PRICING A SELECT AND ULTIMATE ANNUAL RENEWABLE TERM PRODUCT

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ABSTRACT

This paper discusses the special considerations involved in pricing a select/ultimate annual renewable term product. It covers such areas as expenses, conversion costs, and profit calculation, and devotes particular attention to the relationship between mortality and withdrawals on this type of product. A general equation for computing the extra mortality under various lapse assumptions is developed.

INTRODUCTION

Hɪɢʜ interest rates in recent years have led increasingly to a "buy term and invest the difference" strategy among insureds. Insurance companies, like all competitive businesses, must shape their products to fit the desires of their market. This need, combined with the increasing emphasis on term insurance, has intensified competition for the term insurance sale. Many companies hope that their term sales will result in conversions to permanent plans, but some view the term market as a profitable end in itself. In this market there is no competition keener than that for annual renewable term (ART)—or yearly renewable term (YRT). A few years ago, some of the innovative smaller companies began marketing a select and ultimate ART (S/U ART) product. Having just completed the pricing of such a product, which, we believe, could well become the top term product in most portfolios, we felt it might be helpful to discuss our methods and provoke some discussion within the profession.

PRODUCT DESCRIPTION

The usual ART product (which we call the aggregate ART) has annually increasing premiums that vary by attained age only. As the name implies, S/U ART has annually increasing premiums that vary by issue age and duration since underwriting. For the first few durations after underwriting, premiums are quite low; these durations constitute the select period, which generally lasts four or five years, although,

beginning in late 1979, products with a one-year select period began appearing on the market, and products with select periods of ten or fifteen years are not unheard of. While the logical select period would seem to be the select period inherent in the mortality table used for product pricing, in practice one sees the range mentioned above.

At the end of the select period, the insured will pay premiums from the ultimate rate scale; these premiums vary only by attained age. The insured can seek to avoid paying these higher ultimate rates by exercising the reversion feature usually found in S/U ART plans. This feature gives the insured the opportunity at the end of the select period (or earlier for some of the products with long select periods) to provide new evidence of insurability at the company's expense; if the insured is still a standard risk, a new policy is issued at the select rate for the new issue age. The annually revertible (one-year select period) products of which we are aware have less stringent requirements for reversion. The insured may only have to answer three or four questions about his health in the past year. Presumably, much of the excess mortality over that of a new issue is offset by reduced underwriting costs.

The reversion process can be repeated as long as the insured remains a standard risk and below a specified age (typically 70). In the case of a reversion, the agent usually receives a commission equal to 50-100 percent of the first-year commission for a new issue. In this paper, we will examine S/U ART products that require full evidence of insurability for reversion and that pay a full first-year commission to the agent upon reversion. Table 1 compares aggregate ART and S/U ART rates (five-year select period) for issue age 45.

PRICING ASSUMPTIONS

Mortality and Lapses

These will be treated together, since we believe they are intimately connected and are of fundamental importance in pricing this product. We will assume that lapse and mortality experience is available for an aggregate ART plan and that we are attempting to produce premiums for an S/U ART product. Any lapses in excess of the corresponding aggregate ART lapses will be called "reversions." Insureds who do not revert will be called "persisters." We will start with two examples, which we will then expand into a generalized formula for computing mortality rates.

Suppose, for the first example, that we have a select and ultimate ART product with a five-year select period. In pricing the product, we assume that lapses in years 1-4 and in years 6 and over are the same as

those for aggregate ART (that is, no reversions occur in those years); however, to consider the reversion feature, we assume that there is a single reversion of 50 percent of the survivors at the end of year 5 and that these "reverters" are all standard risks. (These "lapses" due to reversion at the end of year 5 are assumed to be in addition to the normal lapses that might be experienced on an aggregate ART where there is less incentive to lapse to obtain a lower premium.) It should be noted that this is not a realistic assumption, since insureds could revert and obtain a lower premium before the end of the select period either with another company or with the same company if such a practice were allowed. Nevertheless, let us assume now, for simplicity, that the only reversions occur at the end of the fifth year.

TABLE 1

COMPARISON OF RATES FOR S/U ART AND AGGREGATE ART

Issue Age	Aggregate ART	S/U ART						
		Select Years					Ultimate Years	
		1	2	3	4	5	Attained Age	Rate
45	4.86	2.98	4.13	5.70	6.92	8.15	50	9.45
46	5.31						51	10.31
47	5.79						52	11.31
48	6.33						53	12.33
49	6.91						54	13.44
50	7.56	3.83	5.84	8.31	10.42	12.53	55	14.68
51	8.25						56	16.10
52	9.05						57	17.66
53	9.86						58	19.33
54	10.75						59	21.16
55	11.74	5.03	7.87	12.23	16.08	19.96	60	23.18
56	12.88						61	25.33
57	14.13						62	27.68
58	15.46						63	30.10
59	16.93						64	32.65
60	18.54	7.67	11.36	18.59	25.18	31.77	65	35.96
61	20.26						66	39.28
62	22.14						67	42.69
63	24.08						68	46.56
64	26.12						69	51.00
65	28.77	13.19	18.83	29.13	38.60	48.06	70	57.21
66	31.42						71	64.30
67	34.15						72	72.41
68	37.25						73	81.54
69	40.80						74	91.60
70	45.77	21.88	29.24	43.32	54.21	74.96	75	101.71
71	51.44						76	113.08
72	57.93						77	125.74
73	65.23						78	138.44
74	73.28						79	154.88
75	81.37							
76	90.46							
77	100.59							
78	110.75							
79	123.90							

Assume, then, that $q_{[x]+t}$ represents the aggregate ART mortality rate and $(qp)_{[x]+t}$ the mortality rate for the persisters on a select and ultimate ART product. Because it is assumed that those who revert must meet full standard underwriting requirements for a new issue, those who revert at the end of policy year n will "start over" with mortality rate $(qr)_{[[x]+n]+t}$. Under our assumption of full underwriting at reversion, $(qr)_{[[x]+n]+t} = q_{[x+n]+t}$, where $x + n$ is the age at reversion. In addition, we assume that the total deaths experienced by the reverters and persisters will equal the total deaths that would be experienced by a group of the same size on an aggregate ART product. We also assume that the underlying lapse rates (apart from the reversion rate) for reverters and persisters are the same and are equal to those of an aggregate ART plan. These assumptions lead to three conclusions:

1. $(qp)_{[x]+t} = q_{[x]+t}$ for $t < 5$. This follows from the assumption that lapse rates are the same for both the aggregate ART and S/U ART products before the end of the fifth year. It also assumes that there is no additional antiselection on an S/U ART and that underwriting standards are the same for both products.

2. $(qp)_{[x]+t} > q_{[x]+t}$ for $5 \leq t < 5 +$ (select period in pricing mortality table). This follows from the assumption that the 50 percent that leave the population through reversion at the end of year 5 are all standard risks, indicating that the persisters have a mortality rate higher than that of the comparable aggregate class.

3. $(qp)_{[x]+t} = q_{[x]+t}$ for $t \geq 5 +$ (select period in pricing mortality table). This follows from the assumption that total deaths for persisters and reverters equal total deaths under an aggregate ART product. Thus, after the effects of selection assumed in the pricing mortality table have worn off, persisters and reverters experience the same mortality rates.

These conclusions will be seen more clearly in the development of the formulas needed to calculate $(qp)_{[x]+t}$. To develop those formulas, we add the following definitions to those we already have:

$l_{[x]+t}$ = Total number of survivors t years after issue at age x.
$\phantom{l_{[x]+t}}$ = Total number of reverters and persisters at duration t.

$(lp)_{[x]+t}$ = Number of persisters at duration t after issue at age x.

$(lr)_{[[x]+n]+t}$ = Total number of survivors t years after reversion at age $x + n$, where the original issue age was x. Note that a distinction is being made between, say, $(lr)_{[[35]+5]+t}$ (which represents total survivors t years after reversion at age 40 for issue age 35) and $l_{[40]+t}$ (which represents total survivors t years after issue at age 40).

$q^w_{[x]+t}$ = Aggregate ART lapse rate at duration t.

$d_{[x]+t}$ = Total number of deaths between durations t and $t+1$ after issue at age x

= Total number of deaths among reverters and persisters between durations t and $t+1$.

$d^w_{[x]+t} = l_{[x]+t}q^w_{[x]+t}.$

$(dp)_{[x]+t}$ = Number of deaths among persisters between durations t and $t+1$ after issue at age x.

$(dp)^w_{[x]+t} = (lp)_{[x]+t}q^w_{[x]+t}.$

$(dr)_{[[x]+n]+t}$ = Number of deaths between durations t and $t+1$ after reversion at age $x+n$.

$(dr)^w_{[[x]+n]+t} = (lr)_{[[x]+n]+t}q^w_{[x]+n+t}.$

Note that all l_{x+t} functions are calculated as $l_{x+t} = l_{x+t-1} - d_{x+t-1} - d^w_{x+t-1}$, where d^w_{x+t} reflects expected lapses under an aggregate ART product. Similar formulas apply to the calculation of $(lp)_{x+t}$ and $(lr)_{x+t}$. Note also that the rate of lapse, q^w_{x+t}, is assumed to be the same for persisters and reverters.

Given these assumptions and definitions, the following formulas emerge (see Appendix I for a comparison of aggregate ART mortality and S/U ART persister mortality as produced by these formulas).

First,
$$(lr)_{[[x]+5]} = 0.5l_{[x]+5},$$

$$(lp)_{[x]+5} = 0.5l_{[x]+5},$$

$$d_{[x]+5} = (dp)_{[x]+5} + (dr)_{[[x]+5]}.$$

Second,
$$q_{[x]+5}l_{[x]+5} = (qp)_{[x]+5}(lp)_{[x]+5} + (qr)_{[[x]+5]}(lr)_{[[x]+5]},$$

$$(qp)_{[x]+5} = \frac{q_{[x]+5}l_{[x]+5} - (qr)_{[[x]+5]}(lr)_{[[x]+5]}}{(lp)_{[x]+5}}$$

$$= \frac{l_{[x]+5}(q_{[x]+5} - 0.5q_{[x+5]})}{0.5l_{[x]+5}}, \quad \text{since } (qr)_{[[x]+5]} = q_{[x+5]}$$

$$= 2q_{[x]+5} - q_{[x+5]}.$$

Third,
$$d_{[x]+6} = (dp)_{[x]+6} + (dr)_{[[x]+5]+1}.$$
Thus,
$$q_{[x]+6}l_{[x]+6} = (qp)_{[x]+6}(lp)_{[x]+6} + (qr)_{[[x]+5]+1}(lr)_{[[x]+5]+1},$$

where
$$(lp)_{[x]+6} = (lp)_{[x]+5}[1 - (qp)_{[x]+5} - q^w_{[x]+5}]$$
and
$$(lr)_{[[x]+5]+1} = (lr)_{[[x]+5]}(1 - q_{[[x]+5]} - q^w_{[x]+5}).$$

Thus,

$$(qp)_{[x]+6} = \frac{q_{[x]+6}l_{[x]+6} - (qr)_{[[x]+5]+1}(lr)_{[[x]+5]+1}}{(lp)_{[x]+5}[1 - (qp)_{[x]+5} - q^w_{[x]+5}]}$$

$$= \frac{q_{[x]+6}l_{[x]+6} - q_{[x+5]+1}(lr)_{[[x]+5]+1}}{(lp)_{[x]+5}[1 - (qp)_{[x]+5} - q^w_{[x]+5}]},$$

since $(qr)_{[[x]+5]+1} = q_{[x+5]+1}$.

Fourth, in general, for $t \geq 6$,

$$(qp)_{[x]+t} = \frac{q_{[x]+t}l_{[x]+t} - q_{[x+5]+t-5}(lr)_{[[x]+5]+t-5}}{(lp)_{[x]+t}},$$

where

$$(lp)_{[x]+t} = (lp)_{[x]+t-1}[1 - (qp)_{[x]+t-1} - q^w_{[x]+t-1}]$$

and

$$(lr)_{[[x]+5]+t-5} = (lr)_{[[x]+5]+t-6}[1 - (qr)_{[[x]+5]+t-6} - q^w_{[x]+t-1}].$$

Fifth, for $t \geq 5 +$ (pricing mortality select period),

$$q_{[x]+t} = (qr)_{[[x]+5]+t-5} = q_{[x+5]+t-5} = q_{x+t}$$

$$= \text{Ultimate pricing mortality}.$$

Thus,

$$(qp)_{[x]+t} = \frac{q_{x+t}[l_{[x]+t} - (lr)_{[[x]+5]+t-5}]}{(lp)_{[x]+t}}.$$

Since total deaths and withdrawals for reverters and persisters under an S/U ART product are the same as total deaths and withdrawals under an aggregate ART product,

$$d_{[x]+t} = (dp)_{[x]+t} + (dr)_{[[x]+5]+t-5}$$

and

$$d^w_{[x]+t} = (dp)^w_{[x]+t} + (dr)^w_{[[x]+5]+t-5},$$

which implies

$$l_{[x]+t} = (lp)_{[x]+t} + (lr)_{[[x]+5]+t-5},$$

or

$$(lp)_{[x]+t} = l_{[x]+t} - (lr)_{[[x]+5]+t-5}.$$

Thus the equation in the fifth statement above reduces to $(qp)_{[x]+t} = q_{x+t}$.

Two observations are in order. First, it is not necessary to consider further reversions among the reverters when calculating $(qp)_{[x]+t}$. This follows from our basic observation that the total number of deaths for a group of policyholders is the same regardless of how the group is split—

a sort of conservation-of-total-deaths principle. The basic components in the calculation of $(qp)_{[x]+t}$ are (1) deaths in the $(t + 1)$st policy year from the entire group of policies issued at age x, namely, $d_{[x]+t}$, and (2) deaths among reverters, namely,

$$\sum_{n=1}^{t} (dr)_{[[x]+n]+t-n} .$$

The conservation-of-total-deaths principle implies that deaths among persisters in the $(t + 1)$st policy year equals (1) $-$ (2). The point of the observation is that none of the terms of the sum in (2) is affected by reversions after the reversion that gave rise to the term in the first place. This follows from applying the conservation-of-total-deaths principle to each group of reverters, $(lr)_{[[x]+n]+t-n}$, giving rise to the terms in (2). An analogue to the conservation-of-total-deaths principle can be found in the conservation-of-total-momentum principle of physics. Picture a particle, T, moving along with momentum M_T. Suddenly T splits into two particles, R (as in reverter) and P (as in persister), with momenta M_R and M_P, respectively. Then $M_R + M_P = M_T$. If one knows M_T and M_R, one can calculate M_P. If R splits into two or more fragments, we know that the momenta of the fragments add up to the momentum of R, so consideration of the fragments adds nothing but unnecessary complication to the computation of M_P.

The second observation is that the more reverters there are, the higher $(qp)_{[x]+t}$ will be. This is intuitively obvious, since when a closed group loses its better risks, the mortality for those remaining clearly will be worse than that for the group before the loss.

The second observation leads to some real pricing headaches. The S/U ART products currently on the market generally have minimum issue amounts of at least $100,000. Consequently it seems reasonable to assume that the insureds who purchase these products are on the whole fairly sophisticated and likely to take advantage of situations that will decrease their cost. In other words, one would expect reversions before the limiting date specified in the policy form. Thus our simple assumption of 50 percent reversions at the end of year 5 with no earlier reversions is probably unrealistic. These reversions before the end of the select period create higher than aggregate ART mortality in the remaining select years and decrease the number of insureds over which expenses can be amortized, thus leading to a steeper premium scale. But the steeper the premium scale, the greater the advantage to be obtained by applying for a new select rate. In other words, pessimistic assumptions have a tendency to be self-fulfilling (at least on paper—we have no experience

to go by). One way to combat this problem would be to offer an n-year renewable and convertible term product such that, at the end of n years, the product would be renewable at a relatively low rate if satisfactory evidence of insurability were provided, but at a relatively high rate if no evidence were furnished.

Another problem with an S/U ART product is that slightly substandard cases in the ultimate years may prefer to lapse the S/U product and purchase an aggregate ART product rather than pay ultimate premiums. Such lapses further steepen the premium scale and exacerbate the problem mentioned above of steep premiums causing higher lapses leading to higher mortality and yet higher premiums. Let us hope this is a convergent sequence.

If it is assumed that there will be reversions before the end of the select period, the expected mortality rates will be affected. Suppose, as our second example, we assume the following pattern of reversions:

1. No one reverts in policy years 1 and 2.
2. 30 percent of the survivors revert at the end of policy year 3.
3. 10 percent of the survivors revert at the end of policy year 4.
4. 20 percent of the survivors revert at the end of policy year 5.

In the authors' view, this pattern of reversions probably is more realistic, given the mobile nature of the middle- and higher-amount term market. We again assume that these reversions are in addition to the normal lapses that one might expect in the case of an aggregate ART product. We also assume that all of these reversions are standard risks.

Under this set of assumptions the following formulas would emerge (see Appendix II for a comparison of aggregate ART mortality and S/U ART persister mortality produced by these formulas).

First,

$$(lr)_{[[x]+3]} = 0.3 l_{[x]+3}, \qquad (lp)_{[x]+3} = 0.7 l_{[x]+3},$$

where $l_{[x]+3}$ is some convenient radix;

$$(dr)_{[[x]+3]} = (qr)_{[[x]+3]} (lr)_{[[x]+3]},$$

$$(dp)_{[x]+3} = (qp)_{[x]+3} (lp)_{[x]+3},$$

$$d_{[x]+3} = q_{[x]+3} l_{[x]+3}.$$

Since $d_{[x]+3} = (dr)_{[[x]+3]} + (dp)_{[x]+3}$, then

$$q_{[x]+3} l_{[x]+3} = (qr)_{[[x]+3]} (lr)_{[[x]+3]} + (qp)_{[x]+3} (lp)_{[x]+3},$$

$$(qp)_{[x]+3} = \frac{q_{[x]+3} l_{[x]+3} - q_{[x+3]} (lr)_{[[x]+3]}}{(lp)_{[x]+3]}},$$

because $(qr)_{[[x]+3]} = q_{[x+3]}$.

Second,

$$(lr)_{[[x]+4]} = 0.1[(lp)_{[x]+3} - (dp)_{[x]+3} - (dp)^w_{[x]+3}],$$

$$(lp)_{[x]+4} = 0.9[(lp)_{[x]+3} - (dp)_{[x]+3} - (dp)^w_{[x]+3}],$$

$$l_{[x]+4} = l_{[x]+3} - d_{[x]+3} - d^w_{[x]+3},$$

$$(lr)_{[[x]+3]+1} = (lr)_{[[x]+3]} - (dr)_{[[x]+3]} - (dr)^w_{[[x]+3]}.$$

Since $d_{[x]+4} = (dp)_{[x]+4} + (dr)_{[[x]+3]+1} + (dr)_{[[x]+4]}$ for both deaths and withdrawals, then

$$q_{[x]+4}l_{[x]+4} = (qp)_{[x]+4}(lp)_{[x]+4} + q_{[x+3]+1}(lr)_{[[x]+3]+1} + q_{[x+4]}(lr)_{[[x]+4]},$$

because $(qr)_{[[x]+3]+1} = q_{[x+3]+1}$ and $(qr)_{[[x]+4]} = q_{[x+4]}$. We then solve for $(qp)_{[x]+4}$, which is the only unknown.

Third,

$$(lr)_{[[x]+5]} = 0.2[(lp)_{[x]+4} - (dp)_{[x]+4} - (dp)^w_{[x]+4}],$$

$$(lp)_{[x]+5} = 0.8[(lp)_{[x]+4} - (dp)_{[x]+4} - (dp)^w_{[x]+4}],$$

$$l_{[x]+5} = l_{[x]+4} - d_{[x]+4} - d^w_{[x]+4},$$

$$(lr)_{[[x]+4]+1} = (lr)_{[[x]+4]} - (dr)_{[[x]+4]} - (dr)^w_{[[x]+4]},$$

$$(lr)_{[[x]+3]+2} = (lr)_{[[x]+3]+1} - (dr)_{[[x]+3]+1} - (dr)^w_{[[x]+3]+1}.$$

Since $d_{[x]+5} = (dp)_{[x]+5} + (dr)_{[[x]+4]+1} + (dr)_{[[x]+3]+2} + (dr)_{[[x]+5]}$ for both deaths and withdrawals, then

$$q_{[x]+5}l_{[x]+5} = (qp)_{[x]+5}(lp)_{[x]+5} + q_{[x+4]+1}(lr)_{[[x]+4]+1}$$
$$+ q_{[x+3]+2}(lr)_{[[x]+3]+2} + q_{[x+5]}(lr)_{[[x]+5]}.$$

We then solve for $(qp)_{[x]+5}$, which is the only unknown.

Fourth, for $t \geq 6$,

$$(lp)_{[x]+t} = (lp)_{[x]+t-1} - (dp)_{[x]+t-1} - (dp)^w_{[x]+t-1},$$

$$(lr)_{[[x]+5]+t-5} = (lr)_{[[x]+5]+t-6} - (dr)_{[[x]+5]+t-6} - (dr)^w_{[[x]+5]+t-6},$$

$$(lr)_{[[x]+4]+t-4} = (lr)_{[[x]+4]+t-5} - (dr)_{[[x]+4]+t-5} - (dr)^w_{[[x]+4]+t-5},$$

$$(lr)_{[[x]+3]+t-3} = (lr)_{[[x]+3]+t-4} - (dr)_{[[x]+3]+t-4} - (dr)^w_{[[x]+3]+t-4},$$

$$l_{[x]+t} = l_{[x]+t-1} - d_{[x]+t-1} - d^w_{[x]+t-1}.$$

Since $d_{[x]+t} = (dp)_{[x]+t} + (dr)_{[[x]+5]+t-5} + (dr)_{[[x]+4]+t-4} + (dr)_{[[x]+3]+t-3}$ for both deaths and withdrawals, then

$$q_{[x]+t}l_{[x]+t} = (qp)_{[x]+t}(lp)_{[x]+t} + (qr)_{[[x]+5]+t-5}(lr)_{[[x]+5]+t-5}$$

$$+ (qr)_{[[x]+4]+t-4}(lr)_{[[x]+4]+t-4} + (qr)_{[[x]+3]+t-3}(lr)_{[[x]+3]+t-3} \,.$$

Fifth, again, for $t \geq 5 +$ (pricing mortality select period),

$$q_{[x]+t} = (qp)_{[x]+t} = q_{x+t} \,.$$

We are now in a position to develop a generalized formula. Let $q^r_{[x]+t-1}$ be the proportion reverting (all of whom are assumed to be standard), and let $q^w_{[x]+t-1}$ be the normal aggregate ART lapse rate ($t =$ duration since original issue). Note that we are assuming that $q^w_{[x]+t-1}$ depends only on duration since original issue at age x for all persisters and reverters arising from that issue age. We are also assuming that all lapses occur at year-end and that lapses and deaths are independent.

We can then solve for $(qp)_{[x]+t}$ as follows:

$$q_{[x]+t}l_{[x]+t} = (qp)_{[x]+t}(lp)_{[x]+t} + \sum_{n=1}^{t} q_{[[x]+n]+t-n}(lr)_{[[x]+n]+t-n} \,,$$

where

$$l_{[x]+t} = l_{[x]}\prod_{s=0}^{t-1}(1 - q_{[x]+s} - q^w_{[x]+s}) \,,$$

and

$$(lp)_{[x]+t} = l_{[x]}\prod_{s=0}^{t-1}(1 - q^r_{[x]+s})[1 - (qp)_{[x]+s} - q^w_{[x]+s}] \,,$$

where $(qp)_{[x]} = q_{[x]}$; and

$$(lr)_{[[x]+n]+t-n} = \left[l_{[x]}q^r_{[x]+n-1}\prod_{s=0}^{n-2}(1 - q^r_{[x]+s}) \right]$$

$$\times \left\{ \prod_{s=0}^{n-1}[1 - (qp)_{[x]+s} - q^w_{[x]+s}] \right\}$$

$$\times \left\{ \prod_{s=0}^{t-n-1}[1 - (qr)_{[[x]+n]+s} - q^w_{[x]+n+s}] \right\} \,.$$

Until experience develops on this product, estimates of percentages reverting will necessarily be guesses, but the underwriting department might be able to give some assistance in estimating the percentage of potential reverters who would still be standard risks at given ages and durations. An alternative, albeit complicated, method of ascertaining the

proportion of persisters who are still standard at a given duration might be to apply the techniques developed by Richard Ziock in his paper "Gross Premiums for Term Insurance with Varying Benefits and Premiums" (*TSA*, XXII, 19).

It probably would be advisable to price S/U ART using two or three scales of reversion rates. Any such scale probably should have a relative or absolute maximum for the policy year given in the policy form for reversion, since both the agent and the insured have a financial incentive for reversion at that duration. In that year, regardless of the premium-paying mode, total lapses (which equal aggregate ART lapses plus reversions) would be skewed toward the end of the year, since all reversions would tend to occur at year-end. Figure 1 shows some of the possible total lapse patterns that could be assumed. Pattern A assumes total lapse rates equal to those under an aggregate ART product (that is, no reversions) until duration 5, when reversions of 50 percent are assumed. Pattern B assumes the same reversion rate at duration 5 as Pattern A, but with additional reversions in years 1-4. Pattern C assumes that the largest reversion rate will occur in year 3, in spite of the five-year select period, with another large block of reversions at the end of year 5.
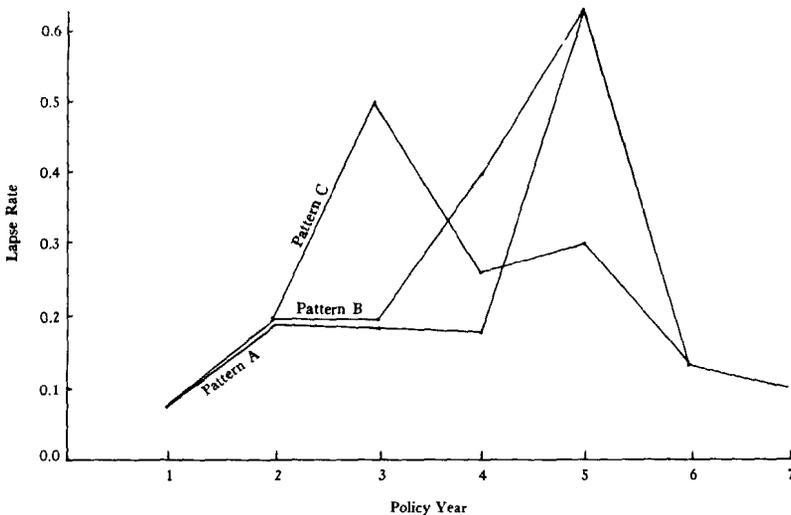
FIG. 1.—Possible total lapse rate patterns for S/U ART

Conversions and Conversion Single Premiums

Our conversion rate assumptions for S/U ART did not differ from those used in pricing an aggregate ART product. An argument could be made for using somewhat higher conversion rates in the ultimate years, since the differential between the premium for a standard permanent product and the ultimate S/U ART premium (which for our product was equivalent to a low substandard aggregate ART premium) might be small enough to induce extra conversions.

We also made the debatable assumption that the mortality of people converting their S/U ART to a permanent plan of insurance would be no higher or lower than the mortality of those continuing with the S/U ART product. This assumption is consistent with our pricing of other term plans. Our reasoning was that the S/U ART is renewable well beyond the last conversion date, at rates significantly below those for a permanent plan; hence it would be cheaper for an insured in very poor health to hold onto the term product. Naturally there are gray areas—people who are in poor health but who are not on their deathbeds might feel that they should convert while they still have the chance. The healthier members of the group, however, could equally well decide that they want permanent coverage, and exercise their conversion options. In any event, until the duration t is such that $(qp)_{[x]+t} = q_{[x]+t}$ (that is, while $t \geq$ (S/U ART select period) + (select period in pricing mortality table), the above mortality assumption produces much higher ultimate-year conversion single premiums (CSPs) than one obtains for an aggregate ART product. These high CSPs can have a significant effect on profits or premium levels in the ultimate years. One possible solution would be to limit the convertibility of S/U ART to the select years only. To ignore the conversion cost is to assume that the extra conversion mortality will be borne by the conversion product, which therefore should be priced accordingly.

Expenses

It is important to account for any extra selection expenses expected in the year of reversion guaranteed by the policy. One would expect that virtually everyone would ask to be underwritten if the financial incentive were great enough (and it probably is for our product, since the company pays the cost). As a result, the company would expect to incur medical and inspection costs for nearly the entire group of insureds at that duration, but only those who are still standard can revert to a new select rate. Those who revert are priced as new issues; their underwriting costs will be more than compensated for, because it is reasonable to

suppose that all who qualify as standard risks will revert, and because new-issue underwriting costs are inflated by the not-taken and declination rates. Thus, in accounting for extra selection expenses, the real question is whether the percentage of those applying to revert who are not standard (and thus not allowed to revert) is greater than or less than the usual not-taken rate for new issues. An additional expense need be added in the pricing only if it is expected that more people will be declined for reversion than would decide not to take the policy if they were new first-time applicants.

Equity

A very real question of equity arises if persisters are required to amortize acquisition expenses incurred by reverters. In any plan of insurance, those who continue under the plan are burdened with the acquisition expense of those who lapse in the early years. However, under an S/U ART product, this condition is aggravated by the contractual provision allowing reversions and by commission and premium scales that encourage reversions before the point called for in the contract.

One approach to this problem would be to discourage early reversions by making reversions less attractive to the agent. This might be accomplished by paying a level commission during the select years. Since the agent's commission (as a percentage of premium) would be the same whether the insured continued on the select scale or reverted early, the agent's incentive to seek early reversions might be reduced. Alternatively, the company could agree to pay only a renewal commission in cases of early reversion. This also would reduce the incentive to the agent to seek early reversions, but it would require that the company be able to detect them. Under this approach, the savings realized by not paying a full first-year commission for those who revert early would be used to offset the unamortized original acquisition expense that the early reverters would otherwise leave behind for the persisters to absorb. (It is assumed that such savings arise because premiums were calculated on the basis of full first-year commissions.) An extension of this idea would be to pay a reduced commission on all reversions, whether early or not.

Another approach would be to make early reversions less attractive to the insured. One could require the insured to supply satisfactory underwriting evidence at his expense in order to apply for a contractual reversion or an early reversion. Here again, if rates were calculated assuming full underwriting expenses, the savings could be used to offset the amount of unamortized initial acquisition expense. This approach

might also reduce the number of those seeking to revert. Alternatively, one could design an S/U ART product with level premiums during the select period. This would eliminate the incentive to revert early, since the level premium rate would be higher for higher issue ages.

Unfortunately, many of these proposed solutions may not seem very practical in the current marketplace. Reducing commissions to the agent on contractual or early reversions could well result in having reversions placed with other companies that pay full first-year commissions on new lives; the company then would realize no savings with which to offset unamortized acquisition expenses. A similar result might follow from having reverters pay for their own underwriting. Designing a product with level select-year premiums is an intriguing idea, but the product might not be attractive to insureds who can obtain a lower rate in the early years by purchasing a nonlevel select-year premium product. Despite these problems, it is the authors' belief that some of the above measures should be instituted in order to emphasize to agent and insured alike that early reversions are not desired by the company.

If we assume that some control can be exercised over early reversions, one further point should be stressed: it is important to preserve equity between those who revert contractually at the end of the select period and those who persist beyond the select period. This can be accomplished in the pricing process by making sure that the asset share at the end of the select period is sufficient to generate a percent-of-premium profit roughly equal to that which will be contributed by the persisters over the expected lifetime of the policy.

Profits

Because of the many uncertainties about the magnitudes of the major variables needed to price this product, it seems reasonable that one would want higher than normal profit margins built into the premiums. Further, we felt that the asset share should be positive at the end of the select period, even for the most pessimistic lapse assumptions.

Calculation of percent-of-premium profit to be earned over, say, thirty years for a closed block of new, first-time issues is complicated by the fact that the policy provides for reversion at the end of the select period, as indicated in Figure 2. It should be noted that Figure 2 assumes that reverters before the end of the fifth year (the date the contract allows reversion) are really lapses and do not contribute to profits.

Let us define the following:

$(AS)_{[x]+t}$ = Asset share per unit in force at the end of policy year t for issue age x;

$P^R_{[y]+5}$ = Probability of reverting at the end of a policy select period of five years;

$_tP^T_{[y]}$ = Probability of surviving all decrements for a t-year period for someone aged y at issue; and

$\pi_{[x]+t}$ = Accumulated premium dollars per unit in force.

Then the total asset share per unit in force after thirty years attributable to a new issue at age x is

$$(AS)_{[x]+30} + {}_5P^T_{[x]}P^R_{[x]+5}(AS)'_{[x+5]+25}$$

$$+ {}_5P^T_{[x]}P^R_{[x]+5} \; {}_5P'^T_{[x+5]}P'^R_{[x+5]+5}(AS)'_{[x+10]+20}$$

$$+ \ldots + {}_5P^T_{[x]}P^R_{[x]+5} \ldots {}_5P'^T_{[x+20]}P'^R_{[x+20]+5}(AS)'_{[x+25]+5} \; ,$$

where the primes indicate that the lapse assumptions entering into the computations for those who have reverted at least once may differ from the lapse assumptions used for new issues. For instance, in the first five-year period, lapses might be greater than those given by a blended pricing assumption pattern; in subsequent (reversion) select periods, lapses might be expected to be somewhat less than those assumed in the pricing. Total accumulated premium for the given closed block of business is computed using the above formula and replacing all AS's by π's.
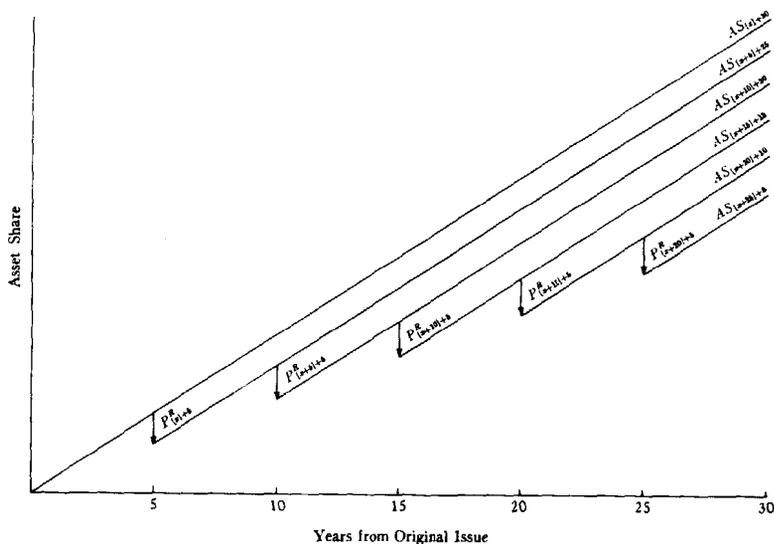
FIG. 2.—Asset shares for S/U ART

Ultimate Premiums at Ages beyond the Last Allowed Reversion Age

If the policy is renewable beyond the last age for which the insured is allowed to revert, then the ultimate premiums for ages beyond the maximum reversion age should reflect the fact that the group of insureds paying these ultimate rates includes an increasing number of standard risks. If the maximum issue age and maximum reversion age coincide, then, from the end of the last premium select period onward, one might expect mortality about equal to that for an aggregate product with the same maximum age at issue.

ADMINISTRATION AND EXPERIENCE MONITORING

Since lapses have such an impact on mortality rates and expense amortization, they must be monitored carefully and discouraged when possible. One question that is certain to arise is, "What do you do if an insured applies for a new select rate before the allowed reversion date?" Since reversions are treated as lapses in the asset share, this behavior increases lapse rates. Our company will not pay the agent a first-year commission on a new policy resulting from a reversion before the end of the select period. However, this system is not foolproof; in a brokerage agency, for example, the broker could have the insured switch back and forth every year or two between two companies offering S/U ART products. The insured would get low rates and the agent would pile up first-year commissions. Discouraging these frequent replacements would seem to be very difficult. Policies with level premiums and level commissions in the select period probably would help a great deal, as would a good program to detect twisting. A company could also emphasize the possible disadvantage to the insured of starting a new contestable period with each replacement.

Although reversions are treated as lapses, it would be useful, for purposes of refining the pricing assumptions, to know what proportions are reverting. One also would need to know (*a*) how many reversion medicals the company is paying for, as compared with the number reverting and (*b*) the usual not-taken rate.

CONCLUSION

We hope that the techniques and considerations presented in this paper give actuaries some useful ideas for pricing S/U ART and an awareness of the very real dangers inherent in marketing these kinds of plans. Emerging lapse experience will take much of the guesswork out of estimating reversion rates and thus will improve the accuracy of future pricing. Meanwhile, in our opinion, the best policy would be to

price defensively by designing the plan so that there are few incentives for early reversion. Some of the defensive measures mentioned in the paper may be hard to market. Given the reversion feature in the contract, the development of ultimate-year mortality rates based on assumed reversion rates is of primary importance. It is the authors' conclusion that, because of this reversion feature, ultimate-year premiums should be substantially higher than either select-year rates or aggregate ART premiums.

APPENDIX I

AGGREGATE ART MORTALITY COMPARED WITH S/U ART PERSISTER MORTALITY: ALL REVERSIONS AT END OF YEAR 5

(Issue Age 45)

DURATION SINCE ISSUE	MORTALITY RATE PER 1,000		MODIFICATION TO AGGREGATE ART MORTALITY {[(2)−(1)]/(1)}	PERCENTAGE REVERTING
	Aggregate ART Mortality (1)	S/U ART Persister Mortality (2)		
0.........	1.70	1.70	0.0	0%
1.........	2.36	2.36	0.0	0
2.........	2.99	2.99	0.0	0
3.........	3.59	3.59	0.0	0
4.........	4.12	4.12	0.0	0
5.........	4.66	7.00	50.2	50
6.........	5.23	7.01	34.0	0
7.........	5.85	7.20	23.1	0
8.........	6.53	7.54	15.5	0
9.........	7.48	8.43	12.8	0
10........	8.51	9.57	12.4	0
11........	9.68	11.00	13.6	0
12........	11.02	12.72	15.4	0
13........	12.55	14.71	17.2	0
14........	14.34	17.10	19.3	0
15........	16.50	19.98	21.1	0
16........	18.01	21.16	17.5	0
17........	19.69	22.25	13.0	0
18........	21.63	23.48	8.6	0
19........	23.81	24.80	4.2	0
20........	26.17	26.17	0.0	0
21........	28.73	28.73	0.0	0
22........	31.40	31.40	0.0	0
23........	34.21	34.21	0.0	0
24........	36.99	36.99	0.0	0
25........	39.92	39.92	0.0	0
26........	43.46	43.46	0.0	0
27........	47.47	47.47	0.0	0
28........	51.73	51.73	0.0	0
29........	56.43	56.43	0.0	0

NOTE.—Test calculations using an underlying lapse rate equal to that for aggregate ART and using an underlying lapse rate of 0% produced mortality rates only slightly different from each other.

APPENDIX II

AGGREGATE ART MORTALITY COMPARED WITH S/U ART PERSISTER MORTALITY: REVERSIONS AT END OF YEARS 3, 4, AND 5

(Issue Age 45)

DURATION SINCE ISSUE	MORTALITY RATE PER 1,000		MODIFICATION TO AGGREGATE ART MORTALITY $\{[(2)-(1)]/(1)\}$	PERCENTAGE REVERTING
	Aggregate ART Mortality (1)	S/U ART Persister Mortality (2)		
0.........	1.70	1.70	0.0	0%
1.........	2.36	2.36	0.0	0
2.........	2.99	2.99	0.0	0
3.........	3.59	4.25	18.3	30
4.........	4.12	4.89	18.8	10
5.........	4.66	5.96	28.0	20
6.........	5.23	6.19	18.4	0
7.........	5.85	6.56	12.1	0
8.........	6.53	7.10	8.8	0
9.........	7.48	8.20	9.6	0
10.......	8.51	9.43	10.8	0
11.......	9.68	10.81	11.7	0
12.......	11.02	12.43	12.8	0
13.......	12.55	14.25	13.5	0
14.......	14.34	16.37	14.1	0
15.......	16.50	18.96	14.9	0
16.......	18.01	19.96	10.8	0
17.......	19.69	20.97	6.5	0
18.......	21.63	22.20	2.6	0
19.......	23.81	24.05	1.0	0
20.......	26.17	26.17	0.0	0
21.......	28.73	28.73	0.0	0
22.......	31.40	31.40	0.0	0
23.......	34.21	34.21	0.0	0
24.......	36.99	36.99	0.0	0
25.......	39.92	39.92	0.0	0
26.......	43.46	43.46	0.0	0
27.......	47.47	47.47	0.0	0
28.......	51.73	51.73	0.0	0
29.......	56.43	56.43	0.0	0

NOTE.—Test calculations using an underlying lapse rate equal to that for aggregate ART and using an underlying lapse rate of 0% produced mortality rates only slightly different from each other.

DISCUSSION OF PRECEDING PAPER

TOM BAKOS:

One reason for the development of a select and ultimate annual renewable term (ART) product that was not mentioned by the authors is to minimize deficiency reserves. Although the expression "deficiency reserve" is not used in the 1976 amendments to the standard valuation law, additional reserves equivalent to what used to be called deficiency reserves are still required if an ART gross premium is less than some statutory minimum. These additional reserves can be minimized by establishing the ultimate premiums of a select and ultimate ART product at a level that is not deficient. Competition demands, and experience mortality permits, select premiums much lower than ultimate premiums. These select premiums probably would be deficient. If the select period were five years, the anticipated premium stream would consist of select (probably deficient) rates for the first five years and ultimate (non-deficient) rates thereafter. Deficiencies would be limited to the first five policy years, not the entire renewal period, producing a significantly smaller surplus strain than would be produced by the alternative, a competitive aggregate ART product.

To make what would otherwise be a very uncompetitive product competitive, the reversion feature is added, which allows insureds to requalify for the select rates as described in the paper. Those companies that adopt a select and ultimate premium structure only as a gimmick to avoid large deficiency reserves probably would impose minimal underwriting requirements so as to requalify nearly 100 percent of their policyholders at each requalification period. Such companies would have what the paper describes as aggregate ART products disguised as select and ultimate products. Those companies that approached the select and ultimate product more naively and attempted more serious underwriting would requalify something less than 100 percent and would incur significant additional expenses that would increase the price of the product. Both types of companies would lose the competitive advantage of guaranteed low rates throughout the renewal period. In this sense, a select and ultimate reversionary product is not as competitive as an aggregate guaranteed premium product.

Although there may have been other needs that caused the development of a select and ultimate ART product, the need to minimize de-

ficiency reserves was the first that I became aware of, and it is reasonable to suppose that these other needs developed in an effort to legitimize the concept. Certainly the need to reduce deficiency reserve requirements has stimulated other forms of product innovation. For example, we also have guaranteed maximum premium ART products and participating ART with an ultimate premium rate structure after, say, the third year. The dividends for this participating ART begin at the end of the third year and are mysteriously equal to the difference between the ultimate and the initial-scale premium.

Another approach to pricing a select and ultimate ART product is described in the remainder of this discussion. This alternative approach starts with the principle of conservation of total deaths described in the paper, but limits its application because mortality and persistency are felt to unfold in a manner different from that assumed in the paper.

Basic Approach

Take the paper's basic formula that expresses the principle of conservation of total deaths at the requalification age $[x] + 5$,

$$q_{[x]+5}l_{[x]+5} = (qp)_{[x]+5}(lp)_{[x]+5} + (qr)_{[[x]+5]}(lr)_{[[x]+5]} ,$$

and rearrange it as follows:

$$q_{[x]+5} = \frac{(lr)_{[[x]+5]}}{l_{[x]+5}} (qr)_{[[x]+5]} + \frac{(lp)_{[x]+5}}{l_{[x]+5}} (qp)_{[x]+5} .$$

We know that $l_{[x]+5} = (lr)_{[[x]+5]} + (lp)_{[x]+5}$, so we can make the following definitions:

$$k_{[x]+5} = \frac{(lr)_{[[x]+5]}}{l_{[x]+5}} \quad \text{and} \quad 1 - k_{[x]+5} = \frac{(lp)_{[x]+5}}{l_{[x]+5}} ,$$

where $k_{[x]+5}$ is the proportion of the age x issues reverting at age $[x] + 5$. Substituting in the rearranged formula yields

$$q_{[x]+5} = k_{[x]+5}(qr)_{[[x]+5]} + (1 - k_{[x]+5}) (qp)_{[x]+5} ,$$

which we can solve for $k_{[x]+5}$ as follows:

$$k_{[x]+5} = \frac{(qp)_{[x]+5} - q_{[x]+5}}{(qp)_{[x]+5} - (qr)_{[[x]+5]}} .$$

In their paper Dukes and MacDonald assume that the proportion reverting is 50 percent and then solve for the persister mortality, (qp). They have also assumed that the reverter mortality, (qr), is equal to new select mortality. Their solution for (qp) beyond duration 5 depends on the assumption that lapse rates for the persister and reverter classes are

identical and that total lapses are the same as for an aggregate ART product. This assumption seems questionable. The reverters, through the reselection process, have been determined to be standard. The persisters, then, are obviously substandard, and the ultimate premium they will be charged includes, implicitly, a substandard extra. Presumably, under an aggregate premium structure none of these persisters would have lapsed for another ART product, since it would be available to them only on a substandard basis. Why give up a standard rate for a sub-standard rate? These same persisters, however, when charged an ulti-mate, implicitly substandard premium, would be prompted to shop. Those better substandard risks in the persister class probably would be successful in finding a better aggregate ART rate, even if it were sub-standard, and they would lapse their policies.

The reverse of this argument could be made for the reverter class, and it could be asserted that their persistency will be better than that for an aggregate product. Thus, it is logical to assume that persistency for the reverter class would be different from that for the persister class and that, in total, persistency would be different from aggregate ART persistency.

It should be noted that the persister mortality is just as dependent upon the assumption made for the proportion reverting as it is upon the lapse assumption. This was demonstrated in the paper. If these assump-tions cannot be made reliably, then the persister mortality cannot be solved for reliably. There is, therefore, no particular advantage in approaching the problem in this way. Instead, one could establish an assumption about the level of persister mortality consistent with the reselection effort planned and the substandard mortality expected, and then solve for $k_{[x]+5}$, the proportion of the age x issues reverting. Thus, the ultimate ART rates could be set in much the same way that sub-standard premium rates are set. Knowing the mortality levels under-lying the ultimate ART rate structure and using $k_{[x]+5}$ as a guide, the underwriter could classify the risk as either a reverter (standard) or a persister (substandard).

In solving for $k_{[x]+5}$, the additional assumption would be made that $(qr)_{[[x]+5]} = q_{[x+5]}$, as was done in the paper, and the formula would become

$$k_{[x]+5} = \frac{(qp)_{[x]+5} - q_{[x]+5}}{(qp)_{[x]+5} - q_{[x+5]}}.$$

All of the assumptions made to solve for $k_{[x]+5}$ would be made only with respect to the duration in which the reversion occurred. The value of

$k_{[x]+5}$ would be useful only as a guide in evaluating the reselection process.

Mortality Assumption

In solving for persister mortality, (qp), the paper invokes the conservation-of-total-deaths principle, assumes that lapse rates for reverters and persisters are the same and are equal to those of an aggregate ART plan, and assumes that reverter mortality is identical with new select mortality. Under these assumptions, persister mortality and reverter mortality are related as shown in Figure 1. The curve labeled q shows the original-issue select and ultimate mortality assumption. The curves (qp) and (qr) show persister and reverter mortality equaling ultimate mortality fifteen years after the reversion period.

However, the assumption that reverter and persister mortality each equal ultimate mortality fifteen years after reversion seems to be only a pricing convenience. The conservation-of-total-deaths principle is not infringed if we assume that persister mortality is always greater than ultimate mortality and that reverter mortality is always less than ultimate mortality, as shown in Figure 2. This assumption seems reasonable because the reselection process can be expected to weed out the poorer risks each time it is exercised on the prior reverter class. The reverters
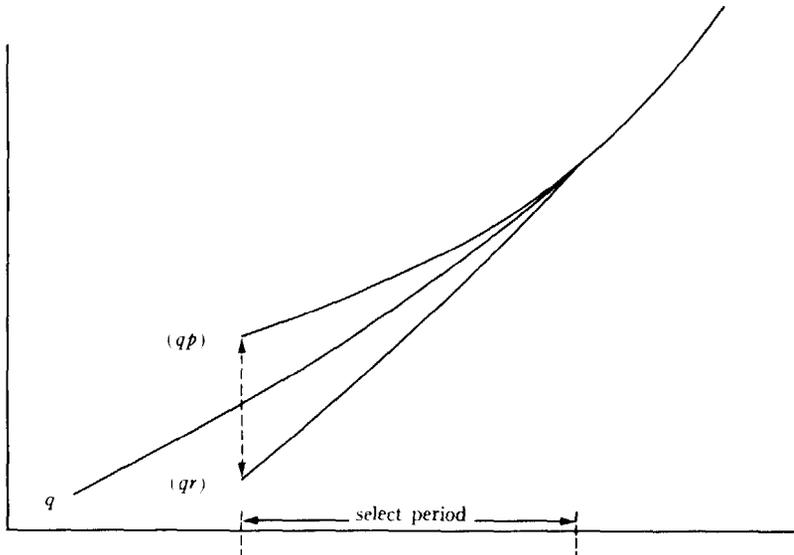
FIG. 1.—Relationship of persister, aggregate, and reverter mortality assumed in paper.
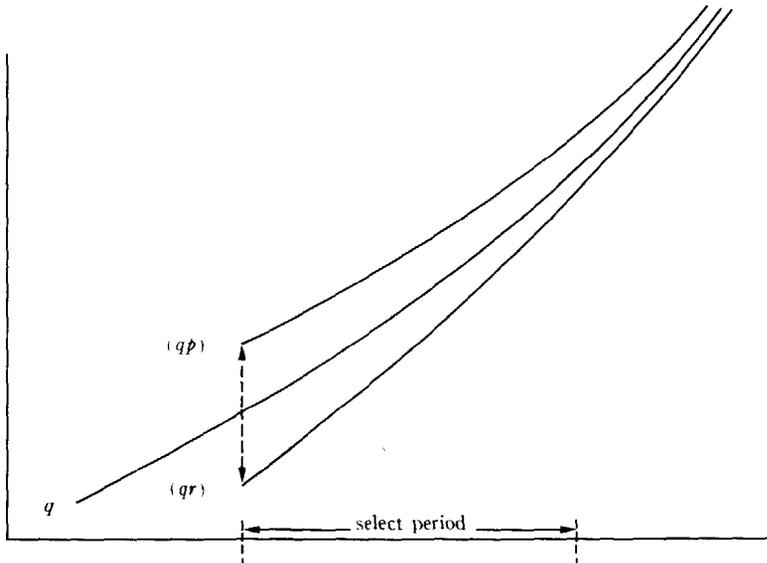
FIG. 2.—Relationship of persister, aggregate, and reverter mortality assumed in discussion.

remaining after continual reselection might exhibit new select mortality at the time of reversion, but this select mortality might wear off more slowly than in an aggregate select class and might settle at a level lower than aggregate ultimate mortality. Thus, the reverter class can be assumed to be superselect in the sense that the select period it exhibits is longer than the select period normally assumed in pricing aggregate ART. To preserve the conservation-of-total-deaths principle, the persister mortality would have to be always greater than the aggregate ART's pricing mortality. This is consistent with the concept that persisters are really substandard risks.

The formula presented in the paper and modified in this discussion gives a relationship among k, q, (qp), and (qr) that holds at all durations only under the lapse assumption made in the paper. If this lapse assumption is assumed not to hold, then the relationship among these four terms is not so simple. The complexities of this relationship can be avoided by assuming that reverter and persister mortality (and persistency) are independent of each other.

If a requirement of a select and ultimate product is that deficiency reserves be minimized, then ultimate premiums for the lowest premium band or class should not be less than $1,000c_x$ computed on the Modern

CSO Table at $4\frac{1}{2}$ percent. This would eliminate deficiencies during the ultimate period in most states. For pricing purposes, the mortality underlying these ultimate rates could be estimated in two ways.

Method 1. The ultimate premium could be "unloaded." Aggregate ART premiums from a current product could be compared at each age with the average pricing mortality rate for that age to compute a "load." This "load" could then be applied in reverse to estimate the persister mortality underlying the chosen ultimate premium.

Method 2. The substandard rating implied by the chosen ultimate premium could be determined by relating the ultimate premium to a current aggregate ART premium. The percentage extra mortality implicit in the ultimate premium could then be multiplied by the average aggregate ART pricing mortality rate for each age to estimate the persister mortality underlying the chosen ultimate premium.

Table 1 shows the solution for the underlying ultimate mortality using method 1, and Table 2 shows the solution using method 2. The results, given the crudeness of the processes, are similar and show that the maximum substandard mortality occurs at age 30 at about the table 4 (200 percent of standard mortality) level. One might want to modify these underlying rates for two reasons: First, at the higher attained ages they are less than the ultimate pricing mortality used for the aggregate ART, and, second, the implied substandard ratio is not level by age.

TABLE 1

CALCULATION OF UNDERLYING ULTIMATE MORTALITY USING METHOD 1

Age (1)	Ultimate Premium $1,000c_x$ Modern CSO $4\frac{1}{2}\%$ (2)	Unloading Ratio* (3)	Underlying Ultimate Mortality $[(2)\times(3)]$ (4)	Average Aggregate Pricing Mortality† (5)	Implied Substandard Ratio $[(4)/(5)]$ (6)
15.	1.30	0.45	0.59	0.52	1.13
20.	1.68	0.45	0.76	0.52	1.46
25.	1.92	0.46	0.88	0.49	1.80
30.	2.03	0.42	0.85	0.45	1.89
35.	2.27	0.50	1.14	0.64	1.78
40.	3.02	0.56	1.69	1.07	1.58
45.	4.41	0.62	2.73	1.87	1.46
50.	6.67	0.64	4.27	3.03	1.41
55.	10.38	0.62	6.44	4.80	1.34
60.	16.07	0.52	8.36	7.06	1.18
65.	26.78	0.42	11.25	10.53	1.07
70.	41.89	0.40	16.76	16.10	1.04

* The unloading ratio for each age is the ratio of average pricing mortality to the lowest premium band rate.

† This is a weighted average or "aggregate" pricing mortality at each attained age, which recognizes the distribution of in-force by duration at each attained age.

TABLE 2

CALCULATION OF UNDERLYING ULTIMATE MORTALITY USING METHOD 2

Age (1)	Ultimate Premium $1,000c_x$ Modern CSO $4\frac{1}{2}\%$ (2)	Aggregate ART Standard Rate (3)	Implied Substandard Ratio* $[\{[(2)/(3)-1]/0.9\}+1]$ (4)	Average Aggregate Pricing Mortality (5)	Underlying Ultimate Mortality $[(4)\times(5)]$ (6)
15..........	1.30	1.03	1.29	0.52	0.67
20..........	1.68	1.03	1.70	0.52	0.88
25..........	1.92	1.03	1.96	0.49	0.96
30..........	2.03	1.03	2.08	0.45	0.94
35..........	2.27	1.23	1.94	0.64	1.24
40..........	3.02	1.85	1.70	1.07	1.82
45..........	4.41	2.90	1.58	1.87	2.95
50..........	6.67	4.57	1.51	3.03	4.58
55..........	10.38	7.49	1.43	4.80	6.86
60..........	16.07	13.11	1.26	7.06	8.90
65..........	26.78	24.19	1.12	10.53	11.79

* Our company's substandard ART premiums are calculated by the following formula:

Substandard premium = $P[0.9(r - 1) + 1]$,

where P is the standard ART premium and r is the substandard ratio (e.g., for table 2 substandard mortality, $r = 1.5$). The values in col. 4 were obtained by solving this equation for r.

In Table 3, $k_{[x]+5}$, the proportion reverting five years after issue, is calculated assuming that persister mortality is equal to table 4 (200 percent) substandard mortality. The value of k is fairly uniform by age, indicating that about 70 percent of the in-force would revert under these assumptions. That is, at the time of reversion the underwriting process would have to be efficient and precise enough to select the 30 percent of the total persisters and reverters who will be persisters with an average mortality equal to table 4. This selection probably would have to be made knowing that some risks placed in the persister class would lapse rather than pay the higher ultimate premium. Therefore, the underwriting target might be to select the, say, 50 percent that average table 2 substandard mortality.

Other Pricing Assumptions and Profits

Under the approach suggested in this discussion, the lapse assumption should reflect the expected additional lapses that probably would occur as insureds lapse their policies rather than accept the ultimate rate. The expense assumption, as pointed out in the paper, would have to include the extra selection expenses in the year of reversion. With higher than normal profit margins built into the product as the authors suggest, the asset share will be positive at the end of the select period. This would

TABLE 3

CALCULATION OF IMPLIED $k_{[x]+5}$

Age $x+5$	Aggregate Select Mortality $1,000q_{[x]+5}$ (1)	Persister Mortality $(qp)_{[x]+5}$ $[2\times(1)]$ (2)	Reverter Mortality $1,000q_{[x+5]}$ (3)	$k_{[x]+5}$ $\{[(2)-(1)]/$ $[(2)-(3)]\}$ (4)
15	0.52	1.04	0.33	0.73
20	0.52	1.04	0.39	0.80
25	0.49	0.98	0.32	0.74
30	0.45	0.90	0.37	0.85
35	0.64	1.28	0.46	0.78
40	1.07	2.14	0.68	0.73
45	1.87	3.74	1.04	0.69
50	3.03	6.06	1.47	0.66
55	4.80	9.60	2.10	0.64
60	7.06	14.12	3.35	0.66
65	10.53	21.06	5.68	0.68
70	16.10	32.20	7.24	0.65

NOTE.—Persister mortality, (qp), is assumed to be equal to 2 times the average aggregate pricing mortality, which is assumed to be equal to $q_{[x]+5}$, the fifth-year select pricing mortality.

assure amortization of initial selection expense before the first reversion. In this situation, the authors suggest that the extra selection expense can be equated to the initial selection expense, with the reverters equivalent to new issues and the persisters equivalent to not-takens. They state that "an additional expense need be added in the pricing only if it is expected that more people will be declined for reversion than would decide not to take the policy if they were new first-time applicants." It seems, however, that some additional "expense" has been implicitly included in the price of the product in the form of higher profit margins and the expectation of a positive asset share at the end of the select period (five years in the example). If persisters are greater in number than not-takens, even *more* additional expense would need to be incorporated if it were expected that the reverter class would amortize all the reselection underwriting expense, including that of the persisters who were declined for reversion.

Marketability of Select and Ultimate ART

Common sense should indicate that the extra selection expense and the additional first-year compensation associated with reversion under a select and ultimate ART product would make it more costly than an otherwise similar aggregate ART product. When we considered introduction of a select and ultimate ART product as a means of reducing

our deficiency reserve requirements, we found that we could not compete with our own recently introduced aggregate ART product.

The nonguaranteed nature of the renewal premiums is a significant problem for select and ultimate ART products. Everyone, no doubt, thinks he will qualify for reversion; however, the assumptions used in this discussion indicate that 30 percent will not, and in the paper it was assumed that 50 percent will not. These nonqualifiers will be expected to pay the higher ultimate premiums. As each reversion period passes, the size of the persister class grows. Ten years after issue, there probably will be more persisters than reverters, assuming that the persisters have not lapsed. After fifteen years, at least two-thirds will be persisters. This large group of people, by opting for a select and ultimate product, will have given up the guaranteed standard renewal premiums they would otherwise be paying for an aggregate ART product.

The nature of a select and ultimate ART would seem to prohibit substandard issues. If, in order to revert, an insured must be a standard risk, he probably should be standard at issue also. Would a company that offered only select and ultimate ART be able to insure a substandard risk?

Summary

The purpose of this discussion was to point out that another approach to pricing a select and ultimate ART product would be to choose an appropriate level for the ultimate-year mortality rates and develop the reversion rates implied by that level of mortality. The discussion was meant to imply that this would be a more practical way of approaching select and ultimate ART product development than the procedure suggested in the paper.

JOHN C. GOULD AND JAMES R. PORTER:

This is a timely paper, since it addresses very real questions in pricing a currently popular and competitive product. This discussion addresses questions raised by the authors' observation that lapse rates had little effect on persister mortality. For their illustrations, the authors assumed that the same lapse rates applied to the persisters and the reverters.

Calculations involving decrements of mortality (q^m) and withdrawal (q^w) commonly employ one of the following expressions for the persistency rate:

$$1 - q_t^m - q_t^w \tag{1}$$

or

$$(1 - q_t^m)(1 - q_t^w) . \tag{2}$$

If the q's are from a double decrement table (deaths and withdrawals) and if both decrements apply simultaneously and continuously, then expression (1) is exact. This expression is used in the paper. However, if the only q's available are from separate mortality and withdrawal tables, formula (14.38) from Jordan's *Life Contingencies* can be used to compute the double decrement rates from the known single decrement rates. The persistency rate, in terms of the single decrement rates, is given by the expression

$$\frac{(1 - q_t'^m)(1 - q_t'^w) - \frac{1}{4}q_t'^m q_t'^w}{1 - \frac{1}{4}q_t'^m q_t'^w}.\tag{3}$$

Expression (2) is a closer approximation to this expression than is expression (1). To illustrate the difference between these expressions, assume a mortality rate of 2 deaths per 1,000 and a 10 percent withdrawal rate. The resulting values of the three expressions are

$$(1)\quad 0.8980;\quad (2)\quad 0.8982;\quad (3)\quad 0.898245\ .$$

If the exposure to withdrawal is on premium due dates (as when there are no cash values) and weighted heavily on the anniversary (as for annual premiums or annual increases in premium), expression (2) is the best approximation to the persistency rate. Given this approximation and the assumption of the same withdrawal rates for persisters and reverters, persister mortality is independent of withdrawal rates:

$$(qp)_t = \frac{q_t l_{t-1}(1 - q_{t-1}) - (qr)_t (lr)_{t-1}[1 - (qr)_{t-1}]}{(lp)_{t-1}[1 - (qp)_{t-1}]}.\tag{4}$$

Table 1 of this discussion compares persister mortality rates computed using the authors' formula with those computed using the formula above under various withdrawal assumptions. The assumed basic mortality is from a five-year select table with select rates equal to the following percentages of the ultimate rates (see Table 3): 85 percent in the first year, then 90, 94, 97, and 99 percent. Reversion rates assumed are 50 percent of in-force at the end of two years, and 30 percent of persisters in force at the end of four years.

The first column of Table 1 shows persister mortality rates assuming no lapses. These will also be the mortality rates assuming equal lapse rates for reverters and persisters and using expression (2) for the persistency rate.

Column 2 of Table 1 shows persister mortality calculated using the authors' formulas with a flat 10 percent lapse rate. A comparison of columns 1 and 2 illustrates the very slight effect of the assumed lapses.

TABLE 1

COMPUTED PERSISTER MORTALITY RATES ($\times 1,000$)

Attained Age	No Lapse (1)	Flat 10% Lapse (2)	Lapses from Table 3 (3)	Lapses from Table 3 (4)
30.............	1.8275	1.8275	1.8275	1.8275
31.............	1.9800	1.9800	1.9800	1.9800
32.............	2.3175	2.3175	2.3175	2.3175
33.............	2.423266	2.423274	2.4775	2.4669
34.............	2.691554	2.691568	2.7923	2.7488
35.............	2.714481	2.714503	2.7911	2.7802
36.............	2.756132	2.756146	2.7918	2.7892
37.............	2.836050	2.836055	2.843824	2.843854
38.............	3.012877	3.012879	3.015653	3.015664
39.............	3.250	3.250	3.250	3.250

NOTE.—Column 3 calculated using expression (1) of this discussion; col. 4 calculated using expression (2).

Columns 3 and 4 show the effect of assuming that the lapse rates from the withdrawal table are combined rates but that the reverters' lapse rates are significantly lower. (It could be argued that the reverters have lower lapse rates because they pay lower premiums, or that the persisters have lower lapse rates because a significant portion have discovered that they are uninsurable or are rated risks.) Column 3 was computed using the authors' formulas, while column 4 uses the expressions in this discussion. (Lapse assumptions used are shown in Table 3.) Persister lapse rates, shown in Table 2, were computed in a manner analogous to the method used for the persister mortality rates.

Our conclusions are as follows:

1. When differing lapse rates can be confidently assumed (from experience) for persisters and reverters, they should be recognized for their effect on both mortality and persistency. Until then, it is reasonable as well as convenient

TABLE 2

PERSISTER LAPSE RATES (PERCENT), COMPUTED USING LAPSE RATES
IN TABLE 3, AND EXPRESSIONS (1) AND (2) OF THIS DISCUSSION

Attained Age	Expression (1)	Expression (2)	Attained Age	Expression (1)	Expression (2)
30..........	0.0	0.0	35..........	9.5864	9.5890
31..........	25.0	25.0	36..........	6.8450	6.8466
32..........	25.0	25.0	37..........	5.0	5.0
33..........	12.5345	12.5361	38..........	5.0	5.0
34..........	15.3813	15.3843	39..........	5.0	5.0

TABLE 3

MORTALITY AND LAPSE ASSUMPTIONS USED IN
THE ILLUSTRATIONS

Attained Age	Ultimate Mortality ×1,000 (1)	Aggregate Lapse (%) (2)	Reverter at Duration 2 Lapse (%) (3)	Reverter at Duration 4 Lapse (%) (4)
30........	2.15	0.0	....	....
31........	2.20	25.0	....	....
32........	2.25	15.0	5.0	....
33........	2.33	10.0	8.0	....
34........	2.40	9.0	7.0	3.0
35........	2.50	7.0	6.0	6.0
36........	2.65	5.5	5.0	5.0
37........	2.80	5.0	5.0	5.0
38........	3.00	5.0	5.0	5.0
39........	3.25	5.0	5.0	5.0

to compute persister mortality ignoring withdrawals. Given the computed mortality table, various withdrawal rates can and probably should be tested.
2. The "conservation of total deaths" concept is a little too handy. It would not be appropriate to adopt it this early as a generally accepted actuarial assumption like the time-honored concept of uniform distribution of deaths. In the meantime, this paper defines an important territory of uncertainty and begins to map it.

COURTLAND C. SMITH:

Messrs. Dukes and MacDonald have presented a timely and interesting paper. For the rational, informed consumer in an inflationary environment, the product is a plus. The select and ultimate annual renewable term (S/U ART) policy gives low-cost insurance. With reversion, the customer's options are increased. Given continuing competition, costs can only come down.

The product would seem to represent a positive development for the rational, informed agent as well. Caught in the squeeze between declining first-year commissions and rising living costs, the agent is forced to make more frequent sales or sell ever larger policies to survive. S/U ART, with its reversion feature, legitimizes frequent resales to existing customers who remain in good health.

For the rational, knowledgeable life insurance company, S/U ART represents an opportunity and a problem. The company needs new business to survive, and the product is attractive. S/U ART can help attract new healthy lives, but the company may not prosper as a result. Much

existing in-force may simply be rewritten. The business written may not persist long enough to amortize first-year costs. The proportion reverting may be greater than anticipated, and both reverters and persisters may then show much higher mortality than was assumed in the original pricing. Thus, the Dukes-MacDonald S/U ART product seems especially vulnerable to lapse by healthy lives at the start of the third, fifth, and sixth durations, and to renewal by lives less healthy than anticipated at the start of the sixth and later durations.

The solution to the life company's problem lies in the fact that there are numerous reinsurance companies in the marketplace that are willing to compete aggressively for new business. By coinsuring the lapse risk as well as the mortality risk at favorable allowances, the direct company can shift the hazards of S/U ART to the reinsurers and remain confidently competitive. I have heard it said that some term policies being sold today are profitable only because of the reinsurance.

It seems to me that the most refined form of S/U ART policy would allow reversion every year. To save underwriting expenses, medical requirements would be reduced each policy year, except perhaps the fifth, tenth, fifteenth, and so on. As cases reach their first anniversary, and the insureds are given the option to revert, the healthiest insureds are likely to submit evidence first, and very little adverse information is likely to be found. I think it would be very tempting, given these early results, for the marketing department to propose that further requests for evidence be waived in the first year in order to reduce both expenses and lapses! Interestingly, if the coinsurance conditions are sufficiently competitive, it could pay the actuary to ask his reinsurers to agree. And it might be difficult for them to refuse!

I have heard the S/U ART policy described as the first life insurance product in history designed to self-destruct. In the present market, I suspect that the policy may survive, but some individual companies may self-destruct instead.

The property-casualty insurance market is based mainly on sales of annual renewable term policies having yearly reentry provisions. With inflation in medical costs and property repair charges, claim costs and coverage limits tend to rise. Premiums tend to exhibit a roller-coaster pattern. Premiums increase faster than claims when catastrophic experience or technological innovation drives excess reinsurance capacity out of the market, but more slowly than claims when a series of profitable years draws insurance and reinsurance capacity back in. In the capacity-contraction phase, it is not unusual for companies to self-destruct or merge.

To some observers, the property-casualty roller-coaster cycle lasts an average of six to seven years. The life insurance industry has all the signs of moving in the same direction. If so, I wonder how long the life cycle will be.

<div align="center">(AUTHORS' REVIEW OF DISCUSSION)</div>

<div align="center">JEFFERY DUKES AND ANDREW M. MAC DONALD:</div>

We were somewhat disappointed that this paper did not generate more written discussion of the merits, viability, and pricing methodology of S/U ART products, especially since so many companies are issuing or reinsuring such plans. However, we did receive three such discussions, and we wish to thank these contributors for taking the time to put their thoughts in writing.

Before we examine each discussion separately, we would like to address one issue that was raised in two responses to our paper, that is, the issue of assuming different lapse rates for the persisters than for the reverters. Mr. Bakos argues convincingly that persister lapse rates will be higher than aggregate ART lapse rates and that reverter lapse rates will be lower. Messrs. Gould and Porter suggest that differing experiential lapse rates should be recognized for their effect on mortality; until such experience is available, they advocate employing a "conservation of total lapses" principle to develop differing lapse rates for each class. On this issue, we would like to make the following points:

1. We realize that there is a case for assuming different lapse rates for the persisters than for the reverters. In the absence of any experience, however, the introduction of different lapse rates greatly complicates the formulas needed to calculate persister mortality. For instance, let us assume that lapse rates for reverters are equal to those for aggregate ART but that lapse rates for persisters are higher than those for aggregate ART. These extra lapses could be viewed as extra reverters in the context of our generalized formula, which allows for annual reversions. The mortality rate for the reverting class then would be a blend of standard mortality for the true reverters and some degree of substandard mortality for the extra persister lapses. The generalized formula would become

$$l_{[x]+t}q_{[x]+t} = (lp)_{[x]+t}(qp)_{[x]+t} + \sum_{n=1}^{t} q^*_{[[x]+n]+t-n}(lr)_{[[x]+n]+t-n} ,$$

where $q^*_{[[x]+n]+t-n}$ is the blended mortality rate referred to above. The clear difficulty is in quantifying the degree of the substandard mortality to be sustained by the extra persister lapses. It probably is safe to say that there will be no extra persister lapses before the first contractually allowed opportunity to revert; thus, for those years, $q^*_{[[x]+n]+t-n} = q_{[[x]+n]+t-n} = q_{[x+n]+t-n}$. After that point, however, there is considerable question as to what will happen. It

could be argued that all extra persister lapses will occur immediately when reversion is denied and that no additional lapses will occur after that point. This would confine the blended mortality problem to one cohort of reverters but would leave the problem of choosing the blended mortality level. The problem expands if one assumes that there will be additional persister lapses in all durations after the first reversion opportunity. Assuming higher persister lapses at the first reversion opportunity and lower persister lapses afterward complicates matters further—those lower lapses could be considered as negative reverters, perhaps in a high-risk class.

In any case, it should be clear that assuming different lapse rates for persisters and reverters poses some serious challenges for the pricing actuary.

2. Mr. Bakos's approach to the above-described complexities is to assume that mortality levels and persistency levels operate independently of each other. If we read his comments correctly, he believes that one can set the persister premium at a level high enough to eliminate deficiency reserves and not worry about the effect of persistency on the viability of those premiums. It seems clear to us that setting persister premiums at a table 4 level as he suggests would expose the company to the same cycle of lapses by the better risks (table 3 or better), leading to higher sustained mortality, which, in turn, would lead to losses or higher persister premiums.

3. A proposed alternative to these complicated formulations is the use of a "conservation-of-total-lapses" principle. Although we had difficulty following Messrs. Gould and Porter's calculation, it appears that this approach solves for the persister-class lapse rate by establishing a lapse rate for the reverter class and assuming that the mortality rate for the two classes is the same (much as we solved for the persister-class mortality rate by assuming a reverter-class mortality rate and equal lapse rates for the two classes). This assumption does not seem appreciably better than our assumption that the lapse rates are the same. Having to assume that mortality rates are the same for both classes in order to arrive at this assumption is one flaw. Also, the conservation-of-deaths principle works because people do not choose to die; so as long as you insure the same class of risk, total deaths should be the same. The conservation-of-lapses principle does *not* work because people can *choose* to lapse depending on the premium scale they are paying; thus, total lapses would not necessarily be the same.

4. In any case, Messrs. Gould and Porter's discussion shows that there appears to be no substantial difference between persister mortality calculated using our admittedly convenient lapse assumptions and that calculated using their approach with separate lapse rates for persisters and reverters. The maximum differential is roughly 7 deaths per 100,000 and is often much less than that. This differential seems especially small in light of the approximate nature of the other assumptions that must be made in pricing this product. These findings corroborate our conclusion that relative lapse rate differentials have only a minor effect on persister mortality.

To a large extent, the second conclusion in the Gould-Porter discussion is included in our paper. Since lapse rates appear to play a relatively small part in determining persister mortality, it would be a better use of time to calculate the effect of different reversion rates. With regard to the third conclusion, while "conservation of total deaths" may be handy, it is also entirely reasonable. We do not comprehend how splitting a group of risks into two subgroups could result in a different number of deaths for the sum of the two subgroups than for the group as a whole. It would have been helpful if the contributors had elaborated on this point.

Mr. Bakos suggests that a primary reason for developing an S/U ART plan is to reduce deficiency reserves while offering competitive rates. A plan developed with this objective would resemble a non-guaranteed-premium aggregate plan if "reversion" underwriting were minimal. Such a plan really could not have select premiums, since mortality would be aggregate. We were not aware that lower deficiency reserves were a major factor in the development of true S/U ART plans—the idea seems reasonable, but they were not a factor at our company. In fact, at the time we priced this plan, the method being used to value deficiency reserves (pre–1976 amendments) produced deficiencies in the early years of the contract comparable to those for our aggregate plan. If a company were to develop an S/U ART plan with the hope of getting immediate surplus relief, it would be disappointed.

Mr. Bakos continues his discussion with the observation that persister mortality is just as dependent on persister lapse rates as on reversion rates and that, since neither is easy to predict, one should not try. He proposes fixing the level of persister mortality at an expected substandard level and calculating appropriate ultimate (substandard) premium rates. Using the fixed persister mortality, aggregate mortality, and conservation of total deaths, one then calculates the theoretical reversion percentage and directs the underwriters to underwrite reversions so that ultimate mortality is as anticipated.

We feel that there are some serious flaws in this approach. For instance, how much control over reversions can the company really exercise via its underwriting program? Mr. Bakos plans to keep the premiums for the persister class in line by restricting the number that can revert, thus increasing the number of persisters. It is not clear where these extra persisters are to be obtained. Presumably they would otherwise have been reverters. If they were standard risks under new-issue underwriting standards, then the underwriter would either be discriminating unfairly by letting some standard risks revert and not others, or he would have to

require that reverters (and new issues?) be superselect. If the latter, then the standard persisters will obtain another policy elsewhere, thereby increasing the mortality of the remaining persisters and defeating the objective. The other possibility is that, if 70 percent are allowed to revert, underwriting on reversion is more lax than at issue, and that you are merely tightening the underwriting when you increase the size of the persister group. Even so, these new persisters must be better risks than the unenlarged group of persisters, and we would think that they would be less willing to pay the high ultimate rate than the original 30 percent, again defeating the objective. So what to do? Lower the ultimate rate and start over? It seems at least possible that the end result of a process like this will be an aggregate plan.

Additionally, one should not be overly preoccupied with the rate of reversion at the contractually allowed point if insureds can revert de facto before that point. Those early reversions may be a real source of loss—so much so that few "persisters" remain by the time the first contractual reversion date arrives.

Also, from a theoretical point of view, it seems dangerous to adopt a pricing philosophy that involves setting the premiums (i.e., ultimate persister premiums) and then deriving the experience assumptions (i.e., reversion rates) that they will support.

Mr. Bakos's hypothesis that the reverter class eventually might become superselect while the persisters' mortality remains above the level of ultimate mortality is conceivable. For example, a group of reverting insureds underwritten as standard at several points over the past few years might be a better class of risk than a group of new issues likewise underwritten as standard. One reason this might happen is that the group of reverting insureds would have had more exposure to the underwriting process than the new issues, thus providing the company with a correspondingly greater chance to discover underlying medical problems. It is not clear whether, or to what degree, this would actually occur. If some assumption could be made as to the improvement in mortality among the reverting group, the conservation-of-deaths principle would provide the mortality assumption for the persister group, which would be higher than ultimate.

In discussing expense considerations, we did not say that persisters are equivalent to not-takens, as Mr. Bakos has suggested, although that would be the case if *everyone* applied for reversion. We did not view the higher-than-normal profit goal as an extra expense but rather as an added risk charge for a very risky product.

We agree that the nature of an S/U ART product would seem to pre-

clude substandard issues. However, the marketplace dictates otherwise, and substandard S/U ART products are available in abundance. We also agree that it is difficult to see how such a product really can be cheaper in the long run for most insureds.

Mr. Smith succinctly raises the question of the viability of S/U ART and suggests that reinsurers are doing much to foster its increasing popularity. He points out the extremely competitive nature of today's reinsurance market, which means that a writing company usually can find at least one reinsurer that will offer competitive coinsurance allowances on virtually any product. Not only do marketing pressures force reinsurers to be superaggressive in pricing, but, as Mr. Smith illustrates with an essentially true-to-life example, they can be pressured into accepting questionable underwriting practices on existing plans as well.

Mr. Smith's comparison with property/casualty products is quite apt. It is interesting to note that these products generally pay a level commission, which may indicate the future direction of the term insurance market.

There is, perhaps, a place for S/U ART, but, in our opinion, not in the form addressed in the paper. Reinsurers have used select and ultimate YRT rates for years—clearly the opportunities for agent and insured selection are not present there. Another possibility that may have merit is to charge select and ultimate premiums for the pure insurance component of a universal life plan. However, we agree with Mr. Smith and dozens of other actuaries with whom we have discussed S/U ART that those companies selling products (be they term or "whole life") with select premiums may be asking for trouble. Time will tell whether trouble will respond, but we expect it will. The purpose of our paper was not to allay the fears of hesitant actuaries about to price this type of plan but rather to point out the huge uncertainties involved and indicate a method whereby one could evaluate results under different scenarios.