MODELING MORTALITY RISK FROM EXPOSURE TO A POTENTIAL FUTURE EXTREME EVENT AND ITS IMPACT ON LIFE INSURANCE

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Abstract

This paper presents the modeling of mortality risk from exposure to a potential future extreme event such as a natural disaster and a terrorist attack, and its impact on life insurance. In addition to its uncertainty, a natural or man-made catastrophe has characteristics that it usually takes place suddenly and lasts only for a short period of time. The outbreak of the associated mortality assumes the same features. In the observation time window of a human's lifetime span, the duration of such an event is so short that it can be treated as an instantaneous case. Based on this concept, the mortality risk is modeled with an extra force of mortality consisting of two random variables—time of occurrence and severity of the sudden hazard. Both of the two parameters are independent of the variable of time to death without exposure to mortality risk. Therefore, the total force of mortality is obtained through superposition of the extra force of mortality and the force of mortality in absence of any tragedy. With this new model, quantities associated with the total force of mortality such as survival function are derived and the impact of mortality risk on life insurance is examined.

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1. Introduction

The existing mortality tables provide useful tools for life insurance and annuities, or other studies where mortality is involved (Bowers et al., 1997). However, the existing old tables are not enough for insurers since future mortality is controlled by many factors, such as socioeconomic factors, health conditions, environmental improvements, and so on (Rogers, 2002), and has been changing all the time. Therefore, insurance companies are exposed to the risk of future mortality uncertainty. Hence, the study of mortality risk becomes important for insurers to mitigate or avoid the associated potential adverse financial consequences.

Mortality rate exhibits an overall trend of a decreasing pattern (Berin et al., 1989; Friedland, 1998). This improvement of mortality is commonly believed to continue in the coming future (Friedland, 1998; Charlton, 1997; Goss et al., 1998), though researchers hold different views towards how mortality will improve. For instance, rectangularization and steady progress are currently the two main alternative views about future mortality improvement (Buettner, 2002). Mortality risk is usually referred to as the risk from the inaccuracy in forecasting future mortality improvements (Blake and W. Burrows, 2001). That it is defined this way is at least partly due to the fact that the study of mortality risk is mainly focused on projecting mortality trend and hedging the associated risk. A relatively rich body of literature can be found on modeling future mortality trend (Brass, 1971; Alho and Spencer, 1985; Lee and L. Carter, 1992; Congdon, 1993; Renshaw et al., 1996; Willets, 1999). Wong and Haberman (2004) have summarized the most important existing models during the past decade in the United Kingdom and the United States and

provided a comparative review of techniques and approaches used in modeling mortality rates.

A decreasing pattern is the overall trend of future mortality, and the study of the corresponding mortality risk is of great importance. However, on top of this main trend, there always exist some "noises" which may not be ignored. These "noises" have a variety of resources such as natural disasters, terrorist attacks, epidemic diseases, and so on. They can cause mortality rate to assume a extremely high value during a very short period of time, and can largely jeopardize an insurance company's profit and may even lead it to bankruptcy. For instance, the Mw 7.4 earthquake struck northwestern Turkey at 3:02 local time on August 17, 1999, and lasted for 45 seconds (Sansal, 2003). It left 19,118 dead and 40,000 injured, and caused total losses of about $20 billion, 10.10% of Turkey's GDP, with insured losses of approximately $2 billion (Swiss Re, 1999). The death rate in Izmit, the epicenter, was beyond 3%. Considering the short duration of the disaster, the force of mortality generated was almost infinitive. Another example is the September 11th terrorist attacks on World Trade Center in New York, which took place within hours and resulted in more than 3000 deaths. The insured losses from the attacks were $50 billion of which two thirds were expected to be paid by insurers (Warfel, 2003). A global example is the influenza pandemic of 1918-1919. It circled the globe, and killed more people than World War I, at somewhere between 20-40 million lives (Billings 1997). The influenza virus had a profound virulence, with a high mortality rate at 2.5% and an extremely high rate at 5% in India (Billings 1997). During October and November, the mortality rate in New York reached its highest value of more than 6%, the rates in Europe such as those in London, Paris, and Belgium also climbed to the peak values

around 5% (National Museum of Health and Medicine). There are also a plenty of other examples, such as Northridge Earthquake and Hurricane Andrew (Warfel, 2000). In addition to their extreme harmful nature, all these natural and man-made catastrophes have common characteristics that they take place all of a sudden, last only for a short period of time, and the resulted mortality rates are hard to predict. In a broad view, the associated risk from the uncertainty of mortality rate generated by such as a catastrophe also belongs to mortality risk. In contrast to the mortality risk from mortality improvements, particularly, the longevity risk, this kind of mortality risk is less studied, though some related work has been done to develop models for natural and man-made catastrophes, such as that done by Applied Insurance Research (AIR), Eqecat, and Risk Management Solutions (RMS) (*Economist,* 2002).

Undoubtedly, these extreme events will cause adverse impact on insurers, especially those who do not consider these potential risks they are exposed to. Therefore, it will be a significant work to investigate the associated mortality risks. To this end, this paper focuses on the study of future mortality under possible effects from tragedies such as terrorism, earthquakes, hurricanes, epidemic diseases, and so on, which have a short duration. In this paper, two things are considered most important in developing the model for mortality risk. The first is the time of occurrence of a sudden hazard. The other one is the severity of the hazard. As they are used to describe future uncertain things, these two parameters are assumed to be random variables, denoted by τ and ξ, respectively.

In the following sections, model developing will be first presented. Following this, the impact of mortality risk on life insurance will be examined based on this new model.

2. Modeling mortality risk from exposure to a sudden extreme event

As mentioned in the introduction, most of the extreme events break out abruptly and last only for a very short period of time. The mortality rate generated in a tragedy should also be a momentary sharp spike. Figure 1 presents the mortality rates from both influenza alone and all causes in Kansas during the 1918 influenza pandemic (US Census Bureau). From the figure it can be seen that the most serious increase in mortality occurs
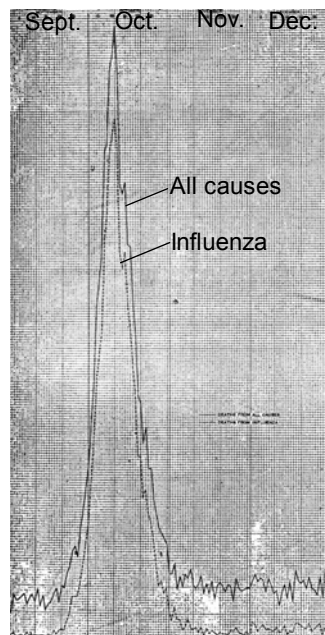

Figure 1: Mortality rates in Kansas during 1918 influenza pandemic.

within approximately one month. For most of other extreme cases the duration may be even far shorter (e.g., the 1999 Turkey earthquake sustained less than one minute). From an insurer's perspective, the exact profile of the associated force of mortality is no more important as long as the overall effect is known, since the duration of the extreme event is so short. In modeling the mortality risk, the sudden extreme event can be treated as an instantaneous case in the observation time window of a human's life span. This concept

is similar to the one employed in physics and engineering when dealing with "point" actions. They are named "point" actions because they are highly localized in space, or time, or both. The most frequently encountered examples in physics and engineering are point forces and couples, point masses, electrical charges, electrical pulses, and so on (Greenberg, 1978; Levan, 1992; Weertman, 1996).

Now assume that a disaster will occur at a future time $\tau \geq 0$ and will generate a force of mortality $\mu_1(t)$ as shown in Figure 2 a. In the reference of a human's life span (e.g., 100 years), the duration of the tragedy $\Delta \tau_1$ (e.g., one hour) is so short that the exact shape of the profile of $\mu_1(t)$ becomes unimportant. It makes almost no difference if another force of mortality with totally different profile, say, $\mu_2(t)$ in Figure 2 b, is generated instead of $\mu_1(t)$. The only requirement is that the total effect remains the same. That is,

$$\int_0^\infty \mu_1(t)dt = \int_0^\infty \mu_2(t)dt = \xi.$$

(1)

a: $\mu_1(t)$ $\tau \; \Delta\tau_1$ t

b: $\mu_2(t)$ $\tau \; \Delta\tau_2$ t
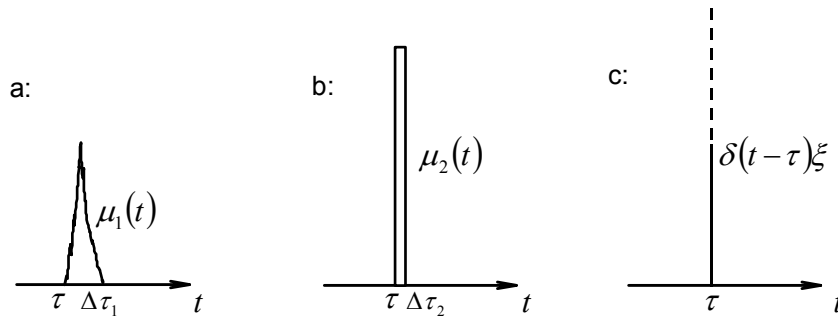
c: $\delta(t-\tau)\xi$ τ t

Figure 2: Modeling the extra force of mortality due to a disaster.

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When $\Delta\tau_2$ goes to zero the event converges to a limiting case, the instantaneous case. Hence, how the force of mortality changes with time can be ignored and there remain only two things to be addressed in the instantaneous case. The first one is the time of occurrence of the event. The other parameter is the severity that describes how severe the situation is. Such an instantaneous case can be well modeled by utilizing a special mathematic function, the delta function $\delta(t)$. The exact expression for the associated force of mortality in the instantaneous case is given by $\delta(t-\tau)\xi$, which has the following properties

$$\delta(t-\tau)\xi = \begin{cases} \infty & for \quad t = \tau \\ 0 & for \quad t \neq \tau \end{cases}, \tag{2}$$

and

$$\int_0^\infty \delta(t-\tau)\xi dt = \xi. \tag{3}$$

In equation (2), the force of mortality is nontrivial only at time τ, meaning that the extreme event appears instantaneously at time τ. This variable is defined as the time of occurrence. Equation (3) shows that the overall effect or impact remains ξ, which is referred to as the severity of the extreme event. The two parameters τ and ξ, together with the time-to-death variable t, are assumed to be independent, so the total force of mortality $\mu_x^*(t;\tau,\xi)$ is obtained by the following superposition

$$\mu_x^*(t;\tau,\xi) = \mu_x(t) + \delta(t-\tau)\xi. \tag{4}$$

In equation (4), the superscript "*" denotes quantity when mortality risk exists. This denotation also applies for other quantities in the following discussion. The first term

$\mu_x(t)$ on the right hand side of equation (4) is the force of mortality under normal situation and the other term $\delta(t - t_F)\mu_F$ is the extra force of mortality from exposure to mortality risk. Essentially, the total force of mortality with exposure to mortality risk is modeled as the sum of two parts, the force of mortality in absence of mortality risk and the extra force of mortality generated by the extreme event.

To continue for a future discussion, it is necessary to give a brief description on two special mathematic functions, the delta function $\delta(t)$ and Heaviside function $H(t)$. They are well suited for the study of "point" cases (referred to as instantaneous case in this paper, as time is involved). These functions are two of the mostly used functions in physics and engineering (Greenberg, 1978; Weertman, 1996). The definitions for the two functions are given below

$$\delta(t) = \begin{cases} \infty & for \quad t = 0 \\ 0 & for \quad t \neq 0 \end{cases}, \tag{5}$$

and

$$H(t) = \begin{cases} 1 & for \quad t \geq 0 \\ 0 & for \quad t < 0 \end{cases}. \tag{6}$$

The delta function is related to Heaviside function by

$$H(t) = \int_{-\infty}^{t} \delta(s)ds. \tag{7}$$

The above two special functions are good for occurrence at time zero. For an extreme event at time τ, the following revision is needed

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$$\delta(t-\tau) = \begin{cases} \infty & for \quad t = \tau \\ 0 & for \quad t \neq \tau \end{cases}, \tag{8}$$

$$H(t-\tau) = \begin{cases} 1 & for \quad t \geq \tau \\ 0 & for \quad t < \tau \end{cases}, \tag{9}$$

and
$$H(t-\tau) = \int_{-\infty}^{t} \delta(s-\tau)ds. \tag{10}$$

With the total force of mortality given in equation (4), the probability of life (x) surviving t years can be readily obtained

$$_t p_x^*(\tau,\xi) = \exp\left[-\int_0^t \mu_x^*(s;\tau,\xi)ds\right]. \tag{11}$$

Inserting equation (4) into equation (11) yields

$$_t p_x^*(\tau,\xi) = \exp\left[-\int_0^t (\mu_x(s) + \delta(s-\tau)\xi)ds\right], \tag{12}$$

or
$$_t p_x^*(\tau,\xi) = \exp\left[-\int_0^t \mu_x(s)ds\right]\exp\left[-\int_0^t \delta(s-\tau)\xi ds\right]. \tag{13}$$

Recalling the relation of the delta function and Heaviside function in equation (10) gives

$$_t p_x^*(\tau,\xi) = \exp\left[-\int_0^t \mu_x(s)ds\right]\exp[-H(t-\tau)\xi], \tag{14}$$

or
$$_t p_x^*(\tau,\xi) = {_t p_x} \exp[-H(t-\tau)\xi]. \tag{15}$$

where $_t p_x = \exp\left[-\int_0^t \mu_x(s)ds\right]$ is the probability of life (x) surviving t years without exposure to mortality risk.

Considering the fact that the extreme event may happen only at a future time of (x), equation (15) can be expressed in terms of survival function as

$$\frac{s^*(x+t;\tau,\xi)}{s(x)} = \frac{s(x+t)}{s(x)}\exp[-H(t-\tau)\xi],\tag{16}$$

or
$$s^*(x+t;\tau,\xi) = s(x+t)\exp[-H(t-\xi)\xi].\tag{17}$$

Similarly, $l_{x+t}^*(\tau,\xi)$, the number of people surviving age $x+t$, can be found to be

$$l_{x+t}^*(\tau,\xi) = l_{x+t}\exp[-H(t-\tau)\xi].\tag{18}$$

If the cohort of people aged 0 is l_0, then the remaining number of people at age $x+t$ is

$$l_{x+t}^*(\tau,\xi) = l_o \exp[-\mu_x(t) - H(t-\tau)\xi].\tag{19}$$

For example, if the original force of mortality $\mu_x(t)$ in absence of mortality risk is constant μ, then there is

$$l_{x+t}^*(\tau,\xi) = l_o \exp[-(x+t)\mu - H(t-\tau)\xi].\tag{20}$$

By equation (19), death rate r due to the disaster can be easily achieved

$$r = 1 - \exp(-\xi),\tag{21}$$

or
$$\xi = \ln\left(\frac{1}{1-r}\right). \tag{22}$$

To better show how this model depicts the impact of a future tragedy on a population, a specific example will be taken. Consider a cohort of 100 people aged 0 subject to a constant force of mortality $\mu_x(t) = 0.02$. Without exposure to risk, the number of survivors decreases with time exponentially from 100 to 0, as indicated by the light thick line in Figure 3. If a disaster occurs at some future time τ with severity ξ, then the number of population changes with time differently. Consider two cases, with $\tau = 20$, $\xi = 0.2$ in case 1, and $\tau = 40$, $\xi = 2$ in case 2. From Figure 3, it can be seen that no matter which case, as long as a disaster appears, there is an instantaneous drop in the
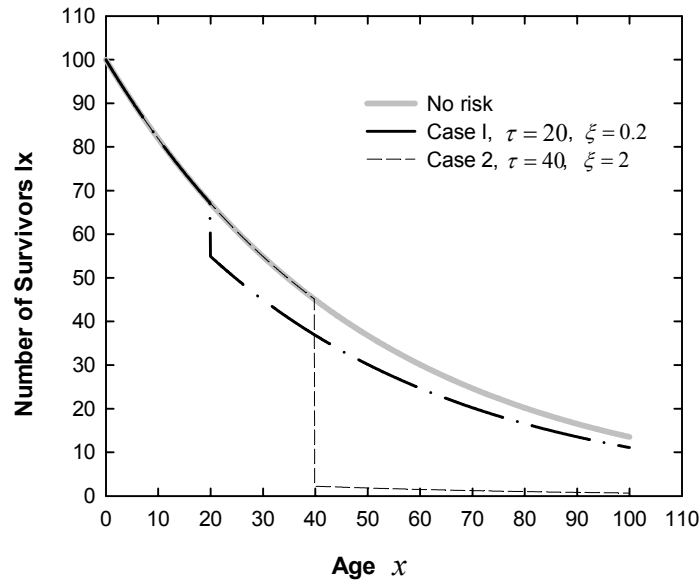

Figure 3: Impact of a disaster on a population.

population at time τ, which is consistent with our understanding of disasters. For the first case the sudden decrease in the population is approximately 12.15, as indicated by the dash-dot line. It can be examined that about 38.85 people lose their lives in the second case, as shown by the dash line. Numbers in the two cases are dramatically different, which is due to the different severity of the disasters, indicated by the different values of ξ. When severity ξ assumes a value of zero, it means that nothing will happen in the future. When ξ goes to infinitive, it means that nobody can survive the tragedy. With the model for the force of mortality under influence of risk, its impact on life insurance and annuities can be studied, and it will be given in the next section.

3. Investigating the impact of mortality risk on life insurance

With the force of mortality known, the impact of the extreme event on various life insurances and annuities can be investigated. Without lost of generality, the effect on continuous whole life insurance and annuity are examined in this paper. It is straight forward that the actuarial present value for a whole life insurance $\overline{A}_x^*(\tau,\xi)$ can be obtained using the following formula

$$\overline{A}_x^*(\tau,\xi) = \int_0^\infty v^t \, _t p_x^*(\tau,\xi)\mu_x^*(t;\tau,\xi)dt, \tag{23}$$

where v is the discount rate; $\mu_x^*(t;\tau,\xi)$ and $_t p_x^*(\tau,\xi)$ are given in equations (4) and (15), respectively. Plugging equations (4) and (15) into equation (23) leads to

$$\overline{A}_x^*(\tau,\xi) = \int_0^\infty v_{t}^{t}\, p_x \exp[-H(t-\tau)\xi]$$
$$\times [\mu_x(t) + \delta(t-\tau)\xi]dt. \tag{24}$$

Using the properties of the delta function and rearranging equation (24) gives

$$\overline{A}_x^*(\tau,\xi) = \int_0^\tau v_{t}^{t}\, p_x \mu_x(t)dt + \int_\tau^\infty v_{t}^{t}\, p_x \exp(-\xi)\mu_x(t)dt$$
$$+ \int_0^\infty v_{t}^{t}\, p_x \exp[-H(t-\tau)\xi]\delta(t-\tau)\xi dt, \tag{25}$$

Because $\delta(t-\tau) = 0$ on $(0,\tau^-) \cup (\tau^+, \infty)$, equation (25) is the same as

$$\overline{A}_x^*(\tau,\xi) = \int_0^\tau v_{t}^{t}\, p_x \mu_x(t)dt + \int_\tau^\infty v_{t}^{t}\, p_x \exp(-\xi)\mu_x(t)dt$$
$$+ \int_{\tau^-}^{\tau^+} v_{t}^{t}\, p_x \exp[-H(t-\tau)\xi]\delta(t-\tau)\xi dt. \tag{26}$$

The term $v_{t}^{t}\, p_x$ is continuous function, so it is equal to a constant $v_{\tau}^{\tau}\, p_x$ over (τ^-, τ^+). Hence, equation (24) becomes

$$\overline{A}_x^*(\tau,\xi) = \int_0^\tau v_{t}^{t}\, p_x \mu_x(t)dt + \int_\tau^\infty v_{t}^{t}\, p_x \exp(-\xi)\mu_x(t)dt$$
$$+ v_{\tau}^{\tau}\, p_x \int_{\tau^-}^{\tau^+} \exp[-H(t-\tau)\xi]\delta(t-\tau)\xi dt. \tag{27}$$

Recalling the fact that the derivative of $H(t-\tau)$ over t is $\delta(t-\tau)$ leads equal (27) to

$$\bar{A}_x^*(\tau,\xi) = \int_0^\tau v^t \, {}_t p_x \mu_x(t) dt + \int_\tau^\infty v^t \, {}_t p_x \exp(-\xi) \mu_x(t) dt$$

$$- v^\tau \, {}_\tau p_x \exp[-H(t-\tau)\xi]_{\tau^-}^{\tau^+} ,$$

(28)

or

$$\bar{A}_x^*(\tau,\xi) = \int_0^\tau v^t \, {}_t p_x \mu_x(t) dt + \int_\tau^\infty v^t \, {}_t p_x \exp(-\xi) \mu_x(t) dt$$

$$+ v^\tau \, {}_\tau p_x [1 - \exp(-\xi)] .$$

(29)

In terms of actuarial present values, equation (29) can be expressed as

$$\bar{A}_x^*(\tau,\xi) = \bar{A}_{x:\overline{\tau|}}^1 + \exp(-\xi) v^\tau \, {}_\tau p_x \bar{A}_{x+\tau}$$

$$+ v^\tau \, {}_\tau p_x [1 - \exp(-\xi)]$$

(30)

The first term on the right hand side of equation (30) is the actuarial present value for a τ year term insurance that is not affected by the mortality risk. The second term on the right hand side is a τ year-deferred whole life insurance, which has a coefficient $\exp[-\xi]$ due to the fact that there are less people alive right after the extreme event. The last term on the right hand side accounts for people who died from the disaster at time τ.

Similarly, the actuarial present value of a continuous whole life annuity can be obtained through the same procedure

$$\bar{a}_x^*(\tau,\xi) = \int_0^\tau v^t \, {}_t p_x^*(\tau,\xi) dt ,$$

(31)

or

$$\bar{a}_x^*(\tau,\xi) = \int_0^\infty v^t \, {}_t p_x \exp[-H(t-\tau)\xi] dt .$$

(32)

If there is a constant force of interest δ and a constant force of mortality $\mu_x(t) = \mu$, then equations (29) and (32) can be simplified as

$$\overline{A}_x^*(\tau,\xi) = \frac{\mu}{\delta+\mu} + \frac{\delta}{\delta+\mu}\exp\left[-(\delta+\mu)\tau\right]$$
$$\times\left[1 - \exp(-\xi)\right], \tag{33}$$

and
$$\overline{a}_x^*(\tau,\xi) = \frac{1}{\delta+\mu} - \frac{1}{\delta+\mu}\exp\left[-(\delta+\mu)\tau\right]$$
$$\times\left[1 - \exp(-\xi)\right]. \tag{34}$$

Given δ and μ, the actuarial present values of both the continuous whole life insurance and annuity depend solely on the two variables τ and ξ of the future extreme event. It can be understood that the earlier and the more severe a disaster is, the larger the actuarial present value of the whole life insurance should be. The reason is due to the time value of money and the fact that the more severe a potential disaster is, the larger the probability is for a person to die. On the contrary, it goes in the opposite direction for the whole life annuity. Equations (33) and (34) are consistent with this understanding as they satisfy the following constrains

$$\begin{cases} \dfrac{\partial}{\partial\tau}\overline{A}_x^*(\tau,\xi) \leq 0 \\[2mm] \dfrac{\partial}{\partial\xi}\overline{A}_x^*(\tau,\xi) \geq 0 \end{cases}, \tag{35}$$

and
$$\begin{cases} \dfrac{\partial}{\partial\tau}\overline{a}_x^*(\tau,\xi) \geq 0 \\[2mm] \dfrac{\partial}{\partial\xi}\overline{a}_x^*(\tau,\xi) \leq 0 \end{cases}. \tag{36}$$

It is also shown graphically in Figure 4 how a future hazard affects the continuous whole life insurance and annuity for the case $\delta = 0.06$ and $\mu = 0.04$. Without the risk of a disaster the actuarial present values of the whole life insurance and annuity are $\overline{A}_x = 0.4$ and $\overline{a}_x = 10$, respectively. It is shown in Figure 4 that $\overline{A}_x^*(\tau,\xi)$ increases with ξ, decreases with ξ, and has an upper bound of 1 and a lower bound of 0.4. On the contrary hand, the adverse event causes $\overline{a}_x^*(\tau,\xi)$ to decrease with τ and increase with ξ.
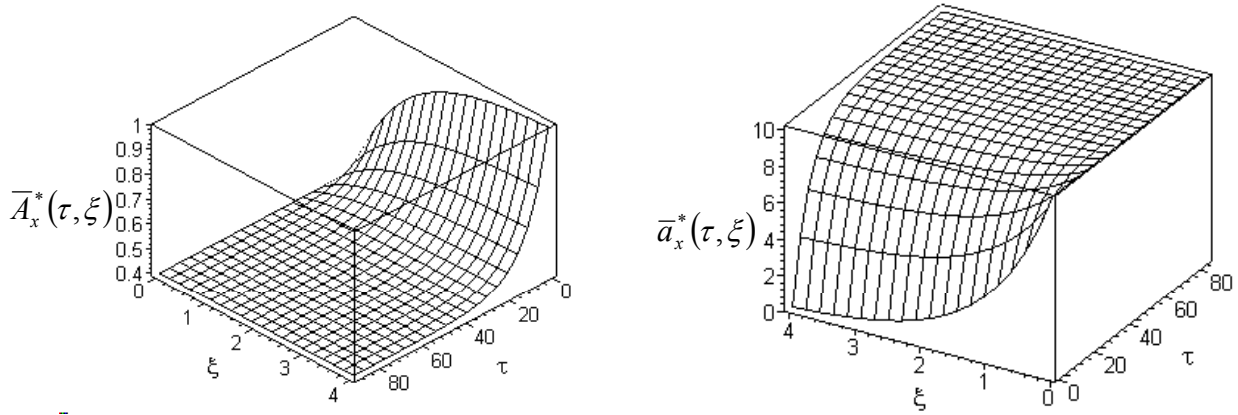

Figure 4: 3D plots for $\overline{A}_x^*(\tau,\xi)$ and $\overline{a}_x^*(\tau,\xi)$

In reality, it is usually uncertain when an extreme event will take place and how treacherous it will be. Hence, $\overline{A}_x^*(\tau,\xi)$ is essentially randomly distributed since its two parameters τ and ξ are random variables. Its expectation can be obtained through simulation or analytically solved if a distribution is given. Assume that τ and ξ are independent and follow distributions $f(\tau)$ and $g(\xi)$, respectively, then the expectation of $\overline{A}_x^*(\tau,\xi)$ can be found by the following integration

$$\overline{A}_x^* = E\left(\overline{A}_x^*(\tau,\xi)\right)$$
$$= \int_0^\infty \int_0^\infty \overline{A}_x^*(\tau,\xi) f(\tau) g(\xi) d\tau d\xi, \tag{37}$$

where $\overline{A}_x^*$ denotes the expected present actuarial value for the continuous whole life insurance $\overline{A}_x^*(\tau,\xi)$.

The expectation of the actuarial present value of whole life annuity $\overline{a}_x^*$ is obtained by

$$\overline{a}_x^* = E\left(\overline{a}_x^*(\tau,\xi)\right)$$
$$= \int_0^\infty \int_0^\infty \overline{a}_x^*(\tau,\xi) f(\tau) g(\xi) d\tau d\xi. \tag{38}$$

If an insurer ignores the mortality risk, then the premium set for a whole life insurance is given by

$$\overline{P}\left(\overline{A}_x\right) = \frac{\overline{A}_x}{\overline{a}_x}, \tag{39}$$

where $\overline{A}_x$ and $\overline{a}_x$ are respectively the actuarial present values of continuous whole insurance and annuity without considering mortality risk; $\overline{P}\left(\overline{A}_x\right)$ the associated premium. However, if there exits exposure to mortality risk, loss occurs for the insurer who ignores it, which is

$$L^* = \overline{A}_x^* - \overline{P}\left(\overline{A}_x\right)\overline{a}_x^*. \tag{40}$$

If $\mu_x(t)$ is a constant μ, force of interest rate is δ, and τ and ξ follow exponential distributions $\beta\exp[-\beta\tau]$ and $\gamma\exp[-\gamma\xi]$, respectively, then, from equations (37) and (38) there are

$$\overline{A}_x^* = \int_0^\infty \int_0^\infty \overline{A}_x^*(\tau,\xi)\beta\exp(-\beta\tau)\gamma\exp[-\gamma\xi]d\tau d\xi, \tag{41}$$

and

$$\overline{a}_x^* = \int_0^\infty \int_0^\infty \overline{a}_x^*(\tau,\xi)\beta\exp(-\beta\tau)\gamma\exp[-\gamma\xi]d\tau d\xi. \tag{42}$$

Finally, utilizing equations (33) and (34) gives

$$\overline{A}_x^* = \frac{\mu}{\mu+\delta} + \frac{\beta\delta}{(1+\gamma)(\delta+\mu)(\delta+\mu+\beta)}, \tag{43}$$

and

$$\overline{a}_x^* = \frac{1}{\mu+\delta} - \frac{\beta}{(1+\gamma)(\delta+\mu)(\delta+\mu+\beta)}. \tag{44}$$

In equation (43) or equation (44), the second term on the right hand side presents the imposed effect by a potential disaster. The two random variables τ and ξ may have other distinct distributions for different future extreme events. How to obtain the individual distributions for these two variables is beyond the scope of this paper and will not be further discussed. In what follows, a numerical example, though not realistic, will be provided as a vehicle to show how to this model works.

Suppose that a potential influenza or SAS may break out in a certain place. The variable τ follows an exponential distribution with mean $E(\tau)=5$ years. Severity ξ also follows exponential distribution with mean $E(\xi)=0.05$ ($\xi=0.05$ corresponds to a death

rate $r = 4.9\%$ by equation (21), which appeared in the 1918 influenza pandemic). Thus, there is $\beta = 0.2$ and $\gamma = 20$. Also, assume that force of interest is $\delta = 0.06$ and the force of mortality in absence of mortality risk is $\mu_x(t) = 0.02$. Then, equations (43) and (44) gives $\overline{A}_x^* = 0.2755$ and $\overline{a}_x^* = 12.0748$. It is easy to obtain the actuarial present values for whole life insurance and annuity without mortality risk. They are given directly $\overline{A}_x = 0.2500$ and $\overline{a}_x = 12.5000$. By equation (39), the corresponding premium is $\overline{P}(\overline{A}_x) = 0.0200$. Therefore, according to equation (40), the loss is $L^* = 0.0340$ for each policy with \$1 benefit. If a company writes 10000 such policies with a benefit of \$100,000 for each one, and it ignores the potential mortality risk that its policy holders are exposed to, then the total loss is \$34 million.

4. Conclusion

In this paper mortality risk from exposure to a potential future sudden extreme event has been studied by modeling the event as a "point" case. Two important variables, time of occurrence τ and severity ξ of a potential future disaster, have been introduced in the model. These two random variables and the time-to-death random variable t in absence of mortality risk are assumed to be independent. By this way, the overall mortality is the sum of the force of mortality in absence of risk and the extra force of mortality $\delta(t - \tau)\xi$ caused by the disaster appearing at instant τ. The associated survival function and the number of population with exposure to mortality risk have also been examined. Formulae for the actuarial present values of continuous whole life insurance and annuity have been obtained. With this new mortality risk model, a numerical case has

also been given. It has been shown that if an insurer ignores the potential mortality risk that its policyholders are exposed to, it may lose money.

The model developed in this paper is for the study of mortality risk from the exposure to a potential future sudden extreme event. The event is not limited to terrorist attacks, earthquakes, or influenza pandemics. It is a general model for any natural or man-made disasters as long as they can be treated as "point" cases (or an instantaneous cases). In other words, the resulted deaths in the event of consideration should take place within a relatively short time in the reference of a human's lifetime span. When using this model to study life insurance and annuity, the exact profile of the force of mortality resulted from the extreme event can be ignored. However, information about τ and ξ is indispensable.

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