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INSURANCE COMPANY GROWTH

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ABSTRACT

The availability of surplus sometimes constrains the growth of an insurance company. To optimize growth, a company under such constraint must develop equivalent profit standards for all opportunities that use surplus, such as sales of insurance products, acquisition of investments, or development of a sales force. This paper defines a concept of equivalence for profit standards. Microeconomic theory provides a technical setting for the definition of profit equivalence, using the concepts of marginal profitability and marginal use of surplus. The central principle advanced is that the company should set prices on all its products to produce marginal profits that are equal in proportion to the products' marginal use of surplus. After considering technical and practical problems in the application of this principle, the paper touches on questions of equity. A new equity principle is proposed for debate.

1. INTRODUCTION

Both stock and mutual insurance companies seek to grow. Different companies measure their growth in different ways; common bases for measurement may include premium income in force, life insurance in force, assets held, and, for stock insurance companies, growth in earnings per share according to generally accepted accounting principles (GAAP). For mutual insurance companies, growth in these measures may indicate growth in successful service to policyholders. Both types of insurance company seek to grow faster than the company's expenses inflate, to keep down unit expenses.[1] Some companies feel that they need to grow enough to maintain their market share. Whatever the growth target, however, the insurance company faces a difficult resource-allocation problem when it seeks to optimize growth. The solution may lie in an application of microeconomic theory to actuarial theory.

[1] *RSA*, V, 36, 1360; VI, 307–8, 310, 644.

A company's growth is limited by its key resources: agents, employees, management, space, computers, cash, and surplus.[2] For any one company, a single resource may be the limiting factor.

For example, in some companies the size and productivity of the agency force may be the limiting factor; increased sales of one insurance product (or other financial service) may come at the cost of reduced sales of another product. Even for a company with an increasing *volume* of sales, the *number* of new sales may be constant. For some companies, certain insurance products act as "door-openers" that lead to additional sales. Some products may be "add-ons" whose sales can be achieved with no additional sales effort. In these situations, productivity of the sales force is not the limiting factor. Before a company can plan a strategy to optimize growth, it must know its own operations well enough to know not only which are its limited resources but also what are the dynamic relationships among its key resources.

Most resources, including productive agents, can be acquired, in time, at the cost of surplus. This makes surplus a kind of common denominator among resources. Furthermore, gross premium price competition has forced insurance companies to rely more on surplus than on premium margins to absorb experience fluctuations.[3] Surplus levels have declined, however, relative to liabilities.[4] A low level of surplus may limit a company's ability to underwrite additional risks. Thus, surplus is not only a kind of common denominator for scarce resources but also a potentially scarce resource itself.

Although in the past surplus usually has not been a scarce resource for mutual insurance companies, capital and surplus have been scarce resources for stock insurance companies. Therefore, the pricing methods developed for stock insurance companies have highlighted the returns on capital invested in underwriting new business. As mutual insurance companies find that current earnings cannot easily increase surplus to match both real and inflation-related growth, they may start to use pricing methods similar to those of stock insurance companies.[5] Such a pricing method could be as follows:

In mutual insurance companies with limited surplus, all gains after dividends to policyholders are used to support additional growth. Thus, such mutual companies can price a block of new insurance contracts assuming

[2] *RSA*, I, 928–29; II, 224–29.

[3] *RSA*, I, 890–91; IV, 181; VI, 657.

[4] *RSA*, III, 33–35, 956; VI, 647–48.

[5] *RSA*, III, 915; VI, 101, 309.

that the block requires a commitment of surplus from the company at issue and is stripped of statutory gains (or other releases of surplus commitments) as soon as they emerge. Such gains support the sale of additional new business. In these mutual companies, growth is limited ultimately by the return on surplus from the business written.

Compare this with the pricing method of stock insurance companies. In stock insurance companies, the profits not retained to support growth generally are paid out as dividends to stockholders. Thus, stock companies can price a block of new insurance contracts assuming that the block requires a commitment of capital and surplus from stockholders at issue and pays out all statutory gains (and other releases of surplus and capital commitments) as returns of capital or as dividends to stockholders as soon as the gains emerge. Stock insurance companies hold surplus in addition to capital to absorb experience fluctuations, just as mutual insurance companies do. Relying on surplus is less disruptive than reflecting such fluctuations in stockholder dividends or going unexpectedly to the capital markets. The growth of stock insurance companies is limited in the long run by their ability to write business whose return on surplus and capital is at least equal to the return on capital sought by stockholders.[6]

Surplus and capital are resources that must be used or relied upon to achieve growth of business (or to change the mix of the company's business). This paper assumes that surplus (taken together with capital) is the *primary* constraint on an insurance company's growth.

If the net worth of an insurance company is defined as the sum of surplus plus the value of the existing business in force plus the productive capacity of its agency force,[7] then optimum growth occurs when the insurance company maximizes its annual additions to net worth, subject to the constraint of limited surplus. This is optimum growth for a stock insurance company because in most instances it maximizes the value of the company to its stockholders. This is optimum growth for a mutual insurance company because in most instances it means that the company's service capacity, now and in the future, is as great as possible. That is, if surplus is the primary constraint on service capacity, then growth in net worth, which over time can emerge as surplus, means growth in future service capacity.

If surplus is the primary constraint on an insurance company's growth in net worth, then surplus must be budgeted correctly if growth is to be optimized.

[6] *RSA*, V, 857; VI, 99. However, see also *RSA*, IV, 812.

[7] *RSA*, III, 52; V, 970.

Section II of this paper discusses four technical problems that arise in the efficient allocation of the surplus budget.[8] Section III presents the paper's central theme: an insurance company can optimize growth only if surplus is budgeted in such a way that all actions are expected to yield equivalent profits in proportion to the amount of surplus used as a result of those actions. Subsequent sections give applications of the methods developed in this paper and identify certain practical problems. The final section discusses questions of equity and encourages debate.

The methods of this paper can be applied even if a company currently is not limited by its surplus but expects to be so limited after a large sales effort.

This paper uses statutory surplus (defined to include capital in a stock insurance company) plus the mandatory securities valuation reserve (to simplify the consideration of investments) as the definition of surplus because this is the quantity that is most often found to be a limiting resource.[9]

II. FOUR TECHNICAL PROBLEMS IN THE ALLOCATION OF THE SURPLUS BUDGET

Profit is the contribution to surplus.[10] A risk venture is some opportunity that an insurance company can pursue in the hope of making a profit. For example, the company may develop and issue certain insurance contracts rather than others; it may make certain investments rather than others; it may choose to develop a larger or more professional agency force in various ways; or it may establish computer networks or effect other administrative reorganizations. All of these are risk ventures.

A. *Use of Surplus*

The first technical problem in the allocation of the surplus budget is that different risk ventures use surplus in different ways.

Consider first the expenditure of a specific amount of surplus to undertake a risk venture. For example, in the case of an insurance risk venture, some money actually is spent for commissions, marketing expenses, and issue expenses. These outlays, plus the amount needed to meet statutory reserve

[8] The allocation of surplus here does not mean the allocation of funds that determines the class of participating policyholders to receive distributable surplus. It means simply the assignment of the surplus budget in the planning process. Of course, this assignment and the resulting growth may have an effect on future distributable surplus amounts by class of policyholders.

[9] Rather than statutory surplus, one could consider surplus plus long-term subordinated debt. For one commercial bank's views on including long-term debt in its capital base see Howard [7]. The quantification of any surplus amount is open to question; see *RSA*, VI, 646.

[10] *RSA*, V, 29.

requirements, produce a surplus expenditure. Until the policies' inherent profit emerges years later, that amount of surplus no longer exists on a statutory basis; therefore, it cannot be used to support any other risk venture. It is unlikely, however, that the policies will require any further expenditure of surplus. Other examples of risk ventures that use surplus in this way are the development of an agency force, the development of a computer system, and an administrative reorganization. A reinsurance agreement that causes an expenditure of surplus by the assuming company is also a risk venture of this type. The ceding company can treat the same reinsurance agreement as a risk venture with an initially negative expenditure of surplus (that is, surplus relief) and with a negative profit thereafter (that is, a cost).

How much surplus is "used" in a year by a risk venture that initially requires a surplus expenditure? The amount used is the cumulative amount of surplus committed to the risk venture. In any year, the amount of surplus committed to the risk venture is the excess of all losses over all gains on the risk venture to date. Let "book profits" be the risk venture's expected year-by-year contributions to surplus. For an insurance risk venture, book profits are defined with investment earnings calculated as if invested assets equaled the statutory reserves.[11] Because surplus is limited, any profits are used immediately by some other risk venture. Hence, book profits exclude any investment earnings on the risk venture's own accumulated profits. Let the sequence $B_1, B_2, B_3, \ldots$ represent the book profits for each policy year, discounted to the beginning of the policy year, for some risk venture of this first type. Then in year t, the amount of surplus committed to the risk venture is the outstanding surplus investment:

$$IS_t = -\sum_{s=1}^{t} B_s .$$

For most risk ventures, IS_t is positive for $t = 1$ and declines to zero with increasing t. In the case of surplus relief obtained by the ceding company under a reinsurance agreement, IS_1 is negative, and IS_t increases to zero with increasing t.

The second major way to use surplus is to rely on it as a buffer against various hazards that might cause a loss. Usually we are able to measure the reliance upon surplus in proportion to one of the following parameters: assets, liabilities, premiums in force, or net amount at risk.[12] Therefore, we

[11] For a formula for book profits see the calculation ‚B' on pp. 374–77 of Anderson [2]. Federal income tax effects are included in the calculation of book profits.

[12] *RSA*, III, 30–31, 954; IV, 173, [8], 175–79, 183–84, 194–95, 198, 813, 826. See also Leckie [8], Pike [11], and Trowbridge [13].

sort all hazards into four groups, identified by the parameter in proportion to which the reliance upon surplus is measured. We label the parameter P_k^t, where, in year t,

P_1^t = Amount of assets;

P_2^t = Amount of liabilities;

P_3^t = Amount of premiums in force; and

P_4^t = Net amount at risk.

We define f_k^t to be the ratio of surplus committed to buffer against all hazards in hazard group k in year t, to the parameter P_k^t. Following Trowbridge ([13], p. 216), we measure the amount of surplus used as a buffer in year t as

$$BS_t = \sum_{k=1}^{4} f_k^t P_k^t .$$

Yearly renewable term reinsurance ceded will produce a reduction in the amount of surplus commitment needed to buffer against the mortality risk. Both BS_t and P_k^t are negative for reinsurance ceded.

Buffer surplus corresponds to Donald Cody's corporate solidity surplus (*RSA*, III, 29–32) or to Robert Link's minimum target strategic surplus (*RSA*, III, 957).

Table 1 subjectively classifies some risk ventures according to their principal use of surplus. The table differentiates among the various parameters that measure the use of surplus as a buffer, on the assumption that one parameter suffices to measure each risk venture's reliance on surplus. Of course, specific insurance product designs, reserve requirements, and markets can produce risk ventures with these generic descriptions that should be classified differently.

For risk ventures that rely upon surplus as a buffer, we seek to define a "buffer book profit" that will be comparable to the book profit defined for risk ventures that require an expenditure of surplus. We treat the change in surplus commitment as a charge against buffer book profits at the start of the year. We assume that any policyholder dividend calculation provides only for those federal income tax amounts in excess of the federal income tax on investment earnings on surplus. Thus, we use the after-tax investment earnings on surplus to calculate a rate, i, earned on surplus. Since the book profit for a risk venture that requires surplus to be expended is defined as

if the invested assets equaled the statutory reserves, we define the buffer book profit to include the after-tax investment yield, i, on the assets underlying the surplus commitment that is used to buffer against various hazards. Thus, we define the "buffer book profit" in year t as

$$BBP_t = d(BS_t) - (BS_t - BS_{t-1}) ,$$

where $d = i/(1 + i)$, and $BS_0 = 0$.[13]

TABLE 1

TYPOLOGY OF RISK VENTURES

USE OF SURPLUS	TIME OF PROFIT EMERGENCE	PROFIT VOLATILITY	
		Predictable Risks	Volatile Risks
Expenditure	In a few years	Computerization of administrative tasks	Mass marketing of group term insurance
	Over many years	Individual immediate annuities with strain	Individual disability income insurance (noncancelable)
Buffer (relate to assets)	In a few years	Commercial paper	Common stocks
	Over many years	Mortgages; bonds	Real estate
Buffer (relate to liabilities)	In a few years	Automobile liability insurance	Group long-term disability insurance; medical malpractice insurance
	Over many years	None	Pharmaceutical product liability insurance
Buffer (relate to premiums in force)	In a few years	Homeowner property insurance	Group term health insurance; aviation reinsurance; commercial property insurance
	Over many years	None	None
Buffer (relate to net amount at risk)	In a few years	Individual term life insurance	Group accidental death insurance
	Over many years	None	Individual revertible term life insurance

[13] *RSA*, IV, 815, 817; VI, 96–97. We might want to redefine d as $i/(1 + j)$ once j has been defined below.

Actually, individual permanent life insurance and most other insurance risk ventures involve both the expenditure of surplus and the subsequent reliance on surplus as a buffer. We define the "augmented book profit" for a general risk venture to be

$$ABP_t = d(BS_t) - (BS_t - BS_{t-1}) + B_t .$$

The total amount of surplus committed to a risk venture in year t is

$$TS_t = -\sum_{s=1}^{t} ABP_s = -d\sum_{s=1}^{t} BS_s + BS_t + IS_t .$$

We follow Trowbridge [13] in notation, but other formulations are possible; see Lee ([9], pp. 528–29).

Reinsurance can be considered an integral part of an insurance risk venture if the reinsurance is included in the original pricing work. In this case, the augmented book profit for the risk venture is net of the reinsurance effects. Alternatively, reinsurance can be considered a separate risk venture to be added to (or rather, "subtracted" from) the existing portfolio of risk ventures. We will use the term "surplus relief" in this case to refer to the combination of the negative invested surplus and buffer surplus.

B. *Time of Profit Emergence*

The second technical problem in the allocation of the surplus budget is that profit emerges over different periods of time. The profit from an individual permanent life insurance policy emerges over an individual human lifetime (or longer for settlement options). The profit from a group term life insurance contract emerges over a few years. The profit from a property insurance contract may emerge within a year. Table 1 subjectively classifies some risk ventures as to their position relative to the extremes of the time spectrum.

Before the surplus budget can be allocated, the relative value to the company of these profits at different times must be compared. A standard discount rate must be agreed upon. If a low rate is agreed upon, the individual permanent life insurance policy may appear to be the most profitable. At a higher rate, the group term life insurance contract may be deemed the most profitable. At an even higher rate, contracts with the least initial investment and earliest profit will be deemed most profitable. Since the

evaluation of relative profitability can be sensitive to the discount rate, it is important to pick a rate, j, that represents the time value of surplus to the company. There are several rates to consider: the investment income rate, the internal rate of return[14] available on the risk ventures that the company is choosing among, the average internal rate of return inherent in the company's business in force, and the desired rate of surplus growth.

If the company were not constrained by surplus, the time value of surplus would be the investment income rate earned on assets, less a full charge for taxes, since there are no offsetting requirements for policyholder interest. However, we assume that the company is constrained by its surplus, so such a net investment income rate probably is too low to represent the time value of surplus.

Finance textbooks suggest using the marginal cost of capital as the time value of money.[15] If the marginal cost of capital for a stock insurance company is less than the internal rate of return available in risk ventures not yet assumed, then the stock insurance company should raise more capital and assume the more profitable risk ventures. This process will either increase the marginal cost of additional capital, or decrease the internal rate of return available from the risk ventures that are not yet assumed, or both. The process stops when the marginal cost of capital equals the internal rate of return for the most profitable risk venture that remains unassumed. This is a special case of a process discussed in Section III. Through this process, the marginal cost of capital becomes the rate for stock insurance companies to use as the time value of surplus. Any risk venture whose present value of augmented book profits is negative when discounted at the marginal cost of capital is not sufficiently profitable to be a good use of capital and surplus.

Unless a mutual insurance company issues subordinated debt, it may not be able to define a marginal cost of capital for itself. Mutual insurance companies limited by surplus cannot undertake all risk ventures that have a positive net present value (discounted at the marginal cost of capital for similar stock insurance companies). Thus, the minimum time value of surplus for a mutual insurance company should be set at least equal to the internal rate of return implicit in the least profitable risk venture that will become available in the future and that the company is willing to assume. This is an internal rate of return on the margin rather than on average. The profitability of risk ventures that the company is willing to assume is dis-

[14] The internal rate of return is defined as the interest rate that, when used to discount augmented book profits, makes their present value at issue equal to zero. For example, see Weston and Brigham ([14], p. 268).

[15] Ibid., p. 275.

cussed in Section III. We propose the internal rate of return on the margin as the appropriate time value of surplus because it will be the most appropriate discriminator between those risk ventures at the margin among which the company must choose.

To give some feel for the results of using an internal rate of return as the time value of surplus, we digress to discuss the properties of an *average* internal rate of return, rather than a *marginal* internal rate of return. The average internal rate of return may be greater than the marginal internal rate of return because of market saturation, or it may be less than the marginal internal rate of return because of fixed expenses not charged on the margin. The main property of an average internal rate of return is that a company can grow at that rate indefinitely. Take a new line of business in which each unit of new business generates a sequence of augmented book profits whose net present value, discounted at the rate j, is zero. The sales of that product can grow at rate j indefinitely on a self-supporting basis after the line of business reaches surplus equilibrium at the end of the lifetime of the contracts issued in the first year. That is, after surplus commitments are made for all contracts issued during the lifetime of the first year's issues, then the surplus commitment required by any contracts issued later is provided by the release of surplus from the in-force business. The surplus committed to the line of business in the lifetime of the first year's issues is not released by the line of business (unless the growth slows below rate j), but the line of business requires no additional surplus commitment from the company. This finite surplus commitment to a growing business is achieved because the internal rate of return equals the growth rate of surplus needed to support the business. A company can grow at rate j indefinitely if the average internal rate of return implicit in its in-force and new business is j.

A company sometimes sets its desired rate of growth (and surplus growth) in relation to the growth rate of the national economy or of the insurance industry. However, a company should not use such a desired rate of surplus growth as the time value of surplus if its desired rate of growth has been set without regard to the internal rates of return available in risk ventures in the marketplace. If an unrealistically high desired rate of growth is used as the time value of surplus, the "reinvestment" rate implicit in comparisons of risk ventures with different times of profit emergence will not be meaningful, and the profitability comparisons will be misleading. We conclude that available internal rates of return set the limits on the choice of a meaningful time value of surplus to be used to compare profitability.

How do we compare profitability? The most surplus-efficient risk ventures for a company to assume are those with the highest ratio of net profits to

surplus commitment used. We propose the following ratio to compare the attractiveness of alternative risk ventures:

$$R = \frac{\sum_{t=1}^{\infty} ABP_t(1 + j)^{1-t}}{\sum_{t=1}^{M} -ABP_t(1 + j)^{1-t}},$$

where we define M to be the earliest duration at which the denominator (if positive) reaches a maximum, or at which the denominator (if negative) reaches a minimum. That is, in the positive case,

$$\sum_{s=1}^{M} -ABP_s(1 + j)^{1-s} \geq \sum_{s=1}^{t} -ABP_s(1 + j)^{1-s} \quad \text{for all } t \neq M .$$

The denominator is the initial surplus commitment, TS_1, plus the present value of any increases in TS_t. The numerator is just the present value of augmented book profits. Thus, a risk venture with a higher ratio R is more profitable in relation to its use of the limited resource, surplus, than is a second risk venture with a lower ratio R.

If all acceptable risk ventures have a common internal rate of return, j, then they are all equally profitable, and the present value of augmented book profits will be zero. A company could use the internal rate of return, rather than R, as the basis for profitability comparisons in most cases. There are cases, however, where profit comparisons cannot be based on the internal rate of return.[16] This paper will use ratio R for profit comparisons, although the methods presented here can be adapted to the use of the internal rate of return as a profit standard.

C. *Profit Volatility*

The third technical problem in the allocation of the surplus budget is the different profit volatilities inherent in the different risk ventures. Table 1 subjectively classifies risk ventures as to position within the volatility spectrum. The profitability demanded of a volatile risk venture should be greater

[16] In general, there are problems with using internal rates of return to set standards. There can be mutually exclusive risk ventures. Because there can be more than one solution to a polynomial equation, there can be more than one internal rate of return that arises from a stream of augmented book profits. This can often occur when a new product will be reinsured and the two ventures are considered as one. Internal rates of return (especially on marginal augmented book profits) can be very high for small risk ventures that cannot be replaced at the same rate. (See [14], pp. 272, 275, and Appendix A.)

than the profitability of a more predictable risk venture. There are several different ways to define the greater profitability demanded of a more volatile risk venture.

When the surplus investment IS_t is negligible, the simplest way to define the greater profitability demanded of a more volatile risk venture is to use the ratio R defined in the preceding section, with the term BS_t reflecting the level of volatility. One way to measure the surplus relied on is to determine, on the basis of ruin theory, how seriously the company's surplus can be impaired by the worst loss expected in a very long period (such as a century). Risk ventures using more buffer surplus will tend to have a lower ratio R and be less attractive.

A second way to define the greater profitability demanded of a more volatile risk venture is most useful if there will be an initial expenditure of surplus and there is no chance that more expenditures will be required. The buffer surplus BS_t is negligible. If q is the probability that the profit sequence ABP_t is interrupted at any duration, and if we define

$$\Delta j = (1 + j)q/(1 - q) ,$$

then the expected present value of augmented book profits is

$$\sum_{t=1}^{\infty} (1 - q)^{t-1} ABP_t (1 + j)^{1-t} = \sum_{t=1}^{\infty} ABP_t (1 + j + \Delta j)^{1-t} .$$

In this case, it is convenient if the rate used to discount ABP_t in the numerator of the ratio R is $j + \Delta j$, where the increment Δj recognizes the risk of not recovering the surplus already spent. The lower rate j is still used in the denominator to discount the sequence of changes in surplus commitments, $-ABP_t$. Because of the compounding of the discount rate, this method discounts more heavily those profits that lie farther in the future and seem, therefore, less certain. The increment Δj equals zero for any risk venture whose future profits seem certain.

A third way to define the greater profitability demanded of a more volatile risk venture is a combination of the first two ways. The buffer surplus term BS_t reflects the risk that additional surplus may be required, and the increment Δj to j in the numerator reflects the risk that the surplus investment IS_1 may not be fully recovered and that the buffer surplus will not be released as soon (or may never be released if an experience fluctuation wipes it out).

A fourth way to define the greater profitability demanded of a more volatile risk venture involves the calculation of the numerator of the ratio R. It is useful whether or not additional surplus may be at risk. A probabilistic

model (using deterministic or Monte Carlo methods) is used.[17] Multiple possible profit outcomes are considered. In the deterministic model, each possible outcome is assigned a relative weight for probability of occurrence. In either the deterministic model or the Monte Carlo model, each profit outcome is assigned a relative weight for the utility of the outcome. These weights will differ from unity if utility does not vary linearly with dollars. For example, a conservative company (or a small company with a low surplus) assigns poor outcomes greater relative weights, since poor outcomes are of major concern. This method directly considers the size of a potential loss as well as the frequency of losses. It is particularly valuable when a new insurance product will require large administrative expenses in the future whether or not the new product sells well, such as, for example, individual variable annuities where a company sets up its own separate account requiring a maintenance expense for an indefinite period.

A fifth way to define the greater profitability demanded is a variation on the Monte Carlo model. A volatile risk venture is assumed only if R is greater than a given value R^{high} with a certain frequency (in the universe of simulated outcomes). Further, a risk venture is not assumed if the ratio R is less than a given value R^{low} with a certain frequency.

D. *Marginal Profitability and Use of Surplus*

The fourth problem in the allocation of the surplus budget is to recognize *marginal* profitability and use of surplus as distinct from *average* profitability and use of surplus.

Marginal calculations help a company make decisions on the best course of action or on the best price at which to offer insurance. An insurance company may face at different times as many as four distinct types of decisions to make concerning risk ventures:

1. *Buy/introduce*. The insurance company must decide whether it wants to
   - *a*) Buy a certain investment risk venture or buy some other investment risk venture with less risk;
   - *b*) Go ahead with a computer, administrative, or agency-development risk venture or find an alternative that costs less;
   - *c*) Assume a block of reinsurance business;
   - *d*) Introduce a new insurance product or even enter a wholly new insurance line of business;
   - *e*) Buy back some of its outstanding stock or otherwise reduce its capital and surplus.
2. *Pricing new insurance sales*. The insurance company must decide what price level to set for new insurance risk ventures to be sold in the following year (or years).

[17] *RSA*, III, 222–23, 231; V, 34–41. See also Beekman and Fuelling [3].

By *price* level we mean the premium rate scale net of the policyholder dividend scale, experience-refund scale, rate credit scale, or scale of excess interest declared in advance.

3. *Repricing in-force insurance.* In certain situations, the insurance company can change the price level on insurance business in force without regard to the price level on new insurance being sold because no comparable product is currently being sold and because the price level was never guaranteed on the insurance business in force. In these situations, the insurance company must decide whether to change the price level and, if so, to what new price level. The insurance business in force in some markets responds more slowly to a change in price level than does the new business being sold. Therefore, the change in profit caused by a change in price level on in-force business differs from the change in profit caused by a change in price level for new business.
4. *Sell/cede/discontinue.* The insurance company must decide whether it wants to
   a) Sell an investment risk venture;
   b) Cancel some computer, administrative, or agency-development risk ventures;
   c) Cede an in-force block of insurance business to a reinsurer;
   d) Discontinue offering a particular insurance product, leave a particular insurance product line, or cancel certain portions of its business in force that can be canceled;
   e) Raise additional capital and surplus by an offering of stock and subordinated debentures.

   All of the risk ventures in the "sell/cede/discontinue" decision group may release surplus rather than commit surplus.

No matter which type of decision it faces, the insurance company wants to know what profit level and what degree of surplus commitment are implied by each of the alternative solutions. Therefore, the net present value of augmented book profits defined above, the present value of the sequence of changes in surplus commitments defined above, their ratio R defined above, and the internal rates of return mentioned above should all be understood to be calculated using marginal book profits, B_t, and marginal buffer surplus, BS_t.

How do we calculate marginal book profits? For a buy/introduce decision, we calculate marginal book profits as the difference between the book profits if the company decides yes, to go ahead, and the book profits if the company decides no, not to go ahead, with the risk venture that requires a surplus commitment. In certain situations, the "no" decision means that the company plans to go ahead with a close alternative that requires little or no surplus commitment.

For example, an investment in real estate can be considered in comparison with a standard, say an alternative investment in mortgages. The difference between initial investments, the year-by-year excess of net rental income

over mortgage income, and the excess of real estate sales value over return of principal under the mortgage, all create the sequence of marginal book profits for an investment in real estate.

The marginal book profits of a new computer system are made up of the initial developmental expense (a loss) and then the excess of the expense of processing under the old system over the expense of processing under the new system (a series of gains).

An agency development risk venture is best compared with an alternative sales-development strategy that requires less surplus commitment. If different sales-development strategies will result in different products being sold that will, in turn, produce different amounts of profit in comparison with the surplus commitment required, then the marginal book profits of an agency development risk venture are best calculated as the difference in book profits between the types and volumes of products sold, less the difference between the expenses of agency development and the expenses of alternative sales development.

The marginal book profits of a block of reinsurance assumed are obtained by comparing the initial surplus expenditure (a loss) and the later book profits (gains) with the earnings situation assuming no reinsurance. The tax effects of these two alternatives sometimes differ and should be included in the calculation of marginal book profits.

The marginal book profits from a new product line involve a comparison with results assuming the new product line had not been developed. The calculation uses the best estimates for sales volume, premium income at the planned price level, claim costs, interest rates, and persistency. The assumption in the calculation of marginal book profits that differs the most from the corresponding assumption in the calculation of average book profits is the expense assumption. Certain expenses vary with volume: commissions and premium taxes with premium income, and issue expenses with number of contracts or with size of risk underwritten. These are *variable* expenses. Certain other expenses directly related to the product line recur periodically but do not vary with volume. These are *direct-periodic* expenses. Certain other expenses directly related to the introduction of the product line do not recur. These are *direct-fixed* expenses. Variable, direct-periodic, and direct-fixed expenses are all charged in the calculation of marginal book profits for a new product line. When a comparison is made with the alternative of additional sales in existing product lines, the new product line is charged for any direct-periodic or direct-fixed expenses (as well as variable expenses) it will cause, but additional sales in existing product lines are not charged for their direct-periodic expenses because the company is already incurring those expenses on existing sales. There is,

however, the question of how the additional sales in the existing line are to be achieved. Is the price for the existing product line reduced, or do we consider only sales in the existing product line that would be replaced by sales in the new product line because the agency force can sell only so much business in a year? Finally, certain other expenses are not directly related to any product or product line and are not charged against any marginal book profits. These are *indirect-periodic* expenses.[18]

It is not always easy for a given company to decide which expenses are periodic and which are variable. A cross-sectional study today would suggest that most expenses are variable over the long run; large companies have large expenses, somewhat in proportion to their business in force; small companies have smaller expenses. However, some companies, for the intermediate term, cannot afford to view their expenses as variable; even the variable expense of commissions is judged periodic for a company with a large branch manager and career agency system.[19]

The marginal book profits for a new product line are in a sense equal to the difference between the two *average* book profit figures that are expected if the company either does or does not introduce the product line at a particular price level. Once the decision to introduce the product line is made, there is a second marginal book profit calculation made to determine the optimum price level. We discuss below the marginal book profit calculated using a change-of-price comparison.

The marginal book profits from buying back stock are composed of the initial cost (a loss) and the future dividend payments saved (a series of gains). If a company does not feel constrained by surplus, it probably should buy back stock, to improve the earnings per share of stock outstanding, until it does feel reasonably constrained by surplus.[20]

Once an insurance company has made the decision to introduce an insurance risk venture, and until it has made the decision to discontinue selling such a venture, it must decide on the optimum price and commission level. The considerations in setting a commission scale are similar to, and interrelated with, the considerations in setting a price level. We shall speak only of setting a price level, to simplify the discussion of the marginal book profits for a decision on pricing new insurance sales.

Consider a particular price level and the resulting volume of new business that will be sold in a year at that price level. If the company decreases the price on the insurance risk venture, then, usually, the sales volume increases. All units sold during the year, not just the additional units, get the

[18] *RSA*, III, 32, 38; IV, 206–8; VI, 98–100, and 645. See also LOMA [10].

[19] *RSA*, IV, 208.

[20] *RSA*, IV, 825.

reduced price. The price reduction may also extend to some in-force business. We calculate the marginal book profit for an insurance risk venture at a particular price level as the total net change in book profit that would result from a price reduction. Marginal book profit from additional units sold because of a price reduction is decreased by the change in book profit corresponding to the price reduction on all the units that would have been sold at the original price and on any affected units in force.[21] The change in book profit corresponding to a price reduction on units that would have been sold or on units in force should reflect any persistency improvement caused by the price reduction.

It is advantageous for the insurance company to recognize and use natural market differentiation so that it can change prices only in the specific segments of the market that have the greatest (net) marginal augmented book profits for a given amount of additional surplus commitment. Such natural market differentiation may be based on age, sex, occupation, income, geographic area, marketing method, tax qualification status, size of case, or client sophistication.

Sometimes an insurance company expands its sales by promotion and underwriting changes rather than by price reductions. This avoids the loss of incremental premium on the business that would have been sold at the original price, but the marginal book profits calculated must recognize the expenses of additional promotion or the poorer claim cost and persistency experience on the additional sales if the underwriting standards are relaxed.

For a decision on repricing in-force business when this is distinct from pricing new business, the marginal book profits at a particular price level are equal to the changes in premium income (a loss) less the changes in premium-related variable expenses caused by a slight reduction in price level, plus the marginal book profits on contracts that would have lapsed but now do not because of the slight change in price level.

For a sell/cede/discontinue decision the company calculates marginal book profits in much the same way as it would calculate marginal book profits for a buy/introduce decision. The decision is based on the facts and options available at the time the decision is made. One point worth mentioning relates to the decision whether to discontinue sales in an insurance product line. If the insurance company is not able to cancel existing business in force, direct-periodic expenses should not be charged against the marginal book profits calculated for the choice of continuing to write new business, because such expenses will not be saved if the company chooses to stop writing new business. The sign convention assumed in this paper for marginal book profits for a sell/cede/discontinue decision is that the marginal

[21] Bragg [4] gives some estimates for life insurance.

book profits are positive if the decision to sell, cede, or discontinue creates additional profit or reduces losses.

How do we calculate marginal buffer surplus? The comparisons should be made on the same bases as are used to calculate marginal book profits; that is, the same alternatives are compared, the same amount of price reductions from the same original price level is assumed, and the same commission changes are compared.

For a buy/introduce decision, we calculate the marginal buffer surplus required as the difference between the buffer surplus required if the company decides to go ahead, and the buffer surplus required if the company decides not to go ahead with the risk venture that requires a surplus commitment.

The calculation of the marginal factor $f_k = \Delta BS/\Delta P_k$ requires judgment. We consider not only the volatility of the new risk venture itself but also the volatility of the other risk ventures already assumed by the company.[22] For example, some insurance product lines are less profitable if unemployment rises. If the company insures several of these lines and is considering entering another, the marginal-buffer-surplus factor is at a high level similar to the average-buffer-surplus factor. Similarly, some types of assets and some insurance product lines are adversely impacted by inflation. If the company already is exposed to many such risks, any additional risk ventures that are adversely impacted by inflation have a high marginal-buffer-surplus factor.

With the exception of reinsurance ceded, it is fairly difficult to find risk ventures that contribute to surplus just when other risk ventures are incurring losses that must be paid out of surplus. Sometimes, income-paying annuities and life insurance are such opposites with regard to the mortality risk. If a company can identify a new risk venture that reacts to changes in the national economy in a direction that is opposite to the direction in which all other risk ventures of the company react, then the new risk venture's marginal buffer surplus can be considered to be less than or equal to zero, because it can be considered in combination with other risk ventures.[23] The company may want to assume this risk venture because it releases buffer surplus to allow additional growth in other product lines.

For the pricing-new-insurance-sales decision, we examine the volatility of book profits on individual additional new sales in comparison with the volatility of the in-force business in that same product line and of similarly varying risk ventures, as we do in the buy/introduce decision. For example,

[22] *RSA*, I, 899, 904; III, 30, 34, 38; V, 41; and Pike ([11], p. 260).

[23] For discussion of a related idea outside the insurance industry see Weston and Brigham ([14], pp. 321–23).

let us contrast additional group life sales with additional group health sales. Suppose an insurance company has a large amount of group life insurance. Since the claim costs on an incremental piece of group life business are largely independent (based on probability theory) of the average group life claim costs, and since there already is a significant buffer surplus committed to the existing group life business, the *marginal* buffer surplus per dollar of new group life premium may be less than the *average* buffer surplus per dollar of existing group life premium. Percentage deviations from the mean for the whole line of business are smaller because many independent units are exposed (central limit theorem). On the other hand, suppose the company has a large amount of group health insurance. The variation in claim costs on an incremental piece of group health business depends, in part, on health care inflation, as does the variation in claim costs on existing group health business. The *marginal* buffer surplus factor for group health may be no less than the *average* buffer surplus factor for group health. The company does not derive much benefit from insuring a large number of people, because the deviations of individual claim costs from expected claim costs are not independent.

After we have determined the buffer surplus committed to an additional individual unit sold at a particular price level, we calculate marginal buffer surplus as the change in total buffer surplus that results from a reduction in price level (or from an increase in commissions, if that is the basis for change assumed in the calculation of marginal book profits). The marginal buffer surplus committed to the additional units sold at the lower price is added to any increase in buffer surplus due to the price reduction, not only with respect to all the units that would have been sold at the original price, but also with respect to all the units already in force (if the price level on in-force business moves with the price level of new business).

For the repricing-in-force-insurance decision, when this is distinct from pricing new business, the marginal buffer surplus at a particular price level is the change in buffer surplus needed by the in-force business because of a slight reduction in price level, plus the marginal buffer surplus on contracts that would have lapsed but now do not lapse, because of the slight reduction in price level.

Even if a risk venture's use of buffer surplus is measured in proportion to premium income in force, we must recognize that buffer surplus increases if there is a reduction in premium (price level). Marginal f_3 is unrelated to average f_3.

For a sell/cede/discontinue decision the calculation of marginal buffer surplus follows the principles outlined above. The sign convention for mar-

ginal buffer surplus committed is that when a decision is made to sell, cede, or discontinue, the marginal buffer surplus committed is negative, because surplus is released.

III. STRATEGY FOR OPTIMUM GROWTH GIVEN A SURPLUS CONSTRAINT

A. *Basic Microeconomic Principle*

A basic principle of economics is that a company maximizes the production of a good, subject to a single limited resource, when marginal production divided by marginal use of the limited resource is equal across the various ways of producing the good. No reallocation of the limited resource from one way to another way of producing the good will increase the total output of the good.[24]

B. *Strategy of Equal Marginal Results*

We have defined optimum growth as a maximization of annual additions to net worth, subject to the constraint of limited surplus. To obtain optimum growth, a company with limited surplus can adopt the following strategy. First, the company estimates the surplus that will become available from augmented book profits on business in force, year by year, over the next several years. Since the company is looking for just the net amount of surplus becoming available, these amounts are reduced by overhead expenses and other expenses that are charged ultimately against none of the marginal book profits on risk ventures that the company will assume over the next several years. Next, the company estimates the surplus that would be committed, year by year, for each of the risk ventures expected to be available in the next several years. For insurance risk ventures, the surplus commitment will depend on the price level. The company then examines, for each risk venture, the marginal ratio R, defined above to be the ratio of the present value of marginal augmented book profits to the marginal surplus commitment required for the risk venture. Finally, the company goes through a three-step iterative process.

To start the iterative process, the company tentatively plans to assume only those most profitable risk ventures (and to cede or discontinue those least profitable risk ventures) such that, in aggregate, exactly the amount of surplus becoming available in the year (or over several years) is committed. Those risk ventures with ratio R greater than or equal to a threshold value R^1 are tentatively accepted, and those with ratio R less than R^1 are discontinued or tentatively ceded. We define R^1 to be the value of R such that the summation of the first-year surplus commitments over all risk ven-

[24] Henderson and Quandt ([6], pp. 64–65).

tures (other than reinsurance ceded and other risk ventures in the sell/cede/discontinue decision group) with marginal ratios R greater than or equal to R^1, less the summation of the first-year surplus relief over all reinsurance-ceded risk ventures (or surplus released on other risk ventures in the sell/cede/discontinue decision group) with R less than or equal to R^1, is equal to the surplus that will become available to support growth in the next one, or several, years. (If many risk ventures require increasing surplus commitments in later years, the company must check that the surplus that will become available in the next several years can cover these additional demands from the risk ventures assumed in the first year.) George Dinney has called a process of selecting the most profitable risk ventures first a "decision-tree process."[25]

The second step in the iterative process is to reprice all *insurance* risk ventures so that, on the margin, their ratios R will equal R^1. We will discuss what this means in the various types of decision situations.

For an insurance risk venture requiring a buy/introduce decision there are two cases; either the initial ratio R is equal to or greater than R^1, or the initial ratio R is less than R^1. If the risk venture's ratio R (calculated using a yes/no comparison) is equal to or greater than R^1, then the company calculates the risk venture's marginal ratio R using a change-of-price comparison. The company changes the price level so that the marginal ratio R (calculated using a change-of-price comparison) equals R^1. The ratio R (using a yes/no comparison) will still exceed R^1, either because additional units were added at a marginal profit greater than R^1 if the price level was reduced, or because units were removed at a loss of marginal profit less than R^1 if the price level had to be increased. It is not efficient to reduce the ratio R calculated using a yes/no comparison to the R^1 level, because the additional units sold do not produce enough profit to justify the additional surplus commitment. The ratio R calculated on a yes/no basis is really the comparison of two *average* profits, rather than a true *marginal* ratio. If R is less than R^1, the company must examine different strategies for the risk venture to find a strategy (perhaps involving price level, commissions, marketing focus, product design, etc.) that produces a ratio R (calculated on a yes/no basis) greater than or equal to R^1. If such a strategy is found, then this second case has been reduced to the first case. The company alters the price level so that the marginal ratio R (calculated using a change-of-price basis) equals R^1.

For an insurance risk venture requiring a pricing-new-insurance-sales decision, the price level generally should be reduced if the initial ratio R

[25] *RSA*, II, 227. Staley [12] suggests a similar maximization without the surplus constraint.

(calculated using a change-of-price comparison) is greater than R^1. Alternatively, if the initial ratio R is less than R^1, the price level should be increased so that there are fewer sales and more profit on each sale. When R reaches R^1, the next increase in price would have eliminated an acceptable profit in comparison with the surplus used, so the price is fixed at that level.

For an insurance risk venture requiring a repricing-in-force-insurance decision, there should be a price level with maximum total profit; at a higher price level the lapses erode the total profitability, and at a lower price level the reduced premium margin erodes the total profitability. This price level may not, however, be the most efficient use of surplus. At a high price level, the marginal surplus commitment is negligible, perhaps only equal to the surplus commitment required incrementally by the new units that will not lapse if there is a slight price reduction. Because the marginal profitability is high for a slight price reduction, the ratio R at a high price level is greater than R^1. As the price level declines, the marginal profitability drops through zero at the point of maximum total profitability. In the meantime, as the price level drops, the marginal surplus commitment increases. At some price level, the ratio R will equal R^1, and this is the price level to use.

For an insurance risk venture requiring a discontinue decision, there are two cases; either the initial ratio is less than R^1, or the initial ratio is greater than R^1. If the initial ratio R (using a yes/no comparison) is less than R^1, this indicates that the company should discontinue the product line if no better result can be obtained by changing such things as product design, market strategy, price level, commission scale, and so forth. The average profit lost is not very great in comparison with the surplus that is released.[26] If the company does manage to find a new strategy that produces a ratio R (calculated using a yes/no comparison) greater than R^1—this is the second case—then the company should not discontinue the product line.

Because the prices of insurance risk ventures are set in the second step of the iterative process using, among other things, marginal expenses, we digress here to discuss pricing on a marginal basis. Expense charges on a marginal basis are made for comparison purposes only. It is assumed that

[26] The company does not simply cut commissions or increase premiums on a relatively unprofitable product without an analysis of whether this improves the ratio R.

If an insurance risk venture sells easily but uses too much surplus, it is not efficient simply to limit insurance sales by setting a quota. To impose a surplus budget, it is more efficient to raise the premium charged for the insurance. This will reduce use of surplus, increase profits, and reduce sales. For the same number of sales allowed under a quota (or more), a higher profit will be earned. See also *RSA*, V, 44; VI, 315.

The marketing strategy dictates a price level in relation to the competition. See *RSA*, IV, 206–7. The marketing strategy may be based on current market share and desired market share, which in turn is related to ratio R and to growth prospects for that insurance market.

the marginal book profits are sufficient, in total, to cover expenses that are not charged on a marginal basis. If the overhead expenses and other expenses not charged on a marginal basis are greater than the change in net worth in a year on a marginal basis after we have finished optimizing growth, then the company is in trouble. The trouble does not arise from pricing on a marginal basis; rather, the company must find a way to cut the overhead (or other) expenses.

Once all insurance products are repriced or redesigned so that their ratios R (calculated on a price-change basis) equal R^1, the surplus sought by all risk ventures with R greater than or equal to R^1 will be greater than the surplus available (including the surplus released by risk ventures in the sell/cede/discontinue decision group whose ratios R are less than or equal to R^1). This leads us to the third step in the iterative process.

The third step is to increase the price levels on all insurance risk ventures until the following two conditions are achieved:

1. All *insurance* risk ventures have on the margin their ratios R (calculated using a change-of-price comparison) equal to a new threshold value, R^2, higher than R^1.
2. The summation of the first-year surplus commitment over *all* risk ventures (excluding those from the sell/cede/discontinue decision group) whose ratios R (calculated using a yes/no comparison) are greater than or equal to the new R^2, less the summation of the first-year surplus released over all risk ventures from the sell/cede/discontinue decision group whose ratios R (calculated using a yes/no comparison) are less than or equal to R^2, is equal to the surplus that will become available. The company does assume those risk ventures that enter the first summation and sells, cedes, or discontinues those risk ventures that enter the second summation.

At this stage, the company has completed the iterative process. No reallocation of the surplus budget will increase the change in net worth from the year's business. The company does not assume noninsurance risk ventures that are less profitable, in comparison with the required surplus commitment, than the threshold value R^2. The company does not sell product lines if the difference between selling and not selling is less profitable, in comparison with the required surplus commitment, than the threshold value R^2.

Figure 1 is a schematic representation of the new risk ventures available on the margin to a company with limited surplus. If the company has set the price on new insurance risk ventures efficiently and has chosen new risk ventures efficiently, the risk ventures assumed will all have a ratio R equal to or greater than R^2. The incremental risk ventures on which the company has price discretion will all lie on a straight line that passes through the origin and has a slope equal to the common ratio R^2. If risk venture A has

a more attractive ratio R, it will lie to the left of, and above, the line in Figure 1. The company should assume risk venture A and bid aggressively for similar risk ventures. If the company has price control and cannot assume other risk ventures similar to A without assuming A at a lower profit, then the company should lower the price (and profit) to assume more of these risk ventures until, on the margin, $R = R^2$.

Figure 1 illustrates why the insurance company seeks a low ratio R on reinsurance ceded (and other risk ventures from the sell/cede/discontinue decision group) but a high ratio R on all risk ventures assumed. Both rules describe the region to the left of, and above, the line in Figure 1.

If risk venture B has a less attractive ratio R, it will lie to the right of, and below, the line in Figure 1. The company should decline risk venture B (unless it is an insurance risk venture that the company could redesign or reprice). It is easier to decline a risk venture that is a possible investment, an administrative reorganization, or a reinsurance proposal. It may be more difficult to decide to leave an insurance product line if this decision leaves a gap in the insurance company's product portfolio. If it is impossible to

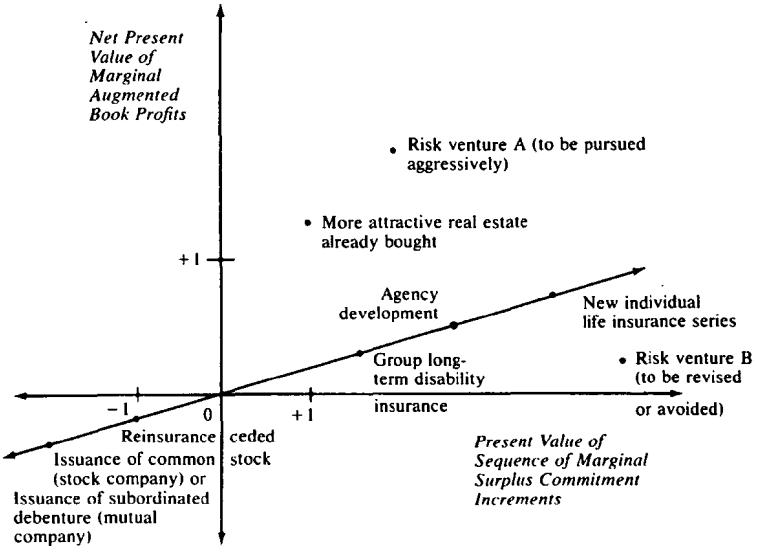

FIG. 1.—Analysis of potential risk ventures, arbitrary scale. Relative placement on line depends on size of incremental change of surplus commitments and of incremental change of augmented book profits. Slope of line equals lowest acceptable ratio R^2. All risk ventures above and to the left of the line should be assumed in preference to those below and to the right of the line.

get risk venture B's ratio up to R^2, and if failure to keep selling or failure to introduce the product line is of concern to the insurance company, then the company can quantify what value product line B has for the other lines of business. This value may be added to the marginal augmented book profits of product line B and deducted from the marginal augmented book profits of the lines helped by the maintenance or introduction of product line B.[27] The marginal profits here are those calculated on the basis of a yes/no comparison.

For some insurance companies, the marketing force is as much the limiting resource as is the surplus. One goal, therefore, of a career agency company is to help new agents survive their first few years. Certain simple insurance products may help because they are more easily sold by new agents than are insurance products directed at sophisticated markets. If these simple products help new agents survive, then the calculation of profitability for the simple products should include a recognition that these products contribute more than near-term statutory profits—they help develop the agency force.[28] Expense allocations that allocate the heavy expenses incurred on financing new agents to the products that new agents sell should be reconsidered in this regard.

C. *Miscellaneous Comments on Strategy*

Theoretically, the company need only undercut the competition slightly in extremely price-sensitive markets to capture the whole market. At that point in pricing, the marginal ratio R is discontinuous. The next reduction in price produces a reduction in profit but no additional sales. The ratio R goes from a value above R^2 to a value below R^2. In such a price-sensitive market, the company should anticipate that the competition will respond by cutting its price to recapture the business. The company may consider undercutting the competition immediately to the ultimate level to which it would be willing to go in a price war. This ultimate level is the lowest price level at which the competition's prices could be set and at which the insurance company's marginal ratio R would then be equal to R^2. If the competition is willing to go below that price, then the insurance company drops out of the competition without having wasted a lot of money, time,

[27] These additions and deductions are for strategic planning, not for statutory accounting.

Other theoretical benefits of a particular corporate policy or strategy may be difficult to quantify in practice. Thus, the growth optimization may be done subject to management-specified constraints. For example, management may believe the company should always maintain a large presence in the ordinary life insurance business. However, to optimize growth rationally, it is best to quantify wherever possible. See *RSA*, III, 223.

[28] *RSA*, III, 45.

and effort capturing the business, losing it, recapturing it, and then losing it again in a progressively less profitable price war. This analysis assumes that the competition *will* react aggressively and assumes that the insurance company's agents will not lose interest in such an aggressively low-priced insurance product (on which their commission income is proportionately low).

The concept of ratio R on a marginal basis may help give a more reliable foundation to two practices: balanced growth and the segregation of surplus as a corporate line.[29]

1. Balanced growth is optimal when the ratio R is equal, on the margin, in all the insurance lines of business.
2. For strategic planning purposes and for management reporting purposes, the investment income after taxes earned on assets corresponding to surplus is allocated not to the insurance lines of business that generated the surplus but rather to the lines of business that require the surplus to be held as a buffer, unshielded from taxes. For the insurance product line that generated the surplus, such investment income affects only profits on a statutory basis (that is, average profits), not marginal profits.

IV. TWO APPLICATIONS OF THE METHODS

There are several strategic questions that can be answered by using the methods described in this paper. Donald Cody has listed elsewhere eight matters requiring corporate decision when surplus is limited.[30] Two of these concern the value of conservative reserve bases. A third concerns the holding of common stock. This section illustrates the application of the first step of the iterative process described in Section III to these matters. The overall repricing effort in the second and third steps of the iterative process will not change the conclusion in the first example but might allow some revision to the conclusion in the second example.

A. *Conservatism in Statutory Reserves*

The actuary reports an opinion on whether insurance reserves held in the annual statement make adequate provision for future contingencies. If the actuary (or a statutory valuation standard) errs, it is better to err on the side of a reserve more conservative than necessary. However, this conservative bias reduces surplus. How does the marginal net present value of augmented book profits from this use of surplus compare with available alternatives? There is some value to safety resulting from conservatism, but

[29] *RSA*, III, 36, 38, 45.

[30] *RSA*, III, 31; see also VI, 646–47.

its marginal value probably declines after a point. Suppose a reduction in reserves did not create the need for additional buffer surplus. Then the only tangible value of conservatism is in the federal income tax treatment of reserves in the United States.

Suppose that a company is taxed marginally on investment income but is given tax deductions on the policyholder's share of investment income. Suppose that the reserves cannot easily be destrengthened in an emergency and that the conservative reserve is a life insurance reserve. We will show that the net present value of augmented book profits from holding a conservative reserve is negative if the time value of surplus is positive and is greater than the sum of the after-tax investment income on surplus, i, and marginal income tax credit on life insurance reserves accumulated to the end of the policy year.[31] The surplus commitment, however, is positive. Hence, a conservative reserve basis has a negative ratio R and should be avoided. That the net present value of augmented book profits is negative can be demonstrated as follows. Buffer surplus is zero, so book profits equal augmented book profits. The net present value of book profits is

$$NPV(B) = (\Delta \text{ Reserves})\left[-1 + \frac{i+c}{(1+j)^{1/2}} + \frac{i+c}{(1+j)^{3/2}} + \cdots + \frac{i+c}{(1+j)^{n-1/2}} + \frac{1}{(1+j)^n}\right],$$

where Δ Reserves is the conservative increment to reserves, which is assumed constant by duration; c is the marginal income tax credit (per dollar of life insurance reserve) earned in the middle of a policy year; j is the rate representing the time value of surplus; and n is the policy year at the end of which the conservative increment to reserves is released.

One can show that

$$NPV(B) = (\Delta \text{ Reserves})\left[\frac{(1+j)^n - 1}{(1+j)^n}\right]\left[\frac{(i+c)(1+j)^{1/2}}{j} - 1\right].$$

By assumption,

$$j > (i+c)(1+j)^{1/2} \quad \text{and} \quad j > 0,$$

so $NPV(B) < 0$. That is, the net present value of marginal book profits is negative.

[31] Fraser [5].

At first glance, this conclusion may contradict common sense, since the tax deduction permanently saves cash while the surplus strain is temporary and "just on paper." However, the assumption that surplus is a limited resource means that there are more profitable insurance products not being sold or more profitable investments not being made because the company's surplus is in short supply. Thus, the company is losing future surplus growth by holding conservative reserves and passing up risk ventures.

Likewise, for a company taxed on gains from operations, the net present value of book profits from holding a conservative reserve is negative unless the reserve is released at a time when the company is being taxed on investment income rather than on gains from operations.

B. *Group Term Insurance Compared with Common Stocks*

Both group term insurance and the ownership of common stock use buffer surplus. If a company has a chance to expand either its group insurance marketing effort or its holdings of common stock, the company must compare the marginal augmented book profits of group insurance with those of common stock, both in relation to their use of surplus. The volatility of either venture determines how much buffer surplus it uses; volatility need not enter the calculation in any other way.

Suppose that both risk ventures use a constant amount of buffer surplus in perpetuity. The net present value of marginal augmented book profits from common stock in relation to marginal surplus commitment can be calculated as

$$R = \{-BS_1 + B_1[1 + (1 + j)^{-1} + (1 + j)^{-2} + \ldots]\}/BS_1$$

$$= \{-BS_1/CS + B_1(1 + j)/[(CS)(j)]\}/(BS_1/CS) ,$$

where

$CS = P_1^1$ = Market value of incremental investment in common stocks;
$BS_1 = f_1^1(P_1^1)$ = Incremental buffer surplus on additional assets invested in common stock; and
B_1 = Marginal investment return on additional assets invested in common stock, including the amount $d(BS_1)$ in each year, all discounted to the start of the year.

We treat the mandatory securities valuation reserve as a part of surplus. Suppose that $d = 0.045$. Suppose that the annual investment return on common stock, $[B_1 - d(BS_1)]/CS$, is about 4 percent more than on some alternative investment that requires no surplus commitment. Suppose that

the company considers common stock so volatile a risk that the buffer surplus used equals the investment in common stock,[32] that is, $BS_1/CS = f_1^1 = 1$. Then for common stock:

$$R = -1 + (0.04 + 0.045)(1 + j)/j$$
$$= -1 + 0.085(1 + j)/j .$$

Suppose that the marketing effort for group insurance does not require any price reduction. The net present value of marginal augmented book profits from group insurance in relation to marginal surplus commitment can be calculated as

$$R = \{-BS_1 + B_1[1 + (1 + j)^{-1} + (1 + j)^{-2} + \ldots]\}/BS_1$$
$$= \{-BS_1/AP + B_1(1 + j)/[(AP)(j)]\}/(BS_1/AP) ,$$

where

$AP = P_3^1$ = Annual premium of incremental group term insurance;
$BS_1 = f_3^1 P_3^1$ = Surplus increment used as a buffer on additional group term premium; and
B_1 = Annual incremental profit on additional group term premium, including the amount $d(BS_1)$ in each year, all discounted to the start of the year.

Suppose that $d = 0.045$. Suppose that on the margin $[B_1 - d(BS_1)]/AP$ is of the order of 1 percent and BS_1/AP is of the order of one-tenth.[33] Then for group insurance, in this company,

$$R = \{-0.1 + [0.01 + 0.045(0.1)](1 + j)/j\}/0.1$$
$$= -1 + 0.145(1 + j)/j .$$

[32] This might be the conclusion drawn from *RSA*, II, 284. See also *RSA*, III, 960. Of course, fixed-income investments have proved to be volatile in market value as well, but statutory accounting allows most fixed-income investments to be carried at book value until sold. This accounting makes stocks seem a more volatile risk venture. However, this accounting assumes that the companies continue to attract positive cash flows at affordable costs. The statutory accounting for mutual savings banks and for mutual savings and loan associations has similar rules for fixed-income investments because of a similar assumption. The assumption has proved wrong in their case lately. Therefore, some insurance companies may consider common stock less volatile than 100 percent at risk, at least in comparison with other investments, especially at low price/earnings ratios. These insurance companies would judge the buffer surplus to be only a fraction of the investment in common stock. See *RSA*, II, 266, for estimates of this fraction (m) as a function of the price/earnings ratio. Because earnings themselves are volatile, I suspect it would be more useful to examine the market value/book value ratio.

[33] *RSA*, I, 904. These may be average, rather than marginal, estimates, so the conclusion may differ for an actual company.

This is higher than the ratio for common stock, if the time value of surplus is positive. Therefore, the company may choose to expand its marketing of group term insurance rather than its common stock holdings until the group insurance business is no more profitable on the margin than is the holding of common stock.

V. PRACTICAL DIFFICULTIES IN THE ALLOCATION OF THE SURPLUS BUDGET

The practical difficulties of calculating both marginal book profits and marginal buffer surplus are more troublesome than are the theoretical difficulties. It may be impractical for some companies to attempt to compare marginal book profits and marginal buffer surplus among risk ventures for the following reasons:

1. It is expensive to make the calculations if the quantities cannot be derived from current financial reporting systems.
2. There is a large opportunity cost associated with the diversion to this project of the time of those who can make the best-informed calculations of the marginal quantities. For example, in the case of an insurance product, these people might be the product manager, the market strategist, or the actuary who sets premium and dividend scales for the product. These people have other valuable work to do.
3. The optimization requires that new prices be set on risk ventures on which the company has price control, perhaps every year, to equalize R.
4. The results of an initial study may not be fully credible. If seemingly unreliable results call for a radical change in the status quo, the project may be dropped. On the other hand, if the study results support the status quo, the project may be deemed a waste of time because it simply seems to tell the company what it already knows.
5. Any changes from the status quo may involve costs that are difficult to quantify. For example, if an insurance product becomes less competitively priced because a higher profit standard is imposed on it, the morale of trained sales and product development personnel may suffer.
6. If investment professionals, product managers, and actuaries are assigned the job of producing the marginal augmented book profit estimates and the marginal surplus commitment estimates upon which their surplus budget is set, they may become advocates for their assigned risk ventures. Should they overestimate the net present value of marginal augmented book profits at a particular price level, or should they underestimate the marginal surplus commitment at that price level, there is no accountability unless actual marginal results can be separated from the actual average results and unless actual results can be expected to emerge within a very few years.
7. For the project to be of most value, marginal augmented book profits and surplus

commitments must be studied company-wide. It may be difficult to compare insurance risk ventures with investment risk ventures, and these, in turn, with other types of risk ventures.

For insurance companies aggressively seeking to optimize growth, these practical difficulties may be overcome to varying degrees. Even where the theoretical approach cannot be followed exactly, it gives a standard against which an actual growth-optimization project may be judged.

VI. QUESTIONS OF EQUITY FOR MUTUAL INSURANCE COMPANIES

The American Academy of Actuaries has recently adopted guidelines for the equitable distribution of surplus to participating policyholders of mutual insurance companies.[34] The guidelines generally endorse the contribution principle as a basis for dividend philosophy: distributable surplus is allocated to policyholders in proportion to their classes' contributions to surplus.

Historically, mutual insurance companies have endorsed the contribution principle, and this formulation of equity has gone hand in hand with their growth. When the public was not too sensitive to the initial premium level, and when the mutual insurance company was not constrained by surplus, the promise of "insurance at cost" was sufficient to ensure efficient growth. The mutual insurance company could avoid the risk of losses on most, if not on all, product lines because of the comfortable dividend margins. Valuable sales effort was not wasted on products that eventually caused losses that had to be made up by other product lines, so the growth was reasonably efficient.

Some modifications to the contribution principle have, however, been necessary in certain situations:

1. Some actuaries believe that the first generations of policyholders did, and later generations should, make a permanent contribution to surplus to fund an ongoing and growing service capacity.[35]
2. Some actuaries believe that a permanent risk charge is required to reserve for catastrophes that may occur infrequently.[36]
3. Some actuaries believe that general equity principles are not appropriate to situations where their application will produce losses or prevent the recovery of losses.[37]

[34] American Academy of Actuaries ([1], p. 4).

[35] Leckie [8] and *RSA*, V, 36; VI, 646.

[36] A permanent risk charge is a standard feature of participating group insurance dividend formulas. This is equitable, for claim fluctuations cannot be recovered when the group is able to switch carriers.

[37] *RSA*, VI, 312–13. A self-imposed restriction on the company's freedom to react, because of equity principles, may fail to benefit the remaining policyholders. But see *RSA*, VI, 649–50.

4. Some actuaries believe that competition is a form of equity.[38]
5. Some actuaries believe that older policyholders should get the same dividend interest rates as new policyholders if the companies are to avoid replacements.[39]
6. Some actuaries believe that overhead expenses cannot be allocated on the basis of general equity principles without an analysis of whether or not the lines of business will be able to support the allocated expenses.[40]

Some, but not all, of these modifications can fit within the guidelines easily.

The situation of mutual insurance companies in the marketplace is changing, and it is changing at different rates in different product lines. The mutual insurance companies have less surplus; it is becoming a critical resource. Because of inflation, buyers of insurance are seeking lower initial premiums and are seeking to pass more of the risk to the insurance company. There are more risks yielding low profits but requiring high buffer surpluses.[41] Because of increasingly price-sensitive buyers and because of the expense of experience studies, some companies are increasingly pricing to the market.[42]

Perhaps the extreme case can be illustrated by the group pension guaranteed-interest contracts (GICs) sold in the last ten years. The GICs may produce a large profit or they may bankrupt a company, depending on swings in interest rates, because the individual risks are not independent. Existing policyholders, say in the individual life line of business, should have the right to expect management to seek the greatest profit available in the marketplace on GICs in relation to the reliance of the GICs upon the surplus accumulated by the existing policyholders. Long before there is distributable surplus, the company must decide what interest rate to guarantee. If resulting sales come too quickly, the company cannot accept all risks because it does not have enough surplus. Setting a quota is not fair, either to existing policyholders or to those groups who sought to buy after the quota was filled. It seems more equitable to set a higher price and to obtain the fewer sales needed to stay within the surplus budget for the year.[43]

When mutual insurance companies face a new limiting resource, surplus, it seems appropriate for the actuarial profession to raise the question of whether or not we need new modifications to the contribution principle, or

[38] *RSA*, VI, 312–13. They recognize that, no matter what they decide in the name of equity, adjustments will occur anyway in a free market. One need only look at the effect of high interest rates, and possibly of money market funds, on mutual savings banks and on mutual savings and loan associations.

[39] *RSA*, VI, 310.

[40] *RSA*, VI, 645.

[41] *RSA*, VI, 650.

[42] *RSA*, VI, 651.

[43] *RSA*, V, 44.

even new equity principles that may be inconsistent with the contribution principle. We propose for discussion the following equity principle:

> A mutual insurance company can seek to serve its risk-bearing function equitably now and in the future by pricing all its insurance risk ventures so that the expected marginal permanent contributions to surplus (that is, marginal augmented book profits) are equal across all risk ventures for equal marginal surplus commitments.

If this principle is judged inequitable for a mutual company to use, even if its use is restricted to new business, then a company may want to consider demutualization if it wishes to optimize growth. Demutualization may be accomplished either directly for the whole company, or for new business only by selling through a stock subsidiary.

A. *Whose Interests Would the Principle Serve?*

If a mutual insurance company directs its growth to less competitive insurance markets that allow it to retain maximum profit after dividends to policyholders for the least use of surplus, there is a question of equity. Whom does such a strategy serve? Existing policyholders? New policyholders in product lines that are less profitable on the margin in relation to their use of surplus? New policyholders in product lines that will be aggressively pursued by the company under the new strategy? Company personnel?

The existing policyholders can be served by the greater strength and safety that come with the maximum surplus attainable in the long run. However, if the strategy suggests that there be a reduction in their dividends just because they will not respond adversely, the equity of such a reduction is not evident. The harder equity question arises when an increase in dividends that is theoretically justified is not implemented because of the new strategy. However, given the linkages between current dividends and illustrated dividends, the strategy may not suggest too much be held back from dividends.

The new policyholders in product lines that will grow more slowly under the new strategy may be well served, since the alternatives are either less security or a quota that may have excluded them. Those that would have been included within the quota will pay a higher price. New policyholders in these product lines are asked by the existing policyholders (the current owners) to be as profitable in proportion to their use of surplus as all other new policyholders. If they wish, the potential policyholders in these product lines can buy from another company without the same surplus constraint, perhaps at a lower price. If the buyer is not price-sensitive at the time of purchase, actuaries will differ on what the company owes the buyer in

equity. At a minimum, disclosure of a planned deviation from the historical contribution principle may be appropriate.

The new policyholders in product lines aggressively pursued can be better served by the lower price and greater competition in the heretofore less competitive market.

The general public can be served by the greater economic efficiency that arises from improved competition because more goods are then available to be shared. In the future, the company will have greater risk-bearing capacity than it would have had otherwise.

The employees are served well by working for a more vigorous and profitable company.

Perhaps the people most directly inconvenienced by such a strategy will be the management, marketing, and product development employees in the product lines restrained by the new strategy. Although these employees can be reassigned to other product lines, the change will be unsettling. Whether the concentrated good of the affected employees outweighs the diffuse (net) good of the other groups is both a quantitative and a philosophic question, which this paper does not address.

B. *Dividends and Expense Analysis*

The strategy for optimum growth described in this paper sometimes leads to setting the price of insurance in relation to the price in the marketplace. It may be argued that it is not equitable for a mutual insurance company to set the price of insurance (after dividends) at the price in the marketplace.[44] Should not expenses, for example, be analyzed and allocated fairly, and should not the dividends be set to reflect this analysis? Perhaps the problem is more complex. Unlike claim costs, many expenses are not directly attributable to any single contract of insurance, and the current allocation basis can seem arbitrary. An alternative allocation basis for fixed expenses may produce a more competitive net cost after dividends in segments of the market that are more price-sensitive. If marginal expenses are low and the use of surplus is not too great, then it is important to sell as much insurance as is possible. By definition, sales are more responsive to changes in price in price-sensitive segments of the market, so these are the segments of the market in which the company may want to be most competitive.

Of course, any alternative allocation basis for expenses that may be developed as a result of the growth-optimization project must meet legal requirements of fairness. Each company should check the laws of the various

[44] *RSA*, III, 37–39; IV, 206–8; V, 44; VI, 308–10, 312–13, 646, 651.

jurisdictions and decide whether or not it is legal for the company to seek an equal marginal profit for equal marginal uses of its limited resource, surplus. But within legal constraints, the company may optimize growth by pricing to produce such equality.

We want to emphasize that a line of business whose *marginal* profitability (in relation to the surplus used) is equalized is not necessarily more or less profitable than any other line on the average (on the legal, or statutory accounting, basis) after fixed expenses are charged and optimum sales volumes are achieved.

While the strategy may lead the company to price to the market in price-sensitive markets, the company does not simply price to the marketplace automatically everywhere. The company may charge more than the usual market price in relatively unprofitable markets, in relatively risky markets, in relatively surplus-intensive markets, or in non-price-sensitive markets. The strategy does *not* advocate the inconsistent practice of marginal pricing in price-sensitive, but low-profit, markets combined with average pricing in other markets.

C. *Equity among Generations*

The profits available in various markets may change over time, and this may limit or expand the growth possible for a company with a given strategic position; however, there may be little a company can do other than grow when it has the chance. Varying profit goals seem more realistic than expecting a fixed annual contribution rate to surplus from each generation and type of policyholder to fund growth.[45] A fixed annual contribution rate breaks down unless indexed to inflation; possibly this contribution rate should be changed after issue if inflation picks up, because a higher rate will be sought from new sales. But even if the fixed contribution is indexed to inflation, this misses consideration of whether the body of policyholders will be worse off because of future responses to such a fixed rule. The question is, what is most equitable? From the point of view of some existing policyholders, varying profit goals (at least for new business) may be more equitable; their company always uses their surplus to build the strongest and largest company possible without too much risk for surplus to bear.

VII. CONCLUSION

The net worth of an insurance company includes not only the value of the surplus but also the value of the existing business in force and the

[45] Leckie [8].

productive capacity of the agency force. Existing business in force for many companies has greater profit margins both in the premiums and in the reserves than has the new business being written.[46] For example, new life insurance company sales involve less permanent life insurance but more term life insurance and more deferred annuities, both of which have smaller profit margins. The future profits inherent in conservative reserve bases and high gross premiums are being released, but observers of only statutory surplus cannot detect the change. When all hidden margins are released, profits will drop. Profits from business in force will no longer be able to support inflation-related growth in new business. A similar problem may exist with regard to some companies' agency forces. Are the expenses for developing additions to the agency force supportable by the present value of the profits attributable to those additions to the agency force?

If surplus, future profit margins on business in force, and the value of the agency force are all decreasing relative to inflation and to the expanding market for insurance, the companies' service capacity is shrinking relative to inflation and relative to the marketplace. Companies not seeking to grow efficiently may not be able to grow as much over the long run as companies that do seek to grow efficiently. Efficient growth means efficient use of a company's most limited, but critical, resource. This paper has assumed that the limited resource is surplus. The methods discussed in this paper may help insurance companies, both stock companies and mutual companies that resolve the equity question in favor of using those methods, to develop a strategy for the efficient use of surplus (or related limited resources) to optimize growth.

The interrelationships among pricing, growth, competition, and surplus needs have long created discussions of equity. This paper seeks to encourage debate on what is equitable for a company with limited surplus.

VIII. ACKNOWLEDGMENTS

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DISCUSSION OF PRECEDING PAPER

ABRAHAM HAZELCORN:

The Society of Actuaries owes Mr. Hagstrom a great debt of gratitude. Many of our papers delve into both broad and narrow aspects of the conduct of a life insurance company. Mr. Hagstrom's paper deals with most of the major items in successful planning for a life insurance company.

The efficient and proper use of surplus is something the management of a company should consider constantly. This paper discusses the many factors involved in deciding on how to use surplus. I agree with what Mr. Hagstrom calls microeconomic approaches; however, in many instances, I would classify his recommendations as good business sense and perhaps macroeconomic theory.

With the recent introduction of indeterminate premium policies, there is a pricing consideration that also directly affects the equitable treatment of policyholders, that is new to this arena. Some states, like New York, require that any pricing changes for indeterminate premium policies be done only prospectively; no consideration should be given to gains or losses up until the point of changing the premium, subject, of course, to the maximum premium in the policy. This would require a different formulation from that used heretofore. In a sense, it may bring the equity concerns of a dividend formula closer to stock company considerations. Or, because of the ability to look back over the whole history of a class of participating policyholders, the cleavage may be sufficiently different so that a new theory should be introduced: one of linking surplus needs and equity on a pure prospective, almost a new-business, approach. Then, too, a regulatory requirement that the profit level remain the same upon changing premiums for a class of indeterminate-premium policyholders in effect brings up the matter of equity to shareholders. We may have a special case of the author's equitable marginal surplus commitment. We may be brought to the position of adversary actuaries in the same life insurance company, one representing policyholders (participating) and the other shareholders. I am told that this situation exists in some countries.

As more and more life insurance companies become part of a holding company system, the use of surplus becomes more critical. The minimum and maximum surplus for the life insurance company will change depending on the status of the entire system. Therefore, the author's subject is at the

center of an even greater "macro" consideration. Competition for claims on surplus could come from many sources beyond the theoretical optimizing of book profits of one life insurance company. They would include the competing demands of sister companies (insurance and noninsurance) in a holding company structure; regulatory minimum requirements, both by statute and by administrative fiat; stability (vitality) beyond Mr. Hagstrom's buffer surplus; redefinition of goals based on a new GAAP/SAP position; and maximizing Best's rating of the life insurance company.

As an aside, I wonder why most of the contributors, other than consultants and academicians, of papers on the subject are from mutual companies. One would think that a stock company that does not issue much, if any, participating business would be more interested in the essence of this subject. Perhaps there is not enough time for such stock company executives. Or perhaps the competition, which applies to smaller life companies, mainly stock companies, does not permit as many stock company actuaries to take an active interest on a Society basis. This may be the author's second point under Section V, "Practical Difficulties . . . ," pertaining to the large opportunity cost involved in diverting people from other valuable work.

The introduction of universal life may well take us further in the direction of determining what indeterminate premium policies require. Since some companies apparently are building substantially all, or most, of their portfolios on a universal life product, it is well to emphasize the basic considerations in Mr. Hagstrom's paper. Again, I wish to thank Mr. Hagstrom for his fine contribution. I do not intend to belittle or in any way lessen the importance of his technical treatment of the problems he discusses. I find that as a profession, actuaries, generally, are good at quantifying problems in various ways once they are recognized. Mr. Hagstrom's paper allows us once again to recognize some broad overall problems and take a "macro look" despite his repeated reference to microeconomic principles.

DAVID N. INGRAM:

I read Mr. Hagstrom's paper with great interest, since we are currently in the process of applying a similar method of surplus management to a mutual company. We have encountered several areas of concern that may be of interest to those considering using this system.

1. *Determining a Marginal Cost of Capital*

This is one of the most critical steps in using this method to evaluate products that have a fairly long payback period for initial surplus committed. Mr. Hagstrom suggests that the cost be set at the "internal rate of return for the least profitable risk venture . . . that the company is willing to

accept." We have operated using the converse of that statement, not being willing to accept a risk venture that does not return at least the cost of capital. In setting the cost of capital, we have looked at two considerations:

a) *Fair return for use of company surplus.* All companies should expect a minimum rate of return on all projects requiring a capital investment. The level of this return is set by the marketplace and is affected by interest rates, inflation, and the risks of the project. In this respect, the only difference between a stock and a mutual company is in the ultimate distribution of this profit.
b) *Growth without surplus depletion.* The rate of return must be at least as high as the long-term growth rate expected. If it is not, there will be a continuing need for additional surplus. If the rate of return is high enough, the product will generate the surplus needed for continued growth after the start-up capital is provided. The growth rate, and therefore the rate of return, of a product must also be at least as high as the inflation rate to maintain the relative size of the company. Also, since the fixed costs are increasing at the inflation rate, keeping the per-unit allocation of the fixed costs level requires that the units grow at the inflation rate. Otherwise, prices would have to rise as a result of the increase of allocated expenses per unit, decaying the competitive position.

Currently the "fair return" is the larger and is being used as our cost of capital. The pre-tax earnings rate on new investments is our measure of "fair return." This means that borrowed capital would have a pre-tax investment income equal to the cost of borrowing, a convenient equality.

2. *Determining Volume of Sales*

The strategy of equal marginal results presupposes the ability to determine accurately the variations in sales volume that result from changes in price. In reality, the problem is not just the traditional supply/demand question, but a three-variable supply/demand/incentive question.

Even with traditional commission rates, the gross level of incentive is a function of price. With new products being introduced with nontraditional commissions, and with changes of commission rates being predicted for the near future on traditional products, the incentive variable assumes an unknown and potentially volatile impact on sales volume. Company marketing management will play a major role in this step, but may not currently be accustomed to thinking in these terms. Time, and extensive interaction with the actuarial staff, will be necessary to develop properly the needed skills.

3. *Other Constraints*

In maximizing the current year's contribution to net worth, it is possible to reach a conclusion that is not justified upon reanalysis of the underlying assumptions. For example, when products are repriced to obtain the desired

level of marginal surplus needs and *R* ratios, the resulting total volume and commissions may not be sufficient to support the agency force. This would mean either that the nonmarginal expenses will rise in the future or that the value of the productive capacity of the agency force will decline. To avoid this problem, a constraint may be placed on the total commissions produced. Management may want to place similar constraints on cash flow, assets, surplus, or home office salary.

FRANK S. IRISH:

The fact that the size of surplus is a limitation on the growth of an insurance company has long been recognized, but the actuarial profession (at least the life branch of it) has been slow to make this concept explicit. The concept of a capacity for acceptance of risk that is related to the size of the company's surplus is gradually coming into use. Within an insurance company, a venture that utilizes some of this capacity, by increasing the aggregate risk of the company's in-force business, is using up a scarce resource just as much as a project that produces a negative cash flow or "surplus strain" in the early years. There seems to be a growing need within the life insurance industry to recognize these facts at the decision-making level and to create a basis of measurement that recognizes within one formula all types of utilization of our scarce surplus resources. This paper is most welcome because it advocates these concepts and shows how they may be used in the decision-making process.

Mr. Hagstrom's formulas for augmented book profits achieve the objective of providing a unified approach to the problem of relating profitability to capacity utilization. In effect, this calculation is carried out very much as if one were going to adjust the accounting results for the interest on the difference between the actual surplus held and the amount of buffer surplus, that is, the amount of surplus needed to support the activities of the segment of business. (This is achieved by, at one point, defining book profits as excluding the interest on existing surplus. At another point, one then adds back the interest on the buffer surplus. The net effect is to adjust results so that the segment of business is exactly capitalized to its needs.)

As a result of the explicit recognition of capacity utilization in the rate of return formula, the company can allocate its resources more wisely knowing that it is optimizing the way it uses its scarcest and most limiting resource, its surplus. Likewise, operating managers within the company can make decisions based on this approach in the full knowledge that their decisions will not be in conflict with the overall objectives and resources of the company.

At various points in the paper, Mr. Hagstrom discusses the application of his concepts to the pricing of business within a mutual insurance company. His basic formulas for rate of return are quite appropriate as a guide to pricing within a mutual company. They state, in effect, that each policyholder should contribute to surplus in proportion to the extent to which he made use of the surplus. Up to now, there has been no effective means of achieving this kind of equity among vastly different kinds of insurance coverages within a mutual insurance company. It seems to me there has been no effective means of comparing the margins on, say, ordinary life insurance, with the margins on group term insurance or group pension products. It is necessary to do so in an effective manner, not only so that the company may allocate its resources wisely among the various lines of business but also so that the company may be confident of maintaining equity among the various lines of business.

At one point in the paper the author discusses the choice of a minimum acceptable rate of return, and I generally agree with his approach. It is also true that, as implied later in the paper, a company will want to reconsider its minimum acceptable rate of return in the light of implications for company action, and will want to modify the rate in order to correct any imbalance between growth and financial soundness. However, as the author points out, a company does not want to let its minimum acceptable rate get too far above the rate to be earned on risk capital in the open market or the marginal cost of capital; this might lead to unrealistic criteria. And certainly one would not want to choose between a project that returned 25 percent and a project that returned 30 percent purely on the basis of rate of return. If one had to make the choice, other factors might well influence it. There is always a problem when an organization has to choose among projects, all of which are acceptable according to its minimum rate of return. This is a technical problem that is much dealt with in the literature, and I do not think there is any really satisfactory solution to the problem. The author of this paper suggests a solution that emphasizes net present value rather than rate of return, and I am in favor of this emphasis because I feel that, in general, present value reveals more about a project than rate of return. I can see that his proposed solution to the problem may contain some difficulties in particular unusual situations, but it probably is nonetheless a better approach than ranking projects purely on the basis of rate of return.

I would hope that, in general, it would not be necessary to develop a technique for ranking projects. The situation in which a company has to ration its capital so severely that it must reject some very attractive projects is, one hopes, an unusual and temporary situation in most companies. In

normal times the company should be able to undertake all of the projects that meet its minimum acceptable rate of return, or, at the very least, it should be able to modify that rate within reasonable limits and avoid the necessity of resorting to the extreme types of capital rationing. The emphasis in the paper is on the ratio R, whereas I would prefer that the emphasis be on the minimum rate j, and on making a choice thereof to fit the company's needs.

It is when Mr. Hagstrom comes to his discussion of marginal expenses that I first feel I have to disagree significantly with the application of his analysis to participating business. It is a very attractive thesis that a new line of business can be priced on a marginal basis because the existing policyholders will be better off (marginally) if the company goes into the new line of business. However, once the company is in the new line of business (if it is a participating line of business), the policyholders in that line must be treated the same as policyholders in other lines of business. It would be improper in a mutual company to treat a participating line of business as being marginal to other lines of business.

I realize that Mr. Hagstrom tries to avoid the trap of imbalance and inequities between lines by suggesting that all lines should, in essence, be treated as marginal to the company, and that overhead costs and fixed expenses should not be charged to any line of business but rather should be recouped out of the rate of return. To my way of thinking, this is merely a different way of allocating overhead costs and fixed expenses among policyholders, and it is a less satisfactory way of allocating these expenses than the standard approaches. My major objection to the use of marginal expenses, however, is a very practical one, namely, that marginal expenses are very subject to misuse and misunderstanding. It is very tempting to think of a new venture as being carried out without any increase in fixed expenses, but this is frequently an unrealistic assumption. I would much rather the new venture bore its share of the new expenses from the outset.

At one point in the paper Mr. Hagstrom points out that most expenses are variable in the long run, and I would agree. I strongly suspect that projects do tend to add to overhead and fixed expenses, over a period of time, and that it would usually be unsound to depend on the marginal advantages derived from "spreading over a larger base." Hence it is simply to reflect what I feel is closer to the long-run reality, as well as to avoid some possible traps for management, that I would prefer to avoid any marginal approach to expenses. The only possible exceptions that might be permitted are those that involve a very short-term payback (that is, there is no assumption that the marginal advantage will continue in the long term)

and are such that the decisions will have no impact on the pricing of participating business.

The concept of a change-of-price comparison is another type of marginal concept, and although I find it helpful to distinguish this clearly from marginal expenses, I still find it unacceptable for participating business in a mutual company (perhaps, however, it is acceptable in a stock company). This particular type of marginality might be called a "marginal receipts" concept, since it seems to be very similar to the marginal receipts curve in microeconomic theory. This marginal receipts concept is equivalent to giving more favorable pricing treatment to segments of the business that are price-sensitive, and this, it seems to me, is wholly unacceptable for participating business. Again, I would admit to the possibility of exceptions in situations where the marginal revenue from an expansion is so great that all policyholders benefit from it in the not-very-long run. But I think the burden is on the mutual company actuary to prove to his own satisfaction that such a situation really does exist. Such situations must be relatively rare, and it would be preferable in a mutual company to avoid such an invitation to inequity.

If this leads a mutual company to adopt a strategy of less than optimal growth, according to some definition of optimal growth, then I have to say so be it. In any case, I suspect very strongly that most mutual companies would not want to make the addition to net worth the defining characteristic of optimum growth.

I would suggest a different set of strategy steps to be used within a mutual company in applying the rate of return formula. As the very first step, immediately after a trial value of the minimum acceptable rate of return has been chosen, I would suggest pricing each segment of the business to the minimum rate of return without any resort to marginal approaches. Thus, we start the process from a basis of product price that is both adequate and equitable. As the next step, I would analyze all proposed projects (and possible discontinuance of old projects) in terms of meeting the standards set by the minimum acceptable rate of return. The question arises in this step as to how to evaluate projects that may have an effect on the amount of business done, such as an agency expansion or a sales training program or a new computer system that improves the attractiveness of the product. One cannot assume that the business will be priced "optimally" in order to take advantage of the new venture. As a matter of fact, in a mutual company, price should not be a variable at this stage in the strategic process. Rather, one should let any new venture stand or fall on the basis of assuming that the product will continue to be priced as it was in the first step of the

strategic process. In other words, if an agency expansion, for example, cannot be profitable (in the long run) under the existing level of product prices, and requires a price increase in order to make it profitable, that program should not be undertaken in a mutual company.

The next step is to make some long-term financial projections based on the preceding steps of the process. In effect, one is reviewing the impact of the proposed level of prices and the proposed level of capital expenditure upon the long-run growth and financial soundness of the company. At this point, one might judge that either growth or financial soundness is inadequate. And, by the way, I do not think that financial soundness necessarily has to be treated as an absolute. Although the concept of buffer surplus (which I consider to be a very useful concept) implies that there is a particular amount of surplus needed to support a particular level of business, and that any surplus beyond this is immediately available for some other purpose, I think that in actual fact we have to treat the dividing line between adequate and excessive surplus in a little less distinct fashion; other considerations have to enter. Most mutual company actuaries would prefer to keep more surplus than the absolute minimum required by the buffer surplus process, and in such a situation the company might then be willing to let a price or growth comparison with other companies enter the reasoning process, and let this influence the extent to which it carried more than the absolute minimum necessary surplus.

In the final step, the company might make moderate adjustments in its minimum acceptable rate of return if the long-run projection demonstrated that growth was inadequate (in which case it would lower the rate) or financial soundness would be inadequate (in which case it would raise the rate). Such a change in "cutoff rate" would then lead to a revision of the pricing level adopted in an earlier step and, at the same time, a reconsideration of project approval decisions. New projections resulting from these changes would (presumably) exhibit an outlook that is more acceptable in the judgment of management.

(AUTHOR'S REVIEW OF DISCUSSION)

DALE S. HAGSTROM

I wish to thank Abe Hazelcorn, David Ingram, and Frank Irish for their discussions of my paper. There may be additional related discussions appearing in the *Record* from the Orlando and perhaps the Colorado Springs meeting held in 1982. Panel discussions on the subject "The Future Outlook for Stock Company Profitability and Mutual Company Surplus Position" are currently scheduled for those two meetings.

Mr. Hazelcorn brings a stock company perspective to the discussion. He suggests some additional constraints on surplus. He points out that indeterminate premium life and universal life products will require prospective analyses that may be tied in with the analyses proposed in the paper.

Mr. Ingram reports that his mutual company uses a planning process similar to that outlined in the paper. His suggestions on setting the marginal cost of capital may prove to be practical. I would only caution that a company be careful not to use a rate too far different from the rate of return on surplus achievable at the margin that will fully use available surplus. The greater the difference between the rate used and the achievable rate, the greater the chance for an inefficient choice between short-term and long-term risk ventures.

Mr. Ingram's comments on determining sales volume are quite true. This is a difficult part of the process suggested in the paper. When the company considers commissions to be paid, a process of comparing trade-offs at the margin similar to that used for pricing will bring the company to the most efficient choice.

Mr. Ingram's comments on setting constraints suggest that I should clarify the paper. On a conceptual basis, if a price reduction causes a problem in the agency force, that agency problem should enter the marginal-profitability calculations directly. To the extent practical, direct recognition is preferable to arbitrary constraints. On a practical basis, it may be convenient to use such constraints, but the company should at some point consider the effect of variations in the level of the given constraints. Some improvement may be possible.

Mr. Ingram pointed out, in a separate communication, that the term (c) in the preprint should have been $(i + c)$, in the example of conservative life reserves. This correction has been made in the final version. I thank Mr. Ingram for catching the error.

Mr. Irish brings his rich experience at the John Hancock to the discussion. He agrees with the concept of charging policyholders in various lines for their proportional use of surplus, if surplus is the main limitation on growth. He prefers to avoid the use of marginal concepts and suggests a strategy that avoids using marginal concepts. His process is well designed and requires fewer difficult judgments.

Mr. Irish's discussion brings to mind several thoughts, as follows. It is true that the concepts of marginal expenses and marginal profitability are subject to misuse, misunderstanding, and possibly inequity. This is particularly true if a mutual insurance company uses marginal concepts only in certain situations, that is, only where competition seems to require it. If a mutual insurance company does not use marginal concepts at all, I rec-

ommend Mr. Irish's planning process to that company. But if the company does use marginal concepts in any situation, I recommend a correct theoretical understanding of their application. More favorable pricing treatment is given to the more price-sensitive segments of the business *only to the extent that the incremental result is still more profitable than the alternative uses of surplus that are precluded.* This means that the company must look at *everything* marginally. Seeking price-sensitive business may, in fact, be a bad use of surplus, so an incomplete application of marginal concepts should not be used to justify seeking such business.

Mr. Irish suggests that a mutual insurance company may want to adopt a goal, defined as growth but not directly defined in relation to additions to net worth. If the company can also identify a critical resource such as surplus that limits the attainment of this goal, then the microeconomic principle used in the paper can still be applied to help the company achieve its goal in the most efficient way, with the least risk. However, if the company seeks only growth, without concern for net worth, it may be even more likely to seek price-sensitive business in ways that both Mr. Irish and I find unwise.

Mr. Irish points out that most mutual insurance company actuaries prefer to keep more surplus than the absolute minimum required by the buffer surplus calculation. This extra surplus provides for corporate vitality or strategic flexibility. This concept is developed by Donald Cody (*RSA*, III, 29–32) and by Robert Link (*RSA*, III, 957). Companies may choose to think about this layer of surplus as a fixed amount, in which case the extra surplus is set aside in the calculation of surplus available, and does not enter the calculations of marginal surplus used. Alternatively, companies may choose to think of this layer of surplus as a potential replacement of buffer surplus to be used when the misfortune anticipated in the buffer surplus calculation occurs. In this case, all buffer surplus amounts otherwise calculated should be increased to include their share of the surplus that stands ready to replace them in the event the contingency happens. The corporate vitality surplus then enters the calculations of marginal surplus used as a part of buffer surplus.

Let me thank Messrs. Hazelcorn, Ingram, and Irish once again for their comments and suggestions.

To conclude the discussion, let me return to the equity question: Is it fair for a mutual insurance company to price its insurance products so that the present values of expected marginal book profits (augmented to reflect reliance on surplus) are equal across all products for equal marginal surplus commitments? For the sake of discussion, the paper assumes that the only

alternative pricing principle is a strict formulation of the contribution principle.

The pricing principle proposed in the paper requires the company to reach the point where only no-gain trade-offs are possible, so the principle may be characterized as producing economically *efficient* results. It is essentially prospective. The alternative pricing principle, the contribution principle in its strictest form, is essentially retrospective. Actuaries agree that the contribution principle produces *equitable* results, viewed retrospectively. The question for actuaries to consider is whether the proposed pricing principle is equitable, at least viewed prospectively.

If pricing according to a strict formulation of the contribution principle is fair retrospectively but is a disaster prospectively, what is the value of such a strict formulation? In fact, such a strict formulation is like a straw man set up to be knocked down. No mutual insurance company should use such a strict, purely retrospective, form of the contribution principle. A prospective test of whether the dividend scale can be continued is usually performed. Surely the actuary must blend both retrospective and prospective analyses. The two perspectives can be blended to include the following points:

1. If a mutual insurance company wishes to use marginal concepts in any part of its strategic planning, then it should use marginal concepts throughout.
2. In most cases the resulting prices can be restated in terms of the contribution principle.
3. If we interpret the pricing principle proposed in the paper as a guide to the allocation of fixed expenses, then the fixed expenses are allocated in such a way that the expenses are supportable. The distribution of business among lines of business and among plans of insurance does not shift away from the distribution projected in the planning process; such a shift would have left the fixed expenses unsupported. The Academy recommendation says that indirect costs should be allocated using sound principles of expense allocation. One might argue that traditional principles of expense allocation when applied to indirect (i.e., fixed) expenses are not sound if they do not recognize such a possible shift in distribution of business.
4. In the few cases where a deviation from the contribution principle is needed to avoid prospective disaster, the recommendations adopted by the Academy seem to allow this protection to the mutual insurance company and its policyholders. Even if the actuary uses the most narrow interpretation of the Academy's recommendations, the actuarial report can disclose a deviation, the need for the deviation, and the effect of the deviation, from whatever formulation of the contribution principle the actuary uses.

Once we have blended the two perspectives on equity, it is reasonable to ask finally what other points of view on fairness exist. One such point of

view, not presented in the paper or in the discussions, may be termed the *equality* principle. Its main proponents would be people active in the political arena who are concerned about discrimination on the basis of age, sex, health status, handicap, smoking and other habits, or geographic location. Under the equality principle, such discrimination is always unfair. Under the contribution principle, such discrimination is fair to the extent that price reflects experience factors. The contribution principle produces results that are far more efficient economically than the results produced by the equality principle.