum_{s=1}^{n-t} \frac{1}{12} q_{[x]+(t+s-1)/12} v^{s/12}(n-t-s+1)M}{\sum_{s=1}^{n} \frac{1}{12} q_{[x]+(s-1)/12} v^{s/12}(n-s+1)M},$$

where M is the monthly repayment amount, n is the original term, and t is the number of months elapsed at date of prepayment.

On this basis, if we assume that $q_x = K$ and that interest is at 0 per cent (two simplifying assumptions that have opposite, but not equally offsetting, effects on the calculations) the formula becomes

$$\frac{\sum_{s=1}^{n-t}(n-t-s+1)}{\sum_{s=1}^{n}(n-s+1)} = {}_tRF_n^{78}\,.$$

Thus, if it is assumed that the gross premium is to be refunded in proportion to mortality incidence, then it would follow that the rule of 78 is the proper basis for developing the unearned life insurance premium on a decreasing term life insurance policy.

So far we have only been discussing how the net premium is earned over the life of a credit insurance contract. An equally important consideration is whether it is applicable to use as a cash value the unearned gross premium. If we do, we are ignoring recovery of acquisition expenses. For ordinary life insurance this is recognized in the Standard Nonforfeiture Law by permitting a company to deduct an expense allowance when determining the "unearned" portion of its premiums received. There should be a similar allowance under credit life and health regulations.

Measurement of Experience

The rule of 78 is also used to obtain the earned premium that is used as the denominator in the loss ratio. Here the effect of any understatement in the unearned premium reserve because of the use of the rule of 78 rather than a more proper reserve is to increase the earned premium and hence artificially reduce the loss ratios. While this is unimportant for a company that has a stable block of business, it has important meaning to the rapidly growing company and to the company writing a new case. This is particularly true under legislation such as that in California which requires mandatory downward rate adjustment on a case-by-case basis.

The above analysis of refunds relates only to reserves on decreasing credit life. Where the curve of costs per claim is more nearly flat, as it is in accident and health insurance (Mr. Hunt agrees with this in his discussion of refunds of credit accident and health premiums), the error in measurement of experience is far more significant.

As an example of the error in accident and health loss ratios, I have used Mr. Hunt's 144-month case with the same net single premium of $1,832 that he shows for life insurance (see Table 1 of this discussion). If we assume a 50 per cent loss ratio based on the true exposure, and if we assume a pattern of level claim costs (which would be approximately true for accident and health), we find that in the first year we would expect to experience \$76.33 of assumed claims (one-twelfth of \$916). However, if we use the rule of 78 for determining earned premiums, we find that we would earn in the first year \$291.65 of the net single premium (15.9 per cent), producing a loss ratio of *26.2 per cent*, a far cry from the true ratio, and even further from the ratio in the last year of the contract, when \$76.33 of losses will be divided by \$13.69 of premium to produce a loss ratio of

TABLE 1

ILLUSTRATION OF DISTORTION IN LOSS RATIO PRODUCED BY RULE OF 78 PREMIUM EARNINGS WITH (*a*) LEVEL CLAIM COSTS AND (*b*) ILLUSTRATIVE MORBIDITY CLAIM COSTS

Year	Earned Premium		Level Claim Costs		Morbidity Claim Costs	
	Unearned Premium Beginning of Year (1)	Earned Premium Rule of 78 (2)	Annual Claim Costs (3)	Loss Ratio [(3)÷(2)] (4)	Annual Claim Costs* (5)	Loss Ratio [(5)÷(2)] (6)
1	\$1,832.00	\$ 291.65	\$ 76.33	26.2%	\$ 76.04	26.1%
2	1,540.35	266.37	76.33	28.7	79.07	29.7
3	1,273.98	241.11	76.33	31.7	81.22	33.7
4	1,032.87	215.84	76.33	35.4	83.35	38.7
5	817.03	190.57	76.33	40.1	85.78	45.0
6	626.46	165.30	76.33	46.2	87.37	52.9
7	461.16	140.03	76.33	54.5	87.07	62.2
8	321.13	114.77	76.33	66.5	83.80	73.0
9	206.36	89.49	76.33	85.3	77.67	86.8
10	116.87	64.23	76.33	118.8	70.05	109.1
11	52.64	38.95	76.33	196.0	59.32	152.3
12	13.69	13.69	76.33	557.6	45.26	330.6
Total	....	\$1,832.00	\$916.00	50.0%	\$916.00	50.0%

* Age 53 at issue, coverage to age 65, 14-day elimination period normalized to produce a 50 per cent loss ratio, based on a recent claims study.

558 per cent. Since the true accident and health pattern of claim costs does not conform to a horizontal line, the final years will not be as poor as in the level case. The ratios developed in columns 5 and 6 illustrate the progression of loss ratios using more realistic disability claim costs (in this case for age 53 at issue, coverage to age 65). It is easy to see how companies are "hurt" during the "runoff" where they have reserved for accident and health on a rule of 78 basis.

Measure of Solvency

The rule of 78, when used for reserves at younger ages, produces a slightly conservative pattern for life insurance. This should be comforting to insurance departments, who should be more concerned with solvency than with the relative equities of terminating policyholders. I use the term "relative equities," since the companies are often required to adjust rates based on experience which would gradually adjust rates to offset profits on refunds.

On the other hand, the use of the rule of 78 unearned *gross* premium for accident and health reserves is completely inappropriate. However, it is usually justified by assuming that the deficiency in the "benefit reserve" is offset by the absence of a capitalized "prepaid acquisition expense" asset (terms usually found in GAAP descriptions). But consider the problem if such a system is used for the valuation of a disability policy with a very low loading formula.

Conclusions

The author has properly identified a problem in the refunding pattern used by finance-type institutions. However, this is only a small segment of that entire business and cannot be considered without viewing the entire question of interest rates, expenses, defaults, extensions of payments, and other factors. Similarly, the author has focused on one small aspect of the highly regulated credit life insurance rate-making system, while ignoring the problems of experience studies, solvency, and expense allowances on termination. Hopefully future papers and discussions will explore these areas more fully, particularly with respect to patterns of claim costs on accident and health and the resultant reserve patterns.

DAVID CRESWELL* AND JAMES L. BERGIN:

Mr. Hunt's paper clearly demonstrates the flaw inherent in estimating the "actuarial method" for calculating earned interest by the rule of 78. However, a great deal more needs to be said concerning the use of the rule of 78 to estimate the earned premium in credit disability insurance.

Two problems arise when the rule of 78 is used in credit disability insurance. The first problem is that unearned premiums on cancellation are not refunded in an equitable manner. The second occurs when experience statistics used for establishing rates are developed using the rule of 78. Seriously inadequate rates can result when this is done.

A proper unearned premium reserve would be one that resulted in a loss ratio that would remain constant over the entire benefit period of a credible block of business. The Committee on Insurance Accounting and Auditing of the American Institute of Certified Public Accountants, in its *Audits of Stock Life Insurance*, recognized this when it required in the case of credit accident and health that "gross premiums should be recog-

* Mr. Creswell, not a member of the Society, is an actuarial student.

nized as revenues over the stated period of the contract in reasonable relationship to claims." The rule of 78 satisfies this only when benefit costs are proportional to the remaining term, and this would occur only in the absence of recoveries. Since recoveries during the first three years include a vast majority of those disabled, some other method is called for.

The premiums earned in the ith month of an n-month contract should be the same percentage of the total gross premium as the benefits incurred in the ith month are of the total benefits paid over the entire n months. The benefits should include both claim costs and refunds on cancellation. If refunds are made on unearned premiums calculated using appropriate claim costs, then the refund costs can be ignored.

Many approximations can be used in calculating these earned premiums without sacrificing accuracy to any great extent. For example, since the pattern of incidence of claim costs does not vary by age nearly as drastically as do actual morbidity rates, much wider age groups are justified than would be the case for active life reserves for disability income. Groupings of different retroactive or elimination periods also can be used in many instances without resulting in appreciable distortion. Also, factors from a single term can often be used to calculate factors for many terms with surprising accuracy. As an example, in a theoretical portfolio of 7 per cent 24-month, 82 per cent 36-month, 6 per cent 48-month, and 5 per cent 60-month benefit periods, minimum accuracy of 99.8 per cent was achieved for GAAP earnings by taking the proportion of the term elapsed for an n-month term and using the unearned premium factor for the same proportion elapsed on a 36-month term.

Although the preceding estimates can eliminate much work with minimal sacrifices in accuracy, use of the rule of 78 cannot be considered even a rough approximation of true benefit cost incidence. For example, in a three-year contract, the rule of 78 will generally indicate an amount of premium earned in the last year of less than half of the true amount. The difference between the true unearned premium and the rule of 78 unearned premium is, of course, zero at the beginning and end of the contract, but near the middle of the contract it can rise to 20 per cent of the original gross premium. Thus, when the rule of 78 is used for refunds on credit health due to cancellation, the effect is similar to a surrender charge which starts at zero, ends at zero, and peaks out at around 20 per cent of the gross premium. It would seem more equitable to return true unearned premium minus a surrender charge which would decrease throughout the life of the contract as initial expenses are recovered.

The distortion in reported earned premium can have a substantial effect on loss ratios submitted to the various states in experience reports. In a state like California, which requires a rate reduction when the loss ratio on a credible block of business over a two-year period is less than 50 per cent, rate reductions could be required even when the true loss ratio is greater than 50 per cent and the rates are, thus, appropriate.

Since the proportional understatement of the premium earned to date, using the rule of 78, is greater for newer business, a block of new business would show even greater distortion in the loss ratio than an evenly distributed block. In a block with an equal amount of business in each of the first 24 policy months, if we assume that all contracts are 36 months, the true loss ratio will be close to 40 per cent greater than that currently being reported by the rule of 78. With the situation even worse for longer-term contracts, the anticipated mandatory rate reductions are sure to discourage companies from going into new credit lines.

Use of the rule of 78 thus produces unreasonable refunds on cancellation and can discourage companies from experimenting with new coverages and markets. The convenience to the creditor, in being able to deduct an equal proportion of the credit insurance premium and the finance charge when a loan is repaid early, does not justify a system which, through the resulting effective surrender charge, benefits those canceling their insurance very early at the expense of those canceling in the middle of the term. Nor is it sound insurance practice to have a surrender charge based not on initial expenses but on the effect of an arbitrary understatement of unearned premium. Finally, use of the rule of 78 distorts earnings without having a compensatory positive effect on efficiency of operations.

WILLIAM C. CUTLIP:

I read with interest Mr. Hunt's paper on the rule of 78. We have been doing a great deal of review and research on this refund method in connection with our credit accident and health business.

Most of Mr. Hunt's paper seemed to deal with financial transactions where only interest is involved. I agree with his analysis that using both the rule of 78 and an added acquisition charge is redundant for these transactions. I take exception to his statement that both systems are penalties. Some form of acquisition charge is not unreasonable as long as it bears a close relation to the cost of establishing and setting up a loan. In addition, the interest charges on the loan itself should be reasonable.

I was quite disappointed with Mr. Hunt's treatment of the refund techniques for credit life and credit accident and health single premiums. His examples demonstrated the very worst practices which are used in the credit insurance field. He also seemed to imply that the examples used represented the practices of all credit insurers, which is certainly not the case. At the same time, I do not feel that his discussion got to the real crux of the problem—that is, development of an equitable refund method for single premium insurance.

There are three basic refund methods in general use in single premium credit insurance. First is the rule of 78, which Mr. Hunt dismisses as being inappropriate and inequitable. Second is the New Hampshire method, which is specified in the statutes and to which Mr. Hunt gives brief and indirect reference. The third method has been suggested by the Pennsylvania Insurance Department to the NAIC Task Force on Long Term Credit Insurance.

Mr. Hunt's discussion of the rule of 78 method centers around a comparison of the amount retained by an insurer using this method and the amount which would be retained using the actuarial method. He has used the term "actuarial method" to describe the application of the monthly interest rate to the outstanding balance of the loan each month. This technique, of course, is not at all directly applicable to the refund of credit life or accident and health premiums, since obviously they involve more than merely interest. While the rule of 78 is not based strictly on premium development theory, it does serve two purposes. First, it is an administrative convenience, especially when the rule of 78 is being used for interest refunds. Second, it serves to allow for amortization of acquisition expenses. The use of an outstanding balance formula generally does not allow for these expenses. It is interesting to note that, after Mr. Hunt's paper was written, Tennessee, which he mentioned as being one of the three states not using the rule of 78, switched to that method effective May, 1974.

Mr. Hunt states that for him an appropriate refund method for credit life would be that method which is also appropriate for calculating the unearned finance charge. This describes the refund method which is used in the state of New Hampshire. This method is designed to refund what is called the "unused" portion of the single premium. The inference to be drawn from this method is that the amount of premium retained by the insurer would represent the "used" portion of the single premium. However, that is not the case, because of the way in which the method is applied.

The New Hampshire refund is calculated by determining the single premium which would be charged for a loan of an amount equal to the remaining outstanding balance on the original loan and a period equal to the remaining period of the original loan. The effect of this is that there is no amortization of initial acquisition costs which occur when the loan is first written. Further, the New Hampshire regulations strictly disallow any deduction from the refund. This precludes the use of a flat acquisition charge. According to this refund method, the acquisition expenses which are built into the single premium cannot be recovered if the loan is terminated before the end of the period.

The Pennsylvania Insurance Department has devised a refund method which it describes as a modified rule of 78 method. The refund is calculated by the rule of 78, and a factor is then applied to this to come up with the final refund. The algebraic solution of this factor application produces very interesting results. The refund turns out to be the ratio of refund interest to total interest, where refund interest is the total interest payments (calculated on the outstanding balance approach) that would have been made from point of refund to the end of the original period, and total interest is the total interest payments which would have been made if the loan had gone to expiry. The interest payments used are based on whatever the lender's interest rate happens to be.

If this ratio were applied to the finance charges for the loan itself, this would produce a retention for the lender equal to the total interest payments which would have been made on the outstanding balance approach. This technique is based on interest only, and it uses the lender's interest rate, which is in no way related to the insurer. This refund method has no theoretical application to the insurance premiums or to the cost of insurance for the period covered.

Of the three methods discussed, I believe that the rule of 78 best meets the criteria of a refund method. There should be equity between the insurer and the insureds. This criterion is not met by the other two methods, since there is not enough allowance for expenses. The refund method should also provide equity among different insureds, both those who terminate before expiry and those who stay to the end of the term. Neither the New Hampshire method nor the Pennsylvania method can offer this equity. Neither allows for the insurer to recoup the acquisition expenses of those who terminate before the end of the period. This cost then has to be borne by those insureds who stay to the end of the period.

A new refund method which would provide the necessary equity is needed. The best approach, I think, would be to apply nonforfeiture techniques, taking account of the underwriting assumptions with respect to mortality/morbidity, interest, expenses, and, perhaps, withdrawals.

DONALD R. SONDERGELD:

When I read Mr. Hunt's paper, my first impression was that more attention should have been given to the "error as a percentage of the actuarial refund" rather than the "error as a percentage of the amount financed or borrowed." This discussion will summarize the results of my analysis and the calculations that were run on one of our time-sharing computer terminals by Tom Corcoran, A.S.A.

I have related the error as a percentage of the amount financed to a $1,000 loan, so this will be referred to as a "dollar error." The error as a percentage of the actuarial refund will be referred to as a "percentage error."

Table 1 of this discussion compares errors at all durations for a 24-month contract. Also shown in Table 1 is P, the dollar error as a percentage of the maximum dollar error. Table 2 shows similar information for P on a 144-month contract. Table 1 illustrates the fact that the percentage errors increase approximately linearly from month to month

TABLE 1

End of Month	Error as a Percentage of the Actuarial Refund			Error in Dollars per $1,000 of Amount Borrowed					
	12%*	24%*	36%*	12%*		24%*		36%*	
				Error	P†	Error	P†	Error	P†
1	0.32%	0.61%	0.87%	$0.38	25%	$1.51	25%	$ 3.37	24%
2	0.64	1.21	1.74	0.70	46	2.79	45	6.22	45
3	0.95	1.82	2.60	0.96	63	3.83	62	8.58	61
4	1.27	2.42	3.46	1.17	76	4.67	76	10.48	75
5	1.59	3.02	4.32	1.32	86	5.31	86	11.93	85
6	1.90	3.62	5.17	1.43	93	5.76	94	12.98	93
7	2.22	4.22	6.03	1.50	98	6.04	98	13.64	98
8	2.53	4.81	6.87	1.53	100	6.16	100	13.96	100
9	2.85	5.41	7.72	1.52	99	6.15	100	13.95	100
10	3.16	6.00	8.56	1.48	97	6.01	98	13.66	98
11	3.48	6.59	9.39	1.42	93	5.75	93	13.12	94
12	3.79	7.18	10.23	1.33	87	5.41	88	12.35	88
13	4.10	7.77	11.06	1.22	80	4.98	81	11.41	82
14	4.42	8.35	11.88	1.10	73	4.49	73	10.31	74
15	4.73	8.93	12.70	0.97	63	3.96	64	9.11	65
16	5.04	9.52	13.52	0.83	54	3.39	55	7.83	56
17	5.35	10.10	14.34	0.68	44	2.82	46	6.52	47
18	5.66	10.67	15.15	0.54	35	2.25	37	5.21	37
19	5.97	11.25	15.96	0.41	27	1.70	28	3.96	28
20	6.28	11.82	16.76	0.29	19	1.20	19	2.80	20
21	6.59	12.40	17.56	0.18	12	0.76	12	1.78	13
22	6.91	12.97	18.36	0.10	7	0.40	6	0.94	7
23	7.25	13.55	19.16	0.03	2	0.14	2	0.33	2

* Initial APR.

† P = Dollar error as a percentage of the maximum dollar error.

(e.g., 7.25 per cent ÷ 23 = 31.5 per cent), with the largest percentage error occurring in the last month. I had percentage errors calculated for all months for each contract term and annual percentage rates (APR's) listed in the paper. The pattern displayed above occurred in all of the calculations.

TABLE 2

End of Month	P		
	12%*	24%*	36%*
6	23%	22%	20%
12	43	40	38
18	60	56	54
24	74	70	67
30	84	81	78
36	92	89	86
42	97	95	93
48	100	99	97
54	100	100	100
60	98	99	100
66	94	97	98
72	89	92	95
78	82	87	90
84	75	79	83
90	66	71	75
96	56	62	66
102	47	52	56
108	37	42	46
114	28	32	35
120	19	22	25
126	12	14	16
132	6	7	8
138	2	2	2

* Initial APR.

The error as a percentage of the actuarial refund, for a prepayment after t months, is

$$ {}_tE_n = 100\left\{1 - \frac{1}{n(n+1)}\left[\frac{(n - a_{\overline{n}|})(n-t)(n-t+1)}{(n-t) - a_{\overline{n-t}|}}\right]\right\}. $$

As indicated in Table 1, ${}_tE_n > {}_{t-1}E_n$ for $1 \leq t < n$. This can be confirmed for all values of n by proving that

$$ -\frac{(n-t)(n-t+1)}{(n-t) - a_{\overline{n-t}|}} > -\frac{(n-t+1)(n-t+2)}{(n-t+1) - a_{\overline{n-t+1}|}} $$

and
$$\frac{(n-t+1)+1}{(n-t+1)-1} > \frac{(n-t+1)-a_{\overline{n-t+1}|}}{(n-t+1)-(1+i)a_{\overline{n-t+1}|}}.$$

Let $m = n - t + 1$, and prove for $1 < m \leq n$ that

$$\frac{m+1}{m-1} > \frac{m-a_{\overline{m}|}}{m-(1+i)a_{\overline{m}|}}, \quad m > \left[1+\left(\frac{m+1}{2}\right)i\right]a_{\overline{m}|},$$

$$\frac{m-a_{\overline{m}|}}{i} > \left(\frac{m+1}{2}\right)a_{\overline{m}|}, \quad (Da)_{\overline{m}|} - \left(\frac{m+1}{2}\right)a_{\overline{m}|} > 0,$$

$$\left(\frac{m-1}{2}\right)v + \left(\frac{m-3}{2}\right)v^2 + \ldots - \left(\frac{m-3}{2}\right)v^{m-1} - \left(\frac{m-1}{2}\right)v^m > 0.$$

By examining terms having the same coefficients, we see that this is true, since $v > v^m$, $v^2 > v^{m-1}$, etc. If m is odd, the middle term will be zero.

I now believe that the errors to look hardest at are the dollar errors rather than the percentage errors. It is therefore interesting to examine the distribution of the dollar errors as related to the maximum dollar error. Figure 1 of this discussion is a graph of P, the dollar error expressed as a percentage of the maximum dollar error, for two of the examples shown in Tables 1 and 2. The *average* dollar error is about 60 per cent of the maximum dollar error.

Table 3 of this discussion shows the maximum percentage error for the three APR's from Table 1 and the seven contract terms illustrated in the

TABLE 3

Term of Indebtedness in Months	Maximum Percentage Error of Actuarial Refund*		
	12%†	24%†	36%†
24.	7.25%	13.55%	19.16%
36.	10.69	19.52	26.97
60.	17.02	29.67	39.36
84.	22.67	37.90	48.60
120.	30.07	47.57	58.56
144.	34.39	52.68	63.45
180.	40.10	58.86	69.06

*Occurs if refund is made one month before the end of the term.

†Initial APR.

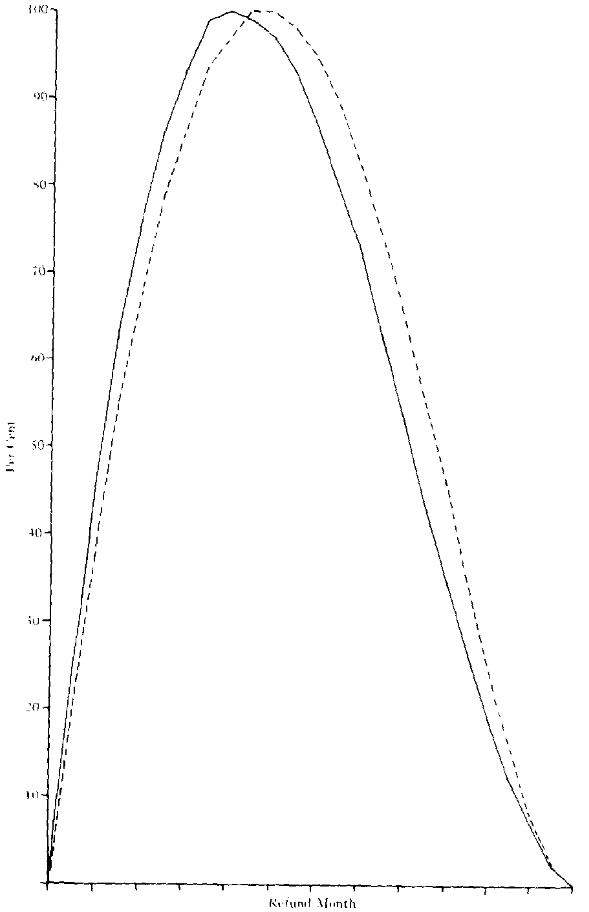

FIG. 1.—Graph of P for two examples. *Solid line:* $n = 24$, APR = 12 per cent; *broken line:* $n = 144$, APR = 36 per cent.

TABLE 4

TERM OF INDEBTEDNESS IN MONTHS	MAXIMUM DOLLAR ERROR PER $1,000 OF AMOUNT BORROWED*			
	12%†	24%†	36%†	Month of Maximum Dollar Error
24	$ 1.53	$ 6.16	$ 13.96	8, 8, 8
36	3.40	13.80	31.38	12 and 13, 13, 13
60	9.44	38.54	87.40	21, 21 and 22, 22
84	18.61	75.97	170.29	29 and 30, 31, 32
120	38.35	154.78	337.28	43, 45, 47
144	55.51	221.03	470.63	52, 55, 58
180	87.14	337.80	693.50	66, 71, 74

* Occurs about one-third or more of the way through the term.

† Initial APR.

paper. The statement made by Mr. Hunt regarding the maximum dollar error also applies to the maximum percentage error: "increases as the interest or term of indebtedness, or both, increase." However, as Mr. Hunt mentioned, the "dollar" errors are not very large when the term of the loan is only 24 months. Table 4 displays the maximum dollar error for the same APR's and contract terms illustrated in Table 3.

Although it is of little practical value, I thought it would be of interest to extend Table 4 to determine when the maximum dollar error per $1,000 of amount borrowed equals $1,000 and $2,000. Tables 5 and 6 give the results.

TABLE 5

n*	APR		n*	APR	
	$1,000 Error	$2,000 Error		$1,000 Error	$2,000 Error
24	347%	564%	120	67%	109%
36	229	372	144	57	91
60	137	221	180	45	73
84	97	157	......	......	......

* n = Term of indebtedness in months.

TABLE 6

$1,000 Error		$2,000 Error	
n*	APR	n	APR
676	12%	1,089	12%
338	24	544	24
226	36	364	36

* n = Term of indebtedness in months.

Mr. Hunt is to be congratulated for a most interesting paper.

RALPH E. EDWARDS:

Mr. Hunt's obvious objective is to discredit the rule of 78. In the course of doing so, he has explained very well how both it and consumer credit operate. Despite this biased presentation, I am not convinced that the rule is other than remarkably efficient and surprisingly accurate. I also find the hidden implications of the paper quite disturbing.

It appears that the business of installment credit at one time was living in a world of add-ons and the rule of 78. Many customers were unaware that the rate of interest on the initial principal amount is roughly doubled when repayment is by installments. It is likely that few cared and possible that some would prefer not to know. Typically, instead of the consumers being educated as to the broad significance of what they were encountering, the burden was shifted to the lender. Despite the complexities of his business, he was required to compress all the facts into a single figure, the equivalent annual rate of interest, and required as well to provide it with great accuracy. If this has a familiar ring to those in the life insurance industry, that is precisely what I intend to convey.

TABLE 1

EFFECTIVE ANNUAL PERCENTAGE RATE

Month	For the Month	From Initial Date*	Month	For the Month	From Initial Date*
1	13.9%	13.9%	96	9.0%	11.5%
6	13.5	13.7	108	8.6	11.4
12	13.1	13.4	120	8.2	11.2
24	12.3	13.0	132	7.9	11.2
36	11.6	12.7	144	7.6	11.1
48	10.9	12.4	156	7.3	11.0
60	10.4	12.1	168	7.1	11.0
72	9.9	11.9	174	6.9	11.0
84	9.4	11.7	179	6.6	11.0

* Slightly approximated.

Mr. Hunt's *enfant terrible* is a fifteen-year loan with a \$7 per year per \$100 add-on, having an annual percentage rate of 11.04 per cent. If we review the nature of this contract, the three factors of influence on the borrower are (1) the 11.04 per cent rate to final installment, (2) the rate to the date of early repayment, and, perhaps, (3) the rate for the current month just past (representing the cost of not paying off the loan a month earlier). For factors 2 and 3 we have prepared Table 1 of this discussion.

Clearly, the rule of thumb that "double the add-on approximates the interest rate" is an overstatement. In these days of two-figure commercial loan rates, the rates for the current month might have significance to the borrower, but given the premise that one and only one figure must represent the array, and lacking any statistical evidence to know where the average borrower fits, is 11.04 per cent reasonable? Would a change to a specific higher rate be more meaningful to the borrower, especially if it did not apply specifically to him?

Actuarial science is more than compound interest (and other) theory, so the practicalities of the situation should be considered. The rule of 78 may truly be the only reason for the table values being shown. This should not blind us to the possibility that a higher rate for early redemptions may be fully justified in fact. Money market considerations are one possibility. Another is that the borrowers may form distinct classes, with those who repay earlier having characteristics different from those of the full-termers. If the former class is more frequently delinquent or otherwise creates more expense, then the higher rate may be quite fair. The earlier part of the loan period may be one of higher expense, which can be amortized over the full period for some borrowers but could be more equitably recognized by a higher rate for those repaying early. Without such a study it would seem better to withhold judgment. Further, there are possibilities which might show that the rate should increase with duration, such as the value of collateral or a high rate of expense per dollar of interest income. In this light it seems highly unlikely that a constant rate throughout is at all appropriate.

Before going on, it may be worth mentioning that Table 1 has the peculiarity that the loan amount increased during the first four months before the interest rate decreased to the point where the interest cost was less than the monthly installment.

Mr. David Sachs, F.S.A., has prepared Table 2 of this discussion for the rule of 78, giving the periodic interest rate from the original date of loan at representative interest rates and repayment periods.

The formula used here is

$$a^{i}_{\overline{n}|} = a_{\overline{t}|} + v^t\left[(n - t) - \frac{(n - t)(n - t + 1)}{n(n + 1)}\,(n - a^{i}_{\overline{n}|})\right],$$

where the interest rate sought was that for $a_{\overline{t}|}$ and v^t. The portion in brackets is an approximation to $a_{\overline{n-t}|}$. When $a_{\overline{n-t}|}$ is expressed in terms of d (the rate of discount) and the terms expanded, the first two terms are correct, and the difference in the rule of 78 first appears in the third terms, where $d^2(n - t + 1)(n - t)(n - 1)/6$ is substituted for $d^2(n - t + 1)(n - t)(n - t - 1)/6$. If the difference in these is differenced with respect to t and set equal to zero, we get

$$t = \frac{1}{3}\left[2n + 1 \pm (n + \tfrac{1}{2})\left(1 + \frac{3}{4n^2 + 4n + 1}\right)^{1/2}\right] \approx \frac{n}{3}.$$

Substituting in the formula for the difference and dividing by n (not $a_{\overline{n}|}$) gives percentages corresponding to those of Mr. Hunt's Table 1 and approximating $\frac{200}{81}n^2d^2$. The results are of the same order of magnitude as those in Mr. Hunt's Table 1. For eight periods when $n = 24$, using d

equal to 0.01, 0.02, and 0.03 gives results of 0.14, 0.57, and 1.3, whereas the author (using i, not d) gets 0.153, 0.616, and 1.395. For 60 periods when $n = 180$ and $d = 0.01$, the result is 8.0, whereas the author gets 7.322 for i equal to 0.092.

One use of the tables in this discussion is to derive the add-on which is equivalent. For example, the 144-month table at 1 per cent reads 1.12 for 36 months. Multiplying this by 12 gives 13.44 per cent. The author's Table 1, C, gives add-ons of $7 for an annual rate of 12.83 per cent and $9 for 16.24 per cent. Interpolating gives an add-on of $7.35. Using Table 1 of this discussion, which is based on an add-on of $7, the 12.7 per cent interest rate for 36 months is less than the 12.83 per cent the author shows for an add-on of $7 for a 36-month contract. This means that the rule of 78 does not yield a result on repayment as unfavorable

TABLE 2

PERIODIC INTEREST RATE FROM ORIGINAL DATE OF LOAN

Installment basis	24 Periods			36 Periods				
Full-term rate	1%	2%	3%	¾%	1%	1¼%	1½%	2%
Number of Periods to Repayment								
1	1.04%	2.15%	3.34%	......	......	......	......	......
6	1.03	2.10	3.22	0.78%	1.04%	1.32%	1.60%	2.17%
12	1.01	2.05	3.10	0.77	1.03	1.30	1.57	2.11
18	1.00	2.02	3.03	0.76	1.02	1.28	1.54	2.07
24	......	......	......	0.76	1.01	1.26	1.52	2.03

Installment basis	84 Periods				144 Periods		
Full-term rate	¾%	1%	1¼%	1½%	¾%	1%	1¼%
Number of Periods to Repayment							
6	0.82%	1.12%	1.44%	1.76%	0.87%	1.21%	1.57%
12	0.81	1.10	1.41	1.72	0.86	1.19	1.54
24	0.79	1.07	1.36	1.65	0.84	1.15	1.48
36	0.78	1.05	1.32	1.60	0.82	1.12	1.43
48	0.77	1.03	1.29	1.55	0.81	1.07	1.39
60	0.76	1.01	1.27	1.52	0.79	1.05	1.35
72	0.75	1.00	1.25	1.51	0.78	1.03	1.32
96	......	......	......	......	0.77	1.01	1.28
120	......	......	......	......	0.75	1.00	1.26

to the borrower as if the same add-on had been used for the repayment period. Thus a lender can state to the borrower that the 180-month contract based on a \$7 add-on has a lower interest rate, if paid off after 36 months, than if the contract had been written on a 36-month basis using a \$7 add-on and, further, that the 180-month contract has smaller installments.

We might also note that the tables chosen by Mr. Hunt suggest that short-term loans may have higher interest rates than long-term loans. It is hardly necessary to point out that when a long-term loan is paid off early, it has certain characteristics of a short-term loan and may thus warrant a higher interest rate.

I suspect that the rule of 78 grew up in a period of longhand arithmetic. The add-on determined the interest charge quite handily, straight division yielded the installment amounts, and a simple table or calculation for the rule of 78 handled repayments. This efficient system may someday be improved on, but there should be sound practical reasons for a change.

Mr. Hunt shows a preference for fractions with a maximum numerator and a minimum denominator. He also chooses pejorative words such as "error," "unearned," "penalties," and "unfair." I think these unfortunate in an actuarial presentation, where a broader viewpoint would seem to be required before judgment is rendered. Actuarial theory does not, in itself, support the notion of a constant, unvarying interest rate, or the philosophy of necessarily taking sides with the debtor as against the creditor or with the consumer as against the business enterprise.

Finally, Mr. Hunt suggests to me that regulation, purportedly merely determining an index, now seeks to enforce that index throughout the structure of the business. The sculptor is conforming the subject to the statue and not vice versa. Is this to be the fate of the loan business (and other businesses as well)?

(AUTHOR'S REVIEW OF DISCUSSION)

JAMES H. HUNT:

Those who discussed my paper have contributed much to the enlightenment of those who are interested in the rule of 78. Messrs. Cutlip, Kayton, Creswell, and Bergin were quick to point out that I dealt inadequately with the insurance ramifications. As this discussion will reveal, I hope, it was not entirely out of ignorance but rather a result of trying to focus primarily on the rule of 78's shortcomings in calculating refunds of unearned finance charges, where many more consumer dollars are involved, and on truth-in-lending implications.

Mr. Kayton's discussion includes a justification of the rule of 78 for refunds of life insurance premiums. The justification rests on two assumptions: (1) that mortality is constant for all ages and (2) that interest is 0 per cent. Assumption 1 is reasonable because of the prevailing practice of charging all debtors under age 65 the same rate. Assumption 2 is the reason why I wrote the paper; interest is not 0 per cent in the real world of consumer finance, but 9 per cent and up. There is one application of the rule of 78 to which Mr. Kayton's justification would apply. Some creditors, by far the minority and, to my knowledge, generally banks, pay premiums to the insurance company monthly, based on a rate applied to outstanding balances. At the inception of the contract, the aggregate of these payments to be made is calculated by the formula

$$\text{Identifiable charge} = \frac{(n+1)}{20}(\text{MOB})\frac{(\text{Initial indebtedness})}{\$100},$$

where MOB = premium rate per \$1,000 per month. If no finance charge is added to the identifiable charge, then the debtor repays a level amount to the creditor during the term of his indebtedness, which, at the end of the term, aggregates the amount in dollars paid by the creditor to the insurer. At any point short of the end of the term, the creditor has advanced more to the insurer than he has collected from the debtor, because the monthly premiums decrease as the indebtedness decreases. In such event, a 0 per cent interest assumption is realistic and the rule of 78 thereby appropriate. However, *if a finance charge is added to the identifiable charge,* the 0 per cent assumption is no longer appropriate and the rule of 78, therefore, may be questioned. Of course, in this latter event, the rule of 78 may still be used if its use produces reasonable refunds or if its financial value to the creditor is understood in setting maximum credit life rates.

In addition, Mr. Kayton usefully illustrates the danger for a credit accident and health insurer in earning premiums by the rule of 78. I believe there is general agreement in credit accident and health that GAAP earned premiums ought to be calculated by using premium reserves which are roughly halfway between pro rata reserves and rule of 78 reserves.

Messrs. Creswell and Bergin have contributed an excellent discussion on the inappropriateness of the rule of 78 in credit accident and health insurance which, with one qualification, I would be very happy to have substituted for my short paragraph on this subject. The qualification would be with regard to their comment that "it would be more equitable to return true unearned premium less a surrender charge which would decrease throughout the life of the contract as initial expenses are recovered." The comments which follow also apply to Mr. Cutlip's and Mr. Kayton's observations that the error in the rule of 78 serves as a kind of surrender charge.

Most credit life and accident and health insurance is group insurance or is administered using group techniques. I see no more reason for a surrender charge concept to apply in such circumstances than when an employee drops out of an employer's group life or group health plan. It can be argued further that, if monthly premiums based on the outstanding balances during the term of his loan, rather than a single premium, were charged the debtor, there would be no surrender charge. Single premiums are for the convenience of the creditor and insurer who like the investment income. Why should the debtor pay a surrender charge under these circumstances, especially when commissions and premium taxes are paid only on the premiums net of refunds? Further, the creditor will end up with most of any surrender charge anyway, bargaining it away from the insurer in negotiating the insurance arrangement. In this connection, is there need to couple an insurance surrender charge to the twin deductions from the return of finance charge—the rule of 78 error and the acquisition charge of, say, \$25 permitted in most states' finance laws? Last, why should there be a penalty at all upon a refinancing when the creditor undertakes a new deal with more insurance sold and greater finance charges levied? In short, should not the surrender charge concept on credit life and accident and health be limited to an outright prepayment in an environment where the insurance transaction bears more resemblance to the sale of an individual, rather than group, policy?

Messrs. Creswell and Bergin also point out the problems in credit accident and health insurance when the rule of 78 is used to determine premium reserves for loss-ratio purposes in California, where a low loss ratio means that premium rates must be lowered. Let me state this problem in a different regulatory context.

A very large national finance company with an increasing volume of credit accident and health business in a particular state had been reporting statistics to that state using pro rata reserves (even though it kept rule of 78 premium reserves in its financial statements). If pro rata reserves are used on an increasing volume of accident and health business, the result is, of course, opposite to that illustrated by Messrs. Creswell and Bergin; premiums are "earned" too slowly, producing higher loss ratios. Such higher loss ratios had actually been used by this finance company to secure "deviated," higher credit accident and health rates in that state. However, legislation was enacted making it more difficult to write new business, with the result that the finance company's volume dropped. Its loss ratio using pro rata reserves in the new situation was only 63 per cent—not enough to substantiate the deviated rate. So it switched to rule of 78 reserves, the loss ratio jumped to 94 per cent based on the same data, and a substantially higher rate was secured. This was not one of the finest hours in state regulation.

Mr. Cutlip suggests that my premium refund examples demonstrate "the very worst practices." I will state what I believe to be fact: "Whenever single premiums are collected at the inception of a credit transaction, the rule of 78 is virtually certain to be the premium refund technique used, in credit life and in credit accident and health."

To clarify his comments a bit, I know of no state, including New Hampshire, which does not permit the rule of 78 to be used for refunds of credit life insurance premiums.

Mr. Sondergeld has provided some interesting additional calculations which serve to fill some gaps in my paper. In particular, he shows that the "error as a percentage of the actuarial refund" leaves one looking for a better way to analyze the rule of 78. His graph of the dollar errors as a percentage of the maximum dollar errors (Fig. 1 of his discussion) suggests visually in the resulting bell-shaped curves that the rule of 78 would not be the choice of an actuary given the task of devising a method for determining a rational surrender charge. Such a charge, if required at all, should be at a maximum at the inception of a credit transaction (after a reasonable "cooling-off" period) and decline thereafter.

Mr. Edwards has taken a different look at the rule of 78, which I find very perceptive. He computes, in his first table, the effective annual interest rate for each month of a loan based on "outstanding balances" computed by the rule of 78. In his phrase, "the cost of not paying off the loan a month earlier" is seen to decline more or less uniformly from a level substantially above the disclosed APR at the early durations of a loan to a level substantially below it at the later durations. Mr. Edwards suggests that this is not an illogical schedule. If the disparity between extremes (13.9 and 6.6 per cent) were narrowed, I would tend to agree with him in the case of a prepayment (as opposed to a refinancing) which did not involve the deduction of an "acquisition charge" from the finance charge prior to applying the rule of 78. In the case of a refinancing debtor (who is probably more valuable to the creditor), or in the usual sales finance situation where an acquisition charge *is* deducted from the finance charge, I see no need for the twin "penalties" of a declining interest rate schedule *and* an acquisition charge deduction from the finance charge prior to computing the refund.

It is also interesting to reflect on whether the debtor would like to know that, in the later stages of his contract, the effective interest rate is considerably lower than disclosed. He might make a different decision about prepayment.

The remarkable drop in the effective APR as the duration of the loan increases would seem to be a corollary of the rapid increase in the error in the rule of 78 as a percentage of the actuarial refund, which was demonstrated by Mr. Sondergeld. For the 180-month contract analyzed by Mr. Edwards, such error reaches 10 per cent at duration 45, 20 per cent at duration 90, 30 per cent at duration 139, and 38 per cent at duration 179.

As a concluding comment, I would like to mention that a few banks are beginning to write consumer loans on an outstanding balance basis. At inception of the contract, the arrangement is similar to a conventional "add-on" loan, except that the "mysterious rule-of-78," as one bank calls it in its advertising of "loans with a simple difference," is omitted. In practice, the monthly finance charge is computed by applying one-twelfth of the APR to the average outstanding balance during the month. Thus there is a trade-off between the rule of 78 "breakage" which is forgone and the expected extra income from using actual balances, debtors generally being a little late in making monthly payments. Unfortunately, the rule of 78 is much more advantageous, so banks will move in this new direction slowly. One could guess, remembering Mr. Edward's declining APR's, that there will be important implications for truth in lending if this movement toward simple-interest consumer loans catches on.

It is my hope that the paper may hasten the day when the rate you see is the rate you get in the loan and finance business.