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PROFITABILITY AS A RETURN ON TOTAL CAPITAL

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ABSTRACT

It has been said that the primary job of management is to manage capital, and that all other management functions should be related to that goal. The return on GAAP capital is, in fact, an index used in measuring the job that management is doing.

The return on total capital is another measuring rod used to compare the profitability of insurance and noninsurance operations. The techniques suggested in this paper can be used to develop a figure representing the return on total capital for each insurance line of business so management can understand the contribution each makes to the corporation's total return.

The GAAP capital and surplus for each line of business includes an item referred to as "benchmark surplus." It is the amount of statutory surplus that is set aside, in addition to reserves, to support in-force business. This essential item generally has been ignored in actuarial techniques used in the past to measure profitability of products sold by life insurance companies.

A method of calculating the unlevel GAAP returns that can be expected to emerge annually from a closed block of new business is developed in this paper. The paper shows how those GAAP returns are related to the internal rate of return the actuary may have calculated to describe expected profitability on a statutory basis. The paper also outlines a method of determining the actual GAAP return on total capital each calendar year on the in-force business for a specific line of business.

1. INTRODUCTION

Each year, *Forbes* magazine publishes an *Annual Report on American Industry*. That report contains a wealth of information on a number of industries and a number of companies within each industry. Various financial yardsticks are used. One of the measuring rods for profitability is return on total capital (ROTC), which is defined as GAAP net income for the year, divided by GAAP capital and surplus at the beginning of the year.

The purposes of this paper are (*a*) to describe a method of measuring the contribution each line of business in a multiline insurance company makes to profitability each year, and (*b*) to describe an approach for determining the expected annual contribution to future profitability from a block of new business. Although, in theory, the ROTC is a measuring rod only for stock companies, the concepts discussed should be useful to the management of mutual companies.

There are three basic parts to the paper. Section II discusses the concept of "benchmark surplus"—which is the amount of statutory surplus needed to support a block of in-force business. Section III describes the measurement of the actual ROTC in a given calendar year for each line of business. Section IV provides examples and formulas that show the contribution a closed block of new business is expected to make to the ROTC in the future. The unlevel ROTC for each policy year also is related to the statutory internal rate of return (IRR) profitability measuring rod. The statutory IRR is based on book profits described by James C. H. Anderson in the paper "Gross Premium Calculation and Profit Measurement for Nonparticipating Insurance" (*TSA*, XI, 357).

II. BENCHMARK SURPLUS

The 1981 *Record of the Society of Actuaries* (VII, 67) contains an excellent discussion of the subject "Effective Use of Capital." This includes comments on the need for what I refer to as benchmark surplus. Benchmark surplus is the amount of statutory surplus that is needed for each line of business in order to reduce the probability of insolvency to management's comfort level. It is based upon the various risks associated with in-force business. It is a basic premise of this paper that benchmark surplus must be included in determining the profitability of a company, a line of business, or a product within a line of business.

For example, consider a one-year policy with no statutory drain that is expected to generate an after-tax margin of 2 percent of premium. If the benchmark surplus needed is 50 percent of premium (which can earn 8 percent interest after tax), the return on total capital is 12 percent.[1] If this is too low a return, an effort should be made to increase the after-tax margin and/or to reexamine the benchmark surplus formula to see if 50 percent of premium is too conservative.

In an example in Section IV of this paper, 3 percent of statutory reserves plus 3 percent of premium is chosen as the benchmark surplus. This may not be the right amount, but it is used for illustration. The method of

[1] $12\% = [0.02P + (0.08)(0.50P)]/0.50P$.

determining the correct amount is outside the scope of this paper. However, the A. M. Best Company annually publishes a financial and operational overview of life insurance companies in the United States and Canada. It is my understanding that Best's has used the following benchmark surplus guidelines for rating "large" companies. More stringent requirements are used for "small" companies.

Individual life:	3% of statutory reserves
	+ $4.80 per $1,000 of permanent insurance in force
	+ $2.40 per $1,000 of term insurance in force
Group life:	3% of statutory reserves
	+ $1.20 per $1,000 of insurance in force
Annuities:	3% of general account reserves
Health:	24% of premium

More information on the extremely important subject of required surplus is given in the discussion draft prepared by the Society of Actuaries' Committee on Valuation (*RSA*, V [1979], 256). That report discusses the surplus needed to cover three risks: the C_1 (asset depreciation) risk, the C_2 (pricing inadequacy) risk, and the C_3 (interest rate change) risk.

III. ACTUAL RETURN IN A CALENDAR YEAR FOR A LINE OF BUSINESS

It is easy to calculate the return on total capital for a given company. This is simply GAAP net income for the year divided by GAAP capital and surplus. Some investment analysts use "beginning-of-year," some use "end-of-year," and some use "average" GAAP surplus in the denominator. However, in order to make the ROTC an effective management tool, it is necessary to know the ROTC for each profit center or line of business in the company. A method of determining these returns for each line of business will be outlined in this section of the paper.

The process is to allocate both GAAP net income and GAAP capital and surplus for the company (referred to hereafter as GAAP surplus) to each line of business. Those items that do not relate to an operating line of business can be allocated to a corporate line of business.

We are all familiar with the following equation on a total company basis:

Statutory capital and surplus + GAAP adjustments
= GAAP capital and surplus.

In the above equation, statutory capital and surplus for any operating line of business equals the benchmark surplus for the line. At my company, benchmark surplus (and GAAP adjustments) are redetermined for each

operating line of business each month. The difference between total company statutory capital and surplus and the sum of the benchmark surplus for each operating line of business is allocated to a corporate line of business.

Most GAAP adjustments can be related directly to the operating lines of business. The major GAAP adjustments are the prepaid acquisition expense asset, the difference between GAAP benefit reserves and statutory benefit reserves, and deferred taxes. Those GAAP adjustments that cannot be allocated directly to the operating lines of business can be allocated to the corporate line of business. The GAAP surplus for any operating line of business will therefore equal the statutory benchmark surplus for the line plus the GAAP adjustment for the line. The GAAP surplus for the corporate line of business is the excess of total company GAAP surplus over the GAAP surplus for all the operating lines of business.

The GAAP after-tax earnings for each operating line of business equal the earnings from the product sold and include investment income on statutory benchmark surplus. The ratio of GAAP after-tax earnings to GAAP surplus is the ROTC for the line of business being measured.

IV. INTERNAL RATE OF RETURN VERSUS UNLEVEL ROTC

A. *Methodology*

For purposes of analyzing GAAP earnings, GAAP and statutory surplus each may be separated into three categories, as described in my paper "Earnings and the Internal Rate of Return Measurement of Profit" (*TSA*, XXVI, 617): insurance surplus account, benchmark surplus account, and corporate surplus account. Statutory surplus consists of required benchmark surplus for each operating line of business, with any excess (or deficit) being defined as corporate (or free) surplus, which is allocated to a corporate line of business. The year-end surplus in the insurance surplus account for each operating line of business is zero under statutory accounting; under GAAP accounting, it is equal to the algebraic excess of GAAP surplus over statutory surplus (i.e., GAAP adjustments).

The technique I have used is to transfer an amount from the corporate surplus account to the statutory insurance surplus account to cover statutory losses and to transfer statutory gains from the statutory insurance surplus account to the corporate surplus account. The initial transfer from corporate surplus is made at the *beginning* of the first policy year because usually losses are generated by acquisition expenses in the first policy year. In renewal years, statutory earnings are transferred from the stat-

utory insurance surplus account to the corporate surplus account. These renewal-year transfers are made at the *end* of the policy year. They can be retained in the corporate surplus account, reinvested in the new business of any operating line of business, or paid out as a dividend to stockholders. In this way, the statutory insurance surplus account for any operating line of business becomes zero at the end of each policy year, irrespective of whether there are statutory gains or losses. Similarly, at the *beginning* of the first policy year and at the *end* of all other policy years, a transfer is made between the benchmark surplus account and the corporate surplus account so that the benchmark surplus account is at the appropriate level.

The same transfers that are made between the corporate surplus account and the statutory insurance surplus account are made between the GAAP insurance surplus account and the corporate surplus account, that is, statutory after-tax earnings. Thus, the amount in the GAAP insurance surplus account is the GAAP adjustment and equals the excess of GAAP after-tax earnings over statutory after-tax earnings. The sum of the amounts in the GAAP insurance surplus account and the benchmark surplus account represents the GAAP capital and surplus associated with the block of business being analyzed.

The technique of making the surplus transfers between the corporate surplus account and the insurance surplus account at the beginning of the first policy year and at the end of renewal year is, in my opinion, a reasonable basis to use for analysis. Many products generate a statutory surplus drain at the beginning of the first policy year, and surplus is needed then. The statutory earnings of policy years after the first year are transferred at year-end. This is the basis illustrated in this paper. Some actuaries may, in practice, prefer to use a surplus transfer at the beginning *and* end of policy years—especially the first year. Some may prefer to make transfers monthly or even continuously.

B. *Actuarial Model*

To demonstrate the techniques of relating the internal rate of return (IRR) to the unlevel ROTCs, as described in this paper, I have developed a simple model that illustrates how the ROTC may emerge for a five-year plan of insurance. Earnings and surplus over a five-year period are compared in Table 1 on both a statutory and a GAAP basis. For both bases, total results are shown according to the three sources: insurance surplus account, benchmark surplus, and corporate surplus.

Table 1 starts with the amount of corporate (free) surplus needed to write new business. The new business generates an after-tax statutory

TABLE 1

SURPLUS AND EARNINGS EMERGENCE

	STATUTORY BASIS					GAAP BASIS				
	Insurance Surplus Account (*a*)	Benchmark Surplus (*b*)	Total (*a*) + (*b*) (*c*)	Corporate Surplus (*d*)	Total Surplus (*c*) + (*d*) (*e*)	Insurance Surplus Account (*f*)	Benchmark Surplus (*g*)	Total (*f*) + (*g*) (*h*)	Corporate Surplus (*i*)	Total Surplus (*h*) + (*i*) (*j*)
1. Surplus at end of year 0, before transfer	$ 0	$ 0	$ 0	$129,382	$129,382	$ 0	$ 0	$ 0	$129,382	$129,382
2. Surplus transfer at end of year 0	123,441	5,941	129,382	− 129,382	0	123,441	5,941	129,382	− 129,382	0
3. Surplus after transfer [(1) + (2)]	123,441	5,941	129,382	0	129,382	123,441	5,941	129,382	0	129,382
4. After-tax earnings for year 1	− 123,441	179	− 123,262	0	− 123,262	17,435	179	17,614	0	17,614
5. Return [(4) ÷ (3)]	.....	.....	.....	.....	.....	14.1%	3.0%	13.6%	.....	13.6%
6. Surplus transfer at end of year 1	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0	$ 0
7. Surplus after transfer [(3) + (4) + (6)]	0	6,120	6,120	0	6,120	140,876	6,120	146,996	0	146,996
8. After-tax earnings for year 2	68,124	183	68,307	0	68,307	16,940	183	17,123	0	17,123
9. Return [(8) ÷ (7)]	.....	.....	.....	.....	.....	12.0%	3.0%	11.6%	.....	11.6%
10. Surplus transfer at end of year 2	− $ 68,124	− $1,353	− $ 69,477	$ 69,477	$ 0	− $ 68,124	− $1,353	− $ 69,477	$ 69,477	$ 0
11. Surplus after transfer [(7) + (8) + (10)]	0	4,950	4,950	69,477	74,427	89,692	4,950	94,642	69,477	164,119
12. After-tax earnings for year 3	46,655	148	46,803	2,084	48,887	11,521	148	11,669	2,084	13,753
13. Return [(12) ÷ (11)]	.....	.....	.....	.....	.....	12.8%	3.0%	12.3%	3.0%	8.4%
14. Surplus transfer at end of year 3	− $ 46,655	$ 152	− $ 46,503	$ 46,503	$ 0	− $ 46,655	$ 152	− $ 46,503	$ 46,503	$ 0
15. Surplus after transfer [(11) + (12) + (14)]	0	5,250	5,250	118,064	123,314	54,558	5,250	59,808	118,064	177,872
16. After-tax earnings for year 4	50,645	158	50,803	3,542	54,345	8,687	158	8,845	3,542	12,387
17. Return [(16) ÷ (15)]	.....	.....	.....	.....	.....	15.9%	3.0%	14.8%	3.0%	7.0%
18. Surplus transfer at end of year 4	− $ 50,645	− $ 308	− $ 50,953	$ 50,953	$ 0	− $ 50,645	− $ 308	− $ 50,953	$ 50,953	$ 0
19. Surplus after transfer [(15) + (16) + (18)]	0	5,100	5,100	172,559	177,659	12,600	5,100	17,700	172,559	190,259
20. After-tax earnings for year 5	13,356	153	13,509	5,177	18,686	756	153	909	5,177	6,086
21. Return [(20) ÷ (19)]	.....	.....	.....	.....	.....	6.0%	3.0%	5.1%	3.0%	3.2%
22. Surplus transfer at end of year 5	− $ 13,356	− $5,253	− $ 18,609	$ 18,609	$ 0	− $ 13,356	− $5,253	− $ 18,609	$ 18,609	$ 0
23. Surplus after transfer [(19) + (20) + (22)]	0	0	0	196,345	196,345	0	0	0	196,345	196,345

NOTE.—Internal rate of return (using formulas in Section F) is 13.2 percent excluding statutory benchmark surplus, and 12.6 percent including statutory benchmark surplus.

drain of $123,441 at the beginning of the first policy year. Since we also need $5,941 of benchmark surplus at the beginning of year 1, this example begins with the sum of these two items, or $129,382 of corporate surplus.

The formulas and definitions used in developing data for the statutory and GAAP insurance surplus accounts and the benchmark surplus account are shown in Sections D and E. These formulas are on a per-original-issue basis, not on a per-survivor basis. The formulas assume all acquisition expenses are deferrable and that GAAP pretax profits are a uniform percentage of premium. Also, it is assumed there are no margins for adverse deviations contained in the GAAP assumptions. These would, of course, change the size and incidence of expected reported GAAP earnings.

Section F develops formulas for calculating the GAAP ROTC and statutory IRR. Section G contains the actuarial assumptions used for the plan of insurance and the development of the financial information contained in Table 1.

C. *Comments on Table 1*

As expected, statutory and GAAP surplus are equal at the beginning and end of the five-year period—and total earnings are equal on the statutory and GAAP bases over the five years.

An analysis of the relationship between the ROTCs and the IRR can be made by applying the methods found in S. David Promislow's paper "A New Approach to the Theory of Interest" (*TSA*, XXXII, 53). Promislow defines a transaction $T = (c_0, c_1, c_2 \ldots c_n)$, where c_k represents a net payment of c_k units at the end of k periods and $P_i(T)$ represents the present value of the payments at interest rate i; that is, $P_i(T) = \sum_{k=0}^{n} c_k (1 + i)^{-k}$. Then i is the yield rate if $P_i(T) = 0$. This transaction can be decomposed into a series of transactions.

For example, consider five one-year transactions from column h of Table 1. Using Promislow's nomenclature (a loan is treated as a negative number, and the pay-back as a positive number), we have the following:

Transaction A = (−$129,382; $146,996; 0; 0; 0; 0), with ROTC = 13.6%
Transaction B = (0; −$146,996; $164,119; 0; 0; 0), with ROTC = 11.6%
Transaction C = (0; 0; −$94,642; $106,311; 0; 0), with ROTC = 12.3%
Transaction D = (0; 0; 0; −$59,808; $68,653; 0), with ROTC = 14.8%
Transaction E = (0; 0; 0; 0; −$17,700; $18,609), with ROTC = 5.1%

This can be interpreted as investing $129,382 and earning 13.6 percent for one year. This is followed by an increased investment in the second year of $146,996, which earns 11.6 percent. In the third year, the investment is reduced to $94,642, and this earns 12.3 percent. In the fourth

year, the investment is reduced to $59,808, and this earns 14.8 percent. Finally, in the fifth year, the investment is reduced to $17,700, which earns 5.1 percent. If we sum the five transactions into one transaction, we get Transaction (A + B + C + D + E) = (−$129,382; 0; $69,477; $46,503; $50,953; $18,609), which yields a statutory IRR of 12.6 percent. Each item in this transaction is a surplus transfer. Thus, if the actuary tells management that a plan of insurance will produce a 12.6 percent statutory IRR, he should also tell management that the GAAP ROTC that is expected to appear, which is equivalent to the 12.6 percent IRR, is 13.6 percent in the first year, 11.6 percent in the second year, 12.3 percent in the third year, 14.8 percent in the fourth year, and 5.1 percent in the fifth year.

GAAP accounting takes the statutory transaction (A + B + C + D + E) and decomposes it into the five GAAP transactions shown above. Promislow's paper indicates that there is often more than one way to decompose a transaction into a series of transactions. Some other accounting system would produce a different decomposition. Also, if there is some flexibility in how GAAP benefit reserves or prepaid acquisition amortization schedules are developed, there will be other decompositions and different ROTCs.

D. *Symbols and Formulas Used in the Statutory and GAAP Insurance Surplus Accounts*

The GAAP insurance surplus account for each policy is the GAAP adjustment for the policy. It represents the accumulated excess of GAAP earnings over statutory earnings. This account equals zero at the end of the policy term. This section develops formulas for pretax statutory book profits and GAAP pretax book profits as a uniform percentage of premiums. The present values of pretax statutory and GAAP book profits are equal. The sums of pretax statutory and GAAP earnings are equal over the policy term. Statutory and GAAP federal income taxes (and the resultant after-tax earnings) are assumed to be equal over the policy term.

(1) ${}_{t}i$ = Pretax interest rate assumed in gross premium calculation for year t.

(2) ${}_{t}^{s}R$ = Statutory reserve at end of year t.

(3) ${}_{t}^{s}\tilde{B}$ = Pretax statutory insurance cash flow in year t, discounted to beginning of year t.

(4) ${}_{t}^{s}B$ = Pretax statutory book profit at beginning of year t

$$= {}_{t}^{s}\tilde{B} + {}_{t-1}{}^{s}R - \frac{{}_{t}^{s}R}{1 + {}_{t}i}.$$

(5) ${}_tS$ = Pretax statutory insurance surplus account at end of year t
$= 0$.

(6) ${}_tE$ = Pretax earnings in statutory insurance surplus account in year t
$= (1 + {}_ti)({}_tB)$ for $t > 1$, and ${}_tB$ for $t = 1$.

(7) ${}_tT$ = Statutory tax for year t.

(8) ${}_tE'$ = After-tax earnings in statutory insurance surplus account in year t
$= {}_tE - {}_tT$.

(9) ${}_tS'$ = After-tax statutory insurance surplus account at end of year t
$= 0$.

(10) ${}_tX'$ = Surplus transfer *from* corporate surplus account to insurance surplus account
$= -{}_1E'$ at beginning of year 1
$= -{}_tE'$ at end of year t, for $t > 1$.

(11) ${}_tP$ = Premium income at beginning of year t.

(12) k = Uniform ratio of GAAP pretax book profits to premium

$$= \frac{\sum_{t=1}^{m}\left[{}_tB\prod_{s=0}^{t-1}(1+{}_si)^{-1}\right]}{\sum_{t=1}^{n}\left[{}_tP\prod_{s=0}^{t-1}(1+{}_si)^{-1}\right]} = \frac{\sum_{t=1}^{m}\left[{}_tB\prod_{s=0}^{t-1}(1+{}_si)^{-1}\right]}{\sum_{t=1}^{n}\left[{}_tP\prod_{s=0}^{t-1}(1+{}_si)^{-1}\right]} = \frac{\sum_{t=1}^{m}\left[{}_t^GB\prod_{s=0}^{t-1}(1+{}_si)^{-1}\right]}{\sum_{t=1}^{n}\left[{}_tP\prod_{s=0}^{t-1}(1+{}_si)^{-1}\right]},$$

where m = coverage period, and n = premium-paying period.

(13) ${}_t^GB$ = Pretax GAAP book profit at beginning of year $t = (k)({}_tP)$.

(14) ${}_t^GS$ = Pretax GAAP insurance surplus account at end of year t
$= {}_t^GE - {}_tE$, for $t = 1$
$= {}_{t-}{}^GS + {}_t^GE - {}_tE$, for $t > 1$.

(15) ${}_t^GS = \sum_{j=1}^{t}({}_j^GE - {}_jE) = \sum_{j=t+1}^{m}({}_jE - {}_j^GE)$.

(16) ${}_t^GE$ = Pretax earnings in pretax GAAP insurance surplus account in year t.

(17) ${}_t^GE = (1 + {}_ti)({}_t^GB) - ({}_ti)({}_tE)$, for $t = 1$
$= (1 + {}_ti)({}_t^GB) + ({}_ti)({}_{t-}{}^GS)$, for $t > 1$.

(18) ${}_t^GT$ = GAAP tax for year t.

(19) ${}_t^GE'$ = After-tax earnings in GAAP insurance surplus account in year t
$= {}_t^GE - {}_t^GT$.

(20) ${}_t^GS'$ = After-tax GAAP insurance surplus account at end of year t
$= {}_t^GE' - {}_tE'$, for $t = 1$
$= {}_{t-}{}^GS' - {}_tE' + {}_t^GE'$, for $t > 1$.

(21) ${}^{g}_{t}S' = {}^{g}_{t}S - \sum_{j=1}^{t} ({}^{g}_{j}T - {}^{s}_{j}T)$.

(22) ${}^{g}_{t}S' = \sum_{j=1}^{t} ({}^{g}_{j}E' - {}^{s}_{j}E') = \sum_{j=t+1}^{m} ({}^{s}_{j}E' - {}^{g}_{j}E')$.

(23) $\sum_{t=1}^{m} {}^{g}_{t}E = \sum_{t=1}^{m} {}^{s}_{t}E$
(i.e., GAAP and statutory pretax earnings are equal over the policy term).

(24) $\sum_{t=1}^{m} {}^{g}_{t}T = \sum_{t=1}^{m} {}^{s}_{t}T$,
which assumes GAAP deferred tax = 0 when $t = m$.

(25) $\sum_{t=1}^{m} {}^{g}_{t}E' = \sum_{t=1}^{m} {}^{g}_{t}E - \sum_{t=1}^{m} {}^{g}_{t}T = \sum_{t=1}^{m} {}^{s}_{t}E - \sum_{t=1}^{m} {}^{s}_{t}T$.

(26) $\sum_{t=1}^{m} {}^{g}_{t}E' = \sum_{t=1}^{m} {}^{s}_{t}E'$
(i.e., GAAP and statutory after-tax earnings are equal over the policy term).

E. *Symbols and Formulas Used in the Benchmark Surplus Account*

There is no benchmark surplus applicable to a policy until it is issued. The actuary determines the amount that is needed while the policy is in force. Like policy reserves, the benchmark surplus is zero at the end of the policy term. Formulas for benchmark surplus and earnings thereon are shown in this section.

(27) ${}_{t}j$ = Pretax interest rate earned on benchmark surplus and on corporate surplus for year t.

(28) ${}_{t}f$ = Federal income tax rate applicable to interest earned on benchmark surplus and on corporate surplus in year t.

(29) ${}^{b}_{t}S'$ = Needed benchmark surplus at end of year = ${}_{t-}{}^{b}_{1}S' + {}^{b}_{t}E' + {}^{b}_{t}X'$.

(30) ${}^{b}_{t}X'$ = Surplus transfer *from* corporate surplus account to benchmark surplus account.

(31) ${}^{b}_{t}X' = \dfrac{{}^{b}_{1}S'}{1 + (1 - {}_{t}f)({}_{t}j)}$ at beginning of year 1, for $t = 1$

$= {}^{b}_{t}S' - [1 + (1 - {}_{t}f)({}_{t}j)]({}_{t-}{}^{b}_{1}S')$ at end of the year, for $t > 1$.

(32) ${}^{b}_{t}E$ = Pretax earnings in benchmark surplus account in year t

$= \dfrac{({}_{1}j)({}^{b}_{1}S')}{1 + (1 - {}_{1}f)({}_{1}j)}$, for $t = 1$

$= ({}_{t}j)({}_{t-}{}^{b}_{1}S')$, for $t > 1$.

(33) ${}_t^bE'$ = After-tax earnings in benchmark surplus account in year t

$$= \frac{(1 - {}_1f)({}_1j)({}_1^bS')}{1 + (1 - {}_1f)({}_1j)}, \quad \text{for } t = 1$$

$$= (1 - {}_tf)({}_tj)({}_{t-1}^bS'), \quad \text{for } t > 1.$$

F. *Return on Total Capital and Internal Rate of Return Formulas*

In this section, formulas are displayed for ROTC and IRR, using the symbols defined earlier.

The return on total capital is simply GAAP after-tax earnings divided by GAAP surplus at the beginning of the policy year. Two ROTCs are calculated for comparative purposes: formula (34) includes benchmark surplus and earnings thereon, and formula (36) excludes these items.

The internal rate of return is the yield rate at which the present value of the surplus transfers equals zero. Again, two formulas are shown: formula (38) includes benchmark surplus transfers, and formula (39) excludes them.

(34) ${}^{Gb}_t$ROTC = Return on total capital using GAAP insurance surplus account and benchmark surplus

$$= \frac{{}_t^GE' + {}_t^bE'}{{}_{t-1}^GS' + {}_{t-1}^bS'}, \quad \text{where } {}_0^GS' = {}_1X' \text{ and } {}_0^bS' = {}_1^bX'.$$

(35) $${}_1X' + {}_1^bX' + \sum_{t=2}^{m} ({}_tX' + {}_t^bX') \prod_{r=1}^{t} (1 + {}^{Gb}_r\text{ROTC})^{-1} = 0.$$

(36) ${}^G_t$ROTC = Return on total capital using GAAP insurance surplus account

$$= \frac{{}_t^GE'}{{}_{t-1}^GS'}.$$

(37) $${}_1X' + \sum_{t=2}^{m} {}_tX' \prod_{r=1}^{t} (1 + {}^G_r\text{ROTC})^{-1} = 0.$$

(38) ^{sb}IRR = Internal rate of return including statutory benchmark surplus, satisfies the equation

$${}_1X' + {}_1^bX' + \sum_{t=2}^{m} ({}_tX' + {}_t^bX')(1 + {}^{sb}\text{IRR})^{-t} = 0.$$

(39) ^{s}IRR = Internal rate of return excluding statutory benchmark surplus, satisfies the equation

$${}_1X' + \sum_{t=2}^{m} ({}_tX')(1 + {}^{s}\text{IRR})^{-t} = 0.$$

(40) $$\sum_{t=2}^{m} ({}_tX' + {}_t^bX')(1 + {}^{sb}\text{IRR})^{-t} = \sum_{t=2}^{m} ({}_tX' + {}_t^bX') \prod_{r=1}^{t} (1 + {}^{Gb}_r\text{ROTC})^{-1}.$$

Formula (40) illustrates that a given set of statutory values (${}_t^sX'$, ${}_t^bX'$, and ^{sb}IRR) can lead to many possible solutions to this equation involving ${}_t$ROTCs. The solutions are a function of the GAAP method and the GAAP assumptions being used.

The above formulas regarding the IRR assume the classic case where there is a statutory loss (investment) in the first policy year, followed by a string of statutory gains. In their separate discussions of the Promislow paper mentioned earlier, Marjorie V. Butcher and James C. Hickman referred to a technique to use in calculating an IRR when the outstanding investment becomes negative during the life of the policy. The technique they referred to, which is to use two yield rates, was outlined in two papers by Teichroew, Robichek, and Montalbano.[2] This overcomes the problem of solving an IRR equation that has multiple roots. It also eliminates the problem of one root that represents not only the yield that one gets on his "investment," but also the rate one pays when the outstanding investment changes sign one or more times during the life of the transaction, and the investor becomes a borrower. The technique is summarized below and uses two yield rates. The rate one is willing to pay to borrow money is determined first, and then a unique IRR can be derived. The modification to formula (39) is shown below.

$$
\begin{aligned}
{}^sOB_t &= \text{Outstanding balance at end of year } t \\
&= {}^sOB_{t-1}(1 + {}_tr) + {}_t^sX', \quad \text{where} \\
{}_tr &= {}^s\text{IRR if } {}^sOB_t > 0, \quad \text{and} \\
{}_tr &= \text{Borrowing rate if } {}^sOB_t < 0 .
\end{aligned}
$$

G. *Actuarial Assumptions*

A five-year plan of insurance is chosen. Premiums are payable at the beginning of the year, and amount to $200,000, $150,000, $140,000, and $130,000 in years 1, 2, 3, and 4, respectively. Pretax cash flow excludes investment income and occurs at the beginning of each year. Defined as premiums less expenses less benefits, pretax cash flow is −$172,640, $100,000, $80,000, $70,000, and −$22,000 in years 1, 2, 3, 4, and 5, respectively. Statutory reserves are $4,000, $15,000, $35,000, $40,000, and 0 at the end of years 1, 2, 3, 4, and 5, respectively. Year-end benchmark surplus is 3 percent of reserves plus 3 percent of premium. Pretax interest (${}_ti$) is 8 percent the first two years, 7 percent the next two years, and 6

[2] "Mathematical Analysis of Rates of Return under Certainty," *Management Science*, Vol. XI (January 1965), and "An Analysis of Criteria for Investment and Financing Decisions under Certainty," *Management Science*, Vol. XII (November 1965).

percent the last year. Pretax interest ($_rj$) is 6 percent in all years on benchmark surplus and on corporate surplus.

It is further assumed that all acquisition expenses are deferrable, so that GAAP profits are a uniform percentage of premium income. In practice, GAAP pretax profits are not a uniform percentage of premium, because some acquisition expenses are not deferrable.

Statutory federal income taxes are assumed to be 30 percent of statutory pretax earnings. GAAP (current plus deferred) federal income taxes also are assumed to be 30 percent of GAAP pretax earnings. Federal income taxes are assumed to be 50 percent of the interest earned on needed benchmark surplus and on corporate surplus on both the statutory and GAAP bases.

The financial comparisons shown in Table 1 were developed from the above data, using the formulas of the preceding sections. This development is shown in Table 2.

H. *Summary*

Formulas for ROTC and IRR have been developed on an after-tax basis, both including and excluding benchmark surplus. The important formulas are those that include benchmark surplus, because it is essential that benchmark surplus be included in defining profitability. The formulas excluding benchmark surplus are displayed only for those who wish to determine the impact that benchmark surplus has on the IRR and ROTC.

Some people have criticized the use of an IRR as a profit-measuring rod because it is not applicable to those lines of business that do not have a statutory drain in the first policy year. When benchmark surplus is included in the IRR calculation, one finds that formula (38) can be used to calculate an IRR for all lines of business. However, a statutory IRR that may appear to be reasonable for a new product should be converted to an unlevel ROTC on a GAAP basis to see if the yield pattern is also satisfactory.

As mentioned at the beginning of this paper, the job of management is to manage capital. This paper has developed tools for calculating the contribution that new business is expected to make to the GAAP return on total capital in the future. It also relates the unlevel GAAP returns to the statutory internal rate of return.

TABLE 2

FINANCIAL DATA

YEAR	STATUTORY DATA								
	Given				Derived				
	$_{t}i$	$_{t}P$	$^{s}_{t}\bar{B}$	$^{s}_{t}R$	$^{1}_{t}B(4)$	$^{2}_{t}E(6)$	$^{s}_{t}T^{*}$	$^{1}_{t}E'(8)$	$^{s}_{t}X'(10)$
1	8%	$200,000	−$172,640	$ 4,000	−$176,344	−$176,344	−$52,903	−$123,441	$123,441
2	8	150,000	100,000	15,000	90,111	97,320	29,196	68,124	− 68,124
3	7	140,000	80,000	35,000	62,290	66,650	19,995	46,655	− 46,655
4	7	130,000	70,000	40,000	67,617	72,350	21,705	50,645	− 50,645
5	6	0	− 22,000	0	18,000	19,080	5,724	13,356	− 13,356
Total	.....	$620,000	$ 55,360	.....	$ 61,674	$ 79,056	$23,717	$ 55,339	−$ 55,339

NOTE.—Numbers in parentheses indicate the formula used.

* Additional assumptions: $_{t}T$ = 30% of $_{t}E$; $^{g}_{t}T$ = 30% of $^{g}_{t}E$; $^{b}_{t}T$ = 50% of $^{b}_{t}E$ (i.e., $_{t}f$ = 50%); $^{s}_{t}S'$ = 3% of $_{t}P$ + 3% of $^{s}_{t}R$.

TABLE 2—*Continued*

Year	GAAP Data					
	${}^{G}_{t}B(13)$†	${}^{G}_{t}E(17)$	${}^{G}_{t}S(14)$	${}^{G}_{t}T$*	${}^{G}_{t}E'(19)$	${}^{G}_{t}S'(20)$
1	$10,000	$24,907	$201,251	$ 7,472	$17,435	$140,876
2	7,500	24,200	128,131	7,260	16,940	89,692
3	7,000	16,459	77,940	4,938	11,521	54,558
4	6,500	12,410	18,000	3,723	8,687	12,600
5	0	1,080	0	324	756	0
Total	$31,000	$79,056	.....	$23,717	$55,339	.....

Year	Benchmark Surplus Data					
	Given		Derived			
	i	${}^{b}_{t}S'$*	${}^{b}_{t}E(32)$	${}^{b}_{t}T$*	${}^{b}_{t}E'(33)$	${}^{b}_{t}X'(31)$
1	6%	$6,120	$ 357	$178	$179	$5,941
2	6	4,950	367	184	183	− 1,353
3	6	5,250	297	149	148	152
4	6	5,100	315	157	158	− 308
5	6	0	306	153	153	− 5,253
Total	.....	.....	$1,642	$821	$821	−$ 821

NOTE.—Numbers in parentheses indicate the formula used.

* Additional assumptions: ${}^{s}_{t}T$ = 30% of ${}^{s}_{t}E$; ${}^{G}_{t}T$ = 30% of ${}^{G}_{t}E$; ${}^{b}_{t}T$ = 50% of ${}^{b}_{t}E$ (i.e., ${}_{t}f$ = 50%); ${}^{b}_{t}S'$ = 3% of ${}_{t}P$ + 3% of ${}^{s}_{t}R$.

† k = 5% from formula (12).

DISCUSSION OF PRECEDING PAPER

HENRY B. RAMSEY, JR.:

I found Mr. Sondergeld's paper to be very interesting and amazingly consistent with the manner of profit analysis now in use in my company. We use the term "required surplus" for what Mr. Sondergeld refers to as "benchmark surplus," but the meaning seems to be essentially the same. Each company is "required" to carry the amount of "required surplus" and it is treated in the same manner as statutory reserves in determining book profits.

We are in the early stages of using a form of financial statement based upon GAAP principles, which we call "the management basis statement." As a mutual life insurance company, we have had to develop our own definitions of terms to be consistent with the principles applicable to the GAAP statement for stock life companies. Not being constrained by the same external requirements, we have exercised some latitude in the method of calculating contract reserves. Contract reserves are calculated as the statutory reserve less the present value of future after-tax statutory earnings, treating the required surplus as a reserve. Thus, using the example in Table 1 of the paper, our management basis statement reserve would be equal to the statutory reserve plus benchmark surplus less the present value of the amounts shown on lines 6, 10, 14, 18, and 22 of column *d*. The interest rate used in the present value calculation would be an appropriate cost of capital, which for this example might be 12 percent. An additional modification is that the present value at issue of that stream, including the initial investment shown on line 2, is spread over the life of the contract in proportion to premium income. This is a relatively simple way to calculate the management basis statement reserve and has some particular advantages from the standpoint of the incidence of earnings.

For this example, I have calculated the following "surplus after transfer" amounts (col. *h* of Table 1) at the end of years 0–4, respectively: $129,382, $145,760, $94,413, $59,834, and $16,615. The corresponding "after-tax earnings" amounts (col. *h*) for years 1–5, respectively, are: $16,378, $18,129, $11,925, $7,734, and $1,994. These amounts are equal to 12 percent of the beginning-of-the-year capital plus 0.426 percent of annual premiums.

It is of interest to note the values of the "GAAP ROTC" as defined in the next-to-the-last paragraph of Section IV(C) of your paper. The percentages are as follows for years 1–5, respectively: 12.7, 12.4, 12.6, 12.9, and 12.0 percent. This technique produces quite level percentages of return and thus no special interpretation is needed in order to relate the rate of return demonstrated in the total financial statement to the expected rate of return for the in-force products.

We apply this technique to participating products by using experience assumptions consistent with the experience assumptions underlying the current dividend scale determination, treating current dividends as a disbursement in the book profit calculation. This technique is consistent with that now proposed for indeterminate premium products.

(AUTHOR'S REVIEW OF DISCUSSION)

DONALD R. SONDERGELD:

I would like to thank Mr. Ramsey for his comments. In general, I like his technique of working directly with total after-tax statutory earnings, which include after-tax earnings on benchmark (required) surplus, and redistributing those earnings for management reporting purposes.

Mr. Ramsey's approach can be illustrated using the following formulas:

(41) ${}_tc$ = Cost-of-capital rate in year t.

(42) PV = Present value of surplus transfers, discounted at the cost-of-capital rate

$$= {}_1^sX' + {}_1^bX' + \sum_{t=2}^{m} ({}_t^sX' + {}_t^bX') \prod_{r=1}^{t} (1 + {}_rc)^{-1} .$$

(43) k' = Uniform ratio of PV to the present value of premiums, also discounted at the cost-of-capital rate

$$= PV \Big/ \left[\sum_{t=1}^{n} {}_tP \prod_{s=0}^{t-1} (1 + {}_sc)^{-1} \right] .$$

(44) ${}_t^RS'$ = Ramsey after-tax surplus (insurance surplus and benchmark surplus combined)

$$= {}_1^RE' - {}_1^sX' - {}_1^bX' \quad \text{for } t = 1$$

$$= {}_{t-1}{}_1^RS' - {}_t^sX' - {}_t^bX' + {}_t^RE' \quad \text{for } t > 1 .$$

(45) ${}_t^RE'$ = Ramsey after-tax earnings

$$= (1 + {}_1c)(k')({}_1P) + ({}_1c)({}_1^sX' + {}_1^bX') \quad \text{for } t = 1$$

$$= (1 + {}_tc)(k')({}_tP) + ({}_tc)({}_{t-1}{}_1^RS') \quad \text{for } t > 1$$

$$(46)\quad {}_{t}^{R}\mathrm{ROTC} = {}_{t}^{R}E'/{}_{t-1}^{R}S' = \frac{(1 + {}_{t}c)(k')({}_{t}P)}{{}_{t-1}^{R}S'} + {}_{t}c\ .$$

Mr. Ramsey used 12 percent for ${}_{t}c$, his cost-of-capital assumption. Had he used the IRR rate of 12.6 percent for ${}_{t}c$, then k' would be zero, rather than 0.426/1.12 percent. This is the internal rate of return method of accounting (IRRMA) described in my earlier paper (*TSA*, XXVI, 617).

As chief actuary for a stock life insurance company, I must use GAAP and envy the latitude Mr. Ramsey has at a mutual company. For example, under GAAP, deferred taxes are undiscounted, or discounted at 0 percent. Mr. Ramsey, in effect, discounts deferred taxes using the cost-of-capital rate. Also, under GAAP, we generally are not able to capitalize all acquisition expenses, an accounting restraint not imposed on mutual companies.

I wonder what Mr. Ramsey would do if the cost of capital were larger than the 12.6 percent IRR. Would the negative present value be written off the first year, or would it be spread over premiums? I suspect he might revise the cost-of-capital rate downward to a number like 12.6 percent or less.

It can easily be seen from the equation (46) that the ROTC will equal the cost-of-capital rate whenever the policy is paid up. Further, the ROTC will be very close to the IRR if ${}_{t}c$ is close to the IRR—which was the case in Mr. Ramsey's example.