

EARNINGS AND THE INTERNAL RATE OF
RETURN MEASUREMENT OF PROFIT

DONALD R. SONDERGELD

ABSTRACT

The internal rate of return is an important index that is often used in measuring expected profitability. This paper develops a financial tool that may be useful in explaining the differences between earnings that are reported under GAAP and profits that are expressed as an internal rate of return. This financial tool can also be used to compare annually actual and expected earnings and to calculate a company's aggregate internal rate of return.

INTRODUCTION

THE general purpose of this paper is to discuss the measurement of the profitability of individual life insurance policies and the reporting of earnings. However, the concepts are applicable to product lines other than individual life.

Most actuaries use more than a single measuring rod for profit objectives. For example, expected profit can be expressed as a flat dollar amount per \$1,000 of face amount or as a uniform percentage of premium income. The present value of expected profits can also be determined and related to some base, such as first-year premium or the present value of commissions.

Specific profit margins commensurate with the mortality, interest, expense, and persistency risks assumed can be built into the gross premium assumptions. However, the individual life product line of business is a capital-intensive line of business. Thus the expected return on the investment that is made in the first policy year should be known.

This paper discusses the internal rate of return (IRR) profitability measuring rod, or the return on investment concept. The IRR is based upon principles described by J. C. H. Anderson.¹ For our discussion of the IRR method of profit measurement, the negative statutory book profit of the first policy year of an individual life insurance policy is looked upon as an investment. Statutory book profits in renewal years represent a repayment of that investment plus a return. The IRR is the rate at which

¹ "Gross Premium Calculation and Profit Measurement for Nonparticipating Insurance," *TSA*, Vol. XI (1959).

statutory book profits in renewal years must be discounted so that their present value equals the first-year investment. Expressed another way, the present value of first-year and renewal statutory book profits equals zero when discounted at a rate equal to the IRR.

When reported earnings are based upon statutory accounting, it is difficult to explain to an investor the meaning of a particular pattern of emergence of expected profits related to individual life insurance. This is because statutory accounting is concerned with the measurement of solvency and a conservative balance sheet is developed.

The deficiencies of statutory accounting as related to the reporting of earnings are supposedly eliminated when reported earnings are based upon generally accepted accounting principles (GAAP), which are oriented more toward the income statement. GAAP accounting suggests that reasonably conservative rate-making assumptions be used, so that the risks of adverse deviations are provided for. GAAP accounting, as currently defined, develops earnings that are related largely to one of the measuring rods for profits used by some actuaries. That is, if we assume that margins for adverse deviations are needed, expected profits should emerge as a uniform percentage of premium income.

Under GAAP the new-business drain is treated essentially as an investment through the capitalization of acquisition expenses. GAAP accounting redistributes statutory earnings over the life of a policy, but the pattern of emergence of GAAP earnings is not necessarily related to the investment.

Some actuaries, however, use the IRR as a profit objective, instead of using one of the methods referred to above. One purpose of this paper is to examine the inconsistencies of using the IRR to define profits and using GAAP accounting to report earnings. This paper develops a tool that can be used to help explain the difference between the incidence of reported earnings under GAAP and the profitability of a closed block of business measured as an IRR. The tool is a method of accounting, where the "investment" in a life insurance product is treated like a security. The IRR on the investment is the earnings that appear each year. Under the internal rate of return method of accounting (IRRMA), expected annual earnings related to a closed block of business emerge as a uniform percentage of IRRMA surplus, where the uniform percentage is the IRR.

STRUCTURE OF PAPER

This paper is divided into two parts. In Part I a simple model is used to illustrate the expected emergence of earnings associated with either investing statutory surplus in the writing of new life insurance or not

investing in new life insurance. The emergence of expected earnings is then compared on three accounting bases: statutory, GAAP, and IRRMA. It is important to note that statutory reserves were assumed to be zero in the examples and formulas to simplify the illustrations.

To assist in making comparisons of these different methods of accounting, the excess of IRRMA surplus and GAAP surplus over statutory surplus is shown as a separate surplus account. Part I begins with a discussion of this separation and then lists IRRMA and GAAP formulas related to the surplus accounts.

After the illustrations of the expected emergence of earnings under various accounting methods, there is an example of how IRRMA could measure the IRR as a result of actual experience differing from that which was expected. Actual IRRMA surplus is equal to actual statutory surplus plus the present value (discounted at the IRR) of future statutory book profits.

Part II begins with a discussion of the impact statutory reserves have on the IRR. An example is given which points out the fact that the statutory reserve basis affects the incidence of earnings under statutory and IRRMA accounting but has no effect upon reported GAAP earnings.

I

SURPLUS ACCOUNTS (ASSUMES ZERO STATUTORY RESERVES)

For purposes of comparing earnings that are expected to arise under IRRMA, statutory, and GAAP accounting, total surplus has been separated into two categories: (a) noninsurance surplus account and (b) insurance surplus account. Under all three methods of accounting, the noninsurance surplus account is equal to statutory surplus. The amount in the noninsurance surplus account represents real dollars and usually is invested at a rate lower than the IRR or is paid out as dividends to stockholders or may be reinvested in the insurance surplus account.

The surplus in the insurance surplus account equals (i) zero (under statutory accounting), (ii) the algebraic excess of GAAP surplus over statutory surplus (under GAAP accounting), or (iii) the present value of future statutory book profits, discounted at the IRR (under IRRMA accounting). Thus total IRRMA surplus is equal to total statutory surplus plus the present value of future statutory book profits discounted at the IRR. Formulas that describe the development of the IRRMA and GAAP insurance surplus accounts are displayed in the following sections.

At the time a statutory book profit occurs, a transfer is made from the

insurance surplus account to the noninsurance surplus account. In this way the statutory insurance surplus account will always equal zero.

Investment in new business produces no immediate change in total IRRMA surplus. There is simply a transfer from the noninsurance surplus account to the insurance surplus account. IRRMA surplus remains at the same level as if surplus were invested in cash or in some security. Hopefully the future earning power of the surplus so invested in new business is increased.

SYMBOLS AND FORMULAS USED IN THE IRRMA
INSURANCE SURPLUS ACCOUNT

Note that these formulas are based on the assumption that statutory earnings occur at the *beginning* of the policy year. Similar formulas can be developed, based upon other assumptions.

$${}_tE = \text{Statutory earnings in year } t \text{ (i.e., statutory book profits);} \quad (1)$$

$${}^{\text{IRR}}{}_tE = \text{IRRMA earnings in year } t; \quad (2)$$

$${}^{\text{IRR}}{}_tS = \text{IRRMA insurance surplus account at end of year } t; \quad (3)$$

$${}^{\text{IRR}}{}_tS = \sum_{j=t}^{\infty} \frac{{}_jE}{(1 + \text{IRR})^{j-t}}, \quad \text{where } {}^{\text{IRR}}{}_0S = 0; \quad (4)$$

$${}^{\text{IRR}}{}_tE = (\text{IRR})({}^{\text{IRR}}{}_{t-1}S - {}_tE); \quad (5)$$

$${}^{\text{IRR}}{}_tS = {}^{\text{IRR}}{}_{t-1}S - ({}_tE - {}^{\text{IRR}}{}_tE); \quad (6)$$

$${}^{\text{IRR}}{}_tS = \sum_{j=1}^t ({}^{\text{IRR}}{}_jE - {}_jE); \quad (7)$$

$${}^{\text{IRR}}{}_tS = (1 + \text{IRR})({}^{\text{IRR}}{}_{t-1}S - {}_tE); \quad (8)$$

$${}_tE = \frac{(\text{IRR})({}^{\text{IRR}}{}_{t-1}S) + ({}^{\text{IRR}}{}_{t-1}S - {}^{\text{IRR}}{}_tS)}{1 + \text{IRR}}. \quad (9)$$

Formula (4) indicates that IRRMA surplus equals the present value of future statutory earnings, discounted at the IRR. Formula (5) indicates that IRRMA earnings are equal to the IRR on the unpaid investment. The insurance surplus account is reduced each year by a partial repayment of the original investment. This is the excess of statutory earnings over IRRMA earnings. The reduction is ${}_tE - {}^{\text{IRR}}{}_tE$, from formula (6). Statutory earnings in each renewal year can be split into two pieces—the IRR yield on the unpaid investment plus a partial repayment of the original investment (see formula [9]).

SYMBOLS AND FORMULAS USED IN THE GAAP

INSURANCE SURPLUS ACCOUNT

${}_tV$ = Statutory reserve at end of year t ; (10)

${}_t\bar{E}$ = Statutory cash-flow earnings in year t (i.e., ${}_tE + {}_tV - {}_{t-1}V$); (11)

${}_tP$ = Premium income in year t ; (12)

k = Uniform ratio of GAAP profits to premium: the present value of premiums less expenses less benefits divided by the present value of premiums (present values are based upon the interest rates assumed in the gross premium calculation); (13)

i = Interest rate assumed in the gross premium calculation for year t ; for illustration in the tables the interest rate was assumed to be level from year to year and also to equal the rate earned on invested surplus: $(1 + i)^t = (1 + i)(1 + {}_1i)(1 + {}_2i) \dots (1 + {}_t i)$; (14)

${}_t^G S = \sum_{j=1}^t (k {}_jP - {}_j\bar{E})(1 + i)^{t-j+1}$, where ${}_0^G S = 0$; (15)

${}_t^G E = (1 + i)k {}_tP + i({}_{t-1}^G S - {}_t\bar{E})$; (16)

${}_t^G S = {}_{t-1}^G S - ({}_t\bar{E} - {}_t^G E)$; (17)

${}_t^G S = (1 + i)({}_{t-1}^G S - {}_t\bar{E} + k {}_tP)$; (18)

${}_t\bar{E} = {}_t^G E + ({}_{t-1}^G S - {}_t^G S)$. (19)

Formula (16) indicates that GAAP earnings equal GAAP profits plus interest on GAAP profits and GAAP surplus. GAAP profits are a uniform percentage of premium income. The insurance surplus account is reduced each year by the excess of cash-flow earnings over GAAP earnings. The reduction is ${}_t\bar{E} - {}_t^G S$ from formula (17). Cash-flow earnings in each renewal year can be split into two pieces—GAAP earnings plus the amount by which the insurance surplus account is reduced (see formula [19]).

ACTUARIAL MODEL AND ASSUMPTIONS (ASSUMES

ZERO STATUTORY RESERVES)

The plan of insurance, the assumptions, and the method used are simple ones that were chosen to demonstrate IRRMA. Care should be taken to avoid generalizing after making any comparison of IRRMA's numerical results with those produced by statutory or GAAP accounting. Federal

income taxes have been excluded from the model. The allocation of expected taxes and tax credits by policy and between the insurance and noninsurance surplus accounts would have complicated the model unduly.

A four-year plan of insurance was chosen. Premiums are collected at the beginning of each year and amount to \$200,000, \$150,000, \$130,000, and \$120,000 in years 1, 2, 3, and 4, respectively. Cash flow excludes investment income and occurs at the beginning of each year. It is defined as premiums less expenses less benefits. Cash flow is —\$202,886, \$100,000, \$80,000, and \$70,000 in years 1, 2, 3, and 4, respectively. This produces an IRR of 12 per cent.

The interest assumption related to cash flow in the noninsurance surplus account is equal to the rate earned on invested surplus. The examples in Table 1 are based upon the assumption that noninsurance surplus is invested at 5 per cent. Table 2 is based upon a 12 per cent interest rate, the same rate as the IRR yield rate. Table 3 is based upon a 15 per cent interest rate. Statutory reserves are zero. Each table displays the emergence of expected earnings and surplus (*a*) if insurance was not written (versus writing insurance) and using (*b*) statutory, (*c*) IRRMA, and (*d*) GAAP accounting related to such investment of surplus.

The GAAP earnings displayed in Tables 1-3 are derived from the material contained in Tables 4 and 5.

The earnings each year in the noninsurance surplus account are identical under statutory, IRRMA, and GAAP accounting as they represent interest earnings on statutory surplus. They emerge at the *end* of each year. The earnings in the insurance surplus account are equal over the life of the policy using statutory, IRRMA, and GAAP accounting. However, the examples have statutory earnings occurring at the *beginning* of each year and IRRMA earnings at the *end* of each year. GAAP earnings consist of GAAP profits (which occur at the *beginning* of the year) and interest (which occurs at the *end* of the year) on GAAP profits and the GAAP insurance surplus account. Although the various assumptions that were made generated these timing differences, they were not considered to be material for purposes of illustrating IRRMA.

COMMENTS ON TABLES 1-5

The growth in surplus that emerges in Tables 1-3 is shown in the tabulation on page 623. If no insurance is written, the IRR is 5, 12, or 15 per cent—the assumed yield on surplus in Tables 1, 2, and 3, respectively. Statutory results are shown first.

Under IRRMA the aggregate yield on surplus is the weighted average of the 12 per cent IRR related to the surplus in the insurance surplus

POLICY YEAR	TOTAL SURPLUS AT END OF YEAR ÷ SURPLUS AT BEGINNING OF YEAR											
	If Insurance Is Not Written			Accounting Basis Used if Insurance Is Written								
				Statutory			IRRMA			GAAP		
	Table 1	Table 2	Table 3	Table 1	Table 2	Table 3	Table 1	Table 2	Table 3	Table 1	Table 2	Table 3
1.....	1.05	1.12	1.15	0	0	0	1.12	1.12	1.12	1.10	1.12	1.10
2.....	1.05	1.12	1.15	∞	∞	∞	1.09	1.12	1.13	1.08	1.12	1.15
3.....	1.05	1.12	1.15	1.85	1.92	1.95	1.07	1.12	1.14	1.08	1.12	1.15
4.....	1.05	1.12	1.15	1.43	1.48	1.51	1.05	1.12	1.15	1.07	1.12	1.15

account and the 5, 12, or 15 per cent assumed yield on the noninsurance surplus account.

Care should be taken in drawing any conclusions regarding the GAAP results, which are affected by the incidence of premium income and the interest rates assumed in the gross premium calculation. If cash flow remained the same, but a different pattern of premium income had been chosen for the examples, then statutory earnings and IRRMA earnings would not have changed, but there would have been a change in GAAP earnings from year to year.

As expected, total surplus is the same under statutory, IRRMA, and GAAP accounting at the end of the four years. It is interesting to note that if the assumed interest rate is equal to the IRR, then IRRMA and GAAP earnings are equal. Also, in this case GAAP profits, expressed as a percentage of premium income, are zero.

MODEL MODIFIED TO REFLECT ACTUAL EXPERIENCE

Total IRRMA surplus is equal to total statutory surplus plus the present value (discounted at the IRR) of future statutory book profits. The actual amount could be determined each year as follows: First, it would be necessary to develop a set of "surplus" factors per unit of in-force (e.g., face amount) for each plan, age, and duration. These factors would represent the present value (discounted at the appropriate IRR) of future statutory book profits. The actual face amounts in force at year end could be multiplied by these factors and the result added to total statutory surplus to arrive at total IRRMA surplus. The increase in total IRRMA surplus from year to year would be total IRRMA earnings.

For the same reason that actual and expected mortality, interest, lapse,

TABLE 1
SURPLUS AND EARNINGS EMERGENCE: ASSUMED INTEREST RATE = 5 PER CENT

	ALTER- NATE INVEST- MENT*	STATUTORY†			IRRMA‡			GAAP§		
		Insurance Surplus Account	Noninsur- ance Surplus Account	Total Surplus Account	Insurance Surplus Account	Noninsur- ance Surplus Account	Total Surplus Account	Insurance Surplus Account	Noninsur- ance Surplus Account	Total Surplus Account
Initial surplus at end of year 0.....	\$202,886	\$ 0	\$202,886	\$202,886	\$ 0	\$202,886	\$202,886	\$ 0	\$202,886	\$202,886
Surplus transfer at beginning of year 1.....	0	202,886	- 202,886	0	202,886	- 202,886	0	202,886	- 202,886	0
Surplus after transfer.....	202,886	202,886	0	202,886	202,886	0	202,886	202,886	0	202,886
Earnings in year 1.....	10,144	- 202,886	0	- 202,886	24,346	0	24,346	19,588	0	19,588
Surplus at end of year 1.....	213,030	0	0	0	227,232	0	227,232	222,474	0	222,474
Surplus transfer at beginning of year 2.....	0	- 100,000	100,000	0	- 100,000	100,000	0	- 100,000	100,000	0
Surplus after transfer.....	213,030	- 100,000	100,000	0	127,232	100,000	227,232	122,474	100,000	222,474
Earnings in year 2.....	10,652	100,000	5,000	105,000	15,268	5,000	20,268	13,208	5,000	18,208
Surplus at end of year 2.....	223,682	0	105,000	105,000	142,500	105,000	247,500	135,682	105,000	240,682
Surplus transfer at beginning of year 3.....	0	- 80,000	80,000	0	- 80,000	80,000	0	- 80,000	80,000	0
Surplus after transfer.....	223,682	- 80,000	185,000	105,000	62,500	185,000	247,500	55,682	185,000	240,682
Earnings in year 3.....	11,184	80,000	9,250	89,250	7,500	9,250	16,750	8,921	9,250	18,171
Surplus at end of year 3.....	234,866	0	194,250	194,250	70,000	194,250	264,250	64,603	194,250	258,853
Surplus transfer at beginning of year 4.....	0	- 70,000	70,000	0	- 70,000	70,000	0	- 70,000	70,000	0
Surplus after transfer.....	234,866	- 70,000	264,250	194,250	0	264,250	264,250	5,397	264,250	258,853
Earnings in year 4.....	11,743	70,000	13,213	83,213	0	13,213	13,213	5,397	13,213	18,610
Surplus at end of year 4.....	246,609	0	277,463	277,463	0	277,463	277,463	0	277,463	277,463

* Alternate investment earnings equal the interest at the assumed rate on noninsurance surplus at the beginning of each year.

† Statutory earnings equal earnings from the insurance operations (which have been defined as -\$202,886, \$100,000, \$80,000, and \$70,000) plus interest at the assumed rate on statutory surplus.

‡ IRRMA earnings equal earnings from the insurance operations (these are the IRR, or 12 per cent, on the outstanding insurance investment balance as of the beginning of the year) plus interest at the assumed rate on statutory surplus.

§ GAAP earnings equal earnings from the insurance operations (see Tables 4 and 5) plus interest at the assumed rate on statutory surplus.

TABLE 2
SURPLUS AND EARNINGS EMERGENCE: ASSUMED INTEREST RATE = 12 PER CENT

	ALTER- NATE INVEST- MENT*	STATUTORY†			IRRMA‡			GAAP§		
		Insurance Surplus Account	Noninsur- ance Surplus Account	Total Surplus Account	Insurance Surplus Account	Noninsur- ance Surplus Account	Total Surplus Account	Insurance Surplus Account	Noninsur- ance Surplus Account	Total Surplus Account
Initial surplus at end of year 0.....	\$202,886	\$ 0	\$202,886	\$202,886	\$ 0	\$202,886	\$202,886	\$ 0	\$202,886	\$202,886
Surplus transfer at beginning of year 1.....	0	202,886	- 202,886	0	202,886	- 202,886	0	202,886	- 202,886	0
Surplus after transfer.....	202,886	202,886	0	202,886	202,886	0	202,886	202,886	0	202,886
Earnings in year 1.....	24,346	- 202,886	0	- 202,886	24,346	0	24,346	24,346	0	24,346
Surplus at end of year 1.....	227,232	0	0	0	227,232	0	227,232	227,232	0	227,232
Surplus transfer at beginning of year 2.....	0	- 100,000	100,000	0	- 100,000	100,000	0	- 100,000	100,000	0
Surplus after transfer.....	227,232	- 100,000	100,000	0	127,232	100,000	227,232	127,232	100,000	227,232
Earnings in year 2.....	27,268	100,000	12,000	112,000	15,268	12,000	27,268	15,268	12,000	27,268
Surplus at end of year 2.....	254,500	0	112,000	112,000	142,500	112,000	254,500	142,500	112,000	254,500
Surplus transfer at beginning of year 3.....	0	- 80,000	80,000	0	- 80,000	80,000	0	- 80,000	80,000	0
Surplus after transfer.....	254,500	- 80,000	192,000	112,000	62,500	192,000	254,500	62,500	192,000	254,500
Earnings in year 3.....	30,540	80,000	23,040	103,040	7,500	23,040	30,540	7,500	23,040	30,540
Surplus at end of year 3.....	285,040	0	215,040	215,040	70,000	215,040	285,040	70,000	215,040	285,040
Surplus transfer at beginning of year 4.....	0	- 70,000	70,000	0	- 70,000	70,000	0	- 70,000	70,000	0
Surplus after transfer.....	285,040	- 70,000	285,040	215,040	0	285,040	285,040	0	285,040	285,040
Earnings in year 4.....	34,205	70,000	34,205	104,205	34,205	34,205	34,205	34,205	34,205	34,205
Surplus at end of year 4.....	319,245	0	319,245	319,245	0	319,245	319,245	0	319,245	319,245

* Alternate investment earnings equal the interest at the assumed rate on noninsurance surplus at the beginning of each year.

† Statutory earnings equal earnings from the insurance operations (which have been defined as -\$202,886, \$100,000, \$80,000, and \$70,000) plus interest at the assumed rate on statutory surplus.

‡ IRRMA earnings equal earnings from the insurance operations (these are the IRR, or 12 per cent, on the outstanding insurance investment balance as of the beginning of the year) plus interest at the assumed rate on statutory surplus.

§ GAAP earnings equal earnings from the insurance operations (see Tables 4 and 5) plus interest at the assumed rate on statutory surplus.

TABLE 3

SURPLUS AND EARNINGS EMERGENCE: ASSUMED INTEREST RATE = 15 PER CENT

	ALTER- NATE INVEST- MENT*	STATUTORY †			IRRMA ‡			GAAP §		
		Insurance Surplus Account	Noninsur- ance Surplus Account	Total Surplus Account	Insurance Surplus Account	Noninsur- ance Surplus Account	Total Surplus Account	Insurance Surplus Account	Noninsur- ance Surplus Account	Total Surplus Account
Initial surplus at end of year 0	\$202,886	\$ 0	\$202,886	\$202,886	\$ 0	\$202,886	\$202,886	\$ 0	\$202,886	\$202,886
Surplus transfer at beginning of year 1	0	202,886	- 202,886	0	202,886	- 202,886	0	202,886	- 202,886	0
Surplus after transfer	202,886	202,886	0	202,886	202,886	0	202,886	202,886	0	202,886
Earnings in year 1	30,433	- 202,886	0	- 202,886	24,346	0	24,346	19,609	0	19,609
Surplus at end of year 1	233,319	0	0	0	227,232	0	227,232	222,495	0	222,495
Surplus transfer at beginning of year 2	0	- 100,000	100,000	0	- 100,000	100,000	0	- 100,000	100,000	0
Surplus after transfer	233,319	- 100,000	100,000	0	127,232	100,000	227,232	122,495	100,000	222,495
Earnings in year 2	34,998	100,000	15,000	115,000	15,268	15,000	30,268	18,374	15,000	33,374
Surplus at end of year 2	268,317	0	115,000	115,000	142,500	115,000	257,500	140,869	115,000	255,869
Surplus transfer at beginning of year 3	0	- 80,000	80,000	0	- 80,000	80,000	0	- 80,000	80,000	0
Surplus after transfer	268,317	- 80,000	195,000	115,000	62,500	195,000	257,500	60,869	195,000	255,869
Earnings in year 3	40,248	80,000	29,250	109,250	7,500	29,250	36,750	9,131	29,250	38,381
Surplus at end of year 3	308,565	0	224,250	224,250	70,000	224,250	294,250	70,000	224,250	294,250
Surplus transfer at beginning of year 4	0	- 70,000	70,000	0	- 70,000	70,000	0	- 70,000	70,000	0
Surplus after transfer	308,565	- 70,000	294,250	224,250	0	294,250	294,250	0	294,250	294,250
Earnings in year 4	46,285	70,000	44,138	114,138	0	44,138	44,138	0	44,138	44,138
Surplus at end of year 4	354,850	0	338,388	338,388	0	338,388	338,388	0	338,388	338,388

* Alternate investment earnings equal the interest at the assumed rate on noninsurance surplus at the beginning of each year.

† Statutory earnings equal earnings from the insurance operations (which have been defined as ~\$202,886, \$100,000, \$80,000, and \$70,000) plus interest at the assumed rate on statutory surplus.

‡ IRRMA earnings equal earnings from the insurance operations (these are the IRR, or 12 per cent, on the outstanding insurance investment balance as of the beginning of the year) plus interest at the assumed rate on statutory surplus.

§ GAAP earnings equal earnings from the insurance operations (see Tables 4 and 5) plus interest at the assumed rate on statutory surplus.

TABLE 4
GAAP PROFITS THAT ARE A UNIFORM PERCENTAGE OF PREMIUM INCOME

YEAR	PREMIUM INCOME	CASH FLOW = PREMIUM MINUS EXPENSES AND BENEFITS	PROFITS THAT ARE A UNIFORM PERCENTAGE OF PREMIUM			
			0%*	5%*	12%*	15%*
1.....	\$200,000	-\$202,886	\$15,705	\$ 8,994	\$0	-\$ 3,708
2.....	150,000	100,000	11,778	6,746	0	- 2,781
3.....	130,000	80,000	10,208	5,846	0	- 2,410
4.....	120,000	70,000	9,423	5,397	0	- 2,225
Total..	\$600,000	\$ 47,114	\$47,114	\$26,983	\$0	-\$11,124

* Assumed interest rate.

Assumed Interest Rate	Present Value of Premium Income	Present Value of Cash Flow	Profits as a % of Premium
0%.....	\$600,000	\$47,114	7.85233%
5%.....	564,431.5	25,383.09	4.49711
12%.....	522,977.4	0	0
15%.....	507,635.4	- 9,411.85	-1.85406

TABLE 5
EARNINGS FROM THE INSURANCE SURPLUS ACCOUNT

YEAR	ASSUMED INTEREST RATE = 5%			ASSUMED INTEREST RATE = 12%			ASSUMED INTEREST RATE = 15%		
	Statutory	IRRMA	GAAP	Statutory	IRRMA	GAAP	Statutory	IRRMA	GAAP
1....	-\$202,886	\$24,346	\$19,588	-\$202,886	\$24,346	\$24,346	-\$202,886	\$24,346	\$26,169
2....	100,000	15,268	13,208	100,000	15,268	15,268	100,000	15,268	16,160
3....	80,000	7,500	8,921	80,000	7,500	7,500	80,000	7,500	7,010
4....	70,000	0	5,397	70,000	0	0	70,000	0	- 2,225
Total	\$ 47,114	\$47,114	\$47,114	\$ 47,114	\$47,114	\$47,114	\$ 47,114	\$47,114	\$47,114

NOTE.—The \$47,114 statutory earnings of the insurance surplus account have the following meaning:

- a) Using IRRMA, annual earnings are 12 per cent of the IRRMA insurance surplus account.
- b) Using GAAP, annual earnings are a uniform percentage of premium, plus investment income (using the assumed interest rate) on both the uniform percentage of premium and the GAAP insurance surplus account. (For example, in the first year, using the 5 per cent assumed interest rate, GAAP earnings are \$19,588. This is \$8,994 [see Table 4] + 5 per cent of (\$8,994 + \$202,886) = \$8,994 + \$10,594 = \$19,588.) Refer to formula (16).

Since GAAP would not permit negative profits to emerge each year, the results using 15 per cent should be modified to produce a negative profit of \$9,412 in the first year and 0 per cent of premiums in other years. The GAAP earnings that would result are \$19,609 + \$18,374 + \$9,131 + \$0 = \$47,114, where \$19,609 = -\$9,412 + 15 per cent of (\$202,886 - \$9,412).

and expense rates are compared, the actual and expected IRR might also be developed. Table 6 provides two examples of actual experience differing from the expected experience of Table 1. In both examples, actual experience differed from that which was expected in the second policy year (see the accompanying tabulation).

	TOTAL IRRMA SURPLUS AT END OF YEAR ÷ IRRMA SURPLUS AT BEGINNING OF YEAR			
	Example A		Example B	
	Actual	Expected	Actual	Expected
Policy year 2	1.04	1.09	0.87	1.09

The difference between *actual* total IRRMA earnings and *expected* total IRRMA earnings in any year would be the gain or loss for the year. Let primed symbols denote actual experience. Formulas relating to gains and losses in the IRRMA insurance surplus account in year *t* are as follows:

$${}^{\text{IRR}}_t E = {}^{\text{IRR}}_t S - {}^{\text{IRR}}_{t-1} S + {}_t E, \quad \text{from formula (6)}; \tag{20}$$

$${}^{\text{IRR}}_t E' = {}^{\text{IRR}}_t S' - {}^{\text{IRR}}_{t-1} S' + {}_t E', \tag{21}$$

since ${}^{\text{IRR}}_{t-1} S = {}^{\text{IRR}}_{t-1} S'$ at the beginning of the year;

$$\text{Gain} = {}^{\text{IRR}}_t E' - {}^{\text{IRR}}_t E = ({}_t E' - {}_t E) + ({}^{\text{IRR}}_t S' - {}^{\text{IRR}}_t S). \tag{22}$$

Formula (22) indicates that the gain in the IRRMA insurance surplus account equals the gain from statutory earnings of the current year plus the present value of the expected gain in future statutory earnings.

$${}^{\text{IRR}}_t E' = {}^{\text{IRR}}_t E + ({}_t E' - {}_t E) + ({}^{\text{IRR}}_t S' - {}^{\text{IRR}}_t S), \tag{23}$$

from formulas (20) and (21);

$${}^{\text{IRR}}_t E' = (\text{IRR})({}^{\text{IRR}}_{t-1} S - {}_t E) + ({}_t E' - {}_t E) + ({}^{\text{IRR}}_t S' - {}^{\text{IRR}}_t S); \tag{24}$$

$$\begin{aligned} {}^{\text{IRR}}_t E' &= (\text{IRR})({}^{\text{IRR}}_{t-1} S - {}_t E') \\ &+ (1 + \text{IRR})({}_t E' - {}_t E) + ({}^{\text{IRR}}_t S' - {}^{\text{IRR}}_t S). \end{aligned} \tag{25}$$

TABLE 6

TABLE 1 MODIFIED BECAUSE OF ACTUAL EXPERIENCE DIFFERING FROM EXPECTED EXPERIENCE

	EXAMPLE A						EXAMPLE B					
	Statutory			IRRMA			Statutory			IRRMA		
	Insurance Surplus Account	Non-insurance Surplus Account	Total Surplus Account	Insurance Surplus Account	Non-insurance Surplus Account	Total Surplus Account	Insurance Surplus Account	Non-insurance Surplus Account	Total Surplus Account	Insurance Surplus Account	Non-insurance Surplus Account	Total Surplus Account
Surplus at end of year 1	\$ 0	\$ 0	\$ 0	\$227,232	\$ 0	\$227,232	\$ 0	\$ 0	\$ 0	\$227,232	\$ 0	\$227,232
Surplus transfer at beginning of year 2	- 90,000	90,000	0	- 90,000	90,000	0	- 80,000	80,000	0	- 80,000	80,000	0
Surplus after transfer	- 90,000	90,000	0	137,252	90,000	227,232	- 80,000	80,000	0	147,232	80,000	227,232
Earnings in year 2	90,000	4,500	94,500	5,268	4,500	9,768	80,000	4,000	84,000	33,232	4,000	29,232
Surplus at end of year 2	0	94,500	94,500	142,500	94,500	237,000	0	84,000	84,000	114,000	84,000	198,000
Surplus transfer at beginning of year 3	- 80,000	80,000	0	- 80,000	80,000	0	- 64,000	64,000	0	- 64,000	64,000	0
Surplus after transfer	- 80,000	174,500	94,500	62,500	174,500	237,000	- 64,000	148,000	84,000	50,000	148,000	198,000
Earnings in year 3	80,000	8,725	88,725	7,500	8,725	16,225	64,000	7,400	71,400	6,000	7,400	13,400
Surplus at end of year 3	0	183,225	183,225	70,000	183,225	253,225	0	155,400	155,400	56,000	155,400	211,400
Surplus transfer at beginning of year 4	- 70,000	70,000	0	- 70,000	70,000	0	- 56,000	56,000	0	- 56,000	56,000	0
Surplus after transfer	- 70,000	253,225	183,225	0	253,225	253,225	- 56,000	211,400	155,400	0	211,400	211,400
Earnings in year 4	70,000	12,661	82,661	0	12,661	12,661	56,000	10,570	66,570	0	10,570	10,570
Surplus at end of year 4	0	265,886	265,886	0	265,886	265,886	0	221,970	221,970	0	221,970	221,970

NOTE.—Noninsurance surplus account actually earns 5 per cent, which is the expected amount.

Example A: In this example, persistency was exactly as expected in all years, but statutory earnings were lower than expected, because of expense overruns of \$10,000 in the second year only. Thus, actual statutory earnings were \$90,000, \$80,000, and \$70,000 in years 2, 3, and 4 instead of \$100,000, \$80,000, and \$70,000. The reduction of earnings in the IRRMA insurance surplus account in the second year equals the reduction in statutory earnings of \$10,000, plus the present value of the expected reduction in future statutory earnings. As there is no expected reduction in future statutory earnings, the reduction of earnings in the IRRMA insurance surplus account is \$10,000. The reduction of earnings in the noninsurance surplus account is 5 per cent of the \$10,000 reduction in statutory earnings, or \$500. The total reduction in earnings in the second year is \$10,500 (i.e., the difference between actual earnings of \$9,768 and expected earnings of \$20,268 shown in Table 1).

Example B: In this example, only 80 per cent of the policies that were expected to pay premiums at the beginning of the second policy year did so. Future persistency was as expected. Thus, actual statutory earnings were \$80,000, \$64,000, and \$56,000 instead of \$100,000, \$80,000, and \$70,000. The reduction of earnings in the IRRMA insurance account in the second year equals the reduction in statutory earnings of \$20,000, plus the present value of the expected reduction in future statutory earnings (i.e., \$16,000 + \$14,000 = \$30,000). The reduction of earnings in the IRRMA insurance surplus account is therefore \$48,500. The reduction of earnings in the noninsurance surplus account is 5 per cent of the \$20,000 reduction in statutory earnings, or \$1,000. The total reduction in earnings in the second year is \$49,500 (i.e., the difference between actual earnings of -\$29,232 and expected earnings of \$20,268 shown in Table 1).

Formulas (24) and (25), in addition to formula (21), represent different ways of describing actual IRRMA earnings in the insurance surplus account. The gain in the noninsurance surplus account is the difference between actual and expected interest earnings on statutory surplus.

II

RESERVE BASIS

What if a company established a profit objective expressed as a level percentage of each premium (e.g., 10 per cent)? An important limitation associated with such an objective was stated by Jim Anderson on page 363 of *TSA*, Volume XI: "the surplus depletion incurred at issue is recovered, together with interest at the assumed rate, and profit in excess of the interest on surplus depletion is realized, but the additional profit is not directly associated with the amount of surplus expended to produce it." He went on to state on page 374: "Unless the yield rate on invested surplus is exactly equal to the assumed interest rate, the reserve basis will have a significant effect on the value of realized profit." This is very important.

A company could ask its actuary to calculate a set of gross premiums with 10 per cent of premium as its profit objective. Then, after some policies had been issued, the company could decide to reserve on either the net level reserve basis or a modified basis such as the Commissioners Reserve Valuation Method (CRVM). This decision would not alter the statement that its profit margin is 10 per cent of premium. Yet, if an IRR were determined for the statutory book profits that are expected to emerge under each reserve basis, the actuary might note that the IRR was 15 per cent if net level premium reserves were used and 17 per cent if CRVM reserves were used. If the company had a policy of investing its statutory reserves at one rate of interest and its statutory surplus at another (e.g., in new business), we would expect that the total surplus of the company would not be at the same level after these policies had matured, if its reserve basis had been CRVM instead of net level. Thus the statutory reserve basis can affect earnings and surplus.

In determining the IRR for a particular plan of insurance, we might also include as a reserve the amount of statutory surplus that is expected to be maintained in addition to the required statutory reserve on each policy. (For example, this might be 3 per cent of statutory policy reserves on individual life and 30 per cent of premium on group health.)

For a given set of gross premiums, the IRR will generally be lowered by using statutory policy reserves (including deficiency reserves) plus an

TABLE 7

ADJUSTMENTS TO TABLES 1-3 BECAUSE OF ADDITIONAL SURPLUS FOR REQUIRED RESERVES

	ASSUMED INTEREST RATE = 5%				ASSUMED INTEREST RATE = 12%				ASSUMED INTEREST RATE = 15%			
	Alternate Investment	Statutory	IRRMA	GAAP	Alternate Investment	Statutory	IRRMA	GAAP	Alternate Investment	Statutory	IRRMA	GAAP
Additional earnings:†												
Year 1.....	\$ 5,000	*	†	\$ 5,000	\$ 12,000	*	\$ 12,000	\$ 12,000	\$ 15,000	*	†	\$ 15,000
Year 2.....	5,250	*	†	5,250	13,440	*	13,440	13,440	17,250	*	†	17,250
Year 3.....	5,513	*	†	5,513	15,053	*	15,053	15,053	19,838	*	†	19,838
Year 4.....	5,788	*	†	5,788	16,859	*	16,859	16,859	22,813	*	†	22,813
	\$ 21,551	\$ 21,551	\$ 21,551	\$ 21,551	\$ 57,352	\$ 57,352	\$ 57,352	\$ 57,352	\$ 74,901	\$ 74,901	\$ 74,901	\$ 74,901
Surplus at the end of the fourth year:												
Surplus from the \$202,886 investment.	\$246,609	\$277,463	\$277,463	\$277,463	\$319,245	\$319,245	\$319,245	\$319,245	\$354,850	\$338,388	\$338,388	\$338,388
Surplus from the \$100,000 investment	121,551	121,551	121,551	121,551	157,352	157,352	157,352	157,352	174,901	174,901	174,901	174,901
Total surplus from the \$302,886 investment.....	\$368,160	\$399,014	\$399,014	\$399,014	\$476,597	\$476,597	\$476,597	\$476,597	\$529,751	\$513,289	\$513,289	\$513,289

* Cannot be determined until the pattern of the change in reserve from year to year is defined.

† Cannot be determined until the pattern of the change in reserve from year to year is defined. The pattern will affect the IRR.

‡ Earnings from additional \$100,000 of initial surplus required for statutory reserves.

earmarked portion of statutory surplus, unless statutory surplus is invested at a rate equal to or greater than the IRR.

MODEL MODIFIED TO INCLUDE STATUTORY RESERVES

Let us revise the results shown in Tables 1-3 by assuming that an initial statutory reserve of \$100,000 was required. This means that the company needed \$302,886 of surplus, not \$202,886, in order to write the business. The incidence of earnings emerging from this additional \$100,000 is shown in Table 7.

Total expected earnings are increased by the interest earned on this \$100,000. Although the statutory reserve starts at \$100,000 and ends at \$0 over the four-year period, the pattern of the change in reserve from year to year has no effect on alternate investment (since no insurance reserves are needed) or GAAP (which attempts to ignore statutory reserves). The additional interest earned on the statutory reserve is included in GAAP earnings.

The effect on statutory earnings year by year will equal the additional interest earnings pattern displayed under alternate investment, reduced by the change in reserve from year to year.

Under IRRMA, the pattern of the changes in reserve from year to year is needed so that the IRR can be calculated. By general reasoning, the IRR that was 12 per cent using cash-flow book profits will not change if the assumed interest rate is also 12 per cent. However, we need to know what the reserves are from year to year in those cases where the reserves are assumed to earn more or less than the IRR. This is required to calculate statutory book profits to determine the IRR. Thus the reserve level will affect the pattern of emergence of IRRMA earnings. Also, the ultimate level of surplus will be affected if different rates of interest are earned on statutory surplus and statutory reserves.

DISCUSSION OF PRECEDING PAPER

RICHARD S. ROBERTSON:

Mr. Sondergeld has developed a very ingenious accounting method. At the risk of straying somewhat from the subject matter of this fine paper, I am using the opportunity of its presentation to express some concerns that have been bothering me for some time. Stated briefly, it is my opinion that the internal rate of return (IRR) profitability measure is badly overused and overvalued by many actuaries. I find it shocking that some apparently are using this measure as a profit objective. This is not to say that the IRR does not have its uses. And it certainly does no harm to calculate the IRR as one of several profit measures, provided that its significance is not overstated.

The major problem is that the IRR is very sensitive to subjective changes in assumptions or procedures that really do not affect profitability. First, consider allocation of overhead expenses. In most life insurance companies there is a very large volume of operating expense that is not related directly to either new business or in-force business. These "overhead expenses" could just as logically be allocated in different ways between first-year and renewal and would produce significantly different unit expenses. The traditional measures of profitability are not greatly affected by the practices used to allocate these overhead expenses as long as they are allocated in some reasonable manner. The incidence of profits is affected, of course. In practice, the bulk of these expenses probably are allocated to the first year because such allocation might be thought somewhat more conservative.

However, the IRR is affected very substantially by the extent to which overhead expenses are charged to the first year. This is because the expenses charged to the first year are the base against which future profits are offset. To illustrate the sensitivity of the IRR to expense allocation procedures, I prepared profit tests for an ordinary life policy issued at age 40 with a gross premium of \$20. According to the assumptions I used, the expected profits per year were 4.9 per cent of premium and the IRR was 16.26 per cent. It has been my experience that as much as half the expense allocated to the first year might, in some cases, be considered in the "overhead" category. Accordingly, I reduced the amount of first-year home office expense by half and increased renewal expenses by an amount sufficient to keep the expected profit unchanged. However, the IRR increased to 22.52 per cent. This, of course, represents an extreme reallocation of expenses, but it does illustrate the point I am trying to make.

I am also concerned about the effect of the choice of reserving methods on the IRR. If net level reserves are used rather than the Commissioners Reserve Valuation Method (CRVM) for my illustrative profit tests, the IRR is reduced to 10.15 per cent. The expected profit, of course, remains unchanged. Mr. Sondergeld quotes Mr. Anderson to the effect that the choice of reserve basis will influence the profitability of a plan if the assets representing surplus and reserves are invested differently. While this is true, it does not seem reasonable that the difference should be as large as suggested by the IRR. Actually, a more likely premise than different yields on reserves and surplus is the effect of federal income taxes. Taxes are influenced by the reserve choice, so that in this respect profitability is affected by choice of reserves. Again, I think the difference is not nearly as large as suggested by the IRR. I am not sure that the change in profitability resulting from changing from CRVM to net level is even in the same direction as suggested by the IRR. For a company using generally accepted accounting principles (GAAP) accounting, there is even less reason to suggest that the profitability of a block of business should depend on the statutory reserve basis.

Changes in other assumptions can affect the IRR as well. For example, an increase of 0.1 per cent in the assumed interest rate increased the expected profit from 4.9 per cent of premium to 5.5 per cent, and it increased the IRR from 16.26 to 17.09 per cent. While the relationship between the effects on these two measures of profit seems reasonable, I think the degree of leverage the interest assumption has on the IRR is disturbing.

In summary, Mr. Sondergeld's point that it is inconsistent to use the IRR to define profits and GAAP accounting to report earnings is well taken. In my opinion, the fault lies with the IRR—not with the accounting system.

(AUTHOR'S REVIEW OF DISCUSSION)

DONALD R. SONDERGELD:

Let me thank Dick Robertson for his comments, which seem to be directed primarily at the title of my paper. We agree that it is helpful to use more than one measuring rod for describing expected profits. However, Dick questions the value of the IRR as a profit objective and states that it is sensitive to changes in assumptions that really do not affect profitability.

Mr. Robertson gave an example in which certain overhead expenses were reallocated from first to renewal policy years in such a way that expected profit remained unchanged. This reallocation apparently was

accomplished by using his actuarial assumptions regarding interest rates. However, if this reallocation had been made using a 16.26 per cent interest rate, the IRR would have remained at 16.26 per cent rather than increasing to 22.52 per cent, and profits that were expressed as 4.9 per cent of each premium would become a smaller number.

Merely the fact that some expenses are overhead expenses does not mean that reasonable assumptions cannot be made as to their incidence. Assumptions must be made irrespective of the method of measuring profit. These assumptions in turn may affect whichever profit measuring rod is used.

It is desirable to use a number of different assumptions in profit testing in order to learn which assumptions are the key ones and to see how sensitive expected profitability is to certain changes in assumptions. In my opinion, a profit measuring rod should not be chosen on the basis of its being insensitive to changes in assumptions or on the basis of its being traditional.

In his illustration, Mr. Robertson indicated further that, if net level premium reserves are used rather than CRVM, the IRR drops from 16.26 to 10.15 per cent. He agrees that profits are affected if assets representing surplus and reserves are invested differently. However, there is the implication in his comments that the large difference generated in his illustration might be reduced if federal income taxes were taken into account. That effect can be directly determined by calculating the IRR on an after-tax basis.

Similarly, Mr. Robertson could have developed an after-tax per cent of premium index. However, the actual reserve or surplus level is not reflected fully in the per cent of premium index unless the asset share modifications, described by Richard W. Ziock,¹ are used.

Mr. Ziock suggests working with two interest rates. One is j_t , the interest rate charged to the life insurance company on funds loaned by the holding company in year t , to restore surplus to zero. A loan is made whenever the reserve is larger than the asset share. It is equal to the algebraic excess of the reserve over the asset share. The other is i_t , the interest rate earned in year t within the life insurance company. Then, for all t where ${}_{t-1}AS' \leq {}_{t-1}V$, ${}_{t-1}AS'$ is equal to the traditional asset share formula (accumulated at rate i_t in year t) minus $({}_{t-1}V - {}_{t-1}AS')$ ($j_t - i_t$), and, for all t where ${}_{t-1}AS' > {}_{t-1}V$, it is equal to the traditional asset share formula (accumulated at rate i_t in year t).

It is important to note that, irrespective of whether we are dealing with

¹ Richard W. Ziock, "A Realistic Profit Model for Individual Non-Participating Life Insurance," *Journal of Risk and Insurance*, September, 1973.

a holding company complex or a single corporate entity, measuring the return on investment is important in analyzing best uses of capital. In this regard, calculating an IRR is a useful common denominator that can be developed to measure profitability of different insurance products and different lines of business.

Let us turn to Table 4 of my paper, which can also be used to illustrate that profits can be expressed a number of ways. In the examples in my paper the IRR was used to define profit as a return on investment plus nothing else. If the underlying "asset share" interest rate were 5 per cent, then Dick Robertson would probably look at Table 4 and define profits as 4.5 per cent of each premium. What he, in effect, would be saying is that profits represent a 5 per cent return on investment, plus 4.5 per cent of each premium. I suspect that James C. H. Anderson would measure profits by first fixing the desired rate of return and then calculating profits as a percentage of each premium (or of one premium). In this case, if the desired rate of return is 15 per cent, he might examine Table 4 and say that profits represent a 15 per cent return on investment, less 1.85 per cent of each premium. I would prefer to define profits as a 12 per cent return on investment, plus 0 per cent of premium. The important point is that the level of profits is dependent upon the underlying assumptions as to return on investment. You can't have one without the other.