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# A CORPORATE MODEL FOR GAAP, ET CETERA

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1. The problem of proving for the risks of adverse deviation in GAAP reserves.
2. Economic aspects of the problem.
3. The SOFASIM model.
4. Applying SOFASIM to the GAAP problem.
5. Results: GAAP deltas.

MR. JOHN C. WOODDY: It seems reasonable to begin this session by relating the events leading to the appearance of this group of people on this platform.

In the beginning was the word, and the word was "delta". It first appeared, in its present sense, in a paper by Dick Horn entitled, "Life Insurance Earnings and the Release from Risk Policy Reserve System" in the 1971 Transactions. This was the time of the first blooming of the Audit Guide for Stock Life Insurance Companies.

It seemed to a number of senior actuaries during the reign of Tommy Bowles that if we actuaries were to defend our turf in the life insurance industry we had better get moving on deltas, since the accountants had left us precious little else to call our own. Provision for the risks of adverse deviation was, in colloquial terms, the name of the game.

It just so happened that, by a stroke of Providence, a new committee had been created by the Board only a few months earlier, the Joint (with the Casualty Actuarial Society) Committee on Theory of Risk (JCOTOR) under the chairmanship of Ed Lew. The other members were: Charlie Hachemeister, John Beekman, Dave Grady, Paul Kahn, Dale Nelson, Paul Otteson, Walt Stewart, Frank DiPaolo, Dick Horan, Dick Ziock, and myself. By another stroke of Providence Ed was elected to succeed Tommy. I leave you to guess who became chairman of JCOTOR and was tapped for the honor of leading the actuarial delta crusade.

But how did a couple of innocent civilians like Barbara and Harry Markowitz become enmeshed in this Odyssey? In brief, JCOTOR met to discuss our project, decided that interest variation would have to be highly significant, and that guidance from an economist would be necessary.

I presented JCOTOR's report to the Board of Governors in April of 1973 and asked for $50,000 for consulting fees and costs of computer usage. The Board said "yes" and our lives have not been the same since.

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Harry was the consulting economist and, incidentally, the originator of the programming language SIMSCRIPT. In the early days he was putting in only about four times as much work as we were paying him for. After he joined the IBM Research Center in Yorktown Heights, New York, he continued to offer advice and guidance on the project and refused to accept any fees at all, thus reaching infinity, although perhaps not beatitude.

By this time, Harry had recruited Barbara to our cause, thus attaining infinity squared. Also Dale Lamps and Lou Weinstein had replaced Dick Horn and Dick Ziock on the Committee. The computer program for SOFASIM was now essentially complete, except that a certain amount of jockeying around, amendment, substitution, enhancement and plain debugging stretched out over far more time than it took to write SOFASIM in the first instance. This latter activity has occupied Barbara Markowitz and Alice Goldstein for many more moons than I can count, even using toes as well as fingers.

While making all of the necessary modifications, they added numerous small but useful improvements. Finally we had to face the fact that SOFASIM was as ready as it would ever be. Another meeting, this time of COTOR (my colleagues in the CAS had grown restless under the interminable focus on life insurance deltas) was called in May 1977 to lay down specifications for the input with which SOFASIM would solve the GAAP problem. You will hear more of that from Alice Goldstein.

To digress a moment, along the way we came to realize that SOFASIM had acquired a life of its own, being applicable to a far wider variety of problems than GAAP deltas. It does seem to have many useful characteristics for modeling a life insurance company not possessed by other simulation programs. But, then, no other model has had a Harry Markowitz doing the programming.

Back, now, to GAAP, we started with the problem of mortality variation, thinking this the easiest. We were right. Later you will hear something of our findings.

Lapse was our second subject of study. The problems were somewhat different from those of mortality but not drastically so.

As mentioned earlier, interest variation was expected to be more significant than mortality and lapse variation. We were right; more later.

A complicating factor: for interest we had to develop our own probability distribution governing random variation; the Poisson for mortality and the Normal for lapse seemed sufficiently conventional not to require agonizing over. Harry will tell you about our approach to interest variation.

So far we have not done much with expense variation. A couple of runs with constant expenses at higher levels than "standard" and a couple of runs with expenses increasing at 5% and 10% per year compounded are the extent of our work to date.

Now it is time to hear from Harry Markowitz.

## ECONOMIC CONSIDERATIONS IN THE DESIGN OF THE SOFASIM (SOCIETY OF ACTUARIES) SIMULATOR

DR. HARRY M. MARKOWITZ: I propose to review certain major design decisions made in the building of SOFASIM. The first major decision, indeed, was the decision to build an integrated simulation model of a stock life insurance company. Let me trace out how this came about.

The GAAP problem, as explained to me by Ed Lew, then President of the Society of Actuaries and John Wooddy, Chairman of the Joint Committee on the Theory of Risk, may be briefly characterized as follows. Actuaries compute gross premium reserves using mortality rates, policy lapse rates, interest rates and expenses. In attempting to establish Generally Accepted Accounting Principles to apply to life insurance companies owned by stockholders, the accountants proposed that "most likely" values be used for mortality, lapse, interest, and expense rates. The actuaries argued that an increment or "delta" should be added or subtracted from these most likely rates to allow for adverse deviations. The accountants concurred. But this left the problem of how to compute the deltas. I was invited by Ed and John to help the Committee on the Theory of Risk establish a procedure for estimating the deltas for interest rates.

One problem immediately encountered in trying to analyze interest rate deltas is the relationship between the market interest rates which the economist or financial analyst reads in the newspapers, or stores in his database, as compared with the rate of return actually earned on the company's investments. Suppose, for example, that during some relatively short period of time the market rate on some particular type of bond dropped from eight percent to six percent. What does this imply for the earned rate of return, either for the life insurance company as a whole or for the interest rate number which should have been used in computing premiums for a particular policy?

This is an intricate matter. When the market interest rate shifts, there are already moneys invested in bonds at the old rate. These would continue receiving interest payments at the prior coupon rates while new premiums and the cash receipts from existing investments are invested at the new rate. But bonds mature, become cash and then are reinvested at the new rate. Thus, depending on the maturity structure of the company's bond portfolio, the internal earned interest rate of the company would move more quickly or more slowly from the old market rates to the new market rates. Also, some bonds are callable (at the discretion of the issuer). Thus, when interest rates go down, insofar as the company holds callable rather than noncallable bonds, the transition from the higher rates to the lower rates may be accelerated.

It seemed to me (and still does) that the only practical way to take into account with any degree of accuracy this complex linkage between the market rate of interest and the company's earned rate of interest is by means of a dynamic simulation model. It seemed reasonable to Ed, John and me that such a model should also include such other aspects of the life insurance company as are major contributors to the company's cash flow. Many detailed decisions concerning what was to be in and what was to be out of the model, and how these matters were to be handled as outlined below, were made by John Wooddy and me.

It was obvious from the first that the model of the life insurance company (SOFASIM, as it was named) must have an actuarial side as well as an investment side. The actuarial and related activites of the model include the sales of policies of various types and sizes to purchasers of various ages; the paying of commissions on these sales according to appropriate schedules; the paying of death, surrender, and endowment benefits; and the computing of statutory reserves. Since the project was concerned with "adverse deviations", and since it is not at all clear how an unusual event in one part of the business will ramify through the rest, it was decided that the model should allow deaths and lapses (and even sales, when desired) to be drawn randomly. In reality, policyholders die in integral numbers, a given policyholder being either legally alive or dead. Similarly SOFASIM generated integral deaths rather than continuous deaths as is frequently done in nonrandom actuarial models. Of course we do not have to draw individual random numbers for each policyholder to decide whether he dies during some period of time, since a draw from a Poisson distribution to determine the number of deaths for a suitably homogeneous collection of policyholders gives approximately the same effect much more economically.

In some cases (as will be illustrated in a subsequent talk) an analyst may wish to compare the results of a deterministic (nonrandom) run of the model with the results from several random runs. To facilitate this SOFASIM allows the user to specify whether deaths and/or lapses and/or sales are to be generated randomly or deterministically.

In addition to the actuarial and investment side of the life insurance business, the tax aspect of the business seemed of sufficient importance to warrant explicit, detailed representation in the model. In the first instance it was the size of the tax bite on the cash flow, and the fact that this did not spread itself evenly through the year, that made modeling it seem necessary. When SOFASIM became operational the necessity for a detailed tax computation within the model, to accurately answer certain types of questions, became even clearer.

When SOFASIM seemed debugged (as far as Dr. Barbara Markowitz and I could tell) we turned it over to John Woody for testing. John prescribed initial runs whose outcomes could be computed by techniques already available to the actuary. The first of such runs were performed without the tax calculations. (An input to the model allows the tax calculation to be bypassed for any run.) The first runs showed that the model was not sufficiently accurate in certain areas, and we altered the model accordingly. When we made these corrections, found that the model met the initial tests, and turned to similar tests including tax calculations, the model had a surprise for us.

As before, we ran a case where we knew the right answer in order to see if the model would agree with us. As before, the model came up with a different answer. But this time it turned out that the model was right and we were wrong. The inputs had been designed so that, according to conventional actuarial calculations, the simulated life insurance company should have just broken even. It should have ended with the same value of assets as it had at the beginning. The model on the other hand insisted that the life insurance company had lost a substantial amount of money. The reason it gave, if you looked closely at the various balance sheets and P&L statements it prints out for each simulated year, was that the particular simulated company sold policies in its early years, paid out much in the way of commissions and seemed to lose money (at least according to the way in which gain-from-operations is computed for tax purposes). As the years went by this same simulated company had little or no new sales, but seemed to make money by collecting premiums from previously sold policies and paying smaller commissions according to the schedule. But the income tax law limits the amount of time that a loss in one year can be carried forward to cancel a gain in another year.

Thus, in this particular simulated run, the company had to pay taxes on the apparent gains made in subsequent years since the tax law did not allow these to be canceled against the apparent losses made in remote prior years. In other instances as well, unanticipated results of the model turned out to be consequences of the tax laws.

Part of John Wooddy's testing of the SOFASIM model consisted in comparing the amount of tax on income and capital gain paid by the simulated company with the amount which the company should have paid, according to independent tax calculations by life insurance company tax experts, based on the simulated company's various income and expense items as reported by the model. It turned out that my hastily acquired knowledge of life insurance income tax procedures were incomplete, and the model had to be amplified in this area. This was done by Alice Goldstein who (1) read the SOFASIM description of the tax computation (SOFASIM is written in SIMSCRIPT II which has a more English-like syntax than conventional programming languages); (2) learned the tax law with precision, partly from the Society of Actuaries study note which I had also used, partly from reading the tax form and associated instructions, and partly from tax experts such as Barbara Zucker of North American Reassurance; (3) wrote out an amended tax calculation in a SIMSCRIPT-like style, whose syntax Barbara Markowitz and I corrected to reflect the precise SIMSCRIPT II rules; and (4) compiled, ran, tested, and debugged until SOFASIM computed taxes like a tax expert.

One of the fundamental design decisions in the building of a simulator is the choice of time increment. For example, at one extreme we could have assumed in SOFASIM that all cash outflows and inflows--all premium collections, benefit payments, tax payments, interest receipts, etc.--happened once per year. While annual actuarial models are not uncommon, this assumption seemed undesirable for SOFASIM. The inflow to cash might exceed the outflows from cash for a year, yet outflows exceed inflows for a particular month, for example when taxes had to be paid. The annual model might conclude that no sales of bonds from the portfolio was required, while a monthly model might conclude that sales from the portfolio were needed at one point in the year followed by even greater purchases at some other point in the year. At the other extreme the model could have been built with a daily time increment, or even with individual deaths, lapses, coupon payments, etc. happening individually as they occur. While it would not have been difficult to program the computer for such a small time increment, or for continuous time, we believe the cost in computer running time would have been unnecessarily large. As it was, we decided that a monthly time increment would be sufficiently fine for our purposes.

Thus a basic assumption of SOFASIM is that all bond coupon payments are received on a certain day of the month, all operating expenses are paid on a different day of the month, etc. At one point in this monthly cycle we charge the simulated company a short term interest rate if its cash balance is negative, but otherwise we don't worry about temporary negative cash balances during the month, since the simulated company has artifically wide swings in cash because of the model's handling of time.

Quarterly dividend payments are one of the cash outflow items of a stock life insurance company. The simulated company in SOFASIM decides on its dividend rate. The deciding and paying of dividends are included in SOFASIM, not only because they are part of cash flow, but also because they are the "name of the game." The object of Generally Accepted Accounting Principles is to fairly evaluate the worth of a company, that is, the worth of the company to its stockholders. More generally, we consider the potential use of SOFASIM to be to evaluate the effect of possible changes in environment or corporate policy on the value of the company as a whole. If we consider, for this purpose, a stockholder who is committed to holding the stock of the company for an indefinite period of time then the worth of the company to him is given by some function of the future stream of dividend payments (or more precisely the probability distribution of the future stream of possible dividend payments).

One might argue that the investor should look at the stream of earnings reports rather than the stream of dividend payments; and indeed SOFASIM reports discounted earnings as well as discounted dividends. But the computation of earnings requires some estimate of changes in liabilities; which requires some estimate of reserves; and part of the original problem was that it is not completely clear how reserves should be calculated. For this and other reasons earnings is a number subject to question while dividends are the payoff which the stockholder can spend on yachts or campus buildings or new investments.

A principal purpose of SOFASIM is to evaluate the effects of adverse deviations. In SOFASIM these adverse deviations can occur to the simulated company by either or both of two general sources of variability. One broad class of variability is that due to the drawing of random numbers within the program. In particular, an analyst using SOFASIM may specify that deaths and/or lapses and/or sales may be random. Provision is also made for replicating a run with all parameters and conditions the same but with new random drawings of deaths, lapses and sales.

A second method of introducing variation into the model consists of facilities by which the analyst can change conditions or parameters exogenously, that is, from outside the simulation run. For example, some of the initial inputs to the model are various cost coefficients used in computing operating expenses. In preparing the input to a specific SOFASIM run the analyst may specify that these cost coefficients can change at any point in time and as often as the analyst desires. Other parameters which the analyst specifies initially and may change exogenously at any point in simulated time are income tax rates, investment as well as operating expense coefficients, the yields on callable and/or noncallable bonds of various coupons and maturities, the mortality tables and lapse tables (as distinguished from the individual random drawings from these tables), the sales rates on various life insurance policy types by size of policy and age at issue, and various decision rule parameters used by the company.

The fact that the model receives these changes exogenously rather than generating them randomly within the model does not preclude the random generation of interest rates, expenses, etc. For example, in some of the runs which Alice Goldstein will report later in this session a constant interest rate was assumed throughout a run, with different runs having different (but constant through time) interest rates. In other runs the interest rates which were input to SOFASIM were themselves the outputs of a random interest generating model which John Wooddy and I had fit to historical data. Different runs of the SOFASIM model had different interest rate time patterns put in exogenously, as generated by different runs of the interest rate generating model. If someone had a different model of interest rates he could similarly use his model to generate the inputs to SOFASIM. SOFASIM then would show the consequences for the profitability of the life insurance company as a whole of one or another random interest rate generating processes.

Changes in interest rates were made exogenous to SOFASIM since we wanted it to be easy to change the way interest rates are generated; and we are willing to assume that the simulated Company's actions do not perceptibly alter market interest rates. On the other hand, we generate deaths and lapses internally since (among other things) there is no other convenient way to run the simulated Company through time. These two examples illustrate why some sources of variability are handled exogenously and other endogenously.

We have spoken of features that are included in SOFASIM. We could also speak of features which SOFASIM excludes. For example, we judged it sufficient for our purposes to assume that the Company's only investments are bonds and cash. Another potential user of SOFASIM might feel that the model is almost exactly what he needs but that mortgages should also be included. One of the objectives of the SIMSCRIPT II programming language, used in the programming of SOFASIM, was to facilitate the modification of simulation models; since experimentation is central to simulation. An analyst who knows SIMSCRIPT--perhaps having taken a short course in it, and then building a small model of his own--could easily add mortgages to SOFASIM once he had clearly in mind how the simulated mortgage was to work in the model.

It turns out that even without changing the SOFASIM program the existing model can sometimes be used to simulate features not originally envisioned, by "lying" to the model in presenting its input data. John Wooddy has been especially ingenious in this regard, and may have time to give an example or two later.

I have discussed generally why SOFASIM includes the actuarial, investment, tax and dividend sides of the business; why a monthly time increment was chosen; why some sources of variability are handled exogenously while others are handled endogenously. Subsequent speakers will present some concrete details concerning SOFASIM's representation of these facets of the life insurance business; describe how SOFASIM was used to analyze the GAAP problem; and offer some general conclusions concerning the GAAP analysis in particular and the potential uses of SOFASIM in general.

## THE SOFASIM MODEL

DR. BARBARA G. MARKOWITZ: SOFASIM is a model of a stock life insurance company which can issue three basic plans of non-par insurance with level benefits; co-terminous term; co-terminous endowment; limited pay and whole life. All claim settlements are made by lump sum payments.

SOFASIM can best be described in terms of: the inputs which specify the system to be simulated: i.e., the parameters of the simulated company and the environment in which the company is to function; the model within the computer; and the outputs which summarize the performance of the simulated company within the postulated environment.

THE INPUTS

The input parameters are written on twelve different types of SOFASIM input forms. Your handout includes one each of the twelve different input forms containing a sample of the actual inputs used for one of the GAAP simulations. Also included in your handout are the results produced by that run. Because of the volume of the input data these forms are usually given to a keypunch operator who punches their contents onto a set of IBM cards. After a listing of these cards has been checked for errors the cards are read into a computer and stored in a disk file. A SOFASIM run can now be executed and the results produced immediately or stored for later printing.

You specify the initial status of the simulated company, which may be a company already in existence or a company just starting into business. If you are simulating an ongoing company, the initial inputs include the number of policies of each type, size, issue year and age at issue currently in force at the beginning of the simulation; the number of bonds of various maturities, coupons and callability currently in the company's portfolio; and five years of tax-related history and shareholder-dividend-related history.

You must also provide a description of the insurance policies in force and those to be issued by the company in the future. For each policy type, size, age-at-issue combination, both for those in force at the beginning of a simulation and for any that may be issued during a run, you must define all of the characteristics including gross premiums, sales rates, valuation bases and cash value bases. Commission tables, a stockholder dividend policy, and an investment strategy must also be provided.

The environment is described by specifying mortality tables, lapse tables, the interest rate structure, operating expense coefficients and the tax rates. Many of these variables can be changed during the simulation. The tax computation can be suppressed.

Deaths and lapses can be generated randomly using the mortality and lapse table entries as their expected values or they can be generated nonrandomly using these table values to determine the actual integer numbers of deaths and lapses.

The SOFASIM model utilizes these company and environmental specifications to simulate the monthly operations of a stock life insurance company. When the simulation is complete annual reports are produced describing the performance of the company over the time frame specified on one of the input forms.

THE MODEL

The SOFASIM computer model performs three major functions; initialization, the processing of events and the generation of final summary reports.

During initialization the values specified on the input forms are used to set up the initial conditions of the company and the environment. Each input card is error edited. An error message is generated if an error is encountered and the simulation stops so that the error can be corrected before proceeding.

After all of the initial inputs have been correctly entered SOFASIM is ready to begin simulating the operation of your life insurance company. During execution it processes monthly, nonmonthly and external events. One line of output is generated each year but the final reports are printed after the simulation is complete.

Each month SOFASIM processes policies, pays operating expenses, processes existing investments and buys or sells investments, in that order. Other events are executed regularly but not monthly. These include the events which decide the size of the dividend, pay the dividends, calculate estimated and final income taxes, pay taxes or collect refunds as appropriate, and generate the annual reports.

Besides the monthly and other periodic events, external events are executed at the date and time specified on input forms B through K. Because SOFASIM is an event oriented simulation you can change company parameters, commission tables, the desired investment profile, interest rates, mortality, valuation and lapse tables, or certain policy characteristics such as sales rates and gross premiums at any time during the simulation.

A cohort represents a unique combination of policy type, age at issue, policy size and issue year. Each month the Process Policies routine performs the following operations on each cohort. First, sales are generated for cohorts selling policies this month, keeping track of the month and year of issue. If sales are random the number of sales is drawn from the normal distribution using the input values as the means and standard deviations. If sales are nonrandom the actual number of sales is determined by the input on form K1. The first annual premium is collected, commissions are paid and cash is updated. Only annual premiums are permitted.

Next, endowments are matured and term policies are expired for cohorts destined to mature or expire this month. Cash and year to date totals are updated.

Lapses are generated in each anniversary month. Since the first annual premium has already been collected policies cannot lapse in the year of issue. Lapses can be random or not random. If random and the expected number is small a draw is made from the Poisson distribution, if not small a draw is made from the normal distribution. Surrender benefits are paid, cash and year to date totals updated.

Next, premiums are collected for all cohorts with this as their anniversary month except for these new cohorts generated this month who have already paid their premiums. Premiums are added to cash and year to date totals are updated.

Deaths are generated for each and every cohort, each and every month. Each cohort is exposed to twelve months of each policy year's mortality rates.

Monthly mortality rates were calculated from the annual year input. Deaths can be random or nonrandom. If random the number of deaths is drawn from the Poisson distribution. Death benefits are paid, cash and year to date totals are updated.

In December the cash surrender values and reserves to be used in the following year are calculated.

Operating expenses are computed utilizing the factors specified on form B. These include; the first year and renewal years per policy expenses, the first year and renewal years per thousand expenses, and a monthly overhead expense. These costs are subtracted from cash, and year to date totals are updated.

SOFASIM's portfolio consists of cash and callable and/or noncallable bonds over a range of coupon maturity combinations. Each month these existing investments are updated. First, short term interest is collected or paid on the outstanding cash balance. Coupons are clipped for 1/12 of the bonds held in each coupon category for each maturity. One-twelfth of the bonds which mature this year are cashed. Then a calculation is made to see if any bonds should be called and if so what proportion of them should be called. The inputs determine the outcome of this computation, and bonds are called if appropriate. The company's cash balance, capital gains and losses, if any, and the amount invested in each call category and maturity combination are updated.

The last regular monthly event to occur is Buy or Sell Investments. Bonds are bought at par and sold at a price calculated as a function of the interest rate structure specified in the inputs. In this routine, a decision is made each month as to whether bonds need to be bought or sold depending on the ratio of cash to total reserves using input parameters. After this decision is made it is necessary to decide which maturity of bonds to buy or sell. This determination depends in part on what the company already holds for each of the maturities and on the desired investment profile.

After sales and purchases are made, investment expenses are calculated as a function of the fact value of the bonds held and the amount of transaction made. These investment costs were specified on the input forms. Money market changes can be reflected in the model by changing the interest rate structure yields to maturity during the run. Alice will discuss how this was accomplished and its effect on the GAAP runs.

Among the nonmonthly events, the Decide Dividend routine determines whether to change the current dividend rate. The dividend decided upon will be paid quarterly for the next four quarters beginning on the date you specified. The Pay Dividend routine pays the dividend, updates the values of cash balance and dividends paid out, and, if necessary, makes an accounting transfer from the policyholders surplus account to the shareholders surplus account.

The routine which calculates estimated taxes four times a year and the final tax on December 31 of each year is a very complicated program and reasonably duplicates the actual tax computation currently in use. When the taxes are calculated the appropriate values are updated and a tax payment or refund event is scheduled as appropriate. The Pay Tax routine pays the estimated or final tax and updates cash and estimated tax paid. The Refund routine collects any refunds and updates cash.

The last routine to occur each year is the Report and Reset routine. It stores the annual values which are printed in the yearly summaries at the completion of the simulation. It also resets year to date totals to zero.

THE OUTPUTS

A sample output is included in your handout. The first set of yearly data is not generated by the Report and Reset routine. It is generated by the Trace routine which prints one line as each year is completed. Thus, if the run is terminated before its completion, you can see how far the simulation ran and what happened in the years completed. The remainder of the output is printed after the run is finished. All output is in thousands of dollars.

The second table of numbers displays balance sheet information for each year. It lists, for example, the face value of investments held, the cash on hand at the end of the year, total assets, total reserves, the tax due as of year end, total liabilities, the various surpluses (policyholders, shareholders and other), and lastly the year number and date.

The third table shows annual profit and loss data such as the total premiums, investment income, investment expense, death benefits, matured endowments, surrender benefits, commissions, operating expenses, increase in reserve, gain from operations before taxes, federal tax and gain from operations after taxes. The fourth set of data is tax related, the fifth miscellaneous.

A historical summary is printed which shows the mean and standard deviation for gain after taxes and stockholder dividends. The "present value" of dividends and of gain from operations after taxes is computed with three different discount rates, six percent, nine percent and twelve percent.

The final random seeds for deaths, lapses and sales are printed for reference value. They may be used as input to subsequent runs which replicate the same company and environmental parameters but with new, independently drawn, random deaths, lapses or sales.

## APPLYING SOFASIM TO THE GAAP PROBLEM

MRS. ALICE B. GOLDSTEIN: As John mentioned, the Committee on Theory of Risk was given the tasks of determining explicit allowances, or "deltas" to be used when calculating GAAP reserves. These "deltas" are to be incorporated into assumptions underlying reserve calculations in order to provide for the risk of adverse deviations. The basic approach was to analyze frequency distributions of the present worth at issue of the policies under investigation by means of Monte Carlo simulations. SOFASIM was built to generate such simulations of the effects of adverse deviations arising from random fluctuations.

The committee decided to study a closed block of business - one year of issues. This closed block was designed to be reasonably representative of the plan, age at issue, and policy size distributions currently found in the life insurance industry.

We could not study the closed block of business in isolation, however. New issues usually show loses in the first year due to high first year commissions and expenses, and this would, of course, put us in an uncharacteristic tax situation. In order to study the single block of business and still keep a reasonable looking company throughout the simulation, John devised the concept of "foreground" and "background". The foreground is our closed block. The background represents a stable, ongoing company.

The foreground (closed block of business) consists of three different types of plans of insurance: whole life, term, and endowment, at four different issue ages: 25, 35, 45 and 55. The mix of age, policy size and contract type was arrived at with the aid of the 1977 Life Insurance Fact Book. Each simulation was run for 31 years so that all policies except whole life terminated before the end of the run. We used the 1965-70 Basic Select and Ultimate Table for "actual" mortality, the 1958 CSO for valuation mortality, the LIMRA 1971-72 Expected Lapse Table by age, duration and type of plan for lapses, Commissioners reserve valuation method, minimum cash values, and a valuation interest rate of 3 1/2%. These same assumptions were used to calculate the gross premiums used in the simulations. (A table describing the closed block is included in the handout.)

We designed the background so that the total company would be in a Phase II tax position, and have no Phase III tax. Since our company is a stock company, we tried to maintain a reasonable ratio between surplus and assets.

Given the input, it was time to start production runs. To serve as a basis for comparison, the first two runs were completely nonrandom. The first run was of the background only; the other was of the Total Company, i.e., the closed block of business superimposed on the background.

For these two runs, as well as for all other simulations, we calculated the present worth of the simulated company. Present worth means the present value at the beginning of year one of the 31 years of stockholder dividends paid plus the discounted value of the statutory capital and surplus funds held at the end of the 31st year. We used a 6% discount rate since that was the basic earned investment rate assumed throughout the simulation. The difference between the worth of each total company run, be it deterministic or random, and the worth of the background was denoted as the "incremental present value of the foreground".

In order to see the effects of randomness in mortality on present worth, we executed 25 runs with random mortality, but with deterministic lapses, interest rate and expense factors. To see the incremental effect of randomness in lapses, we ran 25 simulations with random lapses as well as random mortality, but again with deterministic interest rates and expense factors. Next, to test the incremental effect of randomness in interest we did an additional 25 runs with interest, mortality and lapses all random. Finally, to test the hypothesis that these effects are additive, we executed 25 simulations with random interest, but with deterministic mortality, lapses and expenses.

Our analysis also required a number of deterministic runs. We duplicated our deterministic Total Company run but changed one variable. For example, we did several nonrandom runs, each with a different level of mortality.

Let us consider some of our results. To begin with, let's look at the findings of the mortality runs starting with the deterministic cases. This graph shows the incremental present value of the foreground, in thousands, plotted against mortality level. Note that in this and the two following graphs, the independent variable is plotted on the vertical axis and the dependent variable is plotted on the horizontal axis.

We see from the run with a mortality level of 120% of standard that the extra 20% mortality results in a loss of $3.7 million for our closed block of business. The line is the least squares fit. We conclude from the closeness of the actual observations to the line that there is essentially a linear relationship between mortality and present worth for the range from 80% to 120% of standard mortality.

The next graph summarizes the results of the 25 runs where mortality was random, but lapses were deterministic and interest was level.

The incremental present values of the 25 runs are marked on the horizontal axis ranging from a low of -$687,000 to a high of $568,000, with various percentiles noted. The vertical axis shows mortality level. The line shows the least squares regression from the previous graph of incremental present value on mortality.

Of course, we recognize that this is a sample; we have only point estimates of each of the population percentiles; and there is really a confidence interval around each point. However, for simplicity of discussion at this time let us assume that these are the population percentiles.

Under this assumption, suppose we wanted to know what increase in mortality assumption would completely protect us 92% of the time against the effects of adverse deviation due to random mortality. According to the graph, the answer is 102% of standard. Therefore, for a 92% level of protection, mortality "delta" equals 2%. On the other hand, if we are willing to live with an 80% level of protection, the mortality "delta" equals 1.5%.

We also studied the effects of adverse deviation due to lapses, expenses and interest, but we will not have time today to delve into all of them. There will be documentation available later this year. What I'd like to discuss now are the effects of interest variability.

SOFASIM has the facility to internally generate random variation in mortality and lapses, but changes in interest rates must be entered as input. Therefore, a separate model was constructed to generate twenty-five sets, each of 372 interest rates - i.e., 31 years of rates changing monthly. The distribution function underlying the random interest was devised by analyzing monthly interest rates of Corporate Long-Term Bond Yields from 1899 to 1976 as found in the Durand Series.

Two GAAP runs were executed for each set of interest rates - one holding mortality and lapse deterministic, and the other incorporating random mortality and random lapses. In the latter case deaths and lapses occurred identically to those in the random mortality and lapse simulations mentioned earlier.

We also ran several cases with constant rates of return. Let's look at some of the results of these simulations, again starting with the deterministic runs.

The vertical axis of this graph shows interest rates. The horizontal axis shows the incremental present value of the foreground in millions (not thousands, as we used in the mortality graphs). The line is a least squares fit to the observations between 4% and 10% interest rates. The closeness of actual observations to the line again leads us to conclude that there is essentially a linear relationship between interest rates and the incremental present values, at least within the range between 4% and 10%.

When we extended the least squares line to include 2% interest and 12% interest, we found that the actual observations at these rates were perceptibly different from the line. I believe that this divergence is caused by the 10-for-1 rule of the tax calculation. This graph (deterministic interest) shows the relationship between incremental present value and deterministic interest rates for different levels of interest. Earlier we saw a graph relating incremental present value to deterministic mortality for different levels of mortality. It is interesting to put these results together.

This next graph displays three curves - one for interest, one for mortality and one for steady inflation rates investigated in our expense study. The vertical axis shows the incremental present value of the foreground. The horizontal axis differs for the three curves, showing: 80% to 120% of standard for mortality; 0% to 10% rates of inflation for expenses; and 4% to 8% for interest. The choice of scales for the three different cases is, of course, subjective, but the curves would not drastically change with any reasonable choice of scale.

There are many ways of verbalizing what this graph shows concerning relative magnitudes. For example, there is approximately a $7.5 million difference in incremental present value when we assume 120% of standard mortality compared with an 80% mortality rate. The same $7.5 million difference will occur when we assume 6.2% interest rather than 6% interest.

Obviously, it is much more likely that we will experience 6% interest when 6.2% has been assumed than that we will experience 120% of '65 - '70 mortality when 80% seemed an appropriate assumption. Clearly, errors in interest assumptions have a much more dramatic impact than errors in level of mortality.

One of the fundamental assumptions of our 'delta' approach is that 'deltas' are additive. The same interest delta should emerge whether we add random interest to a model with deterministic mortality and lapses or to one with random mortality and lapses. To test this assumption, as well as to derive interest 'deltas', we generated two sets of random runs - one set with only interest random, the other with interest, mortality and lapse all random.

We computed interest 'deltas' for each of the sets of runs, separately, using the same levels of protection (i.e., 92%, 80%, 76% and 60%) as we did for mortality. We found interest 'deltas' were almost identical for the two sets of runs. For example, if you want a 92% level of protection against random fluctuations in interest, the set assuming interest alone random produced an interest 'delta' of .172 of "most likely" interest; the set assuming everything random produced a 'delta' of .170 of "most likely".

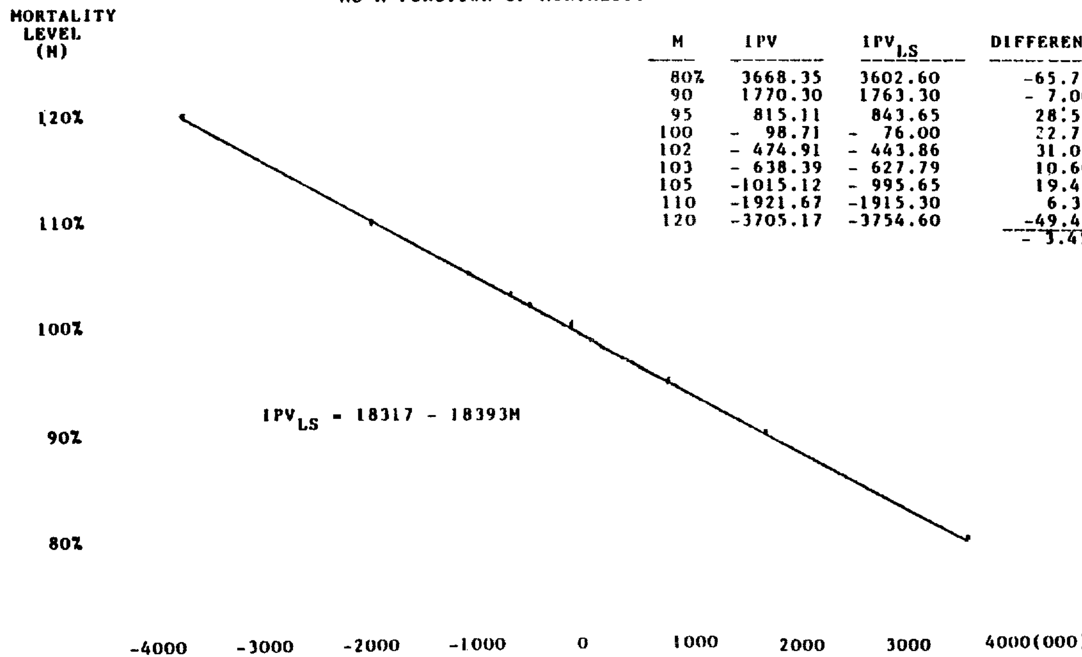
This next graph further analyzes the additivity assumption. The y - axis shows the difference between the present value for simulations having random mortality and lapses and the present value of a total company run done completely deterministically. In other words it shows the effect on our company of having random mortality and random lapses. To get the values for the X - axis, we subtracted the present value of simulations having interest alone random from the present value of runs with interest, mortality and lapses all random. This shows the effect on our company of having random mortality and random lapses when our company also has random interests. Thus, each point really is the result of four runs, whose present values we have denoted on the graph as : $PV_0$, $PV_{ML}$, $PV_{IML}$, and $PV_I$. For a given point on the graph, the ML run has exactly the same deaths and lapses as the IML run. The IML run and the I run for that same point have the identical interest rates month by month. Therefore, each point on the graph represents the effect of randomness of mortality and lapse arrived at from two different directions.

If the deltas were truly additive, i.e., if the effect on present value of making mortality and lapse random was independent of whether interest was random or not, then the observations should be along a $45^o$ line through the origin. The line on the graph is the least squares fit to the actual observations. We see that the slope of the least squares line is quite close to $45^o$ and its intercept is not significantly different from zero. This graph certainly suggests that, on the average, the effects are additive.

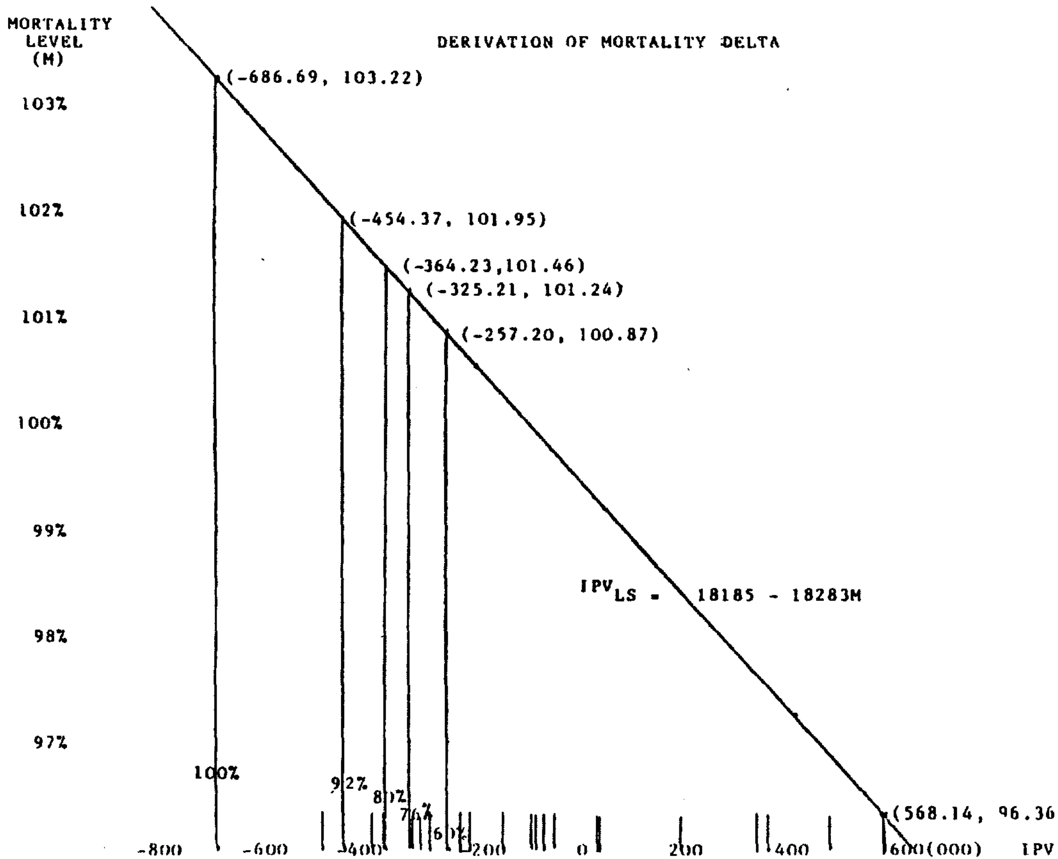
We have seen that the input for our stock life insurance company was based on a closed block of business, "foreground", and a "background" required to simulate an ongoing company. We saw how random runs with this input were used to compute the sought-for deltas. We saw that the deterministic runs showed (and incidentally the random runs confirmed) that the effect of uncertainty in interest was of a greater order of magnitude than were mortality, lapses or expenses. We also reported that, in so far as we could tell from our results, the effects on incremental present value of randomness in mortality and/or lapse and/or interest are essentially additive.

INCREMENTAL PRESENT VALUE OF FOREGROUND

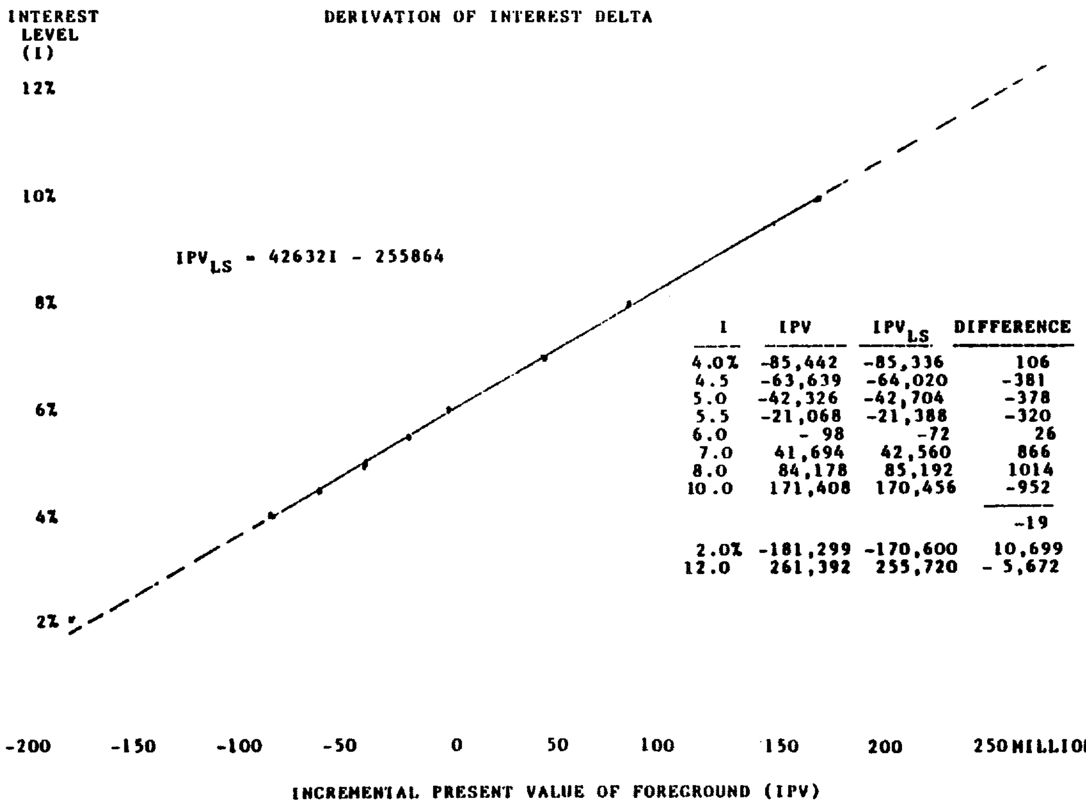
AS A FUNCTION OF MORTALITY



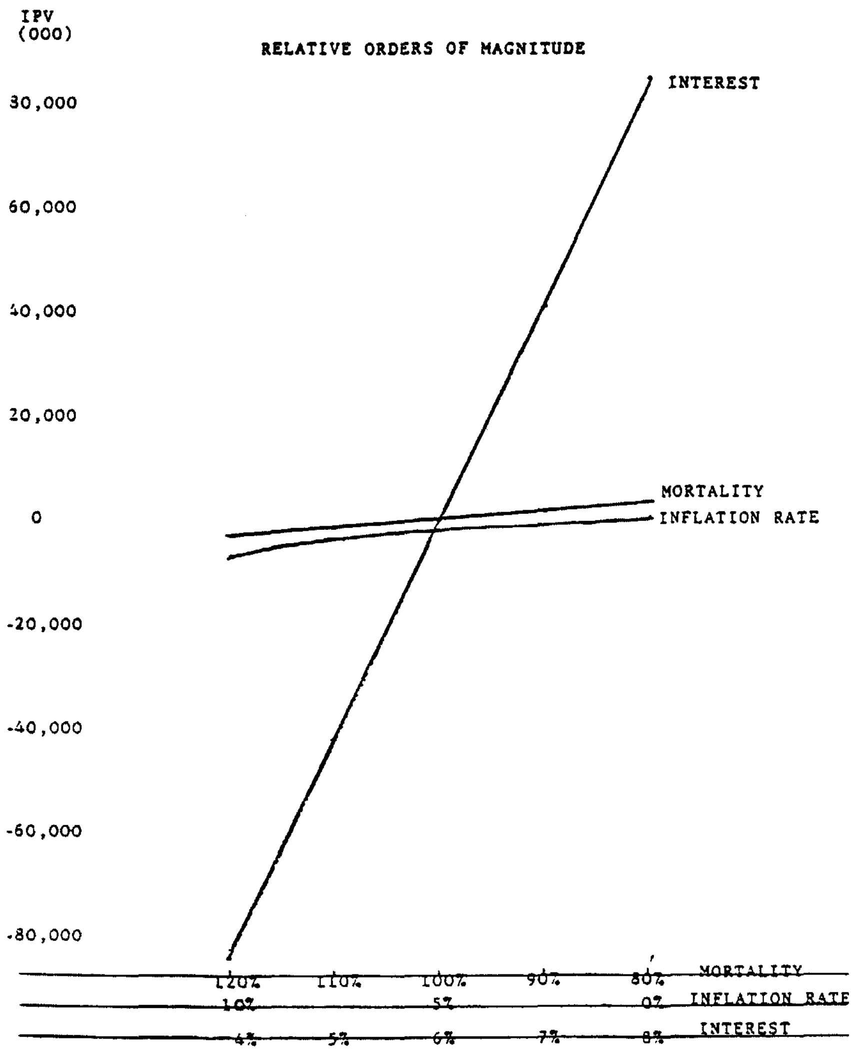

| M | IPV | $IPV_{LS}$ | DIFFERENCE |
|---|---|---|---|
| 80% | 3668.35 | 3602.60 | -65.75 |
| 90 | 1770.30 | 1763.30 | - 7.00 |
| 95 | 815.11 | 843.65 | 28.54 |
| 100 | - 98.71 | - 76.00 | 22.71 |
| 102 | - 474.91 | - 443.86 | 31.05 |
| 103 | - 638.39 | - 627.79 | 10.60 |
| 105 | -1015.12 | - 995.65 | 19.47 |
| 110 | -1921.67 | -1915.30 | 6.37 |
| 120 | -3705.17 | -3754.60 | -49.43 |
| | | | - 3.45 |

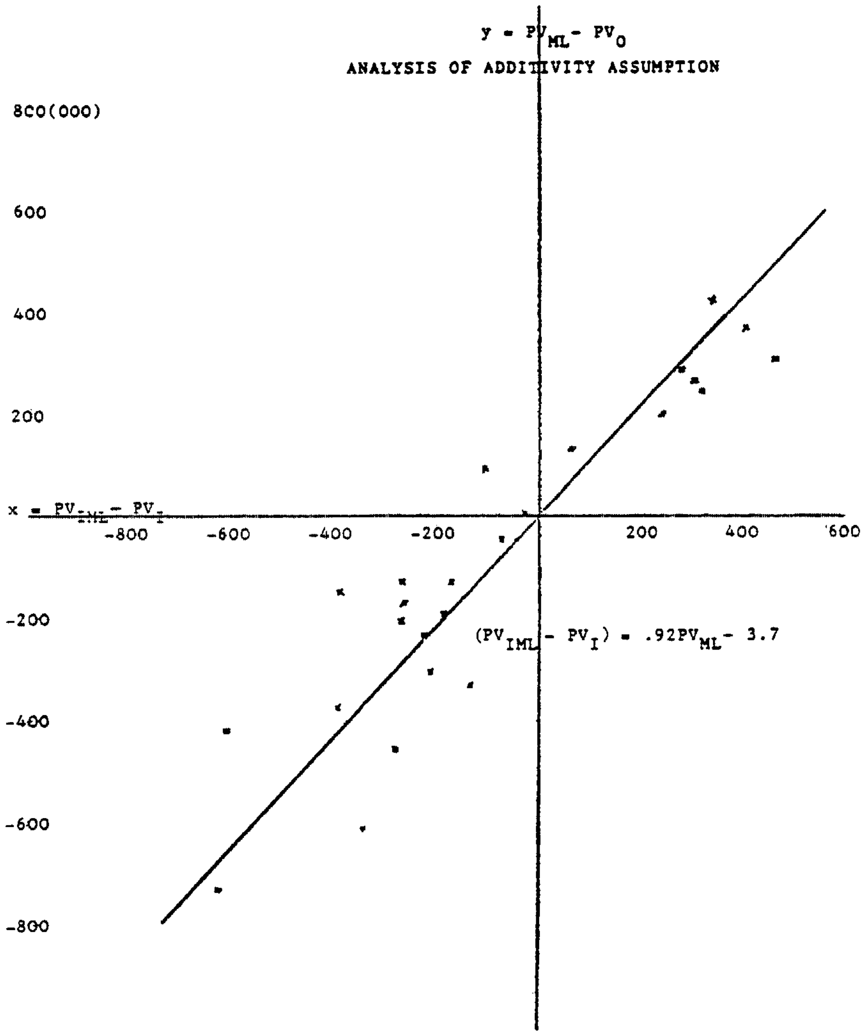

MORTALITY LEVEL (M)
DERIVATION OF MORTALITY DELTA
(-686.69, 103.22)
(-454.37, 101.95)
(-364.23,101.46)
(-325.21, 101.24)
(-257.20, 100.87)
(568.14, 96.36)
IPV<sub>LS</sub> = 18185 - 18283M
103%
102%
101%
100%
99%
98%
97%
100%
92%
80%
70%
60%
-800
-600
-400
-200
0
200
400
600(000)
IPV

## DERIVATION OF INTEREST DELTA




| I | IPV | $IPV_{LS}$ | DIFFERENCE |
|---|---|---|---|
| 4.0% | -85,442 | -85,336 | 106 |
| 4.5 | -63,639 | -64,020 | -381 |
| 5.0 | -42,326 | -42,704 | -378 |
| 5.5 | -21,068 | -21,388 | -320 |
| 6.0 | - 98 | -72 | 26 |
| 7.0 | 41,694 | 42,560 | 866 |
| 8.0 | 84,178 | 85,192 | 1014 |
| 10.0 | 171,408 | 170,456 | -952 |
| | | | -19 |
| 2.0% | -181,299 | -170,600 | 10,699 |
| 12.0 | 261,392 | 255,720 | - 5,672 |


IPV
(000)
RELATIVE ORDERS OF MAGNITUDE
80,000
60,000
40,000
20,000
0
-20,000
-40,000
-60,000
-80,000
INTEREST
MORTALITY
INFLATION RATE
120% 110% 100% 90% 80% MORTALITY
10% 5% 0% INFLATION RATE
4% 5% 6% 7% 8% INTEREST



y = PV_ML - PV_O
ANALYSIS OF ADDITIVITY ASSUMPTION
800(000)
600
400
200
x = PV_IML - PV_I
-800
-600
-400
-200
200
400
600
-200
-400
-600
-800
(PV_IML - PV_I) = .92PV_ML - 3.7

MR. WOODDY: Where are we now? From the standpoint of GAAP we have the means to finish the job. From the standpoint of the actuary seeking to model a real or hypothetical life company, we have the tool. Actuaries have something to call their own!

For the GAAP project, we are in the process of preparing a detailed description of the techniques used to calculate deltas and of how our results may be applied to specific situations. We plan to provide all the information necessary to replicate our experiments for anyone so inclined. Of course, this implies access to the requisite hardware and software. The analysis of our results, barely sketched in Alice's talk, will be presented in full detail.

Considering the economics of GAAP, I think it was a fair assumption that the objective was some sort of uniform way of looking at life company income statements. Today I think it may fairly be said that the objective has not been achieved. With the coming availability of a mechanism for calculating explicit deltas, at least it will be possible to require that the actuary responsible for the GAAP statement declare his unloaded assumptions and the level at which he is making provision for random adverse deviations. This procedure will isolate the judgmental determination of provisions for non-random adverse deviations.

The ability to use SOFASIM to analyze the investment results of a particular portfolio and investment strategy under specified market conditions should contribute to the process of choosing an unloaded interest assumption. In this connection I might pick up Harry's comment about creative lying to the computer. If the investment results are of primary concern at a particular time, and underwriting results are needed only to supply the appropriate cash flow, a very few large cohorts, maybe only one, can be designed to give the desired premium, commission, cash value, death claim, maturity, reserve increase pattern, thus holding down the cost - and complications - of doing many runs just to see the effects on investment results of different hypotheses.

Another example of creative lying, while I am on the subject, is to use the category of "callable" bonds as simply another grade of bonds with different yield curves for both purchases and sales from the yield curves for the non-callable bonds. The supposedly callable bonds can be made effectively non-callable by choice of call parameters which will never take effect.

The main thing to realize in trying to figure out what kind of lies will cause SOFASIM to perform some function not thought of during its gestation is that it is essentially a model of cash flows governed by different kinds of rules. True, it goes through accumulation processes to generate reserve and cash value factors, and it does a U.S. Federal Income Tax calculation; but mostly it counts money.

Also, the fact that the insurance portfolio as well as the investment portfolio may be compartmentalized and the different sections made subject to different parameters -- although the same forms of rules - gives scope for ingenuity. For example, some issue ages, or all, may be made to experience a more or less severe epidemic during the course of the simulation. Any degrees and combinations of secular and cyclical variation in mortality, lapse, interest and expense may be hypothesized and quantified as input. Without need to resort to auxiliary models, once the input has been determined, SOFASIM will integrate all of the implications (subject, of course, to its basic constraints) and produce an output which takes account of every transaction and calculation called for by the input.

We have some time for audience participation in the form of comment and/or question on the subject of GAAP and/or SOFASIM. Leading off is Ron Emery, who has applied SOFASIM to develop some results for United Life and Accident.

MR. RONALD H. EMERY: At United Life and Accident we have been using SOFASIM for about a year, and we have been very pleased with it. It seems to me that SOFASIM is a very general purpose model building type of thing in that we have the stochastic ability on several of the parameters, and we are able to change not only by policy year duration but also by calendar year duration. This allows us to have a great facility in this model that I have not seen in some of the other models. We have used this for both statutory and GAAP projections and for handling non-level premiums. This is something that the SOFASIM Manual says it cannot do, but SOFASIM is very accommodating and allows you to do a lot of things it says you cannot do with it. Among these are: handling non-level premiums; putting reinsurance into the model; we can test retention limits; and we have been testing different methods of expense allocation. We are also able to trick the machine into thinking that we are offering some par policies although I do not believe that this would actually come out correctly in the tax calculations. We can simulate par which is not allowed for the model, we can simulate non-level premiums which are not allowed, and we can put in reinsurance. In general, I find it a very powerful kind of tool, and I am glad that it is available for us to use.