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## PRICING IN A RETURN-ON-EQUITY ENVIRONMENT

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#### Abstract

Return on invested statutory surplus (ROI) is a pricing objective used frequently within product development areas of stock life insurance companies. ${ }^{1}$ This pricing objective implies that statutory surplus is the limiting resource within a company. ${ }^{2}$ Return on equity (ROE), defined here as generally accepted accounting principles (GAAP) income divided by prior year's ending GAAP equity, is a corporate profitability measure often used within the insurance industry today. ${ }^{3}$ Additionally, growth in the amount of new business produced is a corporate objective found in many insurance companies' strategic plans. This paper will use a simple example to examine the relationship between the pricing objective of ROI and the corporate profit measure of ROE. The example also will be used to examine the relationship between ROE and after-tax ROE. Additionally, the paper will examine the effect on ROI and ROE of growth in the amount of new business; current and past accounting practices; reinvestment of retained earnings and stockholder dividend practices; and the age of the existing block of business as well as its size in relation to the amount of new business to be produced.


## I. EXAMPLE

A simple example will illustrate the complexity of attempting to manage corporate objectives. The following example assumes a new company is capitalized with $\$ 10$ million on December 31, the last day of its accounting period. It has one product to sell, and it uses the entire $\$ 10$ million to acquire a block of business (on December 31) that was priced to return the initial investment plus an annual pretax return on its investment of 15 percent. The company's financial position, both before and after the acquisition of the block of business, is shown in table 1 .

[^0]TABLE 1
Initial Financial Position
December 31, Year 00

| 12-31-00 | Before Acquisition of Block of Business ( 000 's omitted) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Statutory <br> Surplus | GAAP <br> Equity | Investable Assets | DAC* |
|  | \$10,000 | \$10,000 | \$10,000 | \$0 |
| 12-31-00 | After Acquisition of Block of Business (000's omitted) |  |  |  |
|  | Statuory Supplus | GAAP <br> Equity | Investable <br> Assets | DAC* |
|  | \$ 0 | \$10,000 | \$ 0 | \$10,000 |

*Deferred acquisition cost (DAC) in this and all subsequent examples will represent all Statutory-to-GAAP pretax differences, that is, DAC (traditional definition) + (Statutory Reserves - GAAP Bencfit Reserves).

Statutory Book Profits ${ }^{4}$ (at the end of the year) in this simple example will be $\$ 1,992,520$ annually.

$$
1,992,520 a_{\left.\overline{10}\right|_{15 \%}}=10,000,000
$$

Thus, the 15 percent pretax return on the initial investment is assured. Obviously the emergence of statutory profit differs greatly from product to product and company to company. This simple emergence of earnings nonetheless will help illustrate the difficulty involved in managing corporate objectives.

As the introduction of federal income tax adds even more complexity, the examples that follow are all pretax. Later examples will illustrate the effect of federal income tax. Table 2 shows the emergence of earnings, assuming a 10 percent pretax investment earnings rate and the amortization of the DACs, using interest only, at 10 percent annually.

## Formulas

$$
\begin{aligned}
& \text { Stat Surplus }_{t}=\text { Stat Surplus }_{t-1}+\text { Stat Income }_{t}-\text { Dividend }_{t} \\
& \text { GAAP Equity }_{t}=\text { GAAP Equity }_{t-1}+\text { GAAP Income }_{t}-\text { Dividend }_{t} \\
&{\text { Stat } \text { Income }_{t}}=1,992.52+i\left(\text { Stat Surplus }_{t-1}\right)
\end{aligned}
$$

[^1]PRICING IN A RETURN-ON-EQUITY ENVIRONMENT

TABLE 2
Statutory Earnings Reinvested at Ten Percent Pretax Rates Dollar Amounts in Thousands
$\mathrm{ROI}=15 \%, i=10 \%, g=10 \%)$

| Year <br> Ending | Statutory <br> Surplus | GAAP <br> Equity | Statutory <br> Income | DAC | GAAP <br> Income | Dividends | ROE |
| :---: | ---: | ---: | :---: | ---: | ---: | :---: | :---: |
| $12-31-00$ | $\$ r$ | 0.00 | $\$ 10,000.00$ | - | $\$ 10,000.00$ | - | - |
| $12-31-01$ | $1,992.52$ | $11,365.07$ | $\$ 1,992.52$ | $9,372.55$ | $\$ 1,365.07$ | $\$ 0.00$ | $13.65 \%$ |
| $12-31-02$ | $4,184.29$ | $12,866.64$ | $2,191.77$ | $8,682.35$ | $1,501.57$ | 0.00 | 13.21 |
| $12-31-03$ | $6,595.24$ | $14,518.37$ | $2,410.95$ | $7,923.13$ | $1,651.73$ | 0.00 | 12.84 |
| $12-31-04$ | $9,247.29$ | $16,335.27$ | $2,652.04$ | $7,087.99$ | $1,816.90$ | 0.00 | 12.51 |
| $12-31-05$ | $12,164.54$ | $18,333.87$ | $2,917.25$ | $6,169.33$ | $1,998.59$ | 0.00 | 12.23 |
| $12-31-06$ | $15,373.51$ | $20,532.32$ | $3,208.97$ | $5,158.81$ | $2,198.45$ | 0.00 | 11.99 |
| $12-31-07$ | $18,903.38$ | $22,950.62$ | $3,529.87$ | $4,047.24$ | $2,418.30$ | 0.00 | 11.78 |
| $12-31-08$ | $22,786.24$ | $25,610.75$ | $3,882.86$ | $2,824.51$ | $2,660.13$ | 0.00 | 11.59 |
| $12-31-09$ | $27,057.39$ | $28,536.89$ | $4,271.14$ | $1,479.50$ | $2,926.14$ | 0.00 | 11.43 |
| $12-31-10$ | $31,755.65$ | $31,755.65$ | $4,698.26$ | 0.00 | $3,218.76$ | 0.00 | 11.28 |

NOTE: ROI represents return on investment
$i$ represents pretax investment earnings rate
$g$ represents amortization rate for deferred acquisition costs
where $i$ is the investment earnings rate.

$$
D A C_{t}=D A C_{t-1}-\frac{\left(D A C_{0}\right) g\left[1 /(1+g)^{11-t}\right]}{1-[1 /(1+g)]^{10}}
$$

where $g$ is the interest rate used to generate the DAC amortization schedule.

$$
\begin{aligned}
\text { GAAP Income }_{t} & =\text { Stat Income }_{t}-D A C_{t-1}+D A C_{t} \\
R O E & =\text { GAAP Income } / \text { GAAP Equity }
\end{aligned}
$$

The ROE starts at 13.65 percent and falls to 11.28 percent after ten years. This occurs for two reasons:

1. Each year's statutory income is reinvested at 10 percent, thus bringing the ROE down progressively through the years.
2. The DAC, part of the GAAP equity, is assumed to be earning only 10 percent. Therefore, more is amortized in year one than would be amortized if the amortization schedule used a 15 percent interest rate assumption. This is the reason the ROE in year one is below 15 percent. This will turn around in later years, resulting in a ROE in those years greater than 15 percent (see Table 3).
Table 3 eliminates the problem discussed in (1). Dividends, either paid to stockholders or moved to another line of business where they can be reinvested effectively, equal to the statutory income for that year, are paid
in the year they are earned. Statutory surplus remains zero, and GAAP equity is equal to the unamortized deferred acquisition cost. The ROE starts below 15 percent, since the DAC is "earning" less than 15 percent, but turns greater than 15 percent in the fifth year, reflecting the lesser expense to be amortized in the later years and the resulting lower GAAP equity.

Table 4 illustrates the effect of amortizing the DAC using an amortization schedule with an inherent 15 percent interest rate assumption. In this example, however, dividends are not paid but are reinvested at an investment earnings rate of 10 percent. The ROE in year one has increased to 15 percent, reflecting the lower amount of amortization occurring in year one. However,

TABLE 3
Dividends Equal Statutory Earnings Dollar Amounts in Thousands
$(\mathrm{ROI}=15 \%, g=10 \%)$

| Year <br> Ending | Statutory <br> Surplus | GAAP <br> Equity | Statutory <br> Income | DAC | GAAP <br> Income | Dividends | ROE |
| :---: | :---: | ---: | :---: | ---: | :---: | :---: | :---: |
| $12-31-00$ | $\$ 0.00$ | $\$ 10,000.00$ | - | $\$ 10,000.00$ | - | - | - |
| $12-31-01$ | 0.00 | $9,372.55$ | $\$ 1,992.52$ | $9,372.55$ | $\$ 1,365.07$ | $\$ 1,992.52$ | $13.65 \%$ |
| $12-31-02$ | 0.00 | $8,682.35$ | $1,992.52$ | $8,682.35$ | $1,302.32$ | $1,992.52$ | 13.90 |
| $12-31-03$ | 0.00 | $7,923.13$ | $1,992.52$ | $7,923.13$ | $1,233.30$ | $1,992.52$ | 14.20 |
| $12-31-04$ | 0.00 | $7,087.99$ | $1,992.52$ | $7,087.99$ | $1,157.38$ | $1,992.52$ | 14.61 |
| $12-31-05$ | 0.00 | $6,169.33$ | $1,992.52$ | $6,169.33$ | $1,073.87$ | $1,992.52$ | 15.15 |
| $12-31-06$ | 0.00 | $5,158.81$ | $1,992.52$ | $5,158.81$ | 982.00 | $1,992.52$ | 15.92 |
| $12-31-07$ | 0.00 | $4,047.24$ | $1,992.52$ | $4,047.24$ | 880.95 | $1,992.52$ | 17.08 |
| $12-31-08$ | 0.00 | $2,824.51$ | $1,992.52$ | $2,824.51$ | 769.79 | $1,992.52$ | 19.02 |
| $12-31-09$ | 0.00 | $1,479.50$ | $1,992.52$ | $1,479.50$ | 647.52 | $1,992.52$ | 22.92 |
| $12-31-10$ | 0.00 | 0.00 | $1,992.52$ | 0.00 | 513.02 | $1,992.52$ | 34.67 |

TABLE 4
Amortizing Deferred Acquisition Costs at Initinl Investment R^te Dollar Amounts in Thousands (ROI $=15 \%, g=15 \%, i=10 \%$ )

| Year <br> Ending | Statutory <br> Surplus | GAAP <br> Equiry | Statutory <br> Income | DAC | GAAP <br> Income | Dividends | ROE |
| :---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: |
| $12-31-00$ | $\$$ | 0.00 | $\$ 10,000.00$ | - | $\$ 10,000.00$ | - | - |
| $12-31-01$ | $1,992.52$ | $11,500.00$ | $\$ 1,992.52$ | $9,507.48$ | $\$ 1,500.00$ | $\$ 0.00$ | $15.00 \%$ |
| $12-31-02$ | $4,184.29$ | $13,125.37$ | $2,191.77$ | $8,941.08$ | $1,625.37$ | 0.00 | 14.13 |
| $12-31-03$ | $6,595.24$ | $14,884.97$ | $2,410.95$ | $8,289.72$ | $1,759.59$ | 0.00 | 13.41 |
| $12-31-04$ | $9,247.29$ | $16,787.95$ | $2,652.04$ | $7,540.66$ | $1,902.98$ | 0.00 | 12.78 |
| $12-31-05$ | $12,164.54$ | $18,843.78$ | $2,917.25$ | $6,679.24$ | $2,055.83$ | 0.00 | 12.25 |
| $12-31-06$ | $15,373.51$ | $21,062.12$ | $3,208.97$ | $5,688.60$ | $2,218.34$ | 0.00 | 11.77 |
| $12-31-07$ | $18,903.38$ | $23,452.76$ | $3,529.87$ | $4,549.37$ | $2,390.64$ | 0.00 | 11.35 |
| $12-31-08$ | $22,786.24$ | $26,025.50$ | $3,882.86$ | $3,239.26$ | $2,572.74$ | 0.00 | 10.97 |
| $12-31-09$ | $27,057.39$ | $28,790.01$ | $4,271.14$ | $1,732.63$ | $2,764.51$ | 0.00 | 10.62 |
| $12-31-10$ | $31,755.65$ | $31,755.65$ | $4,698.26$ | 0.00 | $2,965.63$ | 0.00 | 10.30 |

the ROEs in subsequent years are less than 15 percent due to the reinvestment each year of profits at the 10 percent investment earnings rate.

Table 5 is the same as table 4 except it assumes that dividends equal to each year's statutory income are paid. A level 15 percent ROE emerges.

A few observations are appropriate. Ignoring the after-tax nature of the typical Return on Equity (ROE) measurement, at least two things prevent a block of business priced using a return on investment (ROI) objective equal to the ROE objective demanded by the company from achieving such an ROE:

1. The necessity of having statutory surplus available that is not being used to acquire new business will hurt the company's performance if such surplus is not earning an annual return equal to the ROE objective.
2. GAAP for life insurance companies requires some degree of conservatism. This eliminates the use of the annual ROI pricing objective (which should be larger than the investment earnings rate assumed when pricing the product) as the investment earnings rate assumption in the development of the schedule used to amortize deferred acquisition costs and in the development of GAAP benefit reserves. Thus earnings will be deferred to later years resulting in a nonlevel (increasing) emergence of return on equity.

TABLE 5

> | Amortizing Deferred Acquisition Costs at Initial Investment Rate |
| :--- |
| with Dividends Equal Statutory Earnings |
| Dollar Amounts in Thousands |
| (ROI $=15 \%, g=15 \%$ ) |

| Year <br> Ending | Statutory <br> Surplus | GAAP <br> Equity | Statutory <br> Income | DAC | GAAP <br> Income | Dividends | ROE |
| :---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| $12-31-00$ | ,$\$ 0.00$ | $\$ 10,000.00$ | - | $\$ 10,000.00$ | - | - | - |
| $12-31-01$ | 0.00 | $9,507.48$ | $\$ 1,992.52$ | $9,507.48$ | $\$ 1,500.00$ | $\$ 1,992.52$ | $15.00 \%$ |
| $12-31-02$ | 0.00 | $8,941.08$ | $1,992.52$ | $8,941.08$ | $1,426.12$ | $1,992.52$ | 15.00 |
| $12-31-03$ | 0.00 | $8,289.72$ | $1,992.52$ | $8,289.72$ | $1,341.16$ | $1,992.52$ | 15.00 |
| $12-31-04$ | 0.00 | $7,540.66$ | $1,992.52$ | $7,540.66$ | $1,243.46$ | $1,992.52$ | 15.00 |
| $12-31-05$ | 0.00 | $6,679.24$ | $1,992.52$ | $6,679.24$ | $1,131.10$ | $1,992.52$ | 15.00 |
| $12-31-06$ | 0.00 | $5,688.60$ | $1,992.52$ | $5,688.60$ | $1,001.89$ | $1,992.52$ | 15.00 |
| $12-31-07$ | 0.00 | $4,549.37$ | $1,992.52$ | $4,549.37$ | 853.29 | $1,992.52$ | 15.00 |
| $12-31-08$ | 0.00 | $3,239.26$ | $1,992.52$ | $3,239.26$ | 682.41 | $1,992.52$ | 15.00 |
| $12-31-09$ | 0.00 | $1,732.63$ | $1,992.52$ | $1,732.63$ | 485.89 | $1,992.52$ | 15.00 |
| $12-31-10$ | 0.00 | 0.00 | $1,992.52$ | 0.00 | 259.89 | $1,992.52$ | 15.00 |

## II. EFFECT OF FEDERAL INCOME TAX

Introducing federal income tax into the ROE calculation complicates the relationship between return on investment and return on equity. Generally ROI is affected minimally, if at all, when pricing a product on an after-tax basis. If we assume that all expenses, acquisition and maintenance, are incrementally deductible and that all revenues are incrementally taxable, the after-tax ROI will be equal to the pretax ROI. This will not be true to the extent that tax reserves differ from statutory reserves and overall company tax benefits such as tax-exempt interest, dividends-received deductions, and investment-tax credits are allocated by product or line of business in the after-tax ROI calculation.

ROI and ROI' are defined such that

$$
\sum_{t=0}^{n} B P_{t} /(1+R O I)^{t}=0
$$

and

$$
\sum_{t=0}^{n} A T B P_{t} /\left(1+R O I^{\prime}\right)^{t}=0
$$

Assuming no companywide tax benefits are allocated to each product or line of business in the after-tax ROI calculation,

$$
\begin{aligned}
B P_{t} & =P_{t}+I_{t}-E_{t}-C_{t}-{ }_{t} V+{ }_{t-1} V \\
\text { and } A T B P_{t} & =(1-T)\left(P_{t}+I_{t}-E_{t}-C_{t}\right. \\
& \left.-{ }_{t} T V+{ }_{t-1} T V\right) \\
\text { if } t V & ={ }_{t} V \text { for all years } \\
\text { then } A T B P_{t} & =(1-T)\left(P_{t}+I_{t}-E_{t}-C_{t}\right. \\
& -{ }_{t} V+{ }_{t-1} V \\
& =(1-T) B P_{t}
\end{aligned}
$$

$$
\text { then } \begin{aligned}
\sum_{t=0}^{n} A T B P_{t} /\left(1+R O I^{\prime}\right)^{t} & =(1-T) \sum_{t=0}^{n} B P_{t} /\left(1+R O I^{\prime}\right)^{t} \\
& =\sum_{t=0}^{n} B P_{t} /(1+R O I)^{t}=0
\end{aligned}
$$

If $T$ does not equal one hundred percent and $B P_{t}$ is not zero at all durations, ROI equals $R O I^{\prime}$, where
$B P_{t}=$ book profit in year $t$
$A T B P_{t}=$ after-tax book profit in year $t$
$P_{t}=$ premium income in year $t$
$I_{t}=$ interest on assets equal to the statutory reserve at the end of year $t-1$ and interest on cash flow during year $t$
$E_{t}=$ expenses in year $t$
$C_{t}=$ claims in year $t$
${ }^{V} V=$ statutory reserve held at end of year $t$
$R O I=$ return on invested statutory surplus desired over an $n$-year period
$T=$ tax rate in each of the $n$ years
${ }_{,} T V=$ tax reserve at the end of year $t$
ROI $^{\prime}=$ after-tax return on invested statutory surplus desired over an $n-$ year period.
The effect of introducing federal income tax into the ROE calculation drastically changes its relationship to ROI. Table 6 shows that, even if the amortization schedule is calculated using an investment earnings rate assumption equal to the ROI projected for the product and if all statutory earnings are "dividended," the introduction of a tax (the assumed rate in this example is 36.8 percent) lowers the ROE in the first few years below the 15 percent ROI.

Since the initial equity is not affected by the after-tax nature of the ROE calculation, the first-year ROE is reduced by a factor of $(1-T)$, where $T$ is the applicable tax rate. In the pretax calculation of the ROE, it was shown that the ROE would be equal to the ROI if the GAAP equity at the end of each year were equal to the DAC outstanding at the end of the year, where the DAC was being amortized using an interest assumption equal to the ROI. Additionally, statutory surplus at the end of each year was zero (statutory earnings were dividended out) or was reinvested at a rate equal to the ROI. It follows that for the after-tax ROE to be equal to the ROI, the GAAP

TABLE 6
Effect of Federal Income Taxes with Dividends Equal After-Tax Statutory Earnings
Dollar amounts in Thousands
$(\mathrm{ROI}=15 \%, g=15 \%)$

| $\begin{gathered} \text { Year } \\ \text { Ending } \\ \hline \end{gathered}$ | After-Tax <br> Statutory Surplus | After-Tax GAAP Equity | After-Tax Statutory Income | DAC | $\begin{gathered} \text { After-Tax } \\ \text { GAAP Income } \end{gathered}$ | Dividend | $\begin{aligned} & \text { After-Tax } \\ & \text { ROE } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12-31-00 | \$ 0.00 | \$10,000.00 |  | \$10,000.00 |  |  |  |
| 12-31-01 | 0.00 | 8,955.48 | \$1,992.52 | 9,507.48 | \$948.00 | \$1,992.52 | 9.48\% |
| 12-31-02 | 0.00 | 7,864.27 | 1,992.52 | 8,941.08 | 901.31 | 1,992.52 | 10.06 |
| 12-31-03 | 0.00 | 6,719.36 | 1,992.52 | 8,289.72 | 847.61 | 1,992.52 | 10.78 |
| 12-31-04 | 0.00 | 5,512.71 | 1,992.52 | 7,540.66 | 785.87 | 1,992.52 | 11.70 |
| 12-31-05 | 0.00 | 4,235.04 | 1,992.52 | 6,679.24 | 714.85 | 1,992.52 | 12.97 |
| 12-31-06 | 0.00 | 3,595.20 | 1,273.04 | 5,688.60 | 633.19 | 1,273.04 | 14.95 |
| 12-31-07 | 0.00 | 2,875.20 | 1,259.27 | 4,549.37 | 539.28 | 1,259.27 | 15.00 |
| 12-31-08 | 0.00 | 2,047.21 | 1,259.27 | 3,239.26 | 431.28 | 1,259.27 | 15.00 |
| 12-31-09 | 0.00 | 1,095.02 | 1,259.27 | 1,732.63 | 307.08 | 1,259.27 | 15.00 |
| 12-31-10 | 0.00 | 0.00 | 1,259.27 | 0.00 | 164.25 | 1,259.27 | 15.00 |

equity at the end of each year must be equal to $(1-T)$ times the DAC outstanding at the end of the year, where the DAC is being amortized using an interest assumption equal to the ROI. Likewise, statutory surplus at the end of each year must be equal to zero (table 6) or reinvested at a rate equal to the ROI divided by $(1-T)$ (see table 7). This is illustrated in the formulas and tables that follow.

TABLE 7
Statutory Earnings Reinvested to Yield ROI After-Tax
Dollar Amounts in Thousands
$[\mathrm{ROI}=15 \%, g=15 \%, i=15 \% /(1-.368)]$

| $\begin{gathered} \text { Year } \\ \text { Ending } \\ \hline \end{gathered}$ | After-Tax <br> Statutory Surplus | $\begin{gathered} \text { Afite-Tax } \\ \text { GAAP Equity } \\ \hline \end{gathered}$ | After-Tax <br> Statulory Income | DAC | $\begin{array}{c\|} \text { After-Tax } \\ \text { GAAP Income } \\ \hline \end{array}$ | Dividend | $\begin{gathered} \text { After-Tax } \\ \text { ROE } \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12-31-00 | \$ 0.00 | \$10,000.00 |  | \$10,000.00 |  | - |  |
| 12-31-01 | 1,992.52 | 10,948.00 | \$1,992.52 | 9,507.48 | \$ 948.00 | \$ 0.00 | 9.48\% |
| 12.31.02 | 4,457.95 | 12,148.19 | 2,465.43 | 8,941.08 | 1,200.19 | 0.00 | 10.96 |
| 12-31-03 | 7,508.53 | 13,664.49 | 3,050.58 | 8,289.72 | 1,516.31 | 0.00 | 12.48 |
| 12-31-04 | 10,810.94 | 15,576.64 | 3,302.41 | 7,540.66 | 1,912.14 | 0.00 | 13.99 |
| 12-31-05 | 13,691.86 | 17,913.13 | 2,880.91 | 6,679.24 | 2,336.50 | 0.00 | 15.00 |
| 12-31-06 | 17,004.91 | 20,600.10 | 3,313.05 | 5,688.60 | 2,686.97 | 0.00 | 15.00 |
| 12-31-07 | 20,814.92 | 23,690.12 | 3,810.01 | 4,549.37 | 3,090.02 | 0.00 | 15.00 |
| 12-31-08 | 25,196.43 | 27,243.64 | 4,381.51 | 3,239.26 | 3,553.52 | 0.00 | 15.00 |
| 12-31-09 | 30,235.16 | 31,330.18 | 5,038.74 | 1,732.63 | 4,086.55 | 0.00 | 15.00 |
| 12-31-10 | 36,029.71 | 36,029.71 | 5,794.55 | 0.00 | 4,699.53 | 0.00 | 15.00 |

## Formulas

$$
\begin{aligned}
& \text { After-Tax Stat Surplus, }=\text { After-Tax Stat Surplus } t_{t-1} \\
& + \text { After-Tax Stat Income }_{1} \text { - Dividend }{ }_{r} \\
& \text { After-Tax GAAP Equity }_{t}=\text { After-Tax GAAP Equity }_{t-1}
\end{aligned}
$$

$$
\begin{aligned}
& D A C_{t}=D A C_{t-1}-\frac{\left(D A C_{0}\right) g\left[1 /(1+g)^{11-t}\right]}{1-[1 /(1+g)]^{10}} \\
& \text { After-Tax GAAP Income }{ }_{t}=(1-0.368)[1,992.52 \\
& +i\left(\text { After-Tax Stat Surplus }_{t-1}\right) \\
& \left.+D A C_{t}-D A C_{t-1}\right] \\
& \text { After-Tax ROE }=\frac{\text { After-Tax GAAP Income }_{t}}{\text { After-Tax GAAP Equity }} \\
& \text { After-Tax Stat Income }{ }_{t}=\left[1,992.52+i\left(\text { After-Tax Stat Surplus }_{t-1}\right)\right] \\
& \text { - the larger of }[0.368[1,992.52 \\
& +i \text { (After-Tax Stat Surplus }{ }_{t-1} \text { )] } \\
& + \text { the smaller of }\left\{0.368 \sum_{n=1}^{t-1}[1,992.52\right. \\
& \left.+i\left(\text { After-Tax Stat Surplus }_{n-1}\right)\right] \\
& \text { - 10,000 (0.368) and 0\} and 0】 }
\end{aligned}
$$

The ramifications of this are significant. It is apparent that a quickly growing company that sells business priced to meet its return on investment objective will have a difficult time (impossible if its pricing assumptions hold true) producing an after-tax return on equity equal to its ROI objective. In fact, any company either holding surplus-invested at a rate less than ROI/( $1-T$ )-in excess of that necessary to produce new business and/or producing new business cannot produce a companywide after-tax return on equity equal to its ROI pricing objective (assuming pricing assumptions hold true), unless it owns a block of business with an after-tax return on equity that is greater than the companywide after-tax ROE objective.

This would occur if either (1) past business were priced to produce a ROI greater than the current objective and pricing assumptions were met; (2) experience emerged more favorably than was anticipated in the pricing; or (3) past accounting practices were conservative, thus deferring earnings to later (current) years.

If statutory earnings are reinvested at a pretax investment earnings rate of 10 percent, the resulting ROE is even lower, as shown in table 8.

TABLE 8
Statutory Earnings Reinvested at Pretax Rate of Ten Percent
Dollar amounts in Thousands
$(\mathrm{ROI}=15 \%, g=15 \%, i=10 \%)$

| Year <br> Ending | After-Tax <br> Statutory <br> Surplus | After-Tax GAAP Equity | After-Tax Statutory Income | DAC | After-Tax GAAP Income | Dividend | After-Tax ROE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12-31-00 | \$ 0.00 | \$10,000.00 | - | \$10,000.00 |  | - 0.00 | - |
| 12-31-01 | 1,992.52 | 10,948.00 | \$1,992.52 | 9,507.48 | \$ 948.00 | \$ 0.00 | 9.48\% |
| 12-31-02 | 4,184.29 | 11,975.24 | 2,191.77 | 8,941.08 | 1,027.24 | 0.00 | 9.38 |
| 12-31-03 | 6,595.24 | 13,087.30 | 2,410.95 | 8,289.72 | 1,112.06 | 0.00 | 9.29 |
| 12-31-04 | 9,247.29 | 14,289.98 | 2,652.04 | 7,540.66 | 1,202.69 | 0.00 | 9.19 |
| 12-31-05 | 11,367.99 | 15,589.27 | 2,120.70 | 6,679.24 | 1,299.28 | 0.00 | 9.09 |
| 12-31-06 | 13,345.72 | 16,940.91 | 1,977.73 | 5,688.60 | 1,351.65 | 0.00 | 8.67 |
| 12-31-07 | 15,448.44 | 18,323.64 | 2,102.72 | 4,549.37 | 1,382.73 | 0.00 | 8.16 |
| 12-31-08 | 17,684.05 | 19,731.27 | 2,235.61 | 3,239.26 | 1,407.62 | 0.00 | 7.68 |
| 12-31-09 | 20,060.96 | 21,155.98 | 2,376.91 | 1,732.63 | 1,424.71 | 0.00 | 7.22 |
| 12-31-10 | 22,588.09 | 22,588.09 | 2,527.13 | 0.00 | 1,432.11 | 0.00 | 6.77 |

One potential solution, in addition to the corporate planning and execution obviously necessary, is to price new products using a return on investment pricing objective in excess of the corporate after-tax return on equity objective. Table 9 shows the emergence of earnings for a product priced using a ROI objective of 19 percent. The DAC in this example is amortized using an interest rate assumption equal to the ROI. Statutory surplus at the end of each year is maintained at zero.

TABLE 9
Product Priced for Return on Investment in Excess of Roe objective
Dollar Amounts in Thousands
$(\mathrm{ROI}=19 \%, g=19 \%)$

| Year <br> Ending | After-Tax <br> Statutory <br> Surplus | After-Tax <br> GAAP Equity | After-Tax <br> Statutory <br> Income | DAC | After-Tax <br> GAAP Income | Dividend | Rfter-Tax <br> ROE |
| :---: | :---: | ---: | :---: | ---: | ---: | ---: | :---: |
| $12-31-00$ | $\$ 0.00$ | $\$ 10,000.00$ | - | $\$ 10,000.00$ | - | - |  |
| $12-31-01$ | 0.00 | $8,896.09$ | $\$ 2,304.71$ | $9,595.29$ | $\$ 1,200.80$ | $\$ 2,304.71$ | $12.01 \%$ |
| $12-31-02$ | 0.00 | $7,743.58$ | $2,304.71$ | $9,113.68$ | $1,152.20$ | $2,304.71$ | 12.95 |
| $12-31-03$ | 0.00 | $6,533.23$ | $2,304.71$ | $8,540.56$ | $1,094.37$ | $2,304.71$ | 14.13 |
| $12-31-04$ | 0.00 | $5,254.07$ | $2,304.71$ | $7,858.56$ | $1,025.55$ | $2,304.71$ | 15.70 |
| $12-31-05$ | 0.00 | $4,453.69$ | $1,744.04$ | $7,046.97$ | 943.66 | $1,744.04$ | 17.96 |
| $12-31-06$ | 0.00 | $3,843.31$ | $1,456.58$ | $6,081.18$ | 846.20 | $1,456.58$ | 19.00 |
| $12-31-07$ | 0.00 | $3,116.96$ | $1,456.58$ | $4,931.89$ | 730.23 | $1,456.58$ | 19.00 |
| $12-31-08$ | 0.00 | $2,252.60$ | $1,456.58$ | $3,564.24$ | 592.22 | $1,456.58$ | 19.00 |
| $12-31-09$ | 0.00 | $1,224.02$ | $1,456.58$ | $1,936.73$ | 427.99 | $1,456.58$ | 19.00 |
| $12-31-10$ | 0.00 | 0.00 | $1,456.58$ | 0.00 | 232.56 | $1,456.58$ | 19.00 |

This has two advantages. If the corporate after-tax ROE objective is less than the ROI, the corporate objective ROE will be reached sooner. Additionally, in later years, renewal business will be producing an after-tax ROE equal to the ROI pricing objective, which is greater than the corporate aftertax ROE objective, thus tending to offset the negative effect that the production of new business has on the corporate after-tax ROE.

If the goal of new business were to enable the corporation to achieve its after-tax ROE objective in the first year, the pricing objective would have to be equal to the corporate after-tax ROE objective divided by $(1-T)$. If statutory surplus were maintained at zero, the after-tax ROE would exceed the corporate objective in renewal years, as illustrated in table 10 (the corporate objective in this example is assumed to be a 15 percent after-tax ROE).

The matter is complicated further by the fact that GAAP will not allow the amortization of deferred acquisition costs using an amortization schedule with an interest rate assumption equal to the ROI pricing objective. Table 11 illustrates the emergence of earnings and after-tax return on equity on a product priced for a 15 percent return on investment using an amortization schedule with a 10 percent interest rate assumption. Statutory surplus in this example is assumed to be dividended out at the end of each year.

Since earnings are deferred to later years, the ROE starts below 15 percent and eventually increases to a level in excess of 15 percent.

Finally, table 12 illustrates the emergence of earnings and after-tax ROE in the previous example if statutory income is assumed to be reinvested at a 10 percent pretax interest rate each year.

TABLE 10
Product Priced to Meet Afier-Tax RoE Objective in First Year Dollar Amounts in Thousands $(\mathrm{ROI}=23.73 \%, g=23.73 \%)$

| Year <br> Ending | After-Tax <br> Statutory <br> Surplus | After-Tax <br> GAAP Equity | After-Tax <br> Statutory <br> Income | DAC | After-Tax <br> GAAP Income | (ividend | After-Tax <br> ROE |
| :---: | :---: | ---: | :---: | ---: | ---: | ---: | :---: |
| $12-31-00$ | $\$ 0.00$ | $\$ 10,000.00$ | - | $\$ 10,000.00$ | - | - |  |
| $12-31-01$ | 0.00 | $8,806.37$ | $\$ 2,693.63$ | $9,679.79$ | $\$ 1,500.00$ | $\$ 2,693.63$ | $15.00 \%$ |
| $12-31-02$ | 0.00 | $7,564.70$ | $2,693.63$ | $9,283.57$ | $1,451.97$ | $2,693.63$ | 16.49 |
| $12-31-03$ | 0.00 | $6,263.61$ | $2,693.63$ | $8,793.32$ | $1,392.54$ | $2,693.63$ | 18.41 |
| $12-31-04$ | 0.00 | $5,174.00$ | $2,408.61$ | $8,186.70$ | $1,319.00$ | $2,408.61$ | 21.06 |
| $12-31-05$ | 0.00 | $4,699.63$ | $1,702.38$ | $7,436.12$ | $1,228.01$ | $1,702.38$ | 23.73 |
| $12-31-06$ | 0.00 | $4,112.67$ | $1,702.38$ | $6,507.39$ | $1,115.42$ | $1,702.38$ | 23.73 |
| $12-31-07$ | 0.00 | $3,386.40$ | $1,702.38$ | $5,358.23$ | 976.11 | $1,702.38$ | 23.73 |
| $12-31-08$ | 0.00 | $2,487.76$ | $1,702.38$ | $3,936.33$ | 803.73 | $1,702.38$ | 23.73 |
| $12-31-09$ | 0.00 | $1,375.83$ | $1,702.38$ | $2,176.95$ | 590.45 | $1,702.38$ | 23.73 |
| $12-31-10$ | 0.00 | 0.00 | $1,702.38$ | 0.00 | 326.54 | $1,702.38$ | 23.73 |

TABLE 11

## Product Priced for Fifteen Percent Return on Investment With Ten Percent Amortization Dollar amounts in Thousands

$(\mathrm{ROI}=15 \%, g=10 \%)$

| Year <br> Ending | After-Tax <br> Statutory <br> Surplus | After-Tax <br> GAAP Equity | After-Tax <br> Statutory <br> Income | DAC | After-Tax <br> GAAP Income | Dividend | Afier-Tax <br> ROE |
| :---: | :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 2 - 3 1 - 0 0}$ | $\$ 0.00$ | $\$ 10,000.00$ | - | $\$ 10,000.00$ | - | - | - |
| $12-31-01$ | 0.00 | $8,870.20$ | $\$ 1,992.52$ | $9,372.55$ | $\$ 862.72$ | $\$ 1,992.52$ | $8.63 \%$ |
| $12-31-02$ | 0.00 | $7,700.75$ | $1,992.52$ | $8,682.35$ | 823.07 | $1,992.52$ | 9.28 |
| $12-31-03$ | 0.00 | $6,487.67$ | $1,992.52$ | $7,923.13$ | 779.45 | $1,992.52$ | 10.12 |
| $12-31-04$ | 0.00 | $5,226.62$ | $1,992.52$ | $7,087.99$ | 731.46 | $1,992.52$ | 11.27 |
| $12-31-05$ | 0.00 | $3,912.78$ | $1,992.52$ | $6,169.33$ | 678.68 | $1,992.52$ | 12.99 |
| $12-31-06$ | 0.00 | $3,260.37$ | $1,273.04$ | $5,158.81$ | 620.62 | $1,273.04$ | 15.86 |
| $12-31-07$ | 0.00 | $2,557.85$ | $1,259.27$ | $4,047.24$ | 556.76 | $1,259.27$ | 17.08 |
| $12-31-08$ | 0.00 | $1,785.09$ | $1,259.27$ | $2,824.51$ | 486.51 | $1,259.27$ | 19.02 |
| $12-31-09$ | 0.00 | 935.05 | $1,259.27$ | $1,479.50$ | 409.23 | $1,259.27$ | 22.92 |
| $12-31-10$ | 0.00 | 0.00 | $1,259.27$ | 0.00 | 324.23 | $1,259.27$ | 34.67 |

TABLE 12
Statutory Earnings Reinvested at Pretax Rate of Ten Percent Dollar Amounts in Thousands
(ROI $=15 \%, g=10 \%, i=10 \%)$

| Year <br> Ending | After-Tax Statutory Surplus | After-Tax GAAP Equity | After-Tax Statutory Income | DAC | After-Tax GAAP Income | Dividend | After-Tax ROE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12-31-00 | \$ 0.00 | \$10,000.00 | - | \$10,000.00 |  | - | - |
| 12-31-01 | 1,992.52 | 10,862.72 | 1,992.52 | 9,372.55 | \$ 862.72 | \$ 0.00 | 8.63\% |
| 12-31-02 | 4,184.29 | 11,811.72 | 2,191.77 | 8,682.35 | 948.99 | 0.00 | 8.74 |
| 12-31-03 | 6,595.24 | 12,855.61 | 2,410.95 | 7,923.13 | 1,043.89 | 0.00 | 8.84 |
| 12-31-04 | 9,247.29 | 14,003.89 | 2,652.04 | 7,087.99 | 1,148.28 | 0.00 | 8.93 |
| 12-31-05 | 11,367.99 | 15,267.00 | 2,120.70 | 6,169.33 | 1,263.11 | 0.00 | 9.02 |
| 12-31-06 | 13,345.72 | 16,606.09 | 1,977.73 | 5,158.81 | 1,339.08 | 0.00 | 8.77 |
| 12-31-07 | 15,448.44 | 18,006.29 | 2,102.72 | 4,047.24 | 1,400.21 | 0.00 | 8.43 |
| 12-31-08 | 17,684.05 | 19,469.14 | 2,235.61 | 2,824.51 | 1,462.85 | 0.00 | 8.12 |
| 12-31-09 | 20,060.96 | 20,996.01 | 2,376.91 | 1,479.50 | 1,526.86 | 0.00 | 7.84 |
| 12-31-10 | 22,588.09 | 22,588.09 | 2,527.13 | 0.00 | 1,592.08 | 0.00 | 7.58 |

As the marketplace may not allow the success of an insurance product charging a premium that produces a return on investment of, say, 23.7 percent $[15 /(1-.368)]$, the question of how to produce a corporate after-tax return on equity of 15 percent remains. Many industries can produce such a return by leveraging their equity with medium- and long-term debt, thus increasing their working capital (current assets minus current liabilities) without directly increasing their equity. Statutory accounting principles basically eliminate this as an option for most life insurance companies (ignoring sur-
plus debentures and surplus-relief reinsurance agreements). Equity, not just current assets, is needed to acquire new life insurance business.
$\dot{A}$ different type of leveraging is needed: a leveraging of new business with old. If a corporate after-tax ROE objective equal to or even close to the ROI pricing objective is demanded, controlled growth is required. The growth in new production (or decline in new production) will be dictated by past and current accounting practices, reinvestment and stockholder dividend practices, the age of the existing block of business, and the relative size of the existing block of business compared to the new business to be produced. As illustrated previously, all of these items can have a major impact on the eventual after-tax ROE produced in any given year. Likewise, when evaluating the performance of any company, these items must be considered over a number of years before any significant conclusions can be drawn.

Reinvestment and stockholder dividend practices also must consider the degree of risk associated with the existing block of business.

Statutory surplus in excess of that needed to produce new business must be maintained. The amount to be retained will depend upon the risk elements associated with the particular block of business. Since it is unreasonable to expect the investment of this statutory surplus to produce a pretax return equal to (let alone greater than) the return generated on new business (Otherwise why invest in new business?), the negative effect on after-tax ROE of maintaining this statutory surplus must be considered in the determination of the amount of new business that can be written while reaching the desired corporate after-tax ROE objective.

An appropriate solution would be to calculate the ROI on new products on the basis of a statutory "reserve" equal to the reserve using the desired valuation basis plus the additional amount needed as risk surplus. to ensure the solvency of the line of business. ${ }^{5}$ A ROE equal to the ROI objective used to price the product eventually will emerge even though there are sufficient assets available to ensure the solvency of the line.

## III. GENERALIZED CASE

The preceding examples use a block of business with a specific, rather unrealistic, emergence of statutory profits to illustrate the relationship between ROI and ROE. With the exception of the emergence of statutory profits, the formulas used in these examples are generalized. Additionally, the Statutory-to-GAAP pretax adjustment (referred to here as DAC although

[^2]representing both traditional DAC and the difference between statutory and GAAP benefit reserves) has assumed a level premium income stream throughout the period (that is, DAC was amortized using interest only). However, the conclusions drawn from the examples are not dependent upon the specific emergence of statutory profits assumed. The sum of the present values of each year's statutory book profit is zero when discounted at a rate equal to the ROI (by definition). This implies that the present value of GAAP book profits is zero when discounted at a rate equal to the ROI, since the reserving (benefit and expense reserve for GAAP) technique (GAAP versus statutory) does not affect the ultimate level of the present value of profits (that is, since the ultimate reserve is zero in either case, the present value of the increase in reserve is zero). ${ }^{6}$

If the investment earnings rate assumption used in the development of GAAP reserves is equal to the ROI and all other assumptions are equal to those used in pricing the product (i.e., no explicit margin for adverse deviation from the assumptions used in the pricing process), the GAAP book profit each year (leveled as a percentage of premium) must be equal to zero.

Since GAAP Income $=$ GAAP Book Profit $+i\left(\right.$ GAAP Equity $\left._{t-1}\right)$
and GAAP Book Profit $t_{t}=0$ for all $t$
GAAP Income $_{t}=i\left(\right.$ GAAP Equity $\left._{t-1}\right)$
and GAAP Equity ${ }_{t}=D A C_{t}$ for all $t$ if Stat Surplus $=0$ for all $t$.
But $D A C_{t}$ earns a rate of ROI each year (given)

$$
\text { so } \text { ROE }_{t}=\text { GAAP Income }_{t} / \text { GAAP Equity }_{t-1} .
$$

Therefore ROE $_{t}=i\left(\right.$ GAAP Equity $\left._{t-1}\right) /$ GAAP Equity $_{t-1}$

$$
\text { and } R O E_{t}=R O I\left(D A C_{t-1}\right) / D A C_{t-1}
$$

Therefore $R O E_{t}=$ ROI for all $t$.
Since the present value of statutory profits discounted at a rate equal to the ROI is zero by definition, this proves that the ROE each year will be equal to the ROI, regardless of the emergence of statutory profits, if the Statutory-to-GAAP adjustments assume an investment earnings rate assumption equal to the ROI (along with the other pricing assumptions for persistency, mortality, morbidity, and expense). See table 5 for the specific example.

[^3]
## IV. SPECIAL CONSIDERATIONS

Recent developments within the industry affect the theory as described in this paper. The Deficit Reduction Act of 1984 (DEFRA) "fresh start" had the effect of increasing GAAP income and ROE in 1984, while substantially increasing GAAP equity at the end of 1984. This additional nonearning equity certainly disrupted the achievement of future near-term corporate ROE objectives in many companies.

The interpretation of the value of a block of business upon acquisition by one company from another affects the drain on future earnings and certainly must be considered during the corporate planning process if corporate objectives are to be met. Valuation of this block would fall under the consideration of past accounting practices.

## V. CONCLUSIONS

This paper has attempted to examine both the theoretical and practical relationships between a commonly used pricing objective, return on investment, and a common corporate profit measure, return on equity.

The relationship between ROI and ROE was shown to be dependent upon a number of items:

1. past and current accounting practices,
2. past and future reinvestment and stockholder dividend practices,
3. the risk associated with the existing block of business and the statutory surplus required to insure the company's future solvency,
4. the age of the existing block of business, and
5. the relative size of the existing block of business when compared to the new business to be produced.
All of these items are important and must be considered in the planning process. The pricing process and the objectives used in that process must be consistent with those used in the planning process if corporate objectives of profitability and growth are to be met.

## ACKNOWLEDGMENT

The author would like to thank Pacquitta Davidson for assisting with the development of this paper.

# DISCUSSION OF PRECEDING PAPER 

DONALD R. SONDERGELD:

This is a subject I have been very interested in, as evidenced by two papers I wrote: "Profitability as a Return on Total Capital," TSA, XXXIV, and "Earnings and the Internal Rate of Return Measurement of Profit," TSA, XXVI. Also, Walter S. Rugland and I were the faculty for two seminars given by the Society of Actuaries at Snowbird, Utah in March 1987, titled "Managing Surplus in a Return on Equity Environment."

I have made the following points before, but will emphasize them again:

1. The sum of GAAP earnings and the sum of statutory earnings are equal over the life of the policy.
2. By definition, the present value of GAAP pretax book profits is equal to the present value of statutory pretax book profits when discounted at the GAAP interest rate $g$. If $g=$ ROI, the present value of GAAP pretax book profits is also zero.
3. If there are no federal income taxes, GAAP earnings equal GAAP book profits plus interest (at the rate $g$ ) on GAAP equity. This means that if the GAAP interest rate $g$ is equal to the author's ROI (return on the statutory investment), then GAAP book profits equal zero. The GAAP earnings that result will equal the GAAP interest rate $g$ (or ROI) on GAAP equity. Therefore, the ROE (return on GAAP equity) will equal the ROI each year.
My 1974 paper ( $T S A$, XXVI) displayed GAAP pretax book profits that were generated if the ROI was 12 percent and the GAAP interest rate $g$ was 0 percent, 5 percent, 12 percent, or 15 percent. For $g=0$ percent and 5 percent, the GAAP pretax book profits were positive. For $g=12$ percent, GAAP pretax book profits were zero. In this case, GAAP accounting and the Internal Rate of Return Method of Accounting (IRRMA) were equal. For $g=15$ percent, GAAP pretax book profits were negative.
Most of the following comments relate to taxes:
4. When federal income taxes are introduced, the results are similar, but the incidence is affected if there are tax loss carryforwards, as illustrated by the author in Tables 6-12.
5. In Tables 6 through 12 in Mr. Smith's paper, the difference between "DAC" and "After-Tax GAAP Equity" represents the liability for deferred taxes. To illustrate, the items in Table 6 here relate to Table 6 of the paper.

TABLE 6
Related Deferred Acquisition Cost and Taxes

| Year <br> Ending | DAC | Liability <br> for <br> Deferred Taxes | Deferred <br> Tax <br> $\div$ DAC | Change in <br> Deferred <br> Tax Liability | Current <br> Tax | Total <br> GAAP <br> Tax |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $12-31-00$ | $\$ 1,000.00$ | $\$ 0.00$ | 0.000 | $\$ 0.00$ | $\$ 0.00$ | $\$ 0.00$ |
| $12-31-01$ | $9,507.48$ | 552.00 | 0.058 | 552.00 | 0.00 | 552.00 |
| $12-31-02$ | $8,941.08$ | $1,076.81$ | 0.120 | 524.81 | 0.00 | 524.81 |
| $12-31-03$ | $8,289.72$ | $1,570.36$ | 0.189 | 493.55 | 0.00 | 493.55 |
| $12-31-04$ | $7,540.66$ | $2,027.95$ | 0.269 | 457.59 | 0.00 | 457.59 |
| $12-31-05$ | $6,679.24$ | $2,444.20$ | 0.366 | 416.25 | 0.00 | 416.25 |
| $12-31-06$ | $5,688.60$ | $2,093.40$ | 0.368 | -350.80 | 719.48 | 368.68 |
| $12-31-07$ | $4,549.37$ | $1,674.17$ | 0.368 | -419.23 | 733.25 | 314.02 |
| $12-31-08$ | $3,239.26$ | $1,192.05$ | 0.368 | -482.12 | 733.25 | 251.13 |
| $12-31-09$ | $1,732.63$ | 637.61 | 0.368 | -554.44 | 733.25 | 178.81 |
| $12-31-10$ | 0.00 | 0.00 | - | -637.61 | 733.25 | 95.64 |

After-tax GAAP equity $=$ pretax GAAP equity less deferred taxes
Generally, ROE $_{t}=\frac{(g)(\text { pretax GAAP equity })_{t-1}(1-T)}{\text { (pretax GAAP equity less deferred taxes })_{t-1}}$
6. After-tax GAAP equity does not equal (pretax GAAP equity) $(1-T)$, unless all statutory tax loss carryforwards have expired. In that case,

$$
\text { ROE }_{t}=\frac{(g) \text { (pretax GAAP equity })_{t-1}(1-T)}{(\text { pretax GAAP equity })_{t-1}(1-T)}=g
$$

Examples of this are shown in Table 6 for years 7-10; in Table 7 for years 5-10; in Table 9 for years $6-10$; and in Table 10 for years $5-10$.
7. Tables $2,4,7,8$, and 12 show the effect of not paying dividends. The ROE that occurs each year is, of course, a weighted average of the return on statutory surplus and the return on the GAAP adjustments (that is, the author's DAC less any deferred taxes).
8. Tables $3,5,6,9,10$, and 11 show the results of using various GAAP interest rates. To summarize:
a. Table 3 excludes taxes. As $g$ is 10 percent, GAAP income (earnings) equals 10 percent of GAAP equity plus $\$ 365.07$ of GAAP book profits:

$$
\left.\$ 365.07=\left[-10,000+(1,992.52) a_{\overline{10} \mid 10 \%}\right)\right] \div a_{\overline{10} \mid 10 \%} .
$$

In this case, the ROE goes from less than the ROI to greater than the ROI over the life of the policy.
b. Table 5, which also excludes taxes, demonstrates that if $g=$ ROI $=15$ percent, then ROE will be a level 15 percent, using what is referred to as the Internal Rate of Return Method of Accounting (IRRMA) in my 1974 paper (TSA, XXVI). In this example, GAAP income is 15 percent of GAAP equity plus zero GAAP book profits.
c. In Table 6, although the pretax $\mathrm{ROI}=g=15$ percent, the after-tax ROI $=$ 11.36 percent. Comment 5 above indicates why the ROE is less than 11.36
percent in the early years and 15 percent in the later years.
d. Since the after-tax ROI' in Table 6 is 11.36 percent, I thought it instructive to develop an amortization schedule to produce ROEs equal to 11.36 percent in all years. This was accomplished by using the following formulas for developing Table 6A:

$$
\begin{aligned}
(\text { After-Tax GAAP Income })_{t}= & (\text { After-Tax GAAP Equity })_{t-1}\left(\text { ROI' }^{\prime}\right) \\
(\text { After-Tax GAAP Equity })_{t}= & (\text { After-Tax GAAP Equity })_{t-1} \\
& + \text { (After-Tax GAAP Income }) \\
& - \text { (After-Tax Statutory Income) }
\end{aligned}
$$

$$
(\text { Pretax GAAP Income })_{t}=(\text { After-Tax GAAP Income })_{t} \div(1-T)
$$

$\mathrm{DAC}_{t}=\mathrm{DAC}_{t-1}+(\text { Pretax GAAP Income })_{t}-(\text { Pretax Statutory Income })_{t}$
$g_{t}=\left[(\text { Pretax GAAP Income })_{t}-(\text { pretax GAAP book profits })_{t}\right] \div$ DAC $_{t-1}$.
Since pretax GAAP book profits equal zero in all years, we have:

$$
g_{t}=(\text { pretax GAAP income })_{t} \div \mathrm{DAC}_{t-1} .
$$

The after-tax statutory surplus is zero in all years, as in Table 6. The GAAP income results in Table 6A can be produced by defining pretax GAAP book profits as $\$ 123$ and solving for $g_{t}$ using the following equation:

$$
(\text { pretax GAAP income })_{t}=\left(g_{t}\right)\left(\text { DAC }_{t-1}\right)+\$ 123 .
$$

TABLE 6A
Dollar amounts in Thousands

$$
\left(\text { ROI }=15 \%, \text { ROI }^{\prime}=11.36 \%, g \text { is variable }\right)
$$

| Year <br> Ending | $g$ | After-Tax <br> GAAP Equity | After-Tax <br> Statutory <br> Income | DAC | After-Tax <br> GAAP Income | Dividend | After-Tax <br> ROE |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $12-31-00$ | - | $\$ 10,000.00$ | $\$-10,000.00$ | $\$ 10,000.00$ | - | $\#$ | - |
| $12-31-01$ | $17.97 \%$ | $9,143.04$ | $1,992.52$ | $9,804.25$ | $\$ 1,135.56$ | $\$ 1,992.52$ | $11.36 \%$ |
| $12-31-02$ | 16.76 | $8,188.77$ | $1,992.52$ | $9,454.53$ | $1,038.25$ | $1,992.52$ | $11.36 \%$ |
| $12-31-03$ | 15.56 | $7,126.13$ | $1,992.52$ | $8,933.34$ | 929.88 | $1,992.52$ | $11.36 \%$ |
| $12-31-04$ | 14.33 | $5,942.82$ | $1,992.52$ | $8,221.22$ | 809.21 | $1,992.52$ | $11.36 \%$ |
| $12-31-05$ | 12.99 | $4,625.14$ | $1,992.52$ | $7,296.48$ | 674.84 | $1,992.52$ | $11.36 \%$ |
| $12-31-06$ | 11.39 | $3,877.32$ | $1,273.04$ | $6,135.00$ | 525.22 | $1,273.04$ | $11.36 \%$ |
| $12-31-07$ | 11.36 | $3,058.35$ | $1,259.27$ | $4,839.16$ | 440.30 | $1,259.27$ | $11.36 \%$ |
| $12-31-08$ | 11.36 | $2,146.38$ | $1,259.27$ | $3,396.17$ | 347.30 | $1,259.27$ | $11.36 \%$ |
| $12-31-09$ | 11.36 | $1,130.85$ | $1,259.27$ | $1,789.31$ | 243.74 | $1,259.27$ | $11.36 \%$ |
| $12-31-10$ | 11.36 | 0.00 | $1,259.27$ | 0.00 | 128.42 | $1,259.27$ | $11.36 \%$ |
| Total | - | - | $\$ 6,272.72$ | - | $\$ 6,272.72$ | - | - |

\# Can be thought of as $\$-\$ 10,000$.

The resulting $g_{t}$ 's, for $t=1,2, \ldots 10$, are $0.1674,0.1550,0.1426,0.1296,0.1149$, $0.0970,0.0935,0.0881,0.0773$, and 0.0448 , respectively.
As pretax GAAP book profits are increased, the $g_{t}$ 's are reduced, and conversely.
e. It may be of interest to modify Table 6 on the assumption that a tax credit is allocated to this product in the first year and taxes are paid in each of the next ten years. This is shown in Table 6B. The after-tax GAAP income each year, the sum of the after-tax GAAP income for all years, and the sum of the aftertax statutory income for all years remain the same as in Table 6 . However, in this case ROI $=$ ROI $^{\prime}=g=$ ROE $=15$ percent. This was mentioned by Mr. Smith in Section II of the paper.
In Table 6B, deferred taxes equal 36.8 percent of DAC in all years.
TABLE 6B
Dollar Amounts in Thousands
$\left(\mathrm{ROI}=\mathrm{ROI}^{\prime}=g=15 \%\right)$

| Year <br> Ending | After-Tax <br> Statutory <br> Surplus | After-Tax <br> GAAP Equity | After-Tax <br> Statutory <br> Income | DAC | After-Tax <br> GAAP Income | Dividend |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | :---: | | After-Tax |
| :---: |
| ROE |

\# Can be thought of as $\$-6,320$.
f. In Table 9, although the pretax ROI $=g=19$ percent, the after-tax ROI' $=$ 14.63 percent. Pretax GAAP book profits equal zero.
g. In Table 10, although the pretax ROI $=g=23.73$ percent, the after-tax ROI ${ }^{\prime}$ $=18.54$ percent. Pretax GAAP book profits equal zero.
h. In Table 11, although the pretax $\mathrm{ROI}=15$ percent, the after-tax $\mathrm{ROI}^{\prime}=11.36$ percent. Pretax GAAP book profit equal $\$ 365.07$, as $g=10$ percent.
i. One minor point is that each of the tables should have included $-10,000$ as the statutory income for the year which ended on 12-31-00.

The modifications to Table 6 further emphasize the point that, by starting with a statutory after-tax ROI', the accounting method (in this case GAAP) and the actuarial assumptions chosén affect the after-tax ROE incidence -
but the weighted average ROE is still the ROI'. Table 11 is also a modification of Table 6 . That is, $g=15$ percent in Table 6 and 10 percent in Table 11.

In my 1982 paper (TSA, XXXIV), I defined 'book profits" as of a point in time, whereas "earnings" were for a period of one year. In that paper, I chose to define statutory book profits as of the beginning of the first year and as of the end of renewal years. This choice is a matter of individual preference.

If pricing actuaries at two different companies used the same assumptions in calculating an ROI (or IRR) for a product, but one actuary defined statutory pretax book profits as of the beginning of each year and the other actuary defined statutory pretax book profits as of the end of each year, they would produce different ROIs unless $I_{t}$ (the interest earned on the statutory reserve at the end of the previous year and on cash flow of the current year) was based upon the ROI.

What if the financial actuaries for these two companies projected GAAP pretax earnings and wished to demonstrate to their respective managers that the sum of statutory pretax book profits equaled the sum of GAAP pretax earnings over the life of the policy? Further assume the GAAP interest rate $g$ is the interest rate expected to be earned on assets. They both can calculate GAAP pretax earnings, which will appear to differ, but can be internally consistent. That is, in the company where statutory pretax book profits were defined at the beginning of the policy year by the pricing actuary, the financial actuary, at least for analysis purposes, can assume those amounts are paid as dividends at the beginning of the policy year. Similarly, the financial actuary for the other company can treat the statutory pretax book profit as a dividend at the end of each policy year.

For example, Table 3 can be revised by defining renewal statutory book profits at the beginning of the year, instead of at the end of the year, and assuming that dividends of $\$ 1,732.63$ are paid at the beginning of the year. Thus,

$$
\$ 10,000=\$ 1,992.52 \ddot{a}_{\overline{10} \overline{10}_{15 \%}}=\$ 1,732.63 \ddot{a}_{\bar{o}_{10}}{ }_{15 \%} .
$$

If we define GAAP book profits at the end of the year, they become $\$ 278.44$ :

$$
\$ 278.44=\left[-\$ 10,000+(\$ 1,732.63)\left(\ddot{a}_{\overline{10} 10 \%}\right)\right] \div a_{\overline{1 q} 10 \%} .
$$

The statutory surplus is zero in all years as in Table 3.

TABLE 3A
Dollar Amounts in Thousands
( $\mathrm{ROI}=15 \%, g=10 \%$ )

| Year <br> Ending | GAAP <br> Equiry | GAAP <br> Equity* | Staturory <br> Income | DAC | GAAP <br> Income | Dividends | ROE*** |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $12-31-00$ | - | $\$ 10,000.00$ | $\$-10,000.00$ | $\$ 10,000.00$ | - | $\#$ | - |
| $12-31-01$ | $\$ 8,267.37$ | $9,372.55$ | $1,732.63$ | $9,372.55$ | $\$ 1,105.18$ | $\$ 1,732.63$ | $13.37 \%$ |
| $12-31-02$ | $7,639.92$ | $8,682.35$ | $1,732.63$ | $8,682.35$ | $1,042.43$ | $1,732.63$ | 13.64 |
| $12-31-03$ | $6,949.72$ | $7,923.13$ | $1,732.63$ | $7,923.13$ | 973.41 | $1,732.63$ | 14.01 |
| $12-31-04$ | $6,30.50$ | $7,087.99$ | $1,732.63$ | $7,087.99$ | 897.49 | $1,732.63$ | 14.50 |
| $12-31-05$ | $5,355.36$ | $6,169.34$ | $1,732.63$ | $6,169.34$ | 813.98 | $1,732.63$ | 15.20 |
| $12-31-06$ | $4,436.71$ | $5,158.82$ | $1,732.63$ | $5,158.82$ | 722.11 | $1,732.63$ | 16.28 |
| $12-31-07$ | $3,426.19$ | $4,047.25$ | $1,732.63$ | $4,047.25$ | 621.06 | $1,732.63$ | 18.13 |
| $12-31-08$ | $2,314.62$ | $2,824.52$ | $1,732.63$ | $2,824.52$ | 509.90 | $1,732.63$ | 22.03 |
| $12-31-09$ | $1,091.89$ | $1,479.52$ | $1,732.63$ | $1,479.52$ | 387.63 | $1,732.63$ | 35.50 |
| $12-31-10$ | -253.11 | 0.00 | $1,732.63$ | 0.00 | 253.11 | $1,732.63$ | -100.00 |
| Total | - | - | $7,326.30$ | - | $7,326.30$ | - | - |

${ }^{*}$ At beginning of year, after dividend paid.
${ }^{* *}$ At end of year.
***GAAP Income $\div$ GAAP Equity at beginning of year.
\#Can be thought of as $\$-10,000$ at end of year.
Near the end of Section III of the paper, the author states
The sum of the present values of each year's statutory book profit is zero when discounted at a rate equal to the ROI (by definition)."
I agree. He then states
This implies that the present value of GAAP book profits is zero when discounted at a rate equal to the ROI, since the reserving (benefit and expense reserve for GAAP) technique (GAAP versus statutory) does not affect the ultimate level of the present value of profits (that is, since the ultimate reserve is zero in either case, the present value of the increase in reserve is zero). This is true only if, when calculating the increase-in-reserve component of book profit in a particular year, the reserve at the end of the year is discounted at a rate equal to the ROI (or if the reserve at the beginning of the year is accumulated at a rate equal to the ROI) rather than the investment earnings rate for that year.
However, the statement I agreed with does not imply that the present value of any set of GAAP book profits is zero when discounted at a rate equal to the ROI. This is true if the $g$ used to calculate GAAP book profits was equal to the ROI , or if variable $g_{\prime}$ 's were chosen that were equivalent to the level ROI. Each GAAP book profit is zero if the GAAP interest rate $g=$ ROI, and the present value of a string of zeros is zero. For example, the ten $\$ 365.07$ GAAP book profits from Table 3 discounted at the 15
percent ROI will not produce a zero present value. Furthermore, the parenthetical expression in the previous paragraph should have included cash flow during the year, accumulated at the ROI.

At the end of Section III of the paper, the author shows that $\mathrm{ROE}_{t}=$ ROI for all $t$ if there are no taxes. It is also true if deferred taxes are a uniform percentage of DAC, as shown below:

$$
\frac{\left[\left(g_{t}\right)\left(\mathrm{DAC}_{t-1}\right)+(\text { pretax GAAP book profits })_{t}\right](1-\mathrm{T})}{\text { DAC }_{t-1}-(\text { deferred taxes })_{t-1}}
$$

If (deferred taxes) $t_{t-1}=(T)\left(\mathrm{DAC}_{t-1}\right)$, then the uniform percentage is

$$
\frac{\left(g_{t}\right)\left(\mathrm{DAC}_{t-1}\right)+(\text { pretax GAAP book profits })_{t}}{\text { DAC }_{t-1}}
$$

If, in addition, $g_{t}=$ ROI, pretax GAAP book profits equal zero, and after-tax $\mathrm{ROE}_{6}=$ pretax statutory ROI.

## DAVID N. BECKER:

Many actuaries use a statutory based return on equity (ROE) computation to determine whether or not a given product design achieves the minimum return level set by senior management. This ROE profit measure is a theoretical level return over the period of the pricing horizon. When an acceptable configuration is attained, the pricing process is terminated.

The actuarial and/or financial departments must set up the GAAP accounting for the product once its sales reach a material volume. As its sales and in-force business grow, the products contribution to reported ROE emerges. Somewhere in this process the reported ROE is examined, and it may well be different from the minimum level used for pricing. It is then important to understand the differences and to be able to explain them to senior management.

Mr. Smith's paper is an excellent and comprehensive discussion for actuaries who have not yet experienced this situation or have not probed it in depth. It furthers understanding of the relationships among profit criteria. The list of items presented in Section V is a very valuable, ready-access checklist. Additionally, the concepts are well communicated through the many numerical examples.

I would like to add a discussion of certain aspects of the statutory return on investment/equity computation and a collection of observations and thoughts - not conclusions - on ROE.

Many actuaries follow the classical book-profit approach described by James C. H. Anderson in his article "Gross Premium Calculations and Profit Measurement for Nonparticipating Insurance," TSA, XI. This approach may lead to another difficulty, described below, in reconciling the statutory return on investment/equity to the accounting return on equity actually reported.

In Mr. Smith's paper the "product" has a statutory return on investment of 15 percent. It consists of an initial investment of $\$ 10,000,000$ and subsequent pretax book profits of $\$ 1,992,520$ for ten years. This example correctly presents the return on investment over the ten-year period because it has an initial investment followed by ten book profits. Using the notation of S. David Promislow in "A New Approach to the Theory of Interest," TSA, XXXII, the example in the paper can be represented by the sequence $(-10,000,000,+1,992,520,+1,992,520, \ldots,+1,992,520)$. For the tenyear period this series of 11 numbers generates the return of 15 percent.

In the classical Anderson book-profit approach for a ten-year product there would be ten such book profits, representing the book profits at the end of each of the ten years. But this does not adequately describe the return on investment. This is because the Anderson method essentially establishes its "investment" at the end of the first year, while a true return on investment requires a "flow" at the beginning of the first year followed by the later flows. Two other items intrude. First, the company must be statutorially solvent at all times, including the time the product is issued. Second, the classical approach holds a reserve only for those who persist into the second policy year. To remain solvent, the company needs to establish the appropropriate liability at time zero for all policyholders.

This difficulty can be resolved by splitting the first-year Anderson book profit into two components: an initial investment at time zero and an adjusted book profit for the end of the first year. These two items, when combined with the remaining Anderson book profits, provide the appropriate set of values over which to compute a ten-year return on investment.

Unfortunately, there are many ways this split can be made. Described below are four potential splits. These will be applied to the book profits for a hypothetical ten-year endowment. The resulting returns on investment for the Anderson approach and the four modifications will be seen to be very different. Therefore, the method used in calculating the return on investment must be included in an analysis of possible differences between statutory return on investment/equity and the accounting return on equity.

Let $I_{0}=$ the initial investment and $A B P_{1}$ be the classical Anderson book profit measured at the end of the first year. One might define $I_{0}$ to be the
premiums less expenses, commissions, and the liability to be held. Let $A A B P_{1}$ be the "adjusted" Anderson book profit at the end of the first year. The four modifications represent different ways of replacing $A B P_{1}$ by $I_{0}$ and $A A B P_{1}$.

The results of the modifications will replace $\left(A B P_{1}, A B P_{2}, \ldots, A B P_{10}\right)$ by ( $I_{0}, A A B P_{1}, A B P_{2}, \ldots, A B P_{10}$ ).

Let (A) be the "null" split, representing the classical Anderson bookprofit return on investment.

Let (B) be the split defined by $I_{0}=A B P_{1} /\left(1+i_{1}\right), A A B P_{1}=0$.
Let (C) be the split defined by $I_{0}=P_{1}-E_{1}-C_{1}-L_{0}$, $A A B P_{1}=A B P_{1}-I_{0}\left(1+i_{1}\right)$.

Here, the $L_{0}$ is the initial liability and the items with subscript 1 are the premiums, expenses, and commissions incurred at the beginning of the first year ( $i_{1}$ is the asset investment rate in year 1 ).

Let (D) be the split defined by $I_{0}=P_{1}-E_{1}-C_{1}-L_{0}, A A B P_{1}=A B P_{1}$ $-I_{0}+\left(i_{1}\right) L_{0}$.

Let (E) be the split defined by $I_{0}=P_{1}-E_{1}-C_{1}-L_{0}, A A B P_{1}=\mathrm{ABP}_{1}$ $-I_{0}$.
Other than (A), split ( E ) is the most straightforward. It merely divides the first-year Anderson book profit into two pieces: the initial investment and the balance. The algebraic sum is the Anderson book profit.

Split (D) recognizes the fact that if the company is charged for the liability $L_{0}$ at time zero in computing the return on investment, then, as that liability is backed by an interest-earning asset, the company should receive the benefit of the investment income on this liability at the end of the year. (Note that the "liability" may include both reserve and minimum, or target, surplus components.)

Split (C) duplicates a property possessed by the Anderson book profits, that is, if the Anderson book profits are discounted at an interest rate that equals the investment rate assumed to be earned by the assets, then the resulting present value is independent of the reserve basis. For split (C) this is achieved by setting $A B P_{1} /\left(1+i_{1}\right)=I_{0}+A A B P_{1} /\left(1+i_{1}\right)$ and solving for the adjusted Anderson book profit.

Split (B) was used by Donald R. Sondergeld in his paper "Profitability as a Return on Total Capital," TSA, XXXIV.

Consider the following book profits for a hypothetical ten-year endowment:

|  | Book Profit for Split |  |  |  |  |  |
| :---: | :---: | ---: | ---: | ---: | ---: | :---: |
| Year | (A) | (B) | (C) | (D) | (E) |  |
| 0 | - | $\$-49.76$ | $\$-104.50$ | $\$-104.50$ | $\$-104.50$ |  |
| 1 | $\$-54.24$ | 0.00 | 59.66 | 55.95 | 50.26 |  |
| 2 | 14.87 | 14.87 | 14.87 | 14.87 | 14.87 |  |
| 3 | 16.19 | 16.19 | 16.19 | 16.19 | 16.19 |  |
| 4 | 15.72 | 15.72 | 15.72 | 15.72 | 15.72 |  |
| 5 | 16.11 | 16.11 | 16.11 | 16.11 | 16.11 |  |
| 6 | 16.93 | 16.93 | 16.93 | 16.93 | 16.93 |  |
| 7 | 16.89 | 16.89 | 16.89 | 16.89 | 16.89 |  |
| 8 | 17.66 | 17.66 | 17.66 | 17.66 | 17.66 |  |
| 9 | 19.18 | 19.18 | 19.18 | 19.18 | 19.18 |  |
| 10 | 21.44 | 21.44 | 21.44 | 21.44 | 21.44 |  |

The returns on investment for the various splits are as follows:

| Split | Return on Investment |
| :---: | :---: |
| (A) | $26.4 \%$ |
| (B) | 22.6 |
| (C) | 20.2 |
| (D) | 19.1 |
| (E) | 17.5 |

As can be readily seen, different measures of the statutory return on investment can produce markedly different numerical results. This can be a source of confusion when reconciling the ROI to the accounting ROE.
A pervasive concern is the choice of an acceptable value (hurdle rate) for ROE. A theoretically correct approach for a stock company would be to poll the shareholders to determine their collective preference between a dollar today and a dollar one year in the future, a dollar two years in the future, and so on. Such a canvass would not result in a single hurdle rate, but would yield a set of spot rates that could be applied to the profit stream.

However, a polling of stockholders is obviously not practical, so approximate methods are employed. One method is to survey the ROEs reported by successful competitors with similar markets and products. By comparing ROEs and corresponding share prices, one can set a hurdle rate. A second method is to "build"' a hurdle rate from components based on the risk-free rate of return, inflation, and risk assumed. A third method is to determine a rate that makes the board of directors comfortable.

There are pitfalls in relying solely on the statutory-return-on-investment approach to anticipate the accounting return on equity for products. Mr. Smith's exposition describes many of them. In addition, some aspects of the statutory pricing process can add complications. The statutory return on investment is a mathematical solution to a polynomial, which can have multiple roots (solutions). The potential for multiple real roots increases with
the changes in sign in the sequence of Anderson book profits. Next, the return on investment is very sensitive to the initial loss. Products with a small initial investment may have large apparent returns but low profits as measured by other criteria as, for instance, term insurance. And, with some flexible premium and single premium designs, it is possible to have an initial gain followed by losses and then gains reemerging. Any or all of these factors can result in a statutory return on investment that may mislead the unwary into believing that the product is adequately priced and will provide the desired dividends to shareholders when, in fact, it may do neither.

One can avoid being misled by always considering additional profit measures: present value of profit as a percent of present value of premium; present value of profits; break-even year; profit per unit of in-force business. These measures are often computed on a "profits released" basis, that is, assuming that negative flows are provided by the shareholders and that positive flows are paid back to the shareholders. Analogous profit measures can be computed on a "profits retained" basis, that is, where the profits (including the initial loss) are retained and accumulated with the product. Another paper by Bradley M. Smith, "The Choice of the Proper Profit Objective," $T S A$, XXXV, along with its discussion, is also a valuable reference.

After a hurdle rate has been chosen, it should be compared to the cost of capital on a periodic basis. If the cost of capital shifts substantially, then either adjustment of the hurdle rate or a different strategy may be indicated. If the cost of capital becomes very much less than the hurdle rate, then perhaps it is appropriate to lower the hurdle rate. It may not be possible to keep the prior level, remain competitive, and grow. Another possibility is to keep the rate up by leveraging. If the cost of capital is low enough, funds to support the investment can be borrowed. However, this may require greater use of surplus relief reinsurance or changes in statutory accounting practices.

One way of pricing for the borrowing situation is to view the initial loss as being offset by a loan that is repaid over some number of years representing the pricing horizon. This approach produces an initial net loss of zero. (Note that this outcome makes computing a return on investment impossible as there is no investment. The method also assumes that there are no strange book profit patterns in later durations.) Subsequent book profits are reduced by the after-tax cost of the "loan" repayment. If $P$ represents the aftertax effect of the loan repayment, then in S. David Promislow's notation, the sequence of book profits is ( $0, A A B P_{1}-P, A B P_{2}-P, \ldots, A B P_{n}-P$ ). This sequence then can be investigated to determine various profit measures. (The
sequence also can represent a situation where a stock company might want to use a profits-retained approach using the cost of capital.)

As an interesting note, the sequence ( $I_{0}, A A B P_{1}, A B P_{2}, \ldots, A B P_{n}$ ) contains most of the financial information about the product. Most profit criteria are determined by analyzing this sequence. A similar result is true for profitsretained measures where the sequence contains the after-tax earnings on surplus.

The pricing actuary must not neglect analysis of after-tax financial results, because of statutory solvency requirements and the fact that a stock company cannot pay out shareholder dividends in excess of its after-tax statutory gain from operations. Previous comments demonstrate that other statutory profit measures also should be included in the analysis. But for publicly traded stock companies, return on GAAP equity is currently of principal importance. Whether or not the product is priced on a statutory basis, the actuary needs to examine GAAP returns as part of the pricing process. It may be too late to ensure adequate pricing if GAAP earnings are analyzed only at the time of earnings projections made for planning purposes or after the product has actually gone to market.

In his/her pricing analysis, the actuary needs to include not only statutory profit measures and cash flow analyses, but also the returns that the product provides, as measured on a GAAP basis. The challenge to the actuary is to synthesize the key items to present to senior management for decision making.

## CHARLIE T. WHITLEY:

Mr. Smith is to be congratulated on his choice of the simple example used in his paper. This choice enables him to illustrate various influences with apparent ease. I believe the relationship between ROI and ROE has been successfully explored.

The flow of this paper is such that a "revelation" seems to be presented when Mr. Smith observes

A different type of leveraging is needed: a leveraging of new business with old. If a corporate after-tax ROE objective equal to or even close to the ROI pricing objective is demanded, controlled growth is required.
I find it surprising that this revelation is not more directly evident in the paper's Conclusions. It is observed that the five relationship dependencies may each reflect this revelation indirectly, but greater attention could have been focused on it.

After some deliberation, I conclude that the author is yet laboring in the controlled growth vineyard when he introduces bench-mark surplus as a solution:
An appropriate solution would be to calculate the ROI on new products on the basis of a statutory "reserve" equal to the reserve using the desired valuation basis plus the additional amount needed as risk surplus to ensure the solvency of the line of business.

Mr. Donald R. Sondergeld has adequately discussed the appropriateness of using bench-mark surplus in his (TSA, XXXIV) paper, to which Mr. Smith has made reference. However, I suggest that the practical result of injecting positive risk surplus into the calculation of ROI will be the reduction of the ROI objective. I believe this observation of reduction in ROI objective to be of the same practical nature as Mr. Smith's recognition, earlier in the paper, of the market's role in pricing:
...the marketplace may not allow the success of an insurance product charging a premium that produces a return on investment of, say, 23.7 percent ...
This discussion has been written because I am convinced that my observation of reduction in ROI is needed to balance the author's suggestion of a 23.7 percent ROI. Absent this observation, the casual reader is free to conclude that no problems are expected from introducing risk surplus (using bench-mark surplus) in a pricing environment in which it has not previously been used, notwithstanding that the ROI objective remains unchanged and is, in fact, set equal to the ROE objective.

I congratulate Mr. Smith on a very interesting and worthwhile paper.

## DAVID N. INGRAM:

This paper provides a clear example of the relationships between ROI and ROE in several different circumstances. I believe, however, that the aftertax illustration is somewhat misleading. The magnitude of the difference between after-tax ROE and pricing ROI shown in table 6 is specific to the situation that has been assumed. The difference is caused by the treatment of the interest lost on the tax loss carry-forward. The magnitude of this lost interest is maximized (relative to equity) by the assumption of a completely new company, which has no gains to offset the losses created by the new business.

In any event, this is a real loss and should be reflected as such on the financial statement, as has been done. However, this expected tax situation also should affect ROI since the following statement is not always true:
...ROI is affected minimally, if at all, when pricing a product on an after-tax basis.
The example in the paper provides an illustration of a situation where this statement does not hold. The ROI calculation assumes immediate usage of tax losses that are not fully used for several years.

Pricing for a loss carry-forward is difficult but necessary for a company in that position, wishing to meet its after-tax return objective. I have seen three methods used to price for loss carry-forwards:

1. Treat each year's issues as a separate company that carries forward its own losses. In an existing company, this tends to overstate the impact of the loss carry-forwards. The ROE will exceed the ROI measured on this basis. For a new, growing company, the impact may be understated if there is a loss expiry situation.
$\backslash 2$. Assume zero taxes in calendar years until taxes are paid. This produces a price spiral as the date of tax payment and the repricing of new business approaches. This method is sometimes adopted by new companies, thinking they never will pay taxes.
2. Model the company operations. This is the only way to estimate the total cost of the loss carry-forward accurately. Then the lost interest can be spread over many years' issues in whatever way is reasonable.
As shown in the other examples, the lost interest and its recovery through pricing margins will emerge in proportion to equity (producing a level ROE) only if the accounting system spreads both the costs and the margins in proportion to equity.

## CLAUDE Y. PAQUIN:

Considering the difficulties inherent in predicting the future, it may appear somewhat presumptuous to set out to calculate precise rates of return in advance or to set objectives that do not seem arbitrary and perhaps pseudoscientific. There are strong misgivings within the actuarial profession over the simplistic yet appealing concepts of return on investment (ROI), return on equity (ROE), and other similar terms.

These misgivings are the consequence of the numerous traps that may be found in the associated computations. The basic concept of return on investment presupposes an initial investment. Yet some life insurance products produce a profit (however measured) in the year of issue: some mail-order
and health insurance business, for instance, or ancillary benefits like accidental death or waiver of premium. With no investment, how does one compute a return on investment?

Some products, such as reducing term life insurance, are notorious for their potential to produce years of negative earnings interspersed with years of positive earnings. When confronted with pluses and minuses, the computer typically solves for "the" rate of return, caring little whether the insurer earns that rate from the insured group or pays it out to that group.

The effect of income tax is also important, as it tends to decrease the amount of first-year investment on which a rate of return is demanded or computed. While income tax will reduce later profits, the computed or predicted rate of return often will rise as a consequence of paying income tax.

From all this, it would appear that the return-on-equity environment for the pricing of life insurance company products is simplistic and dangerous. It is very important that the actuary realize its dangers and educate those who would use the return-on-equity tool as to its limitations. A good stockbroker performs the same kind of function when he educates his client about the limitations of price-earnings ratios.

One more comment may be particularly appropriate on return-on-equity figures. Typically, in the sales process, some equity is spent for expenses, and the insurer hopes to recover it eventually (with a suitable rate of return) from the future income attributed to the sales. But some equity is simply tied up or earmarked into statutory reserves, without really being at risk. The first type of equity can be described as "equity at risk" and the latter as "equity at work." If theory be pursued to its logical end, spent equity, that is, equity at risk, should command a higher return (because of the risk of not getting it back) than equity at work. Surely an insurer's money tied up in deficiency reserves is less at risk than a sales commission already paid out to an agent. When rate-of-return objectives become ambitous, it may be wise to keep the difference in mind; it is analogous to the difference between an investment in U.S. Treasury securities and one in junk bonds. There should not be one rate for all.

In 1974, I had the opportunity to examine many GAAP concepts discussed in this paper and to report my findings in a paper published in TSA, XXV. It is informative to examine certain statements in Mr. Smith's paper in the light of these previous findings (which continue to have validity today).

First of all, it should be made clear that the DAC, or deferred acquisition cost, is a nebulous asset. It is not physical, like a building, a computer, or a desk. It is an intangible weaker than an account receivable, somewhat akin
to good will. Since it definitely and absolutely earns no interest, I find all references in this paper to the interest "earned" by the DAC to be unfortunate. My paper demonstrated that a DAC amortized with interest was equivalent to a sinking fund depreciation schedule. That type of depreciation schedule, favored by the railroads in former times to account for the replacement of their rolling stock, has hardly been seen since the 1930s. Only in life insurance accounting is it how commonly seen, though in a disguised form. The higher the interest rate used for amortizing the DAC, the slower the amortization and the higher the early "reported" earnings.

Mr. Smith's assertion,
GAAP for life insurance companies requires some degree of conservatism. This eliminates the use of the annual ROI pricing objective... [in developing a DAC amortization schedule]
is, in my view, not quite accurate. The Audit Guide (for GAAP) called for charging acquisition expenses "...against income in proportion to premium revenues recognized..." (Audits of Stock Life Insurance Companies, AICPA, 1972, at p. 72), but the words "in proportion to" obviously were not taken literally over the years. The use of interest to amortize the DAC, though resisted on sound theoretical grounds by some, has been common. It accelerates earnings in the early policy years and is obviously not so conservative as an amortization "in proportion to" premium income.

If the concept of amortizing expenses in proportion to premium income has virtually been relegated to the scrap heap of history, could one go so far as to use the ROI to amortize deferred acquisition costs? I merely observe that no authoritative document has been cited as indicating an actuary could not go that far.

The most interesting new wrinkle on DACs, and one to which Mr. Smith has not alluded in his comments on income tax, involves the application of the alternative minimum tax to life insurance company book (that is, GAAP) earnings following the adoption of the Tax Reform Act of 1986 in the United States. While the subject is complex (as is anything that touches upon taxes), the wisdom of preserving a certain degree of freedom in decelerating, rather than accelerating, GAAP carnings may eventually emerge. This is why it is so important to bring pertinent concepts concerning the amortization of DACs to the fore again.

I very much fear that corporate objectives of profitability and growth are, by and large, set arbitrarily and with little understanding of the nuances, exceptions, and pitfalls that inevitably lie ahead. Were I the coach of a
sports team, my objective would be to win every game my team played, or at least as many as possible. What would be wrong with a stock life insurance company whose objective would simply be to make - honestly of course - as much money as possible?

## (AUTHOR'S REVIEW OF DISCUSSION)

BRADLEY M. SMITH:
I would like to thank each of the respondents to this paper as their comments have shed additional life upon the relationship between the pricing objective of return on (statutory) investment (ROI) and the corporate objective of return on (GAAP) equity (ROE). By doing so, each has greatly increased the value of this paper.

Certainly the prior contributions of Mr. Donald Sondergeld to the actuarial literature examining this topic distinguish him as a leading authority in this area. His comments on this paper add very valuable and meaningful insight into the relationship between ROI and ROE, particularly on an after-tax basis. I would like to thank him individually for these comments.

Mr. David Becker makes many excellent points. His discussion of the stream of book profits and the timing of the recognition of such book profits, as well as the example he presents, adds insight to the problem of defining ROI. Recognizing the results obtained at issue of a product (time zero), although not universally accepted within the pricing community of the actuarial profession, clearly has merit. As stated, a company must be statutorily solvent at all times including immediately following issuance of a product. Additionally, stating the stream of book profits in this way eliminates many of the instances where statutory investment appears not to be required in producing the business.

Mr. Becker's method of splitting the year-one book profit into time 0 and end-of-year-one results deserves a few comments. Method A is defined as the Anderson book-profit stream. One technical correction is appropriate. Anderson book profits (as originally defined, TSA, XI) were discounted to the beginning of the year using the investment earnings rate rather than accumulated to the end of the year. Of the remaining possible splits, split C seems to me to be the most appealing because it retains the properties attributable to the Anderson book-profit stream.

As the investment required to produce a block of business usually is incurred in the first few years, it seems appropriate for the hurdle rate required of that block to reflect the existing cost of capital. Reasons against
changing the hurdle rate to reflect changes in the cost of capital generally hinge on the argument that life insurance contracts are long-term contracts and should reflect the average cost of capital over the long term. However, since the initial investment is made over a relatively short period of time, it seems more appropriate for the hurdle rate to reflect the current cost of longterm capital (plus the incremental risk rate associated with investing in life insurance contracts). I totally agree with Mr. Becker's comments on this subject.

The concept of calculating an ROI in which the initial investment is reduced by the amount of borrowed funds is not very appealing, as it generally renders the results meaningless. The result of the ROI calculation is changed by altering the initial "investment." Borrowing funds to produce new business may be appropriate in certain circumstances. However, "reduction" of the statutory strain associated with producing new business in the ROI calculation through the use of a loan is inappropriate. The economics of producing new business have not changed because funds are borrowed (the amount of statutory investment is the same whether it is from an asset or from the assumption of a liability). As stated previously, the hurdle rate should reflect this cost of capital. Calculation of an ROI on a product for which the initial statutory strain was minimized through the use of reinsurance is different from the borrowed funds example and is appropriate since the statutory surplus position of the company (and the investment required by the company) actually is altered when the product is reinsured.

I agree with Mr. Becker that the pricing actuary must anticipate the accounting method to be used with the product and examine the product's GAAP earnings and ROE emergence as well as its incremental effect on the company as a whole. In today's environment, company management may examine numerous decision-making criteria. Not meeting any one of these criteria may limit the acceptability of a given product.

Mr. Charlie Whitley comments on the effect of injecting positive risk surplus into the ROI calculation in the product development process. He believes that doing so will lower the ROI objective because of competitive pressures. Clearly, a company's competitive position in the marketplace has to be examined in the premium formulation process. It seems that premium setting has become as much art as science. I certainly did not mean to imply that premiums should be set in a mechanical fashion so as to return the desired level of return, with or without the injection of positive risk surplus.

Nor did I intend to imply that any desired return, at whatever level, could be obtained merely by setting the premium at the level required to produce such a return in a profit study calculation. Clearly, no (or few) sales of an uncompetitive product will result in a poor return to the company as revenues generally will not recover the expenses associated with the development of the product. Additionally, the persistency of an uncompetitive product most likely will not be as favorable as that of a more competitive counterpart. I agree wholeheartedly with Mr. Whitley that the pricing process is not strictly a mechanical one.

However, I believe that, in the long run, examination of the level of profitability and communication of those results along with information about the risks associated with the issuance of the product (as reflected in the assignment of required surplus when necessary) to senior managements of life insurance companies will prevent the life insurance industry from systematically underpricing its products. Such examination is a necessary step in the process of assuring that anticipated returns on the products offered are consistent with the risks undertaken.

Mr. David Ingram observes that the effect of federal income tax on the ROI calculation is dependent upon the timing of the utilization of tax losses created when issuing the product. Certainly, the ROI and ROE calculations need to be consistent as to timing of the utilization of these initial losses, as Mr. Ingram correctly points out. His comments along with those of Mr. Sondergeld provide insight into the complexities added when examining profitability on an after-tax basis and the variation in results obtained according to the assumed timing of utilization of tax losses.

I agree with Mr. Ingram in his assessment that modeling a company's operation is the only way to accurately estimate the timing of the utilization of these losses. Modeling also will help in the analysis of expense levels to be assumed by the new product as well as the incremental effect that the introduction of the product will have on the financial results of the company as a whole. The consistency of the pricing process with corporate objectives is facilitated by the use of company models in the pricing process, thus helping to eliminate management surprises in the future.

Mr. Claude Paquin has indicated that there are strong misgivings within the actuarial profession over the appropriateness of the use of ROI as a profit objective. He states that the use of ROI presupposes an investment and points out that some products do not require any investment at all. This certainly can be the case, but a more careful examination of a variety of products, particularly in light of Mr. Becker's comments on the timing of results (that
is, calculating profit at issue as well as at the end of each year) will reveal few products that require no initial statutory surplus investment. One example cited by Mr. Paquin, mail-order insurance, generally requires some initial investment, as the cost of mailing a solicitation, no matter what the response, is incurred prior to the receipt of any revenue.

Products or product lines requiring no initial statutory investment generally are those tending to exhibit highly volatile results. Examination of such products generally has led me to the conclusion that the required statutory reserve should be supplemented with some required surplus allocation, which represents an investment in the production of this business.

As discussed in my previous paper referred to by Mr. Becker, ROI should be used as a primary profit objective when a company's limiting resource is statutory surplus. If statutory surplus is not a limiting resource, the value of the company (as measured by the discounted value of future profits) will not necessarily be maximized by using ROI as a pricing objective for new products. A profit objective that maximizes profit per unit of limiting resource should be used. Likewise, if statutory surplus is a limiting resource, the sale of profitable products not requiring a statutory surplus investment should be emphasized.

The purpose of this paper was to examine the relationship between ROI (a measure with which actuaries are generally comfortable and familiar) and ROE, which, whether it is appropriate or not, is used to judge the performance of stock life insurance company management by outsiders (stock, analysts, directors, rating agencies) not necessarily aware of the intricacies and dynamics of life insurance company performance. The purpose was not to advocate the use of ROE as such a measure, but only to understand those items that can affect it.

While it is clear that different products entail different risks and different degrees of risk and therefore should command different rates of return, it is not clear to me that the risk necessarily is related to the amount of "equity at risk" versus the amount of "equity at work," as defined by Mr. Paquin. The use of a reserve that includes any target surplus required to insure the solvency of the line of business greatly blurs this, at best, ambiguous distinction.

I was confused by Mr. Paquin's comments concerning the amortization of DAC. On the one hand, he seems to be arguing that the amortization of DAC with interest is inappropriate. Then he seems to suggest that the amortization of DAC using an investment rate equal to the anticipated ROI on the product could be acceptable. Clearly if acquisition expenses are to be
charged "...against income in proportion to premium revenues recognized..." (Audits of Stock Life Insurance Companies, AICPA, 1972), the time value of money needs to be recognized. In fact, the Audit Guide (page 75) goes on to state the following:
...the interest assumption for each block of new issues should not be inconsistent with such factors as actual yields, trends in yields, porffolio mix and maturities, and a company's overall investment experience generally. Since life insurance involves long-term obligations and investment risks, the assumed interest rate should include provision for the risk of adverse deviation from such estimates. Generally, the interest assumption to be used in computing reserves in conformity with generally accepted accounting principles should be based on the estimate of future interest expected at the time that the policies are issued.
Finally, Mr. Paquin asks
What would be wrong with a stock life insurance company whose objective would be simply to make - honestly of course - as much money as possible?
My response is that this is an acceptable objective if "as much money as possible" is enough to justify the risks undertaken and the capital required in doing so.

## ACKNOWLEDGMENT

I would like to thank Mr. Jim Hawke for helping to clarify the various issues at work in the examination of the relationship between ROI and ROE. Our dialogue helped to clarify the issues involved and proved to be very useful.


[^0]:    ${ }^{1}$ Also referred to as internal rate of return (IRR) and yield rate.
    ${ }^{2}$ For a further discussion of the limiting resource concept and hurdle rate see Bradley M. Smith, "'The Choice of the Proper Profit Objective," TSA, XXXV (1983): 367-391.
    ${ }^{3}$ Defined as return on total capital (ROTC) in Donald R. Sondergeld, "Profitability as a Return on Total Capital," TSA, XXXIV (1982): 415-433.

[^1]:    ${ }^{4}$ See James C.H. Anderson, "Gross Premium Calculation and Profit Measurement for Nonparticipating Insurance," TSA, XI (1959): 357-420.

[^2]:    ${ }^{\text {sch }}$ Calied Benchmark Surplus in "Profitability as a Return on Total Capital," TSA XXXIV and Required Surplus in the discussion that follows it.

[^3]:    ${ }^{6}$ This is true only if, when calculating the increase-in-reserve component of book profit in a particular year, the reserve at the end of the year is discounted at a rate equal to the ROI (or if the reserve at the beginning of the year is accumulated at a rate equal to the ROI ) rather than the investment earnings rate for that year.

