Modeling Capital Market
with
Financial Signal Processing

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Outline

- **Theory and Techniques**
  - **Theoretic Framework of Modeling Capital Markets:**
    Index-Based Composition Methodology
  - **Statistical Procedure of Model Construction and Extension:**
    Wavelets-Based Financial Signal Processing Technique
- **Implications and Applications (ex. S&P 500)**
  - **Measuring Market Uncertainty and Volatility**
  - **Formatting Dynamic Strategies into Strategic Curves:**
    Adaptive Futures Leveraging & Efficient Options Pricing
  - **Monitoring Impacts of Smart-Money Timing Strategies**
  - **Gauging Cyclic Structure & Forecasting Market Crises**
    Converging Patterns towards Market Crashes and Bubbles

Theoretic Framework of Modeling Capital Markets

Index-Based Composition Methodology

$$R_{i+1} = r_i + \Psi(S_i) + \Sigma(S_i)\cdot\varepsilon_{i+1}\ , \quad \varepsilon_i\text{'s i.i.d.} \sim \mathcal{N}(0,1).$$

Strategic Index

$$S_i = \Gamma\,(R_{i-L+1}, \ldots, R_i)$$

static regression

dynamic auto-regression

Non-stationary Correlation
Noise barrier
→ Dimension Curse

$$R_{i+1} = r_i + \mu(R_{i-L+1}, \ldots, R_i) + \sigma(R_{i-L+1}, \ldots, R_i)\cdot\varepsilon_{i+1}\ ,$$

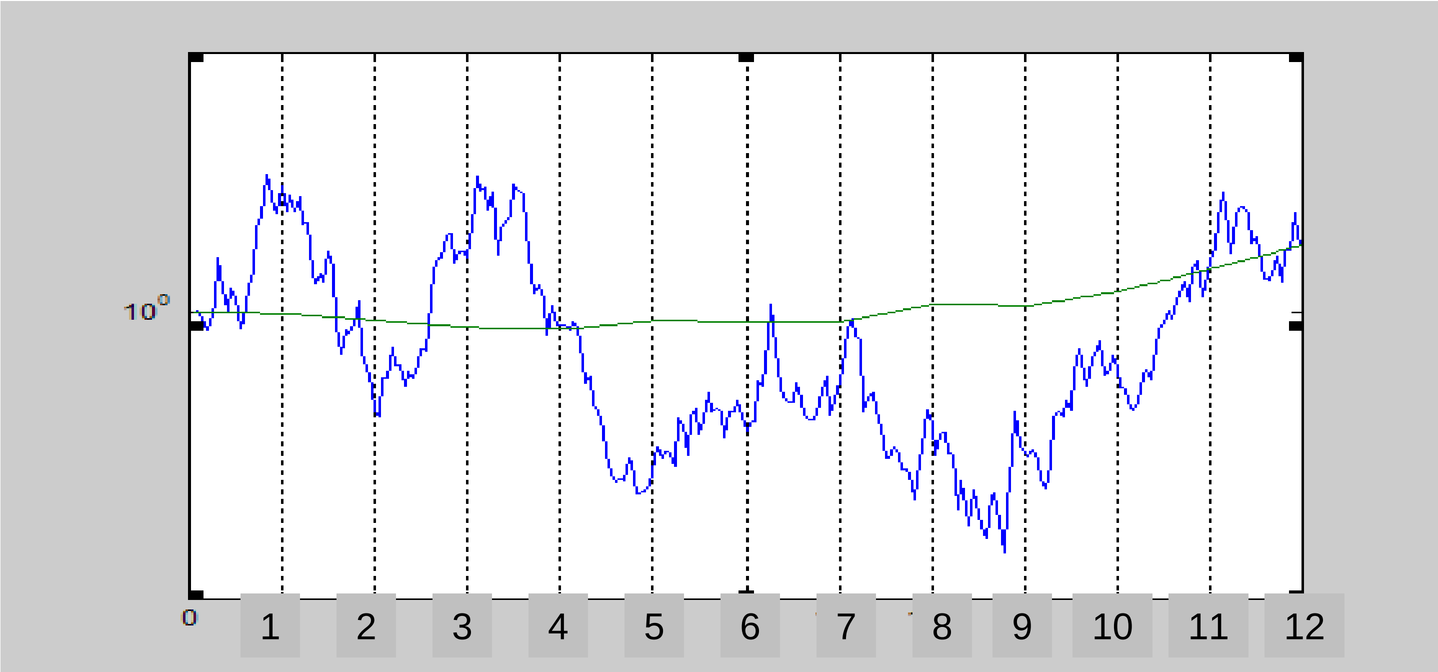
$$\mu = \Psi \circ \Gamma\ ;\ \sigma = \Sigma \circ \Gamma$$

- R_{i+1}: (i+1)-th periodical short-term market return rate - say, S&P 500 monthly;
- r_i: i-th periodical average short-term interest rate - say, FFR monthly average;
- S_i: i-th periodically updated strategic index value – say, STTB Index, shown next

Mission Impossible to De-noise through Dimension Curse

Piecewise (Monthly) Constant Geometric Brownian Motion

$$dX_t/X_t = r_i + \mu_i + \sigma_i \cdot dW_t, \quad \text{for } t_{i-1} \leq t < t_i$$



Key of Long-Term Consistent Profitability: Low-Frequency Component of Market Fluctuation

Patterns about Interactions between μ_i's and σ_i's on Time-Domain

Non-Stationarity & High-Frequency Noise-Barrier

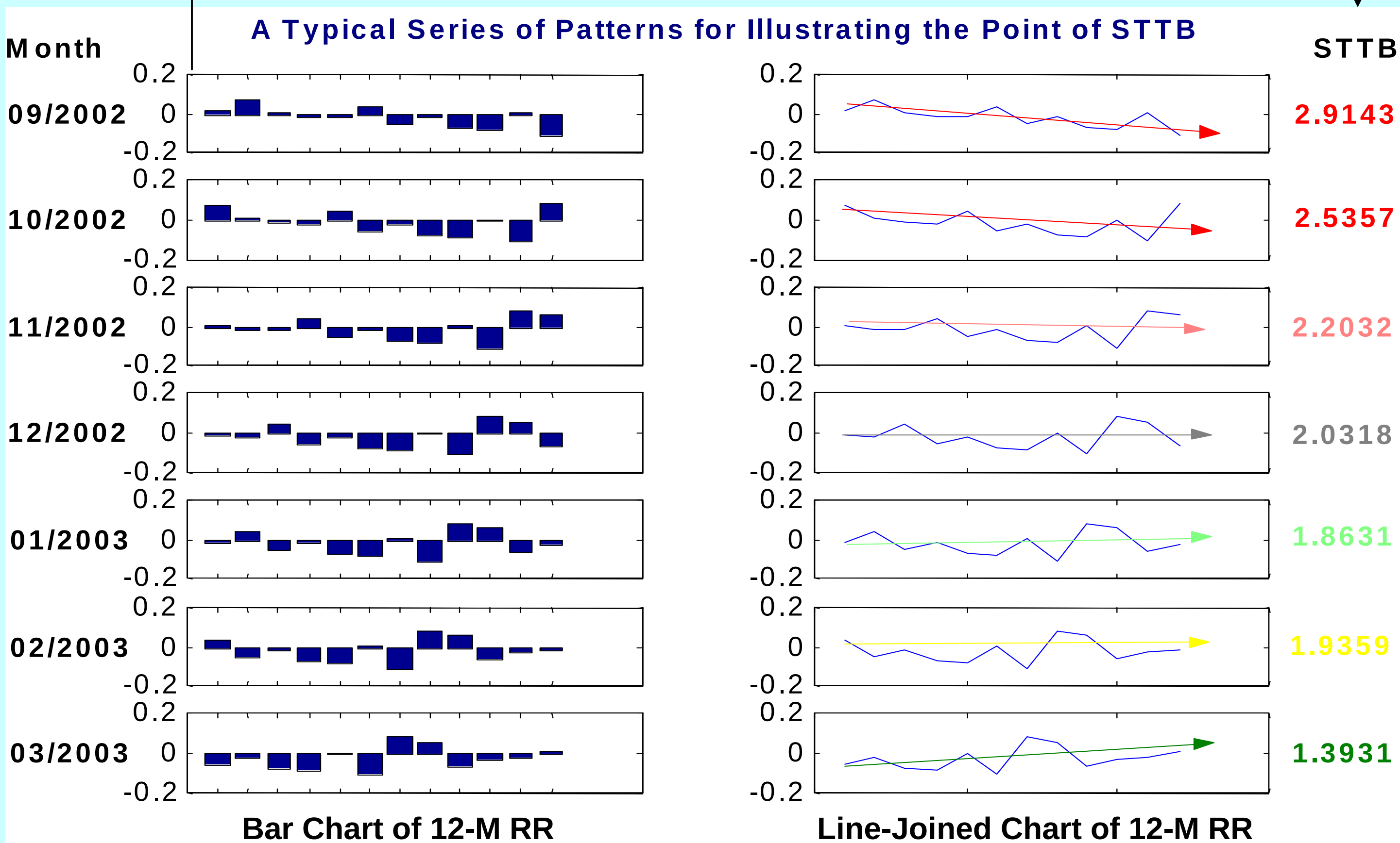
Knowledge in Ψ and Σ on S-Domain

Strategic Index

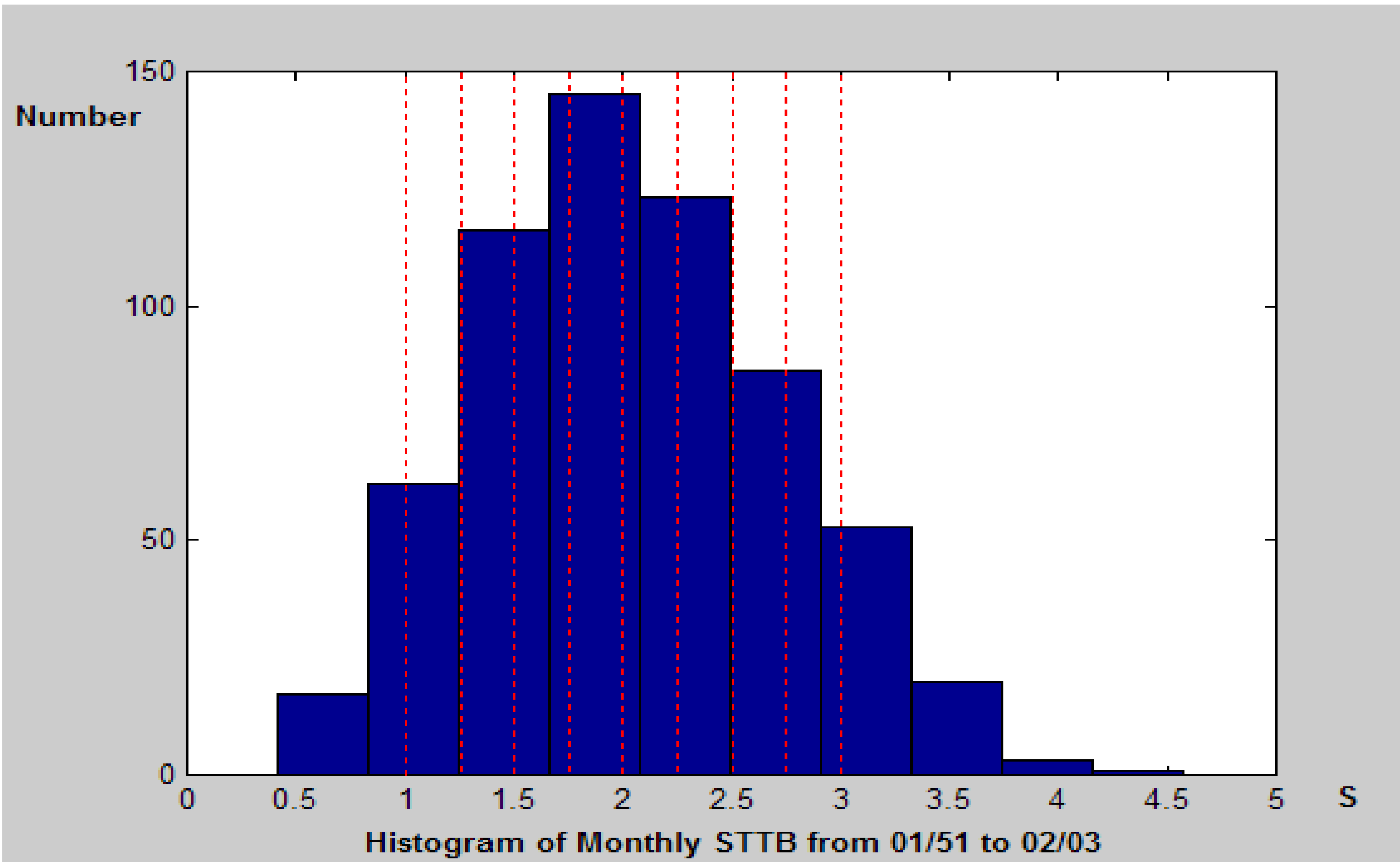
Short-Term Trend Bias

(STTB)

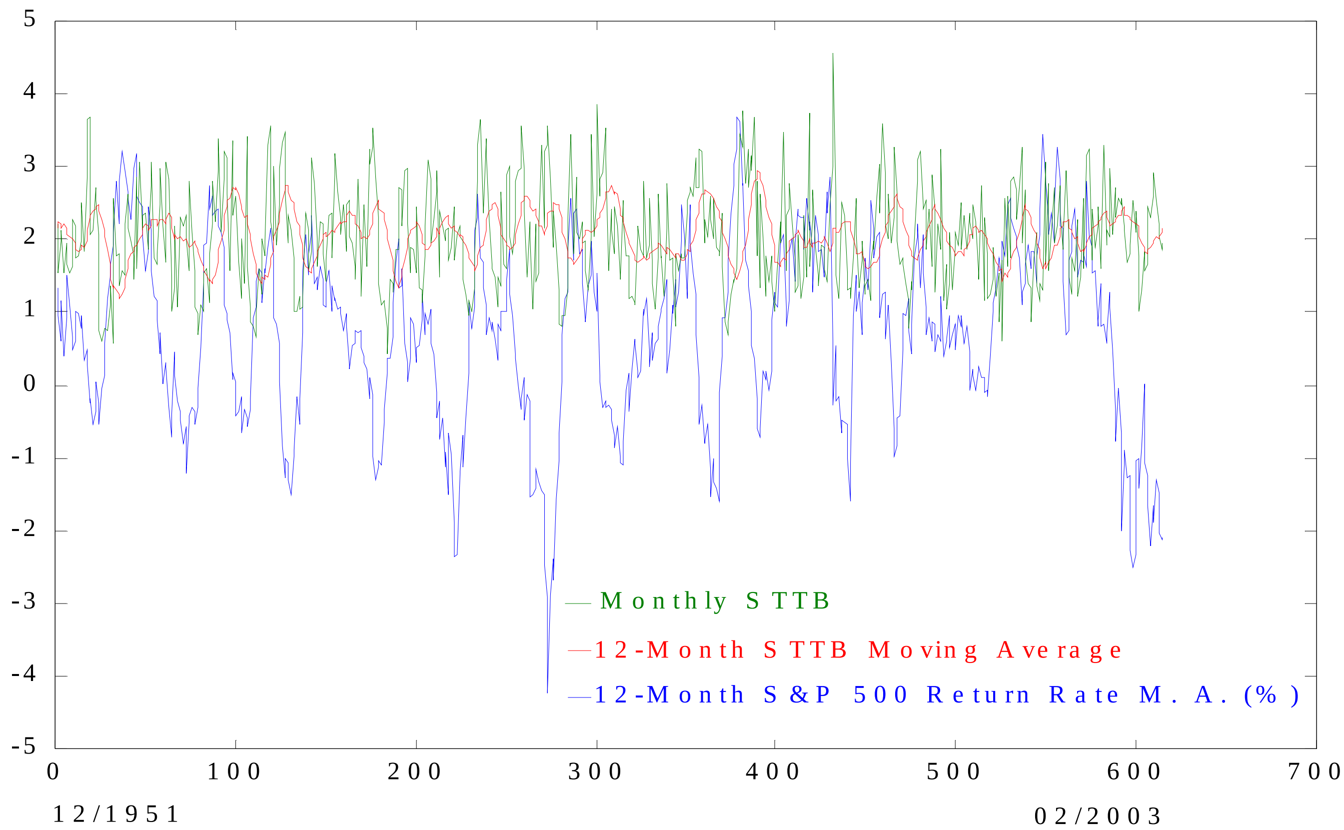
Γ (non-parametric statistic)



Distribution of STTB



Gauging the Structure of a Market Cycle



higher Red up, steeper Blue down; longer Red stays up, deeper Blue sinks down; vice versa

Consistently Leading the Cyclic Trend

Red Series (MA_STTB) is leading Blue Series in turnaround in a smooth conclusive way

The Fundamental Model – Quantitative Psychological Model

Parametric Model: Linear Heteroscedastic Parabolic Model

$$R_{i+1} - r_i = \Psi(S_i) + \Sigma(S_i)\cdot\varepsilon_{i+1},$$

$$\Psi(S) = k\cdot(S-a)^2 + b; \quad \Sigma(S) = c\cdot S + d, \quad \text{for } 1 \le S < 3.$$

a – Maximum Uncertainty Level: MLE = 2.0092

b – Uncertainty Aversion Rate: MLE = -0.0014

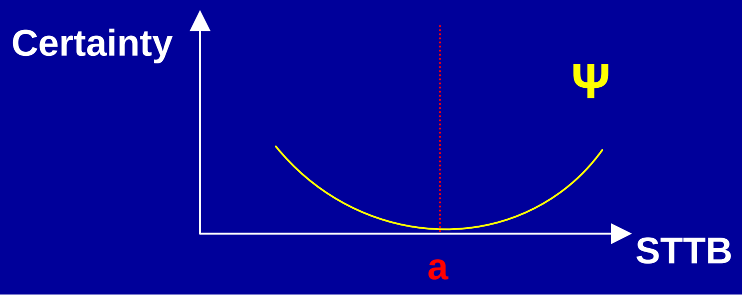
k – Rational Confidence Coefficient: MLE = 0.0107

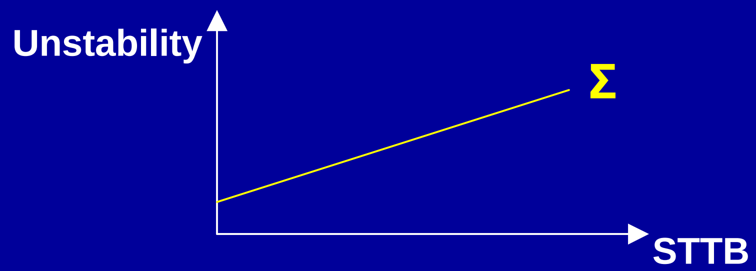
c – Stability Coefficient: MLE = 0.0096

d – Efficient Market Volatility: MLE = 0.0230

*** **MLE results are based on S&P 500 monthly data from 01/1951 to 02/2003** ***

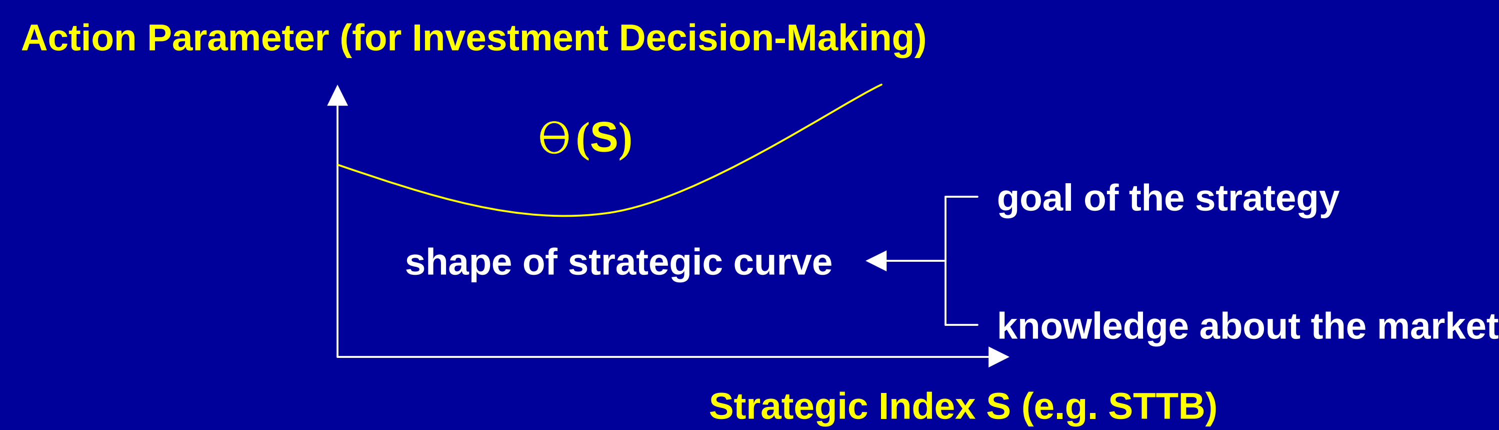
Market Volatility = Dynamic (Low-Freq.: Uncertainty) + Stochastic (High-Freq.: Stability)





Basic Structure of Shaping Dynamic Investment Strategies over Domain of Strategic Index

Strategic Curve



Elementary Examples

- **Future Leveraging Strategy –**
  maximizing cumulative return of a simple portfolio combining **S&P 500 Stock Index Future and Cash**
  (leverage-multiple of the total invested capital)

$$\Theta(S) = \Psi(S) \,/\, \Sigma^2(S)$$

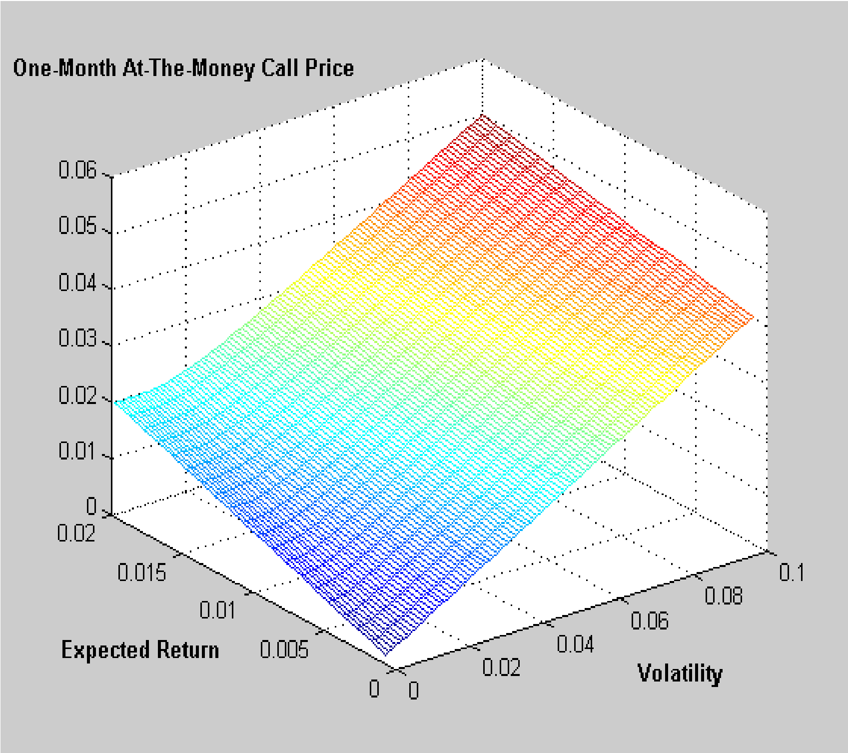
- **Option Pricing Strategy –**
  fairly pricing the value of a **One-Month At-The-Money Call** contract
  (as a fraction of the current value of the underlying asset)

$$\Theta(S) = \Phi[(r/\Sigma^2+\Psi/\Sigma^2+1/2)\cdot\Sigma] - e^{-(r+\Psi)}\cdot\Phi[(r/\Sigma^2+\Psi/\Sigma^2-1/2)\cdot\Sigma]$$

Efficient Options Pricing

Black-Scholes Model

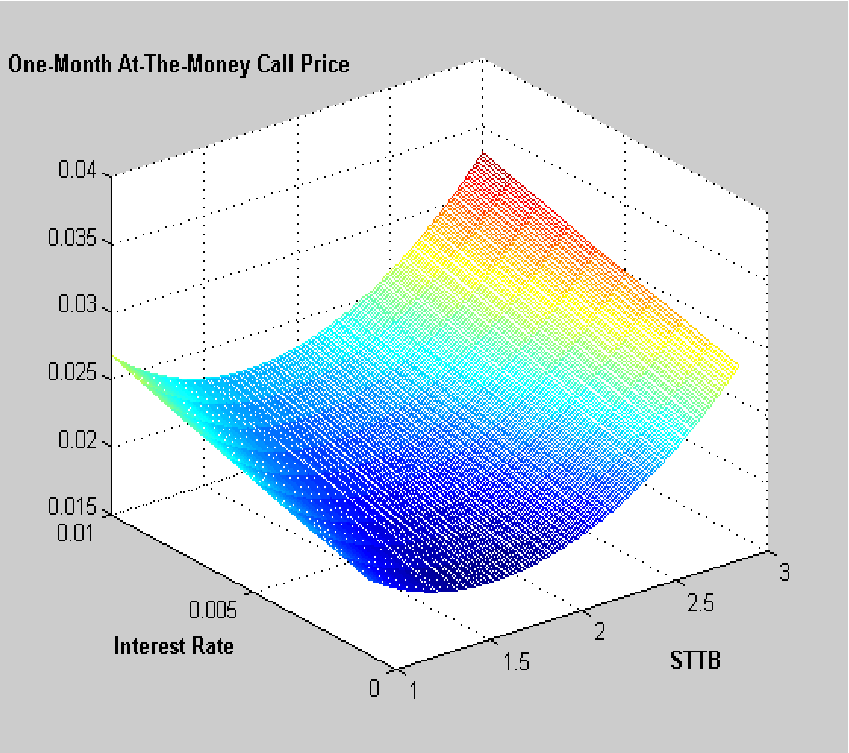
$R_{i+1} = r_i + \mu + \sigma \cdot \varepsilon_{i+1}$



Expected Return = r + μ ?;
Volatility = σ ?

L.H.P. Composite Model

$R_{i+1} = r_i + \Psi(S_i) + \Sigma(S_i) \cdot \varepsilon_{i+1}$

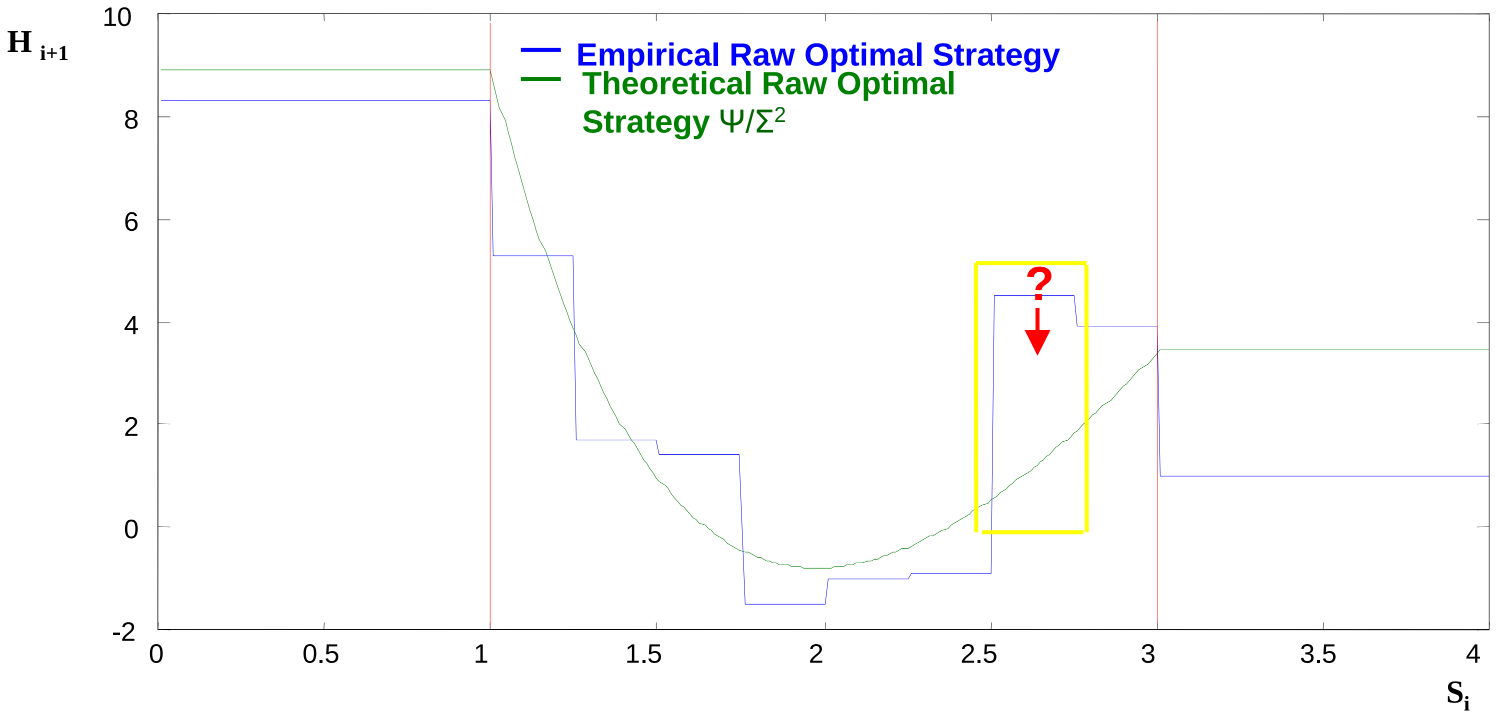


Predict Expected Return & Volatility
by Interest Rate and STTB

Adaptive Leveraging Strategy for S&P 500 Future

in comparison with one via model-free simulation

maximizing cumulative return without risk control



The Mystery of The Missing Bump?

The Complex

extending knowledge beyond the psychological factor

Complex Additive Model

$$R_{i+1} = [\Psi(S_i) + \Delta(S_i) + \Omega(S_i)] + \Sigma(S_i)\cdot\varepsilon_{i+1}\ , \quad \varepsilon_i\text{'s i.i.d.} \sim \mathcal{N}(0,1).$$

- **Psychological Factor**

Rationality-Oriented, such as Uncertainty, Momentum

Ψ, Smooth Curve

- **Strategical Factor**

Discipline-Oriented, such as Contrarian, Hedge Fund Arbitrage

$\Delta = \Delta_0 + \Delta_1$, Δ_0, Concentrated → **Missing Bump**

- **Economical Factor**

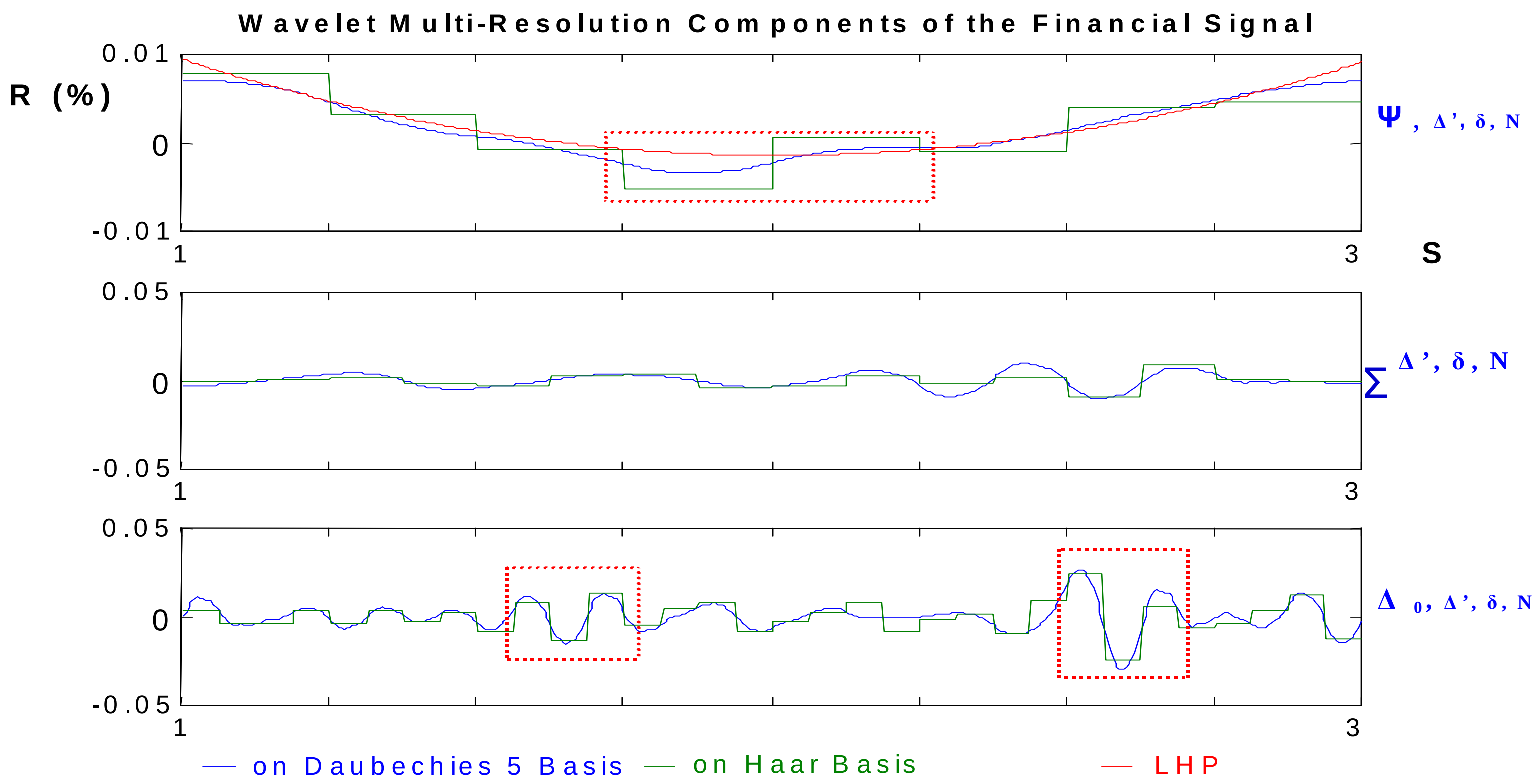
Policy-Oriented, such as Short-Term Interest Rate (Feds Fund Rate)

$\Omega(S_i) = r_i + \delta(S_i)$, δ, asymmetrically distributed

Nonparametric Decomposition to realize Model-Free Simulation
distinguishing and recognizing factors moving the market

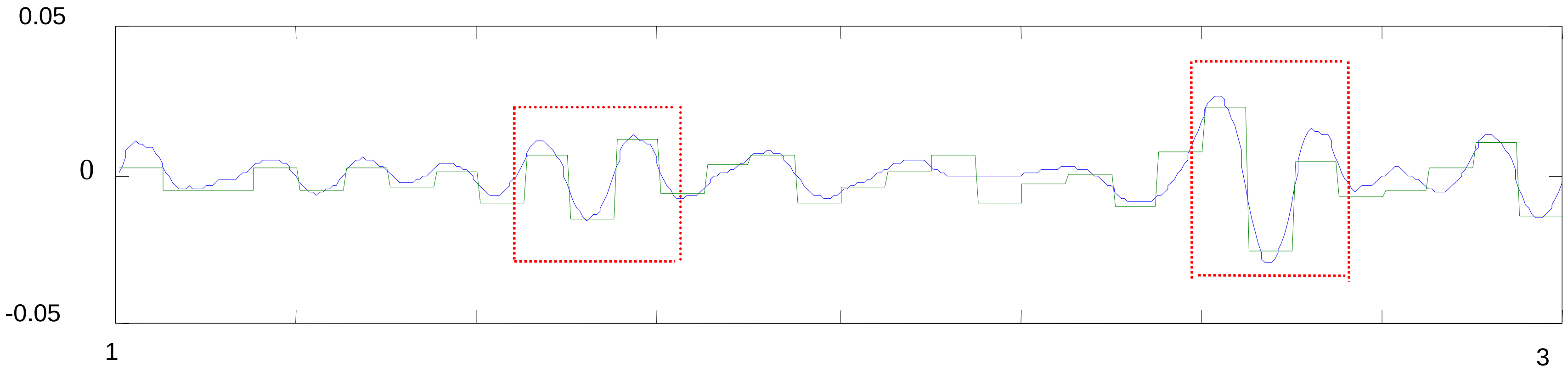
Advanced Financial Signal Processing via Wavelet Technique

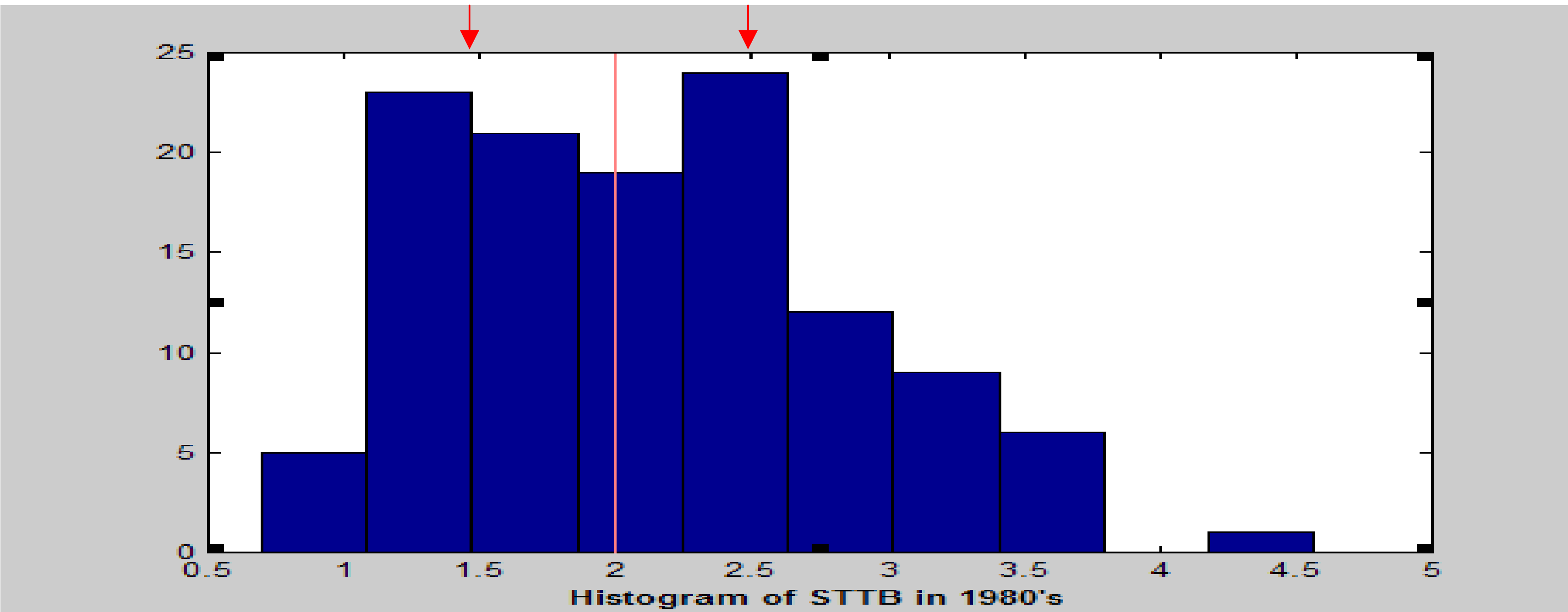
Raw Financial Signal: $R_{i+1} - r_i = [\Psi(S_i) + \Delta(S_i) + \delta(S_i)] + \Sigma(S_i) \cdot \varepsilon_{i+1}$, $i=1, \ldots, n$



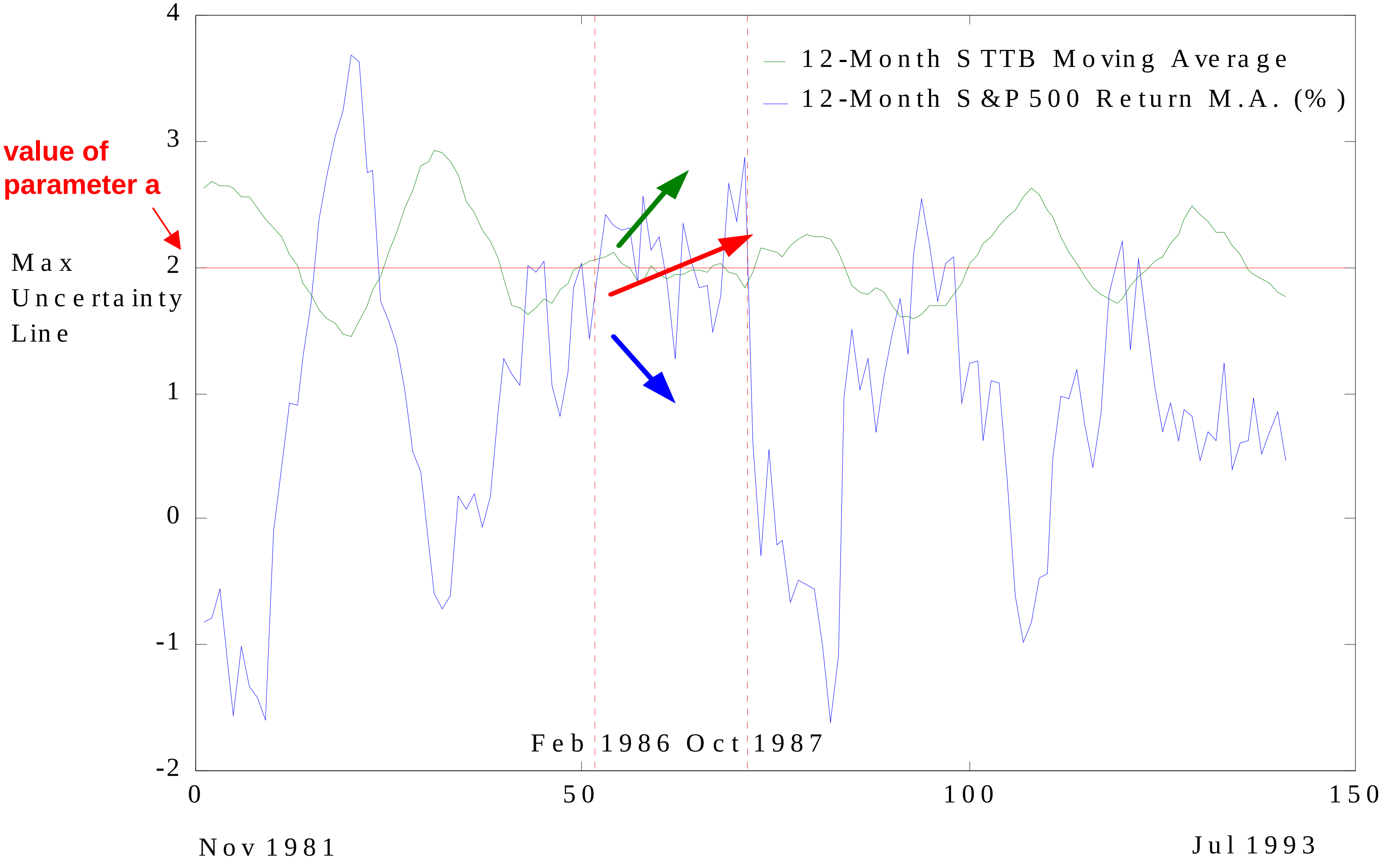
decomposition levels ($\log_2 n$) with higher resolution are ignored – almost nothing living there except for the components of the heteroscedastic white noise

Two-Peak Phenomenon

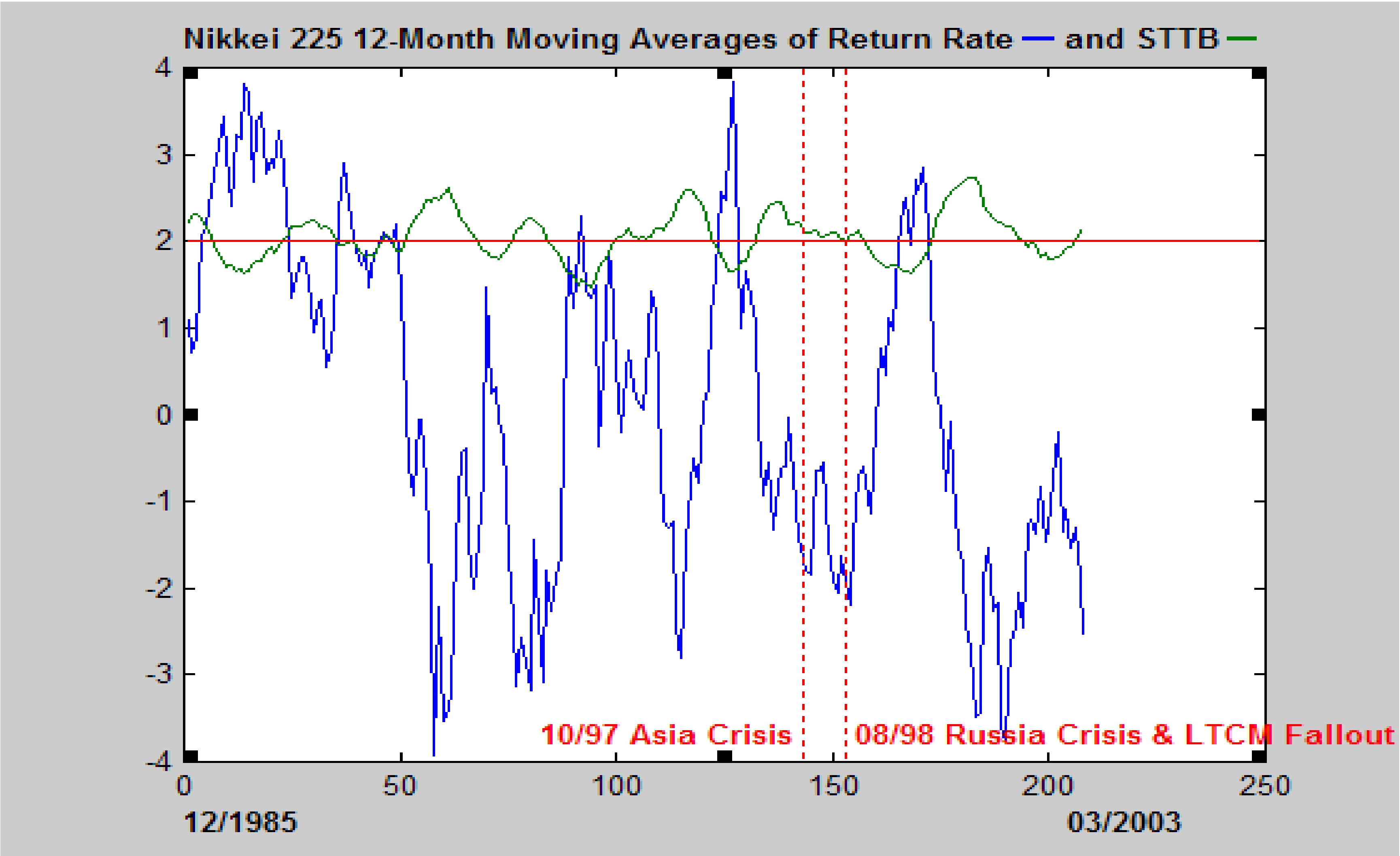




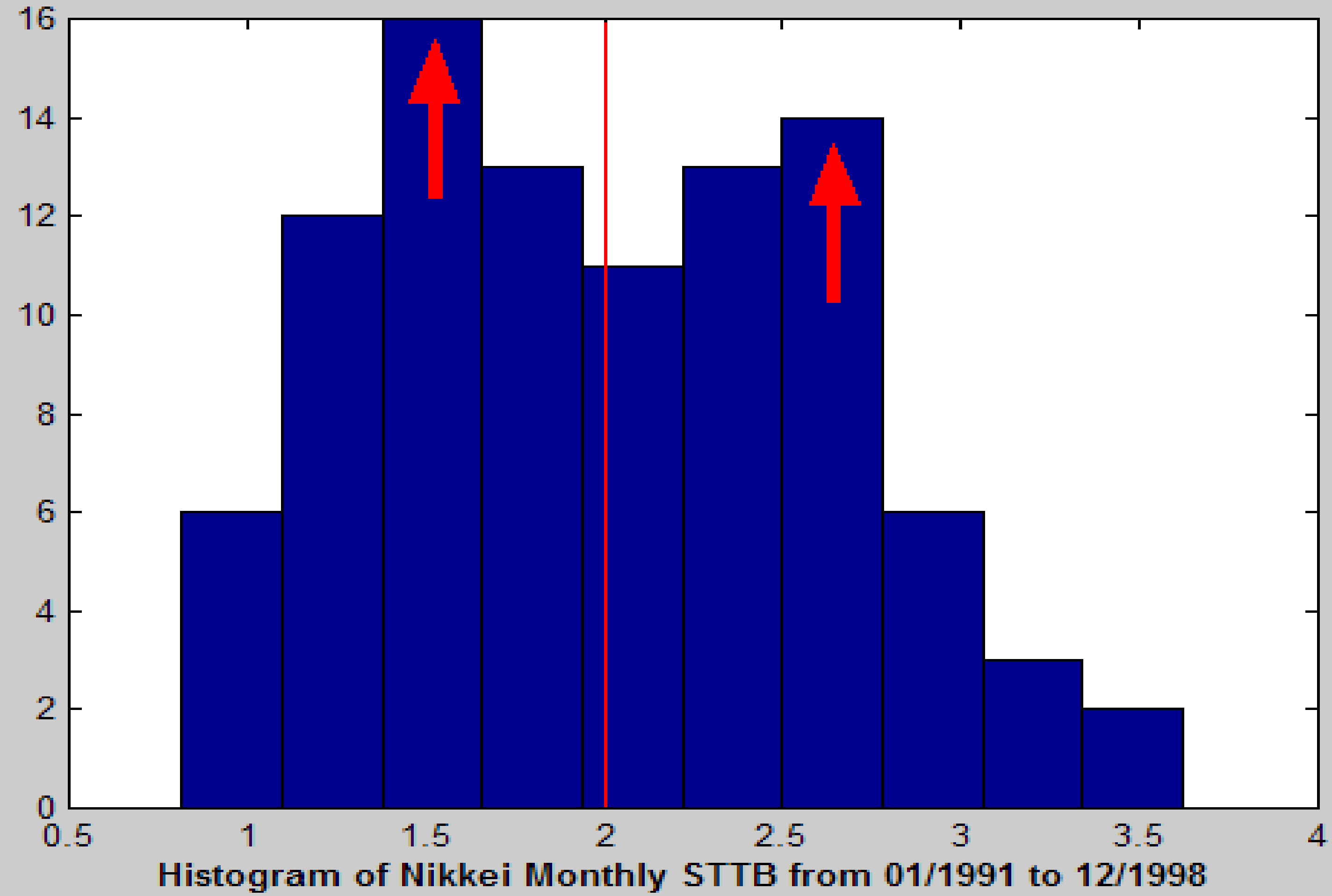
A Clue to the Remarkable Story of the Great Crash



Similar Sign before Another Crash in Another Market



Striking Coincidence



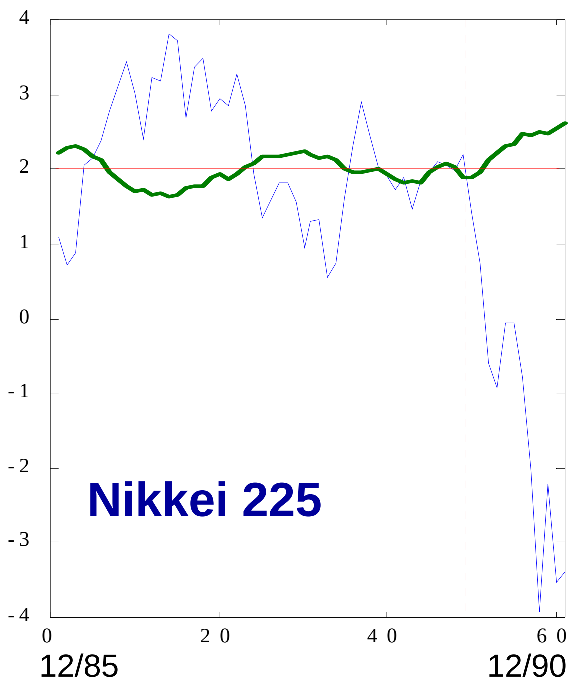
Principle of Cyclic Hazard

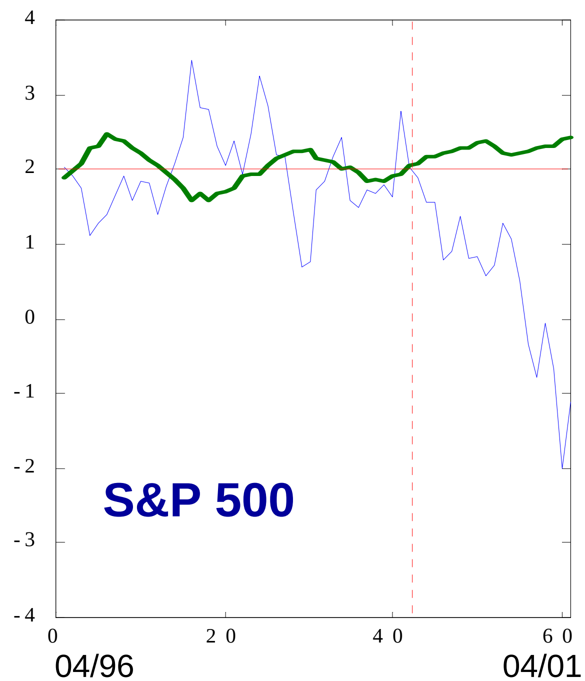
from the above two pre-crash patterns, it is intuitive to perceive the following principle:

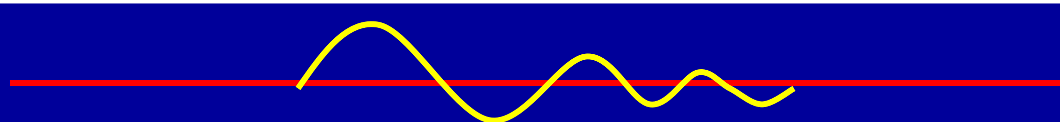
When the market's behavior eventually evolves into a rapid oscillation around the maximum uncertainty level rather than taking a typical cyclic course, the chance for the market to crash and the crash extent will increase day-by-day until that happens.

Bubble Phenomena:

a dynamic picture for the principle of cyclic hazard







as the cycles keep converging to, thus oscillating around, the maximum uncertainty level,

accumulating fear of uncertainty builds up to end up with a market crisis

Underlying Mechanism for Principle of Cyclic Hazard

Ping-Pong Hazard - a physical illustration with a pendulum