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THE FUTURE OF THE STOCK MARKET

Moderator: ROBERT J. JOHANSEN. Panelists: ROGER G. IBBOTSON, GERSHON N. MANDELKER. Discussant: WILLIAM A. DREHER

Dr. Ibbotson will describe methods of projecting long-run stock returns using inflation rates from bond yield curves with incremental returns for risk taking based on historical data.

Dr. Mandelker will discuss the relationship between concurrent, expected and unexpected inflation rates and the rates of return on common stocks and U.S. government bonds and notes. His paper will point out that rates of return on common stocks are negatively related to concurrent as well as to expected and unexpected rates of inflation. The results on U.S. government bonds are mixed.

Following the presentation of the two papers by the participants from the American Statistical Association, the Discussant will present a discussion of the two papers from the actuary's viewpoint. Discussion from the floor will follow.

MR. ROBERT J. JOHANSEN: This is the second of three sessions arranged for by the American Statistical Association. +

Our first speaker is Dr. Roger G. Ibbotson who will discuss The Future of Stock Returns.

Dr. Ibbotson is Senior Lecturer in Finance at the University of Chicago Graduate School of Business. He has an MBA from Indiana and a Ph. D. in finance and economics from the University of Chicago. His teaching interests include corporate finance, investment theory and real estate. His research efforts have covered measurement of wealth and rates of return as well as long range planning and prediction.

A second edition of Dr. Ibbotson's book Stocks, Bonds, Bills and Inflation: Historical Returns (1926-1978) was recently published. He is also a consultant for banks, utilities and other corporations.

The second speaker is Dr. Gershon N. Mandelker, Associate Professor in Finance at the University of Pittsburgh Graduate School of Business.

Dr. Mandelker came to the U.S. in 1968. He has a B.A. from the Hebrew University in Jerusalem. He has a Masters and Ph.D. in Finance from the University of Chicago. He has taught at the University of Chicago, Carnegie-Mellon University and has done research on inflation, mergers and acquisitions of industrial corporations, cost of capital and valuation of firms.

\+ See introduction to Concurrent Session, Recent Advances in Prediction Theory on page 1227.

The Discussant is William A. Dreher, FSA who is well known to many of you. Bill Dreher is a Principal in the Executive Office of Peat, Marwick, Mitchell and is National Director of Employee Benefit Services. Since 1968 he has focused on integrating asset and liability planning for pension plans and fostering an understanding of the implications of the economy and capital markets on investment policies and the choice of actuarial assumptions.

DR. ROGER G. IBBOTSON*: In our book Stocks, Bonds, Bills, and Inflation: Historical Returns (1926-1978), Rex Sinquefield and I show how to use historical returns to forecast the future. The returns of stocks, or any other asset such as bonds or real estate, can be forecast by recognizing two factors. They are (1) the expected rate of inflation and (2) the expected risk return tradeoff.

Inflation can be estimated from bond yield curves, extrapolating historical rates, or studying the economy directly. I personally favor using yield curves which reflect the market's anticipations of what will happen. Thus a yield on a long term bond reflects what both investors and issuers think will happen to inflation rates until the bond matures. Bond holders demand a high enough yield to compensate them for any losses due to inflation. Given zero real rates and no default or maturity premiums on a bond yield, the yield directly measures the bond market's inflation rate forecast.

We will focus our discussion on the risk return tradeoff to stock returns. Thus the expected return for common stocks can be thought of as the sum of the expected real rate of interest, the expected inflation rate, and the expected risk return tradeoff.

The most direct way to estimate the risk return tradeoff for common stocks is to measure what it has been historically. Exhibit 1 shows that $1.00 invested in 1926 would be worth $89.59 by year end 1978 if all dividends were reinvested. This is much greater than the growth in bond portfolios, and much greater growth than if the $1.00 had been invested in U.S. Treasury bills. In fact, investments in U.S. Treasury bills have almost exactly matched the cumulative inflation over the period. This implies that the real (ex-inflation) rate of interest has been zero historically.

However, risky investments such as stocks have had high returns. Exhibit 2 shows how risky stocks are relative to government bonds by plotting total returns year by year.

Exhibit 3 summarizes the annual returns of the various asset categories historically. The geometric mean measures the compound returns and the standard deviation measures the risk.

Note that it is only the high risk stocks that have had high returns. The nearly riskless Treasury bills have almost exactly matched the inflation rate.

*Dr. Ibbotson, not a member of the Society, is Senior Lecturer in Finance at the University of Chicago.

Exhibit 4 breaks the returns into components. It shows that the real rate of interest has been zero historically, so that the only way investors can expect to earn positive returns in real (inflation adjusted) terms is to take on risk. The payoff for risk historically has been 6.2% compounded for investing in common stocks instead of Treasury bills.

Exhibit 5 enlarges the investment universe to include real estate and all three types of bonds (U.S. government, corporate, and municipal). Although common stocks have historically performed the best, real estate has done second best. Municipal bonds have done the worst (before taxes), but this could have been anticipated since their yields are always lower than other bonds in that their coupons are tax exempt.

My forecast for the future is quite simple and follows directly from the historical results. As stated earlier, the expected return on stocks is the sum of the expected inflation rate, the real rate of interest, and the risk premium. I think that both real rates and risk premiums will over the long run equal their historical averages. Thus I believe the real rate of interest will be zero and the risk premium on stocks will be slightly in excess of 6% compounded annually.

This gives us a forecast for common stocks of 6% plus anticipated inflation. Gershon Mandelker, of the University of Pittsburgh, will present evidence that high inflation rates adversely affect real interest rates and risk premiums. If this is so, the forecast would be expected inflation plus something less than 6% when inflation rates are high and expected inflation plus something greater than this 6 percent rate when inflation rates are low.

In sum then, I expect the future annual return on common stocks to be the expected inflation rate plus 6%. Gershon Mandelker may want to adjust this forecast downward because of today's high inflation rates. If we forecast inflation to be 10% long term, then I would be forecasting a 16% compounded return on common stocks. This 16% return would be far in excess of the historical rate of return of 8.9% for common stocks over the period 1926 to 1978. However, inflation rates only averaged 2.5% over that period. The higher anticipated inflation rates get reflected in higher expected returns for all types of securities.

DR. GERSHON N. MANDELKER**: The topic of this presentation is to report on an update of a paper presented before the American Finance Association four years ago in December 1975. It was one of the first papers that dealt with the impact of inflation on returns from common stocks, bills, notes and bonds. Since then this topic has received wide attention in the literature, yet the theoretical and empirical work to date has failed, up to now, to give an appropriate explanation of why returns on common stocks react the way they have to inflation; in a way that is contrary to long-held views and to the widely accepted economic theory. It still remains a puzzle. Why are investments in common stocks not a hedge against inflation? As common stocks represent ownership of real goods their value should increase according to general price inflation.

**Dr. Mandelker, not a member of the Society, is Associate Professor of Finance at the University of Pittsburgh.

A similar phenomenon, though almost in the opposite direction happens when we look at the impact of inflation on returns on bonds, notes and bills. Here it has been believed that returns on fixed income securities would be negatively affected by inflation. As these obligations are fixed, so runs the argument, an increase in the rate of inflation would depress bond prices and bondholders will come out on the losing side. However the empirical results do not seem to be consistent with this view. Again, contrary to perceived wisdom, bonds seem to be a better hedge against inflation than expected.

I will not present data on mean returns for different securities. Ibbotson and Sinquefield have meticulously accumulated and clearly presented these data. My purpose here is not to present data on total returns. I will present research on the topic of the impact of inflation on the returns to an investor in common stocks and on fixed income securities,i.e., bonds, notes and bills.

This analysis may shed some light on the price we have paid and may expect to pay for the high rates of inflation. It is relevant not only to direct investments in the securities markets but also to indirect investors such as investors in most retirement, pension funds and mutual funds as well as investment in human capital. In short, it is relevant to the present and future worth of corporate and noncorporate America, as it is affected by inflation.

We can investigate the effectiveness of investment in common stock as a hedge against inflation by using the following regression.

(1) $R_{mt} = a + b R_{It} + e_t$

where R_{mt} is an index of returns on common stocks and R_{It} is an index of concurrent inflation or expected inflation.

The first table investigates this relationship between 3 indices of common stock each with the current rate of inflation. The first, R_{Et}, is an equally weighted index of rates of return on all common stocks listed on the NYSE during the period 1965-1978. The parameter b shows the sensitivity or the effect of a change in the rate of inflation on the returns on the index of common stocks.

As we can see the relationship is negative and "significantly" so. A one percentage rate of inflation will reduce the return by 5.76 percent, or a rate of inflation greater than Z = 0.625 (7.5% annually) will cause a negative total return! (3.6/5.76 = 0.625 = a). Similar results are obtained with R_{Vt}, a value weighted index of all NYSE stocks, as well as with the NASDAQ index. It seems that the smaller the firms are the more negative is the impact of inflation. The t values are all large. R^2 (coefficient of determination) are relatively small. This only indicates that there are other more important factors affecting the rate of return on common stocks. In table 2 we have included a dummy variable for the period 8/71 - 12/74 during which there were price controls. This somewhat reduces b (but very little); the logic behind it is that during the period of price controls any small increase in inflation has a larger negative impact on common stocks than in other periods. R^2 goes up somewhat as the explanatory power of the regression also increases.

In table 3 we show a similar regression but instead of the rate of inflation we use the holding period returns on treasury bills. The holding period returns are used as a proxy for expected inflation. Presumably the price of a treasury bill will reflect the expected rate of inflation for the length of the period to maturity plus the real rate of interest. If we assume that the real rate of interest changes very little, then the holding period returns on a treasury bill is a good proxy for the expected rate of inflation. b is again negative but more than twice as large (in absolute value). A one percent expected inflation rate will cause the return on common stock to be smaller by 10-14%. (if the return on a bill exceeds Z, .52% (or 6.3% annually) the returns on the index R_{Vt} will be negative!! .065/12.38 = .0052 = a)

In table 4 we have a similar regression but with a dummy variable for the period of price controls. b is lower, R^2 increases, but t (c) is "significant".

Up to now, we have looked at the impact of concurrent inflation and of expected inflation. Now we will investigate the impact of unexpected inflation.

$R_{TB3} - R_{INF}$ denotes expected inflation minus actual inflation (i.e. this is a proxy for unexpected inflation). Tables 5 and 6 show the results of the following equation:

$$R_{mt} = a + b\,(R_{TB} - R_{INF}) + U_t$$

The results are similar. Unexpected inflation has a negative impact on returns on common stock.

An interesting question is how does expected inflation affect the real returns (i.e. after discounting for actual inflation) on common stocks. Real returns are computed by the formula

$$r_{mt} = \frac{1+R_{mt}}{1+R_{INF}} - 1$$

Tables 7 and 8 show results of the regression $r_{mt} = a + b\,R_{TB} + e_t$.

Surprisingly real returns are also affected negatively by expected inflation and even more so than nominal returns.

We deal here with government securities. Following Fisher (1930) we postulate the following relation: $E(R_{bxt}) = a + E(I_t)$ where $E(R_{bxt})$ is the expected holding period return over a month t on a bond maturing in x periods. a is a constant and represents the real rate of interest. $E(I_t)$ is the expected rate of inflation.

As a proxy for $E(I_t)$ we use holding period returns for a one month T-bill. The above equation implies that the expected holding period return on any fixed income security fully reflects anticipated inflation.

The empirical counterpart of the above regression is (1) $R_{bxt} = a+BR_{Bt}+ e_t$ where it is expected that B=1. The upper part of table 9 shows the results of regression (1). It seems that returns on bonds are <u>over-responsive</u> to inflation as $B>1$. In four of the five regressions it is positive (only in the first two is it significantly different from zero). The R^2 is quite high in the first two regressions, which means that the expected rate of inflation is the major determinant of the holding period return. The lower part of table 9 shows results of a similar regression but for the real returns on the above bonds. Where, $r_{bxt} = \frac{1+R_{bxt}}{1+R_{INF}} - 1$

(2) $r_{bxt} = a + BR_{Bt} + U_t$

Here we expect B=0 as the theory would predict no effect of inflation. In four of the five regressions $B>0$ but insignificantly so. Another proxy for E(I) was used with similar results. We can only say that there is no clear impact of inflation on returns on fixed income securities.

Table 10 shows results of a regression on unanticipated rates of inflation. Here nominal returns are not affected but real returns are inversely affected and more so the longer the term to maturity.

Table 11 shows results of a regression on the concurrent rate of inflation. In four of the five regressions $B>0$ and three of them are significantly so. The holding period returns on bonds is positively related to inflation. However $B<1$, so that bondholders are not fully compensated for the rate of inflation ; a result consistent with that in table 10.

In sum, investment in bonds is a better hedge against inflation than common stocks. These results are contrary to long-held views on investments in these two types of securities.

MR. WILLIAM A. DREHER: The composition of this panel appeals to me because it demonstrates the value of having different professions come to grips with important business and social issues, each approaching them from their own perspective. Hopefully the integration of all these views will improve our collective knowledge.

My own work on the relationship between returns on bonds and stocks was privately published in 1974, shortly before the first Ibbotson and Sinquefield paper. We didn't have the breadth nor length of the data base that Ibbotson and Sinquefield were able to draw upon. Our conclusions focused on the broadly based corporate bond market and the stock market, as represented by the S and P 500, for the period 1948 through 1972 but were basically similar. We observed the same standard deviation of short-interval returns in the bond market, but derived a different conclusion about the volatility of common stock portfolios. During those 25 years the annual standard deviation of stock returns was approximately 14 percent, as compared to the 22 percent which Roger and Rex computed for the years 1926 to 1972.

One of the questions I asked myself while listening to our two guests is: how do we, if we can, tie together the two theses they have developed?

Let me outline one way of looking at that issue. I find persuasive the analysis represented by the Ibbotson-Sinquefield paper, which suggests that the marketplace, over extended time intervals, does reward the investor for the type of risk he accepts. That risk can be described as Dr. Ibbotson has, and it can also be associated with the variability of short-term returns. But how does one link that judgment about long-term equilibrium relationships with the conclusions reached by Dr. Mandelker from his research? I looked at that question in a limited way two years ago, by using economic series taken from U. S. Department of Labor data and the total returns in two capital market indices, the S and P 500 stock index and the Salomon Brothers Long Term Corporate Bond Index. We correlated the annual rates of change in those time series with one another and with the change in the consumer price index. We found, as Dr. Mandelker reported, that there is a negative correlation between total returns on the stock market and the CPI. The correlation was -.42.

We also saw an essentially neutral correlation (-.04) between the bond market and the CPI. How does one reconcile the Ibbotson and Mandelker conclusions? In my view one should look at the rate of change in the rate of inflation. If the slope of the inflation curve is rising, then it makes sense to me that the investor would expect an adverse economic climate and one would forecast poorer short-term returns in both the stock market and the bond market.

This supports Mandelker's conclusion about short-term relationships. Ibbotson's conclusion about long-term relationships also survives a common sense test. If real returns for more risky assets are not higher than those for more secure assets, who will buy the more risky (or volatile) assets? When the long term trend of inflation is rising, real returns suffer, but not so severely as Dr. Mandelker's analysis suggests. For example, real returns on corporate bonds were only 1.8% for the period 1928-78 - about 1% less than the 2½% to 3% real return that convention suggests is appropriate for such securities.

When faced with the investment policy decision for a pension fund, we must focus on tax-free returns on assets. I believe we should accept the Ibbotson-Sinquefield premise that there is an equilibrium relationship between total returns on bonds and stocks which is a reflection of the forecast rate of inflation plus the risk premium attached to each class of securities. Let us say we assume that the long-term inflation rate is 9 percent, which is not inconsistent with the reference made earlier to the 10½ to 11% yield to maturity now available on 20-year corporate bonds. Assume roughly a 2 percent real return on bonds and lay on that perhaps a 6 percent equity risk premium. You come up to a 17% equilibrium level of total returns on stocks. Now let us stop and ask what the market is saying today. We know the value of the S and P 500 stock index and the current level of dividends. We can make a projection about the growth rate of earnings over the next several years and the dividend payout ratio which will be applied to those dividends. By combining this data with an assumption about the long term equilibrium return for the market, which carries with it an implied price earnings ratio for the market, we can solve for the expected return on the market from today to any point in the future.

The assumptions which lead to a 17 percent expected total return on the market imply a P/E ratio of about 8.1 on the market. As of September 30th, the P/E on the market was 7.8, rather close to that equilibrium P/E. One set of valuation assumptions we recently tested suggests that the 5-year expected return on the stock market from that date to 1984 is about 16%. This result seems to say that the stock market is reflecting currently about a 8½ to 9 percent rate of inflation, which was consistent with the expectation for common stocks over the long run. It is also roughly consistent with the historically observed risk premium on corporate bonds and common stocks. These relationships tell me that we have a fairly good validation of the premise that the stock market is an efficient predictor. I think we can infer that the market is saying that the long-term inflation rate is going to be somewhere in the 8½ - 9 percent range.

The issue is what do we do today with our capital. What are other people thinking about the future outlook for the inflation rate? Here we come to an interesting puzzle. I mentioned in yesterday's session, when we were discussing the impact of inflation on pension funds, that the consensus view of a group of financial economists for major institutions, as reported in a Peat Marwick survey conducted earlier this year, estimates a long-term inflation rate of about 5.3 percent, using a time frame of around 20 years. If that view, or anything between that estimate and 8½ and 9 percent, is a better estimate of the future behavior of the economy, then we have a rather uniquely interesting opportunity in the common stock markets today. If the equilibrium level of inflation is only 7 percent, the network of equations I have just described would imply a 5 year total return on the stock market of about 20-21%. It would imply long-term bond returns of around 9 percent and it would imply a spread between expected bond returns and expected stock returns that is quite wide in historic terms and very attractive to an investor capable of tolerating the shorter-term volatility that is inherent in the equity markets.

I'd like to make just one or two comments about modeling. My firm's pension fund financial planning model incorporates capital asset pricing theory, in addition to certain elements of the actuarial basis of the pension fund and specific attributes of the pension portfolio, such as its asset mix, beta factor, turnover rate and investment expense. The major capital market variables are the inflation rate, the expected nominal returns on bonds and stocks, the standard deviation of returns around the mean for any particular asset category and the covariance between asset categories. The standard deviation of common stocks is an enormously important assumption. With a smaller standard deviation the risk of failure to achieve any modest expectation in the equity markets drops very sharply and you can easily reach the conclusion that a bond portfolio has a higher risk of failing to achieve, let's say, an 8 or 9% actuarial assumption than a portfolio with a heavy equity component. If you assumed the 22 percent historic standard deviation, for stocks, you will in many cases be cautioned against heavy investment in equities. Is the future volatility of the stock market likely to resemble the past 50 years of history? Or do post-World War II data give a better forecast? There is a significant element of judgement here. Another issue for the pension plan sponsor relates to the investment policy and techniques for reducing portfolio volatility. There are two weapons available to an investor today: one is international diversification and the other is the options market. Both can produce lower portfolio volatility and a lower standard deviation of short term investment performance.

Another attribute of our model is that we incorporate the pension fund's cash flow characteristics, i.e., the cash flows for benefit payments and expected contributions, as well as certain key elements of the actuarial basis and ERISA's funding limits. These refinements can have a significant impact on the plan sponsor's choice of an investment policy.

DR. IBBOTSON: The one thing that is unusual about Gershon's result is that even when inflation is anticipated it seems to be detrimental to equity returns. Now the reason that this is so disturbing is that it doesn't make any economic sense, and Gershon offers no explanation. Don't forget he's got a very low R^2 so he doesn't pretend that this explains a great deal of what's going on but that the result is statistically significant is important in its own right and so I'd just as soon ask Gershon to what extent he really believes his results.

DR. MANDELKER: Well there is no question now, five years after it has been published in at least 10 papers by very well known economists that have duplicated it and found similar results. So there's no question of believing it or not; it seems that it's a fact. But the question is whether you can make money on it.

MR. DREHER: If I could make another comment, I think the issue here is one of time horizons. The focus of the first paper is heavily emphasizing long-term equilibrium relationships. The second concentrates on short interval data. The two concepts can be tied together by linking shorter term economic estimates with the level of inflation implied in current market values to develop intermediate term investment strategies. Let me offer 2 or 3 examples. If you looked at the market in the summer of 1972 and said: "What is the relationship and the relative risk of being in bonds and stocks at this time?" You had to be willing to presume that the inflation rate from that point forward was going to be about 3 percent in order to have a sufficient spread between the then available bond return and the implied stock return in order to justify assuming the stock risk. And it appeared not to be a good bet. That same form of analysis in the Fall of 1974 suggested that the downside risk of the stock decision from that point in the market, at which point the market had fallen some 40 percent from the high of 1972, was in fact an investment risk well worth taking. The annualized total returns on the markets in the 5 years from September 1974 through September 1979, during which time the Consumer Price Index rose at an annual rate of 8 percent, were 9.8% for the Lehman Brothers Kuhn Loeb Corporate Bond Index and 16.8% for the S and P 500 Stock Index. These two examples from recent history offer a rough validation that one can combine the insights from Ibbotson and Mandelker in deriving an intermediate investment strategy decision. And as I mentioned earlier if you feel that the inflation rate over time will be less than 8½ - 9 percent, it would appear that today is a time when the market offers attractive opportunities.

MR. ROBERT F. REDDINGTON: I want to applaud the panel. This is one of the most fascinating discussions that I've attended.

Not too many years ago we attended similar sessions here, we talked about a real rate of return, premium for inflation, and a premium for risk in setting the most critical actuarial assumption, the assumed investment return on the funds. In going through this analysis we added these components and developed interest rates, which in neither the short-term nor long-term have been realized in many large pension funds and I'm wondering what the implications are for us as actuaries in following up what I call actuarial losses. That's one component.

With that in mind I would like Mr. Dreher to comment on what an appropriate asset mix might be.

DR. IBBOTSON: Let me respond to the asset allocation problem. That is, how much to invest in stocks and bonds, etc. First I'd like to say that I don't think this is an actuary's problem.

Second, my belief is that it's purely a matter of the kind of risk a corporation desires to take and in a larger sense it isn't really even an important question because it's only one small part of the asset portfolio of the corporation. It's just purely a question of what kind of risk you want to take and you will probably get paid off for taking risks, but you might not. There is no optimum answer. Even if there was an optimum answer it wouldn't be an actuary's problem.

MR. DREHER: I disagree with Roger's opinion. Actuarial principles, as this audience knows, require that our work focus primarily on the Pension Fund or the Insurance Company as a microcosm. Looking at its network of liabilities and assets, we are forecasting those liabilities in nominal terms and making estimates of the future events affecting the benefit and contribution cash flow. The inevitable and necessary corollary of that approach to forecasting the liabilities is that we should be forecasting the expected nominal total return on the plan's assets. The plan sponsor's judgment about the appropriate investment policy and the results to be derived from it need to be factored into the actuarial equation. My own view, assuming a typical, viable company with positive cash flow etc., is that the pension fund should have a long-term bias toward common stocks.

DR. IBBOTSON: I want to make one final comment. The discount rate that is used in calculating an actuarial liability should be independent of whatever is held on the opposite side.

MR. JERAULD G. SPIGAL: That's actually what my question was. Could you expand a little as to why that's so?

DR. IBBOTSON: Essentially it all stems from the Miller-Madigliani separation principle between the asset side and the liability side of the balance sheet.

The two sides of a balance sheet have the same value, but any particular component need not be of the same risk as any component on the liability side. What you invest on the asset side should have no effect on what the actuary is doing as far as calculating the liability. The value of the liability is exactly the same no matter what the assets are invested in. A liability is a liability and if the firm were to change its financial policy and invest in something else it would have no effect on the present value of a stream of payments - how you pay for that liability is a separate issue and this is the whole Miller-Madigliani separation principle.

MR. SPIGAL: If you're forecasting a certain rate of inflation and a certain risk premium, shouldn't you discount your future payment stream by that rate?

DR. IBBOTSON: No, and let me give you an example. Suppose your liability payments, your pension payments were certain for whatever reason. You knew when everyone was going to die and your turnover rates were exact so there was no risk involved. Now in a situation like that how you pay for it is a separate issue. The liability here is purely the discounted present value of the certain payments and they should be discounted at the riskless rate.

MR. SPIGAL: Why?

DR. IBBOTSON: Because they are certain riskless payments that you have to make. Suppose you were valuing a house. The house shouldn't be worth something different because you happen to be invested in stocks vs. T-bills.

MR. DREHER: You invite another example. I did some work a few years ago for a very large pension fund which was thinking of paying off all of its retired life liabilities. This was a known set of benefits for a closed population. Subject to the accuracy of the mortality tables we could project year by year the dollar cash flow. If we assume that the forecast was accurate, we had an absolute measure of the dollars required year by year. In advising the client, the question I asked myself was: what was the price at which this liability could be transferred from the pension fund to someone else? We looked at it in two contexts. The client could buy a single premium annuity from an insurance company. The annuity rates would reflect the expected nominal return on the mixture of assets purchased with the proceeds of the annuity contract. Or we could assemble a portfolio of marketable securities with a pattern of cash flow, from income and maturities, that matched the cash flows for benefit payments. We tested this second item by using portfolios of government and corporate bonds.

The answer to both questions was a function of nominal returns to the holder of the assets, based upon the chosen investment policy. In neither of the two working models, both of which existed in the real world, was the riskless rate going to give an acceptable answer.

Exhibit 1

WEALTH INDEXES OF
INVESTMENTS IN THE U.S. CAPITAL MARKETS
1926 - 1978

Assumed Initial Investment of $1.00 at Year End 1925
(includes Reinvestment income)

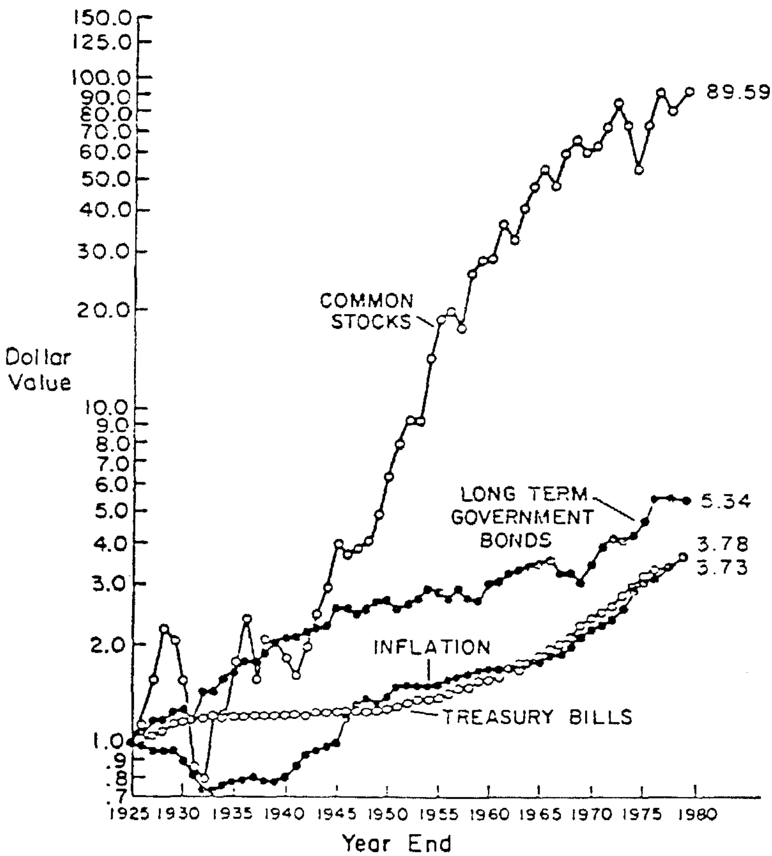

Exhibit 2

VOLATILITY OF ANNUAL RETURNS FROM THE U.S. CAPITAL MARKETS
COMMON STOCKS vs LONG TERM GOVERNMENT BONDS

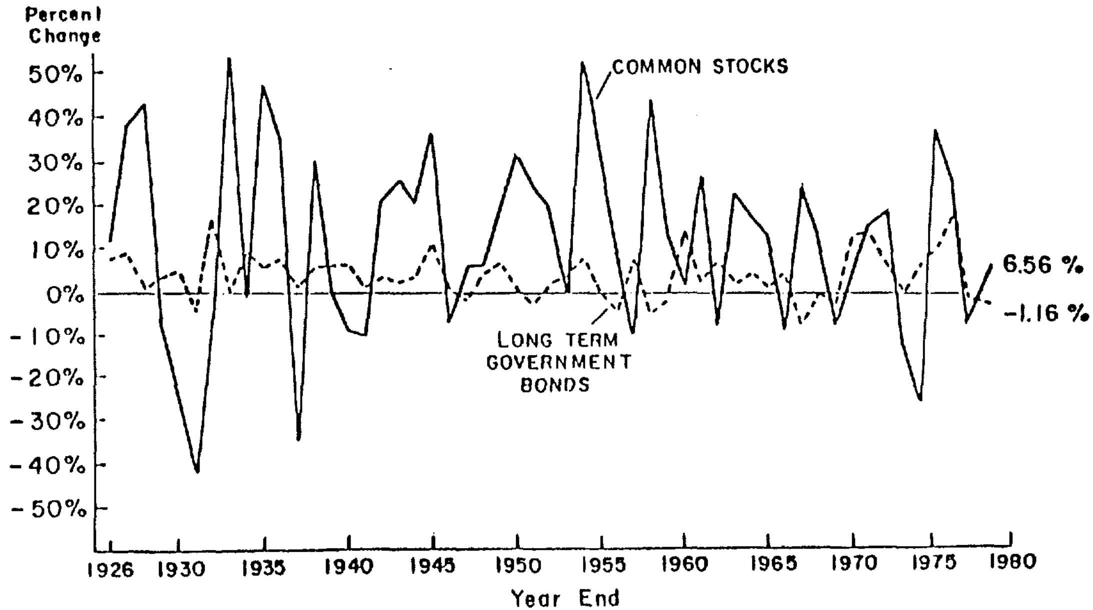

Exhibit 3

BASIC SERIES
INVESTMENT TOTAL ANNUAL RETURNS
1926 - 1978

Series	Geometric Mean	Arithmetic Mean	Standard Deviation	Distribution
Common Stocks	8.9%	11.2%	22.2%	
Long Term Corporate Bonds	4.0%	4.1%	5.6%	
Long Term Government Bonds	3.2%	3.4%	5.7%	
U.S. Treasury Bills	2.5%	2.5%	2.2%	
Inflation	2.5%	2.6%	4.8%	

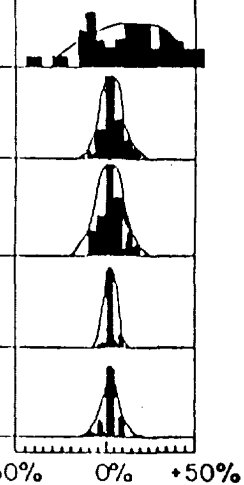

Exhibit 4

COMPONENT ANNUAL RETURNS
1926 - 1978

Series	Geometric Mean	Arithmetic Mean	Standard Deviation	Distribution
Equity Risk Premiums (Stocks-Bills)	6.2%	8.7%	22.3%	
Default Premiums (LT Corps-LT Govts)	0.6%	0.7%	3.2%	
Maturity Premiums (LT Govts-Bills)	0.7%	0.9%	6.0%	
Real Interest Rates (Bills-Inflation)	0.0%	0.0%	4.6%	

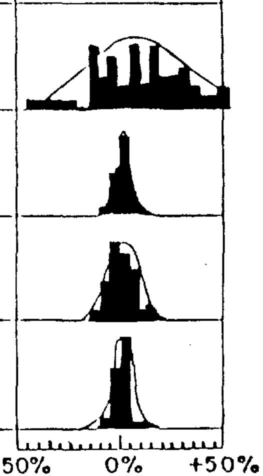

Exhibit 5

Cumulative Wealth Indices of
Capital Market Security Groups
(Year-end 1946 = $1.00)

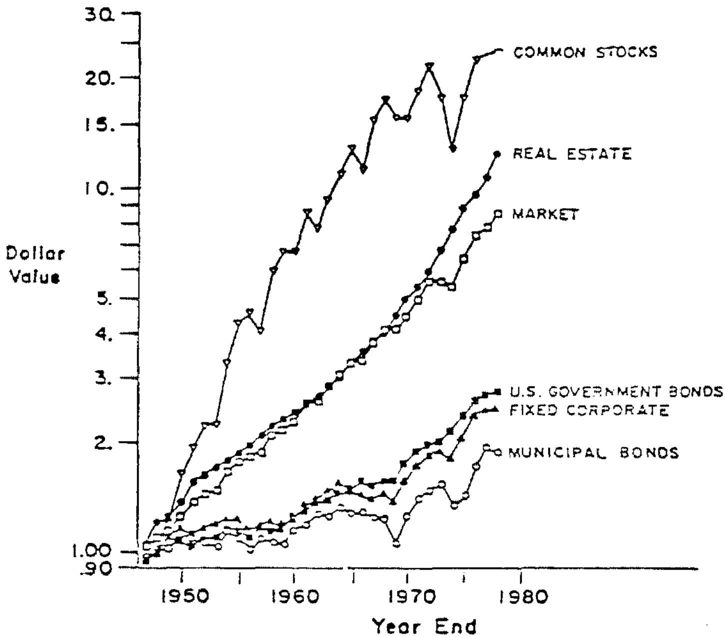

Table 1

$R_t = a + bR_{RINF}$

IND. VAR. R_t	a	b	t(b)	R^2	D.W.	Z Monthly/Annually	
R_{Et}	.036	-5.76	-3.70	.08	1.70	.625	(7.5)
R_{Vt}	.023	-3.96	3.47	.07	1.96	.58	(7.0)
R_{NASDAQ}	.037	-6.32	-4.6	.11	1.64	.58	(7.0)

R_{INF} - Monthly rate of inflation

R_{et} - Monthly rate of return on an equally weighted index of all NYSE common stock

R_{vt} - Monthly rate of return on a value weighted index of all NYSE common stock

R_{NASDAQ} - NASDAQ index of common stock

Z - if the rate of inflation (or the return of a T-bill) exceeds Z the return on the dependent variable will be negative.

Table 2

$R_t = a + bR_{RINF} + cD_t$

IND. VAR. R_t	a	b	t(b)	c	t(c)	R^2	D.W.	Z Monthly/Annually	
R_{Et}	.037	-5.1	-3.2	-.02	-1.6	.09	1.70	.7	(8.7)
R_{Vt}	.020	-3.7	-3.1	-.01	-.8	.07	1.97	.5	(5.2)
R_{NASDAQ}	.037	-5.7	-4.0	-.02	-1.7	.13	1.66	.6	(7.8)

Table 3

$R_t = a + bR_{TB3}$

IND. VAR. R_t	a	b	t(b)	R^2	D.W.	Z Monthly/Annually	
R_{Et}	.065	-12.38	-2.94	.05	1.78	.52	(6.3)
R_{Vt}	.049	- 9.61	-3.11	.05	1.97	.51	(6.1)
R_{NASDAQ}	.073	-14.46	-3.90	.08	1.67	.51	(6.1)

Table 4

$R_t = a + bR_{TB3} + cD_t$

IND. VAR. R_t	a	b	t(b)	c	t(c)	R^2	D.W.	Z
R_{Et}	.063	-10.66	-2.5	-.02	-1.9	.07	1.82	7.1
R_{Vt}	.051	-8.9	-2.8	-.01	-1.1	.06	1.99	6.7
R_{NASDAQ}	.070	-12.87	-3.4	-.02	-2.0	.10	1.73	6.5

Table 5

$$R_t = a + b\ (R_{TB3} - R_{INF})$$

IND. VAR. R_t	a	b	t(b)	R^2	D.W.	Z Monthly/Annually	
R_{Vt}	.005	-3.8	-2.7	.04	1.93	.13	(1.6)
R_{Et}	.010	-5.9	-3.1	.05	1.67	.16	(2.0)
R_{NASDAQ}	.008	-6.3	-3.7	.08	1.59	.13	(1.5)

Table 6

$$R_t = a + b(R_{TB3} - R_{INF}) + cD_t$$

IND. VAR. R_t	a	b	t(b)	c	t(b)	R^2	D.W.
R_{Vt}	.008	-3.5	-2.4	-.009	-1.1	.05	1.94
R_{Et}	.015	-5.1	-2.6	-.020	-1.8	.07	1.70
R_{NASDAQ}	.012	-5.5	-3.2	-.019	-2.0	.10	1.62

Table 7

$R_t = a + b\, R_{TB3}$

R_1 Dependent Variable	a	b	t(b)	R^2	D.W.	Z Monthly/Annually	
RR_{Vt}	.051	-11.2	-3.6	.07	1.97	.46	(5.5)
RR_{Et}	.068	-13.9	-3.3	.06	1.79	.49	(5.8)
RR_{NASDAQ}	.075	-16.0	-4.3	.10	1.67	.47	(5.6)
RR_{AMEX}	.035	-1.7	-.4	.001	2.01		

Table 8

$R_t = a + bR_{TB3} + cD$

R_1 Dependent Variable	a	b	t(b)	c	t(c)	R^2	D.W.
RR_{Vt}	.050	-10.4	-3.3	-.01	-1.2	.08	1.99
RR_{Et}	.065	-12.1	-2.8	-.02	-2.1	.08	1.81
RR_{NASDAQ}	.073	-14.3	-3.8	-.02	-2.1	.12	2.00
RR_{AMEX}	-	-	-	-	-	.00	-

TABLE 9

Summary Statistics of Regressions Between the Holding Period Return on Bonds and the Riskfree Rate of Interest (Expected Inflation).

Type of Bond	a	b	t(b)	t(B-1)[1/]	Coeff. of Determination	DW
		Regression:	$R_{bxt} = a + B\,R_{Bt} + e_t$			
Two Month Bill	.00016	1.039	45.32	1.80	.9009	2.01
Three Month Bill	.00018	1.113	32.30	3.50	.8219	1.72
One-Two Year Bond	.00001	1.234	5.56	1.26	.1203	1.57
Five-Six Year Bond	-.00081	1.452	2.44	1.10	.0257	1.94
15-20 Year Bond	.00023	.776	.78	- .23	.0027	2.13
		Regression:	$r_{bxt} = a + B\,R_{Bt} + u_t$			
Two Month Bill	.00086	.052	.50	Not relevant	.0011	1.73
Three Month Bill	.00087	.126	1.17	"	.0060	1.76
One-Two Year Bond	.00070	.247	.99	"	.0043	1.60
Five-Six Year Bond	.00011	.466	.76	"	.0026	1.94
15-20 Year Bond	.00093	-.208	-.21	"	.0002	2.14

1/ This statistic indicates whether B is significantly different from one. (i.e, $H_0: B=1$; vs. $H_1: B\neq 1$)

TABLE 10

Summary Statistics of Regressions Between Holding Period Returns on Bonds and Unanticipated Inflation

Maturity of Security	a	β	t(β)	Coeff. of Determination	DW
Regression: $R_{bxt} = a + \beta\,[I_t - R_{Bt}] + e_t$					
Two Month Bill	.0029	- .0044	- .0929	.0000	.29
Three Month Bill	.0031	.0039	.0735	.0000	.44
1 to 2 Year Bond	.0031	- .1423	- .931	.0038	1.45
5 to 6 Year Bond	.0026	- .5907	-1.523	.0102	1.93
15 to 20 Year Bond	.0018	- .6520	-1.011	.0045	2.12
Regression: $r_{bxt} = a + \beta\,[I_t - R_{Bt}] + u_t$					
Two Month Bill	-.00027	- .998	-63.0	.946	2.006
Three Month Bill	-.00048	- .990	-40.9	.881	1.996
1 to 2 Year Note	-.00053	-1.12	- 7.36	.193	1.576
5 to 6 Year Bond	.000015	-1.55	- 3.801	.0601	1.948
15 to 20 Year Bond	.000712	-1.50	- 2.196	.0209	2.120

TABLE 11

Summary Statistics of Regressions Between the Holding Period Return on Bond, R_{bxt} , and the Rate of Inflation, I_t

$$(R_{bxt} = a + B\ I_t + e_t)$$

Type of Bond	a	B	t(B)	Coeff. of Determination	DW
Two Month Bill	.00231	.306	8.98	.2629	.77
Three Month Bill	.00246	.334	8.65	.2488	.92
1 - 2 Year Note	.00273	.266	2.08	.0188	1.81
5 - 6 Year Bond	.00298	.012	.04	.00005	1.93
15 - 20 Year Bond	.00271	-.233	- .43	.0008	2.12