BOOK REVIEWS

E. J. Moorhead, *Our Yesterdays: the History of the Actuarial Profession in North America, 1809-1979*, pp. 437, published by Society of Actuaries, Schaumburg, Ill. 60173, 1989; $60.

Every actuary should read this fascinating history, so long overdue. As far back as 1982, Dwight Bartlett, a past Society President, first began voicing his concern about the crying need for a history of our profession on this continent. His efforts finally bore fruit with the Council of Presidents in 1983, and the Society was indeed fortunate that a man as well qualified and dedicated as Jack Moorhead was persuaded to tackle this mammoth job. It is extremely difficult to comprehend the enormity of the task that faced the author, the mass of material that had to be dug out of the archives, the voluminous amount of reading required to uncover long-forgotten facts, and the struggles of so many of our predecessors in the days when there was no Institute of Actuaries, no Society or Faculty, no Journals or *Transactions* to record our storehouse of knowledge about things actuarial here and abroad.

The following extract from a letter from Jack Moorhead to the reviewer not only reveals his frustrations in bringing together the mass of material that is the basis of this book, but also includes some suggestions that deserve consideration by some of us who are retired but can still serve our profession.

"1. It is extraordinary, and distressing, that much of this writing was not done years ago. It's baffling to try to understand why this gap was allowed to widen and widen while eyewitnesses were lost by death and valuable records were lost or destroyed.

"2. By no means do I consider that the job is done. My hope is that the book's division of the subject into fourteen parts may encourage others to explore one or more of those parts more thoroughly than I have managed to do even with all the help I received. (I shall even mention some sources that I know have been inadequately tapped.)

"3. Maybe the fact that a retired member in his upper 70s has done this much may remind the powers-that-be that the ranks of the retired include some people who might be put to work on projects to which they can give time that actuaries still on the active list seem less and less able to spare."

If just a handful of retired actuaries would respond to this appeal, that would be the best way to repay the great debt we owe the author.

Chapter I begins in 1809, reciting the struggles of our predecessors over a fifty-year period. The author's criticism of Fackler's unjustified comment in 1909 is well founded. The section headed "Premiums Charged in Early Days" would have been improved if a brief description of the various mortality tables then in use had been included.

Chapter II covers the next thirty years (1859–1888) before the founding of the Actuarial Society of America, which would probably have been established many years sooner

had it not been for the intense and unbridled competition that developed between the larger companies and involved many unsound practices that eventually resulted in the Armstrong Investigation in 1905. This chapter is a fascinating, and most illuminating, account of the highlights of those rather hectic days in the life insurance business in the United States. Canada, as usual, showed greater moderation and control.

Chapter III covers the thirty years from 1889 to 1918, perhaps the most eventful period in the early history and formative years of the profession. Table III.1, listing charter members of the Actuarial Society, shows an overwhelming concentration geographically, as follows:

East	28
Middle West	4
South	1
Far West	1
Canada	4
Total	38

No wonder the American Institute of Actuaries was formed in 1909, with the following distribution of charter members:

East	1
Middle West	35
South	3
Far West	3
Canada	0
Total	42

The section on "Preliminary Term Valuation" reminds this reviewer that J.B. Maclean, in his Presidential Address in October 1943 (*T.A.S.A.* Vol. 44, p. 223), referred in a positive way to the use of this type of reserve, but in practice he never recommended it to his own company. The attitude of the old established companies was undoubtedly a somewhat snobbish one, the argument being that by changing to that weaker basis, the company's surplus got a one-time shot in the arm and thereafter it made little difference to earnings. Of course, this is true only if the volume and type of new business remain constant, which they never do. Eventually the accounting profession quite justifiably insisted on a more realistic basis for measuring current earnings.

The section "Scottish Influence in Pre-Merger Days" impels this reviewer to make some rather personal comments. Henry Moir, one of the leaders in the early days of the Actuarial Society, left the "Scottish Life Assurance Company" in Edinburgh when he emigrated to the United States in 1901. This reviewer left the Scottish Life in 1931 to emigrate to Canada and had been honored by a visit from Henry Moir in 1930, when Moir came to Edinburgh to play golf with some of his former colleagues. While at his old company, Moir remarked that this reviewer was seated at the same old rolltop desk that Moir used thirty years before in 1901. Moir tried to recruit me for a job in the United States, which I refused, but this visit resulted a year later in my being the last actuarial student to emigrate to North America from the Scottish Life after 1931. In that year I

was also the first candidate to take an examination of the Actuarial Society in Scotland by special arrangement at Edinburgh University and doubt that this has happened since. I had the great privilege of meeting most of the Scottish actuaries referred to in this book and found that, even in the 1930s, this small group of men still had considerable influence in the profession on this continent.

The section headed "World War I" mentions briefly the existence of business in foreign countries, and there are also two references on this subject in the Appendix to Chapter III, viz., papers by the eminent Dr. Arthur Hunter to the Fourth and Seventh International Congresses. The three largest U.S. companies, Mutual Life, New York Life and Equitable, had large foreign operations in Europe early in this century, which they closed down after World War I. A small group of Canadian companies later became an important factor in Great Britain and much of the British Empire, Latin America and even some countries in the Far East. Some of these operations of Canadian companies exist even to the present day.

Chapter V is headed "1949-1964: Expansion and Fresh Directions." It could just as well have been titled "The Relentless Speed of Change," although one must admit that the years 1965–1979, covered in Chapter VI, could equally well qualify for such a title. Indeed there is no sign to this day of any slowing down in the speed of change as we approach the end of this century. This relentless pressure and ever-increasing mechanization are beginning to be a frightening aspect of modern civilization. We do not seem to have enough time to assimilate what we have just completed before we are called upon to tackle something new.

In many ways the multitude of subjects covered in such a masterful way in Chapters V and VI may be regarded as the heart of this book. At the same time, so many other things that occurred at a slower pace before and after this most eventful thirty-year period are most certainly important in the history of our profession.

The section in Chapter V headed "Pensions" deserves special comment. The landmark paper "Fundamentals of Pension Funding" was a most important supplement to the classic British papers. The later Trowbridge paper on funding methods (*TSA*, Vol. XV, p. 151) is another important work. The earlier paper in *TSA*, Vol. XI, p. 920, by Warters and Rae and the author's comments thereon are worthy of special emphasis as they lament the decline of such formal papers and the overemphasis on informal discussions, which cannot possibly have the depth of thought that is so important.

The American Society of Pension Actuaries is referred to in Chapter V, page 187, and in Chapter VI, pages 222 and 268. One of the motivations involved in the formation of this organization was the use of "split-funded" pension plants financed by a combination of individual whole life policies with a "side fund" designed to convert these policies to life annuities at retirement date, using the settlement options. This involved an actuarial calculation of the level premium pure endowment payment required into these trust funds each year, a rather simple calculation. It also required actuarial assumptions for interest and mortality, and the total cost of the plan depended on the amount of these payments plus the net cost under the policies.

These plans involved high commissions to the agent and excessive costs to the employer, especially in businesses with high rates of turnover and large numbers of employees. Many of the companies active in these operations were not in the group pension business, and those that were had to establish limits defining the size of case that required use of the group product with consequent lower cost. Perhaps the actuaries should have taken a stronger stand in defining these limits than seemed practical at the time.

The section in Chapter VI on "Policy Cost Comparisons" gives a frank and competent discussion of a particularly thorny subject. In the opinion of this reviewer, this is an example of a failure of our profession to face up to a serious problem. As was inevitable, the void was filled by others less competent than our profession to deal with a matter so complicated and important to the public relations of our industry. We have not yet seen the end of this story.

The section about elections in the Society of Actuaries mentions the desirability of adequate representation geographically and by type of employment. In the twenty-year period 1967–86, there were twelve Presidents from life companies, seven from consulting firms, and one from government. However, in this period only two Presidents were Canadians, and more attention is needed to remedy this important problem.

The major changes in life insurance reserves, statutory minimum nonforfeiture benefits, and finally changes in statutory valuation requirements with substantial broadening in the responsibility of the actuary certifying the reserves are all very well described in several sections of Chapter VI.

The last section of Chapter VII, "Our Profession in 1979," summarizes the disquieting changes that were happening so rapidly, accompanied by the masses of paper generated by computers, which made it more and more difficult to decide what the answers should be.

Chapter VIII describes the sad story of income disability benefits attached to life insurance policies, surely a long series of disastrous decisions that actuaries would prefer to forget. The account given is much kinder than some of us would have expected.

Chapter IX is a fascinating account of the evolution from simple desk calculators to the modern electronic computer in so short a time. The meeting at Staple Inn Hall in London in 1936, at which Phillips presented his paper "Binary Calculations," was one that this reviewer was privileged to attend and that he will never forget.

It is to be hoped that our members will study and profit from this most excellent work. It should make us proud of our heritage and inspire us to do our part in shaping the future of our profession.

CHARLES F. B. RICHARDSON

Newton L. Bowers, Jr., Hans U. Gerber, James C. Hickman, Donald A. Jones, and
Cecil J. Nesbitt, *Actuarial Mathematics*, pp. 624, published by Society of Actuaries,
Itasca, Ill. 60143, 1986; $65.

From a pedagogical point of view, the presentation of the book is impeccable; inter-
esting examples illustrate each new concept. An interpretation is provided for the most
important formulas. Each chapter concludes with a lengthy series of exercises. (The
solution of most exercises, without derivation, is to be found in an appendix. A worth-
while addition to the textbook is the study manual published annually by ACTEX and
distributed by The Actuarial Bookstore, P.O. Box 318, Abington, Connecticut 06230.
It contains detailed solutions to all the textbook exercises, a lot of supplementary prob-
lems and multiple choice questions, and the solution to recent Society of Actuaries
examination questions. Study manual #150, $42.00, #151, $18.00.) Among the seven
appendixes to this book, Appendix 4 is especially noteworthy, because it presents a
comprehensive survey of the international actuarial notation.

The major innovation introduced by this book is the totally probabilistic approach in
the treatment of the mathematics of life contingencies. This breakthrough is definitely
not going to facilitate the work of actuarial students, but it is long overdue. It is best
illustrated by the very first examples from the life insurance chapter. Example 4.1 reads:
The density function of the time-until-death random variable is assumed to be uniform
over the range (0, 80). At a given force of interest δ, calculate the net single premium,
the variance and the 90th percentile of the claim random variable for a whole life insur-
ance of unit amount issued to (x). Example 4.2 is: Assume that each of 100 independent
lives is age x, is subject to a constant force of mortality of 0.04, and is insured for a
death benefit amount of 10 units, payable at the moment of death. The benefit payments
are to be withdrawn from an investment fund earning 6%. Calculate the minimum amount
at time $t = 0$ so that the probability is approximately 0.95 that sufficient funds will be on
hand to withdraw the benefit payment at the death of each individual.

A knowledge of basic aspects of financial mathematics is assumed at all times, as well
as a solid background in undergraduate calculus and probability theory. A three-page
appendix reminds the reader of the most common probability distributions and of some
formulas from the calculus of finite differences, but, otherwise, many theorems from
calculus and probability theory are routinely used without restatement. The reader should
be prepared for a constant use of conditional expectations, moment-generating functions,
integration-by-parts, and so on. Quite often only the key steps of a mathematical deri-
vation are provided, and some computation is required to "move from one line to the
next."

Before reviewing the work chapter by chapter, it might be useful to mention what the
book does not cover:

a. Stochastic interest rates. The interest rates used to convert future payments to a
    present value are considered deterministic at all times and are usually taken as
    constants.

b. Estimation of parameters. The construction of mortality tables, for instance, is not discussed.

c. Computing methods. Issues like the optimal organization of input data, simulation, and computation in actuarial models are not discussed.

The study of Chapters 3–10, 14 and 15 is required from all students of the Society of Actuaries as preparation for their most important examination on life contingencies. Chapters 2 and 11–13 cover the material of the examination on risk theory. Students of the Casualty Actuarial Society have to study Chapters 3–7 and 9 for their Part 4 examination. Very little of the material is specific to the United States or Canada, so the book could be adopted by other associations of actuaries and by non-American universities.

Chapter 1 provides an introduction to the economics of insurance, using utility theory. It serves as a background for the remainder of the book, but it is not essential, because utility theory is not used in the sequel. Chapter 2 is best read as a preparation to the risk theory material that forms Chapters 11–13. It will be reviewed later on.

Chapter 3, "Survival Distributions and Life Tables," introduces the basic random variables that are used throughout the text: the survival function, the (continuous) time-until-death for a person aged x, the (discrete) curtate-future-lifetime, and the force of mortality. An illustrative mortality table is presented and discussed. It is used in many exercises in the sequel. Assumptions for fractional ages are briefly discussed, as well as the most classical analytical laws of mortality, and the use of select-and-ultimate tables.

Chapter 4, "Life Insurance," develops models for the most common insurance payable at the moment of death: term, whole life, endowment. Classical variants are presented: deferred insurances, varying benefits, insurances payable at the end of the year of death. An important theorem allows, for unit-amount policies, the computation of the j-th moment around the origin of the claim random variable Z: $E[Z^j]$ calculated at force of interest δ equals $E[Z]$ calculated at the force of interest $j\delta$. This result allows among others the computation of the variance of the claim amount for all classical policies. Recursion equations and commutation functions are only briefly introduced at the end of the chapter. Chapter 5, "Life Annuities," presents similar developments for annuities. Some practitioners may regret the rather theoretical presentation of these and other chapters. The continuous approach, based on integrals, is always presented before the discrete approach. Readers need to study several chapters thoroughly before getting some acquaintance with insurance practice.

Chapter 6 deals with net premiums. Its presentation focuses on the financial loss random variable. This probabilistic approach allows the computation of not only net annual premiums, but also variances for the major contracts. It also allows a glimpse at premium calculation principles other than the expected value principle. Several examples illustrate the ruin probability principle, applied directly to the random loss variable, or to a portfolio of independent policies by normal approximation.

Net premium reserves are discussed in Chapter 7. The random variable "prospective future loss on a contract already in force" is investigated, in both the continuous and the discrete case. The reserve is defined as the expected value of this variable and obtained

for all usual policies. The probabilistic approach allows the computation of the variance of the loss in most cases. Important sections deal with classical recursive formulas, reserves as fractional durations, differential equations, allocation of the loss to the policy years (including Hattendorff's theorem and applications).

Chapter 8 is an introduction to multiple life functions. The (two-life) joint-life status (failure upon the first death) and the last-survivor status (failure upon the last death) are defined, and their distributions are obtained, assuming independence. Formulas for net single premiums for annuity and insurance contracts involving two lives are established. Those premiums are then evaluated under specific mortality laws (Gompertz, Makeham), and assumptions (uniform distribution of deaths).

In Chapter 9, "Multiple Decrement Models," a single life is considered but with multiple contingencies like withdrawal, retirement, death, disability. Two random variables, the continuous time-until-termination from a status and the discrete cause of decrement need to be defined, and their distributions characterized. This allows the construction of multiple decrement tables and the associated single decrement tables. This model is applied to calculating actuarial present values of benefits and contributions for a participant in a pension plan in Chapter 10, "Valuation Theory for Pension Plans." All usual cases for the benefit rate function are considered, and numerous examples (step-rate plan, offset plan, add-on plan, . . .) familiarize the reader with the practice of employee benefits plans. Withdrawal and disability benefits are also studied and illustrated by interesting examples.

Expenses are—at last—introduced in Chapter 14, "Insurance Models Including Expenses." The individual model is extended to incorporate acquisition and administrative expenses and accounting requirements. The different loading techniques and modified reserve methods are discussed, mainly through examples.

Nonforfeiture benefits and dividends form the subjects of Chapter 15. A model derived from multiple decrement theory is applied to the determination of the nonforfeiture benefits, the benefits that will not be lost because of the premature cessation of premium payments. The use of multiple decrement theory is motivated by considerations of fairness between the two classes of policyholders, those who terminate before their contractual obligations are fulfilled and those who do not. The different options in case of cessation of payments (paid-up insurance, extended term, . . .) are described. The same principles are then used to obtain formulas for asset shares and experience adjustments such as dividends. Some parts of Chapters 14 and 15 (valuation laws, regulations for nonforfeiture benefits) are specific to the United States and Canada, hence of lesser value to actuaries that do not practice in North America.

The sequence of Chapters 2–11–12–13 provides an excellent and modern introduction to risk theory, despite the fact that some important recent developments had to be bypassed, being outside the scope of the book. Chapter 2, "Individual Risk Models for a Short Term," provides a welcome survey of important probabilistic concepts, presented in an insurance framework. The computation of the sum of independent random variables

and its approximation by means of the central limit theorem are reviewed through several examples.

Chapter 11, "Collective Risk Models for a Single Period," focuses on the computation of the aggregate claims distribution. The compound Poisson and compound negative binomial models are introduced. For the former, three different methods that allow the computation of the distribution are presented and abundantly illustrated (two methods compute convolutions; the third is the recursive method). Approximations by the normal and the translated gamma distributions conclude the chapter.

The surplus process is analyzed in Chapter 12, "Collective Risk Models over an Extended Period." The adjustment coefficient is defined in the continuous and discrete cases, and the theorems that enable the computation of the (infinite horizon) ruin probability in both cases are stated. The maximal aggregate loss random variable is characterized, as well as the distribution of the first surplus below the initial level.

Some interesting applications of risk theory are outlined in Chapter 13. It is for instance shown how to compute net stop loss premium using the recursive formula of Chapter 12, and how stop loss reinsurance is linked to a dividend formula in group insurance. The effect of reinsurance on the probability of ruin is illustrated by means of examples, both for proportional and nonproportional reinsurance. Those examples naturally lead to a theorem that states the superiority of nonproportional over proportional reinsurance, if the (unrealistic) assumption is made that the reinsurance loadings are the same.

Chapter 16 to 19 develop special topics and are not included as examination material. They are nevertheless of extreme importance for practitioners. Chapter 16, "Special Annuities and Insurance," computes actuarial present values, net and gross premiums, and net premium reserves, for a wide variety of policies providing special annuity and insurance benefits: installment refund and cash refund annuities, family income and mortgage protection policies, and disability benefits. Of special importance is section 16.5; it deals with variable products, in which benefit levels and reserves depend on the performance of an investment fund. Also important is section 16.6, which addresses another recent major trend in U.S. life products: flexibility. Options for changing benefit amounts, premium, and plan of insurance (such as the possibility of increasing the death benefit of a term insurance, or of switching from a pure term to a policy with a savings component) are briefly discussed and illustrated.

In Chapter 17, "Advanced Muliple Life Theory," the elementary models for plans involving two lives, analyzed in Chapter 8, are extended to incorporate contingencies based on a larger number of lives and more complicated benefits, such as reversionary annuities. More general statuses than the two-life, joint-life, and last-survivor are defined, and a fundamental theorem is used to express the survival functions of these statuses in terms of only joint-life survival functions, under the usual assumptions of independent lifetimes.

Chapter 18 provides a most welcome introduction to population theory. The Lexis diagram is the main tool for the presentation of general population models, which are then applied to tracing the progress of life insurance benefits provided on a group, or

population, basis. In Chapter 19, "Theory of Pension Funding," similar aggregate models are applied to the evolution of retirement income benefits provided on a group basis. The major actuarial cost or funding methods for defined benefit plans are presented and analyzed.

The length of this review is indicative of the thoughts of the reviewer concerning the book. The Society of Actuaries and the five authors have to be commended for producing a superb textbook that will be used throughout the world by actuarial students for many years.

<div style="text-align: right">JEAN LEMAIRE</div>

George H. Andrews and John A. Beekman, *Actuarial Projections for the Old-Age, Survivors, and Disability Insurance Program of Social Security in the United States of America*, pp. 193, published by Actuarial Education and Research Fund, Itasca, Ill. 60143, 1987; $25.

George Andrews and John Beekman have written a long-needed, monumental monograph describing in detail (and also auditing) the assumptions and methodology underlying the actuarial estimates for the Old-Age, Survivors, and Disability Insurance system (OASDI) that are made by the Social Security Administration (SSA). The only such information available previously was in publications of the Federal Government (such as in the annual reports of the Board of Trustees of the OASDI Trust Funds or the Actuarial Studies issued by the SSA) or in writings of those who had been closely associated with the system (such as this reviewer's book, *Social Security*, 3rd edition, Richard D. Irwin, Inc., Homewood, Ill., 1985). So, an independent overview of the actuarial estimates for this extremely important government program by two distinguished academic actuaries is more than welcome.

The monograph contains seven chapters, an Addendum (which briefly points out possible future studies of a similar nature that might be made—such as for the Medicare program), and an Appendix. The first chapter briefly describes the provisions of OASDI (a more detailed description is given in the Appendix).

The second chapter deals with the demographic assumptions (fertility, mortality, immigration, and family composition) and the resulting projection of the total population of the United States. Then, the next chapter relates the economic assumptions underlying the actuarial estimates—such as labor-force participation, trends of wages and prices, and projections of the Gross National Product (GNP).

The fourth chapter carries forward the contents of the previous two chapters by describing the methodology underlying the long-range (75-year) projections of such elements as (1) number of covered workers, insured persons, retired workes and their benefit amounts, and (2) taxable payroll and GNP. This reviewer believes that the GNP estimates are much more susceptible to question than are the estimates of benefits and contributions (taxes). This is especially so because the latter are best used when considered relative to each other (that is, cost as a percentage of taxable payroll), whereas the former involve

many elements that are not interrelated (for example, proportion of compensation paid in other than cash, and not taxable under OASDI). This chapter also discusses the past experience and future assumptions as to disability incidence and termination rates.

The fifth chapter describes the special features of the short-range (five-year) estimates, while the following chapter deals with the long-range (75-year) estimates. The latter goes into detail as to how the auxiliary (spouse and child) and survivor beneficiaries and benefit disbursements are derived. Dealt with are such complexities as dual entitlement adjustments (in which persons are eligible for more than one type of benefit—for example, as a worker and a surviving spouse—but, in essence, receive only the largest). Finally, the concept of "actuarial balance" is defined and illustrated.

The final chapter gives an overview of the short-range and long-range actuarial estimates, explaining why they are needed and describing the several measures of actuarial soundness. Selected projection items, such as the fund balances at the end of the following year and the fifth year, and the long-range actuarial balances (and their components), are presented for OASI and DI separately for the annual Trustees Reports in 1978–1985. The results of sensitivity tests (changing one cost element at a time) for the long-range estimates are given. The chapter concludes with a strong endorsement of the work of the SSA actuarial staff over the years, both as to its quality and as to the "absolute wealth of excellent material" that is made publicly available to interested parties. As is stated, "one of the purposes of this monograph has been to help condense the enormous amounts of material into a smaller study." May the monograph serve as the "key" to unlock this store of riches to many people!

It is "standard operating procedure" for reviewers of books to make at least a few adverse comments and point out some factual errors; otherwise, it will appear that a thorough perusal was not done! As to factual accuracy, this reviewer had the opportunity to review in detail the final manuscript and would be most embarrassed to find any factual errors now. The monograph is truly excellent and should be made "required reading" for any actuary or non-actuary who is interested in the OASDI system, especially if writing a paper or book in the field of OASDI financing. In fact, editors of scholarly journals should require prospective authors in that field to certify that they have diligently and thoroughly studied the Andrews-Beekman monogaph!

ROBERT J. MYERS

J. J. McCutcheon and W. F. Scott, *An Introduction to the Mathematics of Finance*, pp. x + 463, published for the Institute of Actuaries and the Faculty of Actuaries by Heinemann, London, 1986.

The description on the back cover of this comprehensive, well-written book places it in context: "In today's money markets interest rates are all-important. This book, which is intended as a successor to D.W.A. Donald's *Compound Interest and Annuities-Certain*, develops the classical theory of compound interest (in which the force of interest is constant) as a special case of a more general model.

"There is a concise but thorough treatment of the basic compound interest functions, nominal rate of interest, and the yield (or internal rate of return) and there are many examples on discounted cash flow. Also discussed are applications of the theory to capital redemption policies (including a discussion of reinvestment rates), the valuation of stock market securities (with allowance for income tax, capital gains tax and index-linking), and consumer credit calculations. The final chapter provides a simple introduction to stoachastic interest rate models."

The book, primarily written for the actuarial student in the United Kingdom, like its predecessors by Donald (1953, 1970) and Todhunter [1], sets a high standard of expectation—as to theory and especially the solving of involved, practical problems. The McCutcheon and Scott text is noteworthy for containing clear solutions, not only to its 100⁺ well-chosen examples, helpfully distributed throughout most sections of the various chapters, but also (in 130 pages near the end of the book) to all the 176 exercises, stated at the end of the chapters. The near-quadrupling of the number of exercises overcomes a previous criticism of Donald. In this reviewer's opinion, the instructive solutions enhance the value of the book to qualified, serious readers.

Professors McCutcheon and Scott are both Fellows of the Faculty of Actuaries and teach at Heriot-Watt University in Edinburgh, in the Department of Actuarial Mathematics and Statistics. John McCutcheon has prior actuarial experience in life insurance and consulting and on the University of Manitoba faculty. William Scott has taught at the Universities of Glasgow and Michigan. That both have U.K. and North American experience brings a valuable extra dimension to their broad interpretation of the mathematics of finance. U.S. and Canadian works are well-represented in their list of 58 references, which includes books on capital investment and managerial finance.

In the light of recent developments in computation and of interdisciplinary endeavors, *Mathematics of Finance* is timely and up-to-date, particularly as to U.K. practice. The books by Donald, by S.G. Kellison, and by M.V. Butcher and C.J. Nesbitt [2], all of which go back to the early 1970s, are slightly dated and, in some important respects, less complete. Although likewise presupposing primarily a calculus background, the new book is, in general, more intensive and more difficult than the American books, especially in many (but not all) of its realistic applied problems. While purposely greatly influenced by Donald, Professors McCutcheon and Scott have contributed significantly to their subject through their selection of topics (exclusions as well as inclusions), articulate mathematical development, and choice of problems; they have collaborated very effectively.

The introductory chapter justifies accumulation at *compound* interest. Chapter 2 covers valuing (1) single sums at *variable* discrete and continuous rates of interest, with emphasis on varying forces of interest, and (2) cash flows, whether discrete or continuous. Thus, this chapter with its general model of interest rates anticipates Chapters 3 and 4, in which the key rates are constant (δ and $i^{(p)}$, respectively), and Chapter 5, devoted to discounted cash flow.

The third and fourth chapters develop the classical model for compound interest. Chapter 3 discusses the basic compound interest functions and ideas, such as effective rates of interest and discount, equations of value, the yield of a transaction (including criteria for its existence), simple annuities of most standard types, and loan (that is, amortization) schedules. Correspondingly, Chapter 4 deals with nominal rates and more general annuities. There are some adroit verbal interpretations, which become mnemonic devices. The technique of replacing actual with equivalent annuity payments is stressed. The selection of topics for inclusion, their arrangement, and the concise articulation in these chapters seem particularly felicitous.

For illustrative numerical work, the book has brief interest tables (at 1, 2, . . . , 15, 20, 25 percent), with $(Ia_{\overline{n}|}$ usefully replacing $1/s_{\overline{n}|}$ (or $1/a_{\overline{n}|})$. The emphasis throughout this text is on basic understanding of ideas, not details of calculations (except for an Appendix on iterative methods).

Chapter 5, "Discounted Cash Flow," is practical and modern. Involving terminology and measures familiar to some economists, business managers, accountants and actuaries, the "chapter is largely concerned with some applications of compound interest theory to the financial assessment of investments and business ventures" (p. 86). Among topics considered are *net* cash flows and their *present values (NPV)*, yield and profitability of a single investment project or competing ones, the effects of inflation, and various measures of rate of return on investments, including on unitized funds.

The capital redemption policy, though now rare even in the U.K., is ably presented in Chapter 6 as an introduction to life insurance mathematics. The chapter ends with an elegant theoretical development of the accumulated amount under varying *reinvestment* rates of (1) a sequence of unit sums and (2) a single unit sum.

The subject of Chapter 7 is prices and yields of fixed-interest securities, with interest income possibly liable to income tax. Chapter 8 studies the effect of capital gains tax on these securities, according to current U.K. practice. Because terminology and sometimes practice may differ from North American [3], the reader should note the authors' descriptions and definitions. Like their predecessors, McCutcheon and Scott focus on net (after tax) yield and Makeham's formula. Typically the formula also applies (if necessary, adjusted by additive terms) to a security redeemable in installments [4]. In solving numerous complicated problems for price or yield, the authors show how ingenious, possibly repeated, use of Makeham's formula may be efficient. There is a nice discussion of the effect of the term to redemption on the yield, applicable in the case of optional redemption dates. Chapter 7 ends with a section on *real* (that is, inflation-adjusted) yields and index-linked investments, illustrated by U.K. practice.

Many of the concepts and applications of Chapters 7–10 are of interest to the institutional investor, such as an insurance company or pension fund. Chapter 9 is about a particular kind of fixed-interest security once common in the U.K., namely, a loan repayable by a cumulative sinking fund. The American reader finds this equivalent to amortization, with complications (for example, taxes and payments on principal at premium prices such as 110 percent).

Chapters 5 and 10–12 supplement the traditional material of compound interest books and were outside the scope of Donald's book. Some of the topics of Chapter 10 relate to fixed-interest securities: yield curves; discounted mean term, $T(\delta)$, of a project; and the volatility (when interest rates change slightly) of a project. From the definition of volatility (at force δ) as $-NPV'(\delta)/NPV(\delta)$, it follows easily that volatility $= T(\delta)$; the greater its magnitude for a fixed small change in interest rates, the greater the relative profit or loss on an investment. Other topics of Chapter 10 relate to assets and liabilities: matching and immunization, to protect an investor against changes in interest rates. The text presents Redington's theory (1952) of immunization, which protects against *small* rate changes; one consequence is that (at the current force δ_0) the discounted mean terms of total assets and total liabilities are equal. The Butcher and Nesbitt book also discusses the Redington theory, but McCutcheon and Scott's book goes further in presenting a theory of "full" immunization, in which the investor profits from *any* immediate rate changes. A requirement of this theory—the linking with *each* item of liability-outgo of *two* items of total assets, one due before and the other after the liability—may in reality be difficult to attain [5].

Chapter 11 on consumer credit is mainly concerned with recent U.K. legislation and regulations respecting (1) disclosure of the total charge for credit and the annual percentage rate of charge and (2) the minimum interest rebate on early repayment. For some comparison with the U.S., one might refer to Kellison and to Butcher and Nesbitt.

The final chapter, an introduction to stochastic interest rate models, will be appealing to readers of the Society's *Actuarial Mathematics* (1986). To allow for uncertainty in the future, one uses probability theory to provide variations in interest rates and relies on computers and simulation techniques. The authors state (p. 270): "Stochastic interest rate models offer a powerful tool for the analysis of financial problems. . .fundamentally different from the deterministic approach" of their previous eleven chapters. Assuming that annual yields, i_t, are *independent* random variables, they consider the random variables S_n and A_n, which denote the accumulation of a unit sum and of an annuity-due of n unit payments, respectively, and they derive the mean and variance of each. Then, based on assumptions about the distribution of the i_t, probabilities concerning S_{15} and A_{15} are calculated. For example, if the $1 + i_t$ are assumed to have identical, specific log-normal distributions, then $\log(1 + i_t)$ and $\log S_n$ have specific normal distributions, so exact probability statements about S_{15} can be made; since the distribution of A_{15} is not apparent, the authors carried out 10,000 simulations to approximate it and estimate probabilities. Next, they briefly discuss simple forms of *dependent* interest rates and illustrate A_{15}. The chapter ends with an application of Brownian motion to a single premium investment.

This attractive book proceeds smoothly and carries the reader along, but—if one is not careful—the reader who completes it may find the extent of the coverage and the amount of detail somewhat overwhelming. Nevertheless, the main ideas, because of the lucid presentation and repetition, should be very clear. All in all, this new *Mathematics*

of Finance should be valuable and interesting to North American actuarial students, educators, and Society members.

MARJORIE V. BUTCHER

END NOTES

1. TODHUNTER, RALPH. *Interest Including Annuities-Certain. Part 1 of Institute of Actuaries Text-Book of the Principles of Interest, Life Annuities, and Assurances, and Their Practical Application.* London: Charles and Edwin Layton, 1901; 3d ed. rev. and enl. by R.C. Simmonds and T.P. Thompson. Cambridge: Ulrich, Institute of Actuaries, 1931.
2. Reviewed in *TSA*: VI, 617 and XXIII, 629; XXI, 627; and XXIII, 628, respectively.
3. For example, bonds in U.S. nomenclature are often called stocks in the U.K.
4. This might be a whole bond issue or a loan being amortized.
5. The text is substantially error-free, but has two errors here in its first printing, corrected in later printings. In equations (10.8.3) and (10.8.4), the first factor should read $\exp(-\delta t_1)$, and on p. 248 for $x<0$, $f'(x)<0$. An errata sheet is available from the authors.

Annie N. and Lloyd K. Friedman: *On the Trail of Actuaries in Texas, 1844–1964*, pp. 216, published by Watercress Press, San Antonio, Texas, 1988.

In the fall of 1983, the Actuaries' Club of the Southwest set an example that other clubs and writers would do well to follow when its president, Edwin E. Hightower, proposed as one of its projects a "history of the Club and actuarial practices in the Southwest." The hour produced not just the man but the married couple—Lloyd K. Friedman, F.S.A. 1937, and his wife, Annie. Would that other laborers in the historical vineyard could be half as fortunate! These authors, with the acknowledged aid of other enthusiasts, required less than five years to produce a thoroughly entertaining, informative, and even inspiring volume.

The book is in three chronological parts of four chapters each, ending with an epilogue and three appendixes descriptive of the 1949 foundation of the sponsoring club. The volume's frontispiece portrays the two actuaries whose impacts upon the profession's maturing in the Southwest were exceptional—Lawrence M. Cathles (1877–1958), an immigrant from Scotland, and Texas-born Paul V. Montgomery (1886–1980).

Annie and Lloyd made extensive use of taped interviews, some of them giving the words of widows of the actuaries concerned, and also dug deeply into documentary accounts for the biographies. Readers are thus favored with an appreciation of the activities, mind-sets, and eccentricities of more actuaries than most outsiders would have guessed contributed to the significance and reputation of our profession in Texas. More than sixty men and women received greater-than-passing mention.

Readers should feel even further indebted to the Friedmans for devoting ten pages to a charming O. Henry story about the chivalry of Luke Coonrod Standifer, Texas Commissioner of Insurance, Statistics, and History in the 1880s. This is icing on the cake.

The North American actuarial profession, although rightly considering itself one of the learned societies, has a sadly spotty record when it comes to putting on paper the story of its own heritage. The founders of the Actuarial Society of America in 1889 behaved almost as though no profession existed before their time; a few personal reminiscences and trade paper news items are all that we have today about eighty years of struggle and experimentation. And the Actuarial Society placed a low valuation upon its library; sporadic appropriation to buy books were voted, but in general the quantity now on the shelves is far less than it should be, especially considering the vigor for many years of the book review sections of actuarial journals.

Curiously, the Friedman's book is not for sale. The largest supply of copies sits at the *Yearbook* address of Jack A. Rollier. Requests accompanied by a check for $15 per copy payable to the Actuaries' Club of the Southwest to help cover publication costs will be cheerfully received. If a desire for good reading about the profession prevails among today's actuaries, Mr. Rollier's supply surely will be quickly exhausted.

E. J. MOORHEAD