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THE INTERRELATIONSHIP AMONG INFLATION RATES, SALARY RATES, INTEREST RATES, AND PENSION COSTS

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ABSTRACT

The salary rate and the interest rate are the two most important actuarial assumptions to be made in connection with a salary-based pension plan valuation. Because these two assumptions have counter-balancing effects on the determination of liabilities and normal costs, there is a tendency to consider them in combination rather than separately when evaluating their appropriateness. With the passage of ERISA, pension actuaries may have to revise their criteria for judging whether the assumptions do indeed represent their best estimate of anticipated experience. In this regard, this paper examines tandem levels of salary and interest rates and finds that the absolute level of each assumption as well as the differential are significant cost determinants. Accordingly, it is concluded that pairs of salary and interest rate assumptions cannot be justified as best estimates solely on the basis of their relative levels.

In forming best-estimate salary and interest rate assumptions, the actuary must reconcile the difference, if any, between the currently observed rates and the valuation assumptions. Common practice is to choose expected long-term rather than observed rates for valuation purposes. The error in this procedure is analyzed and found to be tolerable for strictly temporary discrepancies between observed rates and valuation assumptions. For situations where these discrepancies are expected to persist for some time, a need is shown for nonuniform, calendar-year assumptions (or their equivalent) in order to develop best-estimate assumptions.

I. INTRODUCTION

The Employee Retirement Income Security Act (ERISA) of 1974 requires that pension plan actuarial assumptions, in the aggregate, represent best estimates. Undoubtedly the two assumptions most often used as a "combination" best estimate rather than as best estimates individually are the salary rate and interest rate parameters. For example,

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a common procedure is to assume a lower (more conservative) interest rate assumption relative to the expected yield on the plan's investment portfolio, while simultaneously assuming a lower (more liberal) salary rate assumption relative to the expected salary experience of the plan. Since a conservative interest rate increases pension costs, while a liberal salary rate reduces costs for salary-based pension plans, the underlying rationale for underestimating both parameters is that the errors induced are offsetting, thus leaving pension costs relatively unaffected. As a matter of fact, the argument is raised frequently that the critical concern should be the differential between salary and interest rate assumptions and that their absolute levels are relatively less important. If this is the case, best-estimate combinations can be constructed much more easily than if both the absolute and relative levels of these pension plan assumptions have significant impacts on costs. One of the purposes of this paper is to investigate the interrelationship among the salary rate assumption, the interest rate assumption, and pension costs.

A second consideration which compounds the problem of selecting best-estimate salary and interest rate assumptions is the significance which should be attached to the current experience of the plan relative to the expected long-run experience. In this context the paper considers the use of nonuniform, time-dependent salary and interest rates that run from various current levels to expected ultimate levels. Since nonuniform salary and interest rates may not be a practical assumption, the analysis also considers the error associated with assuming flat salary and interest rates instead of the more exact nonuniform pattern.

Although the paper deals primarily with analyzing pairs of assumptions with respect to salary and interest rates, it is also instructive to view the analysis as consisting of a change in the rate of inflation, since inflation is a common theoretical component of both long-term salary and long-term interest rates. In the context of valuation assumptions, the interest rate may be viewed as consisting of a pure rate of interest plus a risk premium plus an inflation premium. Similarly, the salary assumption may be viewed as consisting of a merit component, a productivity component, and an inflation component.[1]

Let the total interest rate assumption be denoted by $I + R$, where I represents an assumed rate of inflation and R denotes the remaining two components of the interest rate. Similarly, let $I + P$ represent the inflation and productivity components of the salary assumption. If y is the

[1] The merit component denotes the individual-based productivity gains achieved by the employee over his working career, while the so-called productivity component of the salary assumption represents labor's share of company-based (or group-based) productivity gains.

youngest entry age into the plan, r the retirement age, and x the attained age, the merit component of the salary assumption can be represented by the function s_x ($y \leq x \leq r$), where 100 ($s_z/s_x - 1$) yields the percentage increase in salary from x to z due to merit. Thus the total rate of salary increase during age x is equal to $I + P + (s_{x+1}/s_x - 1)$. If the salary and interest rates are separated into their various components, it is clear that equal changes in both assumptions can be viewed as a change in their common component I.

Assumptions

The subsequent mathematical analysis and numerical illustrations are based on a plan providing k per cent of the employee's final average n-year salary per year of service. The merit component of the salary assumption used in the numerical illustrations is given in the Appendix (Table A1), and the productivity component, P, is assumed to be 1 per cent. The noninflationary parts of the interest rate assumption, R, are taken to be 3 per cent. The termination rates associated with active employees are also given in Table A1, and the mortality assumption is the 1971 Group Annuity Mortality Table. The single retirement age used for the cost illustrations is age 65. The hypothetical plan population used in connection with the cost illustrations has an average age of 40.3 and an average service period of 10.1 years. Finally, retired plan members represent 13.1 per cent of active employees for this pension plan population.

In the interest of simplicity the numerical illustrations are based on the retirement-related cost of the plan, excluding the cost of vesting and other ancillary benefits that might be provided in a typical plan. The analysis is confined to the aggregate projected benefit cost method (with or without frozen initial supplemental liability) with contributions designed to be a constant percentage of salary, an actuarial cost method commonly used in connection with a final average salary benefit formula.

II. SENSITIVITY ANALYSIS

Mathematical Analysis

The annual contribution, expressed as 100C per cent of salary, for a plan using the aggregate projected benefit cost method may be written as

$$C = \frac{\Sigma\,(PVFB)_x - (\text{Assets} + USL)}{\Sigma\, S_x\ {}^s\ddot{a}^{aa}_{x:\overline{r-x}|}}, \tag{1}$$

where

$\Sigma\,(PVFB)_x$ = Present value of future benefits for all active and nonactive plan members;

USL = Unfunded supplemental liability, if any;

S_x = Salary at age x;

${}^s\ddot{a}^{aa}_{x:\overline{r-x}|}$ = Present value of future salary from age x to age r, based on a unit salary at age x.

The present value of future salaries is practically insensitive to equal changes in the salary and interest rate assumptions. This can be seen by writing the salary-based annuity for an employee at attained age x in terms of its basic components.

$$ {}^s\ddot{a}^{aa}_{x:\overline{r-x}|} = \sum_{j=x}^{r-1} \frac{(1+P+I)^{j-x}}{(1+R+I)^{j-x}} \frac{s_j}{s_x} \, {}_{j-x}p^{aa}_x . \tag{2} $$

Observe that the first ratio in equation (2) can be approximated by

$$ \left(\frac{1+P+I}{1+R+I}\right)^j = \left(\frac{1+P}{1+R} + \frac{I(R-P)}{(1+R)(1+R+I)}\right)^j \doteq \left(\frac{1+P}{1+R}\right)^j , \tag{3} $$

an approximation which is excellent for reasonable values of I, P, and R. Thus a constant change in the salary and interest rates, in the above case brought about by extracting the inflation component from each assumption, has practically no effect on the present value of a participant's future salary. Since this holds for all active employees, the cost effects of using different inflation assumptions or changing the salary and interest rates by the same amount can be analyzed by considering only the numerator of equation (1).

Let us now examine the present value of future benefits for a participant who entered the plan at age y and is currently aged x, assuming $x \leq r - n$.

$$ (PVFB)_x = S_x \frac{\sum_{j=r-n}^{r-1} (1+P+I)^{j-x} s_{j-x}}{n(1+R+I)^{r-x} s_x} k(r-y) \, {}_{r-x}p^{aa}_x \, \ddot{a}^{R+I}_r , \tag{4} $$

where

$\ddot{a}^{R+I}_r$ = Retirement age annuity evaluated at an interest rate equal to $R+I$;

${}_np^{aa}_x$ = Probability of an employee aged x surviving in the service of the employer for n years.

The ratio in expression (4), using the approximation given in formula (3), can be written as

$$ \left(\frac{1+P}{1+R}\right)^{r-x} \frac{1}{n} \sum_{j=r-n}^{r-1} (1+P+I)^{-(r-j)} \frac{s_{j-x}}{s_x} . \tag{5} $$

Moreover, if the salary scale over the final n years of the participant's working career is relatively flat, as is often the case, the following is a good approximation to expression (5):

$$\left(\frac{1+P}{1+R}\right)^{r-x}\frac{s_r}{s_x}\frac{1}{n}\,a_{\overline{n}|P+I}\,, \tag{6}$$

where $a_{\overline{n}|P+I}$ is an n-year annuity-certain evaluated at an interest rate equal to $P + I$. An employee's $(PVFB)_x$ function, using the approximations noted above, can be written as

$$(PVFB)_x \doteq S_x\left(\frac{1+P}{1+R}\right)^{r-x}\frac{s_r}{s_x}\frac{1}{n}\,k(r-y)\;{}_{r-x}p_x^{aa}\;a_{\overline{n}|P+I}\;\ddot{a}_r^{R+I}\;. \tag{7}$$

In this equation $a_{\overline{n}|P+I}$ and $\ddot{a}_r^{R+I}$ are the only terms involving inflation, implying that the effect of a change in the inflation component of the salary and interest rate assumptions (or simply a simultaneous change in both assumptions) on the $(PVFB)_x$ function for all active employees whose age is less than $r - n$ can be approximated by analyzing the change in the product of these two annuities. If the inflation assumption is increased, then both $a_{\overline{n}|P+I}$ and $\ddot{a}_r^{R+I}$ decrease, so that their product and hence $(PVFB)_x$ also decrease. Similarly, if the inflation assumption is decreased, $(PVFB)_x$ must increase.

The corresponding factor to be analyzed for employees within the n-year final average period would be the product of $a_{\overline{r-x}|P+I}$ (assuming actual salary histories) and the retirement annuity $\ddot{a}_r^{R+I}$. Finally, the effect of a change in the inflation assumption on retired employees can be approximated by observing the impact on $\ddot{a}_x^{R+I}$ for $x \geq r$. Since the annuity $\ddot{a}_x^{R+I}$ is less sensitive to an inflation change than $\ddot{a}_r^{R+I}$, changes in the inflation assumption have a smaller impact on the retired sector than on the active sector of a pension plan.

Finally, it is important to note that, if the plan were to provide a cost-of-living increase in the retirement benefit equal to the assumed rate of inflation, then the retirement annuity ($\ddot{a}_r^{R+I}$ for actives and $\ddot{a}_x^{R+I}$ for retired employees) in the $(PVFB)_x$ function is practically insensitive to the rate of inflation. This result follows from application of equation (3). Consequently, the effect of a change in the inflation rate for a plan of this type can be approximated by observing the change in $a_{\overline{n}|P+I}$ for actives whose ages are less than $r - n$ and, for those older, by observing the change in $a_{\overline{r-x}|P+I}$. In this case there would be no impact on $(PVFB)_x$ for retired employees, other than that brought about by the approximation in formula (3).

Numerical Illustrations

Table 1 shows the results of different inflation rate assumptions on the $\Sigma\ (PVFB)_x$ function for a hypothetical group of pension plan members, where benefits are based on the final five-year average salary. The data are expressed as a percentage of the $\Sigma\ (PVFB)_x$ function evaluated at a 4 per cent rate of inflation, that is, an interest rate of 7 per cent and a salary rate of 5 per cent plus the merit component. It can be seen that a fairly substantial change in the $\Sigma\ (PVFB)_x$ function occurs when the inflation component is altered by 2 percentage points, the relationship of this function to the inflation assumption being inverse. In other words, a simultaneous change in the salary and interest rate assumptions does not leave the $\Sigma\ (PVFB)_x$ function relatively unaffected. In order to maintain a constant $\Sigma\ (PVFB)_x$ function while raising salary and interest rates, their differential must be decreased. Moreover, if the plan has assets and/or if an unfunded supplemental liability exists, the effect on the plan's normal cost is magnified. This is shown in Table 2, where assets plus the unfunded supplemental liability total 0, 20, 30, and 40 per cent of the $\Sigma\ (PVFB)_x$ function.[2]

TABLE 1

EFFECT OF INFLATION OR OF EQUAL CHANGES IN THE SALARY AND INTEREST RATE ASSUMPTIONS ON THE $\Sigma\ (PVFB)_x$ FUNCTION

INFLATION	PRESENT VALUE OF FUTURE BENEFITS		
	Active	Retired	Total
0%	146%	127%	143%
2%	120	112	119
4%	100	100	100
6%	85	90	86
8%	73	82	74
10%	63	76	65
12%	56	70	58

Tables 1 and 2 clearly show that the *absolute* level as well as the *relative* level of the salary and interest rate assumptions has a significant impact on costs. This fact makes it difficult to select best-estimate combinations of these two assumptions.

Table 3 has been constructed to illustrate the accuracy of $a_{\overline{n}|P+I}\ \ddot{a}_r^{R+I}$ for approximating the change in $\Sigma\ (PVFB)_x$ for equal changes in the salary and interest rate assumptions. Observe that the product of the two

[2] If one were to recalculate the unfunded supplemental liability at the time of a change in the inflation component, USL in equation (1) would move in the same direction, although probably not the same amount, as the $\Sigma\ (PVFB)_x$ function, thereby mitigating the leveraged effect of a change in $\Sigma\ (PVFB)_x$ on the plan's annual contribution.

TABLE 2

EFFECT OF INFLATION OR OF EQUAL CHANGES IN THE SALARY AND INTEREST RATE ASSUMPTIONS ON NORMAL COST

INFLATION	ASSETS PLUS *USL* AS PER CENT OF $\Sigma\ (PVFB)_x$			
	0%	20%	30%	40%
0%........	143%	154%	161%	172%
2%........	119	124	127	132
4%........	100	100	100	100
6%........	86	82	80	77
8%........	74	68	63	57
10%.......	65	56	50	42
12%.......	58	47	40	30

TABLE 3

APPROXIMATING THE EFFECT OF INFLATION OR OF EQUAL CHANGES IN THE SALARY AND INTEREST RATE ASSUMPTIONS ON THE $\Sigma\ (PVFB)_x$ FUNCTION

INFLATION	$a_{\overline{5}\vert P+I}$	$\ddot{a}_{65}^{R+I}$	$a_{\overline{5}\vert P+I}\ \ddot{a}_r^{R+I}$	$\Sigma\ (PVFB)_x$	
				Active	Retired
0%.........	112%	132%	148%	146%	127%
2%.........	106	114	121	120	112
4%.........	100	100	100	100	100
6%.........	95	89	84	85	90
8%.........	90	80	72	73	82
10%........	85	73	62	63	76
12%........	81	67	55	56	70

annuities is an excellent approximation to the change in $\Sigma\ (PVFB)_x$ for active employees and that $\ddot{a}_r^{R+I}$ is a good approximation to the change in $\Sigma\ (PVFB)_x$ for retired employees. These relationships, therefore, could be most helpful in estimating the effect on $\Sigma\ (PVFB)_x$ of an equal change in the salary and interest rates.

III. NONUNIFORM SALARY AND INTEREST RATES

Constant Differential in Salary and Interest Rates

When the current rate of inflation is believed to be different from the ultimate rate, it is appropriate to consider nonuniform rates of future inflation. In developing the intermediate rates of inflation between the current level and the ultimate level, it is necessary to specify the expected length of time before the ultimate level is reached and the pattern of rates during this time interval. In this section we consider nonuniform rates of inflation, which is tantamount to considering nonuniform rates of salary and interest where the differential between the two is constant.

Table 4 shows the results of assuming nonuniform (calendar-year) inflation rates in calculating $\Sigma\ (PVFB)_x$ for the hypothetical pension plan used in Table 1. Seven different current year's inflation rates are assumed, ranging from 0 to 12 per cent, and each is graded linearly to an ultimate level of 4 per cent over five different grading intervals. The zero-year grading period, which represents use of the ultimate rates throughout, and the infinite-year grading period, which represents use of the current rates throughout, are included for comparison purposes. Finally, all the data in Table 4 are expressed as a percentage of the $\Sigma\ (PVFB)_x$ function calculated on a uniform 4 per cent inflation assumption.

TABLE 4

EFFECT OF NONUNIFORM INFLATION RATES ON THE $\Sigma\ (PVFB)_x$ FUNCTION

CURRENT INFLATION	GRADING INTERVAL (YEARS)				
	0	5	15	30	Infinite
0%.........	100%	103%	108%	116%	143%
2%.........	100	101	104	108	119
4%.........	100	100	100	100	100
6%.........	100	99	97	93	86
8%.........	100	98	94	88	74
10%........	100	96	91	83	65
12%........	100	95	88	78	58

The results are quite interesting and show that if the ultimate inflation rate is reached in less than fifteen years, the use of nonuniform rates of future inflation has little effect on the $\Sigma\ (PVFB)_x$ function. This conclusion can be reached, at least in part, by considering the formula for approximating the effect of a change in the inflation assumption on the $\Sigma\ (PVFB)_x$ function. This approximation formula for all employees less than age $r - n$ depends on the product of $a_{\overline{n}|P+I}$ and $\ddot{a}_r^{R+I}$; consequently, unless the grading period extends beyond the employee's age $r - n$, it has no effect on his $(PVFB)_x$ function. Moreover, even if the grading period extends beyond age $r - n$, it is necessary that the rates of inflation be far enough away from their ultimate value to have an effect. For example, the fifteen-year grading interval given in Table 4 will have virtually no effect on the $(PVFB)_x$ function of an employee less than age 45 and only a minor effect on the $(PVFB)_x$ function of an employee aged 45–50. The $(PVFB)_x$ function for employees over age 50 will be directly affected by the fifteen-year grading interval, the effect being greatest for those closest to retirement age. However, Table 4 does indicate that at grading intervals of fifteen years and more the effect of nonuniform inflation rates cannot safely be ignored. This is even clearer when one considers that the impact on the plan normal cost of changes in the $\Sigma\,(PVFB)_x$ function is magnified by the existence of assets and the unfunded supplemental liability. Table A2 in the Appendix shows the variation in normal cost for various asset levels, the latter being expressed as a percentage of the $\Sigma\,(PVFB)_x$ function of the plan.

The data in Table 4 and Table A2 suggest that the use of either the current or the ultimate inflation rate is unwarranted when the spread between the current and the ultimate rates is expected to last for fifteen years or more. The error associated with using either the current or the ultimate rates of inflation as the valuation assumption when actual future rates of inflation grade linearly from current to ultimate rates is given in Tables 5 and 6. Table 5 shows the percentage error in the calculation of

TABLE 5

PERCENTAGE ERROR IN THE $\Sigma\,(PVFB)_x$ FUNCTION IF THE CURRENT RATE IS USED INSTEAD OF THE GRADED RATES

CURRENT INFLATION	GRADING INTERVAL (YEARS)				
	0	5	15	30	Infinite
0%	43.1%	39.2%	32.4%	23.1%	0%
2%	18.5	16.9	14.2	10.3	0
4%	0	0	0	0	0
6%	−14.3	−13.2	−11.3	− 8.3	0
8%	−25.5	−23.7	−20.4	−15.2	0
10%	−34.5	−32.1	−27.9	−21.0	0
12%	−41.9	−39.0	−34.2	−25.9	0

TABLE 6

PERCENTAGE ERROR IN THE $\Sigma\,(PVFB)_x$ FUNCTION IF THE ULTIMATE RATE IS USED INSTEAD OF THE GRADED RATES

CURRENT INFLATION	GRADING INTERVAL (YEARS)				
	0	5	15	30	Infinite
0%	0%	−2.8%	− 7.5%	−14.0%	−30.1%
2%	0	−1.4	− 3.7	− 7.0	−15.6
4%	0	0	0	0	0
6%	0	1.3	3.5	7.0	16.7
8%	0	2.5	6.9	13.9	34.3
10%	0	3.7	10.1	20.7	52.8
12%	0	4.9	13.2	27.4	72.0

the $\Sigma\ (PVFB)_x$ function if the valuation assumption coincides with the current rate of inflation instead of with the graded rates. Similarly, Table 6 shows the percentage error incurred if the ultimate rate rather than the actual graded rates is used.

It is evident from Table 5 that use of the current inflation rate in the salary and interest rate assumptions creates intolerable errors unless the current rate coincides with the ultimate rate. Alternatively, Table 6 indicates that use of the ultimate inflation rate generates significant errors for grading intervals of fifteen years or more. However, Tables 5 and 6 do lend support to the prevalent actuarial practice of concentrating on the ultimate rather than the current rate of inflation when formulating valuation assumptions, since the error incurred in the use of the ultimate rate is invariably less than the error incurred with the current rate.

In those situations where use of the ultimate rate may unduly affect the plan's normal cost, the actuary should give appropriate weight to the level of current inflation as well as to the expected ultimate inflation rate. The use of a nonuniform inflation rate assumption would give the best results; however, the use of a flat rate is dictated generally by practical considerations. The problem thus is reduced to selecting an inflation rate between current and ultimate rates which produces tolerable values for $\Sigma\ (PVFB)_x$ compared with those generated by the nonuniform assumption. If $\Sigma\ (PVFB)_x^G$ represents $\Sigma\ (PVFB)_x$ evaluated with a graded inflation assumption and $\Sigma\ (PVFB)_x^I$ represents $\Sigma\ (PVFB)_x$ evaluated with a flat inflation assumption of I, then we desire to solve for I, where

$$\Sigma\ (PVFB)_x^G = \Sigma\ (PVFB)_x^I\,. \tag{8}$$

The solutions of equation (8) for the equivalent flat inflation rate I for the seven current inflation levels and the fifteen- and thirty-year grading intervals were reached through an iterative process. Table 7 displays the equivalent rates.

Variable Differential in Salary and Interest Rates

Thus far we have analyzed the impact of nonuniform salary and interest parameters on the $\Sigma\ (PVFB)_x$ function and normal cost under the assumption that an inherent differential exists between the salary and interest rate assumptions. It was found that the present value of future salaries was insensitive to various salary and interest levels but that the quantity $\Sigma\ (PVFB)_x - (\text{Assets} + USL)$ was not, so that changes were produced in the normal cost for various levels of salary and interest that depended only on the changes in $\Sigma\ (PVFB)_x$. We now dispense with the assumption regarding the inherent differential and consider several diverse sets of current salary and interest rate assumptions, some of which reflect unusual and necessarily temporary situations. The ultimate assumptions remain unchanged, that is, 4 per cent inflation or 7 per cent interest and 5 per cent salary (exclusive of the merit component). The present value of future salary under these conditions is no longer insensitive to changes in the interest and salary assumptions, so we might expect larger or smaller fluctuations in the normal cost than otherwise for the various sets of assumptions. In calculating the change in normal cost, asset levels equal to 0, 20, 30, and 40 per cent of the $\Sigma\ (PVFB)_x$ function calculated under the ultimate assumptions were used. Again,

TABLE 7

FLAT INFLATION RATES EQUIVALENT TO NONUNIFORM RATES GRADED FROM THE CURRENT LEVEL TO THE 4 PER CENT ULTIMATE LEVEL OVER 15 AND 30 YEARS

Inflation		Grading Interval	
Current Level	Ultimate Level	15 Years	30 Years
0%	4%	3.1%	2.2%
2	4	3.5	3.1
4	4	4.0	4.0
6	4	4.4	4.9
8	4	4.8	5.6
10	4	5.2	6.4
12	4	5.6	7.3

the current rates are assumed to grade linearly to the ultimate rates over several grading periods. The results are shown in Table 8.

Generally speaking, current levels of salary and interest rates have a noticeable impact on normal cost, even for a grading period of only five years. Also, the greater the assets attributable to the plan, the more pronounced is the influence of the current rates. The impact for grading periods greater than 5 years is substantial enough in most cases to suggest that use of either current or ultimate rates as a valuation assumption is unwarranted. Consequently, the possibility of selecting flat salary and interest rates intermediate to the current and ultimate rates or of using nonuniform assumptions must again be considered. Before, in the case of a constant differential between the salary and interest rate assumptions, the problem was confined to choosing one parameter judiciously. Now the problem is considerably more difficult, because two parameters must be chosen in such a way that each one alone is reasonable in relation to its respective current and ultimate levels, while the combined effect of the assumptions must generate the appropriate normal cost. It is in these

TABLE 8

EFFECT OF NONUNIFORM SALARY AND INTEREST RATE ASSUMPTIONS ON THE NORMAL COST

ASSETS PLUS *USL* LEVEL	INTEREST RATE	SALARY RATE	GRADING INTERVAL (YEARS)				
			0	5	15	30	Infinite
0%.......	3%	9%	100%	100%	108%	124%	207%
	5	7	100	100	104	111	137
	7	5	100	100	100	100	100
	9	3	100	100	96	90	81
	11	1	100	100	93	82	70
	10	10	100	97	95	93	84
	3	3	100	102	108	118	154
20%......	3%	9%	100%	105%	118%	141%	248%
	5	7	100	102	109	120	154
	7	5	100	100	100	100	100
	9	3	100	97	90	81	67
	11	1	100	95	80	64	45
	10	10	100	98	96	94	84
	3	3	100	104	112	126	171
30%......	3%	9%	100%	108%	125%	153%	278%
	5	7	100	104	113	126	167
	7	5	100	100	100	100	100
	9	3	100	96	86	75	57
	11	1	100	91	71	51	28
	10	10	100	98	97	95	84
	3	3	100	105	115	131	184
40%......	3%	9%	100%	112%	134%	169%	317%
	5	7	100	106	118	135	183
	7	5	100	100	100	100	100
	9	3	100	93	81	66	45
	11	1	100	86	60	33	4
	10	10	100	99	98	96	84
	3	3	100	107	119	138	200

instances that the actuary must display the utmost care in formulating best-estimate salary and interest rate assumptions.

IV. FINAL COMMENT

This paper considers two of the most critical assumptions of a pension plan valuation—namely, salary rates and interest rates. Because of the requirements under ERISA, traditional approaches to selecting salary and interest rates may not be appropriate. For example, simultaneous changes in the valuation rates of salary and interest are not offsetting. To the contrary, there is a significant inverse relationship between equal salary and interest rate changes and pension costs calculated under projected benefit cost methods. Also, persistent differences between current and ultimate rates cannot safely be ignored when best-estimate assumptions are formulated. This is especially true when the usual structure of salary and interest levels is temporarily distorted.

It is our hope that we have illuminated some of the complexities in the relationship of salary and interest rates to pension costs and that actuaries will be better able to select these assumptions in light of our findings.

APPENDIX

TABLE A1

TERMINATION RATE AND MERIT SALARY-SCALE ASSUMPTIONS

Age	Ultimate Termination Rate*	Merit Salary Scale	Age	Ultimate Termination Rate*	Merit Salary Scale
20	.243	1.000	43	.045	2.157
21	.224	1.045	44	.043	2.204
22	.207	1.091	45	.042	2.250
23	.191	1.138	46	.041	2.295
24	.176	1.186	47	.040	2.339
25	.162	1.234	48	.039	2.381
26	.149	1.284	49	.039	2.422
27	.136	1.334	50	.038	2.460
28	.125	1.384	51	.038	2.497
29	.115	1.436	52	.037	2.532
30	.106	1.487	53	.036	2.565
31	.097	1.539	54	.035	2.596
32	.090	1.592	55	.034	2.624
33	.083	1.644	56	.033	2.651
34	.076	1.697	57	.032	2.674
35	.071	1.749	58	.030	2.696
36	.066	1.802	59	.028	2.715
37	.061	1.854	60	.026	2.731
38	.057	1.906	61	.023	2.745
39	.054	1.958	62	.020	2.756
40	.051	2.008	63	.016	2.764
41	.049	2.059	64	.012	2.769
42	.047	2.108			

* Although not shown, select rates were used for employees with less than 5 years of service.

TABLE A2

EFFECT OF NONUNIFORM SALARY AND INTEREST RATE ASSUMPTIONS ON THE NORMAL COST

Assets plus *USL* Level	Inflation Rate	Grading Interval (Years)				
		0	5	15	30	Infinite
0%.........	0%	100%	103%	108%	116%	143%
	2	100	101	104	108	119
	4	100	100	100	100	100
	6	100	99	97	93	86
	8	100	98	94	88	74
	10	100	96	91	83	65
	12	100	95	88	78	58
20%........	0%	100%	104%	110%	120%	154%
	2	100	102	105	109	123
	4	100	100	100	100	100
	6	100	98	96	92	82
	8	100	97	92	85	68
	10	100	95	89	79	57
	12	100	94	85	73	48
30%........	0%	100%	104%	112%	123%	162%
	2	100	102	105	111	126
	4	100	100	100	100	100
	6	100	98	95	91	80
	8	100	96	91	83	64
	10	100	95	87	76	51
	12	100	93	83	69	40
40%........	0%	100%	105%	113%	127%	172%
	2	100	102	106	113	131
	4	100	100	100	100	100
	6	100	98	94	89	76
	8	100	96	89	80	57
	10	100	94	85	71	42
	12	100	92	81	64	30

DISCUSSION OF PRECEDING PAPER

BARNET N. BERIN:

This paper is interesting in that it takes the initial premise and develops it mathematically. However, actuaries would do well to question the validity of the premise, for the following reasons.

1. The track record on forecasting of long-term economic results is dismal whether we consider economists, businessmen, government officials, or econometric experts. Without hesitation one may say that the same is true of short-term economic forecasts. Yet these are made by people with specific training in the field, unlike actuaries.
2. The addition of the same rate of inflation to the valuation interest rate and to the salary-scale assumption is altogether too simplistic and pretends too much. First, it *reduces* plan costs considerably. Second, the accuracy of the constant additive is extremely doubtful.

What, then, do we do about inflation and pension actuarial assumptions? I would suggest that we continue to have dialogues with our clients on this subject and that we adjust our assumptions where we consider this to be appropriate. I also suggest that we concentrate on fundamentals —for example, in emphasizing, by means of the annual valuation, that we step hesitatingly into the future, one year at a time, checking our experience carefully by the gain and loss analysis.

In summary, I feel that the means to cope with the problem of inflation already exist and that we should not overlook the fact that *other* short-term and long-term forecasts, which do not assume inflation, not only are possible but may well be equally likely.

DONALD P. HARRINGTON:

This is a timely paper, and a discussion is warranted. The American Academy of Actuaries has recently issued an exposure draft (April, 1975) expressing a preference for the explicit recognition of inflation in each actuarial assumption. Essentially, the draft hints at the same type of methodology that is employed in this paper. The paper thus presents a logical foundation from which one might proceed to discuss the effect on pension costs if inflation were incorporated in this manner.

Inflation and productivity, to the extent that they are reflected in the actuarial assumptions, are generally assumed to have the effect of increasing both wage levels and the rate of pension fund earnings. The inclusion of such a wage-level adjustment in the salary-scale assumption will have a compound effect over time. If the interest assumption is adjusted by an equal amount, then the two assumptions will be approximately offsetting, provided that they both span the same duration. This is particularly true in the determination of the present value of future salaries and is shown in formulas (2) and (3) of the paper. In this situation, both the salary-scale and the interest assumption are in effect from entry age to retirement age.

When the salary-scale and interest assumptions do not span the same duration, an equal-amount adjustment to both will not produce the offset. As a matter of fact, a common rule of thumb indicates that a percentage change in interest usually exerts about twice the leverage of a percentage change in the wage-level component of the salary-scale assumption with respect to the present value of future benefits. The reason for this is shown in formula (4), which is the formula for the determination of the present value of future benefits. Obviously, the annuity value at retirement, $\ddot{a}_r^{R+I}$, covers the period when the salary-scale assumption no longer has any effect. Therefore, raising or lowering the interest assumption in the period beyond which the salary scale applies will have a significant impact, since the full effect of the change will be reflected in the present-value figures. Thus the duration becomes critical when an equal amount is added to each assumption. The salary-scale assumption applies from entry age to retirement age, and the interest assumption from entry age to the end of the mortality table.

If the salary-scale assumption (adjusted for changes in wage levels) were to apply from entry age to the end of the mortality table, then the leverage exerted by the annuity value would be eliminated. This is not as remote as one might expect! If a retirement benefit is viewed as a deferred wage, then indexing this "wage" for the effects of inflation is appropriate. In other words, the pension plan should not be funded at the expense of the retiree by paying the pensioner in cheap dollars. Thus, if the benefits are indexed by the same inflation rate used in the salary-scale and interest assumptions, the effect of inflation on the present value of future benefits will be offset. The indexing of the retirement benefits is analogous to extending the salary scale from retirement age to the end of the mortality table so that the equal adjustments span the same duration.

Finally, no matter how much actuaries and pension experts claim they are dealing with the long run, they are still affected by the current milieu. Messrs. Allison and Winklevoss are no different. The current philosophy seems to imply that rates of return can be subdivided into a risk-free rate of return plus a risk premium plus an adjustment for inflation and, further, that the inflation component can be added to the basic salary assumption in the form of a wage-level change. In my opinion this is an overly simplified approach. The initial problem is one of definition, in that "risk," "productivity," "inflation," and other terms often have different meanings when used by different groups, such as actuaries, investment experts, statisticians, and economists. Furthermore, the idea that there is a one-to-one correspondence between the amount of inflation added to salaries and the rates of return on the pension fund can be seriously questioned. Historical evidence that I have examined does not support this relationship, but one can refer to various studies and expert opinion to support or attack this concept. Individual preference will no doubt exert a great deal of influence on the final conclusion. In large plans, where the data are statistically significant, the salary-scale assumption will be developed from company experience. Techniques can be employed to separate the scale into merit (including seniority) and wage level. Further subdivision of the wage-level component into productivity and inflation is highly subjective. Even if such an apportionment were made, it is doubtful whether the portion labeled "inflation" could be added to the interest assumption.

I hope that my comments will not be construed as a criticism of this paper. I enjoyed studying the paper and felt that actuaries should participate in the discussion in order to expand the literature on this important and timely topic. Messrs. Allison and Winklevoss have demonstrated, for a variety of situations, the effect of changes in two of the most important actuarial assumptions. All actuaries (and not only pension actuaries) now have a firm basis on which to proceed in future discussions.

BRIAN A. JONES:

This paper will be very useful in an area where many of us will have to be more explicit than previously—in our reports to clients, in reporting to the government, and in our own thinking.

I have one small criticism. There appears to be an implied assumption in the paper that wage increases *will* reflect inflation and productivity increases and that increased yields *will* flow from inflation. It seems to me that, in addition to focusing on the "most likely" assumptions in the way the paper suggests, we should be prepared to illustrate other combinations of wage inflation and yield that are not so neatly in step. In particular, we should examine what happens if wage increases occur without the anticipated higher yields; that is, we should give our clients some measure of the ultimate costs if x per cent inflation occurs in wages and the hoped-for increase in yields is not there or is less than expected.

I have found, in presenting analyses similar to those set out in Section

II of the paper and those suggested above, that traditional valuations are often more useful than year-by-year projections. The main reason for this is that clients tend to be mesmerized by projected dollar figures that include a significant amount of inflation. Of course, the results of alternate valuations, particularly the very pessimistic ones where inflation is factored into the benefits but not into the yields, do not produce figures that have any meaning as projected costs. The results must be presented as "ultimate" costs toward which valuation costs would drift if the pessimistic assumptions were realized, but I still find this explanation easier than relating dollar figures from different projections that incorporate various degrees of inflation. This problem, as mentioned in one of the other papers, can be solved partly by focusing on percentages of payroll rather than on dollar figures.

Another complication that is introduced when alternate costs are calculated is the effect of inflation on social security offsets or integration levels, if applicable. I do not mean to suggest that this should have been considered in the paper, but it is an area one must consider in order to give a client a realistic picture of what may happen under various degrees of inflation.

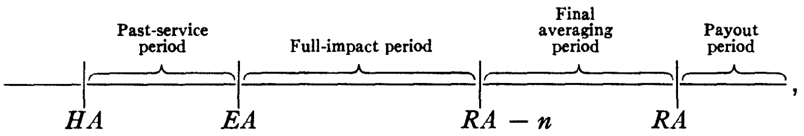
One very valuable effect of the paper is to bring our attention back to the forest rather than the trees. When we see the degree of variation in actual costs that can result from various inflation assumptions—all of which may seem equally reasonable now—it helps to put in perspective some of the intricacies that now take up a good deal of time and effort in designing valuation systems. As an example, in many situations one can get a reasonably good, though rather conservative, approximation to the cost of vesting by suppressing the turnover discount from vesting age onward. Similarly, one can simplify a calculation by using one-year term costs for minor benefits (assuming that the resulting cost is reasonably stable). These approximations are less than perfect, but if, as the result of using them, more actuarial and computer time is available to investigate the larger questions discussed in the paper, I believe they are more than justified.

RICHARD K. KISCHUK:

Mr. Allison and Dr. Winklevoss are to be congratulated on a very valuable addition to the actuarial literature. With interest and inflation rates having reached historically high levels over the past several months, the interrelationship of these factors is of very vital concern to all actuaries working in the pension field.

Although the practice of selecting salary and interest rate assumptions based on their differential rather than on their absolute levels may not work out well in many instances, there is reason to believe that this practice may be appropriate for a large number of plans. The underlying mechanism may be seen by examining the stages through which a given employee progresses during his lifetime.

These progressive stages may be diagramed as follows:



where HA is age at hire, EA is age at entry into the plan, n is the number of years in the final averaging period, and RA is age at retirement.

Assuming a constant differential of, say, 2 per cent between the salary and interest projection rates over the employee's lifetime, then the absolute level of the interest and inflation rate assumptions will be irrelevant during those periods when benefits rise at exactly the same rate as salaries rise. This situation is likely to occur during the full-impact period but can also occur during the payout period if postretirement cost-of-living increases are granted.

The absolute level of the interest and inflation rate assumptions becomes very important where there is not an exact correspondence between the rates at which benefits and salaries rise. This situation occurs during the payout period in plans where there are no cost-of-living increases after retirement. Benefits will also rise at less than the rate of increase in salaries during the final averaging period, which can extend back to the entry age or age at hire in the case of career average plans. Depending on the type of benefit formula, the same situation may occur during the full-impact period. For example, a "dollars times years of service" formula will produce no increase in benefits as salaries increase; a step-rate integrated formula will often produce increases in benefits at more than the rate of increase in salaries.

In plans where benefits rise more or less rapidly than the rate of increase in salaries during some or all of the four stages in the employee's lifetime, the absolute level of the salary and interest assumptions becomes important. In such plans the actuary must use care in selecting the level of these assumptions.

In the case of a career average plan, with no cost-of-living increases after retirement, the absolute level of these assumptions is obviously of great importance. In the case of a final average plan, with cost-of-living increases after retirement, the absolute level of the salary and interest assumptions may be of little importance as long as the correct differential is chosen.

In the example chosen by the authors, the absolute level of the salary and interest assumptions is not of much importance until the final averaging and payout periods are reached. In the case of the younger employees, these stages will be reached so far in the future that the ultimate assumptions are about the best that can be made. For the older employees, the absolute level of these assumptions will not become important until the final averaging period is reached.

Tables 5 and 6 of the paper clearly indicate the error of using either the current or the ultimate rate in valuing this plan and suggest an alternate method of valuation which could be expected to have better results than either of these alternatives. First, the participants are sorted according to the number of years until retirement or since retirement. For those participants who are, say, fifteen or more years from retirement, the ultimate assumptions are used. The remaining active and retired participants are then separated into two or more groups according to the number of years until, or since, retirement. A pair of salary and interest assumptions is then selected for each of these groups.

Depending upon how finely the participant group is divided, and at what point the ultimate assumptions are used, this method should give results much better than those obtained by using either the current or the ultimate rates. This general method can be used for a wide variety of plans, once analysis has been completed in order to determine when the absolute level of the assumptions is important and when merely selecting the appropriate differential is sufficient.

There is another point at which the absolute level of the salary and interest assumptions affects the calculation of plan costs, which is not treated by the authors. This is in the amortization of the unfunded past-service liability. In general, actuarial cost methods assume that, if a plan possesses a given amount of assets equal to the past-service liability and annual payments are made each year in the future equal to the normal cost, the plan will be fully funded as long as the actuarial assumptions are met. To the extent that the plan does not now possess assets in the amount of the past-service liability, the employer must himself make up the investment income which these missing assets would have earned. The rate of interest used to determine the amount of investment earnings that must be made up by the employer is usually the valuation rate of interest. To the extent that the valuation rate of interest understates the actual interest earnings of the plan, the interest-only contribution level is also understated. Thus there is a good argument for using the average rate of investment earnings for the plan assets, rather than the valuation interest rate, in determining the interest-only contribution level. This method can be used to avoid the question of what interest rate to use in determining the amortization portion of the contribution when more than one valuation rate of interest is used, as in the method suggested earlier. Unless this approach is used, it is difficult to see how it can be claimed that excess interest is offsetting excess inflation for any plan which is not fully funded with respect to the past-service liability.

Of course, where there is no systematic understatement of the salary and interest assumptions, the valuation interest rate is probably the appropriate one to use in determining the interest-only contribution level. However, where the actuary purposely is understating the interest assumption in order to offset inflation, the use of the valuation rate of interest understates the interest-only contribution and leads to apparent "actuarial losses" when the assumptions may be operating satisfactorily.

As an alternative to using the average return of invested plan assets, the actuary may prefer to use a realistic estimate of the average rate of return expected to be earned from plan assets in future years. As this estimate changes, the interest rate used in deriving the interest-only contribution changes. This interest rate would be determined separately from the rest of the valuation and could be used when the valuation interest rate is being deliberately understated or when multiple valuation rates of interest are used.

There are many possible variations of this approach. For example, in addition to a separately determined interest rate, a separately determined inflation assumption could be used in order to amortize the past-service liability in terms of "constant dollars." The inflation assumption would probably reflect price inflation rather than wage inflation and could reflect inflation as experienced by the economy as a whole or as experienced by the firm individually.

L. D. LEWIS:*

The authors conclude that the cost of a pension plan as determined by an actuarial valuation depends not only on the differential between salary-scale and interest assumptions but also on the absolute level of each of these assumptions. I would like to point out that this result is in fact well known to actuaries. For example, E. M. Lee, in his book entitled *An Introduction to Pension Funds*, arrives at this result by a process of general reasoning and then demonstrates the result by means of a numerical example. This book is prescribed reading for the examinations of the Institute of Actuaries.

* Mr. Lewis, not a member of the Society, is an Associate of the Institute of Actuaries.

In practice, many consulting actuaries carry out valuations using a number of different bases in order to investigate fully the financial position of the plan. The figures in Table 1 of this discussion are from an actual case study and illustrate clearly the conclusions reached by the authors. Since the liability for pensions in course of payment depends only on the interest rate, this item has been omitted.

The plan from which the figures are derived provides a pension based on final five-year average salary with an offset in respect of benefits from the Canada Pension Plan. The figures shown are the values of the accrued pensions based on salaries projected to normal retirement age.

TABLE 1

LIABILITY FOR PENSIONS OF ACTIVE EMPLOYEES

(Thousands of Dollars)

Salary-Scale Increase per Annum	Valuation Interest Rate per Annum				
	5%	6%	7%	8%	9%
3%	4,122	....	....	....	....
4%	4,802	3,811	....	....	....
5%	5,586	....	3,532	2,872	....
6%	....	5,134	....	3,282	....
7%	....	....	4,734	3,777	3,057
8%	....	....	....	4,377	....
9%	....	....	....	....	4,061

A rule of thumb often used by actuaries is that a change in the salary-scale assumption equal to $1\frac{1}{2}$ times the change (in the opposite direction) in the interest assumption leaves the pension liabilities for active employees approximately unaltered. The accuracy of this rule can be judged by comparing the liability in Table 1 for suitable combinations of salary scale and interest rate, for example (3 per cent, 5 per cent) compared with (9 per cent, 9 per cent); (4 per cent, 6 per cent) compared with (7 per cent, 8 per cent); and (4 per cent, 5 per cent) compared with (7 per cent, 7 per cent).

In this particular example, a 1 per cent increase in the salary-scale assumption raises the liabilities by approximately 17 per cent, while a 1 per cent decrease in the interest rate raises the liabilities by approximately 25 per cent.

An actuary would indeed be naïve if he were to assume that inflation affects salaries and interest rates equally and that an equal increase in assumptions on these two items would not significantly alter his valuation results.

ROBERT F. LINK:

Two developments make the subject of this paper of the most intense and timely interest. First, we have recently experienced unprecedented rates of inflation and associated effects on pay levels and investment yields. Second, the Employee Retirement Income Security Act has reinforced the obligation of pension actuaries to use assumptions representing a best estimate of the real world. This discussion touches on a few aspects of the Allison-Winklevoss presentation.

Under the heading of sensitivity analysis, the following approximate formula is presented as formula (3):

$$\left(\frac{1+P+I}{1+R+I}\right)^j=\left[\frac{1+P}{1+R}+\frac{I(R-P)}{(1+R)(1+R+I)}\right]^j\doteq\left(\frac{1+P}{1+R}\right)^j .$$

Another view would be that the inflation effect should be compounded on top of the pay or interest rate effect. The compounding preserves full purchasing power parity. Under this approach there is no approximation. Formula (3) is replaced by

$$\left[\frac{(1+P)(1+I)}{(1+R)(1+I)}\right]^j=\left(\frac{1+P}{1+R}\right)^j .$$

Table 1 illustrates how the present value of future benefits becomes relatively high under a low inflation assumption and relatively low under a high inflation assumption. I have seen figures indicating that the present value of future benefits would be largely or perhaps entirely unaffected by the inflation assumption if benefits after retirement were indexed to the inflation rate. To the extent that a higher inflation assumption reduces apparent costs, that reduction mirrors a reduction in the purchasing power of the pension payments received by retired persons.

There is an implicit assumption in the paper that salaries and the yield on pension fund assets will respond to inflation in tandem. In fact, the situation is quite complex, and the responses may be quite different. In the case of salaries, there is not only inflation but also other labor market factors. For many years, prevailing pay levels have risen faster than inflation would dictate. More recently, they have risen more slowly. That is, the standard of living has been dropping rather than rising. How this will turn out in the future is unclear.

On the investment side, we have "portfolio drag." I mean by this that the influence of the inflation rate during a specific period of time applies primarily to the investments *made* during that period. Thus, when the inflation rate rises from one prevailing level to a new, higher prevailing level, the yield on the pension fund will rise only gradually to the interest rate associated with the new inflation level. One way of handling this in a pension fund valuation is to do the calculations fully on the basis of the new inflation level and revalue the existing assets downward to a current yield basis. The same problem arises in a much more complex way when one is dealing with a variable inflation rate that settles down at an ultimate level sometime in the future.

The foregoing notwithstanding, let us assume that general salary levels actually do respond primarily to inflation rates (that is, the labor market and standard-of-living factors are effectively neutral). What is the "noninflationary" part of the salary-scale assumption (comparable to the author's assumed noninflationary interest rate of 3 per cent)? This will vary widely by industry and other factors. However, I suspect that there are many organizations in which the average individual salary progression excluding inflation may be at rates as high as 4 per cent, 5 per cent, or even more. This implies the possibility of cases where the salary scale used in a pension valuation should be *higher* than the interest rate. However, I have the impression that few if any pension valuations are actually done on such a basis. Perhaps one saving factor is the possible tendency of salaries in some situations to level off in the years closest to retirement (when the salary scale has its heaviest weight).

MIGUEL A. RAMIREZ:

It is gratifying to see in the literature a paper of this nature that sweeps away a sometime misconception as to how the salary-scale and interest assumptions interrelate. I have a few comments about the authors' approach and results.

First, the results of this paper conform closely to those of some experiments on the same subject done at our office. From our own study the following general observations were made:

1. For a new plan with only active participants, increasing the interest rate and the salary scale by the same factor, say 1 per cent, has, not surprisingly, the same effect as that seen on the value of an immediate annuity at age 65 (or whatever retirement age is assumed) by virtue of increasing the interest rate alone, in this case, about 10 per cent (decrease).
2. The 10-for-1 relationship discovered above also holds for the entry age normal cost and for the entry age accrued liability. The pension expense, however, is another story. If the pension expense is defined as the normal cost plus thirty-year funding on the accrued liability, the effect is more like 5 for 1 as the interest-salary adjustment increases. The reason for this seems to be that the interest obligation works in the opposite direction.
3. The observed 10-for-1 effect holds only for nonintegrated final pay plans. For nonintegrated career average plans, the result is more than 10 for 1

because the salary-scale projection is effective, on the average, for half the period of future service. For integrated final pay plans, however, the result is less than 10 for 1, because the salary influence has a leveraged effect which the interest factor does not have. We did not project the offset or integration level, but I think that if we had the 10-for-1 rule would have been preserved.

Second, the authors demonstrate that, as a rule, equal inflation components are not mathematically equivalent as far as pension costs are concerned. The one general exception seems to be a final pay plan with cost-of-living adjustments after retirement. However, a student of actuarial pension theory could possibly make the inference that for just this exception it is proper to exclude equal inflationary components from both salary-scale and interest assumptions, on the grounds that mathematically the results will be the same. This may be true for the first valuation, but not for the rest.

Gains and losses, as is fairly clear from the Anderson paper (*TSA*, XXIII, 151), arise from the effect of experience deviations on components of the *accrued* liability. Interest gains and losses, on the other hand, emerge from the *funded* accrued liability. Few plans have fully funded past-service liabilities, so that one would expect salary losses on the same component to have a greater effect than interest gains, especially in the first few years of a plan's existence.

Anticipating gains and losses to offset each other may also lead to unrealistic assumptions in another situation. Decreasing salary scales are fairly prevalent ("decreasing" in the sense that the annual increase factor at the low ages is higher than that at the ages closer to retirement). The component of the accrued liability attributable to younger employees is significantly less than their share, as measured by salaries or numbers, because of the longer period of discount for severance and mortality. Thus a sharply pitched salary scale may seem realistic in total, whereas the scale for the important age group is actually on the low side.

Third, the authors analyze the yield assumption into $I + R$ and the salary-scale assumption into $I + P + M$ (the notation is theirs, except that M represents the merit component defined in the paper by $s_{x+1}/s_x - 1$. Although the final conclusion is that the interest and salary inflation components should not be ignored as having offsetting effects, there are hidden assumptions in this presentation which are not fully explored.

The chief problem with this model is that it ought not to be the same for interest and salary. For the former, it should reflect prevailing trends in the cost of credit; for the latter, cost of labor. Although the labor and financial markets are ultimately related, the relationship is not so close that simultaneous or even parallel inflationary trends can be assumed.

Fourth, the authors report that the cancellation of equal inflationary components in inflation-coordinated salary and interest assumptions is close but not quite perfect. The imperfection is due strictly to the theoretical approach. If the problem is analyzed by assuming that the independent forces of interest, salary increase, and inflation correlate momentarily, the problem vanishes.

Let δ be the annualized momentary rate of yield for all but the inflationary component and σ_x the annualized momentary rate of salary increase at age x for all but the inflationary component. Then, in projecting and discounting salaries, the following composite factor is encountered:

$$f(x, y) = \left\{\exp\left[\int_x^y (-\delta)du\right]\right\}\left[\exp\left(\int_x^y \sigma_u du\right)\right].$$

This factor consists of the interest discount multiplied by the salary projection. In computing the present value of future salaries, the factor can be substituted in the authors' formula (2):

$$^{s}\ddot{a}^{aa}_{x:\overline{r-x}|} = \sum_{j=x}^{r-1} f(x, j)\ {}_{j-x}p_x .$$

In computing the present value of future benefits, this factor will also be present if the plan is based on final pay without offsets or integration levels, but for career average and other types of complex plans the factor may undergo considerable transformation (it is these transformations that produce the well-known "leverage" effect).

Assuming that inflation has a constant and continuous effect α on the force of interest and β on the "force" of salary increment, the corresponding annualized momentary rates of yield and salary increase, including the inflationary component, are, respectively,

$$\delta' = \delta + \alpha , \qquad \sigma'_x = \sigma_x + \beta .$$

It is clear that

$$f'(x, y) = f(x, y)\left\{\exp\left[\int_x^y (\beta - \alpha)du\right]\right\} .$$

Thus $f(x, y)$ is invariant if and only if $\alpha = \beta$. Under the authors' approach, however,

$$\alpha = \ln\left(1 + \frac{I}{1 + R}\right), \qquad \beta = \ln\left(1 + \frac{I}{1 + P + M}\right).$$

CHARLES L. TROWBRIDGE:

Mr. Allison and Dr. Winklevoss are to be congratulated for having made a start on a problem that has been too long neglected—the effect of inflation on pension costs. Much more can be said on this important and timely subject, and it is to be hoped that the actuarial literature in this area will develop quickly.

The authors could have made the mathematics much easier if they had introduced the rate of inflation I on a multiplicative, rather than an additive, basis. If the interest rate were assumed to be of the form $(1 + I)(1 + R) - 1$ instead of $I + R$, and the salary-increase rate (exclusive of merit or promotional increase) were treated as $(1 + I)(1 + P) - 1$ instead of $I + P$, formula (3) would fall out without any approximation. Formula (7) follows, as long as the salary scale (reflecting promotional increase only) is nearly constant over the last n years prior to retirement.

Table 1 yields the interesting information that a 1 per cent increase in the assumed rate of inflation cuts the present value of future benefits by about 8 per cent. Since the present value of future salaries is unchanged, the indicated initial contribution under the aggregate method is 8 per cent lower. It is important to realize that this reduction is largely the result of not passing on cost-of-living increases after retirement. As the authors state on page 201, the initial contribution would be almost independent of the inflation rate if cost-of-living increases were granted to those retired.

It also is important to recognize that, while the rate of contribution as a *percentage of payroll* may decrease if higher rates of inflation are assumed, the high-inflation contribution expressed in dollars will overtake and pass the contribution resulting from lower inflation rates. If the total payroll is growing 1 per cent faster, but the contribution starts about 8 per cent lower, we might expect a crossing after about eight years. Similarly, an additional 1 per cent inflation, if it is passed on to retired lives, will after the first year cost the plan an additional 1 per cent annually, because the payroll is rising at a 1 per cent faster rate.

I am not particularly impressed by the part of the paper that implies that a slow grading of salary-increase and investment earnings assumptions from current levels to some assumed ultimate level is more accurate than using the ultimate assumption at once. At several points the paper tries to measure the "error," as if there were something right about the graded assumptions. Inflation rates are erratic rather than smooth, and they never over the short run affect salary levels or investment earnings rates in strict conformance with the assumptions of the paper's theoretical model. The best that the actuary possibly can expect is that his assumptions may be close to the long-term averages; he must stand firm against criticism that his assumptions do not reproduce closely the recent past.

Finally, I feel that a word or two is in order with respect to pension funding terminology. At several points late in the paper the authors use "normal cost" to represent the result of the aggregate projected benefit calculation represented by formula (1). For actuaries familiar with *TSA*, Volume IV, "normal cost" has a quite different meaning. Until pension actuaries learn to define their terms, and to use them consistently, only confusion can be expected.

(AUTHORS' REVIEW OF DISCUSSION):

There are several common points in the discussion presented. We will focus on these areas of general concern first, before turning to the individual responses.

A major premise of our analysis is that the inflation rate is a common theoretical component of both salary and interest rates. There appears to be some discomfort with the assumption that inflation changes are reflected *equally* in salary and interest rates. Certainly no one will deny that economic theory supports the equal-change premise in an idealistic model of an inflationary environment. Lenders in the financial market will demand an interest premium to cover the erosion of invested capital due to inflation, while borrowers will be willing to pay the premium because repayments of principal will be made with cheaper dollars. In the labor market, supply-and-demand considerations determine an equilibrium wage level which can be translated to purchasing power based on the general price level. If the price level changes, the purchasing power of wages changes. But this forces wage levels to adjust in response to the price-level change in order to maintain purchasing power. This means that wage levels, as well as interest rates, change directly with price-level changes.

Our economy, of course, is infinitely more complex than the above description, and the equal-change premise may break down because of various imbalances. Government regulation, structural imperfections in various markets, lagged and serial effects of changes, and interfaces with other economies are just a few of the reasons. Thus, although the equal-change premise applies to an ideal situation, adjustments may be necessary when the transition is made to real life. The actual change in salary or interest rates may be either greater or less than an actual inflation change, particularly in the short run and perhaps even on a long-term basis. However, we feel that our assumption is a good place to start until more sophisticated models of the long-term structure of interest, salary, and inflation rates are verified. Despite the practical problems associated with the equal-change assumption, it is an invaluable tool for analyzing identical changes in the salary and interest assumptions.

Several discussants noted that the approximation in equation (3) could have been avoided by the use of continuous or multiplicative rather than additive expressions involving I. We agree that this would streamline the presentation.

Also, some people observed that a plan providing cost-of-living increases to retirees based on inflation is an exception to the rule that equal changes in salary and interest rates do not leave costs and liabilities unaffected. However, as noted in the mathematical analysis in Section II, there is a differential impact on the liability of active employees depending on $a_{\overline{n}|P+I}$ even when full cost-of-living increases are provided. Table 3 shows the sensitivity of $a_{\overline{5}|P+I}$ to changes in the inflation assumption. Thus, only the liability of a plan based on final salary (not final *average* salary) and providing full postretirement inflation adjustments may properly be regarded as independent of the inflation assumption.

Mr. Berin suggests that the premise underlying the paper is questionable. His first "reason" for questioning the premise is his finding that both short-term and long-term economic forecasts have had dismal success. Apparently Mr. Berin is speaking to the issue of whether the inflation component should be made an explicit component of actuarial assumptions, since his comment does not really speak to any other premise of the paper. Our belief is that inflation must be considered in the actuarial assumptions in order to comply with the "best estimate" requirement of ERISA. Whether the inflation component is explicit or implicit is a matter of personal choice. The point is that expected future inflation cannot be ignored, as Mr. Berin appears to be hinting in his first "reason" and in his emphasis on the gain and loss analysis which deals with inflation on a post facto basis.

His second reason for questioning the underlying premise of the paper is that it is too simplistic. As we pointed out earlier in this response, this indeed may be the case, and we are most anxious to see some research on this difficult problem. Mr. Berin seems to have some problem with the idea that increased inflation under the additive assumption reduces costs. If the additive assumption were correct, it is logical that costs would be suppressed in the current year and in the future years as a percentage of salary. However, at some point, future dollar costs would be higher in an inflationary environment than the costs in a noninflationary environment, a point mentioned in Mr. Trowbridge's comments.

We believe that one of the most serious shortcomings in the area of pension cost analysis can be characterized by Mr. Berin's statement that actuaries should "step hesitatingly into the future, one year at a time, checking [their] experience carefully by the gain and loss analysis." In our view, an actuary is charged with the responsibility of selecting assumptions that are used for estimating contingencies far into the future, and such a responsibility cannot be met by adopting the myopic view indicated in Mr. Berin's statement.

Mr. Harrington has doubts about the additivity of the inflation assumption but recognizes that there is evidence and opinion on both sides of the issue. He also has serious doubts as to whether the productivity component could be accurately measured, and he suggests that this component would be highly subjective. We also feel that measuring the productivity component could be difficult; however, we have no problem with the fact that its inclusion in the construction of a salary-increase rate would be highly subjective. In fact, we feel that the proper selection of each component of both the salary rate and the interest rate (except, perhaps, for the merit component) *must* include a good deal of subjectivity, because historical experience may not be a sound guide to future experience. Finally, Mr. Harrington offers us a rule of thumb that a change in the interest rate has about twice as much impact on the plan's present value of future benefits as an opposite change in the salary rate. We suspect that this rule, like the 6 per cent rule that is often used to describe the effect on liabilities of a $\frac{1}{4}$ per cent change in the interest rate alone, is rather rough.

Mr. Jones believes that actuaries should be prepared to illustrate various combinations of salary rate and interest rate assumptions which are "not so neatly in step" as those in the paper, particularly the case where the full amount of inflation is included in the salary rate but only a portion of it is included in the interest rate. We applaud this suggestion and believe that such illustrations would be a valuable experience for both the actuary and the sponsoring firm. Although we are not particularly enamored of the use of approximations, we agree entirely that the client is better served by using various approximations if the trade-off is more computer and/or man time spent on far more significant pension cost issues.

Mr. Kischuk provides us with an interesting graphic display and qualitative analysis of the interrelationship between salary and interest rate assumptions. He also suggests a novel approach to the problem of giving some recognition to the current level of salary and interest rates in the annual valuation. Rather than using nonuniform salary and interest rate assumptions, or one set of rates weighted between the current and ultimate levels, Mr. Kischuk suggests using ultimate estimates for active employees who are, say, fifteen or more years from retirement, and using estimates with successively more weight given to current experience for correspondingly older attained-age groupings of employees. This approach is tantamount to performing several subvaluations of the plan's liabilities and combining the results to develop the overall valuation. Such a procedure, while undoubtedly producing more accurate results than the exclusive use of either current or ultimate assumptions, may be entirely too complex, possibly for the actuary and certainly for the typical employer. Furthermore, although the procedure reduces the problem of selecting appropriate assumptions weighted by current and ultimate rates to subgroups of the pension population, the selection of particular assumptions for each group is still a difficult problem, especially for subgroups near retirement, where the bulk of the plan's liabilities may be concentrated.

Mr. Kischuk raises an issue not discussed in the paper regarding the appropriate interest assumption to use with respect to the plan's past-service liability. In particular, he points out that a combination low salary and interest rate assumption, which may be appropriate for future-service costs under the plan, may cause the interest-only contribution on the past-service liability to be too low. Under ERISA, of course, the interest-only contribution is too low by definition, but we suspect that Mr. Kischuk would make the same point regarding the forty-year amortization period of the past-service liability. This raises what the authors believe is an even more fundamental point in regard to low combination assumptions which are used as surrogates for more realistic, higher assumptions. We have concluded that, if one considers a time horizon of more than one year, then it is *impossible* to devise a low set of salary rate and interest rate assumptions that is equivalent to a more realistic and higher set. Although the actuarial gains and losses may be offsetting in the first year, they cannot continue to be offsetting as the plan's funding status increases over time, a point noted also by Mr. Ramirez. This conclusion, if correct, rules out the possibility of a so-called combination best-estimate set of assumptions, which in turn implies that ERISA rules out the use of such a procedure. While we believe that combination best estimates will be permitted under ERISA, actuaries should appreciate the fact that there is no such thing if one considers a time horizon of more than one year—a seemingly appropriate view for an actuary to take.

We would indeed be pleased to know that our paper states the obvious, and that actuaries have never made the mistake of thinking that only the differential between the salary and interest rate is important, as Mr. Lewis implies in his discussion. We can hardly accept this proposition, however. Lest one get the impression from Mr. Lewis' remarks that all actuaries hold the same opinion regarding the interaction of the interest rate and salary rate, it is of interest to point out that he states that "a rule of thumb often used by actuaries is that a change in the salary-scale assumption equal to $1\frac{1}{2}$ times the change (in the opposite direction) in the interest assumption leaves the pension liabilities for active employees approximately unaltered," while Mr. Harrington states that, "as a matter of fact, a common rule of thumb indicates that a percentage change in interest usually exerts about twice the leverage of a percentage change in the wage-level component of the salary-scale assumption with respect to the present value of future benefits." Mr. Lewis provides data to support his "often used" rule, while Mr. Harrington does not. Yet we feel confident that Mr. Harrington's "common" rule applies to some plans that he has analyzed.

It seems apparent that there does not exist a universal rule in this regard, and undue reliance on any such rule is unjustified. We suggest the use of basic principles to analyze the impact of salary and interest rate changes, as demonstrated in the article.

Mr. Lewis states: "An actuary would indeed be naïve if he were to assume that inflation affects salaries and interest rates equally." We apologize for our naïveté and hope that Mr. Lewis will share with us the data he may have on the long-run relationship among inflation, interest, and salary rates.

Mr. Link touches on the mathematical simplification achieved with compounded rather than additive inflation rates, the neutralized impact of inflation on a plan having a full cost-of-living provision, the differential impact of inflation on salaries and investment returns in the short run, and the noninflationary component of the salary rate. With regard to the last item, he concludes, and we agree, that it would be possible in some cases for the salary rate to *exceed* the interest rate. If the merit scale were extremely steep, as it may well be for some firms, this result would obtain without much difficulty. Moreover, this condition can exist while at the same time a firm's total salary increase may be *less* than its interest rate assumption. This situation occurs, of course, because the merit scale affects the total salary increase only to the extent that a shift in the underlying population occurs during the year. Although a plan's population may mature, typical deviations in this respect for large plans generally have a minor effect on the total salary increase. If employers tend to think in terms of average salary increases from year to year for all active employees, they should be cognizant of the fact that some, if not all, of the merit component of the salary scale should be abstracted from the valuation salary assumption in order to arrive at the actuary's estimate of future increases in the average salary of all active participants.

Mr. Ramirez offers us a 10-for-1 rule for nonintegrated, final pay plans —that is, a 1 per cent change in both salary rate and interest rate changes liabilities by 10 per cent in the opposite direction. This is in contrast to the well-known interest-only rule, which states that a 1 per cent change in the interest rate affects costs by about 22 per cent in the opposite direction (6 per cent for each $\frac{1}{4}$ per cent change in interest). If these two rules were valid for a given plan, it suggests that a 1 per cent change in the salary rate alone affects costs in the *same* direction by about 12 per cent. These results, interestingly, fall between Mr. Harrington's 2-for-1 rule and Mr. Lewis' 3-for-2 rule. Again, we urge the utmost caution in using such approximations.

Mr. Ramirez points out the fact that gains and losses will not be completely offsetting after the first year under combination assumptions which consist of unrealistically low salary and interest rates. He also makes a good point in reminding us that the heavy concentration of a plan's liability is associated with those participants nearest retirement. Thus, unless the age- and/or service-dependent merit component of salary increases is explicitly recognized, implicitly understating merit increases at the younger ages and overstating merit increases at the older ages will not produce offsetting gains and losses because of the preponderant weight of the older ages in total liabilities.

Mr. Trowbridge observes that, even though an increase in inflation which is fully reflected in the salary and interest assumptions will reduce initial contributions, this reduction may be more apparent than real because cost-of-living increases are not granted to retirees. Moreover, dollar costs will be higher at some point in the future than if there were no increase in inflation. He estimates that this crossover may occur in about eight years for a 1 per cent increase in inflation. This conclusion was drawn from Table 1. To the extent that the plan has assets, the depression in costs per increase in inflation is magnified, so that the actual crossover date may be later.

It is clear from Mr. Trowbridge's comments that he favors the traditional use of ultimate salary and interest rate assumptions irrespective of their current levels. Our results lend support to the use of ultimate rather than current assumptions; however, we do not agree that the use of ultimate assumptions is inherently more appropriate than a grading procedure that recognizes current levels of salary and interest rates. The graded assumptions are intended not to reproduce the recent past but rather to remove the bias inherent in the use of ultimate rates that are different from current rates. For these ultimate assumptions to be an unbiased estimate of future experience, the long-term average rates should be the ultimate rates. But starting at a point, say, higher than the long-term average requires that the average of all points after the current point be lower than the ultimate, and if the ultimate really is a fair statistical average, we would not expect this to happen. There are two possible resolutions of this dilemma. If it is assumed that the ultimate assumption does not vary from year to year—that is, estimates of long-term averages are not revised in light of current experience—then a graded assumption is a first step toward removing an inherent bias. On the other hand, if the ultimate rates represent a long-term average based on a prospective view from the current year forward, then the current rates have been implicitly recognized in the ultimate rates. This second possibility is tantamount to selecting a single flat rate equivalent to a graded rate. Thus, current rates must be considered either explicitly or implicitly in determining unbiased assumptions. Since ERISA mandates the use of best-estimate assumptions, it is appropriate to strive for unbiased assumptions by recognizing current experience in some reasonable manner in selecting assumptions. Assumptions could be biased for other reasons—for example, conservatism—but that is a separate issue.

We would like to thank each of the reviewers for taking time to comment on our paper.