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GAAP ACQUISITION EXPENSE AMORTIZATION METHODOLOGY

JOE B. PHARR

ABSTRACT

Two methods seem to be rather popular for use in the deferral of life insurance company GAAP acquisition expenses. The most popular is apparently the "standard unamortized cost factor" or "expense reserve factor" method; in this paper it is described as being dynamic and actuarially oriented. Another method that enjoys popularity is the "prescheduled amortization" method, which is sometimes referred to as an accountant's worksheet approach; in this paper it is referred to as being static. The AICPA audit guide contains an admonishment that "if actual experience differs significantly from that assumed, the factors should be recomputed." The author attempts to show through selected (but hopefully not biased) numerical demonstrations the reason for such a warning about deviations of actual persistency experience from actuarial expectations. However, cautions about such deviations need not be confined solely to adverse experience, since better-than-expected experience produces results that likely would be considered unsatisfactory. A modification of the two amortization approaches currently being used is developed through an aggregate revenue approach that seems to produce superior results based on selected demonstrations that are outlined in the body of the paper and in an appendix.

I. INTRODUCTION

THE purposes of this paper are (1) to demonstrate the relative effects of common GAAP deferred acquisition expense amortization techniques under selected conditions and (2) to present a variation of current amortization techniques that is unique in many respects and that appears to produce superior results.

Three amortization methods are considered: (1) a *static* approach where the prescheduled amortization process and inherent results do not vary from original assumptions and intentions, (2) a *dynamic* technique where amortization is a function of both original assumptions and actual experience based on inventories of premium revenue in force, and (3) a

third technique that is described herein as an *aggregate (or accumulated) premium revenue* method and that is suggested as an alternative to the two methods above that are in popular use. Both the dynamic and aggregate premium methods depend on original assumptions that remain unchanged, since factors are applied to measures of inventory that often vary, perhaps significantly, from original expectations. As a measure of the relative effectiveness of the three methods, results are compared against those that would have been obtained if the developer of the amortization technique had been precognitive with respect to actual persistency and the estimated incidences of collection of premium revenue following original issue.

The author has concluded that neither the static (prescheduled) amortization process usually implemented through an accountant's worksheet nor the dynamic, actuarially oriented "expense reserve factor" approach produces a reasonable pattern of amortization of deferred GAAP acquisition expenses "if actual experience differs significantly from that assumed."

II. AICPA INDUSTRY AUDIT GUIDE REFERENCES AND COMMENTARY

The AICPA industry audit guide *Audits of Stock Life Insurance Companies* reviews the amortization of deferred acquisition expenses in the section "Recognition of Costs" (pp. 71–74). Specific references to methodology, however, seem confined primarily to Appendix B (pp. 139–46). The following extracts from the audit guide are not intended as an exhaustive, researched listing on the subject, but are pertinent points regarding the methodology of acquisition expense amortization:

1. "Acquisition expenses should be deferred and charged against income in proportion to premium revenues recognized" (p. 72).
2. "Actual acquisition expenses, as distinguished from those assumed, should be used in the calculations as long as it can be shown that the gross premiums charged are sufficient to cover the actual expense" (p. 73).
3. "The deferral and amortization of acquisition costs represents a significant change in accounting practices for life insurance companies. Such deferral and amortization will generally represent a substantial portion of the difference between stockholders' equity and net income presented in conformity with generally accepted accounting principles" (p. 74).
4. "The magnitude of deferred acquisition costs and their effect on reported earnings will be of significant interest to the users of life insurance company financial statements. . . . [T]he Committee has concluded that, because of the magnitude of such amounts, complete disclosure requires their separate presentation and that, because of their nature, fair presentation requires classification of unamortized acquisition costs as a deferred charge" (p. 74).

The following points from the above quotations are to be emphasized: acquisition expenses should be deferred and charged in proportion to premium revenues; actual acquisition expenses are preferable to actuarial estimates of acquisition expenses; and unamortized deferred acquisition expenses should be disclosed separately in financial statements.

Appendix B ("Accounting for Unamortized Acquisition Costs") of the audit guide contains discussions of points regarding the methodology to be used in amortizing deferrable acquisition costs. Statements that are deemed pertinent to the purposes of this paper are as follows:

1. Prescheduled amortization worksheet (static technique): "A more refined approach would involve the use of an amortization schedule. . . . Amortization would be prescheduled to coincide with the expected premium revenue. . . . The method could be modified so that annual or periodic adjustments could be made to give effect to actual terminations."
2. Standard unamortized cost factor or expense reserve factor (dynamic technique): "Another method approximates the technique used by actuaries in the determination of reserve valuation factors. . . . This method uses a 'standard unamortized cost factor' or 'expense reserve factor' which is applied to the insurance in force at the end of each period. . . . [T]he method tends to provide some degree of self-correction in that it causes the rate of amortization to increase or decrease as actual persistency is lower or higher than initially estimated."
3. "If actual experience differs significantly from that assumed, the factors should be recomputed." (It is presumed by the author that, if a worksheet approach were translated to be a function of revenue or amount of insurance in force, it also would be appropriate to recompute such a dynamic worksheet.)

Two general methods are therefore permissible—an amortization schedule (prescheduled worksheet) and the expense reserve factor (or standard unamortized cost factor) method. Although it is stated that the actuarially oriented expense reserve factor approach tends to provide some degree of self-correction, the factors (and it is presumed the amortization schedules) should be recomputed if actual experience differs significantly from that assumed.

An issue considered in this paper is whether the actuarial factors (applied to the appropriate insurance in force inventories for the accounting periods) really provide "some degree of self-correction" or produce results that are any better (any closer to "actual") than results obtained by the amortization schedule/worksheet approach. Is there yet a better method that would avoid any need to recompute either factors or schedules "if actual experience differs significantly from that assumed"?

III. THE UNDERLYING PRINCIPLES

The audit guide indicates that acquisition expenses should be deferred and charged against income in proportion to premium revenues. In other words, acquisition expenses should be matched ratably against the revenue produced by these expenses. This seems to be the essential—even the critical—principle against which methods of acquisition expense amortization should be judged realistic and reasonable.

The dynamic approach to acquisition expense amortization is supported by many actuaries because of "release from risk" considerations. It is argued that use of the dynamic approach on both benefit and expense sides of the balance sheet results in offsets. Where the dynamic method overcorrects one way with respect to expenses (assets), there is a tendency to overcorrect in the opposite direction with respect to benefits (liabilities).

There would seem to be sufficient need for actuaries to consider the methodology of acquisition expense amortization without necessarily insisting upon an analysis of any possible offsets on the benefit (liability) side of the balance sheet. In other words, any conclusions or observations concerning the amortization of deferred acquisition expenses that are developed from the demonstrations outlined in this paper should merit consideration by actuaries without alluding necessarily to suspected or implied offsets on the benefit cost (liability) side of GAAP financial statements.

Some thoughts that seem to support this perspective include the following:

1. One of the basic accounting principles in the audit guide concerns the matching of acquisition expenses ratably with premium revenue.
2. GAAP acquisition expense amortization charges affect directly commission and general insurance figures when shown as separate items in GAAP financial statements.
3. The uniquely accounting-oriented concept relating to the definition of acquisition expenses controls the amount of acquisition expenses deferred.
4. It seems preferable not to offset an overstatement or understatement of an asset (unamortized acquisition expenses) with an overstatement or understatement of a liability (GAAP benefit reserves).
5. For some lines of business such as term insurance and variable annuities, relatively small benefit adjustments are required, so this suggested simplification of viewpoint does not detract from the reality of the situation.

Hence, a critical question concerns the standard against which the results of any GAAP acquisition expense amortization method should be measured. What is the intended result of any GAAP acquisition expense

amortization technique? A hypothesis of this paper is that an appropriate standard of measurement is the pattern of GAAP acquisition expenses that would have been observed if the developer of the method had been possessed of precognitive powers.

IV. EXPECTED UNAMORTIZED ACQUISITION EXPENSES

Actuarial expectations of unamortized GAAP acquisition expenses are developed in Table 1. Expected persistency factors in column 1 are used to develop a projected premium revenue stream starting with $10,000 (col. 2). (The twenty-year cutoff period reflected in column 2 likely would be deemed too conservative in actual practice but is used here to simplify the demonstrations.) Acquisition expense amounts are shown in columns 3 and 4 on an incurred and a GAAP basis, respectively. The GAAP basis charges acquisition expenses in proportion to the expected premium

TABLE 1

PERSISTENCY ASSUMPTIONS AND EXPECTED ACQUISITION EXPENSES

Year	Expected Persistency Factor	Projected Premium Revenue	Expected Acquisition Expenses		
			Incurred in Year	Charged in Proportion to Revenue (GAAP)	Unamortized Deferred
	(1)	(2)	(3)	(4)	(5)
1	.800	$ 10,000	$ 9,500	$ 1,075	$8,425
2	.880	8,000	1,000	860	8,565
3	.925	7,040	500	757	8,308
4	.935	6,512	0	700	7,608
5	.940	6,089	0	655	6,953
6	.945	5,723	0	615	6,338
7	.950	5,409	0	582	5,756
8	.955	5,138	0	552	5,204
9	.960	4,907	0	528	4,676
10	.965	4,711	0	507	4,169
11	.970	4,546	0	489	3,680
12	.970	4,409	0	474	3,206
13	.970	4,277	0	460	2,746
14	.965	4,149	0	446	2,300
15	.960	4,004	0	431	1,869
16	.955	3,843	0	413	1,456
17	.950	3,671	0	395	1,061
18	.945	3,487	0	375	686
19	.940	3,295	0	354	332
20	N/A	3,098	0	332	0
1–20	.....	$102,308	$11,000	$11,000	.....

NOTE.—N/A = Not applicable. Column 4 = Col. 2 × ($11,000 ÷ $102,308) = Col. 2 × 0.107518. Column 5 = Summation of col. 3 minus summation of col. 4 through a given year. Column 4 = Col. 3 minus col. 5 for current year plus col. 5 for prior year.

revenue of column 2. The amounts in column 4 are calculated for each year by multiplying the premium revenue for the year from column 2 by the ratio of (*a*) the sum of the incurred acquisition expenses of column 3 to (*b*) the sum of the premium revenue of column 2. The unamortized acquisition expenses (asset figures) shown in column 5 are developed by accumulating the differences between the figures in columns 3 and 4. The acquisition expense figures in column 4 are the type of expenses shown in GAAP financial statements and may be developed by taking the column 3 incurred acquisition expenses minus the year-to-year changes in the acquisition expense asset figures of column 5; this serves as a check of the figures in column 5. Notes on the arithmetic computations are shown at the foot of the table.

Interest (investment yield) has been left out of the calculations solely for reasons of simplicity in developing supportive illustrations. The author supports the audit guide admonishment in Appendix B that "the rate of amortization should give effect not only to estimated persistency, but to the interest assumed." A twenty-year amortization period has been used in this paper, since it provides a reasonable balance between reality and a desire to develop a limited number of calculations that may be tested readily for accuracy and conceptual understanding.

V. UNAMORTIZED ACQUISITION EXPENSE FACTORS

Using projected premium revenue from column 2 of Table 1, accumulated revenue from the same column, and ratios of unamortized acquisition expenses of column 5 to both such revenue projections, unamortized acquisition expense factors or percentages may be developed that are appropriate for application to actual premium figures on both an in-force inventory and an accumulated revenue basis. These data and results are shown in Table 2. Column 1 of Table 2 is the same as column 2 of Table 1. Aggregate premium revenue figures in column 2 of Table 2 are developed simply by accumulating the revenue figures from column 1 of Table 2.

The unamortized acquisition expense figures in column 3 of Table 2 (and col. 5 of Table 1) are representative of a *static* acquisition expense amortization technique, sometimes referred to as an accountant's worksheet approach or a prescheduled amortization approach. When the unamortized acquisition expense factors in column 4 of Table 2 are applied to appropriate inventories of in-force revenue, the method may be described as a *dynamic* approach, sometimes referred to as an actuarial expense reserve factor approach. An *aggregate (or accumulated) revenue* approach involves the use of the factors in column 5 of Table 2; the factors would be multiplied by the accumulated premium revenue initially issued

in a single year and collected subsequently as business persists and premiums are paid. (For practical reasons these aggregate revenue figures might represent summations of periodic inventories of in-force revenue; of course the factors should be based on the same type of revenue measurement assumptions.)

The factors in columns 4 and 5 of Table 2 are developed by dividing the unamortized acquisition expenses in column 3 by *average* premium revenue figures from columns 1 and 2, respectively. Average premium revenue figures are used here to avoid the implication that the illustrations presented in this paper are valid only if premiums are paid annually. Hopefully, the averaging approach suggests that there is considerable flexibility in choosing a series of points in time at which to inventory premium revenue in force for dynamic GAAP acquisition expense amortization

TABLE 2

UNAMORTIZED ACQUISITION EXPENSES AS PERCENTAGES OF PROJECTED REVENUE

YEAR	PROJECTED IN-FORCE PREMIUM REVENUE (1)	PROJECTED AGGREGATE PREMIUM REVENUE (2)	UNAMORTIZED ACQUISITION EXPENSES (3)	UNAMORTIZED ACQUISITION EXPENSES AS PERCENTAGE OF: Average In-Force Revenue (4)	Average Aggregate Revenue (5)
1	$ 10,000	$ 10,000	$8,425	93.611%	60.179%
2	8,000	18,000	8,565	113.896	39.800
3	7,040	25,040	8,308	122.609	29.361
4	6,512	31,552	7,608	120.752	21.991
5	6,089	37,641	6,953	117.728	17.167
6	5,723	43,364	6,338	113.870	13.758
7	5,409	48,773	5,756	109.150	11.211
8	5,138	53,911	5,204	103.614	9.233
9	4,907	58,818	4,676	97.234	7.644
10	4,711	63,529	4,169	90.072	6.336
11	4,546	68,075	3,680	82.189	5.236
12	4,409	72,484	3,206	73.820	4.296
13	4,277	76,761	2,746	65.179	3.483
14	4,149	80,910	2,300	56.421	2.774
15	4,004	84,914	1,869	47.636	2.152
16	3,843	88,757	1,456	38.754	1.607
17	3,671	92,428	1,061	29.645	1.127
18	3,487	95,915	686	20.230	0.703
19	3,295	99,210	332	10.386	0.329
20	3,098	102,308	0	0.000	0.000
1–20	$102,308				

NOTE.—Column 3: "static" unamortized acquisition expenses. Column 4: "dynamic" unamortized acquisition expense factors. Column 5: "Aggregate revenue" unamortized acquisition expense factors.

purposes. Of course, the assumptions underlying the development of GAAP acquisition expenses as percentages of point-in-time aggregate revenue must be generally consistent with inventory methods for the data to which the percentages will be applied to produce GAAP acquisition expense asset figures.

Attention is called to the slope of the factors in columns 4 and 5 of Table 2. The factors that are a function of inventories of in-force revenue increase during the early years of the projection period, staying well above 100 percent and then dropping off rather rapidly in the later years. Factors that are a function of aggregate revenue by year of issue start well below 100 percent, drop very quickly during the early years (when estimates and actual experience might be close together), and then decline rather slowly over the remainder of the amortization period. Hence it might be expected intuitively that the aggregate revenue method is somewhat insensitive to deviations in experience or significant fluctuations from the underlying actuarial assumptions.

VI. UNAMORTIZED ACQUISITION EXPENSES BASED ON ACTUAL (ADVERSE) EXPERIENCE

The figures and data in Table 3 are developed in the same manner as were the expected unamortized acquisition expenses in Table 1, except that the persistency assumptions of Table 3 are adverse (not as good as expected). The persistency figures in Table 3 are taken as presenting the actual experience (the results if one were able to be precognitive) for comparative purposes using the static, dynamic, and aggregate revenue approaches. The Appendix to this paper contains demonstrations based on actual persistency experience that is better than the expected persistency used in Table 1.

Since one of the principal purposes of GAAP is to produce a reasonable matching of acquisition expenses with premium revenue, the unamortized acquisition expenses in column 5 of Table 3 are considered the ideal or "true" acquisition expenses against which one should compare results obtained from other acquisition expense amortization approaches. They also might be described as the unamortized acquisition expenses one would obtain if truly able to predict future events. In subsequent sections of this paper, comparisons of the actual or "true" unamortized acquisition expenses in column 5 of Table 3 are made with the following:

1. Static or prescheduled unamortized deferred acquisition expenses in column 5 of Table 1.
2. Unamortized deferred acquisition expenses calculated by applying dynamically the factors in column 4 of Table 2 to the average (actual for these illustrative purposes) premium inventories from column 2 of Table 3.

3. Unamortized deferred acquisition expense asset figures developed by applying the aggregate revenue approach factors from column 5 of Table 2 to the average accumulated revenue figures developed from column 2 of Table 3.

VII. DEVELOPMENT OF UNAMORTIZED DEFERRED ACQUISITION EXPENSE ASSETS: STATIC, DYNAMIC, AND AGGREGATE REVENUE

Actual persistency experience as presented in Table 3 has been described as adverse in that the total amount of premium revenue is significantly less than expected from the projections in Table 1 (compare col. 2 of Tables 1 and 3). Actual premium revenue totals $77,404, while expected total revenue is $102,308.

In Table 4, column 1 shows the actual premium revenue in force and column 2 shows the aggregate revenue developed from the figures in

TABLE 3

UNAMORTIZED ACQUISITION EXPENSES (ADVERSE PERSISTENCY EXPERIENCE)

Year	Actual Persistency Factor	Actual Premium Revenue	Acquisition Expenses		
			Incurred in Year	Charged in Proportion to Revenue (GAAP)	Unamortized Deferred
	(1)	(2)	(3)	(4)	(5)
1	.700	$10,000	$ 9,500	$ 1,392	$8,108
2	.810	7,000	875	975	8,008
3	.875	5,670	403	790	7,621
4	.905	4,961	0	691	6,930
5	.930	4,490	0	625	6,305
6	.935	4,176	0	581	5,724
7	.940	3,904	0	544	5,180
8	.945	3,670	0	511	4,669
9	.950	3,468	0	483	4,186
10	.955	3,295	0	459	3,727
11	.960	3,146	0	438	3,289
12	.965	3,021	0	421	2,868
13	.970	2,915	0	406	2,462
14	.975	2,827	0	394	2,068
15	.970	2,757	0	384	1,684
16	.960	2,674	0	372	1,312
17	.950	2,567	0	357	955
18	.940	2,439	0	340	615
19	.930	2,292	0	319	296
20	N/A	2,132	0	296	0
1–20	.....	$77,404	$10,778	$10,778	.....

NOTE.—N/A = Not applicable. Column 4 = Col. 2 × ($10,778 ÷ $77,404) = Col. 2 × 0.139243. Column 5 = Summation of col. 3 minus summation of col. 4, through a given year.

TABLE 4

UNAMORTIZED ACQUISITION EXPENSE PROJECTIONS: ACTUAL, STATIC, DYNAMIC, AND AGGREGATE REVENUE (ADVERSE PERSISTENCY EXPERIENCE)

YEAR	ACTUAL PREMIUM REVENUE (1)	AGGREGATE PREMIUM REVENUE (2)	UNAMORTIZED DEFERRED ACQUISITION EXPENSES			
			Actual (3)	Expected—Static Method (4)	Expected—Dynamic Method (5)	Expected—Aggregate Revenue Method (6)
1	$10,000	$10,000	$8,108	$8,425	$7,957	$8,124
2	7,000	17,000	8,008	8,565	7,215	7,894
3	5,670	22,670	7,621	8,308	6,517	7,384
4	4,961	27,631	6,930	7,608	5,706	6,570
5	4,490	32,121	6,305	6,953	5,101	5,873
6	4,176	36,297	5,724	6,338	4,600	5,262
7	3,904	40,201	5,180	5,756	4,134	4,713
8	3,670	43,871	4,669	5,204	3,698	4,211
9	3,468	47,339	4,186	4,676	3,288	3,745
10	3,295	50,634	3,727	4,169	2,901	3,308
11	3,146	53,780	3,289	3,680	2,534	2,895
12	3,021	56,801	2,868	3,206	2,191	2,503
13	2,915	59,716	2,462	2,746	1,871	2,129
14	2,827	62,543	2,068	2,300	1,575	1,774
15	2,757	65,300	1,684	1,869	1,294	1,434
16	2,674	67,974	1,312	1,456	1,016	1,113
17	2,567	70,541	955	1,061	742	809
18	2,439	72,980	615	686	479	521
19	2,292	75,272	296	332	230	251
20	2,132	77,404	0	0	0	0
1–20	$77,404	.....	.....	.....	.....	.....

NOTE.—Illustrative calculations or references for year 10: Column 3: $3,727 is from Table 3, col. 5, year 10. Column 4: $4,169 is from Table 1, col. 5, year 10. Column 5: $2,901 = ($3,295 + $3,146) × 0.5 × 0.90072 (see Table 2, col. 4). Column 6: $3,308 = ($50,634 + $53,780) × 0.5 × 0.06336 (see Table 2, col. 5).

column 1. Unamortized deferred acquisition expense figures are shown in columns 3–6 and are developed as follows:

Actual—column 3: These unamortized deferred acquisition expenses are based on the actual revenues from column 1 (also see col. 5 of Table 3). These are the actual policyholder deferred acquisition expenses that would be shown as an intangible asset in GAAP financial statements if the developer of the amortization method had precognitive powers and all assumptions were realized.

Static—column 4: These estimated amortized acquisition expenses are based on the original actuarial assumptions of Table 1 (see col. 5 of Table 1). These figures also might be referred to as static acquisition expense assets developed through the use of an accountant's worksheet or amortization schedule method.

Dynamic—column 5: Estimated unamortized acquisition expenses here are based on original actuarial assumptions and are calculated by multiplying the expense reserve factors in column 4 of Table 2 by the average in-force premium revenue figures from column 1 of Table 4. Thus, these asset figures are developed through use of an expense reserve factor technique.

Aggregate revenue—column 6: These estimated unamortized acquisition expenses are based on original actuarial assumptions and are developed by multiplying the factors in column 5 of Table 2 by the average accumulated premium revenue figures in column 2 of Table 4.

VIII. UNAMORTIZED DEFERRED ACQUISITION EXPENSE ASSETS: RATIOS OF ACTUAL TO EXPECTED

Ratios of actual to expected (static, dynamic, and aggregate revenue) unamortized acquisition expenses are shown in Table 5. The underlying data are from columns 3–6 of Table 4.

Ratios of actual to expected under the static method are less than 100 percent, and during the first year the deviation from 100 percent is larger than for the other two methods. The ratios stabilize rather quickly at a level of 90 percent and continue at this general level for the remainder of the twenty-year amortization period.

TABLE 5

RATIOS OF ACTUAL TO EXPECTED UNAMORTIZED ACQUISITION EXPENSES (ADVERSE PERSISTENCY EXPERIENCE)

Year	Static Method (1)	Dynamic Method (2)	Aggregate Revenue Method (3)
1	96%	102%	100%
2	93	111	101
3	92	117	103
4	91	121	105
5	91	124	107
6	90	124	109
7	90	125	110
8	90	126	111
9	90	127	112
10	89	128	113
11	89	130	114
12	89	131	115
13	90	132	116
14	90	131	117
15	90	130	117
16	90	129	118
17	90	129	118
18	90	128	118
19	89	129	118

Ratios of actual to expected under the dynamic approach are reasonably close to 100 percent in the first year but rather quickly show greater deviations from 100 percent than ratios under the other two methods.

Aggregate revenue ratios of actual to expected are rather close to 100 percent in the early years of the amortization period but thereafter progress upward to a level about midway between 100 percent and the level of the ratios under the dynamic method.

The dynamic approach seems to produce the least satisfactory results over the entire amortization period. The static method produces ratios of actual to expected that seem within acceptable tolerances but that are always less than 100 percent. This indicates too slow a rate of amortization of acquisition expenses relative to actual premium revenue, so the method may not be deemed conservative enough from a GAAP viewpoint. (In practice the usual procedure is to truncate static amortization schedules at the end of twenty to thirty years. This builds some conservatism into the methodology, since on the whole life type of coverage some revenues realistically are expected beyond the end of the selected truncation period.) The aggregate revenue method produces good results during the early years, does not react violently to changes in persistency, and produces conservative results, since ratios of actual to expected exceed 100 percent.

IX. GAAP ACQUISITION EXPENSES

In addition to analyses with respect to unamortized acquisition expense asset figures, it also is informative to consider results in terms of the associated acquisition expenses that would occur as charges against revenue in GAAP financial statements of earnings.

GAAP acquisition expenses chargeable against income are developed in Table 6. Column 1 contains the actual (adverse persistency) incurred acquisition expenses (see col. 3 of Table 3) prior to any adjustment for changes in the deferred acquisition expense assets. If the appropriate changes in the four sets of unamortized acquisition expense asset figures (cols. 3–6 of Table 4) are subtracted from the incurred expense figures, GAAP acquisition expenses are developed separately for the methods under consideration. Column 2 of Table 6, showing actual acquisition expenses, is identical with column 4 of Table 3, as would be expected.

Referring to Table 6, the static method produces low acquisition expense charges against income in the early years and relatively high charges in the later years. Under the dynamic approach, the results exceed the actual expenses significantly for the first several years of the projections but are less than actual in later years. Aggregate revenue

results are close to actual in the early years but lower than actual later on.

Since the expenses that appear in GAAP financial statements reflect the experience of blocks of business issued in past years as well as in the present year, it seems appropriate to analyze ratios of actual to expected expenses by comparing accumulated years of expenses rather than by looking solely at a single year's results. Hence, accumulated GAAP acquisition expenses and corresponding ratios of actual to expected are shown in Table 7. The static method produces the poorest results by understating acquisition expense charges against income in the early years. The dynamic method is a close second to the static method in the opposite direction by overstating acquisition expenses, especially in the early years. The aggregate revenue method produces results that appear to be within reasonable tolerances; the differences develop rather slowly

TABLE 6

GAAP ACQUISITION EXPENSE COMPARISONS
(ADVERSE PERSISTENCY EXPERIENCE)

YEAR	INCURRED ACQUISITION EXPENSES (1)	GAAP ACQUISITION EXPENSES*			
		Actual (2)	Static Method (3)	Dynamic Method (4)	Aggregate Revenue Method (5)
1	$ 9,500	$ 1,392	$ 1,075	$ 1,543	$ 1,376
2	875	975	735	1,617	1,105
3	403	790	660	1,101	913
4	0	691	700	811	814
5	0	625	655	605	697
6	0	581	615	501	611
7	0	544	582	466	549
8	0	511	552	436	502
9	0	483	528	410	466
10	0	459	507	387	437
11	0	438	489	367	413
12	0	421	474	343	392
13	0	406	460	320	374
14	0	394	446	296	355
15	0	384	431	281	340
16	0	372	413	278	321
17	0	357	395	274	304
18	0	340	375	263	288
19	0	319	354	249	270
20	0	296	332	230	251
1–20	$10,778	$10,778	$10,778	$10,778	$10,778

* Columns 2, 3, 4, and 5 equal col. 1 minus the change in acquisition expense asset from cols. 3, 4, 5, and 6, respectively, of Table 4.

and in this example the results stay within 10 percent of actual with the definite tendency to overstate somewhat the charges against income.

X. UNDERSTATEMENTS AND OVERSTATEMENTS OF UNAMORTIZED ACQUISITION EXPENSE ASSETS AND ACQUISITION EXPENSES

Table 8 contains a summary of (i) ratios of actual to expected unamortized deferred acquisition expenses, from Table 5, and (ii) ratios of actual to expected accumulated acquisition expenses charged against income, from Table 7. The static method overstates by up to 10 percent the GAAP acquisition expense assets. Accumulated acquisition expenses are understated by about 30 percent during the early years and by gradually declining percentages thereafter. Dynamic method results show understatements of acquisition expense assets, with a peak range on the order of 30 percent. Accumulated acquisition expenses are overstated, by as much as about 25 percent during the early years. Aggregate revenue method results show a pattern of gradual change. Unamortized acquisition expenses are understated eventually on the order of some 15–20

TABLE 7

ACCUMULATED GAAP ACQUISITION EXPENSES AND RATIOS OF ACTUAL TO EXPECTED (ADVERSE PERSISTENCY EXPERIENCE)

YEAR	ACCUMULATED GAAP ACQUISITION EXPENSES				RATIOS OF ACTUAL TO EXPECTED		
	Actual (1)	Expected—Static Method (2)	Expected—Dynamic Method (3)	Expected—Aggregate Revenue Method (4)	Static Method (5)	Dynamic Method (6)	Aggregate Revenue Method (7)
1.....	$ 1,392	$ 1,075	$ 1,543	$ 1,376	129%	90%	101%
2.....	2,367	1,810	3,160	2,481	131	75	95
3.....	3,157	2,470	4,261	3,394	128	74	93
4.....	3,848	3,170	5,072	4,208	121	76	91
5.....	4,473	3,825	5,677	4,905	117	79	91
6.....	5,054	4,440	6,178	5,516	114	82	92
7.....	5,598	5,022	6,644	6,065	111	84	92
8.....	6,109	5,574	7,080	6,567	110	86	93
9.....	6,592	6,102	7,490	7,033	108	88	94
10.....	7,051	6,609	7,877	7,470	107	90	94
11.....	7,489	7,098	8,244	7,883	106	91	95
12.....	7,910	7,572	8,587	8,275	104	92	96
13.....	8,316	8,032	8,907	8,649	104	93	96
14.....	8,710	8,478	9,203	9,004	103	95	97
15.....	9,094	8,909	9,484	9,344	102	96	97
16.....	9,466	9,322	9,762	9,665	102	97	98
17.....	9,823	9,717	10,036	9,969	101	98	99
18.....	10,163	10,092	10,299	10,257	101	99	99
19.....	10,482	10,446	10,548	10,527	100	99	100
20.....	10,778	10,778	10,778	10,778	100	100	100

percent, although the early-year ratios seem well within acceptable tolerances. Accumulated acquisition expenses are overstated, generally by about 5–10 percent.

XI. CONCLUSION

The static method does not produce conservative results, since unamortized deferred acquisition expense assets are overstated and corresponding expenses are understated. Dynamic method results are always conservative in that the unamortized deferred acquisition expense assets are understated and expenses are overstated; however, the degree of understatement and overstatement, respectively, seems excessive where it is of the order of 20–30 percent. An important question to ponder is whether such conservatism can be considered reasonable according to audit guide principles. The aggregate revenue method also produces

TABLE 8

RATIOS OF ACTUAL TO EXPECTED (ADVERSE PERSISTENCY EXPERIENCE)

Year	Unamortized Deferred Acquisition Expenses			Accumulated GAAP Acquisition Expenses		
	Static Method (1)	Dynamic Method (2)	Aggregate Revenue Method (3)	Static Method (4)	Dynamic Method (5)	Aggregate Revenue Method (6)
1	96%	102%	100%	129%	90%	101%
2	93	111	101	131	75	95
3	92	117	103	128	74	93
4	91	121	105	121	76	91
5	91	124	107	117	79	91
6	90	124	109	114	82	92
7	90	125	110	111	84	92
8	90	126	111	110	86	93
9	90	127	112	108	88	94
10	89	128	113	107	90	94
11	89	130	114	106	91	95
12	89	131	115	104	92	96
13	90	132	116	104	93	96
14	90	131	117	103	95	97
15	90	130	117	102	96	97
16	90	129	118	102	97	98
17	90	129	118	101	98	99
18	90	128	118	101	99	99
19	89	129	118	100	99	100
Summary	Over-stated	Under-stated	Under-stated	Under-stated	Over-stated	Over-stated

NOTE.—References to overstatement and understatement indicate the relationship between the results expected from the method and the actual results based on adverse persistency experience.

conservative results, since the assets are understated and the expenses are overstated. The degree of conservatism seems tolerable, however, and would be expected to be in the 5–10 percent range under the adverse persistency assumptions outlined in these illustrations. This approach thus seems to produce results that could be considered reasonable and realistic for GAAP reporting purposes. The results lie between those obtained from the static and dynamic approaches, which are in general use in the life insurance industry.

On the basis of observations of the results under both the static and the dynamic method, those responsible for acquisition expense amortization methodology would be well advised to consider seriously the audit guide admonishment in Appendix B (p. 142), "If actual experience differs significantly from that assumed, the factors [expense reserve factors] should be recomputed." This warning seems valid regardless of whether the dynamic or the static method is used or whether expense reserve factors or amortization schedules are used. It seems to the author that the adoption of the aggregate revenue method of amortizing deferred (GAAP) acquisition expenses would alleviate significantly any need to recompute.

APPENDIX

ADDITIONAL DEMONSTRATIONS

This Appendix furnishes additional support for the observations in the body of this paper. It includes tables that correspond to Tables 3–8 of the paper, except that the persistency assumed is favorable instead of adverse (better instead of worse than expected).

The conclusions as to the relative results produced by the static, dynamic, and aggregate revenue methods still seem valid, although the overstatement and understatement situations are now reversed. The aggregate revenue method still produces reasonable results. The ratios of actual to expected for this method fall between those of the static and dynamic methods and seem to be within acceptable deviation tolerances (generally less than 5 percent). With these relatively minor deviations, there is not the need to recompute the amortization factors or schedules, which the audit guide suggests should be done if the differences between actual and assumed experience are significant (p. 142).

The author has observed similar results on other occasions where tests were made assuming differences between actual and expected persistency. The development of a general mathematical theory as to why such results should be expected would be a very valuable contribution.

TABLE A1

UNAMORTIZED ACQUISITION EXPENSES (FAVORABLE PERSISTENCY EXPERIENCE)

(Corresponds to Table 3 of Paper)

YEAR	ACTUAL PERSISTENCY FACTOR (1)	ACTUAL PREMIUM REVENUE (2)	ACQUISITION EXPENSES		
			Incurred in Year (3)	Charged in Proportion to Revenue (GAAP) (4)	Unamortized Deferred (5)
1	.850	$ 10,000	$ 9,500	$ 961	$8,539
2	.900	8,500	1,060	817	8,782
3	.940	7,650	540	736	8,586
4	.945	7,191	0	691	7,895
5	.950	6,795	0	653	7,242
6	.955	6,456	0	621	6,621
7	.960	6,165	0	593	6,028
8	.965	5,919	0	569	5,459
9	.965	5,711	0	549	4,910
10	.965	5,512	0	530	4,380
11	.965	5,319	0	511	3,869
12	.965	5,132	0	493	3,376
13	.965	4,953	0	476	2,900
14	.965	4,780	0	460	2,440
15	.965	4,612	0	443	1,997
16	.965	4,451	0	428	1,569
17	.965	4,295	0	413	1,156
18	.965	4,145	0	399	757
19	.965	4,000	0	385	372
20	.965	3,860	0	372	0
1–20	.....	$115,446	$11,100	$11,100	.....

NOTE.—Column 4 = Col. 2 × ($11,100 ÷ $115,446) = Col. 2 × 0.096149. Column 5 = Summation of col. 3 minus summation of col. 4, through a given year.

TABLE A2

UNAMORTIZED ACQUISITION EXPENSE PROJECTIONS: ACTUAL, STATIC, DYNAMIC, AND AGGREGATE REVENUE (FAVORABLE PERSISTENCY EXPERIENCE)

(Corresponds to Table 4 of Paper)

YEAR	ACTUAL PREMIUM REVENUE (1)	AGGREGATE PREMIUM REVENUE (2)	UNAMORTIZED DEFERRED ACQUISITION EXPENSES			
			Actual (3)	Expected—Static Method (4)	Expected—Dynamic Method (5)	Expected—Aggregate Revenue Method (6)
1	$ 10,000	$ 10,000	$8,539	$8,425	$8,659	$8,576
2	8,500	18,500	8,782	8,565	9,197	8,885
3	7,650	26,150	8,586	8,308	9,098	8,734
4	7,191	33,341	7,895	7,608	8,444	8,079
5	6,795	40,136	7,242	6,953	7,800	7,337
6	6,456	46,592	6,621	6,338	7,186	6,834
7	6,165	52,757	6,028	5,756	6,595	6,246
8	5,919	58,676	5,459	5,204	6,025	5,681
9	5,711	64,387	4,910	4,676	5,456	5,132
10	5,512	69,899	4,380	4,169	4,878	4,597
11	5,319	75,218	3,869	3,680	4,295	4,073
12	5,132	80,350	3,376	3,206	3,722	3,558
13	4,953	85,303	2,900	2,746	3,172	3,054
14	4,780	90,083	2,440	2,300	2,650	2,565
15	4,612	94,695	1,997	1,869	2,159	2,086
16	4,451	99,146	1,569	1,456	1,695	1,628
17	4,295	103,441	1,156	1,061	1,251	1,189
18	4,145	107,586	757	686	824	770
19	4,000	111,586	372	332	408	373
20	3,860	115,446	0	0	0	0
1–20	$115,446	.....	.....	.....	.....	.....

NOTE.—Illustrative calculations or references for year 10: Column 3: $4,380 is from Table A1, col. 5, year 10. Column 4: $4,169 is from Table 1, col. 5, year 10. Column 5: $4,878 = ($5,512 + $5,319) × 0.5 × 0.90072. Column 6: $4,597 = ($69,899 + $75,218) × 0.5 × 0.06336.

TABLE A3

RATIOS OF ACTUAL TO EXPECTED UNAMORTIZED ACQUISITION EXPENSES (FAVORABLE PERSISTENCY EXPERIENCE)

(Corresponds to Table 5 of Paper)

Year	Static Method (1)	Dynamic Method (2)	Aggregate Revenue Method (3)
1	101%	99%	100%
2	103	95	99
3	103	94	98
4	104	93	98
5	104	93	99
6	104	92	97
7	105	91	97
8	105	91	96
9	105	90	96
10	105	90	95
11	105	90	95
12	105	91	95
13	106	91	95
14	106	92	95
15	107	92	96
16	108	93	96
17	109	92	97
18	110	92	98
19	112	91	100

TABLE A4

GAAP ACQUISITION EXPENSE COMPARISONS
(FAVORABLE PERSISTENCY EXPERIENCE)
(Corresponds to Table 6 of Paper)

YEAR	INCURRED ACQUISITION EXPENSES (1)	GAAP ACQUISITION EXPENSES*			
		Actual (2)	Static Method (3)	Dynamic Method (4)	Aggregate Revenue Method (5)
1	$ 9,500	$ 961	$ 1,075	$ 841	$ 924
2	1,060	817	920	522	751
3	540	736	797	639	691
4	0	691	700	654	655
5	0	653	655	644	742
6	0	621	615	614	503
7	0	593	582	591	588
8	0	569	552	570	565
9	0	549	528	569	549
10	0	530	507	578	535
11	0	511	489	583	524
12	0	493	474	573	515
13	0	476	460	550	504
14	0	460	446	522	489
15	0	443	431	491	479
16	0	428	413	464	458
17	0	413	395	444	439
18	0	399	375	427	419
19	0	385	354	416	397
20	0	372	332	408	373
1–20	$11,100	$11,100	$11,100	$11,100	$11,100

* Columns 2, 3, 4, and 5 equal col. 1 minus the change in acquisition expense asset from columns 3, 4, 5, and 6, respectively, of Table A2.

TABLE A5

ACCUMULATED GAAP ACQUISITION EXPENSES AND RATIOS OF ACTUAL TO EXPECTED (FAVORABLE PERSISTENCY EXPERIENCE)

(Corresponds to Table 7 of Paper)

YEAR	ACCUMULATED GAAP ACQUISITION EXPENSES				RATIOS OF ACTUAL TO EXPECTED		
	Actual (1)	Expected—Static Method (2)	Expected—Dynamic Method (3)	Expected—Aggregate Revenue Method (4)	Static Method (5)	Dynamic Method (6)	Aggregate Revenue Method (7)
1.....	$ 961	$ 1,075	$ 841	$ 924	89%	114%	104%
2.....	1,778	1,995	1,363	1,675	89	130	106
3.....	2,514	2,792	2,002	2,366	90	126	106
4.....	3,205	3,492	2,656	3,021	92	121	106
5.....	3,858	4,147	3,300	3,763	93	117	103
6.....	4,479	4,762	3,914	4,266	94	114	105
7.....	5,072	5,344	4,505	4,854	95	113	104
8.....	5,641	5,896	5,075	5,419	96	111	104
9.....	6,190	6,424	5,644	5,968	96	110	104
10.....	6,720	6,931	6,222	6,503	97	108	103
11.....	7,231	7,420	6,805	7,027	97	106	103
12.....	7,724	7,894	7,378	7,542	98	105	102
13.....	8,200	8,354	7,928	8,046	98	103	102
14.....	8,660	8,800	8,450	8,535	98	102	101
15.....	9,103	9,231	8,941	9,014	99	102	101
16.....	9,531	9,644	9,405	9,472	99	101	101
17.....	9,944	10,039	9,849	9,911	99	101	100
18.....	10,343	10,414	10,276	10,330	99	101	100
19.....	10,728	10,768	10,692	10,727	100	100	100
20.....	11,100	11,100	11,100	11,100	100	100	100

TABLE A6

RATIOS OF ACTUAL TO EXPECTED (FAVORABLE PERSISTENCY EXPERIENCE)

(Corresponds to Table 8 of Paper)

YEAR	UNAMORTIZED DEFERRED ACQUISITION EXPENSES			ACCUMULATED GAAP ACQUISITION EXPENSES		
	Static Method (1)	Dynamic Method (2)	Aggregate Revenue Method (3)	Static Method (4)	Dynamic Method (5)	Aggregate Revenue Method (6)
1	101%	99%	100%	89%	114%	104%
2	103	95	99	89	130	106
3	103	94	98	90	126	106
4	104	93	98	92	121	106
5	104	93	99	93	117	103
6	104	92	97	94	114	105
7	105	91	97	95	113	104
8	105	91	96	96	111	104
9	105	90	96	96	110	104
10	105	90	95	97	108	103
11	105	90	95	97	106	103
12	105	91	95	98	105	102
13	106	91	95	98	103	102
14	106	92	95	98	102	101
15	107	92	96	99	102	101
16	108	93	96	99	101	101
17	109	92	97	99	101	100
18	110	92	98	99	101	100
19	112	91	100	100	100	100
Summary	Understated	Overstated	Overstated	Overstated	Understated	Understated

NOTE.—References to understatement and overstatement indicate the relationship between the results expected from the method and the actual results based on favorable persistency experience.

DISCUSSION OF PRECEDING PAPER

DONALD R. SONDERGELD:

The purpose of this discussion is to provide formulas for analyzing the different amortization methods described in the paper, and to make some observations. First, let me comment on the formulas contained in Table 1 of this discussion.

1. To simplify the presentation, premiums and acquisition expense are taken as of the beginning of the policy year. As a further simplification, acquisition expense is assumed to occur only in the first policy year.

2. A prime is used over a symbol to represent actual persistency, whereas unprimed symbols represent the persistency expected when the amortization schedule was constructed (e.g., ${}_tp'_x$ versus ${}_tp_x$ and $\ddot{a}'_{x:\overline{n}|}$ versus $\ddot{a}_{x:\overline{n}|}$; ${}_mp'_x = 0$ and ${}_np_x = 0$ for the first time at durations m and n, respectively).

3. The determination of "persistency" takes into account all decrements such as mortality, lapse, and surrender.

4. Mr. Pharr used the premium for the current year in the static method, an average premium for the current and following years under the dynamic method, and an average of the aggregate accumulated premium for the current and following years under the aggregate revenue method. The author indicated that this average approach was introduced into the paper to avoid the implication that the illustrations are valid only if premiums are paid annually. It seems to me this introduced an unnecessary inconsistency into the comparisons. My formulas for unamortized expense treat premiums consistently in all three methods.

5. Items (7), (8), (10), (12), (13), and (15*b*) of my table include a reference to tables and columns of the paper. This is to help relate my formulas to the tables and columns of the paper. Although my formulas relate to the tables in the paper, they must be modified to account for the differences mentioned in items 1 and 4 above if one wishes to reproduce the numbers in the paper.

6. I use the words *expected*, *actual*, *reported*, and *true* somewhat differently than does Mr. Pharr. According to my usage, the word *expected* implies the use of persistency assumptions developed before the event happens (that is, before the policy was issued). *Actual* and *true* persistency represents what actually happens. *Reported* unamortized expense is the result of *expected* unamortized expense being modified

TABLE 1

Definitions, Formulas, or Questions	Static Method	Dynamic Method	Aggregate Revenue Method
(1) Acquisition expense initially capitalized	A	A	A
(2) First-year premium	P	P	P
(3) Expected premium in year $t+1$	Not needed	$({}_tp_x)(P)$	Not needed
(4) Actual premium in year $t+1$	Not needed	$({}_tp'_x)(P)$	Not needed
(5) Expected accumulated premium through year $t+1$	Not needed	Not needed	$\sum_{i=0}^{t} ({}_ip_x)(P)$
(6) Actual accumulated premium through year $t+1$	Not needed	Not needed	$\sum_{i=0}^{t} ({}_ip'_x)(P)$
(7) Expected unamortized expenses at time t (Table 2, col. 3)	$\frac{A}{\ddot{a}_{x:\overline{n}\rceil}} \ddot{a}_{x+t:\overline{n-t}\rceil}$	$\frac{A}{\ddot{a}_{x:\overline{n}\rceil}} \ddot{a}_{x+t:\overline{n-t}\rceil}$	$\frac{A}{\ddot{a}_{x:\overline{n}\rceil}} \ddot{a}_{x+t:\overline{n-t}\rceil}$

TABLE 1—*Continued*

Definitions, Formulas, or Questions	Static Method	Dynamic Method	Aggregate Revenue Method
(8) Amortization factors: F_t (Table 2, cols. 3, 4, and 5	(7)	(7) ÷ (3)	(7) ÷ (5)
(9) What should (8) be multiplied by to get reported unamortized expense at time t?	1	$({}_tp'_x)(P)$	$\sum_{i=0}^{t} ({}_ip'_x)(P)$
(10) Reported unamortized expense at time t: (8) × (9) (Table 4, cols. 4, 5, and 6)	$\frac{A}{\ddot{a}_{x:\overline{n}\rceil}}\,\ddot{a}_{x+t:\overline{n-t}\rceil}$	$A\left(\frac{\ddot{a}_{x+t:\overline{n-t}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right)\left(\frac{{}_tp'_x}{{}_tp_x}\right)$	$A\left(\frac{\ddot{a}_{x+t:\overline{n-t}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right)\left(\frac{\sum_{i=0}^{t} {}_ip'_x}{\sum_{i=0}^{t} {}_ip_x}\right)$
(11) What if ${}_tp'_x < {}_tp_x$?	(10) = (7)	(10) < (7)	?
(12) True unamortized expense at time t (Table 4, col. 3)	$A\left(\frac{\ddot{a}'_{x+t:\overline{m-t}\rceil}}{\ddot{a}'_{x:\overline{m}\rceil}}\right)$	$A\left(\frac{\ddot{a}'_{x+t:\overline{m-t}\rceil}}{\ddot{a}'_{x:\overline{m}\rceil}}\right)$	$A\left(\frac{\ddot{a}'_{x+t:\overline{m-t}\rceil}}{\ddot{a}'_{x:\overline{m}\rceil}}\right)$

TABLE 1—*Continued*

Definitions, Formulas, or Questions	Static Method	Dynamic Method	Aggregate Revenue Method
(13) How does reported unamortized expense at time t compare with the true amount? The ratio of reported to true unamortized expense: $R_t = (10)_t \div (12)_t$ (reciprocal of Table 8, cols. 1, 2, and 3)	$\dfrac{\ddot{a}_{x+t:\overline{n-t}\rceil}/\ddot{a}_{x:\overline{n}\rceil}}{\ddot{a}'_{x+t:\overline{m-t}\rceil}/\ddot{a}'_{x:\overline{m}\rceil}}$	$\dfrac{(\ddot{a}_{x+t:\overline{n-t}\rceil}/\ddot{a}_{x:\overline{n}\rceil})({}_tp'_x/{}_tp_x)}{\ddot{a}'_{x+t:\overline{m-t}\rceil}/\ddot{a}'_{x:\overline{m}\rceil}}$	$\dfrac{(\ddot{a}_{x+t:\overline{n-t}\rceil}/\ddot{a}_{x:\overline{n}\rceil})\left(\sum_{i=0}^{t}{}_ip'_x \Big/ \sum_{i=0}^{t}{}_ip_x\right)}{\ddot{a}'_{x+t:\overline{m-t}\rceil}/\ddot{a}'_{x:\overline{m}\rceil}}$
(14) Same question as in item (13), but compare the difference between reported and true unamortized expense: $D_t = (10)_t - (12)_t$	$A\left(\dfrac{\ddot{a}_{x+t:\overline{n-t}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}} - \dfrac{\ddot{a}'_{x+t:\overline{m-t}\rceil}}{\ddot{a}'_{x:\overline{m}\rceil}}\right)$	$A\left[\left(\dfrac{\ddot{a}_{x+t:\overline{n-t}\rceil}}{\ddot{a}'_{x:\overline{n}\rceil}}\right)\left(\dfrac{{}_tp'_x}{{}_tp_x}\right) - \dfrac{\ddot{a}'_{x+t:\overline{m-t}\rceil}}{\ddot{a}'_{x:\overline{m}\rceil}}\right]$	$A\left[\left(\dfrac{\ddot{a}_{x+t:\overline{n-t}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right)\left(\dfrac{\sum_{i=0}^{t}{}_ip'_x}{\sum_{i=0}^{t}{}_ip_x}\right) - \dfrac{\ddot{a}'_{x+t:\overline{m-t}\rceil}}{\ddot{a}'_{x:\overline{m}\rceil}}\right]$

TABLE 1—*Continued*

Definitions, Formulas, or Questions	Static Method	Dynamic Method	Aggregate Revenue Method
(15) Given that actual persistency is different from expected through t years, *a*) What is the expected write-off in year $t+1$, if we anticipate that persistency in that year will be p''_{x+t}?	$(10)_t - (10)_{t+1}$	$(10)_t - A\left(\frac{\ddot{a}_{x+t+1:\overline{n-t-1}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right)\left[\frac{({}_tp'_x)(p''_{x+t})}{{}_{t+1}p_x}\right]$ $= \left[\begin{matrix}\text{(Write-off on}\\ \text{static method)}_{t+1}\end{matrix}\right]\left(\frac{{}_tp'_x}{{}_tp_x}\right)$ $+ A\left(\frac{\ddot{a}_{x+t+1:\overline{n-t-1}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right)\left[\frac{({}_tp'_x)(p_{x+t}-{}_1p''_{x+t})}{{}_{t+1}p_x}\right]$	$(10)_t - A\left(\frac{\ddot{a}_{x+t+1:\overline{n-t-1}\rceil}}{\ddot{a}_{x:\overline{i}\rceil}}\right)$ $\times\left[\frac{({}_tp'_x)(p''_{x+t}) + \sum_{i=0}^{t}{}_ip'_x}{\sum_{i=0}^{t+}{}_ip_x}\right]$ $=\left[\begin{matrix}\text{(Write-off on}\\ \text{static method)}_{t+1}\end{matrix}\right]\left[\frac{\sum_{i=0}^{t}{}_ip'_x}{\sum_{i=0}^{t}{}_ip_x}\right]$ $+ A\left(\frac{\ddot{a}_{x+t+1:\overline{n-t-1}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right)$ $\times\left[\frac{\sum_{i=0}^{t}{}_ip'_x}{\sum_{i=0}^{t}{}_ip_x} - \frac{({}_tp'_x)(p''_{x+t}) + \sum_{i=0}^{t}{}_ip'_x}{\sum_{i=0}^{t+1}{}_ip_x}\right]$

TABLE 1—*Continued*

Definitions, Formulas, or Questions	Static Method	Dynamic Method	Aggregate Revenue Method
b) Reported write-off in year $t+1$ is $(10)_t - (10)_{t+1}$ (Table 6, cols. 3, 4, and 5)	$\frac{A}{\ddot{a}_{x:\overline{n}\rceil}} \times (\ddot{a}_{x+t:\overline{n-t}\rceil} - \ddot{a}_{x+t+1:\overline{n-t-1}\rceil})$	$\left[\begin{matrix}\text{(Write-off on}\\ \text{static method)}_{t+1}\end{matrix}\right]\left(\frac{{}_tp'_x}{{}_tp_x}\right) + A\left(\frac{\ddot{a}_{x+t+1:\overline{n-t-1}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right)\left(\frac{{}_tp'_x}{{}_tp_x} - \frac{{}_{t+1}p'_x}{{}_{t+1}p_x}\right)$ $= \left[\begin{matrix}\text{(Expected write-off on}\\ \text{dynamic method)}_{t+1}\end{matrix}\right] + A\left(\frac{\ddot{a}_{x+t+1:\overline{n-t-1}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right) \times \left[\frac{({}_tp'_x)(p_{x+t} - p'_{x+t})}{{}_{t+1}p_x}\right]$	$\left[\begin{matrix}\text{(Write-off on}\\ \text{static method)}_{t+1}\end{matrix}\right]\left(\frac{\sum_{i=0}^{t} {}_ip'_x}{\sum_{i=0}^{t} {}_ip_x}\right) + A\left(\frac{\ddot{a}_{x+t+1:\overline{n-t-1}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right)\left(\frac{\sum_{i=0}^{t} {}_ip'_x}{\sum_{i=0}^{t} {}_ip_x} - \frac{\sum_{i=0}^{t+1} {}_ip'_x}{\sum_{i=0}^{t+1} {}_ip_x}\right)$ $= \left[\begin{matrix}\text{(Expected write-off on}\\ \text{aggregate revenue method)}_{t+1}\end{matrix}\right] + A\left(\frac{\ddot{a}_{x+t+1:\overline{n-t-1}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right) \times \left[\frac{({}_tp'_x)(p_{x+t} - p'_{x+t})}{\sum_{i=0}^{t+1} {}_ip_x}\right]$

TABLE 1—*Continued*

Definitions, Formulas, or Questions	Static Method	Dynamic Method	Aggregate Revenue Method
c) Reported minus expected write-off in year $t+1$ = (*b*) − (*a*)	0	$A\left(\frac{\ddot{a}_{x+t+1:\overline{n-t-1}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right)\left(\frac{{}_{t+1}p'_x}{{}_{t+1}p_x}\right)\left(\frac{p''_{x+t}}{p'_{x+t}}-1\right)$ $=\left[\begin{array}{l}\text{(Reported unamortized expense}\\ \text{on dynamic method)}_{t+1}\end{array}\right]$ $\times\left(\frac{p''_{x+t}-p'_{x+t}}{p'_{x+t}}\right)$	$A\left(\frac{\ddot{a}_{x+t+1:\overline{n-t-1}\rceil}}{\ddot{a}_{x:\overline{n}\rceil}}\right)\left[\frac{\sum_{i=0}^{t+1}{}_ip'_x}{\sum_{i=0}^{t+1}{}_ip_x}\right]$ $\times\left[\frac{({}_tp'_x)(p''_{x+t})+\sum_{i=0}^{t}{}_ip'_x}{\sum_{i=0}^{t+1}{}_ip'_x}-1\right]$ $=\left[\begin{array}{l}\text{(Reported unamortized expense on}\\ \text{aggregate revenue method)}_{t+1}\end{array}\right]$ $\times\left[\frac{{}_tp'_x}{\sum_{i=0}^{t+1}{}_ip'_x}(p''_{x+t}-p'_{x+t})\right]$
d) Another way of expressing (*c*) is as follows:	0	$\left[\begin{array}{l}\text{(Expected unamortized expense}\\ \text{looking at year } t+1 \text{ from}\\ \text{year 0 on all methods)}_{t+1}\end{array}\right]$ $\times({}_tp'_x)\left(\frac{p''_{x+t}-p'_{x+t}}{{}_{t+1}p_x}\right)$	$\left[\begin{array}{l}\text{(Expected unamortized expense}\\ \text{looking at vear } t+1 \text{ from}\\ \text{year 0 on all methods)}_{t+1}\end{array}\right]$ $\times({}_tp'_x)\left(\frac{p''_{x+t}-p'_{x+t}}{\sum_{i=0}^{t+1}{}_ip_x}\right)$

by actual experience if one of the dynamic amortization methods is chosen. *True* unamortized expense is the *expected* unamortized expense schedule that would have been developed had the actuary, in the words of the author, had precognitive powers.

Observations and General Comments

1. Look at formulas (13) and (14) in Table 1. Ignoring the static method (S), there are six relationships possible at every duration between *true* unamortized expense (T), *reported* unamortized expense using the dynamic method (D), and *reported* unamortized expense using the aggregate revenue method (AR):

Case	Unamortized Expense Relationships	AR versus D	
		Which Undercapitalizes?	Which Is Relatively Better?
I.....	$T \geq AR \geq D$	AR and D	AR
II.....	$D \geq AR \geq T$	Neither	AR
III.....	$D \geq T \geq AR$	AR	AR*
IV.....	$T \geq D \geq AR$	D and AR	D
V.....	$AR \geq D \geq T$	Neither	D
VI.....	$AR \geq T \geq D$	D	D*

* This choice is based on the not necessarily correct premise that it is better to undercapitalize considerably than to overcapitalize slightly.

The illustration in the paper produced an example of Case I at all durations. The appendix to the paper contained an example of Case II at all durations.

It is easy to develop examples of other cases. The reader may want to experiment with formula (14) and vary the actual persistency to see what happens. I have not found many generalizations that can be made. The reasons are similar to the reasons why Lidstone's theorem is limited. The aggregate revenue method is not always superior to the dynamic method. It depends on what actually happens. For example, assume that we use the expected persistency of the illustration in the paper, and use actual persistency equal to expected in the first two years and then 30 percentage points worse in each succeeding year (e.g., 35 percent versus 65 percent, 31 percent versus 61 percent, 27 percent versus 57 percent, etc.). In that event, T, D, and AR unamortized expenses are equal for two years and then Case VI results thereafter,

with the overcapitalization under AR being much larger than the undercapitalization under D.

2. Now examine formula (10). At every duration there are six relationships between *reported* unamortized expenses under S, D, and AR methods.

Case	Unamortized Expense Relationships	Persistency Relationships*		
		${}_tp'_x - {}_tp_x$	$\Sigma_{i=0}^{t} ({}_ip'_x - {}_ip_x)$	${}_tp'_x/{}_tp_x - \Sigma_{i=0}^{t} {}_ip'_x/\Sigma_{i=0}^{t} {}_ip_x$
A.....	$S \geq AR \geq D$	N	N	N
B.....	$D \geq AR \geq S$	P	P	P
C.....	$D \geq S \geq AR$	P	N	P
D.....	$S \geq D \geq AR$	N	N	P
E.....	$AR \geq D \geq S$	P	P	N
F.....	$AR \geq S \geq D$	N	P	N

* N = negative or zero; P = positive or zero.

If we are concerned only with the relative size of D and AR, there are four different roads to follow. That is, ${}_tp'_x - {}_tp_x$ can equal N or P, and $\Sigma_{i=0}^{t} ({}_ip'_x - {}_ip_x)$ can equal N or P. If both of these differences are P, however, either Case B or Case E can result. The point to be made is that the actual persistency through year t will determine whether the *reported* unamortized expense is larger under D or AR. How that compares with T can be determined only when the actual persistency of the future is known.

3. Under what conditions is the *reported* unamortized expense (formula 10) under AR greater than under D? This happens when

$$\frac{\sum_{i=0}^{t} {}_ip'_x}{\sum_{i=0}^{t} {}_ip_x} \geq \frac{{}_tp'_x}{{}_tp_x}$$

or

$$\frac{\sum_{i=0}^{t} {}_ip'_x}{{}_tp'_x} \geq \frac{\sum_{i=0}^{t} {}_ip_x}{{}_tp_x}.$$

One situation where this is true arises when $p_{x+t} \geq p'_{x+t}$ for *all* t, which means that ${}_tp_x \geq {}_tp'_x$. This is true for $t = 0$ and $t = 1$. This can be

proved by induction if we assume it is true for $t = n$ and then examine $t = n + 1$.

$$\frac{\sum_{i=0}^{n+1} {}_ip'_x}{{}_{n+1}p'_x} = \frac{\sum_{i=0}^{n} {}_ip'_x}{(p'_{x+n})({}_np'_x)} + 1 \geq \frac{1}{p'_{x+n}} \frac{\sum_{i=0}^{n} {}_ip_x}{{}_np_x} + 1$$

$$\geq \frac{1}{p_{x+n}} \frac{\sum_{i=0}^{n} {}_ip_x}{{}_np_x} + 1 = \frac{\sum_{i=0}^{n+1} {}_ip_x}{{}_{n+1}p_x}.$$

This condition always will produce Case I, Case V, or Case VI. The converse is obviously true. That is, if $p_{x+t} \leq p'_{x+t}$ for *all* t, Case II, Case III, or Case IV will result.

4. It does not appear to me that the aggregate revenue method will produce a good result when the actual premiums in force are much less than expected. Let me elaborate.

A comparison of *reported* unamortized expense with what *true* would have been can be made only after the block of business on which the comparison is being made is no longer in force. Any dynamic amortization method used will adjust the *expected* unamortized expense by actual experience. Under the dynamic method, a set of amortization factors is developed. These factors are

$$F_t = \frac{A(\ddot{a}_{x+t:\overline{n-t}|}/\ddot{a}_{x:\overline{n}|})}{({}_tp_x)(P)}.$$

They are zero after n years.

Let us assume that actual experience differs from expected and that there is no longer any business in force after m years. If m is less than n, F_m will be multiplied by $({}_mp'_x)(P)$, or zero. If m is greater than n, the amortization factor itself is zero. This means that acquisition expense will be written off in n years if $m > n$, and in m years if $m < n$, under the dynamic method.

Under the aggregate revenue method, which is also a dynamic method, there also are n amortization factors. Again, if m is greater than n, the amortization factors are still zero after n years. However, if m is less than n, I presume that there is a constant aggregate revenue amount during the period from m to n, and this method would continue to generate an unamortized expense item when there should not be any. I suppose the amount could be written off at that time. However, instead of assuming the loss of the entire in-force earlier than anticipated, let us assume that the actual reduction in in-force is steeper than

expected. The aggregate revenue method then would produce a slower write-off than is prudent.

5. A final comment concerns formula (15) in Table 1. I have found formula (15*a*) for the dynamic method useful in answering "what if" questions. Once a block of business is in force, what is the impact on earnings if persistency for the coming year is x, y, or z percent?

It also is interesting to note that if $p'_{x+t} \neq p'_{z+t}$ the ratio for year $t+1$ of (15*d*) on the aggregate revenue method to that on the dynamic method equals ${}_{t+1}p_x / \Sigma_{i=0}^{t+1} {}_ip_x$

CLAUDE Y. PAQUIN:

Mr. Pharr's presentation is interesting, and I commend his effort and willingness to share his insights into the subject of amortizing life insurance company acquisition expenses under GAAP.

The many statements in the audit guide *Audits of Stock Life Insurance Companies* given in support of Mr. Pharr's allegations and conclusions should be taken in the context of that entire guide and considered in the light of the guide's acknowledged purpose "to provide guidance to independent auditors in examining and reporting on financial statements of life insurance companies" (p. 1).

The guidance the guide provides is incomplete and at times contradictory. The best, or worst, instance is where the guide provides for expenses to be recognized *in proportion to* premium revenue, a feat that can be accomplished (as demonstrated in *TSA*, XXV, 459–84) only when expenses are amortized at a 0 percent interest rate, a technique that Mr. Pharr condescends to use here but, unlike me, expressly disavows. Like the Delphic oracle, the audit guide can be found to provide support for many positions.

How does one proceed to use the author's aggregate revenue method? It appears from the paper that the procedure involves the following steps:

1. Calculate the projected aggregate premium revenue (using appropriate projections as to persistency or renewal).
2. Calculate the projected unamortized acquisition expenses ("unrecovered acquisition costs").
3. Calculate item 2 as a percentage of item 1 at various equivalent times during the projected premium-payment period.
4. Calculate the actual aggregate premium revenue up to the time of the actuarial valuation for GAAP purposes.
5. Apply the appropriate percentage from item 3 to the amount from item 4 to produce the unrecovered acquisition cost or deferrable acquisition cost yet to be recovered.

Mr. Pharr is impressed with the fact that this technique seems to work pretty well, especially when measured against the standard: "What would the unrecovered acquisition cost have been if we could have known what the persistency would turn out to be?"

I find this approach bothersome in two respects. First, we are simply faced with a technique that seems to work, and in the appendix to his paper the author is frank to acknowledge that "the development of a general mathematical theory as to why such results should be expected would be a very valuable contribution." In short, it seems that no rationale has yet been developed for this method other than that it seems to work well. Straight-line depreciation, sum-of-the-digits depreciation, double declining balance depreciation all work, and they are all acceptable to the accountant. They are used with no pretense of intellectual superiority over one another. They are typical accountants' methods. Why, then, would it not be possible to present the aggregate revenue method as being simply in the same category as these other accountants' methods?

More bothersome, perhaps, is the fact that the aggregate revenue method aims to smooth out fluctuations when experience does not turn out as expected. If lapses turn out to be 20 percent worse than "expected" (although in some cases the expression "hoped for" might turn out to be more accurate), why should an insurer's earnings not reflect the loss immediately, and vice versa? Why is it considered desirable to smooth out the gains and the losses; does the presence of a large block of policies of various types not help to smooth out gains and losses by itself? Is the dynamic method not always prospectively accurate?

The aggregate revenue method might deserve a lukewarm welcome as an acceptable method by those charged with calculating GAAP earnings. Unlike the author, I would not extend my support for this method beyond that statement.

ALAN GOLDBERG AND LESTER MOSKOWITZ:

Mr. Pharr has provided the actuarial profession with another excellent paper on the practical aspects of GAAP. When lapse experience does not follow original assumptions, Mr. Pharr offers the aggregate revenue method (ARM) as a practical technique to adjust the unamortized acquisition expense asset. ARM produces results superior to the static and dynamic methods now in common use.

Questions Discussed

This paper has inspired comments that address the following questions:

1. Could ARM be modified to improve the adjustment?

2. Is the effect on surplus acceptable when improved adjustments are made to assets but not to reserve liabilities?
3. What definition of "hindsight" calculation is appropriate as a criterion for measuring the accuracy of GAAP financial statements?

Policy Year versus Calendar Year

Before we comment on the above questions, there are two minor aspects to clarify, both relating to policy year versus calendar year. First, Mr. Pharr develops policy year-end assets using the worksheet method and uses those numbers for test comparisons in a calendar-year environment. We do not take exception to that approach, and in these comments the same approach is used in numerical demonstrations. Further, it is presumed that Mr. Pharr's columns headed "premium revenue" are equivalent to in-force l_x's.

Second, it *appears* that ARM involves an adjustment in year t dependent on experience in years t and $t + 1$, when the latter figure has not yet been experienced. This apparent blemish can be overcome by using l_x's. Appendix A provides further clarification of this point.

Modification of ARM

ARM adjusts the deferred acquisition expense asset by the ratio of aggregate premiums received to date to expected aggregate premiums. A modification (MARM) develops an experience ratio of aggregate premiums *not* collected. The numerator of the ratio is aggregate premiums experienced less those assumed; the denominator is aggregate premiums on a zero lapse assumption less those assumed. The static method asset is precomputed on the basis of zero lapses and assumed lapses, and the asset difference is multiplied by the experience ratio. This product is the adjustment to the static method asset. MARM assets are developed in Table 1 of this discussion.

Table 2 compares dollar asset figures among various approaches. The results seem to indicate that ARM provides a better estimate of "actual" than methods in present use and that MARM is an improved version of ARM. Tests using different lapse assumptions and premium-paying periods produced varying but similar results.

Is a Perfect Asset Enough?

Mr. Pharr alerts the reader that present methods produce asset distortions that may be offset to an extent by distortions in benefit reserve liabilities. Mr. Pharr's view (that a perfect asset is a desirable goal) is not contested, but we believe that any technique for adjusting assets—however effective—should not be adopted if such method introduces distortions in surplus. Hence, it is appropriate to examine the

TABLE 1

DEVELOPMENT OF MARM ASSETS

Year	Average Accumulated Premium			Variation in Premiums between Original Lapse Assumptions and		Ratio (5) ÷ (4)	Static Asset		Difference (7) Minus (8)	Product (6) × (9)	Adjusted Assets [(8) + (10)]
	Zero Lapse Assumption (1)	Original Lapse Assumptions (2)	Actual Experience (3)	Zero Lapse Assumption (4)	Actual Experience (5)	(6)	Zero Lapse Assumption (7)	Original Lapse Assumptions (8)	(9)	(10)	(11)
1	$ 15,000	$ 14,000	$13,500	$ 1,000	−$ 500	−0.500	$8,927	$8,425	$ 502	−$251	$8,174
2	25,000	21,520	19,835	3,480	− 1,685	−0.484	9,604	8,565	1,039	− 503	8,062
3	35,000	28,296	25,151	6,704	− 3,145	−0.469	9,741	8,308	1,433	− 672	7,636
4	45,000	34,596	29,876	10,404	− 4,720	−0.454	9,168	7,607	1,561	− 709	6,898
5	55,000	40,502	34,209	14,498	− 6,293	−0.434	8,595	6,953	1,642	− 713	6,240
6	65,000	46,068	38,249	18,932	− 7,819	−0.413	8,022	6,337	1,685	− 696	5,641
7	75,000	51,342	42,036	23,658	− 9,306	−0.393	7,449	5,756	1,693	− 665	5,091
8	85,000	56,364	45,605	28,636	− 10,759	−0.376	6,876	5,203	1,673	− 629	4,574
9	95,000	61,173	48,986	33,827	− 12,187	−0.360	6,303	4,676	1,627	− 586	4,090
10	105,000	65,801	52,207	39,199	− 13,594	−0.347	5,730	4,169	1,561	− 542	3,627
11	115,000	70,279	55,291	44,721	− 14,988	−0.335	5,157	3,681	1,476	− 494	3,187
12	125,000	74,622	58,258	50,378	− 16,364	−0.325	4,584	3,207	1,377	− 448	2,759
13	135,000	78,835	61,129	56,165	− 17,706	−0.315	4,011	2,747	1,264	− 398	2,349
14	145,000	82,912	63,922	62,088	− 18,990	−0.306	3,438	2,301	1,137	− 348	1,953
15	155,000	86,835	66,637	68,165	− 20,198	−0.296	2,865	1,870	995	− 295	1,575
16	165,000	90,592	69,257	74,408	− 21,335	−0.287	2,292	1,457	835	− 240	1,217
17	175,000	94,171	71,760	80,829	− 22,411	−0.277	1,719	1,062	657	− 182	880
18	185,000	97,562	74,126	87,438	− 23,436	−0.268	1,146	687	459	− 123	564
19	195,000	100,758	76,338	94,242	− 24,420	−0.259	573	333	240	− 62	271
20	200,000	102,307	77,404	97,963	− 24,903	−0.255	0	0	0	0	0

distortions that exist currently in reserve liabilities and to compare the surplus distortions that result under various methods.

For the purpose of our comparison, we have assumed that Mr. Pharr's data relate to a twenty-year endowment. Additional assumptions were introduced in order to compute GAAP benefit reserves (see Appendix B). Asset figures were recomputed using interest at 6 percent, and a $40 gross premium was assumed to convert the assets to a per thousand basis. The resulting asset and liability figures are shown in Table 3. Using "actual" as a standard for comparison, we show the distortions in assets, liabilities, and surplus in Table 4.

With respect to asset distortion, ARM represents a clear improvement over present methods, and a further improvement is suggested by MARM. However, because of the offsetting liability distortions, net distortions of surplus resulting from the dynamic method are not improved by ARM and deteriorate further under MARM.

In our opinion, an improved technique to adjust benefit reserves (reducing liability distortion) is needed, without which the suggested modifications in adjusting assets are unacceptable.

TABLE 2

UNAMORTIZED DEFERRED ACQUISITION EXPENSES

Year	Actual (1)	Static Method (2)	Dynamic Method (3)	ARM (4)	MARM (5)
1	$8,108	$8,425	$7,957	$8,124	$8,174
2	8,008	8,565	7,215	7,894	8,062
3	7,621	8,308	6,517	7,384	7,636
4	6,930	7,607	5,706	6,570	6,898
5	6,305	6,953	5,101	5,872	6,240
6	5,724	6,337	4,600	5,262	5,641
7	5,180	5,756	4,134	4,713	5,091
8	4,669	5,203	3,698	4,210	4,574
9	4,186	4,676	3,288	3,744	4,090
10	3,727	4,169	2,901	3,308	3,627
11	3,289	3,681	2,535	2,896	3,187
12	2,869	3,207	2,191	2,503	2,759
13	2,463	2,747	1,872	2,130	2,349
14	2,069	2,301	1,576	1,774	1,953
15	1,685	1,870	1,294	1,435	1,575
16	1,313	1,457	1,016	1,114	1,217
17	956	1,062	743	809	880
18	616	687	479	522	564
19	297	333	230	252	271
20	0	0	0	0	0

We attempted to design a method of adjusting benefit reserves. Briefly described, the method was patterned after the MARM adjustment to assets using the same experience ratio developed in Table 1 and applying that ratio as an adjustment to the difference between precalculated benefit liabilities based on zero lapses and assumed lapses. The method produced good results for liability distortion and a near-perfect fit with "actual" surplus when applied to Mr. Pharr's example. However, acceptable results were not produced consistently when applied to a variety of plans and lapse assumptions; hence, the method is not presented in these remarks.

What Is "Actual"?

What definition of hindsight calculation is appropriate as a criterion for measuring the accuracy of GAAP financial statements? Mr. Pharr has used a *prospective* hindsight that requires a preconception of future variations from assumptions based on experience to date. Up to this

TABLE 3

ASSETS AND LIABILITIES (PER $1,000 ISSUED)

YEAR	ASSETS					LIABILITIES	
	Actual (1)	Static Method (2)	Dynamic Method (3)	ARM (4)	MARM (5)	Actual (6)	Dynamic Method (7)
1.....	$31.84	$33.56	$31.70	$32.36	$32.20	$ 27.12	$ 26.01
2.....	31.56	34.44	29.01	31.74	31.87	43.22	40.60
3.....	30.37	33.89	26.59	30.13	30.49	54.49	51.40
4.....	28.01	31.55	23.66	27.25	27.89	64.48	61.35
5.....	25.90	29.35	21.53	24.79	25.59	74.13	71.15
6.....	23.94	27.27	19.79	22.64	23.52	83.61	80.83
7.....	22.08	25.27	18.15	20.69	21.62	92.54	89.97
8.....	20.31	23.33	16.58	18.88	19.83	101.16	98.83
9.....	18.60	21.43	15.07	17.16	18.11	109.71	107.63
10....	16.94	19.56	13.61	15.52	16.44	118.39	116.57
11....	15.30	17.67	12.17	13.90	14.77	127.39	125.85
12....	13.67	15.77	10.78	12.31	13.10	136.90	135.62
13....	12.03	13.84	9.43	10.73	11.43	147.12	146.05
14....	10.36	11.89	8.14	9.16	9.74	158.24	157.30
15....	8.66	9.91	6.86	7.60	8.06	169.54	168.68
16....	6.92	7.92	5.52	6.06	6.39	179.60	178.84
17....	5.17	5.93	4.15	4.52	4.74	187.72	187.12
18....	3.43	3.94	2.75	2.99	3.13	193.71	193.31
19....	1.70	1.96	1.36	1.49	1.54	197.44	197.25
20....	0	0	0	0	0	207.19	207.19

TABLE 4

DISTORTION IN BALANCE SHEET
STANDARD OF COMPARISON: ACTUAL EXPERIENCE

YEAR	ASSET DISTORTION				LIABILITY DISTORTION	SURPLUS DISTORTION			
	Static (1)	Dynamic (2)	ARM (3)	MARM (4)	(5)	Static (6)	Dynamic (7)	ARM (8)	MARM (9)
1.....	$1.72	−$0.14	$0.52	$0.36	−$1.11	$2.83	$0.97	$1.63	$1.47
2.....	2.88	− 2.55	0.18	0.31	− 2.62	5.50	0.07	2.80	2.93
3.....	3.52	− 3.78	0.24	0.12	− 3.09	6.61	− 0.69	2.85	3.21
4.....	3.54	− 4.35	− 0.76	− 0.12	− 3.13	6.67	− 1.22	2.37	3.01
5.....	3.45	− 4.37	− 1.11	− 0.31	− 2.98	6.43	− 1.39	1.87	2.67
6.....	3.33	− 4.15	− 1.30	− 0.42	− 2.78	6.11	− 1.37	1.48	2.36
7.....	3.19	− 3.93	− 1.39	− 0.46	− 2.57	5.76	− 1.36	1.18	2.11
8.....	3.02	− 3.73	− 1.43	− 0.48	− 2.33	5.35	− 1.40	0.90	1.85
9.....	2.83	− 3.53	− 1.44	− 0.49	− 2.08	4.91	− 1.45	0.64	1.59
10.....	2.62	− 3.33	− 1.42	− 0.50	− 1.82	4.44	− 1.51	0.40	1.32
11.....	2.37	− 3.13	− 1.40	− 0.53	− 1.54	3.91	− 1.59	0.14	1.01
12.....	2.10	− 2.89	− 1.36	− 0.57	− 1.28	3.38	− 1.61	− 0.08	0.71
13.....	1.81	− 2.60	− 1.30	− 0.60	− 1.07	2.88	− 1.53	− 0.23	0.47
14.....	1.53	− 2.22	− 1.20	− 0.62	− 0.94	2.47	− 1.28	− 0.26	0.32
15.....	1.25	− 1.80	− 1.06	− 0.60	− 0.86	2.11	− 0.94	− 0.20	0.26
16.....	1.00	− 1.40	− 0.86	− 0.53	− 0.76	1.76	− 0.64	− 0.10	0.23
17.....	0.76	− 1.02	− 0.65	− 0.43	− 0.60	1.36	− 0.42	− 0.05	0.17
18.....	0.51	− 0.68	− 0.44	− 0.30	− 0.40	0.91	− 0.28	− 0.04	0.10
19.....	0.26	− 0.34	− 0.21	− 0.16	− 0.19	0.45	− 0.15	− 0.02	0.03
20.....	0	0	0	0	0	0	0	0	0

point, these remarks have used prospective hindsight for comparisons, using the label *actual* just as Mr. Pharr did in his paper.

It is our view that *retrospective* hindsight would be more appropriate. Retrospective hindsight bases the calculation of the criterion for desired results on experience to date and the original assumption of the future. This definition would seem to stem from the audit guide's "lock-in" approach and would limit the effect of departures from original assumptions to those differences experienced to date.

Proponents of the prospective view might hold that audit guide theory should be limited to the determination of acceptable methods and should not include the criterion for measuring the accuracy of results. We would suggest, however, that approximate adjustment methods cannot be judged properly unless the criterion for acceptability follows the principles of the audit guide.

Table 5 shows the assets and reserve liabilities resulting from a retrospective hindsight calculation. It should be noted that each duration is calculated independently using terminations experienced up to that duration and original assumptions thereafter.

Table 6 shows asset, liability, and surplus distortions of the various adjustment methods when measured against retrospective hindsight as the criterion. Table 6 indicates that the proposed adjustment methods (ARM and MARM) yield results superior to existing methods not only for assets but for surplus as well. While MARM produces a better fit than ARM for assets, ARM is superior for surplus. Other plans and assumptions were tested and resulted in varying but similar results.

It should be noted that all the methods are techniques of approximating results that reflect experience. If retrospective hindsight is considered

TABLE 5

ASSETS AND LIABILITIES (PER $1,000 ISSUED)
BASED ON REDEFINED ACTUAL

Year	Asset	Liability	Year	Asset	Liability
1	$32.88	$ 27.49	11	$15.22	$127.47
2	32.61	43.44	12	13.52	137.04
3	31.17	54.53	13	11.87	147.27
4	28.58	64.42	14	10.28	158.31
5	26.43	74.04	15	8.66	169.53
6	24.40	83.50	16	6.95	179.55
7	22.46	92.43	17	5.20	187.67
8	20.58	101.07	18	3.44	193.68
9	18.76	109.66	19	1.70	197.44
10	16.98	118.40	20	0	207.19

TABLE 6

DISTORTION IN BALANCE SHEET
STANDARD OF COMPARISON: ACTUAL EXPERIENCE (REDEFINED)

YEAR	ASSET DISTORTION				LIABILITY DISTORTION	SURPLUS DISTORTION			
	Static (1)	Dynamic (2)	ARM (3)	MARM (4)	(5)	Static (6)	Dynamic (7)	ARM (8)	MARM (9)
1.....	$0.68	−$1.18	−$0.52	−$0.68	−$1.48	$2.16	$0.30	$0.96	$0.80
2.....	1.83	− 3.60	− 0.87	− 0.74	− 2.84	4.67	− 0.76	1.97	2.10
3.....	2.72	− 4.58	− 1.04	− 0.68	− 3.13	5.85	− 1.45	2.09	2.45
4.....	2.97	− 4.92	− 1.33	− 0.69	− 3.07	6.04	− 1.85	1.74	2.38
5.....	2.92	− 4.90	− 1.64	− 0.84	− 2.89	5.81	− 2.01	1.25	2.05
6.....	2.87	− 4.61	− 1.76	− 0.88	− 2.67	5.54	− 1.94	0.91	1.79
7.....	2.81	− 4.31	− 1.77	− 0.84	− 2.46	5.27	− 1.85	0.69	1.62
8.....	2.75	− 4.00	− 1.70	− 0.75	− 2.24	4.99	− 1.76	0.54	1.49
9.....	2.67	− 3.69	− 1.60	− 0.65	− 2.03	4.70	− 1.66	0.43	1.38
10.....	2.58	− 3.37	− 1.46	− 0.54	− 1.83	4.41	− 1.54	0.37	1.29
11.....	2.45	− 3.05	− 1.32	− 0.45	− 1.62	4.07	− 1.43	0.30	1.17
12.....	2.25	− 2.74	− 1.21	− 0.42	− 1.42	3.67	− 1.32	0.21	1.00
13.....	1.97	− 2.44	− 1.14	− 0.44	− 1.22	3.19	− 1.22	0.08	0.78
14.....	1.61	− 2.14	− 1.12	− 0.54	− 1.01	2.62	− 1.13	− 0.11	0.47
15.....	1.25	− 1.80	− 1.06	− 0.60	− 0.85	2.10	− 0.95	− 0.21	0.25
16.....	0.97	− 1.43	− 0.89	− 0.56	− 0.71	1.68	− 0.72	− 0.18	0.15
17.....	0.73	− 1.05	− 0.68	− 0.46	− 0.55	1.28	− 0.50	− 0.13	0.09
18.....	0.50	− 0.69	− 0.45	− 0.31	− 0.37	0.87	− 0.32	− 0.08	0.06
19.....	0.26	− 0.34	− 0.21	− 0.16	− 0.19	0.45	− 0.15	− 0.02	0.03
20.....	0	0	0	0	0	0	0	0	0

the correct answer, it is possible (though cumbersome) to report exact results on that basis, thereby eliminating any distortion.

Conclusion

Mr. Pharr's imaginative paper inspires our profession to apply fresh thinking to GAAP financial statements. These remarks are intended merely to continue the direction indicated by Mr. Pharr, and we agree that present methods can be improved. Further work in this area should produce exciting and worthwhile results.

APPENDIX A

MEAN AGGREGATE PREMIUM

Assuming that a block of business is issued in 1978 (average issue date of July 1), we will have the following information as of December 31, 1978:

Date	In-Force
July 1, 1978	$10,000
December 31, 1978	8,500

At the end of 1979, we will have the following information available:

Date	In-Force
July 1, 1978	$10,000
December 31, 1978	8,500
July 1, 1979	7,000
December 31, 1979	6,335

The following table shows Mr. Pharr's technique to obtain mean aggregate premium and our equivalent method.

Date	In-Force	Aggregate Premium (Pharr)		Equivalent Method	
		Terminal	Mean	Expected Premium to Next December 31	Mean Aggregate Premium
	(1)	(2)	(3)	(4)	(5)
July 1, 1978	$10,000	$10,000	.....	$5,000	.....
December 31, 1978	8,500	.....	$13,500	8,500	$13,500
July 1, 1979	7,000	17,000	.....	.....	.....
December 31, 1979	6,335	.....	19,835	6,335	19,835
July 1, 1980	5,670	22,670	.....	.....	.....

Column 3 of the above table is equal to the mean of the adjacent figures in column 2. However, at the December 31 dates the succeeding July 1 figures are unknown, and we fail to see how column 3 can be determined by this method.

Column 5 of the table is simply the cumulative sum of column 4 and is dependent only on the figures known at each December 31. As we indicated,

the results are identical, but we feel that the description of the technique might be somewhat clearer than the description given in the paper.

APPENDIX B

ASSUMPTIONS USED FOR ASSET AND LIABILITY CALCULATION (TWENTY-YEAR ENDOWMENT)

Interest rate: 6 percent (all years)
Gross premium per thousand: $40
Percent-of-premium expense for years 1–3: 95, 12.5, and 7.1 percent
All other expenses: 0
Mortality rates, lapse rates, and cash values as follows:

Year	Mortality Rate	Lapse Rate		Cash Value
		Expected	Actual	
1	0.00079	0.19921	0.29921	$ 1
2	0.00104	0.11896	0.18896	41
3	0.00131	0.07369	0.12369	82
4	0.00151	0.06349	0.09349	124
5	0.00172	0.05828	0.06828	167
6	0.00195	0.05305	0.06305	211
7	0.00226	0.04774	0.05774	257
8	0.00256	0.04244	0.05244	304
9	0.00293	0.03707	0.04707	352
10	0.00336	0.03164	0.04164	402
11	0.00384	0.02616	0.03616	453
12	0.00431	0.02569	0.03069	506
13	0.00480	0.02520	0.02520	561
14	0.00545	0.02955	0.01955	617
15	0.00622	0.03378	0.02378	675
16	0.00695	0.03805	0.03305	736
17	0.00766	0.04234	0.04234	798
18	0.00842	0.04658	0.05158	863
19	0.00926	0.05074	0.06074	930
20	0.01018	0.98981	0.98981	1,000

The above lapse assumptions were chosen so that the total decrements would be the same as those used by Mr. Pharr.

THOMAS G. KABELE:

The author is to be congratulated for his fine paper on methods of amortizing the deferred acquisition costs for GAAP accounting. Under the usual factor method the reported deferred acquisition cost (DAC) is computed by

$$DAC = V(\text{beginning in-force}),$$

where V is the "reserve factor." The author shows that, more generally, the deferred acquisition cost can be computed as

$$DAC = V(f), \tag{1}$$

where f is some function of the beginning in-force. The factor V is computed by

$$DAC' = V(f'), \tag{2}$$

where DAC' and f' are computed using the *expected* lapse rates rather than the *actual* lapse rates used to compute f. The author uses as examples

f_2 = Mean in-force and f_3 = Mean cumulative in-force

in place of the more familiar

f_1 = Beginning in-force.

In Tables 1 and 2, I have calculated the deferred acquisition cost using

f_4 = Mean doubly cumulative in-force.

TABLE 1

UNAMORTIZED ACQUISITION EXPENSE FACTORS

Year	Using Mean In-Force	Using Mean Cumulative In-Force	Using Mean Doubly Cumulative In-Force
1	$ 9,360.90	$6,017.72	$4,434.11
2	11,389.16	3,979.86	2,113.68
3	12,260.50	2,936.00	1,207.24
4	12,074.78	2,198.94	735.65
5	11,772.48	1,716.66	483.13
6	11,386.12	1,375.67	333.58
7	10,915.14	1,121.11	238.52
8	10,360.30	923.20	174.80
9	9,723.67	764.38	130.30
10	9,008.70	633.64	98.18
11	8,220.18	523.72	74.37
12	7,382.85	429.71	56.30
13	6,519.62	348.41	42.36
14	5,644.01	277.48	31.46
15	4,766.50	215.37	22.86
16	3,877.86	160.82	16.03
17	2,968.24	112.80	10.59
18	2,026.89	70.45	6.25
19	1,041.94	33.05	2.77
20	0	0	0

As shown in Table 2, f_4 produces even better results for the illustrative data than f_3 at every duration except the first. In certain cases, however, $_3$ and f_4 will not produce as good results as f_1 or f_2. For example, if the actual in-force suddenly goes to zero, then the actual deferred acquisition costs will go to zero. The functions f_1 and f_2 will produce zero results, but f_3 and f_4 will produce nonzero results.

It may be possible to produce a function of the in-force that always will give better results than f_1 or f_2. Perhaps the author can comment on this.

One disadvantage of using f_3 or f_4 or some other complicated function is that the factor V calculated by equation (2) is not intuitively meaningful. For f_1 or f_2 the factor V can be interpreted as the reserve per unit of in-force, and V can be compared with the benefit reserve factor.

Application to Benefit Reserves

One might think that one could apply Pharr's method to calculate the benefit or maintenance expense reserves. However, the deferred

TABLE 2

UNAMORTIZED ACQUISITION EXPENSE PROJECTIONS

Year	Actual	Expected—Static Method	Using Mean In-Force	Using Mean Cumulative In-Force	Using Mean Doubly Cumulative In-Force
1	$8,107.61	$8,424.81	$7,956.76	$8,123.92	$8,203.10
2	8,007.93	8,564.65	7,215.04	7,894.05	8,102.81
3	7,621.14	8,307.71	6,517.22	7,384.23	7,664.21
4	6,930.34	7,607.55	5,706.05	6,569.61	6,868.18
5	6,305.16	6,952.89	5,100.76	5,872.53	6,163.28
6	5,723.75	6,337.52	4,599.91	5,261.81	5,531.47
7	5,180.13	5,755.99	4,133.67	4,712.70	4,957.69
8	4,669.13	5,203.53	3,697.63	4,210.24	4,430.43
9	4,186.23	4,675.94	3,287.98	3,744.41	3,940.85
10	3,727.47	4,169.45	2,901.33	3,308.05	3,482.08
11	3,289.36	3,680.69	2,534.71	2,895.70	3,048.59
12	2,868.78	3,206.59	2,191.03	2,503.41	2,635.93
13	2,462.91	2,746.71	1,871.88	2,129.82	2,242.32
14	2,069.22	2,300.63	1,575.85	1,773.69	1,866.31
15	1,685.38	1,870.16	1,294.29	1,435.15	1,508.41
16	1,313.05	1,456.91	1,016.22	1,113.81	1,168.99
17	955.61	1,062.26	742.92	809.47	848.31
18	616.04	687.34	479.47	522.23	546.54
19	296.85	333.04	230.49	252.32	263.77
20	0	0	0	0	0

acquisition cost, also called the acquisition expense reserve, is of a different character. Deferred acquisition cost represents money that mostly has been spent (such as on issue, underwriting, or first-year commissions), whereas benefit and maintenance expense reserves represent money needed in the future. In my opinion, the benefit reserve should be independent of the previous lapse history of the cohort of policies being valued. It should depend only on the present in-force and expected future experience.

Terminology

Rather than the term *static*, I would prefer to use the term *aggregate* to denote the fact that the deferred acquisition cost is computed on an aggregate basis, independent of the amount of insurance. In place of *dynamic* or *aggregate revenue* I would use the term *factor method* with an appropriate modifier, as in *factor method based on mean in-force* or *factor method based on mean cumulative in-force.* In any case one should avoid, as Pharr has done, nondescriptive and prejudicial terminology like *accountant's worksheet* or *actuarial method.*

Tables and Appendix

Tables 1 and 2 are extensions of the author's Tables 2 and 4, respectively, showing in the last column the factors and reserves computed using f_4 (mean doubly cumulative in-force). The computer programs used to compute the data are shown in the Appendix. These programs also work for the case where the interest is nonzero.

APPENDIX

```
      ∇AMORT[□]∇
   ∇ AMORT
[1]   G←10000
[2]   INT←0
[3]   APER←0.95 0.125,(50÷704),17⍴0
[4]   A **** EXPECTED RESERVE *****
[5]   TEMP← 970 970 970 965 960 , 955 950 945 940 900
[6]   QW←1−0.001× 800 880 925 935 940 , 945 950 955 960 965 ,TEMP
[7]   ARES←INT RESERVE QW
[8]   A **** FACTORS = EXPECTED RESERVE ÷ EXPECTED INFORCE ****
[9]   FACTΔM←ARES÷AVE←0.5×(LIFE+1↓LIFE,0)
[10]  CUM1←+ LIFE
[11]  FACTΔCUM1←ARES÷AVE1←0.5×(CUM1+1↓CUM1,0)
[12]  CUM2←+ CUM1
[13]  FACTΔCUM2←ARES÷AVE2←0.5×(CUM2+1↓CUM2,0)
[14]  A ***** ACTUAL RESERVE ******
[15]  TEMP← 960 965 970 975 970 , 960 950 940 930 900
```

```
[16]  QW←1−0.001× 700 810 875 905 930 , 935 940 945 950 955 ,TEMP
[17]  ARES2←INT RESERVE QW
[18]  ⍝ ***** REPORTED RESERVES = FACTORS × ACTUAL INFORCE ****
[19]  ARESΔM←FACTΔM×AVE←0.5×(LIFE+1 ↓ LIFE,0)
[20]  CUM1←+ LIFE
[21]  ARESΔCUM1←FACTΔCUM1×AVE1←0.5×(CUM1+1 ↓ CUM1,0)
[22]  CUM2←+ CUM1
[23]  ARESΔCUM2←FACTΔCUM2×AVE2←0.5×(CUM2+1 ↓ CUM2,0)
      ∇
      ∇RESERVE[□]∇
      ∇ ARES←INT RESERVE QW
[1]   ⍝ SEE POSNAK, GAAP STOCK LIFE INSURANCE COS., P 302
[2]   LIFE←¯1 ↓ × 1,1−QW
[3]   VECT←× 1,(⍴LIFE)⍴÷1+INT
[4]   PVΔPREM←(¯1 ↓ VECT)×PREM←G×LIFE
[5]   PVΔAEXP←(¯1 ↓ VECT)×AEXP←APER×PREM
[6]   ANP←(+/PVΔAEXP)÷(+/PVΔPREM)
[7]   ⍝ ARES←(+ PVΔAEXP−PVΔPREM×ANP)÷1 ↓ VECT
[8]   ARES←(1 ↓ (Φ+ Φ(PVΔPREM×ANP)−PVΔAEXP)÷¯1 ↓ VECT),0
      ∇
```

(AUTHOR'S REVIEW OF DISCUSSION)

JOE B. PHARR:

My personal thanks and appreciation to Donald Sondergeld, Claude Paquin, Alan Goldberg and Lester Moskowitz, and Thomas Kabele for taking the time and showing interest in the subject of GAAP acquisition expense amortization methodology by preparing written discussions. These discussions clarify, expand upon, and correct generalizations in the paper. Since each of these discussions presents rather clearly the views of each of the discussants, they are commended to anyone scanning this author's review. (Perhaps it is most appropriate to caution the reader to consider carefully the views and points made by the discussants.) For the busy reader, it might be helpful if the author were to summarize the principal points made in the discussions, with further direct reference to the discussions suggested as circumstances dictate.

Don Sondergeld provides formulas for analyzing the different amortization methods and makes a number of important observations. He also points out that the illustrative figures in the paper are but two cases from a number of relationships that can exist and indicates that in his view possible generalizations are rather limited.

Claude Paquin no doubt would appreciate the development of formulas by Don Sondergeld as supporting rationale for situations where the different methods of acquisition expense amortization tend to work well or do not work well. However, it hardly seems fair or appropriate for

Mr. Paquin to indicate that an aggregate revenue method that is based on reasonable expectations of life insurance persistency falls in the same category with straight-line, sum-of-the-digits, or double declining balance depreciation assumptions, or would be acceptable to knowledgeable accountants for GAAP reporting purposes.

Alan Goldberg and Lester Moskowitz present a modified accumulated revenue method that provides better results than the other three methods when actual deferred acquisition cost asset figures are compared. Although the rationale for the modified method is not obvious to the author at present, it does work well in the illustration based on the adverse (but not uncommon according to the author's experiences) persistency assumptions used in the body of the paper. The suggestion of a "retrospective hindsight" method of calculating GAAP asset and liability factors is quite interesting, although the practical implementation and auditing aspects are rather overwhelming. Further, consideration of such a method as a standard approach for providing meaningful GAAP financial statements deserves further study by accountants and actuaries.

Tom Kabele has extended the calculations in the paper to illustrate improved results from a "mean doubly cumulative in-force" approach. The difference that he notes in the character of the deferred acquisition cost asset and benefit liability has led the author to believe that generalizations about balance-sheet offsets of asset changes against liability changes should not be depended upon in practice.

Recently, a knowledgeable and nationally recognized accounting partner emphasized to a group of actuaries that the orientation of life company GAAP financial statements was toward a fair, reasonable, and consistent presentation of a stream of earnings. Furthermore, it was emphasized that the components of the earnings stream also must be deemed reasonable, fair, and consistent—the GAAP expense and benefit costs each should show these three characteristics.

Last, after considerable reflection the author continues to be of the opinion that the adverse persistency assumptions used in the body of this paper are encountered frequently in practice. Therefore, the implication that one should proceed with caution in adopting a method of acquisition expense amortization should be heeded if meaningful expense and benefit cost components are to be presented to management and the public via GAAP financial statements.