**TRANSACTIONS OF SOCIETY OF ACTUARIES
1974 VOL. 26 PT. 1 NO. 76**

VARIABLE PREMIUM LIFE INSURANCE

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ABSTRACT

This paper deals primarily with the actuarial aspects of a life insurance product providing complete flexibility in the pattern of premium payments. Of primary concern are questions of profit and compliance with the Standard Valuation Law and Standard Nonforfeiture Law in this environment of complete premium flexibility.

The life-cycle concept has generated considerable discussion recently. For several reasons, a contract completely consistent with the concept seems to be out of reach at this time. The subject here is a product which should be realistic, given the current state of our art, and complies at least partially with the life-cycle concept.

I. INTRODUCTION

One of the most interesting and perhaps most challenging of the actuary's responsibilities is the creation of new insurance products. Certainly this responsibility cannot be ignored amid requests from insurance company managements for increased market shares, requests from agents for "something different," and requests from concerned consumers for "a better buy." The life insurance industry and the actuarial profession are moving to meet this responsibility in an atmosphere of increasing interest in new and different insurance products.

Product changes can occur in only three areas: risks covered, benefit patterns, and payment patterns. An example of a policy covering a different risk would be legal insurance, which pays the insured's legal fees when they are incurred. The most timely example of a policy new in the second area is variable life insurance. This paper is concerned primarily with the third category—payment patterns.

The label "flexible premium life insurance" has been used and will be abbreviated "FPLI" hereafter. Basically, FPLI provides the buyer (or his agent) with the opportunity to design premium scales and resulting benefit scales which are most consistent with the buyer's needs.

The following are some of the more important characteristics of FPLI: (1) the premium can be changed or modified as the policyowner's needs change; (2) the net cost (however defined) can be varied; (3) the policyowner is provided an element of participation; (4) FPLI can be used within the existing legal framework for life insurance; and (5) to some extent, the protection element and the savings element are separated.

II. DEFINITIONS

$_tGP$ = Total gross premium paid at beginning of year t;
$_tNP$ = Net premium assumed received at beginning of year t;
$_tE$ = Endowment amount payable at end of year t;
$_tP$ = Policyowner's outlay in year $t = {}_tGP - {}_{t-1}E$;
$K = {}_tNP \div {}_tP$ (constant for t);
x = Issue age;
$A^{G\ 1}_{x+t-1:\overline{1|}}$ = Term premium for year t and issue age x;
EA_x = Expense allowance for reserves;
C = Factor by which $A^{1}_{x:\overline{1|}}$ is multiplied to produce renewal term premiums
$= 1 \div (1 - J)$;
$_tc$ = All expense and profit charges in year t, excluding J per cent of premium in excess of $FA\ A^{G\ 1}_{x+t-1:\overline{1|}}$;
FA = Face amount;
$_ti$ = Guaranteed interest rate in year t;
$_tv = 1 \div (1 + {}_ti)$;
$_ti'$ = Valuation interest rate in year t;
$_tv' = 1 \div (1 + {}_ti')$;
$_ti''$ = Current interest rate in year t;
$_tv'' = 1 \div (1 + {}_ti'')$;
q^w_{x+t} = Probability of withdrawal used for asset share calculations;
q^d_{x+t} = Probability of death used for asset share calculations;
$p^T_{x+t} = 1 - q^w_{x+t} - q^d_{x+t}$;
$p^d_{x+t} = 1 - q^d_{x+t}$;
$_tj$ = Interest rate used for asset share calculations in year t;
$_1E^G_{x+t-1} = {}_tv'' - A^{G\ 1}_{x+t-1:\overline{1|}}$;
J = Per cent of premium expense;
z = Expiry age;
SP = Present value of benefits provided by an insurance plan.

III. THE FLEXIBLE PREMIUM LIFE INSURANCE POLICY

Before considering FPLI, it is instructive to look again at fixed premium life insurance. Consider an annual premium, nonparticipating, permanent plan with a first-year cash value. This policy provides the owner with two benefits in each policy year: insurance for the face amount and a cash value. In return for these benefits, at the beginning of each year the policyowner must make a payment which is composed of two parts. First, the owner pays the annual premium agreed upon at the time the policy was issued. Obviously, this is inadequate to provide both insurance for the year and a cash value several times greater than this premium in most years. The second piece of the owner's payment, then, is the cash value from the previous year. This principle can be stated in another way. At the end of each policy year the owner must select one of two alternatives. He can exercise the cash-value benefit option and terminate the policy, or he can forgo the cash-value benefit and, in addition, pay the annual premium, thereby purchasing the two benefits for the next policy year. Admittedly, this ignores the policy loan provision, reduced paid-up insurance, and extended term insurance as benefits and alternatives. Therefore, fixed premium insurance has fixed premiums because the benefits are fixed. If the premiums are to become flexible, the benefits must also become flexible.

FPLI is a limited endowment annually renewable to age z. The coverage may be renewed for a period of one year by the payment of an annual premium at the beginning of the policy year. This annual premium is used to purchase one-year term insurance for the face amount plus a limited endowment (i.e., the cash value) payable at the end of the year. At the end of this year the coverage may again be renewed, and the endowment payable is used to pay part of the annual premium. The balance of the premium is the policyowner's outlay. The amount of the endowment will be determined by the size of the annual premium paid.

This can be illustrated by using the following familiar formula connecting successive terminal reserves:

$$ {}_tV_x + P_x = 1{,}000A^{\;1}_{x+t:\overline{1}|} + {}_1E_{x+t}\ {}_{t+1}V_x\,. \tag{1}$$

In terms of the preceding definition of FPLI, ${}_tV_x$ is the endowment payable at the end of year t. The annual premium payable at the beginning of year $t+1$ is ${}_tV_x + P_x$, of which ${}_tV_x$ is provided by the previous year's endowment and P_x must be supplied by the policyowner. The term premium is $A^{\;1}_{x+t:\overline{1}|}$ and defines the endowment premium, since ${}_1E_{x+t} = v - A^{\;1}_{x+t:\overline{1}|}$. Finally, given a value of the *variable* P_x, it is possible to solve for the corresponding value of ${}_{t+1}V_x$.

A FPLI plan is completely defined by a set of one-year term premium rates, a table of guaranteed interest rates, and the formula for calculating the endowment amounts. This formula is

$$ {}_tE = \frac{{}_tGP - FA\ \ A^{G\ 1}_{x+t-1:\overline{1}|} - J({}_tP - FA\ \ A^{G\ 1}_{x+t-1:\overline{1}|})}{{}_tv'' - A^{G\ 1}_{x+t-1:\overline{1}|}} \tag{2} $$

(see Appendix I for derivation).

A FPLI plan defined in this manner is actually a family of plans with different premium scales and corresponding cash values. For example, it is possible to reproduce any fixed premium life insurance plan by selecting an interest rate ${}_ti$, substituting the appropriate gross premium for each policyowner's outlay ${}_tP$, and solving for each term premium $A^{G\ 1}_{x+t:\overline{1}|}$.

The current interest rate ${}_ti''$ in the denominator of formula (2) is guaranteed to be not less than some rate ${}_ti$ but can be declared at a higher rate by the company. This variability is a necessary adjunct to the premium variability. If the policyowner can vary the amount of funds placed in the policy, the company must be able to vary the rate paid on these funds. Proper allocation of investment income then becomes critical. When the new-money rate becomes significantly different from the portfolio rate, it will be necessary to have different current interest rates ${}_ti''$ for old policyholders with substantial cash values and new policyholders. While this variability produces several potential problems, it also creates a possible solution to the investment selection exhibited by policyowners electing policy loans. It is possible to reduce the ${}_ti''$ rate to the policy loan rate for any policy fully loaned and to some appropriate weighted average of the ${}_ti''$ rate and the loan rate for partially loaned policies.

Formula (2) assumes annual premiums, and, therefore, the endowment amount payable at the end of the year is actually determined at the beginning of the year. This has the effect of guaranteeing the ${}_ti''$ rate for one year at a time.

Table 1 illustratively defines a FPLI policy with z equal to 65. Notice that the term rates are in a one-year select format. The larger first-year premiums are needed to offset the first-year expenses. Table 2 illustrates the endowment scales generated for a male at age 45 by several different premium scales. Rates are not graded by policy size and are intended for illustration only.

An effort has been made to present FPLI in general terms; however, at times it is necessary to be specific for illustrative purposes. With this in mind, notice that Table 1 represents only one of the many possible FPLI definitions. This particular FPLI plan will be used for illustration throughout the paper, but the comments apply equally well to other FPLI plans.

TABLE 1

Definition of a FPLI Policy

Age	First-Year Term Rate	Renewal Term Rate	Age	First-Year Term Rate	Renewal Term Rate
25	$11.00	$ 2.07	45	$23.87	$ 5.94
26	11.43	2.10	46	24.86	6.48
27	11.88	2.14	47	25.88	7.07
28	12.33	2.18	48	26.96	7.73
29	12.80	2.24	49	28.13	8.46
30	13.30	2.30	50	29.40	9.26
31	13.81	2.36	51	30.77	10.13
32	14.34	2.43	52	32.22	11.08
33	14.89	2.51	53	33.76	12.11
34	15.47	2.61	54	35.40	13.23
35	16.08	2.74	55	37.17	14.46
36	16.73	2.89	56	....	15.80
37	17.42	3.09	57	....	17.29
38	18.14	3.33	58	....	18.91
39	18.89	3.60	59	....	20.68
40	19.66	3.92	60	....	22.62
41	20.44	4.26	61	....	24.72
42	21.24	4.62	62	....	27.02
43	22.06	5.02	63	....	29.53
44	22.93	5.46	64	....	32.28

NOTE.—Renewal rates: $1.1A^{1}_{x:\overline{1}|}$; i: see rates in Table 5; i': 0.035; i'': 0.05; valuation mortality table: 1958 CSO Male, age last birthday, curtate.

TABLE 2

Sample Premiums and Resulting Endowment Values

(Issue Age 45)

Age	Example I		Example II		Example III	
	Premium	Endowment	Premium	Endowment	Premium	Endowment
45	$23.87	$ 0.00	$24.87	$ 0.98	$24.87	$ 0.98
46	23.87	16.73	24.87	18.73	27.87	18.73
47	23.87	33.87	24.87	36.95	24.87	36.95
48	23.87	51.40	24.87	55.62	24.87	55.62
49	23.87	69.31	24.87	74.75	24.87	74.75
50	23.87	87.59	24.87	94.32	24.87	94.32
51	23.87	106.23	24.87	114.33	24.87	114.33
52	23.87	125.22	24.87	134.79	24.87	134.79
53	23.87	144.55	24.87	155.71	24.87	155.71
54	23.87	164.23	24.87	177.07	24.87	177.07
55	23.87	184.23	24.87	198.89	0.00	174.76
56	23.87	204.54	24.87	221.17	0.00	171.24
57	23.87	225.15	24.87	243.90	0.00	166.30
58	23.87	246.03	24.87	267.10	20.00	179.22
59	23.87	267.18	24.87	290.77	21.00	192.67
60	23.87	288.59	24.87	314.94	22.00	206.62
61	23.87	310.26	24.87	339.64	23.00	221.04
62	23.87	332.18	24.87	364.92	24.00	235.90
63	23.87	354.37	24.87	390.84	25.00	251.15
64	23.87	376.83	24.87	417.45	26.00	266.75

IV. PROFIT CONSIDERATIONS

Before exploring the practical aspects of rate-making and profit measurement for FPLI, it is appropriate to mention the different methods for expressing profit objectives. First, profit can be expressed as a constant amount per $1,000 insured. With this objective the profit load in the term rates must be adequate and desirable for all premium levels. While there may be some arguments in favor of this approach, there is also at least one very practical problem. As a general rule, profit is related to premium rather than to amount of insurance. Requiring that FPLI profit be constant for all premium levels would produce a plan that may be either less competitive or less profitable at some point than similar fixed premium insurance.

A second alternative is to express profit as a percentage of premium. This solves the problem mentioned above, since profit increases in direct proportion to premium level. Care must be taken, however, to see that the profit and commissions taken from the additional premium the policyowner elects to pay are not so great as to discourage the payment of higher premiums.

A third alternative is to split the profit objective into two parts—profit for assuming the mortality risk and profit for assuming the investment risk. This seems to be the most satisfying alternative theoretically. Naturally, there is at least one practical problem. The investment risk is limited to the guaranteed interest rate and therefore is usually quite small, particularly in the early contract years. A strict application might produce a profit amount which varied inversely with the premium amount. This is not necessarily undesirable but certainly should be considered.

The intention here is to demonstrate how the variable interest rate can be used to maintain equitable profit margins. Interest is the only variable considered, because the company should absorb all other experience deviations. For convenience it is assumed that profit is to be constant regardless of premium level. Either of the other two profit objectives could be applied.

Formula (2) can be rearranged to the following form:

$$ {}_tGP = (FA - {}_tE)\, A^{G\,1}_{x+t-1:\overline{1}|} + v'' \; {}_tE + J({}_tP - FA \; A^{G\,1}_{x+t-1:\overline{1}|}) \,. \qquad (3) $$

From this it can be seen that ${}_tGP$ provides two benefits plus an expense margin. First, term insurance for the amount at risk, $(FA - {}_tE)A^{G\,1}_{x+t-1:\overline{1}|}$, and, second, an endowment at the end of the year, ${}_tv'' \; {}_tE$. A profit amount is associated with each of these benefits. The expense provided for is the J per cent of premium assumed to be paid by the company on premium received.

Assume that ${}_tGP = FA\ \ A^{G\,1}_{x+t-1:\overline{1}|}$ for all t. In this case there will never be any endowment amounts, and the policy actually will be yearly renewable term. All profit must come from the one-year term rates. For this reason, it is necessary that the term rates be adequate to support both the company and the distribution system. FPLI commissions are expressed as a percentage of the term premium multiplied by the face amount. The only expense assessed against payments above the term premium is the general expense associated with the payment. This is at the rate of J per cent.

The profit contained in the coefficient ${}_tv''$ is of concern when ${}_tGP$ is greater than $FA\ \ A^{G\,1}_{x+t-1:\overline{1}|}$. As ${}_tGP$ increases, ${}_tE$ increases and, therefore, $(FA - {}_tE)$ decreases. This has the effect of eroding the profit and expense loading contained in the one-year term rates. Since the policyowner has the option to vary ${}_tGP$, it is important that this variation not have any significant effect on profit and expense margins. The question then is what the spread should be between the interest rate actually earned and the one used to define the FPLI policy.

A change of Δ_tP changes ${}_tE$ by $\Delta_tP(1 - J)/({}_tv'' - A^{G\,1}_{x+t-1:\overline{1}|})$ and changes the asset share at the end of year t by

$$\frac{\Delta_tP(1 + {}_tj)}{p^T_{x+t-1}} - \frac{\Delta_tEq^w_{x+t-1}}{p^T_{x+t-1}} - \frac{J\Delta_tP}{p^T_{x+t-1}}\,.$$

If the two changes are to be equal, the following must be true:

$$\frac{\Delta_tP(1 - J)}{{}_tv'' - A^{G\,1}_{x+t-1:\overline{1}|}} = \frac{\Delta_tP(1 + j)}{p^T_{x+t-1}} - \frac{\Delta_tEq^w_{x+t-1}}{p^T_{x+t-1}} - \frac{J\Delta_tP}{p^T_{x+t-1}}\,;$$

$$\frac{\Delta_tP(1 - J)}{{}_tv'' - A^{G\,1}_{x+t-1:\overline{1}|}} = \frac{\Delta_tP(1 + j)}{p^T_{x+t-1}} - \frac{\Delta_tP(1 - J)q^w_{x+t-1}}{p^T_{x+t-1}({}_tv'' - A^{G\,1}_{x+t-1:\overline{1}|})} - \frac{J\Delta_tP}{p^T_{x+t-1}}\,;$$

$$(1 - J) = \frac{({}_tv'' - A^{G\,1}_{x+t-1:\overline{1}|})\,(1 + j - J)}{p^T_{x+t-1}} - \frac{(1 - J)q^w_{x+t-1}}{p^T_{x+t-1}}\,;$$

$$(1 - J)(p^T_{x+t-1} + q^w_{x+t-1}) = ({}_tv'' - A^{G\,1}_{x+t-1:\overline{1}|})(1 + {}_tj - J)$$

$$\frac{(1 - J)\,p^d_{x+t-1} + (1 + {}_tj - J)\,A^{G\,1}_{x+t-1:\overline{1}|}}{(1 + {}_tj - J)} = {}_tv''\,;$$

$$i'' = \frac{(1 + {}_tj - J)}{(1 - J)\,p^d_{x+t-1} + (1 + {}_tj - J)\,A^{G\,1}_{x+t-1:\overline{1}|}} - 1\,.$$

Table 3 illustrates the calculation of maximum interest rates of the FPLI plan defined in this paper.

If it is assumed that the actual interest rate earned is 6 per cent and if it is further assumed that the rates shown in Table 3 are used to calculate the endowment amounts, the size of the policyowner's outlay, ${}_tP$, in any year will not affect the difference between the asset share and the cash value for that year.

TABLE 3

CALCULATION OF MAXIMUM i'' RATE

(Issue Age 45)

t	Mortality Rate q^d_{x+t-1}	Survival Rate p^d_{x+t-1}	Term Rate $A^{G\,1}_{x+t-1:\overline{1}\vert}$	Expense Rate J	Earned Interest Rate ${}_tj$	Current Interest Rate ${}_ti''$
1	.00188	.99812	.02387	.09	.06	.04140
2	.00271	.99729	.00648	.09	.06	.06148
3	.00322	.99678	.00707	.09	.06	.06135
4	.00402	.99598	.00773	.09	.06	.06146
5	.00466	.99534	.00846	.09	.06	.06131
6	.00709	.99291	.00926	.09	.06	.06298
7	.00783	.99217	.01013	.09	.06	.06278
8	.00862	.99138	.01108	.09	.06	.06254
9	.00947	.99053	.01211	.09	.06	.06228
10	.01042	.98958	.01323	.09	.06	.06202
11	.01146	.98854	.01446	.09	.06	.06174
12	.01262	.98738	.01580	.09	.06	.06145
13	.01389	.98611	.01729	.09	.06	.06112
14	.01527	.98473	.01891	.09	.06	.06075
15	.01678	.98322	.02068	.09	.06	.06035
16	.01843	.98157	.02262	.09	.06	.05991
17	.02020	.97980	.02472	.09	.06	.05942
18	.02211	.97789	.02702	.09	.06	.05885
19	.02418	.97582	.02953	.09	.06	.05821
20	.02643	.97357	.03228	.09	.06	.05750

Several interesting observations can be made from Table 3 and the formula for calculating the current interest rates. First, ${}_ti''$ is independent of the magnitude and direction of Δ_tP. Also, it is independent of the withdrawal rates. The ${}_ti''$ rates vary inversely with the loading contained in the term rates. This is not unexpected, since, as mentioned previously, as Δ_tP increases, the loading contained in the term rates must be provided by the current interest rate ${}_ti''$. The fact that the rates in Table 3 are greater than 6 per cent at some ages is an indication of the level of the term rates.

A change of Δ_tP will, of course, affect endowment amounts and asset shares in year t and all subsequent years. To this point, only year t has been considered. It is not difficult to extend the calculation of the ${}_ti''$ rate to years after t, making allowance for the persisting effect of $\Delta_t P$.

It seems desirable to let theoretical exactness give way to practical approximations and establish the current interest rate ${}_ti''$ on some average basis. The average rate can be measured and adjusted, by class if necessary, on the exact basis.

V. THE STANDARD VALUATION LAW

The Standard Valuation Law defines a minimum required reserve for uniform premium, uniform benefit policies. For policies "requiring the payment of varying premiums," the law requires a "method consistent." The phrase "varying premiums" certainly refers to policies with fixed, nonlevel premiums; however, it seems unlikely that policies with premiums that could vary at any time after issue were considered at the time the law was written. Regardless of original considerations, FPLI must comply with the Standard Valuation Law as it is written. In his paper "Commissioners Reserve Valuation Method" (*TASA*, XXXV, 258) Walter O. Menge provided a working interpretation of a "method consistent" for policies with fixed but nonlevel premiums or benefits. While I do not quote directly from this paper, some of its principles are used.

The following is a proof that FPLI complies with both the Standard Valuation Law and the Standard Nonforfeiture Law. This proof is divided into four sections. First, the Standard Valuation Law is expressed as a set of six requirements. Next, it is shown that by proper selection of the gross premium scale, the required terminal reserves will be zero when only term insurance is purchased. The third section contains a proof that, regardless of the proposed premium scale at issue, the policy is not deficient, nor will it become deficient at any point, if the guaranteed interest rate has been properly selected. Finally, the fourth section is a proof that any increase in the premium scale after issue will result in an increase in the reserve (endowment amount) greater than the increase required by the Standard Valuation Law.

Cash values, as defined in the Standard Nonforfeiture Law, are less than reserves defined by the Standard Valuation Law at all but extremely high issue ages. Therefore, for a plan with cash values equal to reserves such as this one, compliance with the Standard Valuation Law demonstrates compliance with the Standard Nonforfeiture Law at these issue ages.

A. *The Six Requirements of the Standard Valuation Law*

1. The Standard Valuation Law defines a mortality table and a maximum interest rate, which, when combined with the calculation method, completely define the minimum reserve. It is assumed hereafter that the 1958 CSO Male Table and a $3\frac{1}{2}$ per cent interest rate are referred to when present values and net premiums are mentioned.

2. The expense allowance equals the net level annual premium for the same policy issued one year later with premium and benefit periods reduced one year, minus the cost of the insurance for the first year. This expense allowance cannot be greater than that produced by a twenty-payment life policy or less than zero:

$$EA_x = (B - \alpha) \quad \text{but} \quad 0 \leq EA_x \leq ({}_{19}P_{x+1} - \alpha) .$$

3. The net premium for any year must equal a constant percentage of the benefits plus the expense allowance:

$${}_tNP = K \ {}_tP .$$

4. The present value of the net premiums equals the present value of the benefits plus the expense allowance:

$$\sum_{t=1}^{\infty} {}_tNP \ {}_{t-1}E_x = SP + EA_x .$$

5. The required reserve at any time equals the present value of future guaranteed benefits minus the present value of future net premiums:

$${}_tV_x = SP - \sum_{s=1}^{\infty} {}_{t+s-1}NP \ {}_{s-1}E_{x+t} .$$

6. If the net premium is greater than the gross premium in any year, the present value of the excess must be held as a deficiency reserve.

B. *Restricting FPLI to Term Insurance*

If the premium actually paid each year is equal to the term premium for that year, it will generate no endowments and the plan will be yearly renewable term to age z. If the term rates have been selected properly, terminal reserves of zero will comply with the Standard Valuation Law.

Assume for the moment that requirement 2 (the expense allowance restriction) is satisfied and that the expense allowance is exactly sufficient to produce a reserve of zero at the end of the first year. The constant ratio K of net premium to gross premium can be calculated from the relationship

$$K = \frac{\sum_{t=2}^{z-x-t} {}_tNP \;\; {}_{t-1}E_x}{\sum_{t=2}^{z-x-t} {}_tP \;\; {}_{t-1}E_x} .$$

If ${}_tP = C \;\; {}_tNP$ for all t, this can be simplified to

$$K = \frac{\sum_{t=2}^{z-x-t} {}_tNP \;\; {}_{t-1}E_x}{C \sum_{t=2}^{z-x-t} {}_tNP \;\; {}_{t-1}E_x} = \frac{1}{C} .$$

Therefore, under this assumption, the net premium in each year after the first will be equal to the cost of insurance for that year, and the terminal reserve will always be zero.

The first-year net premium now becomes $K\overset{G}{A}{}^{1}_{x:\overline{1}|}$. The expense allowance is then $K\overset{G}{A}{}^{1}_{x:\overline{1}|} - A^{1}_{x:\overline{1}|}$, which must be less than the maximum allowed,

$$\frac{A^{\;1}_{x+1:\overline{z-x-1}|}}{\ddot{a}_{x+1:\overline{z-x-1}|}} - A^{1}_{x:\overline{1}|} .$$

Table 4 compares the actual expense allowance to the maximum allowed by law, using the assumptions of this illustrative example. Notice that in all cases the actual is lower than the maximum.

TABLE 4

COMPARISON OF ACTUAL TO MAXIMUM EXPENSE ALLOWANCE

Issue Age	Actual	Maximum	Issue Age	Actual	Maximum
25	8.12	15.43	41	14.71	24.62
26	8.48	15.92	42	15.11	25.24
27	8.86	16.41	43	15.49	25.87
28	9.22	16.93	44	15.88	26.50
29	9.60	17.45	45	16.30	27.14
30	10.00	18.00	46	16.71	27.78
31	10.41	18.56	47	17.10	28.42
32	10.83	19.15	48	17.48	29.05
33	11.26	19.75	49	17.88	29.68
34	11.69	20.36	50	18.31	30.30
35	12.13	20.98	51	18.76	30.93
36	12.58	21.60	52	19.22	31.57
37	13.03	22.21	53	19.68	32.21
38	13.47	22.81	54	20.16	32.86
39	13.98	23.41	55	20.65	33.51
40	14.31	24.01			

C. *Removing the Term Insurance Restriction*

Retrospectively, the minimum required reserve at the end of any year t is the accumulated value of the net premiums minus the accumulated value of the insurance provided minus the accumulated value of the expense allowance. A careful examination of this "retrospective" method will reveal that it actually contains two very important prospective elements. First, the expense allowance is a function of the benefits provided (requirement 2), which, in this case, are a function of the total premium pattern. Second, the net premiums are a function of K (requirement 3), which is also a function of the total premium pattern. The net effect of this is that a change in premium in any year can change reserves required in previous years.

The objective is to prove that the endowment amounts are self-adjusting with premium changes and comply with the Standard Valuation Law. The proof contained in this section will make use of the following terms, some of which may have different meanings when used elsewhere in this paper.

$$
\begin{aligned}
{}_tP' &= \text{Term rate for year } t; \\
\Delta_tP' &= \text{Additional premium paid in year } t; \\
\Delta K &= \text{Change in } K \text{ caused by } \Delta_tP'; \\
{}_tV &= \text{Reserve required at end of year } t; \\
{}_xE &= \text{Endowment amount payable at end of year } t; \\
\Delta_{t-1}V &= \text{Change in } {}_{t-1}V \text{ caused by } \Delta_tP'; \\
\Delta_tV &= \text{Change in } {}_tV \text{ caused by } \Delta_tP'; \\
\Delta_tE &= \text{Change in } {}_tE \text{ caused by } \Delta_tP'; \\
{}_tP &= {}_tP' + \Delta_tP'.
\end{aligned}
$$

Assume that it is expected that the policyowner will pay only the term rates ${}_tP'$. Earlier it was shown that the endowment amounts, which are zero, produced by this premium scale comply with the Standard Valuation Law. It will now be shown that each Δ_uP' paid will increase ${}_tE$ by more than it increases ${}_tV$ for all $t \geq u$.

$$\Delta_tE \geq \Delta_tV\ ;$$

$$\frac{(1-J)\Delta_tP'}{{}_{t-u+1}E^G_{x+t-1}} \geq \frac{\Delta_{t-1}V}{{}_{t-u+1}E_{x+t-1}} + \Delta K\sum_{s=t}^{u}\frac{{}_sP}{{}_{s-u+1}E_{x+s-1}} + \frac{(K+\Delta K)\Delta_tP'}{{}_{t-u+1}E_{x+t-1}}\ ;$$

$${}_{t-u+1}E^G_{x+t-1} = {}_{t-u+1}E_{x+t-1}$$

(see Sec. D below). Therefore,

$$(1-J)\Delta_tP' \geq \Delta_{t-1}V + \Delta K\sum_{s=t}^{u}\frac{{}_sP\ {}_{t-u+1}E_{x+t-1}}{{}_{s-u+1}E_{x+s-1}} + (K+\Delta K)\Delta_tP'\ .$$

Consider the term $\Delta_{t-1}V$, the beginning-of-the-year reserve changes due to the change in K. This change is

$$\Delta_{t-1}V = \sum_{s=2}^{t-1}\Delta K \frac{({}_sP' + \Delta_s P')}{{}_{t-s}E_{x+s-1}} .$$

The summation begins at 2 rather than 1 because the accumulated value of the change in the first-year net premium is offset by the accumulated value of the corresponding change in the expense allowance. Since K can never be greater than $1/C$ (see Sec D below), the expense allowance will never be greater than that shown in Table 4, regardless of the magnitude of $\Delta_t P'$.

Now, back to the formula.

$$(1 - J)\Delta_t P \geq \Delta K \sum_{s=2}^{t-1} \frac{{}_sP + \Delta_s P}{{}_{t-s}E_{x+s-1}}$$

$$+ \Delta K \sum_{s=t}^{u} \frac{{}_sP \ {}_{t-u+1}E_{x+t-1}}{{}_{s-u+1}E_{x+s-1}} + (K + \Delta K)\Delta_t P ;$$

$$(1 - J)\Delta_t P - \Delta K\Big[\sum_{s=2}^{t-1} \frac{{}_sP + \Delta_s P}{{}_{t-s}E_{x+s-1}}$$

$$+ \sum_{s=t}^{u} \frac{{}_sP \ {}_{t-u+1}E_{x+t-1}}{{}_{s-u+1}E_{x+s-1}} + \Delta_t P\Big] \geq K\Delta_t P ;$$

$$K \leq (1 - J) - \Delta K\Big[\sum_{s=2}^{t-1} \frac{{}_sP + \Delta_s P}{{}_{t-s}E_{x+s-1}} + \sum_{s=t}^{u} \frac{{}_sP \ {}_{t-u+1}E_{x+t-1}}{{}_{s-u+1}E_{x+s-1}} + \Delta_t P\Big] .$$

Since $K = 1/C = (1 - J)$ (from Sec. D below), the above will always be true if $\Delta K = 0$. The ratio of the change in the numerator of K to the change in the denominator of K for any $\Delta_t P'$ must equal $1/C$ for this to be true.

$$\frac{\Delta_t P'(1 - J) \ {}_{65-x-t+1}E_{x+t-1}}{\Delta_t P' \ {}_{65-x-t+1}E^{G}_{x+t-1}} = \frac{1}{C} ,$$

$$(1 - J) = (1 - J) .$$

Since K is constant, a change in ${}_tP$ does not affect prior years' reserves. This means that the endowment amounts are always at least as large as the reserve required at that point, regardless of past or future premium levels.

D. *Deficiency Premium Reserve*

The Standard Valuation Law requires a deficiency reserve (requirement 6) whenever the net premium for a year is less than the gross premium. Obviously, this will occur whenever K is greater than unity. When only term insurance is purchased, K is equal to $1/C$, which is less than unity, and can be defined as

$$K = \frac{A^{1}_{x:\overline{z-x}|}FA + {}_{z-x}E_x\ {}_zE + EA}{\sum_{t=1}^{z-x} {}_tP\ {}_{t-1}E_x} .$$

From formula (2) it can be seen that an increase in ${}_tP$ of Δ_tP increases the numerator of the above fraction by

$$\frac{\Delta_tP(1-J)\ {}_{65-x}E_x}{{}_{z-x-t+1}E^G_{x+t-1}}$$

and increases the denominator by $\Delta_tP\ {}_{t-1}E_x$.

The ratio K will always equal $1/C$ if every increase in any ${}_tP$ increases the denominator by the corresponding increase in the numerator times $1/C$. In other words,

$$\frac{C(1-J)\Delta_tP\ {}_{z-x}E_x}{{}_{z-x-t+1}E^G_{x+t-1}} = \Delta_tP\ {}_{t-1}E_x ;$$

$$\frac{\Delta_tP\ {}_{z-x}E_x}{{}_{z-x-t+1}E^G_{x+t-1}} = \Delta_tP\ {}_{t-1}E_x ;$$

$${}_{z-x-t+1}E^G_{x+t-1} = {}_{z-x-t+1}E_{x+t-1} .$$

This condition will be satisfied if, for every x,

$${}_1E^G_x = {}_1E_x , \quad vp^G_x = v'p_x , \quad v(1-q^G_x) = v'p_x ,$$

$$v\left(1 - \frac{Cv'q_x}{v}\right) = v'p_x , \quad i = \frac{1+i'}{p_x + Cq_x} - 1 .$$

Table 5 contains the guaranteed rates produced by this formula.

From time to time it is suggested that the Standard Valuation Law and the Standard Nonforfeiture Law be rewritten in forms that would more easily accommodate "nontraditional" insurance policies. Certainly the table of guaranteed interest rates and the requirement that $C = 1/(1-J)$ seem arbitrary and unnecessarily complicated. Regulations providing comparable consumer protection but with an eye toward life-cycle policies would probably be a welcome improvement to this policy design.

However, the premise here is that FPLI is to exist in the present en-

TABLE 5

GUARANTEED INTEREST RATE

Age	Valuation Rate i'	Guaranteed Rate i	Age	Valuation Rate i'	Guaranteed Rate i
25	.035	.03480	45	.035	.03442
26	.035	.03480	46	.035	.03437
27	.035	.03479	47	.035	.03431
28	.035	.03479	48	.035	.03425
29	.035	.03478	49	.035	.03418
30	.035	.03478	50	.035	.03410
31	.035	.03477	51	.035	.03402
32	.035	.03476	52	.035	.03392
33	.035	.03476	53	.035	.03382
34	.035	.03475	54	.035	.03371
35	.035	.03473	55	.035	.03359
36	.035	.03472	56	.035	.03346
37	.035	.03470	57	.035	.03332
38	.035	.03468	58	.035	.03316
39	.035	.03465	59	.035	.03299
40	.035	.03462	60	.035	.03280
41	.035	.03459	61	.035	.03260
42	.035	.03455	62	.035	.03238
43	.035	.03451	63	.035	.03213
44	.035	.03447	64	.035	.03187

vironment for life insurance. This policy, with its seemingly arbitrary requirements, is dependent upon neither a change in nor a new interpretation of the Standard Valuation Law or the Standard Nonforfeiture Law.

VI. SUMMARY

The life-cycle concept represents a significant departure from traditional life insurance forms. It is intended that this paper present a framework around which a policy of the life-cycle family can be built.

VII. ACKNOWLEDGMENT

Acknowledgment is due Gene Buchter for his valuable contributions to the idea underlying this paper and for his reading of the first draft.

APPENDIX I

As would be expected, pricing the FPLI policy is complicated by the potential variability of the premiums paid. This problem is solved in two dimensions. The first dimension is the establishment of the term rates and the formula for calculating the endowment amounts. These are set before the policy is issued. The second dimension, unique to FPLI, is the calculation of the i'' rate at all durations after issue in response to variations in premiums paid. The calculation of the i'' rate is considered in detail in Section IV ("Profit Considerations"). Proper pricing demands that careful attention be given to the interaction of these two pieces of the pricing problem.

It will now be shown that, when the term rates are properly calculated and the ${}_ti''$ rate equals the ${}_tj$ rate for all t, formula (2) in Section III generates endowment amounts which are approximately equal to asset shares. One somewhat unusual but very important assumption implicitly made in formula (2) is that all contingency and profit charges and expenses other than percentage of premium expenses are assessed against the policy as a function of net amount at risk rather than face amount. The rationale of this is that, when FPLI is pure term (the net amount at risk equals the face amount), all expenses and other charges are properly assessed. As FPLI becomes permanent insurance, the i'' rate is adjusted to keep the fixed expenses (all but J per cent of the additional premium) constant. Lapses are ignored, since they have no effect when cash values equal asset shares.

Assuming that charges represented by ${}_tc$ are assessed as a function of net amount at risk, the following formula can be used to calculate asset shares:

$$
\begin{aligned}
{}_tAS = [({}_{t-1}AS + {}_tP)(1 + {}_tj) &- (FA - {}_tAS)\ {}_tc(1 + {}_tj) \\
&- J({}_tP - FA\ A^{G\,1}_{x+t-1:\overline{1}|})(1 + {}_tj) - FA\ q^d_{x+t-1}]/(1 - q^d_{x+t-1})\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{{}_tAS(1 - q^d_{x+t-1})}{(1 + {}_tj)} &= ({}_{t-1}AS + {}_tP) - (FA - {}_tAS)\ {}_tc \\
&\quad - J({}_tP - FA\ A^{G\,1}_{x+t-1:\overline{1}|}) - \frac{FA\ q^d_{x+t-1}}{(1 + {}_tj)}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{{}_tAS}{(1 + {}_tj)} &= ({}_{t-1}AS + {}_tP) - (FA - {}_tAS)\ {}_tc \\
&\quad - J({}_tP - FA\ A^{G\,1}_{x+t-1:\overline{1}|}) - \frac{(FA - {}_tAS)q^d_{+t-1}}{(1 + {}_tj)}\,.
\end{aligned}
$$

Let $A^{G\,1}_{x+t-1:\overline{1}|} = {}_tc + q^d_{x+t-1}/(1 + {}_tj)$ and ${}_ti'' = {}_tj$. Notice that $A^{G\,1}_{x+t:\overline{1}|}$ must be approximate, since the term rates are set at issue and before the ${}_tj$ rates develop.

$$
\begin{aligned}
{}_tAS\ {}_tv'' = {} & ({}_{t-1}AS + {}_tP) \\
& - (FA - {}_tAS)A^{G\ 1}_{x+t-1:\overline{1}|} - ({}_tP - FA\ \ A^{G\ 1}_{x+t-1:\overline{1}|})J\,,
\end{aligned}
$$

$$
{}_tAS = \frac{({}_{t-1}AS + {}_tP) - FA\ \ A^{G\ 1}_{x+t-1:\overline{1}|} - ({}_tP - FA\ \ A^{G\ 1}_{x+t-1:\overline{1}|})J}{{}_tv'' - A^{G\ 1}_{x+t-1:\overline{1}|}}\,.
$$

This formula for the asset share is equivalent to the formula for the endowment amounts.

Notice that the term premium, $A^{G\ 1}_{x+t:\overline{1}|}$, has been loaded for the percentage of premium expense assuming that ${}_tP$ equals $A^{G\ 1}_{x+t:\overline{1}|}$. The term $J({}_tP - A^{G\ 1}_{x+t:\overline{1}|})$ adjusts for the percentage of premium expense as ${}_tP$ varies.

Clearly, when ${}_tP$ is greater than $FA\ \ A^{G\ 1}_{x+t-1:\overline{1}|}$, this term will deduct for the expenses which must be paid from the excess of the premium received over the required term premium. Should ${}_tP$ fall below $FA\ \ A^{G\ 1}_{x+t-1:\overline{1}|}$ in any year, an interesting situation develops. The term now serves to *increase* the endowment amount by J per cent of the excess of the required term premium over the policyowner's outlay for that year.

In any year the required term premium, $FA\ \ A^{G\ 1}_{x+t-1:\overline{1}|}$, must be paid either from funds submitted by the policyowner in that year, ${}_tP$, or from the previous year's endowment amount, ${}_{t+1}E$, or from some combination of these two sources. For expense purposes there is an important difference, however. The policyowner's outlay, ${}_tP$, is subject to the expense, while the previous year's endowment, ${}_{t-1}E$, is not, since it is generated by previous premium payments which were subject to the expense in the year received. If part or all of the required term premium is paid with expense-free dollars, it is only proper that an appropriate amount of this expense load be removed from the term rate. This last term satisfies this requirement. For example, when ${}_tP$ equals zero, it can be seen by combining terms that the entire expense load is deducted from the term rate for that year.

One final property of formula (2) should be considered. When ${}_tE > FA$, a retirement income situation has developed. The formula must now be adjusted to accumulate the endowment amounts at interest only.

DISCUSSION OF PRECEDING PAPER

PAUL R. MILGROM:

Mr. Polk's interesting paper introduces a product that should be quite exciting to consumer advocates. It involves a sufficient separation of the insurance and savings elements of the policy to make the questions, "What is the rate of return on the savings element?" and "What is the cost of the insurance element?" answerable.

It seems, however, that the policy design could be improved and simplified by a few small changes. Consider Mr. Polk's formula (2), which is used to calculate the endowment amounts. The formula can be rearranged as follows:

$$
\begin{aligned}
{}_tE &= [{}_tGP - A^{G}_{x+\overset{1}{t}-1:\overline{1|}}(FA - {}_tE)(1-J) \\
&\qquad - J({}_tP - {}_tE\ A^{G}_{x+\overset{1}{t}-1:\overline{1|}})](1 + {}_ti'') \\[1em]
&= (1 + {}_ti'')\Big[{}_1GP - A_{\overset{1}{x}:\overline{1|}}(FA - {}_1E) \\
&\qquad - FA\ EA - J\ {}_1P + \frac{J}{1-J}\ {}_1E\ A_{\overset{1}{x}:\overline{1|}}\Big] \quad \text{for } t = 1 \\
&= (1 + {}_ti'')\Big[{}_tGP - A_{x+\overset{1}{t}-1:\overline{1|}}(FA - {}_tE) \\
&\qquad - J\ {}_tP + \frac{J}{1-J}\ {}_tE\ A_{x+\overset{1}{t}-1:\overline{1|}}\Big] \qquad \text{for } t \geq 2\,.
\end{aligned}
\tag{1}
$$

The negative component of the loading element, $[J/(1-J)]\ {}_tE\ A_{x+\overset{1}{t}-1:\overline{1|}}$, is a peculiar and illogical looking animal which gives rise to needless complications in the subsequent development. Perhaps it arises from Mr. Polk's treatment of commissions. Eliminating it gives the following formula for calculating endowment amounts:

$$
\begin{aligned}
{}_tE &= \frac{1}{{}_tv'' - A_{x+\overset{1}{t}-1:\overline{1|}}}[{}_1GP - J\ {}_1P - FA(EA + A_{\overset{1}{x}:\overline{1|}})] \ \text{for } t = 1 \\
&= \frac{1}{{}_tv'' - A_{x+\overset{1}{t}-1:\overline{1|}}}({}_tGP - J\ {}_tP - FA\ A_{x+\overset{1}{t}-1:\overline{1|}}) \qquad \text{for } t \geq 2\,.
\end{aligned}
\tag{2}
$$

Using this formula and assuming that the guaranteed rate is equal to the valuation rate, we see that the change $\Delta_{z-x}E$ in ${}_{z-x}E$ resulting from a change Δ_tP in ${}_tP$ is simply

$$(1 - J)\Delta_t P \;_{z-x-t+1}E_{x+t-1} ,$$

that is, that

$$_{z-x-t+1}E^G_{x+t-1} = {}_{z-x-t+1}E_{x+t-1} ,$$

which, as Mr. Polk has demonstrated, is sufficient to prove compliance with the Standard Nonforfeiture Law and the Standard Valuation Law.

This development permits us to dispense with variable guaranteed interest rates and Mr. Polk's Table 5. It also permits the portion of the commissions paid out of the loading factor, J, to be based on the entire premium. (Anna Rappaport's recent paper "Consumerism and the Compensation of the Life Insurance Agent" seems to suggest that such a treatment of commissions is desirable in order to avoid giving the selling agent a bias between permanent and term insurance.)

In the preceding discussion we were able to eliminate varying guaranteed interest rates by attacking their source, namely, a loading that varied with the size of the limited endowment. We can similarly eliminate any theoretical need for "current rates" that vary by duration by attacking their source, namely, redundant valuation one-year term rates. All that must be done is to calculate a set of nonguaranteed one-year net term rates as follows (assuming deaths at the ends of policy years and $i'' = j$)

$$A^{NG}_{\overset{1}{x+t-1}:\overline{1|}} = q^d_{x+t-1}/(1 + i'') . \tag{3}$$

Then the formula for endowment amounts becomes

$$\begin{aligned} tE &= \frac{1}{v'' - A^{NG}_{\overset{1}{x+t-1}:\overline{1|}}} [{}_1GP - FA(A^{NG}_{\overset{1}{x}:\overline{1|}} + EA) - J \;{}_1P] \text{ for } t = 1 \\ &= \frac{1}{v'' - A^{NG}_{\overset{1}{x+t-1}:\overline{1|}}} ({}_tGP - FA \;\; A^{NG}_{\overset{1}{x+t-1}:\overline{1|}} - J \;{}_tP) \qquad \text{for } t \geq 2 . \end{aligned} \tag{4}$$

Under this arrangement, as under Mr. Polk's, variations in the patterns of premiums do not affect company profits. In fact, if (1) all expenses are incurred at the beginnings of policy years, (2) actual expense and profit charges are exactly equal to the available loading each year, and (3) the mortality rates used to figure the nonguaranteed premiums are equal to the experience rates, then the asset share will be exactly equal to the policy cash value, regardless of the pattern of premiums. To prove this, let ${}_tAS$ be the tth asset share. Then, assuming $P^T_{x+t} > 0$ for all t, so that the asset shares ${}_tAS$ are defined, the proof can proceed by mathematical induction:

$$AS = \frac{{}_1P(1-J)(1+j) - FA[EA(1+j) + q_x^d] - q_x^w \, {}_1E}{1 - q^d - q^w}$$

$$= [{}_1P(1-J) - FA \;\; EA](1+j) - q_x^d(FA - {}_1AS)$$

$$- q_x^w({}_1E - {}_1AS) \,.$$

Similarly,

$${}_1E = [{}_1P(1-J) - FA \;\; EA](1+j) - q_x^d(FA - {}_1E) \,,$$

so that

$${}_1AS - {}_1E = q_x^d({}_1AS - {}_1E) - q_x^w({}_1E - {}_1AS) \,,$$

$$0 = (1 - q_x^w - q_x^d)({}_1E - {}_1AS) \,.$$

Therefore,

$${}_1E = {}_1AS \,.$$

Now suppose ${}_{t-1}E = {}_{t-1}AS$. Then a similar argument establishes that

$$0 = ({}_tE - {}_tAS)(1 - q_{x+t-1}^w - q_{x+t-1}^d)$$

$${}_tE = {}_tAS \,. \qquad \text{Q.E.D.}$$

Among the changes suggested, the only substantive one lies in the treatment of commissions. Even this, however, is not central to the approach I suggest. The guaranteed values could be based on a loading on the entire premium, while the nonguaranteed formula included a credit for loading on the savings element of the premium.

In my opinion, the greatest value of flexible premium life insurance (FPLI) lies in its potential for being explained simply to the agent and the consumer. The elements of the explanation to an agent might be as follows:

1. This product combines a savings and an insurance element in any proportions the buyer may wish. The total death benefit is level, but the company charges the yearly term insurance premium only on the amount at risk.
2. Here is our set of guaranteed one-year term rates. Currently, we are charging this lower, nonguaranteed scale.
3. When the policyholder pays more than the term cost for the year, we credit the excess (less commissions, expenses, premium tax, and a profit charge) to his cash value. It accumulates there at a guaranteed rate of 3 per cent. Currently, we are paying $6\frac{1}{2}$ per cent. The expense charge varies by policy size.
4. In addition to the term insurance charge, there is a setup charge in the first year. The setup charge is expressed as so many dollars per $1,000 of face amount. It is used for underwriting and issue expenses and to pay your commission. As you can see, it too is graded by policy size.

Thus the real value of the changes suggested here is that they make the product easier to explain and more susceptible to simple product comparisons. Perhaps this comparability will stimulate the kind of competition our industry sorely needs.

A few more comments concerning various aspects of the product follow.

1. Financial antiselection: This product provides an obvious opportunity for financial antiselection because it gives the policyowner so much flexibility. Two remedies suggest themselves. First, the reserves arising from these policies could be invested in Treasury bills and other short-term securities. Then the same current interest rate could be used for new and old policies. Second, the policy loan interest rate could be kept equal to the current rate or the current rate plus $\frac{1}{2}$ per cent. Perhaps some clever actuary and attorney could word the current rate clause to give the policyowner the guaranteed rate on his savings (3 per cent) and his policy (5 per cent), or the current rate on both, whichever is more favorable. There must certainly be some way to achieve the desired result.
2. Risk loading: Under the variation of FPLI discussed here, all or a portion of the expense and profit could (and probably should) emerge as a risk loading in the nonguaranteed term and interest rates. In this case, to maintain a level current interest rate, some level loading, i_l (of the order of 0.0008), in the rate is needed. (Then the current rate $i'' = j - i_l$.) To ensure that this profit and expense load each year is independent of the pattern of premiums, the loaded mortality rate ${}_lq^d_{x+t}$ and term premiums can be calculated from the following formulas:

$$ {}_lq^d_{x+t} = \frac{1 - q^d_{x+t}}{1 + j} i_l + q^d_{x+t} , \tag{5} $$

$$ A^{NG}_{\overset{1}{x+t}:\overline{1|}} = {}_lq^d_{x+t}/(1 + i'') , \tag{6} $$

   where q^d_{x+t} is the experience mortality rate. Formula (5) is based on the assumptions that $J \;\; {}_tP$ properly reflects actual percentage expenses incurred at the beginnings of policy years, that formula (4) is used for calculating endowment amounts, and that deaths occur at the ends of years.
3. Federal taxes: One must certainly wonder whether a policy whose savings element is so transparent will be treated the same as other permanent policies for all federal tax purposes. Let us here assume that it will. We are left with these questions: Are the excesses of the actual limited endowments over the guaranteed ones true policy values? or dividends? or amounts in the nature of interest? Can the policy wording be manipulated to affect the answers to these questions?

   All other things being equal (which, of course, they never are), a company taxed solely on G (gain from operations) could afford to offer a better current rate than a company taxed on T (taxable investment income) — $250,000 in an era of high interest rates, if the excess endowments are dividends or policy values. If they are amounts in the nature of interest, I believe the federal tax law treats all companies equally. Further investigation is needed in this area.

4. Whole life options and related problems: If the policyowner were allowed to continue premiums for life and there were no reduced paid-up options, the chances of antiselection would be great. Mr. Polk's method of treating this problem is to make the policy endowment insurance renewable annually to a fixed age.

   A related problem is the treatment of the "extended term" insurance situation that arises when the policyholder pays no premium at all in some year(s). This could be treated in several ways, including (1) the introduction of a minimum premium arrangement and automatic premium loans, (2) the introduction of a higher set of one-year term rates for use in this situation, (3) the introduction of true nonforfeiture options and reinstatement provisions, or (4) including a provision for extended insurance mortality in the loading of the nonguaranteed annual term rates.

   To give this product some of the advantages of whole life insurance, the reduced paid-up whole life option should be available when the cash value is small enough to permit it. When the cash value is larger, the policy could provide the option of a cash payment Y and paid-up insurance $FA - Y$. If this option were elected at the end of year t,

$$Y = \frac{{}_tE - FA \;\; A_{x+t}}{1 - A_{x+t}} . \tag{7}$$

   But then, should the paid-up insurance be participating?

5. Expected mortality: Since the policyowner has some control over the amount at risk, some antiselection should be expected. Curiously, that antiselection will not be reflected in a mortality study either by number of policies or by face amount. It is a study by amount at risk that is needed. Since traditional insurance policies offer the owner much less control over this aspect, the past may not be a reliable guide to the future. A possible indication of the level of mortality possible among those electing to skip payments is given by extended term mortality.

   Perhaps the ideas expressed in Richard Ziock's paper "Gross Premiums for Term Insurance with Varying Benefits and Premiums" are applicable here. In any case, I hope that Mr. Polk will comment on his mortality assumptions and how they were arrived at.

In summary, Mr. Polk's paper presents an intriguing conceptual basis for a FPLI product, but it leaves many aspects of the product not sharply defined and many questions unanswered. I look forward to reading the other discussions of this paper and Mr. Polk's responses in the hope that more answers may emerge.

DONALD R. SONDERGELD:

Let me summarize briefly what I believe to be the main items Mr. Polk has presented in his most interesting paper on flexible premium life insurance.

1. The face amount of the policy is fixed and presumably level.
2. The policyholder's minimum outlay in each policy year must equal the cost of insurance, based upon the net amount at risk. This cost is calculated using a scale of predetermined guaranteed one-year renewable term rates. (Actually, the minimum outlay can be less than the cost of insurance if the difference can be provided out of the cash value at the beginning of the year.)
3. Before the policy is issued, premiums in excess of the minimum outlay can be selected. The cash values generated by these excess amounts can be developed by using a formula that, with the predetermined guaranteed term insurance rates and an interest rate or rates, takes into account the Standard Nonforfeiture Law.
4. After the policy is issued, the policyholder's outlay may be varied from what was determined initially, provided that the outlay equals at least the cost of insurance. Any excess over the cost of insurance provides increased cash values that can be demonstrated to meet the requirements of the Standard Nonforfeiture Law. Presumably the above-mentioned formula, term insurance rates, and interest rates would be included in the policy.

Some people will say that, by using Mr. Polk's technique, a whole life policy can be reproduced and therefore split into its protection and savings elements. It is important to note that there is no unique way of making such a separation. Mr. Polk has predefined what the cost of protection is. The savings element is the balancing item.

Let us examine formula (3) of Mr. Polk's paper. By substituting ${}_tP + {}_{t-1}E$ for ${}_tGP$ in formula (3) and solving for ${}_tP$, we obtain

$$
{}_tP = (v''\ {}_tE - {}_{t-1}E) + (FA - {}_tE)A^{G}_{x+t-1:\overline{1}|}) \\
+ J({}_tP - FA\ \ A^{G}_{x+t-1:\overline{1}|}J\ .
$$

As expected, this indicates that the policyowner's outlay in policy year t covers three items:

a) $(FA - {}_tE)A^{G}_{x+t-1:\overline{1}|}$, the gross premium for term insurance equal to the net amount at risk. This can be defined as the protection element. The sum of the next two items would be the savings element.
b) $(v''\ {}_tE - {}_{t-1}E)$, the increase in the endowment value that results from a contribution in excess of the term insurance cost rather than from interest on the cash value at the beginning of the year.

c) $J({}_tP - FA\ A^{G}_{x+t-1:\overline{1}|}^{1}$, expense and profit loading on that portion of the policyowner's outlay that is not needed for the protection element.

The reading of the paper may be assisted by noting that the first part of the paper is based upon the concept of the annual premium (which I recognize is not the policyholder's outlay) being used to purchase one-year term insurance for the *entire face amount* plus a *pure* endowment. However, by the time we get to formula (3), the annual premium is thought of as purchasing one-year term insurance for the *net amount at risk* plus a regular endowment. The results are essentially equivalent, but in practice the determination of interim cash values (i.e., regular endowment amounts) would need to be defined carefully if identical results are desired.

Although Mr. Polk has provided a direction we might take in developing a FPLI policy, there are many practical problems to overcome that are similar to those that Continental Assurance must have grappled with in developing its Comp-U-Term program. That is, the interest rates, loading, and mortality factors in the premium formula, together with the Standard Nonforfeiture Law, were combined by Continental Assurance in such a way that the agent or policyowner could design a pattern of death benefits and then determine what the gross premium would be. Commission differences and competitive factors for each term insurance product needed to be taken into account in one over-all formula.

Using the technique outlined by Mr. Polk, a company could have one formula for all level benefit life, endowment, and term policies. Perhaps Mr. Polk could make general comments on some of the practical aspects that he may already have considered, such as how his company's current scale of commissions and level premiums would fit into such a formula. I suspect a major item to consider is that, under the method suggested by Mr. Polk, more acquisition cost would actually be charged to the policyowner in the first policy year than is the current practice; I would appreciate Mr. Polk's thoughts on this.

I would like to suggest a more direct method of providing flexible premiums and a level death benefit. A FPLI program can be provided that uses some of the features of split life insurance. That is, a yearly renewable term insurance product, combined with a *flexible* payment annuity product, is all that is needed. The two products could be linked together in one policy via a term insurance rider attached to a flexible premium annuity (or vice versa). If statutory restrictions prevented this, separate contracts could be utilized. However, the federal income tax treatment of the excess of the cash value over the premiums received could be affected by this design, as compared with that suggested by Mr. Polk.

The policyholder would initially choose the total death benefit desired. This then would be the maximum amount provided by the term insurance. If the policyowner wished to have a level death benefit, he could purchase yearly renewable term insurance of an amount equal to the difference between the total death benefit desired and the death benefit provided under the flexible payment annuity.

A product using a flexible payment annuity uses the same necessary ingredients as those described by Mr. Polk: a set of guaranteed annual renewal term rates for the protection element and a cash-value formula for the savings element that takes expense and profit loading and an interest rate into account. However, the end result would seem to be much simpler to design and administer.

PETER L. HUTCHINGS:

This stimulating paper presents an idea whose time may have come; in this discussion, I would like to present a related idea whose time has probably passed if it ever existed at all. One name is Roll Your Own Life, or RYOL for short.

Consider a bond fund carried at market whose contents are fully liquid. Consider a mutual fund that works the same way. Assume that each is no-load and that each has an asset charge somewhat in excess of pure investment expenses. The existence of a yearly renewable term premium scale, with margin, is also presumed. The final requirement is a graded formula for a load on incoming cash.

The insured has the following options open to him daily:

1. Pay any premium he likes, whenever he likes.
2. Change his election rates between fixed and equity for future dollars.
3. Change his mix of existing assets either way, subject to a service charge.
4. Draw down assets from either pool at will, subject to a service charge. (Note: draw-downs will reduce face amounts dollar for dollar.)
5. Apply for additional face amount on an evidence basis.
6. Reduce the face amount.

At the end of each day, the insured's asset balance in each fund will be computed as starting assets, plus portfolio growth, plus net deposits, less yearly renewable term cost, and less asset charges. The yearly renewable term cost will be prorated over the accounts by assets at the end of the day. A unit-value approach will be used for both funds. Reinvestment of realized capital gains and dividend and coupon income will be on a no-load basis.

The yearly renewable term face amount will be that sum which, when added to that day's assets, brings the death benefit up to the total face amount. Where assets have been drawn down, there will be a dollar-for-dollar reduction in face amount as noted above.

The insured will also have options open yearly:

1. On a scheduled basis, face amount increases will be permitted without evidence. One such schedule might be every three years between age 21 and age 42.
2. For those policyholders not in phase with a scheduled increase, or too old, the face amount will be increased automatically by a cost-of-living factor (no evidence). Those who do not want the extra coverage can delete it.
3. Where draw-downs have been taken, there will be a no-evidence restoration provision; the face amount will be brought up by one-fifth of the total of the last five years' draw-downs on a no-evidence basis. Example: In 1980 the insured pulls out $10,000 in assets; by 1986 the face amount is fully restored by five $2,000 steps; cost-of-living features will add to this sum.
4. At the beginning of each year, the insured can send in his financial objectives in terms of savings goals, constant-dollar retirement income, and so on. The computer will lay out a set of suggested deposit levels flowing from a (conservative) set of assumptions for asset growth, social security projections, and other factors. These deposit levels will not be guaranteed, of course, and those clients who choose to ignore them can do so. The computer will generate reminder notices, and preauthorized checking is a possibility.

The RYOL product has many features of conventional life insurance: there is a policy loan counterpart (draw-down), an automatic nonforfeiture option (extended term), a premium analogue (suggested deposit level), and all the kinds of loadings one would expect, per dollar of asset, premium, and face amount. It can become term insurance, variable life, mutual life, whole life, or variable annuity, or, indeed, anything else. The sales force will have to live off the load on incoming dollars, and the home office must get by on the other two sources of margin. It appears unlikely that either party can expect the same net income as provided by a cash-value product.

It is hard to know where to start in identifying the impractical/impossible aspects of RYOL. There would be regulatory problems, for starters. RYOL would be a security to the federal people; it would be insurance to the states. In all likelihood the tax status would differ from that of regular insurance. Increases in "cash value" would not be tax-deferred to the policyholder, but the company would not pay federal tax on investment income. Guarantee of fixed-dollar principal is traded off for flexibility and current return, and this is, of course, a sharp departure from current product design. From a marketing viewpoint, RYOL presupposes a sophisticated buyer and a sophisticated salesman. Furthermore, there is little chance that anything close to cash-value commissions could be paid.

These are all serious disadvantages, and on balance they probably preclude RYOL's ever being introduced. Perhaps the concept is useful as an illustration of what the outer limits of flexibility might be like; it is possible that some of the features identified here can be factored into future products.

(AUTHOR'S REVIEW OF DISCUSSION)

KEN E. POLK:

I would like to thank those who discussed this paper. Without exception the discussions contributed significantly to the paper's content.

Before responding to the discussions, I should note an error in Table 4. The maximum expense allowances shown are not correct for the sample policy described. Had the correct maximum expense allowances been shown, they would not have been larger than actual expense allowances for this policy. This situation is corrected in one of two ways. First, the expiry age can be extended beyond age 65 to an age which will produce the desired expense allowances. Second, the spread between the first-year and renewal gross premiums can be reduced. Regardless of the method used, the objective is to reflect a combination of gross premium scale and expiry age which produces actual expense allowances that are less than maximum expense allowances.

In his discussion Mr. Milgrom has offered several modifications which serve to simplify the design considerably. By expressing the formula for the endowment amounts, formula (2), in terms of net premiums rather than gross premium, he eliminates the variable guaranteed interest rates. The gross one-year term premiums are then calculated directly rather than made a function of the net premium, thus eliminating the variable current interest rates.

This most admirable development rests upon expense assumptions slightly different from those used in the paper. I have assumed that all expenses are in one of two classes. Most expenses, including first-year commissions, are expressed on a per $1,000 of insurance basis. All remaining expenses are expressed as a percentage of premium. These are the expenses which clearly will vary with premium level, such as premium taxes. Notice that it is possible to express a piece of the commission as a function of premium and include this in the percentage of premium expense. The percentage of premium expense factor is denoted by J in the paper. The proof in Section V requires that J be constant for all durations.

My development assumes that the one-year term rates are loaded for the J per cent of premium expense, and, therefore, the J rate must be applied only to premiums in excess of the term rates. Mr. Milgrom applies the J rate to all premiums received. Notice from his formula (4) that, if only the term premium, $FA\ \ A^{NG_t}_{x+t-1:\overline{1}|}$, is paid each year, negative endowment amounts will result. These negative endowments are the amounts by which the one-year term rates are deficient. Under this assumption, the minimum premium required in any year would be $FA\ \ A^{NG_t}_{x+t-1:\overline{1}|}(1 + J)$.

Financial antiselection is a problem after the variable current interest rates are deleted. I find the idea of varying the policy loan interest rate an appealing solution to this problem.

The question of federal income taxes is most important and is impossible to resolve clearly. There are arguments for considering the endowment amounts as any of policy values, dividends, or amounts in the nature of interest. If they are dividends, some states would require standard dividend options, which would be a problem.

Antiselection with respect to the mortality risk is minimized by adequate one-year term rates. In the event of poor health, the policyowner may cease making premium payments and the policy will feed off the endowment amounts. This is similar to the extended term insurance option of fixed premium insurance. In fact, since the available cash value is used to purchase term insurance at net rates using the extended term option, while the endowment amounts must purchase term insurance at gross rates using FPLI, it would appear that the risk is less when the policyowner ceases making premium payments on the FPLI policy than when a fixed premium policy goes to extended term insurance. I see no reason why the mortality assumption needs to be substantially different for the two types of insurance.

Mr. Sondergeld has helped to clarify parts of the paper and suggests split life as a more direct and simple method for providing flexible premiums. As he mentioned, it is possible to charge more acquisition expense in the first year. The amount charged is a function of the level of the first-year term premium, which, in turn, is a function of the level of renewal premiums, since the expense allowance must remain within acceptable bounds. Theoretically at least, it is possible to charge all acquisition expense in the first year and produce endowments equal to asset shares.

I cannot but agree that split life makes for a simpler design. However, it seems that at some point the Standard Nonforfeiture Law must be reckoned with. The design described in the paper is to some extent a product of this legislation. Its complicated nature is the result of combining legislative restrictions with the flexible premium objective.

Mr. Hutchings describes an interesting product he calls "Roll Your Own Life." This seems to carry flexibility to its end. I will leave to the reader the task of considering the possibilities after reading his brief description.

Again, I thank those who took time to respond to the paper.