Modeling of Economic Series Coordinated with Interest Rate Scenarios:

A progress report on research sponsored by the Casualty Actuarial Society and the Society of Actuaries[1]

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Abstract

This paper represents a brief progress report on research, being sponsored by the Casualty Actuarial Society (CAS) and the Society of Actuaries (SOA), in the area of modeling of economic series. In particular, research is being undertaken to provide a foundation for the actuarial community's modeling of a variety of economic variables, including interest rates, equity returns, inflation, unemployment rates, and real estate price levels. The specific objectives of this research are to provide a summary of the literature in the area of economic scenario modeling, identify data sources and methodologies in this area useful to the actuarial profession, and produce working software for economic modeling, including appropriate documentation, to be made available to the general actuarial community in order to allow actuaries to use and build upon this research.

[1] The investigators wish to thank the Casualty Actuarial Society and the Society of Actuaries for providing financial support for this research, as well as guidance and feedback on the subject matter.

Section 1: Introduction

Consider the following activities, each of which is actively engaged in by the 21st-century actuary:

- A property-liability insurance company uses dynamic financial analysis (DFA) to compare alternative strategic and operational decisions, and to identify areas of potential opportunity relative to competitors within the insurance industry.
- A life insurer, both for regulatory and internal decision-making purposes, quantifies the impact on its future solvency and profitability of several alternative financial scenarios.
- An insurer evaluates different asset-liability management (ALM) techniques across a variety of possible future economic environments.

All of these activities – activities which every year are the purview of a greater number of actuaries – have at least one component in common: the necessity of systematically and efficiently modeling future values of, and the interrelationships between, economic and financial variables. These and other activities, critical to strategic decision-making and optimization of insurer performance, should be built upon a solid conceptual and practical foundation with respect to modeling the financial and economic environment, which is the broad context within which insurers operate.

Recognizing this, the Casualty Actuarial Society and the Society of Actuaries, in May 2001, issued a Request for Proposals for research in the area of "Modeling of Economic Series Coordinated with Interest Rate Scenarios." The goal of this solicited research was to provide actuaries with a floor, or foundation, for future work in the modeling of financial scenarios, and to prepare a model for projecting economic and financial indices, incorporating realistic interdependencies among the variables. Specific deliverables from the research project included:

1) A *literature review* of work previously done in this area;
2) Identification of appropriate *data sources and methodologies* to enhance economic modeling efforts relevant to the actuarial profession; and
3) Production of a *working model* of economic series, coordinated with interest rates, that could be made public and used by actuaries via the CAS and SOA websites to project future economic scenarios. Categories of economic series to be modeled included interest rates, equity price levels, inflation rates, unemployment rates, equity dividend yields, and real estate price levels.

As alluded to above, this topic is of considerable value and importance to the actuarial profession and the broader insurance community. For example, a key aspect of the dynamic financial analysis process, which continues to be an area of substantial development and interest among actuaries, is the generation of economic and financial scenarios. These scenarios provide an economic context for the evaluation of an insurer's alternative operating decisions and their potential impact, across a variety of possible future economic conditions, on future corporate value. Such stochastic simulation efforts are predicated upon the ability to probabilistically express possible future economic and financial environments. In addition, an integrated scenario generation capability is critical to recognizing the interdependencies between the various economic and financial series – e.g., consistently modeling the relationships between, say, equity returns and interest rate movements.

Similarly, the generation of scenarios is important for regulatory, rating agency, and internal management tests of an insurer's potential future operating conditions. An example is cash flow testing. By testing across scenarios, an insurer's cash position and liquidity can be evaluated over a variety of alternative future economic and financial environments.

This paper represents a status report, as of August 2003, summarizing the authors' progress on this research and the development of a scenario model. The materials underlying this paper, along with a substantial written report, have recently been provided to committees of the CAS and the SOA for comment. These materials represent a draft of the proposed final report and model. It is anticipated that some enhancements, either necessary and/or desirable, will be made in response to forthcoming comments from these committees.

The remainder of this paper is organized as follows. Section 2 provides an overview of the draft report, including the specific deliverables which comprise the report and our research results. Section 3 discusses the specific modeling approaches currently being taken with respect to the various economic series being modeled. Section 4 describes several specific issues that are illustrative of the kinds of important questions asked, and decisions made, in the process of economic modeling. Section 5 concludes, and Section 6 provides a bibliography.

Section 2: Overview of Draft Report

In its current incarnation, the draft report summarizing progress to-date on this research project includes the following sections and attachments:

Section 1:	Introduction and Overview
Section 2:	Excerpts from Original CAS / SOA Request for Proposals
Section 3:	Excerpts from Proposal of Selected Researchers
Section 4:	Literature Review
Section 5:	Descriptions of Data and Approach
Section 6:	Discussions of Issues
Section 7:	Results of Model Simulations
Section 8:	Conclusions and Acknowledgements
Appendix A:	User's Guide to the Financial Scenario Model
Appendix B:	Presentations on This Research
Appendix C:	Simulated Financial Scenario Data

Appendix D: The Financial Scenario Model

Most of the Section and Appendix titles should be self-explanatory. A few very brief comments on just a few of these components are warranted here.

The *Literature Review* (Section 4) includes brief descriptions of a variety of articles – covering the fields of actuarial science (both life and casualty), finance, and economics – that the researchers believe are relevant, to varying degrees, to this research.[2] Research on the development of financial scenarios, and the analysis of financial and economic time series, is a continually evolving and growing area. We recommend that efforts be made, at least periodically if not continually (e.g., by a formal charge to appropriate CAS / SOA research committees, or by engagement of other interested persons), to provide an ongoing search for and review of relevant new work in this area, in order that the results from this project might be enhanced and kept current.

Discussions of Issues (Section 6) describes and comments upon some of the specific issues encountered during the course of this research. In some cases, these issues involved decisions with which we as researchers were confronted; our thought processes and the rationales for selected approaches are included here.[3]

Simulated Financial Scenario Data (Appendix C) is a spreadsheet database of hundreds of scenarios (i.e., simulation paths) of financial and economic variables, generated as output from the Financial Scenario Model. The intent of this data is to provide an alternative to requiring the @Risk simulation package (an add-on to Excel)[4] in order to run the model. This data can be used directly, in lieu of actually running the model; the "pre-simulated" scenario paths can be used as an input to a DFA model or other analytical program.

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[2] Various CAS and SOA members made several suggestions for articles to be included in this literature review. These suggestions are gratefully acknowledged.

[3] Often, these issues were either provoked or reinforced by questions or comments from members of the sponsoring actuarial committees. Again, this input was greatly appreciated and valued throughout the project.

[4] "@Risk" is a software package produced by the Palisade Corporation. For additional information, please see the company's website at www.palisade.com.

The Financial Scenario Model (Appendix D) is an Excel spreadsheet-based program, designed to be run, as mentioned above, through the @Risk simulation add-on. The (ultimately) downloadable version of the model will contain default values of appropriate parameters – however, these can be changed by the user for purposes of updating for new or additional data, sensitivity testing of parameter values, etc.

Please note that our intention is ultimately to post the completed report, along with the financial scenario model and hyperlinks to various presentations and articles emerging from this research, on the websites of both the CAS and the SOA.

Section 3: Progress Report on Research

After an extensive review of the literature in a number of relevant fields, it was found that three articles in particular provided a strong historical foundation for this economic scenario research. Wilkie (1986) used simulation to model future economic scenarios, for a variety of applications. In his article, inflation was modeled as the "driving" variable, with several other economic processes being driven off of inflation in a "cascade"-type fashion. A first-order autoregressive (AR) process was used for inflation, along with certain other variables. Wilkie (1995) followed up on his earlier paper, expanding upon and enhancing the modeling and econometrics. Finally, Hibbert, Mowbray, and Turnbull (2001) examined issues in modeling a number of economic variables. Included in this paper is a comparison of outputs with the results from the Wilkie model. Taken together, these three articles provide an excellent background for understanding our current research.

For this research, the economic series modeled included the following:

- Inflation
- Real interest rates
- Nominal interest rates
- Equity returns

- Large stocks
- Small stocks

- Equity dividend yields
- Real estate returns
- Unemployment

The general approach used for each of these processes will be briefly summarized below[5]. In the formulas which follow, standard Brownian motion processes are represented by ***dB***, and subscripts refer either to time (***t***) or to the relevant economic variable. The descriptions are all provided in a continuous-time framework, as this is the framework in which most financial research of this nature is done. Of course, parameterization and estimation are performed relative to real, actual data, which is discrete in nature. Mathematically, the discrete forms of these processes are analogous to the continuous-time versions provided below.

<u>*Inflation*</u>

We model inflation (***q***) as an Ornstein-Uhlenbeck process. Specifically, inflation is assumed to be a one-factor, mean-reverting process of the form

$$dq_t = \kappa_q (\mu_q - q_t)\, dt + \sigma_q\, dB_q$$

where q is the short-term inflation rate, κ is the speed of mean reversion, μ is the long-run rate to which the process tends to revert, and σ is a volatility parameter. In discrete format, this amounts to an autoregressive process. The model was parameterized using U.S. Bureau of Labor Statistics Consumer Price Index (CPI) data. Two time periods were examined, in order to test the sensitivity of the parameters to different economic epochs: 1913 to 2001, and 1946 to 2001. Because of the noise in monthly CPI series, annual CPI values and regressions were employed. Using an approach similar to Vasicek (1977) for

[5] Much more detail regarding the modeling, mathematics, parameterizations, etc., is provided in the forthcoming formal report of this research, as well as in one or more planned articles.

interest rates, we then produced a term structure of inflation rates (necessary for modeling nominal interest rates – see below).

Real Interest Rates

Real interest rates (***r***) are modeled according to a two-factor Vasicek model[6]. This structure is similar to the one-factor Vasicek model (Vasicek, 1977), in that it is a mean-reverting process with the short-term real interest rate being a stochastic variable. However, in addition, there is a second stochastic factor: the long-run mean (***l***, below) to which the short-term rate tends to revert. The specific formulas used are

$$dr_t = \kappa_r (l_t - r_t)\, dt + \sigma_r\, dB_r$$
$$dl_t = \kappa_l (\mu_l - r_t)\, dt + \sigma_l\, dB_l$$

These equations were parameterized with monthly Federal Reserve data from 1982 to 2001, using two-stage least-squares estimation techniques. Real interest rates, while not directly observable, were estimated *ex post* by determining the differences between nominal interest rates and inflation rates.

Nominal Interest Rates

Once inflation and real interest rates have been modeled, nominal interest rates (***i***) are determined from them, based on the standard Fisher (1930) relationship:

$$i = \{(1 + q) \times (1 + r)\} - 1$$

The model software developed in coordination with this research will allow the user, if s/he so desires, to toggle a "non-negativity" switch, thereby preventing future modeled nominal interest rates from falling below zero.

Equity Returns

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[6] This is also a simple case of the two-factor Hull-White interest rate model. See Hull and White (1994).

A great deal of attention has been paid to modeling equity returns in the financial, economic, and even physics (under the guise of "econophysics") literatures[7]. One of the empirical observations that is frequently noted regarding historical equity returns is the "fat tails" issue – that actual equity returns, when examined as a historical distribution of returns, tend to have fatter tails than typical theoretical distributions would suggest. To that end, we have used (along the lines of Hardy (2001)) a "regime-switching" model for equity returns, with the two regimes having, respectively, low and high volatility. A Markov Chain framework represents the probabilities of switching between regimes from month to month.

We have modeled equity returns (s_t) by specifying an excess return (x_t) over and above the modeled nominal interest rate:

$$s_t = q_t + r_t + x_t$$

The excess return represents a risk premium attributable to capital appreciation (since the dividend yield of equities is modeled separately – see below). We estimated the processes underlying small stocks and large stocks separately.

Equity Dividend Yields

The equity dividend yield (y) is modeled such that the natural logarithm of the dividend yield follows an AR process:

$$d(\ln y_t) = \kappa_y (\mu_y - \ln y_t)\, dt + \sigma_y\, dB_{yt}$$

The parameterization process for this model is similar to that involved for the inflation model, described above.

Real Estate Returns

[7] For econophysics, see, for example, Sornette (2003).

We estimated two versions of an Ornstein-Uhlenbeck model for real estate returns: including and excluding inflation.[8] We used quarterly National Council of Real Estate Investment Fiduciaries (NCREIF) pricing indices to capture returns on commercial properties. The NCREIF data is generated from market appraisals of various property types, including apartment, industrial, office, and retail.

Unemployment

With respect to the unemployment process, a well-known tool, the Phillip's Curve, posits an inverse relationship between the unemployment rate (u) and inflation (q).[9] We have chosen to build upon that relationship, but to include an AR(1) process:

$$du_t = \kappa_u (\mu_u - u_t)\, dt \;+\; \alpha_u\, dq_t \;+\; \sigma_u\, \varepsilon_{ut}$$

Unemployment data, from 1948 to 2001, from the Bureau of Labor Statistics was utilized.

Section 4: Discussion of Several Issues

This section highlights several of the key issues and questions encountered during this research project. This section is not meant to be a comprehensive survey of all critical issues, but rather illustrative of the kinds of problems encountered, and the thought process of the researchers with regard to their resolution.

Inflation versus Interest Rates

In any effort to model key economic variables, a question which must be resolved early in the modeling process involves which variables come "first" – i.e., which variable(s) is(are) key and causal, and which other variables are functions of those. This is a typical question when dealing with interest rates and inflation. Interest rates are probably more frequently

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[8] While we expected inflation to be a driver of real estate returns, the results to-date have not indicated significance.

[9] While this relationship seems plausible in many ways, it should be noted that more recent data (since the original publication of the Phillips Curve in 1958) has not fit this inverse relationship as well.

directly modeled, and a potential disadvantage of varying from that approach can be an inability to take advantage of, and build upon, extensive prior work of numerous authors in interest rate modeling. On the other hand, in the U.S., the Federal Reserve has a large role in impacting the levels of interest rates, in response to inflationary pressures and general economic conditions. This decision, therefore, is not a trivial one.

We chose to model both inflation and real interest rates as the "driver" variables. This decision led to the modeling structures for those two variables, along with nominal interest rates, mentioned in the preceding section. Under our approach, is it possible for the user of the model to correlate the shock (***dB***) terms of the inflation and real interest rate processes, allowing for a direct, partial connection between these series.

Equilibrium vs. Arbitrage Free Models

One of the primary processes in a financial scenario model is a term structure process. A tremendous variety of term structure models is available for both practitioners and researchers. (For a discussion of many of the available models, see Yan (2001)). No single term structure model has yet proven itself worthy for all possible applications (see the discussion in Chapman and Pearson (2001)). In virtually all cases, the user of a term structure model has one or more tradeoffs to consider – e.g., complexity of the model vs. accuracy. The nature of these tradeoffs depend on the specific application of the term structure model.

There are several important issues to consider when choosing among term structure models. One consideration is related to the theoretical background of the model. Specifically, term structure models are typically categorized as "equilibrium" models and "arbitrage-free" models. Equilibrium models typically begin with an assumption for short-term interest rates, which are usually derived from more general assumptions about the state variables that describe the overall economy. Using the assumed process for short-term rates, one can determine the yield on longer-term bonds by looking at the expected path of interest rates until the bond's maturity. One of the primary advantages of equilibrium models is that the prices of many popular securities have closed-form analytic solutions.

Another advantage is that equilibrium models are fairly easy to use. On the negative side, equilibrium term structure models generate yield curves which are inconsistent with current market prices. While the parameters of these models may be selected carefully, there is no guarantee that the resulting term structure will generate observed market prices.

Contrary to equilibrium models, arbitrage-free term structure models project future interest rate paths that emanate from the existing yield curve. For applications using arbitrage-free term structure models, resulting prices will be based on the concept of arbitrage. Unfortunately, arbitrage-free term structure models are frequently more difficult to use than their equilibrium counterparts.

Although we will not elaborate here, other considerations in selecting a term structure model include pricing accuracy, internal consistency, data issues, intended use of the model, and the time horizon over which simulations will be performed. With regard to the last two considerations mentioned above, it is important to bear in mind that insurance and actuarial applications of such models, such as in DFA models, generally involve long time horizons (e.g., five years of simulation projections), with generally fairly course time intervals (monthly, quarterly, or even annual). This is a very different framework from a short-term (hourly, daily, or weekly) trading horizon. It may well be that different models are justifiable for such different analytical frameworks.

Adequacy of a Two-Factor Interest Rate / Inflation Model

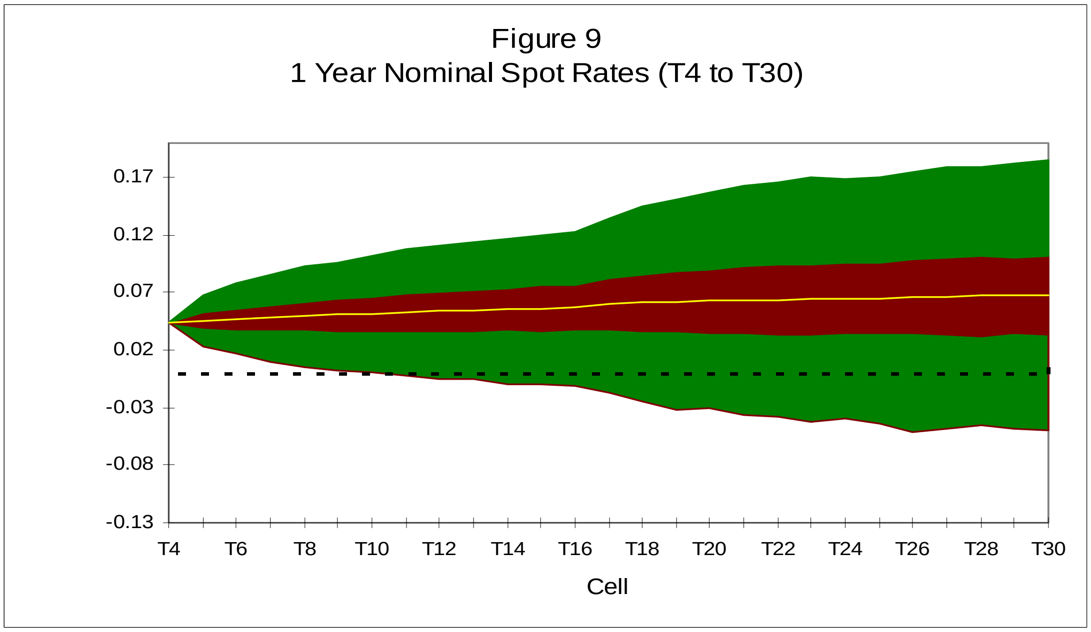
The number of factors to use in modeling interest rates is a decision which frequently elicits passionate debate. Again, as mentioned above, it is important to keep in mind the purpose toward which this research is working: to produce reasonable distributions of future economic values. Our work is ***not*** intended for security-trading purposes. This is a hugely important context to keep in mind – it has implications for the type of interest rate model used, the number of parameters employed, etc. Furthermore, there is often a misunderstanding as to the types and movements of yield curves that are available from two-factor (and with respect to some issues, even one-factor) models. For example, humped curves are indeed possible. (A good paper for considering the types of yield curve movements that predominate historically is Litterman and Schenkman (1991).) We believe that the two-factor model we have employed is a reasonable selection in view of both historical and parsimony considerations.

Section 5: Types of Results

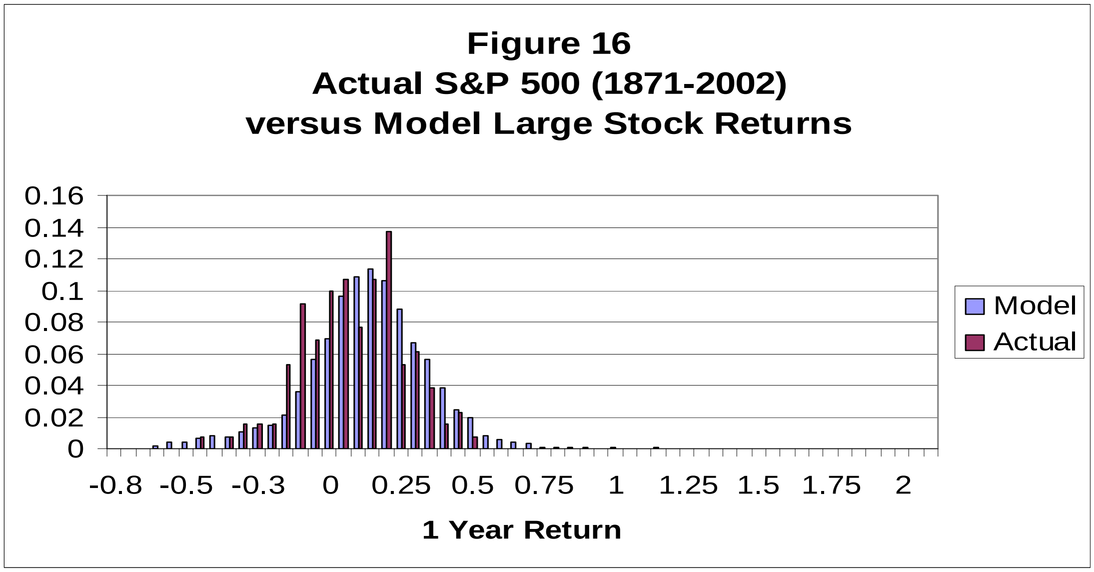
In addition to documentation regarding the analytical infrastructure of this research, our work has provided a number of results in terms of

- *Parameters* for the various economic processes;
- Indications of future *volatilities* for the modeled processes; and
- *Comparisons* of model fits with historical data.

Since our formal report is, as of this writing, currently being reviewed by the sponsoring societies, we have refrained from providing specific quantitative results. These will be provided on the CAS and SOA websites, as part of our formal report, upon completion of this project. However, below are two samples of exhibits from preliminary work. These are included for illustrative purposes only. The first exhibit, immediately below, is a "funnel of doubt" chart, which shows the volatility of simulated future values, over time, of a stochastic variable (in this case, the 1-year nominal spot interest rate). The solid relatively-horizontal line in the middle shows the mean value over time of the (in this case, 5,000) simulations of that stochastic variable. The two shaded areas surrounding the mean line are the 25th to 75th percentile values (the dark shaded section), and the 1st to 99th percentile values (lighter shaded section).



The second illustrative exhibit, immediately below, needs little explanation. This type of exhibit graphically represents the nature of the fit between actual and modeled values (in this case, of the distribution of large stock returns).



Section 6: Conclusion

At the time of writing this article, and the presentation of this material at the Actuarial Research Conference (August 2003, in Ann Arbor, MI), an initial draft of the final report of this research had been submitted to the oversight committees of the CAS and the SOA. While comments from these committees will undoubtedly improve and enhance this research, the current key elements and deliverables of this research include:

- A *literature review* summarizing the relevant literature from several fields, including actuarial science, economics, and finance;
- A *summary of data sources and methodology* providing the analytical underpinnings of our work;
- A *Financial Scenario Model*, programmed in Excel and @Risk, and designed to be available for use by actuaries by downloading from the CAS or SOA website;
- *Simulated financial scenario data*, based on default model parameter assumptions, which can be used in lieu of the model if @Risk software is not available;
- A *User's Guide* to the model; and
- A *discussion of results* from sample implementation of the model.

This paper has presented a survey and highlights of our progress on this actuarial research. Hopefully, it has provided at least a flavor of this work. Significantly more detail, along with specific parameter values and quantitative results, is included in our formal report. The formal report will be available through the CAS and the SOA websites in the near future.

This research, and the accompanying model, should be of wide interest, including to such insurance and financial professionals as insurers, regulators, and pension funds. Specific applications of this research might include parts of dynamic financial analysis, cash flow testing, financial planning, investment analysis, capital budgeting, and analysis of alternative financial risk management solutions.

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