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A POLICY-YEAR MODEL FOR GAAP VALUATION OF COINSURANCE AND MODIFIED COINSURANCE

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ABSTRACT

This paper presents a policy-year model for GAAP valuation of reinsurance accepted on the coinsurance and modified coinsurance bases. Both nonrefunding and refunding situations are considered. Also presented are the interrelationships between the model's methodology and statutory accounting procedures; a further discussion of experience-refunding situations; a demonstration of the model's ability to yield GAAP profits as a level percentage of premium—exactly on a policy-year basis and smoothly level on a quarterly interpolated basis; and a demonstration of the equivalence of the GAAP formulas presented in the paper to those produced by the more usual approach exemplified in Larry Warnock's "GAAP Reserves."

I. INTRODUCTION

THE paper by Richard S. Robertson entitled "GAAP Accounting for Reinsurance Accepted" (*TSA*, XXVII, 375) described a calendar-year model for coinsurance and modified coinsurance that we have been using to value our reinsurance. This calendar-year model assumes a central issue date of July 1. In order to produce quarterly valuations for our financial statements, an approximation to the actual reserves required was made by interpolating between the reserve factors for successive calendar year-ends. Although Mr. Robertson's model yielded appropriate reserve levels at calendar year-ends, it became evident that the quarterly approximations caused uneven results in our financial statements.

An analysis of the uneven results arising from the use of approximations brought to light specific problems that we had to solve. First, since the year-end valuation was not interpolated, the year-end and quarterly valuations were produced on different bases. Second, the assumption of the central issue date of July 1 was not valid. A review of our records demonstrated that a slightly larger percentage of our business had anniversaries in the fourth quarter of the year than in any other quarter. Third, even if the block

of business were such that the assumption of a central issue date was correct, the reserve levels did not progress smoothly from quarter to quarter and the resulting earnings fluctuated significantly.

A review of the financial results and the research we had done demonstrated that a more appropriate model was required for quarterly financial reporting. We then began to consider the alternatives. One solution would have been to develop a calendar-quarter valuation model. However, such a model would have necessitated a cumbersome set of factors and would not have allowed straightforward expansion to monthly valuations if desired in the future. The second solution we considered was to develop a policy-year model and interpolate between the beginning and ending policy-year reserves. With this method, even monthly valuations could be done. This approach, however, required us to maintain a record of the month of issue of the policy.

Although this approach required the solution of certain conceptual problems, we decided to follow it. Under this approach, the same method would be used for both quarterly and year-end valuations. If there were a block of business with a central issue date of July 1, the reserve would progress smoothly, and earnings would be released as a level percentage of premium. In fact, a test of the system showed that, for periodic valuations, all that is required for smoothness in earnings is some uniformity of issues and deaths over the accounting period considered. As this accounting period becomes shorter than the calendar year in the former system, this uniformity is more likely to be achieved.

II. NONREFUNDING COINSURANCE

The following assumptions are used in developing formulas for nonrefunding coinsurance:

1. Premiums are payable annually on the policy anniversary.
2. The unearned premium is returned in the event of death.
3. Terminations, other than by death, are assumed to occur at the end of the policy year.
4. Cash values are paid at the end of the policy year for terminations other than by death.
5. Deaths are assumed to occur centrally during the policy year.
6. The dividend liability, separate from the benefit reserve, is established at the beginning of the policy year.
7. Terminal dividends are paid at the time of death, and at the end of the policy year for terminations other than by death.
8. The premium tax liability, separate from the benefit reserve, is established at the beginning of the policy year.

9. Expenses and extra allowances are paid at the beginning of the policy year, with return of unaccrued amounts in the event of death.
10. Generally accepted accounting principles (GAAP) are defined to be statutorily accepted accounting principles (SAAP) after the lesser of twenty years and the premium-paying period, a convenient approximation to the true GAAP system. In this manner, the GAAP benefit and expense reserves grade over the appropriate period to the statutory mean reserve and zero, respectively.

For coinsurance (and modified coinsurance) the reinsured company is reimbursed according to a prearranged formula for its commissions and a portion of its other expenses. As in Mr. Robertson's paper, these reimbursements are considered acquisition costs for the purpose of determining GAAP reserves and are reserved for separately so that they may be presented on the asset side of the balance sheet. Similarly, dividends do not represent distributions of surplus of the reinsurer but are treated as a contractual benefit, not subject to the discretion of the reinsurer. The original dividend scale of the ceding company is chosen as the best estimate for the expected cost of dividends.

Modified coinsurance is to be treated exactly the same as coinsurance, except that the interest assumption to be used is to be consistent with the interest rate used for the mean reserve adjustment.

The symbols used for nonrefunding coinsurance are as follows:

x = Policy year;
b_x = Death benefit per unit of face amount for policy year x;
P_x = Gross premium per unit of face amount for policy year x;
SP_x = Standard gross premium per unit of face amount for policy year x;
EP_x = Substandard extra premium per unit of face amount for policy year x;
d_x = Dividend per unit of face amount for policy year x;
td_x = Terminal dividend per unit of face amount for policy year x;
c_x = Commission and expense allowance, as a percentage of premium, for policy year x;
u_x^1 = Additional allowance, as a percentage of gross premium, for policy year x;
u_x^2 = Additional allowance, as a percentage of substandard extra premium, for policy year x;
u_x = Additional allowance for policy year x;
CV_x = Cash value per unit of face amount in force for policy year x;
mr_x = Mean reserve per unit of face amount in force for policy year x;
F_x = Face amount in force at the beginning of policy year x;

q_x = Mortality rate for policy year x;
w_x = Termination rate for policy year x;
i_x = Interest rate for policy year x;
s_x = Premium tax, as a percentage of gross premium, for policy year x;
M_x = Premiums earned in policy year x;
A_x = Discount factor for interest from the end of policy year x to the date of issue;
B_x = Net cash income for policy year x after all benefits but before reimbursement of commissions and other expense allowances, accumulated to policy year-end; less a charge equal to the excess of the policy year x statutory mean reserve for those beginning policy year $x + 1$ over the policy year $x - 1$ statutory mean reserve with interest for those beginning policy year x;
E_x = Commissions and other expense allowances in policy year x accumulated to policy year-end;
B = Present value at issue of net cash income;
E = Present value at issue of commissions and other expense allowances;
M = Present value at issue of earned premium;
z = Last policy year of the GAAP adjustment period, that is, the lesser of twenty years and the premium-paying period;
D_x = GAAP benefit reserve at the end of the policy year, all policy benefits having been paid;
S_x = GAAP expense reserve at the end of the policy year;
b, e = Superscripts denoting beginning and end of year, respectively;
t = Valuation quarter indicator; assumes integral values 1–4;
DF, SF = GAAP benefit and expense reserve factors per unit of in-force.

The formulas that define the model are given below. Unless otherwise noted, x is assumed to take on integral values from 1 to z, inclusive.

$$P_x = SP_x + EP_x . \tag{1}$$

$$u_x = u_x^1 P_x + u_x^2 EP_x \quad \text{(used for substandard issues only)} . \tag{2}$$

$$F_1 = 1 ; \quad F_x = F_{x-1}(1 - q_{x-1})(1 - w_{x-1}) \quad \text{for } x > 1 . \tag{3}$$

$$M_x = (1 - \tfrac{1}{2}q_x)F_x P_x . \tag{4}$$

$$A_1 = 1/(1 + i_1) ; \quad A_x = A_{x-1}/(1 + i_x) \quad \text{for } x > 1 . \tag{5}$$

$$
\begin{aligned}
B_x = {} & (1 - s_x)P_xF_x(1 + i_x) \\
& - \tfrac{1}{2}(1 - s_x)P_xq_xF_x(1 + i_x)^{1/2} \\
& - b_xq_xF_x(1 + i_x)^{1/2} \\
& - CV_xw_x(1 - q_x)F_x \\
& - d_x(1 - q_x)F_x(1 + i_x) \\
& - td_xq_xF_x(1 + i_x)^{1/2} \\
& - td_xw_x(1 - q_x)F_x \\
& - mr_xF_x(1 - q_x)(1 - w_x) + mr_{x-1}F_x(1 + i_x) .
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
E_x = {} & c_xP_xF_x(1 + i_x) - \tfrac{1}{2}c_xP_xq_xF_x(1 + i_x)^{1/2} \\
& + u_xF_x(1 + i_x) - \tfrac{1}{2}u_xq_xF_x(1 + i_x)^{1/2} .
\end{aligned} \tag{7}
$$

$$
M = \sum_{j=1}^{z} M_j(1 + i_j)A_j . \tag{8}
$$

$$
B = \sum_{j=1}^{z} B_jA_j . \tag{9}
$$

$$
E = \sum_{j=1}^{z} E_jA_j . \tag{10}
$$

$$
D_0 = 0 ,
$$

$$
\begin{aligned}
D_x = {} & (1 + i_x)D_{x-1} + B_x - M_x(1 + i_x)B/M \\
& + mr_xF_x(1 - q_x)(1 - w_x) - mr_{x-1}F_x(1 + i_x) .
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
S_0 &= 0 , \\
S_x &= (1 + i_x)S_{x-1} - E_x + M_x(1 + i_x)E/M .
\end{aligned} \tag{12}
$$

$$
D_x^b = D_{x-1} + P_xF_x(1 - s_x) - d_x(1 - q_x)F_x . \tag{13a}
$$

$$
D_x^e = D_x + CV_xw_x(1 - q_x)F_x + td_xw_x(1 - q_x)F_x . \tag{13b}
$$

$$
S_x^b = S_{x-1} - c_xP_xF_x - u_xF_x . \tag{14a}
$$

$$
S_x^e = S_x . \tag{14b}
$$

$$DF_x^t = \left[\frac{8-(2t-1)}{8}D_x^b + \frac{(2t-1)}{8}D_x^t\right] \Big/ \left\{\left[1 - \frac{(2t-1)}{8}q_x\right]F_x\right\}. \quad (15)$$

$$SF_x^t = \left[\frac{8-(2t-1)}{8}S_x^b + \frac{(2t-1)}{8}S_x^t\right] \Big/ \left\{\left[1 - \frac{(2t-1)}{8}q_x\right]F_x\right\}. \quad (16)$$

Neither the formulas nor the assumptions exist independent of the accounting system employed in reporting financial results. In developing the formulas, we had to explore the interrelationships between the accounting system and the assumptions in the GAAP system.

Because GAAP is defined to be SAAP at the end of the lesser of twenty years and the premium-paying period, care must be taken not to overstate the GAAP profit during the GAAP adjustment period. Such overstatement could occur, for example, through failure to accrue the liability for the statutory mean reserve to be held following the GAAP adjustment period. This accrual is provided for by charging the net cash income each year for the excess of the statutory mean reserve liability that would be established to provide for policies entering year $x + 1$ (i.e., after all policy benefits of year x have been paid) over the statutory mean reserve liability that was held for those entering policy year x, where interest at the investment yield rate is credited to the latter liability. At the same time, to produce the proper GAAP benefit reserve, the formula for the GAAP benefit reserve should not have any explicit reference to the statutory mean reserve liability.

An examination of formula (11) with regard to the B_x term—recall formula (6)—and the terms involving the statutory mean reserve shows that the terms involving the statutory mean reserve cancel. However, an examination of the B term appearing in (11) demonstrates that there is a residual effect due to the statutory mean reserve charges. The result of this effect is to make the quotient of the total benefit reserve at the end of duration z divided by the in-force at the beginning of duration $z + 1$, D_z/F_{z+1}, equal to the mean reserve per unit. In this way, the GAAP benefit reserve grades into the statutory mean reserve at the end of the GAAP adjustment period. This feature is necessary, since the GAAP valuation system is used to produce adjustments to the statutory valuation, and this grading provides a smooth transition to the post-GAAP period. Thus, the formula for the GAAP benefit reserve, formula (11), is explicitly independent of the statutory mean reserve liability and provides for the accrual of the statutory mean reserve liability. Note that the charge for the increase in the statutory mean reserve liability in formula (6), B_x, will perform a second function if the reinsurance agreement is experience-refunding. This will be seen in Section III.

III. EXPERIENCE-REFUNDING COINSURANCE

The fundamental concept in an experience-refunding reinsurance agreement is that the excess of the statutory gain from operations over the reinsurer's expense charge is shared between the reinsured and the reinsurer. Two items intrude here. The first is that this excess in a given year, or even in an accumulation of years, need not be positive. The second is that, even granting that the excess is positive, contract provisions and SAAP do not contemplate paying out all of the excess (i.e., recognizing a profit) but instead require that a portion be set aside as a contingency reserve against future experience. The reinsurer develops rules for the exact computation of this statutory contingency reserve. Note that, because of the first item, such an accumulation of experience may be zero or negative. This would occur if losses exceeded any positive contingency reserve; in that event, the excess would be accumulated at interest to be charged against future profits, thus producing a negative contingency reserve. If the contingency reserve is negative, then it is reported as zero for the statutory statement. The reinsurer carries in its statutory statement both the ceding company's share and the reinsurer's share of the statutory contingency reserve. The problem now is to develop a restatement of the ceding company's share of the statutory contingency reserve on a GAAP basis, that is, to develop a GAAP contingency reserve corresponding to the ceding company's share of the statutory contingency reserve. The GAAP counterpart of the reinsurer's share of the statutory contingency reserve is zero.

The following are additional symbols and formulas for the experience-refunding situation.

g_x = Expense charge per unit of mean in-force for policy year x;
G_x = Expense charge for policy year x, accrued uniformly;
R_x = Charge for increase in mean reserve for policy year x;
W_x = Experience refund for policy year x;
W = Present value at issue of experience refunds;
T_x = Refund reserve at end of policy year x;
V_x = Present value, at end of policy year x, of future refunds.

$$G_x = g_x(1 - \tfrac{1}{2}q_x)F_x . \tag{17}$$

$$R_x = mr_x(1 - q_x)(1 - w_x)F_x - mr_{x-1}F_x(1 + i_x) . \tag{18}^1$$

$$W_x = \tfrac{1}{2}[B_x - E_x - G_x(1 + i_x)^{1/2}] . \tag{19}$$

[1] Note that R_x is included in the B_x term appearing in formula (19).

$$W = \sum_{j=1}^{z} W_j A_j \,. \tag{20}$$

$$\begin{aligned} T_0 &= 0 \,, \\ T_x &= (1 + i_x)T_{x-1} - W_x + M_x(1 + i_x)W/M \,. \end{aligned} \tag{21}$$

$$\begin{aligned} V_0 &= W \,, \\ V_x &= \sum_{j=x+1}^{z} W_j A_j = (1 + i_x)V_{x-1} - W_x \,. \end{aligned} \tag{22}$$

T_x and V_x represent the refund reserve and the present value of future refunds, respectively. These values are as of the end of the policy year.

In Mr. Robertson's paper, four experience-refunding situations were identified. These situations and the corresponding formulas are given at the end of this section, and a more general discussion of the situations is given in Section IV of this paper.

The formulas involve three quantities: CR, the ceding company's share of the statutory contingency reserve; T, the refund reserve; and V, the present value of future refunds. Mr. Robertson's model was based on a calendar-year approach, which was consistent with the computation of the statutory contingency reserve and the experience refund; therefore, no problems were encountered in combining the various quantities directly. In our policy-year model, however, it is not possible to combine directly the actual statutory contingency reserve and an interpolated policy-year refund reserve or the present value of future refunds, where the beginning and ending policy-year reserves do not reflect the same recognition of the timing of the various financial elements of the actual experience-refund/statutory-contingency-reserve calculation. The problem to be solved was how to adjust the beginning and ending policy-year refund reserve and the present value of future refunds to achieve proper recognition.

Just after the policy anniversary, statutory accounting will reflect the receipt of gross premiums, the disbursement of commissions and allowances, the establishment of the liabilities for premium taxes and dividends, and the increase in the statutory mean reserve. The proper refund reserve (or present value of future refunds) at the beginning of the policy year is then found by reducing the prior policy-year terminal-refund reserve (or present value of future refunds) by the proportion of the above-mentioned items that affect the ceding company's experience-refund/statutory-contingency-reserve computation. Similarly, the refund reserve (or present value of future refunds) at the end of the policy year is found by reducing the current policy-year terminal-refund reserve (or present value of future re-

funds) by that proportion of those items affecting the computation just prior to the policy anniversary, that is, surrenders, terminal dividends, and the statutory mean reserve released upon surrender.

To complete the experience-refunding model, the following additional symbols and formulas are required:

W_x^b = Beginning-of-policy-year adjustment to refund reserve and present value of future refunds;

W_x^e = End-of-policy-year adjustment to refund reserve and present value of future refunds;

T_x^b, T_x^e = Beginning-of-policy-year and end-of-policy-year refund reserves;

V_x^b, V_x^e = Present value of future refunds, as of the beginning and end of the policy year;

TF_x^t = Refund-reserve factor per unit of in-force for policy year x and quarter t;

VF_x^t = Present-value-of-future-refunds factor per unit of in-force for policy year x and quarter t.

$$W_x^b = \tfrac{1}{2}[P_xF_x(1 - c_x - s_x) - u_xF_x - (mr_xF_x - mr_{x-1}F_x) - d_x(1 - q_x)F_x] . \tag{23}$$

$$W_x^e = \tfrac{1}{2}[(CV_x + td_x)w_x(1 - q_x)F_x - mr_xw_x(1 - q_x)F_x] . \tag{24}$$

$$T_x^b = T_{x-1} - W_x^b . \tag{25a}$$

$$T_x^e = T_x - W_x^e . \tag{25b}$$

$$V_x^b = V_{x-1} - W_x^b . \tag{26a}$$

$$V_x^e = V_x - W_x^e . \tag{26b}$$

$$TF_x^t = \left[\frac{8 - (2t - 1)}{8} T_x^b + \frac{(2t - 1)}{8} T_x^e\right] \Big/ \left\{\left[1 - \frac{(2t - 1)}{8} q_x\right] F_x\right\} . \tag{27}$$

$$VF_x^t = \left[\frac{8 - (2t - 1)}{8} V_x^b + \frac{(2t - 1)}{8} V_x^e\right] \Big/ \left\{\left[1 - \frac{(2t - 1)}{8} q_x\right] F_x\right\} . \tag{28}$$

For a given ceding company's account, aggregate values for T and V are computed from formulas (27) and (28). These results are combined with the account's statutory contingency reserve to produce the GAAP contingency reserve. The method used is the same as described by Mr. Robertson in his

paper. For completeness, we have repeated the methodology below. Let CR represent the ceding company's share of the statutory contingency reserve.

Experience-refunding Situation	GAAP Contingency Reserve
A. $V > -CR$ and $V > T$	$CR + T$
B. $V \leq -CR$ and $V > T$	$T - V$
C. $V > -CR$ and $V \leq T$	$CR + V$
D. $V \leq -CR$ and $V \leq T$	0

IV. EXPERIENCE-REFUNDING SITUATIONS

An approach to understanding the experience-refunding situations presented at the end of the last section is to treat the experience refund as a benefit. Thus, we can allocate a portion of the total gross premium to provide for this benefit. As will be seen, this portion of the gross premium, which we will refer to as the experience-refund premium, may be positive, negative, or zero. For reserve and cash-flow purposes, negative values will be treated as zero. For this section of the paper we assume that z equals 20.

Following are some additional symbols and formulas.

GSR_n = GAAP contingency reserve at the end of year n;
CR_n = Ceding company's share of statutory contingency reserve at the end of year n;
W_n = Experience refund for year n;
W = Present value of experience refunds;
V_n = Present value of future experience refunds at end of year n (note that $V_0 = W$);
T_n = Refund reserve at end of year n;
P^{ER} = Experience-refund premium;
$a_{n:\overline{20-n}|}$ = An annuity based on GAAP assumptions for interest, mortality, and withdrawal for the block of business.

The following relationships hold:

1. $a_{n:\overline{20-n}|} \geq 0$ for $0 \leq n \leq 20$.
2. $P^{ER} = W/a_{\overline{20}|}$; P^{ER} and W are either both positive or both less than or equal to zero.
3. $T_n = V_n - P^{ER}a_{n:\overline{20-n}|}$.

If W is positive, then we will refer to the block of business as "valued to be profitable." By "profitable," we mean that the block of business is expected to produce an experience refund. This implies that the block is valued to produce a statutory profit in excess of the expense charge. If W is negative or zero, then we will refer to the block as "valued to be unprofitable." That is, the statutory profit is less than or equal to the expense

charge, and no refunds are expected. We are using the word "profitable" to mean "capable of generating an experience refund," in order to emphasize the situation where W is positive. The word "profitable" is *not* being used to refer to any profit the reinsurer might experience. In fact, in the case where the statutory gain is less than the expense charge, a reinsurer could have a positive statutory gain but not be in a position to pay experience refunds.

If the statutory contingency reserve plus the present value of future refunds is greater than zero at the end of any year n (i.e., $CR_n + V_n > 0$), then we will refer to the block as being profitable at the end of year n. If the statutory contingency reserve plus the present value of future refunds is less than or equal to zero at the end of year n (i.e., $CR_n + V_n \leq 0$), then we will refer to the block as being unprofitable at the end of year n.

Four important deductions result from the preceding definitions.

1. A block is valued to be profitable if and only if $W > 0$; $P^{ER} > 0$; and $V_n > T_n$ for $0 \leq n < 20$. The three conditions are equivalent.
2. A block is valued to be unprofitable if and only if $W \leq 0$; $P^{ER} \leq 0$; and $V_n \leq T_n$ for $0 \leq n < 20$.
3. If there is a value of n such that $0 \leq n < 20$ and $V_n > T_n$, then $W > 0$ and $P^{ER} > 0$, and so $V_n > T_n$ for all n.
4. If there is a value of n such that $0 \leq n < 20$ and $V_n \leq T_n$, then $W \leq 0$ and $P^{ER} \leq 0$, and so $V_n \leq T_n$ for all n.

The first two deductions follow from the definitions and relationship 3 above. The last two follow from the first two and the definitions.

Using the above information, we will consider the situations possible for a single block of business. Earlier, we gave two criteria from Mr. Robertson's paper that could be used to judge the experience-refunding situation. These criteria were (1) a comparison of the present value of future refunds with the negative of the statutory contingency reserve, and (2) a comparison of the present value of future refunds with the refund reserve.

The second criterion depends totally on the assumptions used in the valuation. As was shown, the present value of future refunds, V_n, is either always greater than or always less than or equal to the refund reserve, T_n. That is, if one relationship holds for any specific n, it must hold for all n. Accordingly, the experience-refund premium either is positive or is less than or equal to zero. Thus, the second criterion may be restated as asking whether the block was or was not valued to be profitable.

The first criterion, however, depends on both the actual past experience and the future expected experience of the block of business. Its significance is more readily apparent if it is restated to ask whether the statutory contingency reserve plus the present value of future refunds is greater than, or

less than or equal to, zero; that is, is $CR_n + V_n$ greater than zero, or is $CR_n + V_n$ less than or equal to zero? Using our previous definitions, this criterion is equivalent to asking whether the block of business at year-end n, based on actual past and future expected experience, is profitable (capable of generating a refund) or not.

Assume, for a given block of business, that the sum of the statutory contingency reserve and the present value of future refunds at year-end n is positive, that is, $CR_n + V_n > 0$. In that case, the block is profitable at year-end n, and, because refunds are expected to be paid in the future, a liability should be set up. In fact, the sum of the statutory contingency reserve and the present value of future refunds, $CR_n + V_n$, is an estimate of that liability. Once it is determined that a liability should be set up, the second criterion becomes significant. If the block is valued to be profitable on the GAAP reserving assumptions, then the present value of future refunds will be greater than the refund reserve; that is, $V_n > T_n$. Therefore, the experience-refund premium, P^{ER}, is positive, and the present value of the remaining experience-refund premiums will reduce the liability for future refunds. The liability in this case is the statutory contingency reserve plus the refund reserve:

$$CR_n + V_n - P^{ER}a_{n:\overline{20-n}|} = CR_n + T_n .$$

This is situation A. If, however, the block is valued to be unprofitable ($V_n \leq T_n$), then the experience-refunding premium, P^{ER}, is less than or equal to zero. There is no future experience-refund premium to offset any future refunds; therefore, the liability to be held is equal to the statutory contingency reserve plus the present value of future refunds, $CR_n + V_n$. This is situation C.

Alternatively, let us assume that the block of business is such that the sum of the statutory contingency reserve and the present value of future refunds is less than or equal to zero; that is, $CR_n + V_n \leq 0$. This block of business would be unprofitable at year-end n, and we would not expect to pay experience refunds in the future. Therefore, an estimate of the liability to be held is zero. Again, the second criterion needs to be considered. If the present value of future refunds is greater than the refund reserve ($V_n > T_n$), then the experience-refund premium, P^{ER}, is positive, and the reinsurer can expect to receive future experience-refund premiums while paying no refunds. This is profit that, according to the principles of GAAP accounting, should be leveled over the premium-paying period. In our formula, this is done by setting up a negative liability for the present value of those future experience-refund premiums. Recalling relationship 3, we can determine that the GAAP reserve to be held is the refund reserve minus the present value of future refunds, that is, $T_n - V_n$. This is situation B.

If the present value of future refunds is less than or equal to the refund reserve ($V_n \leq T_n$), then the experience-refunding premium must be less than or equal to zero. Because the first criterion demonstrates that there are no refunds to be paid in the future and the second criterion indicates that no experience-refund premiums will be received in the future, the appropriate liability to be held is zero. This is situation D. Just as the experience refund itself is compiled from the experience of all policies for a ceding company's block of business, so should the GAAP adjustments be aggregated to produce total results.

V. MODEL PLAN

This section contains a sample plan, specifically a ten-year endowment, which will be used to demonstrate the effectiveness of the model in presenting GAAP earnings as a percentage of earned premium. The assumptions underlying the GAAP earnings calculations are shown in Table 1. We have calculated GAAP reserves and financial results on both an annual (Table 2) and a quarterly (Table 3) basis. We will show that, when represented as a percentage of premium, the GAAP profit is exactly level on an annual basis and is very smooth on a quarterly basis. This justifies the use of the interpolated beginning-of-policy-year and end-of-policy-year reserve factors.

Table 3 shows that there is always a decrease in GAAP profits in the first valuation quarter following the anniversary, because during this quarter the end-of-the-prior-policy-year benefits enter the cash flow. Of course, in the very first quarterly valuation there are no prior-policy-year-end benefits to be paid, and the GAAP profit percentage is relatively high. However, despite these uneven cash flows in the first quarter, GAAP profit percentages are quite smooth.

We developed this unrealistic example purposely to illustrate the ability of the system to handle all the variations that a life insurance policy might have. All the parameters except for premium tax have been chosen to be nonlevel in order to demonstrate the capabilities of the model. The plan may be considered to be issued in the middle of the first quarter. Thus, the quarterly valuation following the tenth year will still show a GAAP profit.

It should be noted that for plans that terminate at the end of the premium-paying period, where the premium-paying period is less than or equal to twenty years or the maximum number of years for which GAAP adjustments are to be carried out, the lapse rate in the last year should be equal to unity. If not, the reserves produced will generate a lower GAAP profit percentage during the duration of the policy, and a significant portion of that profit will be allocated to the policy year following termination.

TABLE 1

PLAN ASSUMPTIONS

Year	q_x	w_x	i_x	s_x	g_x	SP_x	EP_x	c_x	u_x^1	u_x^2	d_x	td_x	CV_x	mr_x	b_x
1	.001	.20	.07	.02	0	$240	$40	.80	.10	.10	$ 0	$ 0	$ 100	$ 200	$2,000
2	.002	.15	.07	.02	1	240	40	.20	.05	.05	10	0	200	300	2,000
3	.003	.10	.06	.02	1	240	40	.20	.05	.05	15	0	300	400	2,000
4	.004	.10	.06	.02	1	240	40	.20	.05	.05	15	1	400	500	2,000
5	.005	.10	.05	.02	1	240	40	.20	.05	.05	20	4	500	600	2,000
6	.006	.05	.05	.02	1	200	20	.20	.05	.05	20	8	600	650	1,000
7	.007	.05	.045	.02	1	200	20	.20	.05	.05	25	10	700	750	1,000
8	.008	.05	.045	.02	1	200	20	.20	.05	.05	25	12	800	850	1,000
9	.009	.05	.045	.02	1	200	20	.20	.05	.05	30	15	900	950	1,000
10	.010	1.00	.045	.02	1	200	20	.20	.05	.05	30	0	1,000	1,050	1,000

TABLE 2

GAAP Earnings and Financial Results for Model Plan

(Annual Basis)

Policy Year	GAAP Reserve	Earned Premium	Expenses	Increase in Reserve	Death Benefits	Surrenders	Dividend	Premium Tax	Experience Refund	Investment Income	Profit	Beginning-of-Year Profit	
												Amount	% Earned Premium
1	$ 48.1959	$279.8600	$255.8720	$ 48.1959	$2.0000	$19.9800	$ 0.0000	$5.5972	− $81.1052	$ 1.2189	$30.5389	$28.5411	10.1983%
2	112.3402	223.5522	57.4849	64.1443	3.1968	23.9280	7.9760	4.4710	51.9796	14.0229	24.3945	22.7986	10.1983
3	167.8237	189.5444	48.7400	55.4835	4.0678	20.2778	10.1389	3.7909	40.7942	14.2389	20.4902	19.3304	10.1983
4	212.8486	169.9931	43.7125	45.0249	4.8667	24.2361	9.1515	3.3999	37.9854	16.7605	18.3766	17.3365	10.1983
5	246.4478	152.3054	39.1643	33.5992	5.4531	27.1292	11.0796	3.0461	32.0013	15.4766	16.3092	15.5326	10.1983
6	270.4027	107.1095	27.2642	23.9548	2.9300	14.5619	9.9255	2.1422	30.5164	15.6551	11.4696	10.9234	10.1983
7	301.0950	101.0928	25.7327	30.6923	3.2279	16.0264	11.7087	2.0219	15.7956	14.8865	10.7737	10.3098	10.1983
8	328.4047	95.3180	24.2628	27.3097	3.4800	17.2609	11.0888	1.9064	15.9546	16.1035	10.1583	9.7208	10.1983
9	351.5392	89.7826	22.8537	23.1345	3.6895	18.2816	12.5478	1.7957	14.9955	17.0841	9.5684	9.1563	10.1983
10 ...	0.0000	84.4834	21.5049	− 351.5392	3.8595	382.0856	11.4626	1.6897	24.3995	17.9826	9.0036	8.6159	10.1983

TABLE 3

GAAP EARNINGS AND FINANCIAL RESULTS FOR MODEL PLAN

(Quarterly Interpolated Basis)

POLICY YEAR	QUARTER	GAAP RESERVE	EARNED PREMIUM	EXPENSES	INCREASE IN RESERVE	DEATH BENEFITS	SURRENDERS	DIVIDEND	PREMIUM TAX	EXPERIENCE REFUND	INVEST-MENT INCOME	PROFIT	BEGINNING-OF-QUARTER PROFIT	
													Amount	% Earned Premium
1	1	$105.3207	$279.8600	$255.8720	$105.3207	$0.2479	$ 0.0000	$ 0.0000	$5.5972	−$90.8390	$0.9256	$4.5868	$4.5098	12.8917%
	2	97.5622	0.0000	0.0000	− 7.7585	0.5000	0.0000	0.0000	0.0000	1.4785	1.7798	7.5598	7.4330	10.6239
	3	89.8037	0.0000	0.0000	− 7.7585	0.5000	0.0000	0.0000	0.0000	1.4781	1.6475	7.4279	7.3033	10.4385
	4	82.0452	0.0000	0.0000	− 7.7585	0.5000	0.0000	0.0000	0.0000	1.4777	1.5151	7.2960	7.1736	10.2531
2	1	162.1973	223.5522	57.4849	80.1522	0.6484	19.9800	7.9760	4.4710	48.7776	2.1148	6.1770	6.0734	9.6515
	2	156.4982	0.0000	0.0000	− 5.6992	0.7992	0.0000	0.0000	0.0000	1.6007	2.7465	6.0457	5.9443	10.6361
	3	150.7990	0.0000	0.0000	− 5.6992	0.7992	0.0000	0.0000	0.0000	1.5997	2.6493	5.9495	5.8497	10.4668
	4	145.0998	0.0000	0.0000	− 5.6992	0.7992	0.0000	0.0000	0.0000	1.5988	2.5520	5.8532	5.7551	10.2975
3	1	207.4913	189.5444	48.7400	62.3915	0.9078	23.9280	10.1389	3.7909	37.3268	2.6022	4.9228	4.8516	9.3956
	2	202.9170	0.0000	0.0000	− 4.5743	1.0170	0.0000	0.0000	0.0000	1.4931	3.0263	5.0906	5.0170	10.5874
	3	198.3427	0.0000	0.0000	− 4.5743	1.0170	0.0000	0.0000	0.0000	1.4917	2.9592	5.0249	4.9522	10.4508
	4	193.7683	0.0000	0.0000	− 4.5743	1.0170	0.0000	0.0000	0.0000	1.4902	2.8921	4.9592	4.8875	10.3142
4	1	253.3703	169.9931	43.7125	59.6019	1.1164	20.2778	9.0885	3.3999	31.6689	3.2873	4.4143	4.3505	9.6802
	2	249.5915	0.0000	0.0000	− 3.7788	1.2173	0.0000	0.0000	0.0000	1.6930	3.6966	4.5651	4.4991	10.5866
	3	245.8127	0.0000	0.0000	− 3.7788	1.2173	0.0000	0.0000	0.0000	1.6908	3.6412	4.5119	4.4466	10.4630
	4	242.0339	0.0000	0.0000	− 3.7788	1.2173	0.0000	0.0000	0.0000	1.6887	3.5858	4.4586	4.3941	10.3395
5	1	288.1894	152.3054	39.1643	46.1555	1.2919	24.2361	10.9123	3.0461	27.0822	3.2552	3.6722	3.6277	9.0046
	2	284.8206	0.0000	0.0000	− 3.3688	1.3660	0.0000	0.0000	0.0000	1.4552	3.5195	4.0671	4.0178	10.5519
	3	281.4518	0.0000	0.0000	− 3.3688	1.3660	0.0000	0.0000	0.0000	1.4528	3.4781	4.0282	3.9793	10.4509
	4	278.0829	0.0000	0.0000	− 3.3688	1.3660	0.0000	0.0000	0.0000	1.4504	3.4368	3.9893	3.9409	10.3500

TABLE 3—*Continued*

Policy Year	Quarter	GAAP Reserve	Earned Premium	Expenses	Increase in Reserve	Death Benefits	Surrenders	Dividend	Premium Tax	Experience Refund	Investment Income	Profit	Beginning-of-Quarter Profit	
													Amount	% Earned Premium
6	1	$291.8827	$107.1095	$ 27.2642	$ 13.7998	$ 1.0541	$ 27.1292	$ 9.9250	$2.1422	$26.1432	$3.4950	$3.1468	$3.1086	9.5866%
	2	290.1072	0.0000	0.0000	– 1.7755	0.7384	0.0000	0.0000	0.0000	1.7515	3.5668	2.8525	2.8179	10.5234
	3	288.3317	0.0000	0.0000	– 1.7755	0.7384	0.0000	0.0000	0.0000	1.7487	3.5450	2.8335	2.7992	10.4535
	4	286.5561	0.0000	0.0000	– 1.7755	0.7384	0.0000	0.0000	0.0000	1.7459	3.5233	2.8146	2.7805	10.3836
7	1	323.6925	101.0928	25.7327	37.1364	0.7767	14.5619	11.6416	2.0219	10.1912	3.3703	2.4007	2.3744	9.1234
	2	322.0113	0.0000	0.0000	– 1.6812	0.8151	0.0000	0.0000	0.0000	1.7467	3.5675	2.6870	2.6576	10.5154
	3	320.3301	0.0000	0.0000	– 1.6812	0.8151	0.0000	0.0000	0.0000	1.7434	3.5489	2.6716	2.6424	10.4554
	4	318.6489	0.0000	0.0000	– 1.6812	0.8151	0.0000	0.0000	0.0000	1.7402	3.5304	2.6563	2.6272	10.3954
8	1	351.4357	95.3180	24.2628	32.7868	0.8476	16.0264	11.0170	1.9064	9.7599	3.6996	2.4108	2.3844	9.7119
	2	349.9782	0.0000	0.0000	– 1.4575	0.8805	0.0000	0.0000	0.0000	1.9161	3.8732	2.5341	2.5063	10.5178
	3	348.5207	0.0000	0.0000	– 1.4575	0.8805	0.0000	0.0000	0.0000	1.9122	3.8571	2.5220	2.4944	10.4675
	4	347.0633	0.0000	0.0000	– 1.4575	0.8805	0.0000	0.0000	0.0000	1.9082	3.8410	2.5098	2.4824	10.4172
9	1	374.8919	89.7826	22.8537	27.8286	0.9082	17.2609	12.4467	1.7957	8.4088	3.9843	2.2642	2.2395	9.6789
	2	373.6316	0.0000	0.0000	– 1.2603	0.9362	0.0000	0.0000	0.0000	2.0682	4.1316	2.3874	2.3613	10.5199
	3	372.3714	0.0000	0.0000	– 1.2603	0.9362	0.0000	0.0000	0.0000	2.0635	4.1177	2.3782	2.3522	10.4795
	4	371.1111	0.0000	0.0000	– 1.2603	0.9362	0.0000	0.0000	0.0000	2.0588	4.1038	2.3690	2.3431	10.4390
10 ...	1	395.3704	84.4834	21.5049	24.2592	0.9505	18.2816	11.7673	1.6897	8.1399	4.2285	2.1189	2.0957	9.6208
	2	394.3039	0.0000	0.0000	– 1.0665	0.9649	0.0000	0.0000	0.0000	2.2123	4.3572	2.2466	2.2220	10.5204
	3	393.2374	0.0000	0.0000	– 1.0665	0.9649	0.0000	0.0000	0.0000	2.2068	4.3455	2.2403	2.2158	10.4909
	4	392.1709	0.0000	0.0000	– 1.0665	0.9649	0.0000	0.0000	0.0000	2.2013	4.3337	2.2340	2.2096	10.4615
11 ...	1	0.0000	0.0000	0.0000	– 392.1709	0.4851	382.0856	0.0000	0.0000	10.6470	2.1699	1.1232	1.1109	10.5196

APPENDIX

The purpose of the Appendix is to demonstrate the equivalence of the methodology used in this paper and the more traditional exposition found in the literature. For example, in assembling our system, we reviewed Larry Warnock's "GAAP Reserves" (Society of Actuaries Study Note 79-23-78) and found our methodology to be consistent with his. Notation for benefit and expense reserves will be the same as that used in the body of this paper.

To demonstrate the equivalence, consider the following additional symbols:

PB_x^c = All benefits for policy year x, accumulated to the end of the policy year;

E_x^c = All expenses for policy year x, accumulated to the end of the policy year;

$PV(\ )$ = Abbreviation for the present value of whatever quantity is shown inside the parentheses, such present value extending over all values of any subscript;

BNP, ENP = Benefit natural premium and expense natural premium, respectively, expressed as a percentage of premium.

Because $M = PV(M_x)$ and $B = PV(B_x) = PV$(cash flow after all policy benefits), then $M = B + PV(PB_x^c)$, or $PV(PB_x^c) = M - B$. By definition,

$$BNP = \frac{PV(PB_x^c)}{PV(M_x)},$$

so $BNP = (M - B)/M$. From page 4 of "GAAP Reserves," we have

$$\begin{aligned}(D_x - D_{x-1}) - i_x(D_{x-1} + BNP\ M_x) + PB_x^c &= BNP\ M_x \\ &= [(M - B)/M]M_x \\ &= M_x(1 - B/M)\ .\end{aligned}$$

Expanding and solving for D_x, we obtain

$$D_x = (1 + i_x)D_{x-1} + (1 + i_x)M_x - PB_x^c - M_x(1 + i_x)B/M\ .$$

By observing that $(1 + i_x)M_x - PB_x^c$ is equal to B_x, we can reduce the last equation to $D_x = (1 + i_x)D_{x-1} + B_x - M_x(1 + i_x)M/B$, which was to be shown.

Let $E = PV(E_x)$, so $ENP = E/M$. Then, from page 6 of "GAAP Reserves," we have $S_x - S_{x-1} - i_x(S_{x-1} + ENP\ M_x) + E_x = ENP\ M_x$. Solving for S_x and substituting E/M for ENP, we obtain

$$S_x = (1 + i_x)S_{x-1} - E_x + M_x(1 + i_x)E/M ,$$

which was to be shown.

DISCUSSION OF PRECEDING PAPER

GARY N. SEE:

Messrs. Becker and Eckman are to be congratulated on making a valuable contribution to actuarial literature. This paper, along with Mr. Robertson's, will undoubtedly serve as an important reference to actuaries involved in the financial reporting of reinsurance business.

I would like to make some comments from the viewpoint of a ceding company. As actuaries, we used to think of a coinsurance arrangement as one where the ceding company received allowances equal to its percentage expenses, and normally the ceding company retained the entire policy fee. Thus, the reinsurer was a coinsurer in the true sense of the word.

In recent years, many coinsurance or modified coinsurance arrangements have been consummated that depart from the above, and from what I would label as the "traditional" coinsurance arrangement. The allowances to the ceding company, and their incidence, may be quite different from those actually incurred by the ceding company. Such differences may arise from different assumptions on the part of the reinsurer, particular needs of the ceding company, or as a result of a very competitive reinsurance environment. For whatever reason, when the reinsurance allowances to the ceding company are not based on a "traditional" approach, then in my opinion the GAAP treatment of the business coinsured has to be carefully examined.

One method of allowing for coinsurance on a block of business is to value the whole block, and the coinsured portion of the block, using the same GAAP assumptions, with the ceding company holding the net of the two. However, in the above situation, I do not believe this provides for a proper matching of revenues and costs under GAAP.

Another approach is to view the coinsured business as essentially separate, and requiring unique treatment. Under this approach, the revenues to the ceding company are really the allowances from the reinsurer, and the costs are the expenses of the direct writer. Since the business is coinsured, insurance benefits may be ignored in the matching process. Not only does this result in some simplification, but it also may, in many instances, provide a better matching of revenues and costs, which is the main goal of GAAP.

(AUTHORS' REVIEW OF DISCUSSION)

DAVID N. BECKER AND MICHAEL V. ECKMAN:

Gary See brings up an interesting point that we overlooked in writing the paper. Since we were dealing with reinsurance retroceded from reinsurance accepted, we did not consider the possibility that the commissions and allowances on the retroceded portion could be significantly different from those on the accepted portion. In reinsurance, retrocessions usually go out under the original terms of the accepted business.

As Mr. See points out, since business is coinsured, insurance benefits may be ignored in the matching process. He would, however, consider the coinsurance allowances as revenue and the ceding company's original expenses as the outgo item.

We believe Mr. See's approach deserves further study and possibly a test of its application.