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A REVIEW OF ACTUARIAL COST METHODS FOR DEFINED BENEFIT PENSION PLANS

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ABSTRACT

Financing a defined benefit pension plan can be accomplished in various ways. With the help of a one-life example, this paper gives an elementary actuarial perspective of the problems that arise when a new defined benefit pension plan is initiated, and of the various actuarial concepts and methods developed to ascertain the cost of these pension benefits. The paper shows the range of the cost alternatives conceptually available upon creation of the pension plan. Actuarial formulas are provided in footnotes. The paper is intended as an introduction to defined benefit pension cost determination as well as an elementary example which may be of value in explaining cost alternatives to an actuary's client.

I. INTRODUCTION

A DEFINED benefit pension plan is one in which a benefit has been agreed upon, or "defined." The benefit may have been defined through a simple formula or through a complex one, and the definition may be inherently imprecise (as where the benefit is to be based on future salaries or the cost of living); nevertheless, a benefit has been agreed upon.

After settling upon a pension benefit, the next logical step for the sponsor of a pension plan is to determine how to pay for it. Just like an automobile and a home, a pension plan can be "financed." This paper reviews, in simple terms and with numerical examples, the financing methods, or actuarial cost methods, which are traditional within the actuarial profession. No startling new developments or conclusions should be expected from this paper. Its purpose is to provide, in one place, a complete presentation in modern terms of the concepts and basic formulas which underlie the eminently flexible financing approaches developed to provide pension benefits.

II. A FEW BASIC ASSUMPTIONS

To make the presentation more interesting and meaningful, we shall make use of an example. Consider as a model the case of a pension plan

for one employee (referred to as "Employee"), who is male, aged 45; who joined his employer's business ten years ago when he was 35; and who is expected to retire at age 65 with an annual pension of $100 per year of service[1] from his employer's pension plan. Obviously, then, Employee can expect an annual pension of $3,000. Cost calculations will be based upon the 1971 Group Annuity Mortality Table—Males with 6 per cent interest.[2] The pension available at age 65 shall be payable monthly (at the beginning of each month) as long as Employee lives. Although reference will be made to an annual pension, the calculations will reflect the fact that one-twelfth of that amount is expected to be paid every month. It is also assumed, where pertinent, and only for illustrative purposes, that Employee is now receiving an annual salary of $5,000.

It may be noted that Employee, who is aged 45, was hired at age 35, ten years ago. If he is to receive an annual pension of $100 for each year of service, it is clear that, out of his $3,000 annual pension, $1,000 can be said to be on account of *past service*, while the other $2,000 can be said to be on account of *future service*, although during any one year $100 could be said to be on account of *current service*.

III. A BROAD VIEW OF ACTUARIAL COST METHODS

Under a *pay-as-you-go* plan, an employer could arrange to pay pension benefits directly to retired employees after their retirement. Since no employer is assured of permanent existence, this form of financing is not too reassuring. For Employee's pension, this approach would call for no pension plan contribution or payment by the employer for the next twenty years (until Employee's retirement) and then for a $3,000 annual contribution for Employee's lifetime after age 65 (expected to total $45,462 over 15.154 years, his life expectancy at age 65 when each month entered is counted as a full month[3]).

Another financing method is *terminal funding*, under which an employer waits until an employee retires and then pays a lump sum toward all that

[1] Throughout the paper reference is made to "years of service." The reader may note that many pension plans provide for a waiting period before the employee is eligible to participate in the pension plan. In such a case it would be more proper to speak of "years of participation" than of "years of service." The example assumes that there is no waiting period, so that terms of service and terms of participation in the pension plan are identical. Generally "service" includes only that service eligible to be counted under the terms of the pension plan.

[2] *TSA*, XXIII, 589.

[3] Given by

$$\ddot{e}_{65}^{(12)} = \frac{13}{24} + \frac{1}{l_{65}} \sum_{x=66}^{\omega} l_x .$$

employee's retirement benefits. For Employee's pension, this should call for no contribution for the next twenty years (until his retirement) and then one big contribution of $27,804.98.[4] (Presumably the money would be paid to a life insurer or trustee equipped to pay the $3,000 life annuity out of the amount provided. This is the sort of payment arrangement presumed throughout this paper.)

These two methods, pay-as-you-go and terminal funding, are more theoretical than practical. From an accounting standpoint, they are considered rather unreasonable because they cause a drain on the earnings of an employer quite some time after the years during which the employee gradually earned his pension benefits. From a legal standpoint, pension plans subject to the Employee Retirement Income Security Act of 1974 (ERISA) are not permitted to use them. Hence, in practice, one could begin to separate the basic practical actuarial cost methods available into the broad classes of accrued benefit cost methods and projected benefit cost methods.

An *accrued benefit cost method* is a method which endeavors systematically to match pension costs with the year in which each pension benefit is presumed earned or in which it "accrues." Hence Employee's $3,000 annual pension can be made up, for example, of a $100 pension from service at age 35, of $100 more from service at age 36, and so on to age 64 inclusive (a total of thirty years). As of his age 45, Employee has accrued past-service benefits of $100 a year for ten years, or $1,000 in past-service benefits. For his current service at age 45 he is accruing another $100 current-service benefit.

A *projected benefit cost method*, in contrast, does not consider pension benefits as they may accrue year by year, but considers them as a whole, or in segments such that the division between past-service benefits and future-service benefits might be preserved. In the current example, we are most concerned, when using such a method, with the fact that a $3,000 annual pension benefit needs to be provided for Employee at age 65, although we could also say that Employee can expect a $1,000 past-service benefit and a $2,000 future-service benefit.

Projected benefit cost methods can in turn be divided into *individual level cost methods* and *aggregate level cost methods*. The essential difference between these two types of projected benefit level cost methods is that the individual methods consider the contributions for each employee individually, while the aggregate methods consider the contributions for the group of employees as a whole, with no portion readily ascribable to any specific individual.

[4] $\$3{,}000\, N_{65}^{(12)}/D_{65}$.

IV. A PROBLEM: THE INITIAL ACCRUED LIABILITY

Before beginning to consider the normal actuarial cost methods for pension plans in detail, one should note two problems which customarily confront the new defined benefit pension plan (or any increase in the benefits of an existing plan) when it comes to determining its actuarial cost.

First, the employees are older than they were when they joined the business—the closer they are to retirement, the more, proportionately, their pension benefits for each current year will cost, as compared with benefits the cost of which would have been averaged out over their working lifetime. For instance, the level annual cost for the working lifetime benefits of a 35-year-old employee receiving a pension of $100 a year for each of his thirty remaining working years is $280.12.[5] But for a 45-year-old employee, the average annual cost for his twenty remaining working years is $413.47.[6] Hence the longer one waits to begin a pension plan, the higher the average annual cost of the future benefits ("future-service cost"). If this extra cost were to be expressed as a lump sum (i.e., if we considered the present value of the extra payment of $133.35 required for the next twenty years), an amount[7] of $1,551.96 would be needed.

Second, the employees are often given pension benefits for past service, and this, too, drives up the cost. For instance, the value at age 45 of a pension of $100 a year for each of the ten years of past service of Employee (who is now 45 years old) is $2,406.03.[8]

It can be seen, then, that, if a pension plan had been started when Employee was hired at age 35, an annual cost of $280.12 to his age 65 would have provided him with a pension of $3,000 a year at age 65. Because the employer waited ten years to install the pension plan, (1) the average cost of the future-service benefits (a $2,000-a-year pension) is up, by an amount equivalent to a lump sum of $1,551.96, and (2) the past-service benefits (an extra $1,000-a-year pension) have themselves accrued a substantial liability equal to $2,406.03. In all, then, looking at both components of the liability created by a late start over the normal costs, there is an initial accrued liability of $3,957.99, which includes $2,406.03 for past-service benefits.

[5] $\$100(65 - 35)N_{65}^{(12)}/(N_{35} - N_{65})$.

[6] $\$100(65 - 45)N_{65}^{(12)}/(N_{45} - N_{65})$.

[7] Calculated as

$$\left[\frac{\$100(65 - 45)N_{65}^{(12)}}{N_{45} - N_{65}} - \frac{\$100(65 - 35)N_{65}^{(12)}}{N_{35} - N_{65}}\right]\frac{N_{45} - N_{65}}{D_{45}}.$$

[8] $\$100(45 - 35)N_{65}^{(12)}/D_{45}$.

These extra costs must be absorbed, through contributions, in some reasonable way and over some reasonable period of time. Some actuarial cost methods contemplate the absorption of these costs as part of the total costs of the pension plan, from year to year. These are actuarial cost methods "without supplemental liability." But other actuarial cost methods keep this liability separate and seek to amortize it as a separate item. They can amortize either the whole amount of the initial accrued liability (equivalent to $3,957.99 in the example above), or they can amortize only the liability for past-service benefits (equivalent to $2,406.03 in the example above) and let the regular cost method absorb the rest.

The benefit of having a separate liability to amortize is the flexibility it gives the employer in discharging this separate liability. Even though the accounting profession has certain rules on the minimum and maximum provision for pension costs for those employers who must follow generally accepted accounting principles,[9] the employer is still allowed some flexibility in determining the amount chargeable as pension costs. The United States Internal Revenue Code allows even more flexibility as to the amount of contributions which can be deducted as a business expense for pension costs, although here also there are limitations.[10]

The various defined benefit pension actuarial cost methods are designed to provide a rational way of providing pension benefits, but they seek to account in various ways for (1) the extra cost of starting a plan some time after employees are hired and (2) the extra cost of providing these employees with past-service benefits.

V. ACTUARIAL COST METHODS

For convenience we shall divide the pension actuarial cost methods into the following three basic actuarial families of methods.[11]

1. (*a*) Accrued benefit method with (past-service) supplemental liability.
   (*b*) Accrued benefit method without supplemental liability.
2. (*a*) Individual level cost method with full supplemental liability.
   (*b*) Individual level cost method with partial (past-service) supplemental liability.
   (*c*) Individual level cost method without supplemental liability.

[9] See *Accounting Principles Board Opinion No. 8*, paragraph 17 (American Institute of Certified Public Accountants, 1966).

[10] See, e.g., United States Internal Revenue Code of 1954, as amended, sec. 404(a)(1), Employee Retirement Income Security Act of 1974, sec. 302(b)(2), 29 U.S.C. § 1082 (b)(2).

[11] This classification is a modified and expanded version of that given in D. M. McGill, *Fundamentals of Private Pensions* (2d ed.; Homewood, Ill.: Richard D. Irwin, Inc., 1964), p. 237.

3. (*a*) Aggregate level cost method with full supplemental liability.
   (*b*) Aggregate level cost method with partial (past-service) supplemental liability.
   (*c*) Aggregate level cost method without supplemental liability.

The various pension actuarial cost methods are not known uniformly by these names. The pension literature discloses bewildering confusion as to the meaning of various names by which some methods are known, and, rather than rely upon mere nomenclature, it is far safer to check the essential nature of a method (by whatever name described) and to fit it within the basic actuarial cost methods about to be described.[12]

1. (*a*) *Accrued Benefit Method with (Past-Service) Supplemental Liability*

Under the accrued benefit method, definite pension benefits are deemed to be purchased year by year. Employee's future-service costs could then be accrued through a series of year-by-year current-service costs according to the scale shown in the accompanying tabulation. This will provide a

Beginning of Year	Employee's Age	Annual Pension Amount Purchased	Contribution or Cost
1.....	45	$100	$240.60*
6.....	50	100	328.16
11.....	55	100	453.73
16.....	60	100	638.90
20.....	64	100	857.60

* $\$100N_{65}^{(12)}/D_{45}$, and generally $\$100N_{65}^{(12)}/D_x$, where x is the attained age of the employee.

$2,000 annual pension for Employee on account of future service. At Employee's age 45, there is an initial accrued liability of $2,406.03 on account of his past service. One could possibly amortize this initial accrued liability by annual level contributions[13] of $197.90 over Employee's twenty years of future service, and they could be added to the above figures as constants; but also the liability could possibly be amortized over a shorter or longer period and not necessarily through level amounts.

[12] The reader should note that in May, 1975, a committee of the American Academy of Actuaries, the Committee on Actuarial Principles and Practices in Connection with Pension Plans, made a preliminary recommendation that the term "supplemental present value" be used to designate the quantity variously referred to as "accrued liability," "past-service liability," and "supplemental liability." This paper was prepared before the exposure draft of the committee's recommendations was circulated.

[13] $\$2,406.03/\ddot{a}_{\overline{20}|}$.

(The current amortization rules, under United States tax and pension law, provide that in normal cases the maximum amount which can be deducted in any one year for tax purposes cannot exceed that necessary to amortize the initial accrued liability over ten years, or here \$308.40,[14] and that at least enough must be contributed to amortize the initial accrued liability over, generally, a period of thirty years, or here a minimum of \$164.90.[15] That is, there is a permissible range extending between \$164.90 and \$308.40.)

1. (*b*) *Accrued Benefit Method without Supplemental Liability*

One way to handle employees with past-service benefits would be to spread the benefits in level amounts over the future-service period. Hence, instead of purchasing a \$100 pension per future year of service, one could provide the \$3,000 pension by purchasing \$150 per future year of service (twenty years in total). This would make up the past-service deficiency by retirement time, and there would be no need to make special calculations for the amortization of the supplemental liability for past-service pension benefits.

Cost patterns under the two alternative accrued benefit methods as just described would be as shown in the accompanying tabulation. One

Beginning of Year	Annual Costs under Accrued Benefit Methods			
	With Supplemental Liability			Without Supplemental Liability
	Future Service	Past Service	Total	
1.....	\$240.60	\$197.90	\$ 438.50	\$ 360.90*
6.....	328.16	197.90	526.06	492.24
11.....	453.73	197.90	651.63	680.60
16.....	638.90	197.90	836.80	958.35
20.....	857.60	197.90	1,055.50	1,286.40

* $\$150N_{65}^{(12)}/D_{45}$, and generally $\$150N_{65}^{(12)}/D_x$, where x is the attained age of the employee.

can see readily the different cost patterns that two rational actuarial cost methods can provide for the same benefits.

Accrued benefit methods (with or without supplemental liability) are

[14] $\$2{,}406.03/\ddot{a}_{\overline{10}|}$. See Internal Revenue Code, sec. 404(a)(1)(A)(iii).

[15] $\$2{,}406.03/\ddot{a}_{\overline{30}|}$. See ERISA, sec. 302(b)(2)(B), 29 U.S.C., § 1082(b)(2)(B), and the corresponding section of Internal Revenue Code, sec. 412(b)(2)(B).

also known as "unit benefit,"[16] "unit cost," "unit credit," "single premium," and "step-rate" methods.

2. (*a*) *Individual Level Cost Method with Full Supplemental Liability*

This approach provides the normal level contribution that would have been required had the plan always existed. Hence, as indicated earlier, for Employee, who is now aged 45 and has ten years of service, the normal contributions from his date of entry into service (at age 35) would have been $280.12. This amount will be considered the "normal cost."

Since the plan is starting late, there is an initial accrued liability of $3,957.99 (including $2,406.03 for past-service benefits and $1,551.96 for simply starting the plan ten years after initial employment), and this initial accrued liability can be amortized in various ways; there could possibly be a level amortization over twenty years by additional annual contributions of $325.54,[17] although the amortization could be faster or slower, and not necessarily through level amounts (the possible range being from $271.27 to $507.32, under current amortization rules[18]).

The individual level cost method with full supplemental liability is widely known as the "individual entry age normal cost method."

2. (*b*) *Individual Level Cost Method with Partial (Past-Service) Supplemental Liability*

This approach is very similar to the previous one, except that benefits are first divided into past-service benefits and future-service benefits. The initial accrued liability takes into account past-service benefits only (which here require $2,406.03), and not the fact that the pension plan is being started some years after initial employment. The contributions become the "normal costs" of the future-service benefits, and an annual contribution of $413.47 would be required in the current example.[19] The initial accrued liability (of $2,406.03) could possibly be amortized by level contributions of $197.90 over twenty years, but here again it could

[16] The term "unit benefit," however, can have the meaning of benefit expressed as a percentage of salary for each year of work; it is in this sense that the Internal Revenue Service rules for the integration of pension benefits with social security benefits refer to "unit benefit plans." The accrued benefit method is not necessarily used to determine costs or contributions for a plan with benefits defined in that fashion.

[17] $\$3{,}957.99/\ddot{a}_{\overline{20}|}$.

[18] $\$3{,}957.99/\ddot{a}_{\overline{30}|}$ and $\$3{,}957.99/\ddot{a}_{\overline{10}|}$. See nn. 14 and 15 above.

[19] $\$100(65 - 45)N_{65}^{(12)}/(N_{45} - N_{65})$.

be amortized over a shorter or longer period and not necessarily through level amounts.

The individual level cost method with partial (past-service) supplemental liability is also known as the "individual attained age normal cost method."

2. (*c*) *Individual Level Cost Method without Supplemental Liability*

This approach contemplates that a level contribution will be made for each employee's total pension benefits (past-service and future-service) over the employee's remaining working life. Hence there is no supplemental liability to be handled separately. The level contribution required for Employee's $3,000 annual pension over the next twenty years is $620.20.[20]

This is the method which is inherent in pension plans funded through individual life insurance contracts known as "retirement income plans" and "annual premium deferred annuity plans." The individual level cost method without supplemental liability is also known as the "attained age level premium method" and the "attained age level contribution method."

The following tabulation shows a comparison of the cost patterns which arise under individual level cost methods:

Supplemental Liability	Level Annual Costs under Individual Level Cost Methods		
	Normal Cost	Amortization*	Total
Full.....	$280.12	$325.54	$605.66
Partial.....	413.47	197.90	611.37
None.....	620.20	0	620.20

* Amortization through level amounts for twenty years.

Note that the method with full supplemental liability gives the employer the benefit of knowing what his normal annual cost would be if the pension plan had always been in existence and what it can be when the supplemental liability is extinguished. Note also that the amortization of the supplemental liability can be slower or faster than the twenty years here chosen, and that amortization payments can vary somewhat from year to year according to the employer's ability to pay and other factors.

One might wonder why one method produces level amounts of $605.66 for twenty years, while another produces $611.37, and the third produces $620.20, when the pension benefits are all the same. The reason for this is that normal costs usually are calculated with the expectation that they

[20] $\$100(65 - 35)N_{65}^{(12)}/(N_{45} - N_{65})$.

will be discontinued upon termination of employment from death and other causes, since they are associated with active employees. Amortization payments, in contrast, normally are not associated with active employees, and it has been assumed in our calculations that they would be continued regardless of the death of any of the employees. (Other causes of termination of employment were disregarded to keep the example reasonably simple.) If we had assumed that all amortization payments stopped upon death, they would have been such that the total annual cost under all three methods would have been $620.20. The three methods are actuarially equivalent.

Most people will find the method with full supplemental liability the most informative and, in view of the large liability which can be amortized in various ways, the most flexible.

3. Aggregate Level Cost Methods

It is customary under aggregate level cost methods to seek to finance pension benefits through contributions equal to a level percentage of the employer's payroll. Expressing the contributions on behalf of each employee (as they could be found from the individual level cost method) as a percentage of his salary would normally produce a different percentage of salary for every employee; hence, expressing the required contribution as the same percentage of salary for every employee, as this method essentially does, submerges individual costs into a whole or an aggregate.

Mathematically, the difference between individual and aggregate level cost methods could be illustrated as follows. Costs under the individual method might be regarded as calculated by a formula (disregarding mortality and interest) of the type

$$\tfrac{7}{9}(\$1{,}000) + \tfrac{8}{13}(\$1{,}000) = \$1{,}393.16\,,$$

while costs under the aggregate method are of the type

$$[(7 + 8)/(9 + 13)](\$2{,}000) = \$1{,}363.64.$$

The reader will observe that the individual method example contemplates nine payments of $1,393.16, followed by four more payments of $615.38 ($\frac{8}{13} \times \$1{,}000$), for a total of $15,000 (after correction for rounding); the aggregate method example contemplates nine payments of $1,363.64 ($\frac{15}{22} \times \$2{,}000$) followed by four more payments of $681.82 ($\frac{15}{22} \times \$1{,}000$), for a total of $15,000 (after correction for rounding). This shows that the methods are equivalent in their final result, even though the approach is different.

Essentially, under the aggregate method, one must find a level percentage of the payroll such that it will be sufficient to provide the pension

benefits provided under the pension plan for the employees covered under the plan. This is done by comparing the discounted value of future pension benefits with the discounted value of future payrolls. But where a supplemental liability is used, the supplemental liability is first deducted from the discounted value of the future pension benefits. This supplemental liability can be determined as that produced either by the individual level cost method with full supplemental liability (which takes into account the extra cost of installing the pension plan after employees began their employment with the employer as well as the extra cost of past-service benefits) or by the accrued benefit method with supplemental liability (which takes into account only the extra cost of past-service benefits, referred to as the "past-service liability").

Given our example, the value at Employee's age 45 of a $3,000 pension at age 65 is equal to $7,218.09,[21] while the value of a $5,000 payroll for 20 years is equal to $58,191.49.[22]

If we were to deduct the liability produced under the individual level cost method with full supplemental liability ($3,957.99), we would find that 5.6024 per cent[23] of Employee's salary of $5,000 will be sufficient to finance his pension at age 65, provided that the supplemental liability of $3,957.99 is amortized in some way. (It will be seen that 5.6024 per cent of $5,000 is equal to the $280.12 annual cost produced under the individual level cost method with full supplemental liability; this is so only because the example includes only one employee. The annual contribution provided under this method normally would be somewhat different from that produced by the individual level cost method with full supplemental liability.)

If we were to deduct the liability produced under the accrued benefit method with supplemental liability ($2,406.03), we would find that 8.2694 per cent[24] of Employee's salary of $5,000, or $413.47, will be sufficient to fund his pension at age 65, provided that the supplemental liability of $2,406.03 is amortized in some way.

Finally, if we were to deduct no liability at all, we would find that 12.4040 per cent[25] of Employee's salary of $5,000, or $620.20, will be sufficient to fund his pension at age 65, with no amortization payments to be made.

Note that if a salary scale were used so that each yearly factor going into the calculation of the present value of the payroll were weighted by

[21] $\$3{,}000N_{65}^{(12)}/D_{45}$.

[22] $\Sigma_{x=45}^{64}\ \$5{,}000D_x/D_{45}$.

[23] $(\$3{,}000N_{65}^{(12)}/D_{45} - \$3{,}957.99)/\Sigma_{x=45}^{64}\$5{,}000D_x/D_{45}$.

[24] $(\$3{,}000N_{65}^{(12)}/D_{45} - \$2{,}406.03)/\Sigma_{x=45}^{64}\$5{,}000\,D_x/D_{45}$.

[25] $\$3{,}000N_{65}^{(12)}/\Sigma_{x=45}^{64}\$5{,}000D_x$.

a salary level factor which would reflect increasing salaries, the percentage, while level, could produce a smaller contribution in the early years which would rise gradually with salaries in future years.

For instance, to simulate the potential effect of a salary scale, let us suppose that Employee can expect a raise of $500 in each subsequent year. The value of the payroll is then $101,071.61.[26] We then find that percentages of 3.2255 per cent, or 4.7610 per cent,[27] depending upon the amount of supplemental liability selected, would be appropriate as annual contributions in addition to the amortization payments for the supplemental liability. Correspondingly, 7.1416 per cent would be needed if there were no supplemental liability. This would produce a cost pattern which increases steadily with salaries, and this may be attractive for some employers. The following tabulation illustrates the results:

Beginning of Year	Employee's Age	Payroll	Annual Normal Cost (Level Salary) with a Supplemental Liability of:		
			$3,957.99 Full (5.6024%)	$2,406.03 Partial (8.2694%)	0 None (12.4040%)
1	45	$5,000	$280.12	$413.47	$620.20
6	50	5,000	280.12	413.47	620.20
11	55	5,000	280.12	413.47	620.20
16	60	5,000	280.12	413.47	620.20
20	64	5,000	280.12	413.47	620.20

But where an increasing salary is forecast and incorporated in the cost formula (without a change in benefits), note how initial dollar costs can be lower and total costs shifted to the future:

Beginning of Year	Employee's Age	Payroll	Annual Normal Cost (Increasing Salary) with a Supplemental Liability of:		
			$3,957.99 Full (3.2255%)	$2,406.03 Partial (4.7610%)	0 None (7.1416%)
1	45	$ 5,000	$161.28	$238.05	$ 357.08
6	50	7,500	241.91	357.08	535.62
11	55	10,000	322.55	476.10	714.16
16	60	12,500	403.19	595.13	892.70
20	64	14,500	467.70	690.35	1,035.53

[26] $\Sigma_{t=0}^{19}$ ($5000 + 500t$)D_{45+t}/D_{45}.

[27] The formulas are similar to those in nn. 23–25 above, the denominator being the expression shown in n. 26 above except for one minor adjustment.

Naturally, in either case, the supplemental liability (if any) remains to be amortized.

The aggregate level cost method has also been called the "aggregate method," the "percentage-of-payroll method," and the "reducing cost method." Used with the partial (past-service) supplemental liability, it has also been known as the "attained age normal method with frozen initial liability." Used with the full supplemental liability, it has been known as the "entry age normal method with frozen initial liability." (It is interesting to note that actuarial methods other than the aggregate level cost method are first used to determine the supplemental liability, when one is used. The accrued benefit method defines the partial [past-service] supplemental liability, while the individual level cost method with full supplemental liability defines the full supplemental liability.)

VI. REVIEW OF THE CONTRIBUTIONS CALLED FOR BY THE DIFFERENT DEFINED BENEFIT ACTUARIAL COST METHODS

The annual pension costs for each method, including the amortization of supplemental liabilities through level amounts over twenty years, for Employee's annual pension of $3,000 on retirement would be as shown in the accompanying tabulation. Note that the aggregate level costs

Actuarial Cost Method	Total Cost for Year				
	1	6	11	16	20
Accrued benefit:					
With supplemental liability	$438.50	$526.06	$651.63	$836.80	$1,055.50
Without supplemental liability	360.90	492.24	680.60	958.35	1,286.40
Individual level cost:					
With full supplemental liability	605.66	605.66	605.66	605.66	605.66
With partial supplemental liability	611.37	611.37	611.37	611.37	611.37
Without supplemental liability	620.20	620.20	620.20	620.20	620.20
Aggregate level cost:					
With full supplemental liability	486.82	567.45	648.09	728.73	793.24
With partial supplemental liability	435.95	554.98	674.00	793.03	888.25
Without supplemental liability	357.08	535.62	714.16	892.70	1,035.53

reflect the use of an increasing salary scale (as used in our previous calculations); otherwise these costs would be practically identical with those produced by the individual level cost approach. Note again that twenty-year level amortization payments are included in all figures involving the use of a supplemental liability.

Eventually, an employer could expect pension costs to taper off to the normal costs, without additional payments for the amortization of the

supplemental liability. These costs would average out to some amount equivalent to the $280.12 produced by the individual level cost method. They could, however, take on an upward sloping pattern if the accrued benefit method were used (because of the effect of increasing age) or if the aggregate level cost (percentage-of-salary) method were used (because of the effect of increasing salaries). A continual influx of new employees would generally bring costs down gradually and stabilize them greatly, no matter what method is used.

VII. CONCLUSION

The example given in this paper has been simple. The determination of pension costs, however, is no simple matter, because so much depends upon the timing of the pension plan contributions, the benefits provided, and the actuarial assumptions selected. It is of great importance not only that the actuary have a clear understanding of actuarial concepts himself but also that he be able to communicate them clearly, with careful terminology and reasonably simple demonstrations.

This paper is elementary. It explains matters which have been explained before[28] but not all in one place and not all with the same terminology. It represents an attempt at clarification and synthesis. It will have met its author's objective if it only helps one actuary demonstrate to one client that actuarial science is no imprecise hocus-pocus but a flexible tool to help us achieve socially desirable objectives, such as the financial security of retired workers. It is this very flexibility which I have attempted to emphasize by suggesting that pension plan actuarial cost methods were in the nature of alternate methods of "financing," a term well understood by the layman. It would be easier to choose an actuarial cost method for a pension plan if there were fewer; the actuary who can explain all the choices available and be understood is more apt to generate respect for his professional ability and his profession.

[28] In particular, the contribution made by Charles L. Trowbridge's paper (*TSA*, IV, 17) must be noted.

DISCUSSION OF PRECEDING PAPER

(AUTHOR'S COMMENT ON PAPER)

CLAUDE Y. PAQUIN:

By tradition, every author is afforded an opportunity to review the formal discussions presented on his paper. Although in the case of this paper there are no discussions to be reviewed, a few informal comments by the author might be appropriate and useful.

First, the reader may note the presentation of an additional actuarial cost method in Donald R. Fleischer's paper "The Forecast Valuation Method for Pension Plans," which is printed in this volume of the *Transactions* (p. 93). Fleischer's method involves population projections based upon such factors as hiring rates and, in some respects, may be said to bridge the gap between the traditional pension cost determination methods, based upon originally fixed working populations, and social insurance cost determination methods, based upon "fluid" populations.

Second, it might be proper to alert the reader to the possibility that the costs produced by the accrued benefit method without supplemental liability may not always meet the requirements of the Employee Retirement Income Security Act of 1974. When using that method, one will produce a systematic amortization of the "unfunded past-service liability," but ERISA may indirectly require a minimum contribution of that amount "necessary to amortize in *equal* annual installments" this unfunded past-service liability over a period of thirty years. Thus, in the table in Section V, item 1(*b*), page 499, the contribution required the first year might have to be increased from $360.90 to $405.50 ($240.60 normal cost plus $164.90 minimum amortization).

A third point is that it is quite possible to have an amortization of supplemental liabilities that extends beyond an employee's retirement age. For instance, there is nothing theoretically impossible about amortizing the supplemental liability for Employee's pension over thirty years —what would happen in the last ten years is that the employer would be putting money into the pension fund at the same time that Employee, now retired, is taking some out. Of course, the situation normally should not degenerate to the point where the employer would be putting the money in *after* Employee takes it out, a physically impossible situation unless one considers that the pension fund could possibly borrow its way out of this awkward situation. Pension financing can be so flexible and

imaginative that in considering what can be done or what can happen it is often a better approach to ask, "Why not?" rather than to wonder, "Can I?" Of course, it may be the abuse of that very flexibility that brought about the various legal restrictions we now must contend with.

A comment on aggregate cost methods might also be appropriate. The word "aggregate" generally suggests a method based on salaries, but that is a misleading suggestion. The expression "aggregate cost" in the pension field is analogous to the expression "average premium" in group life insurance; it just does not relate well to any specific individual, being in the nature of an abstraction. Actuaries know that nobody is average and likewise nobody is "aggregate." The word "aggregate" simply should suggest a loss of individual identity, an inability to pin down the cost of *his* pension on any one individual.

Consider an example. Instead of looking at people, mortality, and interest, let us look at two machines in a noninflationary economy, one a "gizmo" and the other a "widget." A gizmo costs $7,000 and lasts nine years, while a widget costs $8,000 and lasts thirteen years. How much should we depreciate or accrue as cost each year? On an "individual" basis, we could depreciate or charge $7,000/9 and $8,000/13 (total $1,393.16) for the first nine years of joint life of our machines, and $8,000/13 for the last four years of solitary life of the widget. On an "aggregate" basis, we could reason that our work force of two machines has a combined life of twenty-two years which is "lived" two years at a time for the first nine years and one at a time for the next four: hence we could depreciate or charge two twenty-seconds of the entire $15,000 combined cost during the first nine years ($1,363.64 a year) and one twenty-second of it in each of the ensuing four years.

However odious the comparison, people, like machines, have different costs (pension benefits) and different life expectancies (working lifetimes). When these things are mixed together and seasoned with interest and other ingredients, what emerges can be an aggregate cost. One can explain how to get there, but once one is there it is hard to explain how he got there (a perhaps apt reflection to make after one has become a Fellow of the Society).

I sincerely hope that this paper will help dispel some of the aura of mystery which enshrouds pension cost determinations for defined benefit pension plans. The reader who has now learned that things are not as complicated as they appear at first glance will soon enough find out, when tangling with ERISA, that neither are they as simple as they could be.