SECURITIZATION OF MORTALITY RISKS IN LIFE ANNUITIES

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ABSTRACT. Securitization of mortality risks is an alternative risk management tool that may have some advantages over reinsurance. The purpose of this paper is to study mortality-based securities, such as swaps and mortality bonds, and to price the proposed mortality securities. We focus on group life annuity data, although the techniques could be applied to other lines of annuity or life insurance.

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1. Introduction

The purpose of this paper is to study the securitization of mortality risks and to price proposed mortality swaps and bonds with embedded mortality options. Mortality based securities will expand the array of tools available to insurers and reinsurers to manage longevity risks. The potential for greater underwriting capacity, innovative long term contracting, and lower costs make securitization worth investigating as a supplement to traditional reinsurance.

The longevity risk is a dynamic phenomenon. Life expectancy throughout the world in recent decades has improved, but that does not necessarily imply that trend can be projected into the future. Mortality analysis has a long tradition in actuarial science because mortality trends can have a profound influence on a life insurer's financial condition. However, since no one can accurately predict the future, risk management of mortality risk is an indispensable part in the insurer's operation. In addition to uncertainty in mortality forecasts, there are economic and policy changes that make management of longevity risk more important than ever.

Ten years ago, Friedman and Warshawsky [7] argued that it was puzzling that so few people avail themselves of the private market for annuities. They listed three potential answers to this puzzle: Firstly, people save not for motives related to the usual life-cycle reasoning but, instead, to leave bequests to their heirs; secondly, most individuals automatically receive life annuities from Social Security and, for a significant fraction of the labor force, employer-sponsored pension plans; lastly, a more plausible explanation deemed by Friedman and Warshawsky is that people shun individual annuities because they are not priced "fairly" in the actuarial sense. Although the individual annuity market is currently quite small, it has attracted substantial interest from researchers and policy makers concerned with the evolving system of retirement income provision. In light of baby boom cohort near retirement, current discussions of Social Security reform and the shift from defined benefit to defined contribution private pension plans, Mitchell *et al.* [11] suggest that there may be increased interest in individual annuity products in the future. They also find evidence that the expected present discounted value of annuity payouts relative to premium payments has increased by approximately 8 percentage points during the last decade. Thus, from the standpoint of potential purchasers, an individual annuity contract appears to be a more attractive product today than ten years ago.

As demand for individual annuities increases, insurer's need for risk management of the potential mortality improvements increases as they write new individual annuity business. As Rappaport [24] describes, insurers manage their risk in issuing these new annuity policies, and are therefore keenly interested in understanding the future course of longevity, as well as the protection provided by hedging, asset allocation strategies, and reinsurance. Securitization of mortality risks is another tool for managing this risk and it has some advantages over reinsurance.

A market for mortality-based securities will develop if the prices and contracting features make the securities attractive to potential buyers and sellers. We illustrate methods for projecting mortality risk and portfolio cash flows so that buyers and sellers can understand and price the proposed securities. If insurers are willing to pay a fair price based on a reasonable projection and they can find counterparties, insurers can eliminate their concerns on the possibilities of longevity risk. Based on the characteristics of the data of the Transactions of the Society of Actuaries Reports, we focus primarily on pricing immediate life annuities.

Section 2 covers the potential expansion of the individual annuity market in the United States. In section 3 we discuss the demand for mortality based securities. In section 4 we describe how insurers can use mortality–based securities and why they may want to sell them. In section 5 we describe the difficulties arising in making mortality projections. We discuss annuity data, including the Individual Annuity Mortality tables and the Group Annuity Experience Mortality (GAEM) reports from Reports of the Transactions of the Society of Actuaries (TSA). We decided to use the GAEM experience for our mortality forecasts with a model by Renshaw *et al.* [26]. We fit a projection model to US annuity data. In section 6 we discuss securitization of mortality risk. We define mortality swaps and show how they can be used to hedge mortality risk. Section 7 reviews the common shock model. In section 8 we introduce mortality risk bonds and price them using the Wang transform. Section 9 is for discussion and conclusions.

2. Individual Annuity Market in the United States

The annuity market, including fixed as well as variable annuities and individual as well as group annuity contracts, has grown sharply in the last decade. With the baby boom cohort approaching retirement, Social Security reform, the decline in the growth of defined benefit pension plans, and the increase in the growth of defined contribution plans, we expect that the individual annuity market will expand dramatically.

Baby Boom. According to Mitchell *et al.* [11], as the baby boom cohort in the United States nears and moves into retirement, analyst, policy makers, and advisors in many nations are devoting increased attention to issues of old-age income security. Increased longevity imposes a greater risk to individuals of outliving their resources who may be forced to substantially reduce their living standards at an advanced age. Life annuities can play an important role in helping people protect against risks arising from dramatic advances in life expectancy. We expect that the baby boom will increase the demand for annuities.

Social Security Reform. In most western nations including the United States, retirees have typically had access to three main sources of annuities: universal publicly provided Social Security payouts, payouts from employer-sponsored defined benefit pension plans, and payouts from privately purchased annuities offered by insurance companies. Social Security has traditionally been the most important of these three.

Mitchell *et al.* [11] argue that Social Security reform discussions in the United States and other nations have the potential to increase the demand for private annuities. These reforms, if enacted, could also substantially affect the structure of the annuity marketplace. The existing Social Security system in the United States is a pay–as–you–go defined benefit plan. In the United States and many other nations, analysts have noted with concern the possibility of Social Security system financial insolvency, if promised benefits come to exceed system revenue. This problem already plagues the underfunded defined benefit systems of many European and Asian nations. While the precise direction of Social Security reform in the United States and other nations may be difficult to predict, most proposals that involve increased reliance on defined contribution-style accounts are likely to result in increased attention to the operation of private annuity markets. Many such proposals could substantially increase the volume of annuity sales. Supplemental or partial replacement of the existing pay–as–you–go public defined benefit system with an individual–accounts defined contribution programs have recently been implemented in the United Kingdom, Argentina, Chile, Mexico, Hong Kong, and elsewhere.

Defined Benefit Pension Plans vs. Defined Contribution Plans. Figure 1 shows the individual and group premium income received by insurance companies for annuity policies over the 1970–99 periods, converted to 1994 dollars using the Consumer Price Index. Poterba [23] notes that although premiums on group policies were three to five times greater than the premiums on individual polices throughout the 1950s and 1960s, individual annuities grew more rapidly from the 1970s until

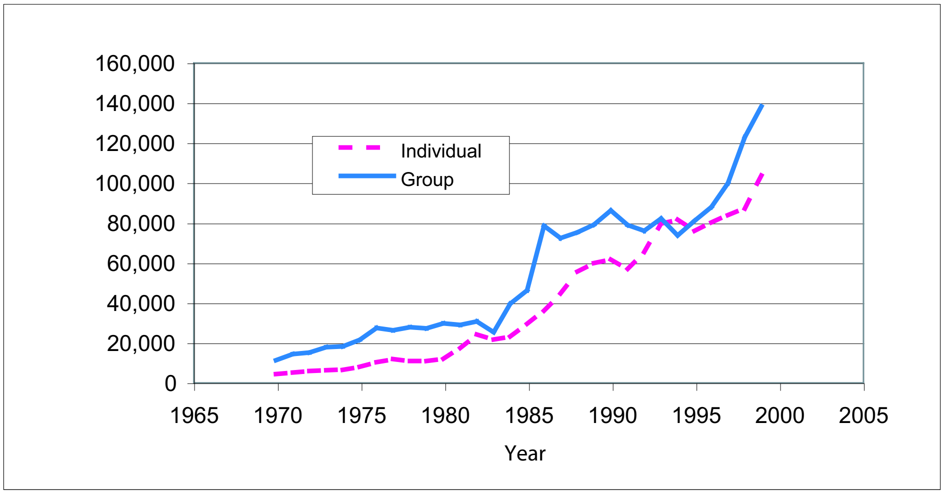

FIGURE 1. Annuity premium annual income of life companies in millions of 1994 dollars from Poterba [23].

the mid–1990s. In 1994, premium income from individual annuities exceeded that from group annuities. By the 1990s, annuity reserves were more than twice the value of life insurance reserves. The long–term growth of individual relative to group annuity premium reflects both the decline in the growth of defined benefit pension plans and the rapid expansion of individual annuity products, particularly variable annuities.

3. Demand for Mortality Based Securities

Insurance risk, usually catastrophic property damage risk, has been successfully passed to bondholders. These are the so-called cat bonds. Cox and Pedersen [3] describe a model for pricing cat bonds and several examples of cat bonds. Cox, Pedersen, and Fairchild [4] discuss the conditions under which a market for cat bonds is viable. They argue that, if cat risks are uncorrelated with the stock and traditional bond markets, then adding cat bonds to the market improves investment opportunities. An investor with traditional high yield bonds will prefer to hold cat bonds of the same investment quality, because of their lower covariance with the market. This helps explain why there were over thirty cat bond transactions reported in the financial press. Mortality risk bonds will be different in several important ways. However, in both cases transactions costs are likely to be relatively high to reinsurance on a transaction basis. For securitization to succeed, it will have to be done on a large scale.

While rationale for a market for cat bonds was (at least in part) based on the notion that cat bonds returns should be uncorrelated with market returns, we have learned from Sven Sinclair that the same argument is not likely to apply to mortality based securities. He directed us to the life cycle theory

> ... which was pioneered a long time ago by Franco Modigliani, and has been generally accepted as a standard macroeconomic model for quite some time, especially since Auerbach and Kotlikoff, in the 1980s, developed Samuelson's overlapping generations framework into a working computational model. The main premise in the model is that workers save for retirement (by themselves or through public or private pensions), and retired people spend down such savings that they accumulated when they were working. The equilibrium asset prices in the economy are then influenced by the demographically–driven supply and demand.

We are not asserting that mortality securities have a zero beta, although investors will want to know beta. The life–cycle theory may imply some relation between mortality securities and equity market returns. However, as far as we can tell, the correlation of unanticipated mortality improvement with the market remains an interesting and open empirical issue. Moreover, while the beta (whatever it is) may help investors determine a price, beta does not have to be zero for a security to be viable. There are in the current market bonds with coupons that depend on interest rates, commodity return, and even equity returns, sometimes called structured notes.

We agree that mortality–based contingent claims may not be zero-beta assets, but the argument in favor of a developing mortality security market do not rely on a zero beta. Investors may buy mortality based bonds as a diversification, even if mortality risk has a positive or negative correlation with the market. The risk–reward relation is what matters. Innovative, long term contracting may bring a premium. Large scale transactions and technological developments may keep costs down.

4. Supply of Mortality Based Securities

The same rationale for hedging longevity risk with reinsurance can be applied to securitization. The advantages of securitization may be lower costs in the long run, more favorable contracts, and elimination of default risk. Reinsurance is sometimes used to provide growth capital. securitization could be used in the same way.

Raising Required Capital. When an insurer sells an immediate annuity, it usually pays a commission (and incurs other costs). Insurance and accounting regulations require that the company hold capital to provide for future annuity benefits. It is possible that the sum of acquisition costs and statutory capital required exceed the premium paid by the annuity owner.

For example, suppose the premium is \$5,000,000 for a male annuitant age 65. The commission and issue expense might be 4% or \$200,000. The total monthly payout is \$39,058 based on the average immediate annuity market quotes in 1995 (about 7.81 dollars per month per 1000 dollars of premium). The statutory reserve is determined by valuation regulations as $39,058 \times 12a_{65}^{(12)}$. With level annual interest rates of 6% and the 1996 US Annuity 2000 Basic annuity table, the liability value for an annuity of one dollar per year is $a_{65}^{(12)} = 10.63566$. The problem for the insurers is that $39,058 \times 12a_{65}^{(12)} = 4,984,891$ which exceeds the net price the company gets after commission (\$4,800,000) by \$184,981. This is about 3.7% of the premium. This does not mean the business is not profitable. On a market valuation basis, the present value of future benefit (PVFB) using realistic mortality and market interest rates is less than the premium. That is, $PVFB + 200,000 < 5,000,000$, so the company adds to its market value on a present value basis. In other words, the company must dedicate capital to the annuity business in order to grow. Reinsurance (or securitization) could be used to address the need for capital as the annuity business grows.

Innovative Contracting. Cummins [5] describes securitization as the repackaging and trading of cash flows that traditionally would have been held on-balance-sheet by financial intermediaries or industries. Securitizations generally involve the agreement between two parties to trade cash flow streams to manage and diversify risk and/or to take advantage of arbitrage opportunities. Reinsurance is a traditional way for insurers to transfer their risks to reinsurers, but securitization may be a viable alternative.

There are some similarities between securitization and reinsurance. For example, both ways improve the balance sheet because the investor's or reinsurer's return is based on the performance of specific collateral as opposed to the overall performance of the cedent or originator; increase actual surplus and decrease required capital; transfer the risks and rewards related to the collateral, which effects the balance sheet and allow the cedent or originator to divest a noncore business line or focus on origination and serving.

Securitization	**Reinsurance**
Publicly traded or private placement	Private placement
Based on liabilities for a cohort defined at issue	Pricing and capacity are cyclical and reflective of recent underwriting results
Bonds carry credit–rating	No credit–rating for the insurance contract
Collateralized bonds have no default risk	Reinsurance buyer bears default risk.
Bonds are loans for tax purposes	Reinsurance transactions can produce taxable income to the buyer.
More regulatory burden	Less regulatory burden
Long-term funding	Short-term funding
Exclude or minimize underwriting risks.	Include underwriting risks.
More regulatory concerns	Less regulatory concerns
High capacity	Low capacity

Table 1. How reinsurance and securitization differ.

Table 1 shows the dissimilarities between securitization and reinsurance. One of the major differences lies in the fact that the capacity of the financial market is much larger than that of primary insurance and reinsurance industries. The bond contact can be customized for the borrower and lender and could be very different from traditional reinsurance contracts. For example, the bond contract might provide for 30 years of coverage. Transactions costs of issuing bonds is expensive relative to buying reinsurance. However, billions of dollars of assets (mortgages, auto loans, and so on) are securitized each year. If the technology used in these securitizations is brought to annuity securitization, and if large numbers of annuitants are involved, then the price per unit may be even less than reinsurance.

While the individual annuity market in the United States is relatively small, there are several reasons to expect that it may grow in the future. One is the rising proportion of older persons holding substantial assets in variable annuities that might someday be annuitized. A second and perhaps more important reason is the recent growth of defined contribution pensions such as 401(k) plans. Projections of future growth presented in Poterba, Venti, and Wise [22] suggest that the average 401(k) balance for individuals retiring in 2030 will be very close to the present discounted value of Social Security benefits, assuming the continuation of current law. If demand for individual annuities increases dramatically, insurers will need to hedge the mortality risk as they write more new individual annuities.

Long-term hedging is another major advantage of the securitization of mortality risks over reinsurance. Reinsurance is normally short-term oriented. Renewing reinsurance frequently may pose higher transaction costs on the primary annuity insurer than those of securitization. Mitchell *et al.* [11] describe dramatic advances in life expectancy in the United States over the last century. Today's typical 65-year-old man and woman can expect to live to age 81 and 85, respectively. Perhaps even more striking is the fact that almost a third of 65-year-old women and almost a fifth of 65-year-old men are likely to live to age 90 or beyond. Long-term hedging is especially important for the annuity insurer. Long-term investment products are common in the financial market. The financial market can tailor a suitable security for the insurer.

Eason *et al.* [6] claim,

> ... The other issue facing reinsurers is that they also, because it is insurance, have the same kind of target capital needs that the insurance company does, which has typically a higher risk-based capital requirement than a pure bank does.

Therefore, reinsurers are likely to charge a higher premium than the financial market, if transaction costs are not considered, simply because they must hold more capital to write the same risk. With greater capacity, better contracting terms (long-terms, for example) and potentially lower cost (more efficient use of capital), securitization will be a better way for an insurer to hedge its mortality risks.

5. Difficulties in Accurate Mortality Projection

General and insured population mortality improves remarkably over the last several decades. At old ages probabilities of death are decreasing, increasing the need for living benefits. The calculation of expected present values (needed in pricing and reserving) requires an appropriate mortality projection in order to avoid underestimation of future costs which will jeopardize an insurer's business solvency. Overcharging customers will lose an insurer's competitiveness. Rogers [27] shows

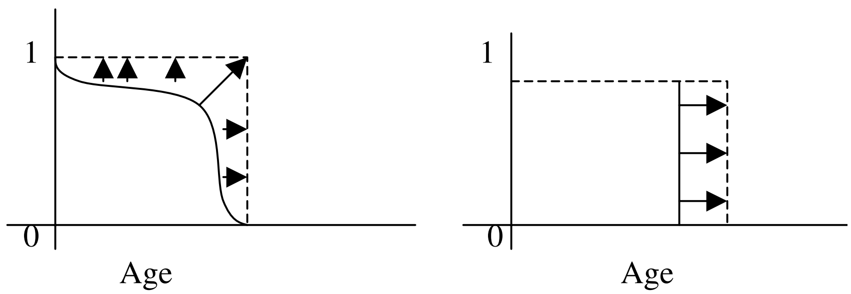

Figure 2. Two views of mortality improvement, rectangularization on the left and steady progress on the right.

that mortality operates within a complex framework and is influenced by socioeconomic factors, biological variables, government policies, environmental influences, health conditions and health behaviors. Not all of these factors improve with time. For example, for biological variables, recent declines in mortality rates were not distributed evenly over the 15 disease categories of underlying and multiple causes of death. Stallard [29] claims that successes against the top three major killers (heart diseases, cerebrovascular diseases and malignant neoplasms) did not translate into successes against many of the lower ranked diseases.

Different Opinions in Mortality Trend.

Improvement. Buettner [2] concludes that there are today two alternative views about the future improvement of mortality at older ages: compression vs. expansion (sometimes also called rectangularization vs. steady progress), illustrated in Figure 2. Mortality compression occurs when age-specific mortality declines over a widening range of adult ages, but meets natural limits for very advanced ages. As a result, the survivor curve would approach a rectangle and mortality across countries may indeed converge to similar patterns.

In the case of steady progress, there are no natural limits to further reductions in mortality at higher ages. The age at which natural limits set in does not exist. Consequently, all age groups, especially higher age groups, would continue to experience declining mortality. The Human Genome Project is producing a rapidly expanding base of knowledge about life processes at their most fundamental level. A growing number of scientists recognize that extension of the maximum life span as a possibility.

Life Table Entropy. Life table entropy refers to a phenomenon that further improvement of already high life expectancies may become increasingly more difficult. The gains in survival a century ago were greater than they have been more recently. For instance, Rogers [27] shows that the survival gains achieved between 1900 and 1920 are large compared to the modest gains realized between 1980 and 1999. Hayflick [9] suggests that,

> ... Those who predict enormous gains in life expectation in the future based only on mathematically sound predictions of life table data but ignore the biological facts that underlie longevity determination and aging do so at their own peril and the peril of those who make health policy for the future of this country.

Deterioration. Although general population mortality has improved over time, the improvement may be overstated. Substantial mortality improvements often come after periods of mortality deterioration. For example, between 1970 and 1975, males aged 30-35 saw annual mortality improvement of over 2%, but this may be an adjustment to the 1.5% annual mortality decline that occurred during the previous five-year period. Moreover there is still a chance for a resurgence of infectious diseases. Deaths due to influenza could increase with the introduction of new influenza strains or with shortages of the influenza vaccine. Rogers [27] argues that although HIV is now controlled, it is not eradicated and could expand, or variants of HIV could develop that could increase mortality. Drug resistant infectious diseases like tuberculosis could increase. Goss, Wade and Bell [8] find that age–adjusted annual death rates for ages 85 and over in the United States actually deteriorated by 0.72% per year for males and by 0.52% for females during the observation period 1990-94.

Technical Difficulties in Mortality Projections.

Quality of Data. Good quality complete data is a prerequisite for a reliable mortality projection. However, in reality it is not easy to obtain data for research. For example, although detailed data on old-age mortality are collected in most countries of the developed world, they are not so commonly available for developing countries. Buettner [2] claims that even in developed countries, the quality of age reporting deteriorates among the very old.

The Society of Actuaries' studies of life annuity experience are of limited value for several reasons. First, it is not timely. Second, it is appropriate only for the products the policy holders owned (whole life, term life, or annuities, for example). So it cannot be used directly to assess mortality for new products or similar products issued on a new basis (e.g. underwriting annuities for select mortality).

Thulin [30] claims that complexity of annuity products nowadays often makes mortality projection difficult. Sometimes, an insurer has to introduce new entries with different mortality assumptions into the insured pool. For instance, trends in the marketplace are blurring traditional distinctions in the following two key areas:

(1) Worksite products sold on an individual basis increasingly show features traditionally associated with group products.
(2) Group products sold on the basis of individual election in the workplace (voluntary products) with minimal participation requirements compete directly with individual products.

They severely limit insurers' ability to underwrite to discern mortality differentials. New sources of underwriting information are becoming a way of life for insurers, as pressure on costs and hastened issue pressure create an underwriting environment with less documentation and information. One solution is making more data available to researchers and making it available sooner.

The Society of Actuaries publishes tables and mortality reports from time to time. The individual annuity mortality (IAM) tables are intended for estimation of insurance company liabilities. While these tables are bases on actual insurance industry experience, the rates are projected or loaded in order to produce conservative estimates of annuity liabilities. Until 1992 the Society published periodic group annuity mortality reports of actual experience. The reports do not contain complete mortality tables and they are no adjustments, so these reports reflect actual industry experience. We will illustrate the dynamics of annuity mortality with the actual experience from *Reports of the Transactions of the Society of Actuaries.*

The *TSA* [12] states,

> . . . In deriving the 1971 IAM Table, the Joint Actuarial Committee based its choice of mortality improvement rates for the period from 1963-71 on the immediate annuity experience from 1958-63 to 1963-67 and the "settlement annuity" experience from 1955-1960 to 1960-65. Annual improvement rates were developed from the combined experience for ages 79 and under (1.6 percent)

> and ages 80 and over (1.1 percent). The same rates were used for males and females. The committee developed a set of improvement rates based largely on the United States white population experience, with some effect given to the medicare experience and the relationship of annuitant to the United States white population improvement rates during the period 1961-65 to 1971-7 . . .
>
> In order to derive a projected 1983 IAM table, a special tabulation of the Society of Actuaries 1971-76 annuity mortality study was prepared for the committee. The committee decided to use the 1971 IAM Table rates had been loaded for use as a valuation table. If these rates were used without adjustment in the 1973 Experience Table, a second loading would be added in the process of deriving the 1983 Table a from the 1983 Basic Table. To avoid this consequence and at the same time provide for a smooth table through all ages, the 1971 IMA Table rates at ages 47 and under were divided by 0.9 to offset exactly the level 10 percent loading adopted by the committee for the 1983 Table a.

Based on the above information, we decided that the loaded or projected IAM tables are not appropriate for our illustration and prediction. We need the actual experience data.

The *1983 Transactions Reports of the Society of Actuaries* [16] present calendar year experience of retired individuals who are covered under insured pension plans in the United States and Canada. The report includes experience of contracts providing insurer–guaranteed annuity benefits to ongoing pension plans and experience of contracts covering closed groups of lives for which purchases are made by a single payment at issue (single–premium close–out business); it excludes contracts which do not contain insurer guarantees of future payments (immediate participation guarantee contract direct-payment benefits). The reports summarize calendar year exposures and deaths in five-year age groups. Male and female data are displayed by number of lives and amount of annual annuity income.

The 1983 *TSA Reports* [16] describes problems encountered in collecting data:

> "The last published report of insured group annuity mortality experience appeared in the 1975 reports covering calendar years 1969 – 71. It was hampered by data

> collection problems during the subsequent ten years: several companies discontinued experience submission, and several others submitted data which were inconsistent or riddled with reporting errors."

The problems remain although the Society is working now to revive its experience studies. In the end, we decided to use the *GAM Experience Reports* since they are based on actual mortality improvement.

The *GAM Experience Reports* [13, 14, 15, 16, 17, 18, 19, 20, 21] describe the mortality improvement from 1951–1992. The *Reports* give the number of deaths observed among a cohort of annuitants in 5–year age groups observed for one year. The observations of deaths and exposures are summarized in the appendix to this paper. The *Reports* provide data, but do not construct mortality tables. We show graphs of this experience in Figures 3, 4 and 5. It is based on retired individuals covered by pension plans in the United States and Canada. We assume that the ratio of the total number of deaths in each group over the total number of exposures in that group (the average death rate in that group) represents the death rate of the middle-point age of that group. We assume the group initially consists of $\ell_{55} = 1,000,000$ lives age 55. The symbol ℓ_x is the number of lives attaining age x in the survivorship group . From the figures, we can tell that the male mortality improves more than the female mortality. The combined the male and female mortality experience is in Figure 5.

Projection Models. Recent changes in mortality challenge mortality projection models. The competitive nature of the insurance market means that an insurer cannot raise its price at will. A sound projection model is crucial to the survival of an insurer. However, the revealed weakness and problems of poor fitting may arise because most projection models do not capture the dynamics of mortality that is changing in a dramatic and fundamental way.

Marocco and Pitacco [10] suggest that a projection procedure requires:

- an appropriate sequence of mortality tables used to express past experience;
- a model representing the mortality trend;
- the estimation of the model parameters.

Renshaw *et al.* [26] suggest a generalized linear model which showed mortality declining over time with the rates of decline not being necessarily uniform across the age range. It incorporates both the age variation in morality and the underlying trends in the mortality rates.

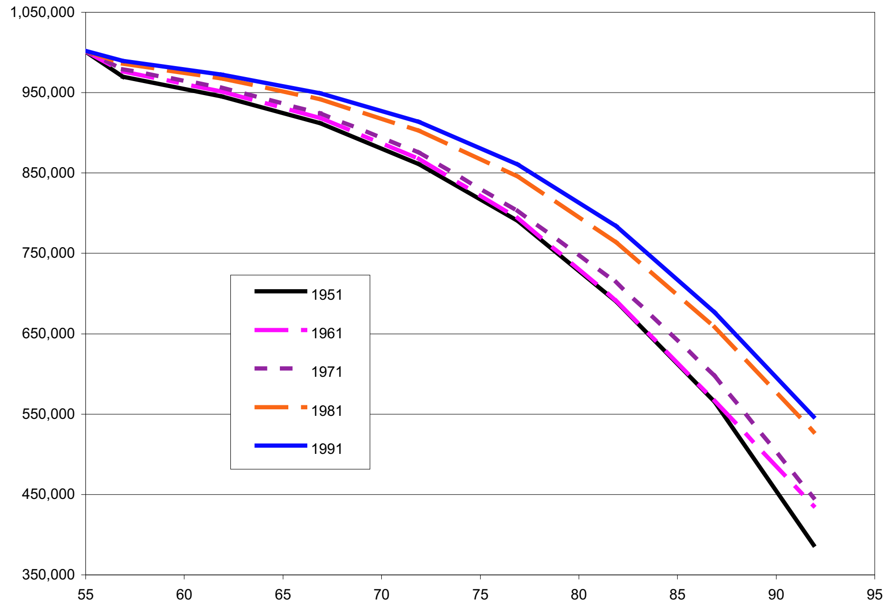

FIGURE 3. Number of survivors of an initial cohort of 1,000,000 male lives at age 55. Based on the Society of Actuaries *TSA Reports* 1951 to 1991 on group annuity experience, without adjustments.

The advantage of this model is that the predictions of future forces of mortality come directly from the model formula. Sithole *et al.* [28] claimed that the mortality improvement model of Renshaw *et al.* [26] can be derived also from the model formula for use with a given set of mortality tables for a given base year. We adopt this model for investigating the performance of mortality derivatives based on a portfolio of life annuities.

During a certain period, the force of mortality, $\mu_{x,t}$, at age x, in calendar year t, is modeled using the following formula:

$$\begin{aligned}\mu_{x,t} &= \exp\left[\beta_0 + \sum_{j=1}^{s}\beta_j L_j(x') + \sum_{i=1}^{r}\alpha_i t'^i + \sum_{i=1}^{r}\sum_{j=1}^{s}\gamma_{i,j}L_j(x')t'^i\right] \\ &= \exp\left\{\sum_{j=0}^{s}\beta_j L_j(x')\right\}\exp\left\{\sum_{i=1}^{r}\left(\alpha_i + \sum_{j=1}^{s}\gamma_{ij}L_j(x')\right)t'^i\right\} \qquad (1)\end{aligned}$$

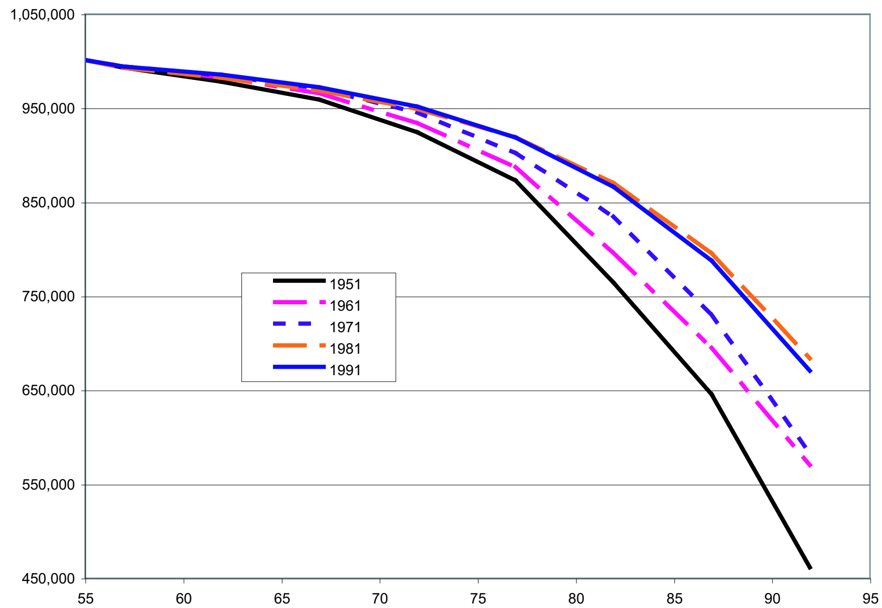

FIGURE 4. Number of survivors of an initial cohort of 1,000,000 female lives at age 55. Based on the Society of Actuaries *TSA Reports* 1951 to 1991 on group annuity experience, without adjustments.

The first of two multiplicative terms is the equivalent of a Gompertz-Makeham graduation term. The second multiplicative term is an adjustment term to predict an age-specific trend. The γ_{ij} terms may be pre-set to 0. The age and time variables are rescaled to x' and t' so that both are mapped onto the interval $[-1, +1]$ after transforming ages and calendar years. $L_j(x)$ is the Legendre polynomial defined below:

$$\begin{aligned}
L_0(x) &= 1 \\
L_1(x) &= x \\
L_2(x) &= x^2 - 1/3 \\
L_3(x) &= x^3 - 3x/5 \\
&\vdots \\
(n+1)L_{n+1}(x) &= (2n+1)xL_n(x) - nL_{n-1}(x)
\end{aligned}$$

where n is a positive integer and $-1 \leq x \leq 1$.

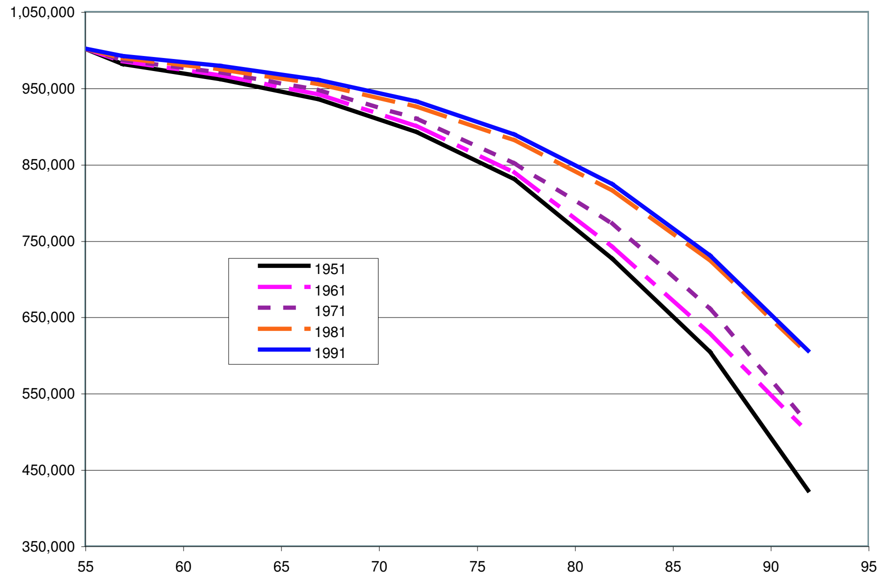

FIGURE 5. Number of survivors of an initial cohort of 1,000,000 female and male lives at age 55. Based on the Society of Actuaries *TSA Reports* 1951 to 1991 on group annuity experience, without adjustments.

The data are the actual group annuity mortality experience for calendar years $t = 1951, 1961, 1971, 1981 - 1992$. Since the GAM Experience Reports are five-year age group results, we assume that the ratio of the total number of deaths in each group over the total number of exposures in that group (the average death rate in that group) represents the death rate of the middle-point age of that group. We use the middle-point age as our observation in the regression. The experience was analyzed for the middle-point age ranges $x = 57$ to 92 years for both male and female with $x' = (x - 74.5)/17.5$, giving a total of 120 data cells for male and female, respectively.

In fitting the equation (1), we found that when the parameter $\gamma_{1,2}$ is excluded from the formula (for male and female), all of the remaining six parameters in the model are statistically significant. Although the six–parameter model which excludes the quadratic coefficient in age effects from the trend adjustment term was next fitted to the data, the revised models seem to be appropriate for making predictions of future forces of mortality. Sithole *et al.* [28] point out that in searching for a model that has a good shape for the purpose of making predictions, there has to be a trade-off between goodness-of-fit and predictive shape. The revised 6-parameter model is as follows:

$$\mu_{x,t} = \exp\left[\beta_0 + \beta_1 L_1(x') + \beta_2 L_2(x') + \beta_3 L_3(x') + \alpha_1 t' + \gamma_{11} L_1(x')t'\right] \tag{2}$$

Details of the revised fit are given in Table 2.

	Male		Female	
	Coefficient	St. error	Coefficient	St. error
β_0	-2.7744	0.0087	-3.3375	0.0111
β_1	1.3991	0.0139	1.7028	0.0179
β_2	0.1579	0.0171	0.2315	0.0219
β_3	-0.2683	0.0318	-0.2181	0.0408
α_1	-0.2719	0.0116	-0.2660	0.0149
$\gamma_{1,1}$	0.0839	0.0178	-0.1294	0.0228
Adjusted R^2	0.99442		0.99301	
Sum of sq. errors	0.07006		0.08990	

TABLE 2. Group annuities, 6-parameter log-link model. All of the coefficients are significant at the 1% level.

Figure 6 shows the the male group annuity predicted forces of mortality based on the 6–parameter model given by (2). All of the predicted forces of mortality progress smoothly with respect to both age and time, and the model naturally predicts a reduction in the rate of improvement in mortality at the old ages. We will use the values predicted by the 6-parameter model to investigate the mortality derivatives performance. We have a model based on experience from 1951 to 1992. There are errors in the estimate which should tell us how confident we can be in projecting mortality into the future, assuming the dynamics of mortality improvement continues as it has in the observation period. This is potentially dangerous. As we have pointed out earlier, there is a good bit of controversy with regard to the dynamics of mortality improvement. We note also that these results are based on group annuity experience. As the market for individual immediate annuities develops, insurers will have to adjust their estimates to reflect the change in the market mortality. They may have to apply underwriting techniques and control for moral hazard when they issue annuities just as they now do for life insurance.

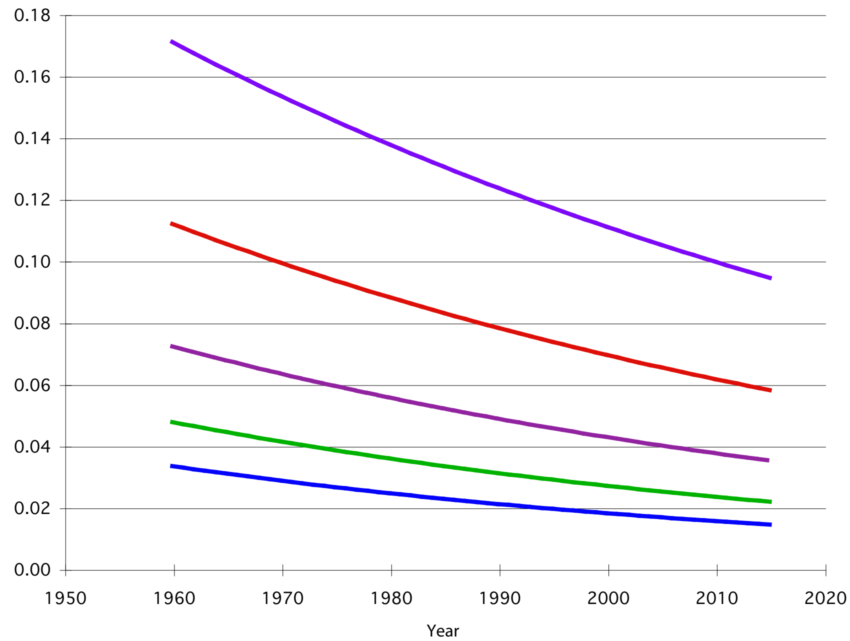

FIGURE 6. Male Group Annuity Mortality, predicted forces of mortality based on 6-parameter log-link model and *TSA Reports* 1951 - 1992. The top curve is the force of mortality for age 85, the one just below it is for age 80, then 75, 70 and the bottom curve is for age 65. The greatest improvement (steepest slope) is at age 85.

6. Mortality Swaps

Insurers need to manage their risk in issuing annuity policies, and are therefore keenly interested in understanding the future course of longevity, as well as the potential uncertainties that they must insure themselves against through hedging, asset allocation strategies, and reinsurance. Recently we have seen that reinsurers use bonds with embedded options (cat bonds, mentioned earlier) and swaps [25] to manage catastrophic property losses. Based on the above discussion, no one can precisely predict future mortality trends and managing mortality risks is always going to be a problem so we expect reinsurers will use mortality swaps and mortality bonds (defined later).

Certainly the dynamics of interest rates plays an important role in pricing and hedging annuity liabilities. However, we are going to focus on mortality and take the interest rate dynamics as given and independent of mortality dynamics. This is a simplifying assumption and one we would like to remove. We became even more skeptical of this approach after a discussion with Sven Sinclair at the University of Waterloo. There should be a long term interaction with longevity and demand for securities (and interest rates). However, for now we ignore the dynamics of interest rates. For example, in pricing a mortality security we discount future cash flows using the yield curve at the time the security is priced. The only random values are due to mortality.

A swap can be regarded as a series of forward contracts, and hence they can be priced using the concept of forwards. We assume the initial number of the survivors is 1,000,000 at age 55. Our idea of mortality swap is motivated by the insurer's desire to pay fixed-level payments for a series of variable-level payments. The characteristics of the mortality swap we propose are very similar to the plain vanilla interest swap. So we call our proposed swap "the plain vanilla mortality swap."

As an example of a mortality swap, consider an insurer that must pay immediate life annuities to N annuitants now all aged x. Set the notional principal at \$1,000 per year per annuitant. The insurer's actual payments could be used, but to keep the concept as simple as possible we fix the principal amount as 1,000 per year per annuitant. Let ℓ_{x+t} denote the number of survivors to year t. The insurer pays (at least) $1,000\ell_{x+t}$ to its annuitants. The swap is designed to hedge this portion of the insurer's payments to its annuitants.

The insurer and its swap counterparty agree on a level X_t for each year. In year t the insurer pays a fixed amount $1000X_t$ (varying only perhaps by duration but not random) to the counterparty and receives $1000\ell_{x+t}$. The insurer and counterparty agree at the beginning as to the annuitant pool in much the same way that mortgage loans are identified in construction of a mortgage–backed security. The insurer and counterparty payments are made on a net basis, so if there are more survivors to year t than expected (relative to the pre-set level) the company gets $1000(\ell_{x+t} - X_t) > 0$. The insurer's net cash flow to annuitants is offset by positive cash flow from the swap: $1000\ell_{x+t} - 1000(\ell_{x+t} - X_t) = 1000X_t$.

Of course, if mortality the other way, the insurer still has a net cash flow of $1000X_t$ since the insurer will pay the excess $1000(X_t - \ell_{x+t})$ to the counterparty. In this way a mortality swap can transform a segment of the insurer's annuity payments into a fixed cash flow.

Under the valuation model we are assuming, the value of the cash flow to the insurer (fixed payor) for an n–year swap is

$$V = 1000 \left[\sum_{t=1}^{n} \mathsf{E}(\ell_{x+t}) d(0,t) - \sum_{i=t}^{n} X_t d(0,t) \right] \tag{3}$$

where $\mathsf{E}(\ell_{x+t})$ denotes the expected number of survivors among the N initial annuitants and $d(0,t)$ is the discount factor based on the current bond market prices. If the counterparties agree to $X_t = \mathsf{E}(\ell_{x+t})$ then $V = 0$ and no initial exchange of cash is required to initiate the swap.

We point out that, given the distribution of survivors, there is very little variance in the cash flows. For example, given the survivor function ${}_tp_x$ of ℓ_{x+t}, we can describe ℓ_{x+t} as a binomial distribution. It is the number of successes in N trials with the probability of a success on a given trial of ${}_tp_x$. The distribution of ℓ_{x+t} is approximately normal with parameters $\mu_t = N {}_tp_x$ and $\sigma_t = \sqrt{N {}_tp_x(1 - {}_tp_x)}$. The coefficient of variation is the ratio of σ_t/μ_t. The graph of the coefficient of variation of the number of survivors for an initial group of 10,000 annuitants, based on the 1994 GAM female (65) survival distribution is shown in Figure 7. Note that for a swap of duration 30-years, the coefficient of variation rises to a maximum of about 1% , so there is little risk, *given the table*. The risk arises from uncertainty in the table. In calculating the swap value, we have to evaluate the expected value $\mathsf{E}(\ell_{x+t})$ carefully. It is not enough to estimate a mortality table and then estimate the expected value. That approach would ignore the uncertainty in the table.

In order to illustrate this further, suppose that the possible tables are labeled with a random variable θ. The the conditional distribution $\ell_{x+t}|\theta$ depends on θ. The unconditional moments are

$$\begin{aligned} \mathsf{E}[\ell_{x+t}] &= \mathsf{E}[\mathsf{E}[\ell_{x+t}|\theta]] = N\mathsf{E}[\mathsf{E}[{}_tp_x|\theta]] \\ \mathsf{Var}[\ell_{x+t}] &= \mathsf{E}[\mathsf{Var}[\ell_{x+t}|\theta]] + \mathsf{Var}[\mathsf{E}[\ell_{x+t}|\theta]]. \end{aligned} \tag{4}$$

Even if, as in Figure 7, there is very little variance in $\mathsf{E}[\ell_{x+t}|\theta]$ for all θ and the range of $t \leq 30$, there is still variance due to table uncertainty (the first term). We have little experience to guide us in estimating the terms $\mathsf{E}[\mathsf{E}[{}_tp_x|\theta]]$ and $\mathsf{E}[\mathsf{Var}[{}_tp_x|\theta]]$. Of course, this uncertainty occurs in all kinds of mortality derivatives, not just swaps.

7. Common Shock Model

In the section 5, we discuss the difficulties in accurate mortality projection. Rogers [27] states that mortality operates within a complex

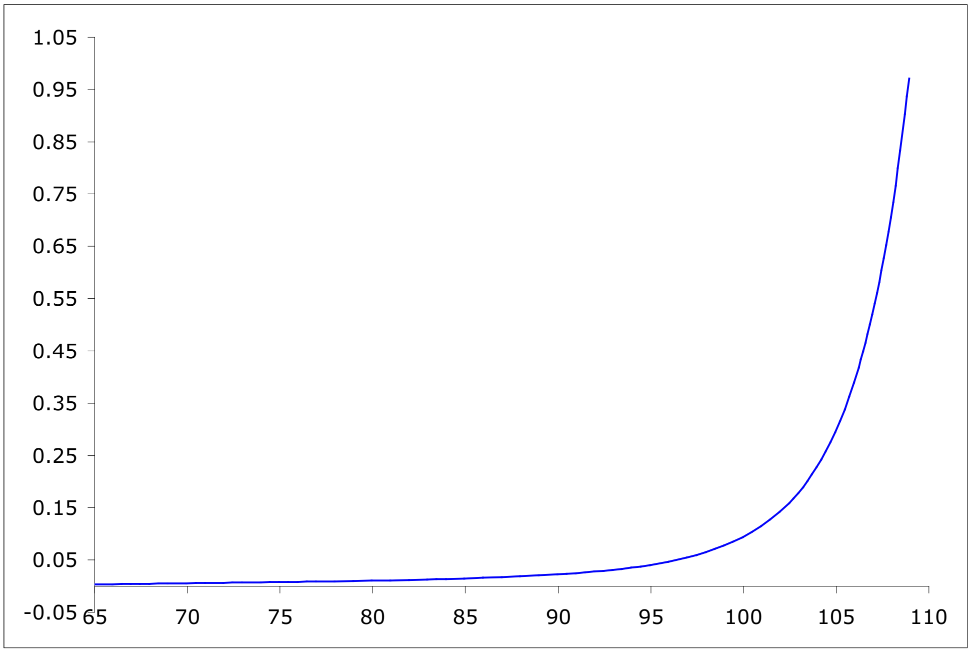

FIGURE 7. The ratio of standard deviation to expected number of survivors of an initial group of 10,000 annuitants, based on the 1994 GAM female (65) mortality distribution.

framework and is influenced by socioeconomic factors, biological variables, government policies, environmental influences, health conditions and health behaviors. Although mortality among the general and insured populations has witnessed remarkable improvements over the last several decades, mortality could continue to improve or it could even worsen. Some events act like a common shock to the entire mortality curve (*e.g.*, an epidemic) and decrease the expected lifetime of all annuitants. We can include a common shock random variable that can affect the joint distribution of annuitants. This common shock random variable is independent of annuitant's future lifetime random variable, in the absence of a shock. This is not the same sense in which Bowers *et al.* [1, Section 9.6] describe a common shock. In order to make it clear we will begin with a discussion of the Bowers *et al.* model of common shock.

Lifetimes deteriorate. Let $T_1^*, \ldots, T_N^*$ denote N independent lifetime random variables, corresponding to N insured lives observed at time 0. The common shock, according to Bowers *et al.*, is a random variable Z which we can picture as "the time of a catastrophe such as an earthquake or aircraft crash." The shock applies to all lives, killing them immediately. Thus the life time of the ith life with a shock becomes

$$T_i = \min(T_i^*, Z).$$

Following Bowers *et al.*, we can derive the joint survivor function:

$$\begin{aligned} S(t_1, t_2, \ldots, t_N) &= \Pr(T_1 > t_1, T_2 > t_2, \ldots T_N > t_N) \\ &= \Pr(\min(T_1^*, Z) > t_1, \ldots \min(T_N^*, Z) > t_N) \\ &= \Pr(T_1^* > t_1, Z > t_1, T_2^* > t_2, Z > t_2 \ldots T_N^* > t_N, Z > t_N) \\ &= S_Z(\max(t_1, \ldots, t_N)) \prod_{i=1}^{N} S_i^*(t) \qquad (5) \end{aligned}$$

In the special case that Z is exponential with mean θ and the lives have identical distributions with $S_i^*(t) = {}_tp_x$, then we get

$$S(t_1, t_2, \ldots, t_N) = e^{-\max(t_1,\ldots,t_N)/\theta} \prod_{i=1}^{N} {}_{t_i}p_x$$

which is a slight generalization of equation (9.6.2) in Bowers *et al.*

Consider a group of N insured lives, with independent and identically distributed lifetimes. Let N_t be the number of survivors to time t. Define the indicator random variables

$$\mathbb{I}_i(t) = \begin{cases} 1 & \text{if } T_i > t \\ 0 & \text{if } T_i \le t \end{cases} \quad \mathbb{I}_i^*(t) = \begin{cases} 1 & \text{if } T_i^* > t \\ 0 & \text{if } T_i^* \le t \end{cases} \quad \mathbb{I}_Z(t) = \begin{cases} 1 & \text{if } Z > t \\ 0 & \text{if } Z \le t \end{cases}$$

so that

$$N_t = \sum_{i=1}^{N} \mathbb{I}_i(t) \qquad N_t^* = \sum_{i=1}^{N} \mathbb{I}_i^*(t)$$

N_t^* denotes the number of survivors in a model with no shock. It has a binomial distribution with number of trials N and success probability ${}_tp_x$. Note that $\mathbb{I}_i(t)|Z \le t = 0$ and $\mathbb{I}_i(t)|Z > t = \mathbb{I}_i^*(t)$ so

$$N_t = \mathbb{I}_Z(t) N_t^*.$$

From this it follows that

$$\begin{aligned} \mathsf{E}[N_t] &= S_Z(t)\,{}_tp_x N \qquad (6) \\ \mathsf{Var}[N_t] &= \mathsf{E}[\mathsf{Var}(N_t|Z)] + \mathsf{Var}[\mathsf{E}(N_t|Z)] \\ &= {}_tp_x\,{}_tq_x N S_Z(t) + {}_tp_x^2 N^2 S_Z(t) F_Z(t) \qquad (7) \end{aligned}$$

Note that for the case that the shock has an exponential distribution, $S_Z(t) = e^{-t/\theta}$, the mean of arrival time of the shock is θ, and the expected number of survivors is $\mathsf{E}(N_t) = e^{-t/\theta}\,{}_tp_x N$. Of course, this is less than the mean of N_t^*, the number of survivors in a model with no shock. The later the mean arrival time θ, the lighter impact the shock has on the group, and the more the moments look like the moments of the model with no shock. This type of shock is good for studying life insurance, but it will not model mortality improvement. We need another type of shock which improves or extends all of the lifetimes.

Lifetimes improve. Let $T_1^*, \ldots, T_N^*$ denote N independent lifetime random variables, corresponding to N annuitants observed at time 0. A common shock is an event, like a cure for a dread disease which applies to all lives and like the previous mortality model it is represented by a positive random variable Z, independent of the lifetimes of the annuitants. The life of the ith annuitant, adjusted by the shock, is

$$T_i = \max(T^*, Z).$$

We suppose that the mortality before a shock has distribution function $F_i^*(t) = {}_tq_{x_i}$ and survivor function $S_i^*(t) = {}_tp_{x_i}$. We can now derive the joint distribution function:

$$\begin{aligned} F(t_1, t_2, \ldots, t_N) &= \Pr(T_1 \leq t_1, T_2 \leq t_2, \ldots T_N \leq t_N) \\ &= \Pr(\max(T_1^*, Z) \leq t_1, \ldots \max(T_N^*, Z) \leq t_N) \\ &= \Pr(T_1^* \leq t_1, Z \leq t_1, T_2^* \leq t_2, Z \leq t_2 \ldots T_N^* \leq t_N, Z \leq t_N) \\ &= F_Z(\min(t_1, \ldots, t_N)) \prod_{i=1}^{N} F_i^*(t) \qquad (8) \end{aligned}$$

In the special case that Z is exponential with mean θ and the lives have identical distributions with $F_i^*(t) = {}_tq_x$, then we get

$$F(t_1, t_2, \ldots, t_N) = \left[1 - e^{-\min(t_1, \ldots, t_N)/\theta}\right] \prod_{i=1}^{N} {}_{t_i}q_x.$$

Now consider a group of N annuitants with independent but identical lifetimes $T_1^*, \ldots, T_N^*$, with distribution function $F(t) = {}_tq_x$, and subject to a common shock Z of mortality improvement. Let N_t be the number of survivors to time t. Define the indicator random variables as we did in the discussion of a mortality deteriorating shock. N_t^* denotes the number of survivors in a model with no shock. It has a binomial distribution with number of trials N and success probability ${}_tp_x$. It is straightforward to show that

$$N_t = N\mathbb{I}_Z(t) + [1 - \mathbb{I}_Z(t)]\, N_t^* \qquad (9)$$

from which one can derive the moments:

$$\begin{aligned}\mathsf{E}(N_t) &= \mathsf{E}[\mathsf{E}(N|Z)] \\ &= \mathsf{E}[N\mathbb{I}_Z(t) + [1-\mathbb{I}_Z(t)]\,\mathsf{E}(N_t^*)] \\ &= NS_Z(t) + N{}_tp_xF_Z(t) \\ &= {}_tp_xN + {}_tq_xNS_Z(t) \end{aligned} \tag{10}$$

and

$$\begin{aligned}\mathsf{Var}(N_t) &= \mathsf{Var}[\mathsf{E}(N_t|Z)] + \mathsf{E}[\mathsf{Var}(N_t|Z)] \\ &= \mathsf{Var}[N\mathbb{I}_Z(t) + [1-\mathbb{I}_Z(t)]\,N{}_tp_x] + \mathsf{E}[(1-\mathbb{I}_Z(t))^2\,N{}_tp_{x\,t}q_x] \\ &= N^2{}_tq_x^2S_Z(t)F_Z(t) + N{}_tp_{x\,t}q_xF_Z(t)\end{aligned}$$

In the case that the shock arrival has an exponential distribution, $S_Z(t) = e^{-t/\theta}$, and the mean arrival time is θ. Since the effect of the shock is to make all the lives survive to the arrival time of the shock, than small values of θ do not have a great impact on the group and the moments of are close to the moments of the model with no shock.

While we can use a projection such as the method of Renshaw *et al.* cited earlier to estimate the pre-shock mortality, we do not have a good way to determine the distribution of Z. We suggest a a more general model is needed too. We might allow each annuitant to receive a shock, but the intensity could vary over the distribution of annuitants. More data and a richer model are needed. We have to leave the matter of estimating and developing the shock models as as an open question. In the next section we consider an entirely different approach.

8. Mortality Risk Bonds

Wang [31, 32, 33] has developed a method of pricing risks that unifies financial and insurance pricing theories. We are going to apply this method to price mortality risk bonds.

Let $\Phi(x)$ be the standard normal cumulative distribution function with a probability density function

$$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$

for all x. Wang defines the distortion operator as

$$g_\lambda(u) = \Phi[\Phi^{-1}(u) - \lambda] \tag{11}$$

for $0 < u < 1$.

Consider an insurer's liability X over a time horizon $[0, T]$. The value or fair price of the liability is the discounted expected value under the distribution obtained from the distortion operator. Omitting the discount for now, we have the formula for the price:

$$H(X, \lambda) = \mathsf{E}^*(X) = \int x dF^*(x)$$

where $F^*(x) = g_\lambda(F)(x) = \Phi[\Phi^{-1}(F(x)) - \lambda]$. The parameter λ is called the market price of risk, reflecting the level of systematic risk. Thus, for an insurer's given liability X with cumulative density function F, the Wang transform will produce a "risk–adjusted" density function F^*. The mean value under F^*, denoted by $\mathsf{E}^*[X]$, will define a risk–adjusted "fair-value" of X at time T, which can be further discounted to time zero, using the risk-free rate. Wang's paper describes the utility of this approach. It turns out to be very general and a generalization of well known techniques in finance and actuarial science. Our idea is to use observed annuity prices to estimate the market price of risk for annuity mortality, then use the same distribution to price mortality bonds.

Market price of risk. First we estimate the market price of risk λ. We defined our transformed distribution F^* as:

$$F^*(t) = g_\lambda(F)(t) = \Phi[\Phi^{-1}({}_tq_{65}) - \lambda] \tag{12}$$

We use the 1995 US Buck Annuity Mortality Tables as initial mortality tables. The 1995 US Buck Annuity Mortality Tables reflect the experience of 25 large industrial clients' pension plans. Then we use the 1995 market quotes of immediate annuities and the 1995 US Treasury yield curve to get the market price of risk λ. We assume a commission rate equal to 4% and get the market price of risk for males and females respectively, as shown in Table 3 and Figure 8. The risk loads are 0.1476 for male annuitants and 0.2024 for female annuitants. Figure 8 shows that the market prices of the annuities are higher than the 1995 pension plan experience and the market curve lies above the 1995 US Buck annuity mortality experience curve. We think of the Buck table as the actual or physical distribution, which requires a distortion to obtain market prices. That is, a risk premium is required for pricing annuities.

Mortality Bond Structure. Like the mortality swap, a designed portfolio of annuities underlies the mortality bond. Suppose that N annuitants are specified, all age $x = 65$ at the time the bond is issued. Mortality bond contracts may specify a mortality table on which both the bondholders and the insurer agree (*e.g.*, the 1995 US Buck Annuity Mortality Table). Moreover, the mortality contract may also set several

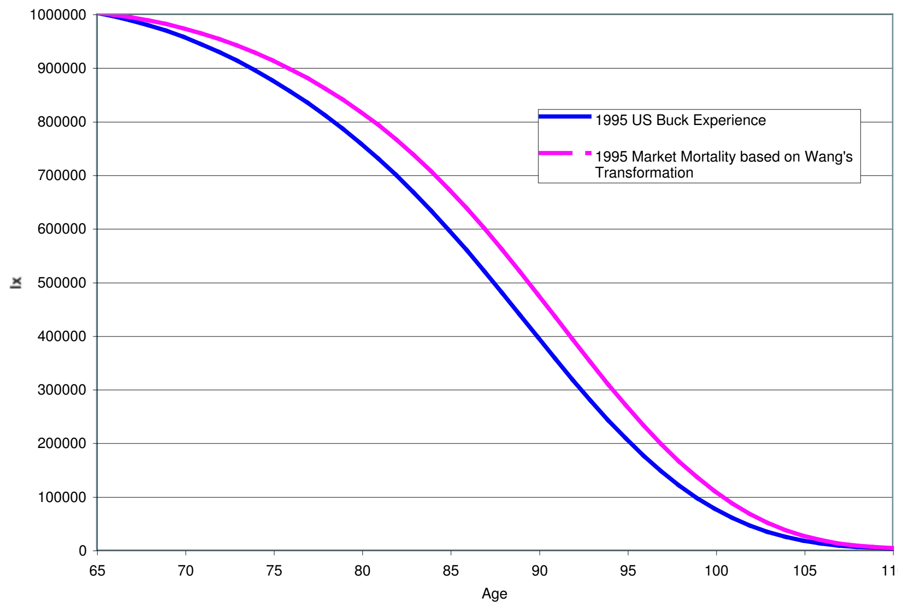

FIGURE 8. The result of applying the Wang transform to the survival distribution based on 1995 US Buck Experience for males (65) and prices from Best's Review, July 1995.

	Payment Rate	Market Value	Market price of risk
Male (65)	7.94	120.91	0.1476
Female (65)	7.17	133.89	0.2024

TABLE 3. The market price of risk, determined by 1995 US Buck annuity mortality, the US Treasury constant maturity interest rate term structure for 1995, and annuity market prices (without commission) from Best's Review (July 1995). The payment rate is the dollars per month of life annuity per $1,000 of annuity premium at the issue age. The market value is the price (net of commission) for $1 per month of life annuity.

improvement levels on the forces of mortality of each age to reflect the future mortality improvement. In our example, we set three different improvement levels for male (65) immediate annuities:

(i) -0.0055 for age from 65 to 74;

(ii) -0.0083 for age from 75 to 84;
(iii) -0.0110 for age from 85 to 94.

In addition, we set another three different improvement levels for female (65) immediate annuities:

(i) -0.0050 for age from 65 to 74;
(ii) -0.0075 for age from 75 to 84;
(iii) -0.0100 for age from 85 to 94.

Including the above improvement factors, the corresponding strike level for each age will be $\overline{\ell}_{65+t}$. The number of survivors ℓ_{65+t} is the number of lives attaining age in the survivorship group set in the contract. We define the bond contract so that the coupons are risky, but the principal is always paid at maturity. The bondholders will get the coupon payment C if the actual number of survivors at time t is smaller than the strike level $\overline{\ell}_{65+t}$. Otherwise, they will get nothing. That is, the bondholder's payment at the end of year t is

$$D_t = \begin{cases} C & \text{if } \ell_{65+t} \leq \bar{l}_{65+t} \\ 0 & \text{if } \ell_{65+t} > \bar{l}_{65+t} \end{cases} \tag{13}$$

for $t = 1, 2, \ldots, T$ where T is the term of the mortality bond, 30 years when the bond is issued.

Suppose we know the survival distribution for the pool of N annuitants upon which the bond is based, so we know the survival probability ${}_tp_{65}$. Then the distribution of the number of survivors has a binomial distribution with number of trials N and success probability ${}_tp_{65}$. Since N is rather large, we can use the normal approximation with parameters $m_t = N{}_tp_{65}$ and $s_t = \sqrt{N{}_tp_{65}(1 - {}_tp_{65})}$ to get the expected value of the bondholder's coupon:

$$\mathsf{E}[D_t] = \Pr(\ell_{65+t} \leq \overline{\ell}_{65+t}) \tag{14}$$

$$\approx \Phi\left(\frac{\overline{\ell}_{65+t} - m_t}{s_t}\right) \tag{15}$$

where $\Phi(z)$ denotes the standard normal cumulative density. Figure 9 shows the $\mathsf{E}[D_t]$ for the Buck mortality (female age 65). This is a calculation that one might perform when the bond is designed. The strike levels $\overline{\ell}_{65+t}$ can be specified at this point. Lower levels provide more protection for the issuer and greater risk to the bondholder. With this mortality bond design, the bondholders are more likely to get the coupons in the earlier years than in the later years. If we assume that the bondholder will get the face value when the mortality bond

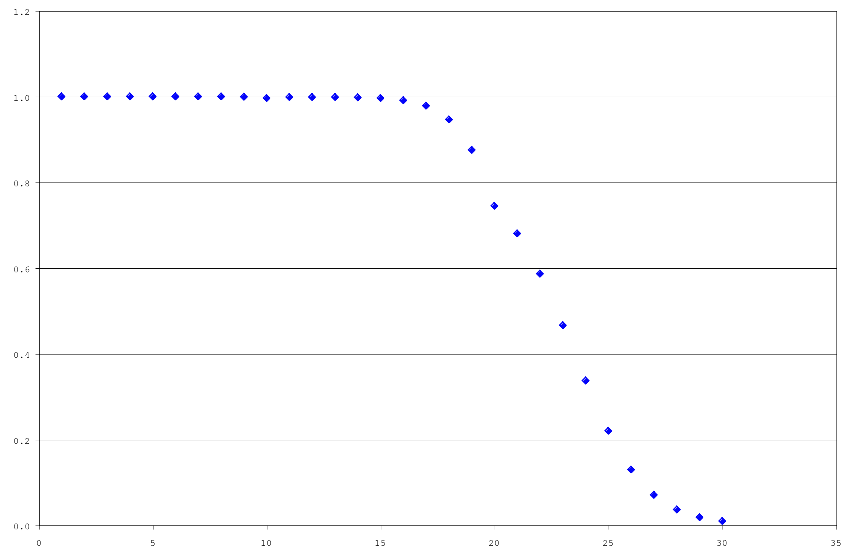

FIGURE 9. The expected values of bondholder's payment $\mathsf{E}[D_t]$ for a coupon rate of $C = 1$, based on the Buck 1995 tables for females age 65.

matures, the price of the mortality bond will be

$$P = Fd(0,T) + C\sum_{t=0}^{T} \mathsf{E}^*[D_t]d(0,t) \tag{16}$$

where $d(0,t)$ is the discount factor based on the risk free interest rate term structure at the time the bond is issued. The face amount F is not at risk; it is paid at time T regardless of the number of surviving annuitants. $\mathsf{E}^*[D_t]$ denotes the expected value based on the market mortality table. The survival distribution in equation (16) is the distribution derived from the annuity market. It is based on the 1995 US Buck Annuity Mortality Tables and the Wang transform (12) with $\lambda = 0.1476$ for male annuitants and $\lambda = 0.2024$ for females. The discount factors are from the 1995 US Treasury interest rate term structure. Table 4 shows prices for a mortality bond for a group of 10,000 male annuitants, with the strike levels defined above and a 7% coupon rate. The price of the mortality bond for a bond based on male (65) immediate annuitants is 930.55 per 1000 of face value. Similarly, we can get the bond price for the female (65) immediate annuitants is 902.34 per 1000.

	Male (65)	Female (65)
Market price of risk (λ)	0.1476	0.2024
Face value	1,000	1,000
Coupon rate	0.07	0.07
Number of annuitants	10,000	10,000
Improvement level age 65 - 74	-0.0055	-0.0050
Improvement level age 75 - 84	-0.0083	-0.0075
Improvement level age 85 - 94	-0.0110	-0.0100
Price	930.55	902.34

TABLE 4. The calculation is based on large enough number of independent annuitant lives that we can be the normal distribution to approximate the distribution of the number of survivors to each age. The underlying survival distribution was derived from the 1995 immediate annuity market using the 1995 US Buck Experience Annuity Mortality Table as a basic table and applying the Wand transform. The discount factors reflect the 1995 US Treasury term structure of interest rates. The straight bond price on the same date is 997.14.

Insurer's mortality bond hedge. The actual annuity payments of an insurer in the future are based on the future actual mortality experience. However, we can study how it might turn out under different scenarios. Assume an insurer has to pay a group of 10,000 annuitants 1,000 per year if the annuitants survive at the end of the year. Suppose also that annuity–based bonds, as described above, are available as a hedge. The insurer insurer sells k bonds for a total face amount of $1,000k$ with each bond based on the same pool of 10,000 annuitants. At the same time the insurer buys k straight bonds with the same coupon rate as the annuity–based bonds. Assuming the annuitants are females, the net cost of the two bond transactions is $997.14k - 902.34k = 94.80k$. The number of bonds can be selected by the insurer and the market. That is, bond contract can be designed for a given annuitant pool, and then the bond can be marketed in units of 1,000 of face value. The annuitant pool plays the role of an index with each bond providing an embedded option on the index. If the insurer creates a hedge involving k mortality bonds and k straight bonds, then the insurer's net cash flow corresponding to \$1,000 of initial annuity liability is random each year. It can be written as the payments to annuitants, plus payments to mortality bondholders, less payments from straight bond issuers:

$$\text{Annuity payments} = 1,000\ell_{x+t}$$

$$\text{Plus coupons to bondholders} = \begin{cases} kC & \text{if } \ell_{x+t} \leq \overline{\ell}_{x+t} \\ 0 & \text{if } \ell_{x+t} > \overline{\ell}_{x+t} \end{cases}$$

$$\text{Minus coupons from bond issuers} = kC$$

$$\text{Equals net cash flow per 1,000} = \begin{cases} 1,000\ell_{x+t} & \text{if } \ell_{x+t} \leq \overline{\ell}_{x+t} \\ 1,000\ell_{x+t} - kC & \text{if } \ell_{x+t} > \overline{\ell}_{x+t} \end{cases}$$

In our example $C = 70$ and $\ell_x = 10,000$. In this case the insurer might issue $k = 10,000$ bonds with a total face value of \$10 million. The cost of the hedge is \$948,000 and the hedge provides coverage in each of 30 future years. The hedge pays \$700,000 in each year in which the number of annuitants exceeds the strike level.

The present value varies with the mortality tables, of course. For example, common shock improvement Z with an exponential distribution, $S_Z(t) = e^{-t/\theta}$ as described in section 7 shifts the distribution of survivors to the right. Figure 10 shows different scenarios for the expected number of survivors for different values of θ for a group of 1,000,000 male (65) annuitants. The expected value is calculated with equation (10).The graph reflects one of the mortality trend opinions – continuous improvement.

One of the most important functions of introducing mortality bonds is to hedge the cash flows of an insurer and reduce the impact of mortality improvement. The following example, illustrated in Figure 11, shows how mortality bonds function as a hedge against improving mortality. Suppose that an insurer sells a \$10,000,000 face value of mortality bonds based on a group of 10,000 male (65) annuitants with a 7% coupon rate and at the same time buys a \$10,000,000 straight bond with a 7% coupon rate. The insurer has to pay the surviving annuitants \$1,000 per year. If the actual number of survivors is less than the strike level $\overline{\ell}_{x+t}$ in the contract, the mortality bond coupons are exactly offset by the coupons from the straight bond. If the actual number of survivors is more than the strike level $\overline{\ell}_{x+t}$, the insurer does not pay the mortality bond coupon so the straight bond coupon reduces the cash outflow. The total cash outflow is shifted down, below is the actual annuity payment level. This is how mortality bonds mitigate the impact of excess mortality improvement relative to the insurer's expectation.

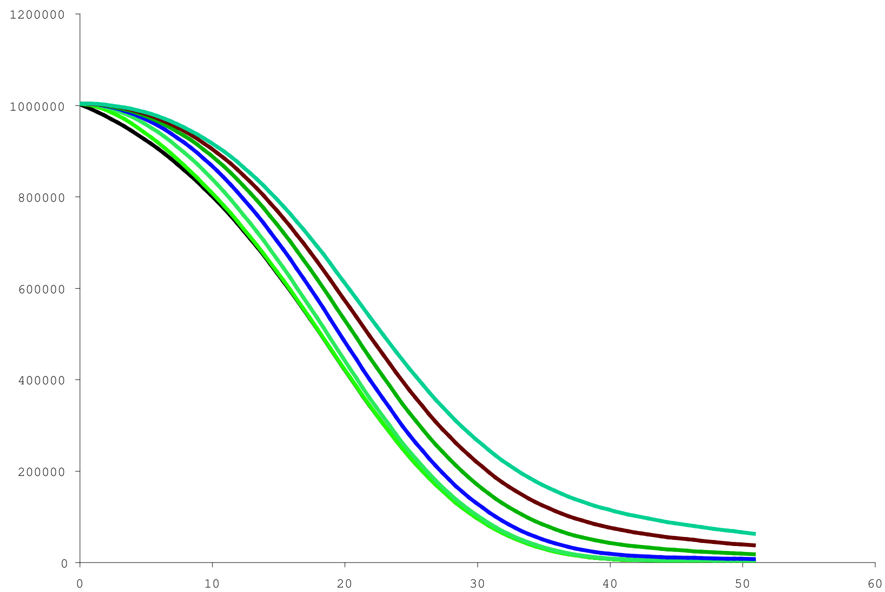

Figure 10. Survivor curves for 1,000,000 males (65) annuitants, illustrating the effect of common shocks to 2000 US Male Basic Annuity table. The shock parameters vary from from 3 to 18 in increments of 3 based equation (10).

9. Discussion and Conclusions

Financial innovation has led to the creation of new classes of securities that provide opportunities for insurers to manage their underwriting and to price risks more efficiently. Cummins and Lewis [5] establish that risk expansion helps to explain the development of catastrophic risk bonds and options in the 1990s. A similar expansion is needed to manage longevity risk. There is a growing demand for a long term hedge against improving annuity mortality. We have shown how innovation in swaps, options and bond contracts can provide new securities which can provide the hedge insurers need.

There is a trend of privatizing social securities systems with insurers taking more longevity risk. Moreover, the trend to defined contribution corporate pension plans is increasing the potential marekt for immediate annuities. This is an opportunity and also a challenge to insurers. Insurers will need increased capacity to take on longevity risk and securities markets can provide it. This will allow life insurers to share this "big cake." Compared with the reinsurance market, securitization of

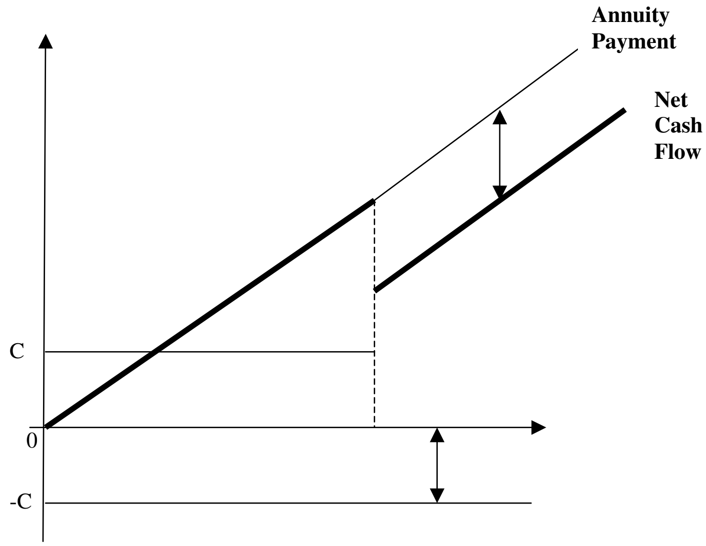

FIGURE 11. The number of survivors ℓ_{x+t} is on the horizontal axis and the insurer's payment on the vertical axis. If the number of survivors is more than the strike level $\bar{\ell}_{x+t}$, the insurer does not pay the mortality bond coupons so the regular bond coupons reduce the cash outflow. If the number of survivors is below the strike level, the coupons are equal and cancel each other. The total cash outflow drops by the coupon amount when the number of survivors exceeds the strike level.

mortality risks has longer duration, higher capacity and possibly lower cost. Demand for new securities arises when new risks appear and when existing risks become more significant in magnitude. And we now have the technology to securitize the mortality risks based on modern financial models. Securitization in the annuity and life insurance markets has been relatively rare, but we have argued that this may change. We explored the securitization of mortality risks showing how it can help solve the difficulties in managing annuity mortality risk.

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Appendix: Summary of data

We collected the data from the Society of Actuaries Transactions Reports for each of the years for which there was data. We used reports for calendar years published for the years 1951, 1961, 1971, and each year from 1981 to 1992. The last report is based on 1992 experience. We understand that the Society of Actuaries is reviving its experience studies.

1951						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	335.70	11.00	1174.25	10.00	1509.95	21.00
60-64	12102.34	308.00	3847.76	57.00	15950.10	365.00
65-69	39871.68	1413.00	4602.89	91.00	44474.57	1504.00
70-74	17218.98	958.00	1737.57	63.00	18956.55	1021.00
75-79	5873.40	484.00	666.00	37.00	6539.40	521.00
80-84	1774.33	226.00	209.00	26.00	1983.33	252.00
85-89	374.08	68.00	51.25	8.00	425.33	76.00
90-94	47.42	15.00	7.00	2.00	54.42	17.00

1961						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	1,371.88	36.00	2,454.63	18.00	3,826.51	54.00
60-64	23,718.46	605.00	9,902.34	116.00	33,620.80	721.00
65-69	96,620.43	3,371.00	19,390.30	333.00	116,010.73	3,704.00
70-74	60,560.45	3,371.00	10,594.01	349.00	71,154.46	3,720.00
75-79	26,772.96	2,275.00	3,901.58	195.00	30,674.54	2,470.00
80-84	7,701.84	1,002.00	1,057.17	109.00	8,759.01	1,111.00
85-89	1,717.08	310.00	275.00	35.00	1,992.08	345.00
90-94	254.42	59.00	39.00	7.00	293.42	66.00

1971						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	3,611.23	85.00	3,574.90	26.00	7,186.13	111.00
60-64	33,806.66	791.00	18,521.74	177.00	52,328.40	968.00
65-69	120,227.85	4,022.00	41,802.04	595.00	162,029.89	4,617.00
70-74	93,795.47	4,955.00	28,542.94	746.00	122,338.41	5,701.00
75-79	63,066.93	5,269.00	16,284.46	747.00	79,351.39	6,016.00
80-84	28,166.41	3,113.00	6,815.79	510.00	34,982.20	3,623.00
85-89	8,022.23	1,315.00	1,699.37	213.00	9,721.60	1,528.00
90-94	1,328.05	338.00	251.95	51.00	1,580.00	389.00

1981						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	26,599.21	440.00	11,124.59	99.00	37,723.80	539.00
60-64	82,756.29	1,568.00	32,978.18	347.00	115,734.47	1,915.00
65-69	185,232.93	4,924.00	73,727.06	1,003.00	258,959.99	5,927.00
70-74	157,276.45	6,571.00	68,210.94	1,397.00	225,487.39	7,968.00
75-79	97,763.34	6,189.00	42,614.73	1,347.00	140,378.07	7,536.00
80-84	48,755.90	4,727.00	20,588.86	1,093.00	69,344.76	5,820.00
85-89	19,601.58	2,719.00	7,936.75	681.00	27,538.33	3,400.00
90-94	4,980.49	990.00	2,087.62	294.00	7,068.11	1,284.00

Group annuity experience 1951, 1961, 1971 and 1981

1982						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	28,631.53	453.00	11,754.62	92.00	40,386.15	545.00
60-64	89,455.43	1,753.00	35,433.49	336.00	124,888.92	2,089.00
65-69	192,308.39	5,097.00	75,640.56	985.00	267,948.95	6,082.00
70-74	162,420.78	6,740.00	72,661.69	1,354.00	235,082.47	8,094.00
75-79	103,419.33	6,465.00	48,058.37	1,540.00	151,477.70	8,005.00
80-84	52,549.11	4,861.00	23,671.10	1,231.00	76,220.21	6,092.00
85-89	21,392.48	2,989.00	9,443.51	832.00	30,835.99	3,821.00
90-94	5,716.77	1,082.00	2,526.42	322.00	8,243.19	1,404.00

1983						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	33,163.65	510.00	13,783.18	117.00	46,946.83	627.00
60-64	98,632.53	1,868.00	41,665.68	435.00	140,298.21	2,303.00
65-69	195,074.64	5,153.00	79,663.64	1,103.00	274,738.28	6,256.00
70-74	170,348.65	6,995.00	72,621.93	1,511.00	242,970.58	8,506.00
75-79	107,213.60	6,964.00	48,482.16	1,613.00	155,695.76	8,577.00
80-84	57,936.04	5,399.00	24,237.52	1,388.00	82,173.56	6,787.00
85-89	22,035.27	3,111.00	9,528.77	895.00	31,564.04	4,006.00
90-94	6,136.86	1,218.00	2,725.40	373.00	8,862.26	1,591.00

1984						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	40,574.69	580.00	16,305.25	132.00	56,879.94	712.00
60-64	119,381.14	2,212.00	48,941.94	448.00	168,323.08	2,660.00
65-69	221,883.84	5,695.00	91,062.97	1,241.00	312,946.81	6,936.00
70-74	200,590.93	8,196.00	86,304.56	1,870.00	286,895.49	10,066.00
75-79	129,357.81	8,141.00	60,361.35	2,106.00	189,719.16	10,247.00
80-84	67,297.97	6,288.00	31,781.28	1,771.00	99,079.25	8,059.00
85-89	26,575.80	3,766.00	12,400.26	1,211.00	38,976.06	4,977.00
90-94	7,743.72	1,574.00	3,681.76	573.00	11,425.48	2,147.00

1985						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	43,299.71	656.00	17,016.15	146.00	60,315.86	802.00
60-64	123,040.09	2,386.00	50,603.92	565.00	173,644.01	2,951.00
65-69	223,999.93	6,226.00	93,571.37	1,368.00	317,571.30	7,594.00
70-74	207,718.42	9,000.00	90,306.94	2,050.00	298,025.36	11,050.00
75-79	137,102.94	9,186.00	65,194.85	2,426.00	202,297.79	11,612.00
80-84	71,953.72	7,141.00	35,412.31	2,137.00	107,366.03	9,278.00
85-89	28,655.87	4,287.00	14,095.45	1,437.00	42,751.32	5,724.00
90-94	8,411.94	1,812.00	4,179.97	671.00	12,591.91	2,483.00

Group annuity experience 1982 – 1985

1986						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	44,010.72	627.00	16,677.86	112.00	60,688.58	739.00
60-64	122,620.42	2,163.00	50,381.10	476.00	173,001.52	2,639.00
65-69	227,995.35	5,699.41	95,512.26	1,261.00	323,507.61	6,960.41
70-74	216,055.50	8,098.29	93,727.78	1,966.00	309,783.28	10,064.29
75-79	146,182.97	8,610.00	68,834.32	2,324.00	215,017.29	10,934.00
80-84	78,070.67	7,153.00	38,836.55	2,108.00	116,907.22	9,261.00
85-89	31,484.42	4,005.00	15,650.49	1,406.00	47,134.91	5,411.00
90-94	9,097.10	1,678.00	4,672.65	690.00	13,769.75	2,368.00

1987						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	47,303.94	598.00	17,781.62	134.00	65,085.56	732.00
60-64	129,028.29	2,138.00	53,226.99	533.00	182,255.28	2,671.00
65-69	238,848.85	5,773.00	101,240.19	1,356.00	340,089.04	7,129.00
70-74	223,665.17	8,714.00	98,442.35	2,054.00	322,107.52	10,768.00
75-79	157,461.29	9,443.00	74,752.64	2,525.00	232,213.93	11,968.00
80-84	83,820.45	7,671.00	43,600.05	2,452.00	127,420.50	10,123.00
85-89	34,094.97	4,590.00	18,036.28	1,677.00	52,131.25	6,267.00
90-94	9,836.78	1,921.00	5,395.54	825.00	15,232.32	2,746.00

1988						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	49,424.32	683.00	18,162.87	141.00	67,587.19	824.00
60-64	132,778.58	2,252.00	53,788.54	513.00	186,567.12	2,765.00
65-69	235,874.82	5,587.00	102,022.53	1,295.00	337,897.35	6,882.00
70-74	221,164.05	8,388.00	99,853.21	2,116.00	321,017.26	10,504.00
75-79	162,202.31	9,530.00	78,542.78	2,630.00	240,745.09	12,160.00
80-84	88,225.65	8,012.00	47,418.51	2,583.00	135,644.16	10,595.00
85-89	35,929.54	4,707.00	20,142.57	1,879.00	56,072.11	6,586.00
90-94	10,484.98	2,002.00	5,926.74	845.00	16,411.72	2,847.00

1989						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	45,167.60	580.00	19,788.90	138.00	64,956.50	718.00
60-64	120,348.84	2,008.00	53,312.98	488.00	173,661.82	2,496.00
65-69	201,223.57	4,827.00	94,345.49	1,235.00	295,569.06	6,062.00
70-74	180,723.00	6,748.00	88,016.87	1,829.00	268,739.87	8,577.00
75-79	134,297.88	7,852.00	70,107.48	2,357.00	204,405.36	10,209.00
80-84	72,524.22	6,606.00	41,921.07	2,353.00	114,445.29	8,959.00
85-89	29,672.14	3,992.00	18,031.93	1,628.00	47,704.07	5,620.00
90-94	8,245.34	1,704.00	5,114.09	820.00	13,359.43	2,524.00

Group annuity experience 1986 – 1989

1990						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	53,375.95	686.00	24,851.00	174.00	78,226.95	860.00
60-64	146,190.29	2,333.00	67,235.53	596.00	213,425.82	2,929.00
65-69	258,735.98	5,949.00	122,669.86	1,562.00	381,405.84	7,511.00
70-74	238,694.07	8,911.00	116,031.28	2,327.00	354,725.35	11,238.00
75-79	189,088.76	11,105.00	95,064.28	3,186.00	284,153.04	14,291.00
80-84	109,583.14	9,912.00	62,967.19	3,520.00	172,550.33	13,432.00
85-89	48,022.47	6,572.00	30,700.37	2,778.00	78,722.84	9,350.00
90-94	14,672.14	2,842.00	10,005.89	1,445.00	24,678.03	4,287.00

1991						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	50,731.54	661.00	22,245.01	158.00	72,976.55	819.00
60-64	137,582.08	2,383.00	60,722.23	543.00	198,304.31	2,926.00
65-69	240,820.91	5,774.00	114,994.74	1,557.00	355,815.65	7,331.00
70-74	230,909.08	8,685.00	115,825.34	2,433.00	346,734.42	11,118.00
75-79	188,317.23	10,961.00	96,727.27	3,360.00	285,044.50	14,321.00
80-84	112,587.59	10,048.00	66,245.62	3,791.00	178,833.21	13,839.00
85-89	48,883.89	6,713.00	33,022.70	2,996.00	81,906.59	9,709.00
90-94	15,033.98	2,901.00	10,909.55	1,624.00	25,943.53	4,525.00

1992						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	47,790.52	689.00	20,925.44	156.00	68,715.96	845.00
60-64	122,033.83	2,143.00	55,616.52	466.00	177,650.35	2,609.00
65-69	216,153.60	5,124.00	107,068.38	1,429.00	323,221.98	6,553.00
70-74	212,415.17	7,526.00	111,099.67	2,260.00	323,514.84	9,786.00
75-79	173,061.53	9,440.00	91,863.84	3,044.00	264,925.37	12,484.00
80-84	106,152.91	9,177.00	63,719.81	3,349.00	169,872.72	12,526.00
85-89	47,214.93	6,190.00	33,278.32	2,984.00	80,493.25	9,174.00
90-94	15,059.41	2,859.00	11,268.86	1,634.00	26,328.27	4,493.00

Group annuity experience 1990 – 1992

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