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CHANGING PREMIUM VALUATION METHOD

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ABSTRACT

This paper discusses the determination of term insurance reserves in the United States for individual life insurance policies. Attention is called to the problems that can arise by using valuation net premiums that are a uniform percentage of the gross premiums when applying either the net level premium reserve method or the Commissioners Reserve Valuation Method (CRVM) to term insurance policies with varying premiums. Such problems are encountered when calculating reserves on renewable term insurance plans, which now are considered by many actuaries and insurance departments to be continuous policies with varying premiums.

A valuation method that appears to produce better results on term insurance policies is suggested. The method is referred to as the changing premium valuation method (CPVM). Under that method, the valuation net premiums are not related directly to the slope of the gross premium scale. Instead, the net premium is level until the gross premium changes, at which time a new net premium is determined. For each period during which the gross premiums do not change, the present value of net premiums equals the present value of benefits. This also means that the gross premium may be less than the net premium in some policy years, in contrast to the "all or nothing" result that occurs if net premiums are a uniform percentage of varying gross premiums.

I. RENEWABLE TERM HISTORY

For many years, renewable term insurance was renewable at the insurance company's option, but today such insurance generally is renewable at the policyholder's option at guaranteed premium rates. This means, for example, that term insurance that is renewable to age 70 at guaranteed premium rates also can be looked upon as varying premium term-to-age-70 insurance.

Nevertheless, the valuation practices that were developed when term insurance was not guaranteed renewable at guaranteed rates have continued to be utilized. A five-year renewable term policy that was renewable at the insurer's option was valued as a five-year term policy, looking only at benefits and premiums of the initial five-year period. If the policy were renewed, it was viewed as a new five-year term policy and valued accordingly. Similarly, a one-year renewable term policy usually was valued as a one-year term policy.

One problem that has developed has been generated by the differing opinions as to how deficiency reserve laws should be applied to renewable term insurance. For instance, in the case of one-year renewable term insurance, are there future net premiums after the first policy year? There are opposing views on this matter. The National Association of Insurance Commissioners and the American Council of Life Insurance currently are attempting to eliminate the confusion and the inconsistent treatment that exists from state to state. Some state insurance departments have indicated that renewable term insurance should be treated for reserve purposes as a continuous contract for the total period during which premium rates are guaranteed. This paper is based on that view.

II. UNIFORM PERCENTAGE METHOD ON RENEWABLE TERM

The CRVM is the valuation method defined in the standard valuation law. It is defined in terms of policies providing a uniform amount of insurance and requiring the payment of uniform premiums. For these policies the law states that the modified net premiums should be a uniform percentage of the gross premiums. The law then indicates that "reserves according to the CRVM for life insurance providing for varying amounts of insurance or requiring the payment of varying premiums . . . shall be calculated by a consistent method." Walter O. Menge's interpretive paper "Commissioners Reserve Valuation Method" (*RAIA*, XXXV, 258) presents a way to apply this uniform percentage method to plans of insurance not characterized by uniform premiums and uniform amounts of insurance. A discussion of that paper is contained in *RAIA*, XXXVI, 101.

III. WHAT'S IN A NAME?

Although many companies have a valuation method that is related to the policy name, it is difficult for me to accept the fact that reserves may be substantially different on policies that have identical benefits and premiums. Consider three companies that file different policy forms that all provide $1,000 of coverage for ten years. The policies differ only in their names. One company calls it a ten-year term policy, the second a five-year renewable term policy, and the third a one-year renewable term policy. Each company has guaranteed *level* gross premiums at issue age 25 of $5 per thousand for ten years. We could have three different results if the valuation method is related to the name of the policy. The following figures are based on a 1958 CSO 3 percent net level reserve basis:

Duration	Ten-Year Term Policy		Five-Year Renewable Term Policy		One-Year Renewable Term Policy	
	Net Premium	Mean Reserve	Net Premium	Mean Reserve	Net Premium	Mean Reserve
1.....	$2.05	$1.12	$1.94	$1.00	$1.87	$0.94
2.....	2.05	1.29	1.94	1.05	1.90	0.95
3.....	2.05	1.44	1.94	1.07	1.93	0.97
4.....	2.05	1.56	1.94	1.06	1.97	0.99
5.....	2.05	1.63	1.94	1.01	2.02	1.01
6.....	2.05	1.66	2.19	1.15	2.07	1.04
7.....	2.05	1.64	2.19	1.25	2.13	1.07
8.....	2.05	1.55	2.19	1.29	2.18	1.09
9.....	2.05	1.40	2.19	1.26	2.22	1.13
10....	2.05	1.17	2.19	1.17	2.33	1.17

It would seem that the reserve should be the same for all three policy forms. Some actuaries will argue that each of these three policies with different names should be valued as a ten-year term policy, irrespective of whether the gross premiums are level or varying. If the gross premiums vary, they would apply the uniform percentage approach to the varying gross premium structure.

IV. IMPACT OF GROSS PREMIUMS ON RESERVES

A number of companies are selling renewable term insurance with premium rates guaranteed to high ages such as age 95. Such a policy can be valued legally using the CRVM with the Menge interpretation (i.e., using net premiums that are a uniform percentage of gross premiums in years after the first). In this case, the policy is considered as a varying premium term-to-age-95 policy. Unfortunately, this permits the level of the basic reserve and deficiency reserve to be manipulated by the choice of the gross premium scale.

Consider a policy with two possible scales of varying gross premiums: Scale A and Scale B. The two scales are identical except that in the last policy year the gross premium is higher under Scale B than under Scale A. If net premiums are a uniform percentage of gross premiums, the reserves associated with Scale B will be less than those for Scale A at all durations except the last, where they become equal. Any deficiency reserves developed under Scale A will be reduced or eliminated by using Scale B.

Examples illustrating this principle are included in the Appendix. It seems to me that these examples demonstrate the need for a valuation method other than the uniform percentage method. Also, as negative terminal reserves can be generated, it is suggested there be a statutory requirement that the year-end reserve equal at least the cost of insurance based upon the net amount at risk for the balance of the policy year.

V. CHANGING PREMIUM VALUATION METHOD (CPVM)

The method that I recommend could apply to all term insurance policies that do not contain cash values. Under this method, the valuation net premiums change when the guaranteed gross premiums change. (For renewable term insurance, this may or may not coincide with the end of a renewal period.) During each period for which the gross premiums do not change, the valuation net premiums for the period are level, and the present value of such net premiums is equal to the present value of the benefits for the period. In fact, this is a method often used for renewable term insurance where the gross premiums change but are level during each term period.

If net premiums are a uniform percentage of gross premiums, deficiency reserves can be reduced or eliminated by increasing the gross premiums in later policy years. However, under the CPVM the gross premium may be less than the net premium at some, but not all, durations. The total reserve under the CPVM would equal the present value of future benefits less the present value of all future "premiums." The "premium" applicable to any contract year would be the changing premium net premium, or the gross premium if smaller.

A *modified* CPVM could be developed to recognize the nonlevel incidence of expenses. The modification period would not extend past the point where renewal premiums first change. For example, if the first ten gross premiums were 5, 5, 5, 5, 7, 7, 9, 9, 9, and 9, respectively, there would be three net premiums on an unmodified basis—one for the first four policy years, one for the next two, and one for the last four. However, on a modified basis (1) the first-year net premium would be the cost of insurance, (2) for the next three policy years a different changing premium net premium would be calculated based on the benefits for those years, and (3) for the last six policy years the unmodified changing premium net premiums would be used. If the second gross premium were different from the first gross premium, there would be no modification period.

APPENDIX

The purpose of this Appendix is to demonstrate how reserves can be manipulated by the choice of the gross premium scale if one-year renewable term is considered a continuous policy with varying premiums and with reserves calculated using what has been referred to as the uniform percentage method.

The CPVM produces better results. (For this plan, the CPVM is identical to the yearly renewable term method that is used by many actuaries to value one-year renewable term, where each year the net premium covers the cost of insurance for that year.) For example, under the uniform percentage method it is possible to adjust the slope of the gross premium scale and the size of later gross premiums in such a way as to reduce or eliminate deficiency reserves and also generate reserves that are less than the cost of insurance. This Appendix compares reserves that were calculated on sixteen different cases using both valuation methods.

The new valuation model regulation or legislation has eliminated the definition of a separate deficiency reserve by substituting the gross premium for the valuation net premium in the prospective reserve calculation whenever the gross premium is less than the valuation net premium. However, to facilitate the comparison of the two methods, a basic reserve, a deficiency reserve, and a total reserve were determined separately under both methods.

When negative terminal reserves were generated under the uniform percentage method, a mean reserve floor of half the net premium was imposed in the basic reserve calculation. Also, the total reserve under the uniform percentage method has a mean reserve floor of half the net premium if there are no deficiency reserves, and half the gross premium if there are deficiency reserves. This assumes that negative terminal reserves are set equal to zero. In many cases, this generates mean reserves that are less than half the cost of insurance. I believe there should be a statutory mean reserve floor of half the cost of insurance, although that floor was not used in the calculations on the uniform percentage method. It is implicit in the yearly renewable term method.

GENERAL

The plan chosen was yearly renewable term (YRT) to age 95 issued at age 35. The mortality table was 1958 CSO, and the interest rate was $3\frac{1}{2}$ percent. The uniform percentage method was applied to all premiums on a net level method, rather than only to renewal net premiums as would have been the case if the CRVM had been used. If the gross premium scales for attained ages and issue ages are the same, the uniform percentage method can be thought of as the CRVM for issue age 34.

Three tables are presented. Table 1 contains a summary of valuation results on sixteen cases that were chosen to illustrate different relationships between the gross premiums and the valuation net premiums. Tables 2 and 3 show details on two of the sixteen cases that are summarized in Table 1.

DEFINITIONS

GP_t = Gross premium for policy year t ($GP_{t+1} \neq GP_t$).

$CPNP_t$ = Changing premium net premium for policy year t. This equals the YRT net premium ($YRTNP_t$), since the gross premium changes each year.

UNP_t = Uniform percentage net premium for policy year t.

UR = Uniform ratio. This is the ratio at issue of the present value of all GPs to the present value of all UNPs. It also is the reciprocal of the uniform percentage referred to in the standard valuation law, applied on a net level basis.

DESCRIPTION OF VALUATION METHODS

Uniform Percentage Method

1. The *basic reserve* uses valuation net premiums that are a uniform percentage of the actual gross premiums for attained ages 35, 36, . . . , 94. The slope of the gross premium scale affects each valuation net premium and therefore the reserve in each renewal year. The *formula mean* was calculated using the normal reserve formula. The *actual mean* is the larger of the formula mean and one-half the UNP.

2. The *deficiency reserve* is zero at all durations if the UR is greater than or equal to 1.00. It is some positive value at each duration if the UR is less than 1.00.

3. The *total mean reserve* is equal to the actual mean, if there are no deficiency reserves. It is equal to the larger of (*a*) one-half the GP and (*b*) the formula mean plus deficiency reserve, if there are deficiency reserves.

Changing Premium Valuation Method

1. The *basic reserve* uses valuation net premiums that equal the one-year term cost of insurance, that is, $C_x \div D_x$. These reserves are not affected by either the slope of the actual gross premiums or their size. The mean reserve is half the net premium.

2. The *deficiency reserve* is the present value of future deficiencies. Any excesses of gross premiums over net premiums are not used to offset deficiencies.

3. The *total mean reserve* is the sum of items 1 and 2.

GENERAL OBSERVATIONS ON NET PREMIUMS AND TERMINAL RESERVES

In the first policy year the basic reserve under the uniform percentage method is greater than that under the CPVM if $\text{UNP}_1 > \text{CPNP}_1$. This also means that $\text{GP}_1/\text{CPNP}_1 > \text{GP}_1/\text{UNP}_1 = \text{UR}$. The basic reserve on the uniform percentage method in the first policy year is negative if $\text{UNP}_1 > \text{CPNP}_1$. In fact, if the first m UNPs are each less than the first m CPNPs, the basic reserves on the uniform percentage method will be negative for the first m years. This becomes obvious when reserves are looked at retrospectively, since net premiums are less than benefits in each of these m years. In this case involving one-year term insurances, the CPNPs are equal to the benefits. Also, if the last n UNPs are each greater than the last n CPNPs, the basic reserves on the uniform percentage method will be negative for these n years. Looking at reserves prospectively, we see that benefits are less than net premiums in each of these n years.

UNPs cannot be all less (or all greater) than corresponding CPNPs, since their present values at issue are equal. The relative size of UNP_t and CPNP_t

therefore must change at least once during the life of the policy if the premiums vary. This excludes the case where UNP_t and $CPNP_t$ are equal each year.

If there are deficiency reserves on the uniform percentage method, they will occur at all durations. This means there must be deficiency reserves at some durations under the CPVM; this is because each of the CPNPs cannot be less than each of the deficient UNPs, since their present values at issue are equal.

If there are deficiency reserves on the uniform percentage method, total reserves are the same under both methods beginning at that duration where all future CPNPs are deficient.

[Tables 1–3 follow]

TABLE 1

COMPARATIVE RESULTS OF 16 CASES

CASE	RATIO OF GP TO CPNP*			RATIO OF GP TO UNP, ALL DURATIONS	FLOOR (DURATIONS)†	UNIFORM PERCENTAGE METHOD RELATIVE TO CPVM:		
	Duration 1	Duration 30	Duration 60			Basic Reserves	Deficiency Reserves	Total Reserves‡
I	1.10	1.100	1.10	1.1000	....	E	N	E
II	0.95	0.950	0.95	0.9500	....	E	E	E
III	1.02	1.060	1.10	1.0607	All	S-31, L-29	N	S-31, L-29
IV	1.10	1.060	1.02	1.0593	....	L	N	L
V	0.98	0.965	0.95	0.9648	....	L	S	E
VI	0.95	0.965	0.98	0.9652	All	S-31, L-29	L	E
VII	1.10	1.025	0.95	1.0238	....	L	S	S-7, L-53
VIII	0.95	1.025	1.10	1.0262	All	S-31, L-29	S-19, N-41	S-31, L-29
IX	1.02	0.985	0.95	0.9844	....	L	S	S-16, E-44
X	0.95	0.985	1.02	0.9856	All	S-31, L-29	S-4, L-56	S-43, L-17
XI	1.05	0.950	1.05	0.9873	Last 29	L-30, S-11, L-19	S-34, L-26	S-45, L-15
XII	1.10	0.950	1.10	1.0060	Last 29	L-30, S-11, L-19	S-39, N-21	S-41, L-19
XIII	1.02	0.980	1.30	1.0495	All	S-37, L-23	S-31, N-29	S-37, L-23
XIV	0.98	1.020	0.70	0.9505	....	L	S	S-31, E-29
XV	0.95	1.020	0.95	0.9939	First 31	S-19, L-41	S	S-38, E-22
XVI	0.95	1.050	0.95	1.0127	First 31	S-19, L-41	S	S-34, L-26

DEFINITIONS.—E: equal at all durations; N: no deficiency reserves under either method; S: smaller at all durations; L: larger at all durations; S-x, L-y: smaller for the first x durations, larger for the last y durations.

* The ratio grades linearly from duration 1 to duration 30 and then from duration 30 to duration 60.

† A mean reserve floor of one-half the uniform percentage net premium was utilized under the uniform percentage method in determining the basic reserve at the durations indicated, since there were negative terminal reserves.

‡ The total reserve was calculated by adding the *unfloored* basic reserve to the deficiency reserve. For the last 26 durations in Case X and the last 23 durations in Case XI, this total then was increased due to the mean reserve floor of one-half the gross premium.

TABLE 2

CASE VIII

t	Gross Premium (GP_t)	Net Premium		CPVM Mean Reserve			Uniform Percentage Method Mean Reserve (UR = 1.0262)			
							Basic		Deficiency	Total
		$CPNP_t$	UNP_t	Basic	Deficiency	Total	Formula	Actual*		
1	2.30	2.43	2.25	1.21	1.47	2.69	1.03	1.12	0.00	1.12
2	2.43	2.55	2.37	1.28	1.40	2.68	0.90	1.18	0.00	1.18
3	2.58	2.71	2.52	1.35	1.33	2.69	0.77	1.26	0.00	1.26
4	2.79	2.91	2.71	1.45	1.26	2.71	0.65	1.36	0.00	1.36
5	3.02	3.14	2.94	1.57	1.18	2.75	0.53	1.47	0.00	1.47
6	3.28	3.41	3.20	1.71	1.10	2.80	0.41	1.60	0.00	1.60
7	3.58	3.71	3.49	1.86	1.01	2.86	0.29	1.75	0.00	1.75
8	3.90	4.03	3.80	2.01	0.92	2.93	0.15	1.90	0.00	1.90
9	4.25	4.38	4.14	2.19	0.82	3.01	0.01	2.07	0.00	2.07
10	4.63	4.75	4.51	2.38	0.73	3.10	−0.14	2.25	0.00	2.25
11	5.04	5.17	4.92	2.58	0.63	3.21	−0.29	2.46	0.00	2.46
12	5.51	5.63	5.37	2.82	0.53	3.35	−0.44	2.69	0.00	2.69
13	6.03	6.14	5.87	3.07	0.43	3.51	−0.60	2.94	0.00	2.94
14	6.60	6.71	6.44	3.36	0.34	3.70	−0.75	3.22	0.00	3.22
15	7.24	7.34	7.06	3.67	0.25	3.92	−0.90	3.53	0.00	3.53
16	7.95	8.04	7.75	4.02	0.17	4.19	−1.05	3.87	0.00	3.87
17	8.73	8.80	8.50	4.40	0.10	4.50	−1.20	4.25	0.00	4.25
18	9.57	9.62	9.32	4.81	0.04	4.86	−1.35	4.66	0.00	4.66
19	10.49	10.52	10.22	5.26	0.01	5.27	−1.50	5.11	0.00	5.11
20	11.49 <	11.50	11.19	5.75	0.00	5.75	−1.64	5.60	0.00	5.60

NOTE.—At issue age 35 the gross premium grades uniformly from 0.95 of the YRT premium at duration 1 to 1.10 of the YRT premium at duration 60.

* Set equal to one-half UNP_t.

TABLE 2—*Continued*

t	Gross Premium (GP_t)	Net Premium		CPVM Mean Reserve			Uniform Percentage Method Mean Reserve (UR = 1.0262)			
							Basic		Deficiency	Total
		$CPNP_t$	UNP_t	Basic	Deficiency	Total	Formula	Actual*		
21....	12.58 >	12.56	12.26	6.28	0.00	6.28	−1.77	6.13	0.00	6.13
22....	13.79	13.73	13.44	6.86	0.00	6.86	−1.88	6.72	0.00	6.72
23....	15.12	15.01	14.73	7.51	0.00	7.51	−1.97	7.37	0.00	7.37
24....	16.58	16.43	16.16	8.21	0.00	8.21	−2.04	8.08	0.00	8.08
25....	18.18	17.96	17.71	8.98	0.00	8.98	−2.07	8.86	0.00	8.86
26....	19.94	19.65	19.43	9.83	0.00	9.83	−2.07	9.72	0.00	9.72
27....	21.86	21.49	21.30	10.74	0.00	10.74	−2.03	10.65	0.00	10.65
28....	23.95	23.49	23.34	11.74	0.00	11.74	−1.94	11.67	0.00	11.67
29....	26.25	25.67	25.58	12.84	0.00	12.84	−1.80	12.79	0.00	12.79
30....	28.76	28.06	28.02	14.03	0.00	14.03	−1.59	14.01	0.00	14.01
31....	31.44	30.68 >	30.64	15.34	0.00	15.34	−1.37	15.32	0.00	15.32
32....	34.49	33.57 <	33.61	16.78	0.00	16.78	−1.06	16.80	0.00	16.80
33....	37.86	36.75	36.89	18.38	0.00	18.38	−0.64	18.45	0.00	18.45
34....	41.59	40.27	40.53	20.14	0.00	20.14	−0.10	20.26	0.00	20.26
35....	45.63	44.07	44.46	22.03	0.00	22.03	0.54	22.23	0.00	22.23
36....	49.93	48.11	48.65	24.05	0.00	24.05	1.26	24.33	0.00	24.33
37....	54.44	52.32	53.05	26.16	0.00	26.16	2.04	26.52	0.00	26.52
38....	59.11	56.67	57.60	28.33	0.00	28.33	2.85	28.80	0.00	28.80
39....	63.91	61.12	62.28	30.56	0.00	30.56	3.69	31.14	0.00	31.14
40....	68.99	65.82	67.23	32.91	0.00	32.91	4.63	33.62	0.00	33.62

Note.—At issue age 35 the gross premium grades uniformly from 0.95 of the YRT premium at duration 1 to 1.10 of the YRT premium at duration 60.

* Set equal to one-half UNP_t.

TABLE 2—*Continued*

t	Gross Premium (GP_t)	Net Premium		CPVM Mean Reserve			Uniform Percentage Method Mean Reserve (UR = 1.0262)			
							Basic		Deficiency	Total
		$CPNP_t$	UNP_t	Basic	Deficiency	Total	Formula	Actual*		
41....	74.49	70.89	72.59	35.44	0.00	35.44	5.74	36.30	0.00	36.30
42....	80.59	76.50	78.53	38.25	0.00	38.25	7.12	39.27	0.00	39.27
43....	87.44	82.80	85.21	41.40	0.00	41.40	8.84	42.60	0.00	42.60
44....	95.18	89.91	92.75	44.96	0.00	44.96	10.98	46.38	0.00	46.38
45....	103.75	97.77	101.10	48.88	0.00	48.88	13.50	50.55	0.00	50.55
46....	113.04	106.26	110.15	53.13	0.00	53.13	16.38	55.08	0.00	55.08
47....	122.97	115.31	119.83	57.66	0.00	57.66	19.58	59.91	0.00	59.91
48....	133.41	124.80	130.00	62.40	0.00	62.40	23.07	65.00	0.00	65.00
49....	144.30	134.67	140.61	67.33	0.00	67.33	26.83	70.31	0.00	70.31
50....	155.68	144.94	151.70	72.47	0.00	72.47	30.93	75.85	0.00	75.85
51....	167.64	155.69	163.35	77.85	0.00	77.85	35.45	81.68	0.00	81.68
52....	180.22	166.98	175.61	83.49	0.00	83.49	40.49	87.81	0.00	87.81
53....	193.52	178.87	188.57	89.43	0.00	89.43	46.20	94.29	0.00	94.29
54....	207.73	191.55	202.42	95.77	0.00	95.77	52.81	101.21	0.00	101.21
55....	223.15	205.28	217.44	102.64	0.00	102.64	60.69	108.72	0.00	108.72
56....	240.19	220.42	234.05	110.21	0.00	110.21	70.40	117.02	0.00	117.02
57....	259.36	237.46	252.73	118.73	0.00	118.73	82.74	126.37	0.00	126.37
58....	281.30	256.94	274.11	128.47	0.00	128.47	98.99	137.06	0.00	137.06
59....	306.75	279.52	298.90	139.76	0.00	139.76	121.21	149.45	0.00	149.45
60....	336.55	305.95	327.94	152.98	0.00	152.98	152.98	163.97	0.00	163.97

TABLE 3

CASE X

t	Gross Premium (GP_t)	Net Premium		CPVM Mean Reserve			Uniform Percentage Method Mean Reserve (UR = 0.9856)			
							Basic		Deficiency	Total
		$CPNP_t$	UNP_t	Basic	Deficiency	Total	Formula	Actual*		
1......	2.30	2.43	2.34	1.21	4.81	6.02	1.12	1.17	4.47	5.59
2......	2.43	2.55	2.46	1.28	4.86	6.14	1.09	1.23	4.60	5.70
3......	2.58	2.71	2.61	1.35	4.91	6.27	1.07	1.31	4.74	5.81
4......	2.77	2.91	2.81	1.45	4.96	6.42	1.06	1.41	4.88	5.94
5......	3.00	3.14	3.04	1.57	5.01	6.58	1.07	1.52	5.02	6.09
6......	3.26	3.41	3.31	1.71	5.05	6.75	1.08	1.65	5.16	6.24
7......	3.55	3.71	3.60	1.86	5.08	6.94	1.09	1.80	5.31	6.41
8......	3.86	4.03	3.92	2.01	5.11	7.13	1.11	1.96	5.46	6.57
9......	4.20	4.38	4.26	2.19	5.13	7.32	1.13	2.13	5.62	6.75
10.....	4.57	4.75	4.63	2.38	5.15	7.53	1.16	2.32	5.77	6.93
11.....	4.97	5.17	5.05	2.58	5.16	7.74	1.19	2.52	5.93	7.12
12.....	5.43	5.63	5.51	2.82	5.16	7.97	1.23	2.75	6.09	7.32
13.....	5.93	6.14	6.01	3.07	5.15	8.22	1.29	3.01	6.25	7.54
14.....	6.48	6.71	6.58	3.36	5.13	8.48	1.36	3.29	6.42	7.78
15.....	7.10	7.34	7.20	3.67	5.10	8.77	1.45	3.60	6.59	8.04
16.....	7.78	8.04	7.90	4.02	5.06	9.08	1.55	3.95	6.76	8.31
17.....	8.53	8.80	8.66	4.40	5.00	9.40	1.68	4.33	6.93	8.60
18.....	9.34	9.62	9.48	4.81	4.94	9.75	1.82	4.74	7.10	8.91
19.....	10.22	10.52	10.37	5.26	4.86	10.12	1.98	5.19	7.27	9.25
20.....	11.19	11.50	11.35	5.75	4.77	10.52	2.16	5.68	7.44	9.60

NOTE.—At issue age 35 the gross premium grades uniformly from 0.95 of the YRT premium at duration 1 to 1.02 of the YRT premium at duration 60.

* Set equal to one-half UNP_t.

TABLE 3—*Continued*

t	Gross Premium (GP_t)	Net Premium		CPVM Mean Reserve			Uniform Percentage Method Mean Reserve (UR = 0.9856)			
							Basic		Deficiency	Total
		$CPNP_t$	UNP_t	Basic	Deficiency	Total	Formula	Actual*		
21	12.24	12.56	12.41	6.28	4.67	10.95	2.37	6.21	7.62	9.99
22	13.39	13.73	13.59	6.86	4.55	11.41	2.62	6.79	7.79	10.41
23	14.66	15.01	14.88	7.51	4.42	11.93	2.90	7.44	7.97	10.87
24	16.06	16.43	16.29	8.21	4.27	12.49	3.23	8.15	8.14	11.37
25	17.58	17.96	17.84	8.98	4.12	13.10	3.61	8.92	8.32	11.92
26	19.26	19.65	19.54	9.83	3.94	13.77	4.05	9.77	8.49	12.53
27	21.09	21.49	21.40	10.74	3.76	14.50	4.54	10.70	8.66	13.20
28	23.08	23.49	23.42	11.74	3.56	15.31	5.09	11.71	8.83	13.92
29	25.26	25.67	25.63	12.84	3.35	16.19	5.72	12.81	8.99	14.72
30	27.64	28.06	28.04	14.03	3.14	17.16	6.44	14.02	9.16	15.60
31	30.22	30.68 >	30.66	15.34	2.87	18.21	7.22	15.33	9.32	16.54
32	33.10	33.57 <	33.59	16.78	2.59	19.38	8.11	16.79	9.47	17.59
33	36.29	36.75	36.82	18.38	2.31	20.68	9.13	18.41	9.63	18.76
34	39.81	40.27	40.39	20.14	2.01	22.14	10.30	20.20	9.77	20.07
35	43.62	44.07	44.26	22.03	1.71	23.74	11.59	22.13	9.91	21.81†
36	47.67	48.11	48.37	24.05	1.41	25.46	12.98	24.19	10.04	23.84†
37	51.91	52.32	52.67	26.16	1.11	27.27	14.44	26.34	10.17	25.96†
38	56.30	56.67	57.12	28.33	0.83	29.16	15.95	28.56	10.29	28.15†
39	60.79	61.12	61.68	30.56	0.57	31.13	17.50	30.84	10.41	30.40†
40	65.54	65.82	66.50	32.91	0.34	33.25	19.17	33.25	10.52	32.77†

† Set equal to one-half GP_t.

TABLE 3—*Continued*

t	Gross Premium (GP_t)	Net Premium		CPVM Mean Reserve			Uniform Percentage Method Mean Reserve (UR = 0.9856)			
							Basic		Deficiency	Total
		$CPNP_t$	UNP_t	Basic	Deficiency	Total	Formula	Actual*		
41.....	70.68	70.89	71.72	35.44	0.16	35.61	21.01	35.86	10.62	35.34†
42.....	76.37	76.50	77.49	38.25	0.04	38.29	23.12	38.74	10.71	38.19†
43.....	82.76 <	82.80	83.97	41.40	0.00	41.40	25.58	41.99	10.80	41.38†
44.....	89.98 >	89.91	91.29	44.96	0.00	44.96	28.44	45.65	10.87	44.99†
45.....	97.95	97.77	99.39	48.88	0.00	48.88	31.69	49.69	10.92	48.98†
46.....	106.59	106.26	108.15	53.13	0.00	53.13	35.27	54.08	10.95	53.30†
47.....	115.81	115.31	117.51	57.66	0.00	57.66	39.16	58.75	10.96	57.91†
48.....	125.49	124.80	127.33	62.40	0.00	62.40	43.29	63.66	10.96	62.74†
49.....	135.57	134.67	137.56	67.33	0.00	67.33	47.65	68.78	10.92	67.79†
50.....	146.09	144.94	148.22	72.47	0.00	72.47	52.28	74.11	10.85	73.04†
51.....	157.11	155.69	159.41	77.85	0.00	77.85	57.25	79.71	10.74	78.56†
52.....	168.70	166.98	171.17	83.49	0.00	83.49	62.60	85.59	10.58	84.35†
53.....	180.94	178.87	183.58	89.43	0.00	89.43	68.43	91.79	10.34	90.47†
54.....	193.99	191.55	196.83	95.77	0.00	95.77	74.90	98.41	10.00	97.00†
55.....	208.14	205.28	211.19	102.64	0.00	102.64	82.26	105.59	9.51	104.07†
56.....	223.77	220.42	227.04	110.21	0.00	110.21	90.87	113.52	8.81	111.89†
57.....	241.35	237.46	244.88	118.73	0.00	118.73	101.24	122.44	7.79	120.68†
58.....	261.46	256.94	265.28	128.47	0.00	128.47	114.15	132.64	6.24	130.73†
59.....	284.77	279.52	288.94	139.76	0.00	139.76	130.75	144.47	3.85	142.38†
60.....	312.07	305.95	316.64	152.98	0.00	152.98	152.98	158.32	0.00	156.04†

NOTE.—At issue age 35 the gross premium grades uniformly from 0.95 of the YRT premium at duration 1 to 1.02 of the YRT premium at duration 60.

* Set equal to one-half UNP_t.

† Set equal to one-half GP_t.

DISCUSSION OF PRECEDING PAPER

THOMAS G. KABELE:

The author is to be congratulated for a timely paper on an important subject. He discusses two reserve methods for annual renewable term (ART) insurance. One method is the uniform percentage method (UPM), which is used by most companies to value permanent coverages. The other method is the changing premium valuation method (CPVM), which treats an ART policy as a series of one-year term policies. In my experience, this is the method now used by most companies in valuing ART policies. The author has given it a new name and added a wrinkle, namely, his method of valuing the deficiency reserve.

The Mortality Table May Be More Important than the Reserve Method

Although the valuation method can have an important impact on the size of the reserve, the mortality table can have an even bigger impact. This is illustrated in my Tables 1–3, which show curtate net level mean reserves for various ART policies under several bases. Shown first is the CPVM reserve, excluding the deficiency reserve (basis 1). Then UPM reserves are shown using the following mortality tables: 1958 CSO (basis 2); 1965–70 Ultimate Basic Table for Males (basis 3); 1965–70 Select and Ultimate Basic Table for Males using Green's modification of the select values (basis 4); and the same as basis 4 with realistic lapse rates added (basis 5).

In Table 3 the reserves produced by basis 3 are about 50 percent larger than those produced by basis 2, and the reserves produced by basis 4 and 5 are almost 100 percent larger than basis 2 reserves. In Tables 1 and 2 the disparities are even greater.

If the Society of Actuaries adopts a very flat "modern CSO" table, the reserves may be even smaller than the 1958 CSO reserves. Since most companies use a basis like 3 or 4 to compute premiums or policyholder dividends, it is questionable whether state regulatory authorities should permit very small reserves. In fact, for ART policies the correct mortality pattern may be select, ultimate, and then antiselect, so that even basis 4 or basis 5 reserves would be inadequate.

UPM May Be More Conservative than CPVM

The author seems to imply that CPVM reserves are more conservative than UPM reserves. In his hypothetical examples the UPM produces

TABLE 1

ANNUAL RENEWABLE TERM TO AGE 95 (CASE VIII)

(Issue Age 35; 3.5 Percent Interest)

YEAR	LIVING*	GROSS PREMIUM	CURATE NET LEVEL MEAN RESERVES*				
			Basis 1	Basis 2	Basis 3	Basis 4	Basis 5
1	1.00000	2.30	1.21	1.03	1.12	1.40	1.13
2	0.79920	2.43	1.28	0.89	1.64	2.46	2.02
3	0.67854	2.58	1.35	0.76	2.18	3.49	2.90
4	0.60990	2.79	1.45	0.65	2.76	4.53	3.79
5	0.54807	3.02	1.57	0.53	3.37	5.58	4.74
6	0.49240	3.28	1.71	0.41	4.01	6.65	5.76
7	0.44227	3.58	1.86	0.28	4.68	7.77	6.87
8	0.39713	3.90	2.01	0.15	5.39	8.91	8.08
9	0.35650	4.25	2.19	0.01	6.14	10.10	9.40
10	0.31991	4.63	2.38	− 0.14	6.93	11.32	10.83
15	0.18494	7.24	3.67	− 0.92	11.38	17.48	19.43
20	0.10512	11.49	5.75	− 1.64	16.30	23.26	29.98
25	0.05828	18.18	8.98	− 2.07	21.38	29.17	43.29
30	0.03107	28.76	14.03	− 1.58	27.46	36.06	59.72
35	0.01559	45.63	22.03	0.54	35.91	45.23	79.43
40	0.00717	68.99	32.91	4.63	54.70	64.52	110.57
45	0.00286	103.75	48.88	13.50	70.68	80.84	142.79
50	0.00091	155.68	72.47	30.92	90.83	100.82	178.27
55	0.00021	223.15	102.64	60.69	84.56	93.28	184.76
60	0.00003	336.55	152.98	152.98	120.21	120.21	120.21
Net premium, duration 1	....	....	2.425	2.241	1.792	1.766	1.423

* Definitions: Living: beginning in-force, calculated using the assumptions of basis 5; basis 1: CPVM, 1958 CSO Mortality Table; basis 2: UPM, 1958 CSO Mortality Table; basis 3: UPM, 1965–70 Ultimate Basic Mortality Table for Males; basis 4: UPM, 1965–70 Select and Ultimate Basic Mortality Table for Males; basis 5: UPM, same mortality as in basis (4), but including lapse rates of 20 percent in the first year, 15 percent in the second year, and 10 percent in the third and later years.

TABLE 2

ANNUAL RENEWABLE TERM TO AGE 70 (CASE VIII)

(Issue Age 35; 3.5 Percent Interest)

YEAR	LIVING*	GROSS PREMIUM	CURTATE NET LEVEL MEAN RESERVES*				
			Basis 1	Basis 2	Basis 3	Basis 4	Basis 5
1	1.00000	2.30	1.21	1.08	1.08	1.33	1.11
2	0.79920	2.43	1.28	1.00	1.54	2.31	1.97
3	0.67854	2.58	1.35	0.93	2.03	3.26	2.81
4	0.60990	2.79	1.45	0.89	2.55	4.21	3.66
5	0.54807	3.02	1.57	0.85	3.09	5.15	4.55
6	0.49240	3.28	1.71	0.81	3.65	6.11	5.51
7	0.44227	3.58	1.86	0.78	4.24	7.09	6.55
8	0.39713	3.90	2.01	0.75	4.85	8.10	7.67
9	0.35650	4.25	2.19	0.73	5.50	9.12	8.88
10	0.31991	4.63	2.38	0.72	6.17	10.16	10.19
15	0.18494	7.24	3.67	0.88	9.80	15.09	17.71
20	0.10512	11.49	5.75	1.78	13.31	18.72	25.71
25	0.05828	18.18	8.98	4.21	15.93	20.90	33.06
30	0.03107	28.76	14.03	9.90	17.62	21.14	35.10
35	0.01559	45.63	22.03	22.03	17.87	17.87	17.87
Net premium, duration 1	....	....	2.425	2.292	1.747	1.697	1.401

* Definitions: see Table 1.

TABLE 3

PARTICIPATING ANNUAL RENEWABLE TERM TO AGE 70

(Issue Age 35; 3.5 Percent Interest)

Year	Living*	Gross Premium	Curtate Net Level Mean Reserves*				
			Basis 1	Basis 2	Basis 3	Basis 4	Basis 5
1	1.00000	3.13	1.21	1.41	1.34	1.58	1.17
2	0.79920	3.25	1.28	1.65	2.05	2.80	2.08
3	0.67854	3.89	1.35	2.30	3.09	4.29	3.21
4	0.60990	4.11	1.45	2.98	4.17	5.78	4.34
5	0.54807	4.35	1.57	3.67	5.27	7.27	5.54
6	0.49240	4.62	1.71	4.35	6.38	8.76	6.81
7	0.44227	4.98	1.86	5.07	7.54	10.30	8.20
8	0.39713	5.36	2.01	5.82	8.76	11.89	9.70
9	0.35650	5.76	2.19	6.59	10.02	13.51	11.32
10	0.31991	6.19	2.38	7.39	11.31	15.15	13.06
15	0.18494	9.21	3.67	12.00	18.35	23.38	23.42
20	0.10512	13.75	5.75	17.15	25.09	30.15	35.04
25	0.05828	21.17	8.98	22.74	30.07	34.63	46.18
30	0.03107	31.54	14.03	27.01	30.59	33.73	49.24
35	0.01559	47.52	22.03	22.03	17.87	17.87	17.87
Net premium, duration 1	....	....	2.425	2.619	2.002	1.945	1.457

* Definitions: see Table 1.

negative terminal reserves and in some cases negative mean reserves. I admit that these can occur, but in most cases the UPM is much more conservative than the CPVM. This is particularly true for participating policies. Table 3 shows the reserves for a participating contract sold by my company. For this contract the UPM reserves are two to three times as high as the CPVM reserves.

The UPM Can Be Manipulated

I agree with the author that the UPM can be manipulated by increasing the gross premiums at later durations. I believe that his suggestion of using as a floor one-half the cost of insurance has merit. It may, however, introduce unwanted complications in the calculation of tax reserves, especially if the floor must be computed using the 1958 CSO Table at a relatively low rate of interest.

As an example of how the reserves can be manipulated, compare Tables 1 and 2. Both are based on the author's Case VIII, but in Table 2 I regard the policy as ART to age 70 rather than ART to age 95. The reserves under basis 2 are much larger under the ART to age 70 approach.

To reduce manipulation, state insurance departments could require the use of realistic mortality rates, or they could require companies to truncate their policies for reserve calculations at either duration 30 or attained age 70, whichever comes later. Under realistic lapse and conversion assumptions there are very few policies in force by duration 30, and companies would not be able to use high premiums (which virtually no one will pay) in the distant future to reduce present reserves. Alternatively, insurance departments could ask companies to reflect lapse rates in their reserve calculation. The extra discounting effect of lapse rates would minimize any gains to be made from very high premiums payable many years in the future.

There Are Parallel Problems with GAAP Reserves for ART Policies

In computing reserves for ART policies, similar problems are encountered under GAAP as under the statutory basis. I have seen five different methods of computing GAAP reserves for ART policies. These are the following:

1. One-year term basis using select and ultimate mortality.
2. One-year term basis using aggregate attained-age mortality.
3. Increasing premium basis with profit a level percentage of the face amount (see Richard S. Robertson, "GAAP Accounting for Reinsurance Accepted," *TSA*, XXVII, 376, 396).
4. Increasing premium basis using the same select and ultimate table as for permanent plans.
5. Same as method 4 but with increased mortality in the select period.

Methods 1, 2, and 3 tend to produce a healthy GAAP profit in the early years, but methods 4 and 5 tend to defer the profit. Of course, as Mr. Sondergeld has indicated to me, the "maintenance reserve" for ART policies is negative (see also Robert Posnak, *GAAP: Stock Life Companies* [New York: Ernst and Ernst, 1974], p. 322). This negative reserve reduces the impact on GAAP earnings of the large death benefit reserve. Still, the GAAP reserves tend to be very high. By using realistic rates of interest and deducting the maintenance reserve, the net reserves that result are about 25 percent lower than those of basis 5 in Tables 1–3. These GAAP reserves are still much higher than statutory reserves computed using the 1958 CSO Table under either the CPVM or the UPM. It is to be hoped that the American Academy of Actuaries will address these problems with GAAP.

GAAP, Tax, and Statutory Reserves Are Interrelated

GAAP and statutory reserves cannot be considered in isolation. First, actuaries must be able to certify that statutory reserves are adequate. If statutory reserves, which should be conservative, are only one-fourth as large as GAAP reserves, which should be more realistic, how can statutory reserves be adequate?

Second, for phase 1- and phase 2-positive companies the GAAP interest rate on reserves is supposed to be net after taxes (see Posnak, *GAAP: Stock Life Companies*, pp. 147–49). If the statutory reserves are only a small fraction of the GAAP reserves, it seems to me that the GAAP interest rate used in calculating benefit and maintenance reserves should be reduced. This would increase GAAP reserves still further.

Finally, if GAAP and statutory reserves are not calculated on a consistent basis (that is, both CPVM or both UPM or both using mortality tables with similar slopes), the company may have substantially inconsistent GAAP and statutory earnings for its ART policies. This may be difficult to explain to stock analysts.

Future Valuation Laws

For valuing ART policies on a statutory basis, I believe that state insurance departments should require companies to use the UPM method with a fairly steep mortality table, such as the 1965–70 Ultimate Table for Males. There should be no maximum limit on the interest rate. The floor should be one-half the cost of insurance, but based on the 1965–70 Ultimate Table rather than the 1958 CSO Table. Deficiency reserves, if any, also should be based on the 1965–70 Ultimate Table.

RICHARD A. COMBS:

Mr. Sondergeld has written an interesting paper on a subject of some concern to those of us involved in reserving for term insurance. That it is of interest to the state insurance departments as well is shown by a recent Texas regulation on the subject; this regulation is essentially the same as a submission by the American Council of Life Insurance to the NAIC Technical Task Force on Valuation as reported in *ACLI General Bulletin No. 2523.*

The appendix to the paper shows reserves by the uniform percentage and changing premium methods for sixteen different annually renewable term cases. Unfortunately, the effects on the reserves of longer-term renewable plans do not appear to have been investigated. Results for one possible case, a ten-year term, are presented in this discussion.

The test case I have chosen to investigate is a ten-year renewable and convertible term plan expiring at age 70. In the eleventh policy year the policy is treated for all purposes as a new issue except that there is no underwriting. Gross premiums are assumed to be equal to the Commissioners Reesrve Valuation Method (CRVM) renewal net premiums during any term period. Note that with these gross premiums, there would be no deficiency reserve required by the changing premium valuation method.

Because of the first-year commission payable at the beginning of each term period, it appears reasonable to set up CRVM modified reserves during each term period. This procedure, referred to in this discussion as the "alternate method," does not appear to be forbidden by the Texas regulation. However, it does not meet the requirements of either the uniform percentage method, which requires that the policy be treated as a single policy until mandatory expiry, or the changing premium valuation method, which would require net level premium valuation in renewal term periods in the absence of deficiency reserves. Tables 1–3 of this discussion show terminal reserves, mean reserves, and net premiums for the alternate, uniform percentage, and changing premium methods (with the CRVM modification in the first year only) for ages 25, 40, and 55.

The basic question appears to be: "How large should be the reserve for renewable term insurance?" From the point of view of the insurance departments, larger reserves can help to ensure a company's solvency. Larger reserves also can have positive federal income tax effects to the company. However, there are possibilities of large surplus strains. This could be a problem for the smaller companies.

PAUL E. SARNOFF:

The Society is indebted to Mr. Sondergeld for presenting a fresh approach to the valuation of renewable term insurance with guaranteed premium rates.

I heartily endorse the observation in the paper that the minimum mean reserve on a policy may not be less than one-half the current year's cost of insurance. This special provision comes into play whenever the traditional minimum of one-half the net annual premium is less than the remainder of the current year's cost of insurance. Clearly, if the policyholder elects not to pay the next annual premium, the minimum reserve must be at least sufficient according to the reserve assumptions to provide the tabular cost of insurance for the remainder of the current policy period.

The Standard Valuation Law prescribes that for policies providing varying premiums, reserves shall be calculated by a method *consistent with the principles* applicable to level premium insurance. The examples of manipulation that have been cited refer to calculation results that

TABLE 1

A. TERMINAL AND MEAN RESERVES FOR TEN-YEAR RENEWABLE AND CONVERTIBLE TERM

(Issue Age 25; 1958 CSO Table [Curtate] with Interest at 3.5 Percent)

DURATION	ALTERNATE METHOD		UNIFORM PERCENTAGE METHOD		CHANGING PREMIUM VALUATION METHOD	
	Terminal Reserve	Mean Reserve	Terminal Reserve	Mean Reserve	Terminal Reserve	Mean Reserve
1	0.00	0.93	0.00	0.93	0.00	0.93
2	0.18	1.12	0.11	1.06	0.18	1.12
3	0.33	1.29	0.20	1.16	0.33	1.29
4	0.45	1.42	0.25	1.22	0.45	1.42
5	0.52	1.52	0.25	1.25	0.52	1.52
6	0.55	1.57	0.20	1.22	0.55	1.57
7	0.52	1.57	0.09	1.14	0.52	1.57
8	0.42	1.50	−0.09	1.00	0.42	1.50
9	0.25	1.37	−0.34	1.00	0.25	1.37
10	0.00	1.16	−0.69	1.00	0.00	1.16
11	0.00	1.21	0.23	1.67	0.93	2.12
12	0.92	2.18	1.06	2.31	1.76	3.01
13	1.72	3.04	1.75	3.07	2.47	3.78
14	2.34	3.75	2.26	3.67	2.99	4.39
15	2.74	4.26	2.55	4.07	3.29	4.80
16	2.87	4.53	2.57	4.22	3.33	4.97
17	2.71	4.51	2.28	4.09	3.05	4.85
18	2.20	4.17	1.64	3.63	2.43	4.40
19	1.31	3.48	0.63	2.80	1.43	3.59
20	0.00	2.38	−0.82	1.67	0.00	2.38
21	0.00	2.58	1.89	4.43	2.64	5.17
22	2.52	5.29	4.23	6.96	4.91	7.63
23	4.61	7.59	6.13	9.08	6.74	9.68
24	6.20	9.43	7.53	10.73	8.06	11.25
25	7.20	10.73	8.33	11.83	8.79	12.28
26	7.53	11.39	8.45	12.30	8.82	12.66
27	7.08	11.33	7.79	12.03	8.07	12.30
28	5.76	10.45	6.24	10.92	6.44	11.11
29	3.45	8.63	3.69	8.87	3.79	8.97
30	0.00	5.75	0.00	5.75	0.00	5.75

B. NET PREMIUMS FOR TEN-YEAR RENEWABLE AND CONVERTIBLE TERM

(Issue Age 25; 1958 CSO Table [Curtate] with Interest at 3.5 Percent)

	DURATION					
	1	2–10	11	12–20	21	22–30
Alternate method	1.86	2.06	2.43	3.44	5.17	8.05
Uniform percentage method	1.86	2.00	3.33	3.33	7.81	7.81
Changing premium valuation method	1.86	2.06	3.32	3.32	7.71	7.71

TABLE 2

A. TERMINAL AND MEAN RESERVES FOR TEN-YEAR RENEWABLE AND CONVERTIBLE TERM

(Issue Age 40; 1958 CSO Table [Curtate] with Interest at 3.5 Percent)

Duration	Alternate Method		Uniform Percentage Method		Changing Premium Valuation Method	
	Terminal Reserve	Mean Reserve	Terminal Reserve	Mean Reserve	Terminal Reserve	Mean Reserve
1	0.00	1.71	0.00	1.71	0.00	1.71
2	1.55	3.38	1.35	3.18	1.55	3.38
3	2.83	4.79	2.43	4.40	2.83	4.79
4	3.80	5.91	3.18	5.31	3.80	5.91
5	4.42	6.71	3.58	5.89	4.42	6.71
6	4.63	7.12	3.56	6.08	4.63	7.12
7	4.37	7.10	3.06	5.82	4.37	7.10
8	3.57	6.57	2.01	5.04	3.57	6.57
9	2.14	5.46	0.32	3.67	2.14	5.46
10	0.00	3.67	− 2.10	2.51	0.00	3.67
11	0.00	4.02	2.04	6.05	4.14	8.07
12	3.91	8.23	5.57	9.85	7.66	11.90
13	7.15	11.81	8.41	13.03	10.50	15.08
14	9.61	14.65	10.44	15.47	12.53	17.52
15	11.16	16.66	11.56	17.04	13.65	19.09
16	11.69	17.70	11.63	17.64	13.73	19.69
17	11.04	17.64	10.49	17.11	12.60	19.17
18	9.02	16.30	7.96	15.27	10.08	17.34
19	5.41	13.49	3.82	11.94	5.96	14.02
20	0.00	8.98	− 2.16	6.87	0.00	8.98
21	0.00	9.83	7.85	17.47	9.84	19.40
22	9.38	19.86	16.52	26.81	18.33	28.57
23	17.23	28.48	23.63	34.70	25.25	36.27
24	23.29	35.43	28.92	40.90	30.35	42.28
25	27.27	40.45	32.10	45.13	33.32	46.32
26	28.80	43.21	32.78	47.06	33.79	48.04
27	27.43	43.29	30.51	46.27	31.29	47.02
28	22.62	40.20	24.75	42.25	25.29	42.77
29	13.72	33.35	14.82	34.41	15.10	34.68
30	0.00	22.03	0.00	22.03	0.00	22.03

B. NET PREMIUMS FOR TEN-YEAR RENEWABLE AND CONVERTIBLE TERM

(Issue Age 40; 1958 CSO Table [Curtate] with Interest at 3.5 Percent)

	Duration					
	1	2–10	11	12–20	21	22–30
Alternate method	3.41	5.20	8.04	12.55	19.65	30.35
Uniform percentage method	3.41	5.01	12.09	12.09	29.24	29.24
Changing premium valuation method	3.41	5.20	12.00	12.00	28.96	28.96

fall short of meeting the statutory prescription for consistency. To then advocate a method that deviates from those principles seems an inappropriate reaction.

A traditional statutory actuarial practice is to base reserve calculations on conservative assumptions, and, where the insured has a variety of choices with differing financial impacts on the company, the most conservative choice is often made. An example is the substitution of zero for negative terminal reserves. Another illustration of this conservative approach is the requirement that an annual statement reserve on a policy may not be less than the corresponding cash value.

TABLE 3

A. TERMINAL AND MEAN RESERVES FOR TEN-YEAR RENEWABLE AND CONVERTIBLE TERM

(Issue Age 55; 1958 CSO Table [Curtate] with Interest at 3.5 Percent)

Duration	Alternate Method		Uniform Percentage Method		Changing Premium Valuation Method	
	Terminal Reserve	Mean Reserve	Terminal Reserve	Mean Reserve	Terminal Reserve	Mean Reserve
1	0.00	6.28	0.00	6.28	0.00	6.28
2	6.09	12.81	5.70	12.43	6.09	12.81
3	11.14	18.38	10.35	17.60	11.14	18.38
4	15.00	22.83	13.77	21.64	15.00	22.83
5	17.47	26.00	15.78	24.35	17.47	26.00
6	18.32	27.66	16.15	25.54	18.32	27.66
7	17.32	27.58	14.63	24.97	17.32	27.58
8	14.17	25.51	10.92	22.35	14.17	25.51
9	8.53	21.11	4.69	17.38	8.53	21.11
10	0.00	14.03	−4.49	9.67	0.00	14.03
11	0.00	15.34	2.62	18.81	6.32	21.46
12	5.12	21.73	7.15	23.69	10.02	26.47
13	7.23	25.35	8.62	26.70	10.60	28.61
14	5.72	25.65	6.45	26.35	7.48	27.34
15	0.00	22.03	0.00	22.03	0.00	22.03

B. NET PREMIUMS FOR TEN-YEAR RENEWABLE AND CONVERTIBLE TERM

(Issue Age 55; 1958 CSO Table [Curtate] with Interest at 3.5 Percent)

	Duration			
	1	2–10	11	12–15
Alternate method	12.56	19.53	30.68	38.34
Uniform percentage method	12.56	19.16	37.62	37.62
Changing premium valuation method	12.56	19.53	36.59	36.59

The Standard Valuation Law does not contemplate the use of persistency assumptions separate and apart from mortality or morbidity assumptions (although in the case of disability and accidental death benefits the use of an ordinary valuation mortality table represents a defined level of persistency). For typical cash-value policies, introduction of lapse assumptions would serve to reduce the reserve level, so the conservative assumption is made that no voluntary terminations occur. A reserve is required to cover the most unfavorable behavior (to the company) of the insured—continuance of the policy in force till maturity. However, the fact that the valuation law does not specify the use of lapse assumptions for typical cash-value policies does not mean that the law totally disregards the matter of voluntary termination; otherwise, why is there the requirement that the reserve be not less than the cash value? In the case of term insurance, the matter of lapse also should be considered by the actuary. In doing so, the actuary would observe that a lapse before the final term expiry date can require a higher reserve than if the contract is assumed to continue till maturity.

A method consistent with the law is to define the terminal reserve at duration t under a renewable term policy as the greatest of the quantities ${}_tV^{1\prime}_{x:\overline{n}|}$, where n ranges from 2 to the greatest duration for which the contract may be renewed, and the prime symbol denotes modified renewal premiums bearing a uniform ratio (not to exceed 1.00) to the contract premiums payable through policy year n. The result is subject to a minimum at duration t of ${}_{t-p}V^{\;1}_{x+p:\overline{r}|}$, where p is the duration of the premium change preceding duration t, r is the number of years the premium remains level, and an appropriate correction is made for any premium deficiency.

My consideration of each respective policy duration for which an insured may choose to continue a policy in force can be reconciled with the author's statement, with which I am in full agreement, that "renewable term insurance should be treated for reserve purposes as a continuous contract for the total period during which premium rates are guaranteed." In explanation of my approach I would observe that, within the *total* period for which rates are guaranteed, it is appropriate in determining the required reserve level for the policy to consider as many individual periods extending from original issue to lapse as there are possibilities for the insured to continue the term policy.

The concept described is quite similar to the reasoning that is used in the new Commissioners Reserve Valuation Method for annuities, which is defined in the 1976 amendments to the NAIC Model Standard Valuation Law as follows:

Reserves according to the Commissioners Annuity Reserve Method for benefits under annuity or pure endowment contracts, excluding any disability and accidental death benefits in such contracts, shall be the greatest of the respective excesses of the present values, at the date of valuation, of the future guaranteed benefits, including guaranteed nonforfeiture benefits, provided for by such contracts at the end of each respective contract year, over the present value, at the date of valuation, of any future valuation considerations derived from future gross considerations, required by the terms of such contract, that become payable prior to the end of such respective contract year.

It is my view that the *concept* outlined above follows uniquely from the general principles applied to determine statutory reserves in accordance with the CRVM for life insurance in the Standard Valuation Law. To be sure, the application of these principles is a matter of considerable difficulty because of the amount of detailed and repetitious work and analysis that it entails.

The paper states that the new valuation regulation or legislation has eliminated the definition of a separate deficiency reserve. With respect to United States business, the Internal Revenue Code section 801(b)(4) defines deficiency reserves as

that portion of the reserve for such contract equal to the amount (if any) by which—

(A) The present value of the future net premiums required for such contract, exceeds

(B) the present value of the future actual premiums and considerations charged for such contract.

The new state laws or regulations will have no effect on that definition.

(AUTHOR'S REVIEW OF DISCUSSION)

DONALD R. SONDERGELD:

My paper called attention to the fact that many insurance departments are viewing renewable term insurance policies with long-term premium rate guarantees as continuous contracts for purposes of calculating deficiency reserves. It also mentioned that some actuaries are using the reserve method for renewable term insurance where net premiums are a uniform percentage of the gross premiums. By the use of that method, the basic reserve and deficiency reserve can be manipulated by the slope the actuary chooses for the gross premium scale. Also, negative reserves can arise.

The purpose of the paper was not necessarily to advocate the use of the changing premium valuation method (CPVM) but to suggest it as a method that may produce better results and is less subject to manipulation than the uniform percentage method (UPM). State regulators, and actuaries who sign the statutory annual statement, should be aware of the results that might be produced using the UPM, as both should be concerned with reserve adequacy.

Mr. Kabele is correct; CPVM is a method that many companies have used in the past for one-year renewable term insurance. However, for five-year renewable term insurance the gross premiums might change once or twice within a five-year period, and the CPVM would produce results different from those produced by companies that use a level net premium during each five-year period.

Mr. Kabele makes an excellent point when he mentions that the mortality table has quite an effect upon the size of the reserve for term insurance. I did not, however, mean to imply that the CPVM necessarily produces more conservative reserves than the UPM. Table 1 of my paper indicates that the UPM produces larger reserves than the CPVM under Case IV.

Mr. Kabele mentions five methods in use for computing GAAP benefit reserves. Some of the methods are the same, but with different mortality assumptions. If an actuary is given the complete set of actuarial assumptions used in developing the gross premiums, he can develop various GAAP accounting entries. The use of those entries should produce GAAP earnings that are equal to GAAP profits (which are a uniform percentage of the premium income) plus interest on the sum of GAAP profits and GAAP surplus. (The reader may wish to refer to my paper "Earnings and the Internal Rate of Return Measurement of Profit," *TSA*, XXVI, 621). It would seem to me that, theoretically, this should produce a unique set of expected GAAP earnings. The actuary should not have five sets of assumptions from which to choose. If a realistic GAAP benefit reserve is larger than the statutory reserve, the actuary should examine the statutory reserve for adequacy.

Mr. Combs suggests that, on a ten-year renewable term plan, the valuation net premiums be modified not only in the first policy year but also in the eleventh, twenty-first, etc., policy years if first-year commissions are payable. This would seem reasonable to me, provided that the resulting statutory reserves exceed the statutory minimum.

Mr. Sarnoff develops a number of interesting points. I wonder whether all actuaries will agree with his argument as to which valuation principles applicable to level premium insurance are also applicable to varying premium insurance. Again, I am advocating adequate statutory reserves—not reserves that meet someone's interpretation of what the statutory minimum might be. The reserve method illustrated by Mr. Sarnoff is a good one, although somewhat complicated.

I agree with Mr. Sarnoff that the elimination of a deficiency reserve in the new statutory legislation may have no effect on the treatment of reserves for federal income tax purposes. There are some who will argue that point.

There seems to be agreement that negative terminal reserves should not be used. One approach is to set them equal to zero. This produces a mean reserve that may be less than half the cost of insurance for the policy year (e.g., if both terminals are negative and the valuation premium is less than the cost of insurance). Some actuaries then impose a mean reserve floor of half the cost of insurance.

It appears to me that neither of these methods necessarily produces adequate reserves at issue. For example, if net premiums and gross premiums are equal and a negative terminal reserve is developed only at duration 5, it would seem that an extra reserve should be set up at duration 0 equal to the present value of the negative fifth-year terminal reserve.

An approach that might be used is to determine at issue those future durations where the terminal reserve is negative, calculate the present value of each of those negatives, and then hold the largest of those present values as an "additional reserve." This process could be repeated at each duration. An example is shown in Table 1 of this review.

It could be argued that, if gross premiums are larger than net premiums, the full additional reserve need not be established out of surplus at every duration. One approach would be to reduce the additional reserve by the present value of any excess of gross premiums over net premiums that occurs between the year of valuation and the year the next negative formula terminal reserve occurs. If that excess is more than sufficient to cover the first negative terminal reserve, any sufficiency could be used to offset the next negative terminal reserve if it had a larger present value, and so on.

I want to thank each of those who discussed my paper, and I hope the thoughts contained in both the paper and the discussions will assist actuaries in developing adequate reserves for term insurance.

TABLE 1—TEN-YEAR DECREASING TERM POLICY
(Male Issue Age 45); 1958 CSO Table, 4 Percent Interest; Net Premium = 4.39)

Duration t (a)	Death Benefit (b)	Cost of Insurance (c)	Formula Terminal Reserve (d)	Additional Terminal Reserve* (e)	Total Terminal Reserve (f)
0	.....	.....	0.00	2.89	2.89
1	971	4.99	−0.63	3.02	2.39
2	904	5.07	−1.37	3.16	1.79
3	830	5.08	−2.15	3.31	1.16
4	750	5.01	−2.90	3.47	0.57
5	662	4.84	−3.50	3.63	0.13
6	565	4.52	−3.81	3.81	0.00
7	460	4.03	−3.62	3.62	0.00
8	344	3.30	−2.65	2.65	0.00
9	277	2.90	−1.22	1.22	0.00
10	277	3.17	0.00	0.00	

t	$\lvert(d)\rvert \times D_{45+t}/D_{45}$ (0)	$(0)\times D_{45}/D_{46}$ (1)	$(0)\times D_{45}/D_{47}$ (2)	$(0)\times D_{45}/D_{48}$ (3)	$(0)\times D_{45}/D_{49}$ (4)	$(0)\times D_{45}/D_{50}$ (5)	$(0)\times D_{45}/D_{51}$ (6)	$(0)\times D_{45}/D_{52}$ (7)	$(0)\times D_{45}/D_{53}$ (8)	$(0)\times D_{45}/D_{54}$ (9)
0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
1	0.6025	†	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
2	1.2525	†	†	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
3	1.8780	†	†	†	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
4	2.4188	†	†	†	†	N.A.	N.A.	N.A.	N.A.	N.A.
5	2.7936	†	†	†	†	†	N.A.	N.A.	N.A.	N.A.
6	2.8914‡	3.02§	3.16§	3.31§	3.47§	3.63§	3.81§	N.A.	N.A.	N.A.
7	2.6175	†	†	†	†	†	†	3.62‖	N.A.	N.A.
8	1.8241	†	†	†	†	†	†	†	2.65	N.A.
9	0.7987	†	†	†	†	†	†	†	†	1.22
10	0	.....	.....	.....	.....	.....	.....	.....	.....	.....

NOTE.—If formula terminal reserves simply are set equal to zero, the initial reserve equals the net premium of 4.39, which is less than the cost of insurance in the first six policy years.
* Additional reserve at duration t is the largest number in col. t in the lower part of the table.
† Not necessary to calculate, since it will be less than the single number shown in this column.
‡ Has the largest present value, therefore is the additional reserve for duration 0.
§ Since the negative at duration 6 has the largest present value at duration 0, it also will have the largest present value at durations 1–6.
‖ The additional reserve for duration 7 must be based on the negative reserve at duration 7, since this has a larger present value at duration 0 than the duration 8 and duration 9 negatives.