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THE APPLICATION OF THE COMMISSIONERS ANNUITY RESERVE METHOD TO FIXED SINGLE PREMIUM DEFERRED ANNUITIES

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ABSTRACT

In the last four years, almost all states have formally or informally adopted the Commissioners Annuity Reserve Method (CARM) as the basis for valuing individual account annuities in statutory statements. While the CARM has been reviewed in the *NAIC Proceedings*, there has not been a discussion of the application of the CARM in the actuarial literature. The paper reviews the CARM and how, in the author's opinion, the various provisions commonly included in modern deferred annuity policies should be reserved to be consistent with the CARM. While the paper concentrates on fixed single premium deferred annuities, the principles are equally applicable in other fixed (as distinguished from variable) annuities.

I. INTRODUCTION

In 1976 the (C3) Technical Task Force of the NAIC recommended the adoption of a standard procedure for valuing fixed (as distinguished from variable) annuities in the Convention Blank. The procedure, termed the Commissioners Annuity Reserve Method (CARM), is described as follows:

> Reserves according to the Commissioners annuity reserve method for benefits under annuity or pure endowment contracts, excluding any disability and accidental death benefits in such contracts, shall be the greatest of the respective excesses of the present values, at the date of valuation, of the future guaranteed benefits, including guaranteed nonforfeiture benefits, provided for by such contracts at the end of each respective contract year, over the present value, at the date of valuation, of any future valuation considerations derived from future gross considerations, required by the terms of such contract, that become payable prior to the end of such respective contract year. The future guaranteed benefits shall be determined by using the mortality table, if any, and the interest rate or rates specified in such contracts for determining guaranteed benefits. The valuation

considerations are the portions of the respective gross considerations applied under the terms of such contracts to determine nonforfeiture values.[1]

With only a handful of exceptions, the states have adopted the CARM as the standard for valuing individual annuities. In the few states that have not yet formally enacted the CARM into law, the method should be acceptable because it produces reserves that are stronger than those produced by most other current methods.

This paper will review how the CARM is applied to a "vanilla" single premium deferred annuity policy. It will then consider the effect of additional or alternative policy provisions that are used in single premium deferred annuity policies.

II. THE BASIC CARM

Volume I of the 1977 *NAIC Proceedings* presented an example of the CARM as it applies to a basic single premium policy. The sample policy had the following characteristics:

1. Single premium: $10,000.
2. Guaranteed interest:

Contract Year	Rate
1	9%
2–5	8
6–10	7
11+	3

3. Surrender charge:

Contract Year	Rate
1	10%
2	9
3	8
4	7
5	6
6–19	5
20+	0

4. Valuation interest rate: 5½ percent.

Note that the sample policy has a back-end load, or surrender charge, as opposed to a front-end load which is immediately subtracted from each premium payment. The CARM, of course, applies to either type of policy.

[1] *Proceedings of the National Association of Insurance Commissioners*, I (1977), 490.

The table in the *Proceedings* (Table 1 in this paper) illustrates the application of the CARM. The CARM reserve is found in the column showing the present values of the tenth-year cash value because this column has the largest present values.

The *Proceedings* also showed a comparison of the \$10,000 single premium accumulated with interest before applying the surrender charge, the cash surrender value, and the terminal reserve. That comparison is shown here as Table 2. This table is very informative because it clearly indicates the principle of the CARM:

> If the combined effect of the guaranteed interest rate plus the reduction in the surrender charge exceeds the valuation interest rate for t years, then the greatest present values will occur by discounting the cash value at the end of the tth contract year. If the combined effect of the guaranteed rate plus the reduction in the surrender charge is sometimes greater and other times less than the valuation interest rate in a variable fashion, then it will be necessary to discount the cash values at most points to find which is the greatest discounted cash value.

In the example in the *Proceedings*, the effective interest rate in the tenth contract year is 7 percent, changing to 3 percent in the eleventh and later contract years. Because the surrender charge is level for years 6 and later, it has no impact on the effective interest rate in the eleventh and later contract years.

If, however, the surrender charge for contract years 11 and later had been 0 percent rather than 5 percent, the effective guaranteed interest rate for the 11th contract year would be 8.42 percent rather than 3 percent. Replacing the illustrated eleventh-year cash surrender value (CSV_{11}) of \$20,352 with the accumulated gross considerations of \$21,423 (which would be the cash surrender value if there were no surrender charge) produces the eleventh-year effective interest rate:

$$\begin{aligned}\text{Eleventh-year effective interest rate} &= \frac{CSV_{11} - CSV_{10}}{CSV_{10}} \\ &= \frac{21{,}423 - 19{,}759}{19{,}759} \\ &= 8.42 \text{ percent}\,.\end{aligned}$$

The CARM reserves would then be equal to the discounted present values at 5½ percent of the eleventh-year cash surrender value of \$21,423 (rather than \$20,352), because these values are greater than the discounted tenth-year cash surrender values. Table 3 compares the discounted values of the cash surrender values available at the end of the tenth and eleventh

TABLE 1

Illustration of Commissioners Annuity Reserve Method

Contract Year of Valuation	Cash Surrender Values Available at End of Contract Year										
	1	2	3	4	5	6	7	8	9	10*	11
	$9,810	$10,713	$11,697	$12,770	$13,939	$15,074	$16,129	$17,259	$18,466	$19,759	$20,352
	Present Value of Above Cash Surrender Values at 5½% Interest										
0	$9,299	$ 9,625	$ 9,961	$10,308	$10,665	$10,932	$11,088	$11,246	$11,405	$11,568	$11,294
1	9,810	10,155	10,509	10,875	11,252	11,534	11,697	11,864	12,032	12,204	11,915
2	....	10,713	11,087	11,473	11,871	12,168	12,341	12,517	12,694	12,875	12,570
3	....	....	11,697	12,104	12,524	12,837	13,020	13,205	13,392	13,583	13,261
4	....	....	....	12,770	13,212	13,543	13,736	13,932	14,129	14,330	13,991
5	....	....	....	....	13,939	14,288	14,491	14,698	14,906	15,118	14,760
6	....	....	....	....	....	15,074	15,288	15,506	15,726	15,950	15,572
7	....	....	....	....	....	....	16,129	16,359	16,591	16,827	16,428
8	....	....	....	....	....	....	....	17,259	17,503	17,753	17,332
9	....	....	....	....	....	....	....	....	18,466	18,729	18,285
10	....	....	....	....	....	....	....	....	....	19,759	19,290
11	....	....	....	....	....	....	....	....	....	....	20,352

Source.—*Proceedings of the National Association of Insurance Commissioners*, I (1977), 541.

* The present values in this column are the largest and are used for the reserves.

TABLE 2

COMPARISON OF CASH SURRENDER VALUES AND CARM TERMINAL RESERVES

End of Contract Year	Guaranteed Interest Rate	Gross Consideration Accumulation	Percentage for Surrender	Cash Surrender Value	CARM Terminal Reserves
0	9%	$10,000	90%	$ 9,000	$11,568
1	9	10,900	90	9,810	12,204
2	8	11,772	91	10,713	12,875
3	8	12,714	92	11,697	13,583
4	8	13,731	93	12,770	14,330
5	8	14,829	94	13,939	15,118
6	7	15,867	95	15,074	15,950
7	7	16,978	95	16,129	16,827
8	7	18,167	95	17,259	17,753
9	7	19,438	95	18,466	18,729
10	7	20,799	95	19,759	19,759
11	3	21,423	95	20,352	20,352
12	3	22,066	95	20,963	20,963
13	3	22,728	95	21,592	21,592
14	3	23,409	95	22,239	22,239
15	3	24,112	95	22,906	22,906

SOURCE.—*Proceedings of the National Association of Insurance Commissioners*, I (1977), 540.

TABLE 3

TABLE OF PRESENT VALUES OF SURRENDER VALUES ASSUMING NO SURRENDER CHARGE AFTER TENTH POLICY YEAR

END OF CONTRACT YEAR OF VALUATION	PRESENT VALUES AT 5½% OF:	
	10th-Year Cash Value	11th-Year Cash Value
0	$11,568	$11,888
1	12,204	12,542
2	12,875	13,231
3	13,583	13,959
4	14,330	14,727
5	15,118	15,537
6	15,950	16,391
7	16,827	17,293
8	17,753	18,244
9	18,729	19,248
10	19,759	20,306
11	....	21,423

years assuming a 0 percent surrender charge for the eleventh and later years.

Once the principle of the CARM is understood, it can be useful in designing products. For example, it is possible to design a policy such that the CARM reserve never exceeds the cash surrender value, given the desired annual decrease in the surrender charge. If, in a particular contract year, the surrender charge of a single premium deferred annuity decreases by 1 percent from 5 to 4 percent, the contract could guarantee 4.4 percent and still use the cash value as the reserve, because the decrease in the surrender charge, plus the guaranteed interest, still results in an effective annual yield of exactly 5½ percent:

$$\begin{aligned}\text{Effective interest rate} &= \frac{(AC_{t+1})(0.96) - (AC_t)(0.95)}{(AC_t)(0.95)}\\ &= \frac{(AC_t)(1 + 0.044)(0.96)}{(AC_t)(0.95)} - 1\\ &= \frac{(1.044)(0.96)}{(0.95)} - 1 = 5.5 \text{ percent },\end{aligned}$$

where AC_t is the accumulated consideration at the end of policy year t. This concept is important if it is desired to use a guaranteed interest rate that does not produce a reserve strain and is other than a very nominal guarantee.

III. CARM RESERVES FOR OTHER BENEFITS

Annuities now contain several ancillary benefits that must be considered when establishing CARM reserves. These benefits definitely change the guaranteed cash values under a policy and, therefore, cannot be ignored when establishing CARM reserves.

Among the more common benefits are a death benefit prior to annuitization, a "bail-out" or "window" option, a money-back guarantee, free partial withdrawal, a no-partial-withdrawal bonus, and no surrender charge on early annuitization. The effect of each of these on CARM reserves will now be discussed.

A. *Death Benefit Prior to Annuitization*

In most single premium deferred annuity policies, there is a preannuitization death benefit equal to the premiums paid or the cash surrender value, if greater. It is also common to pay the account value without any surrender charge in the event of the annuitant's death prior to annuitization. Therefore, it is necessary to establish a reserve in addition to the formula reserve to cover the possibility of preannuitization death.

Theoretically, the death benefit reserve could be calculated as a single premium decreasing term benefit where the assumed death benefits are based on future cash values at the guaranteed interest rates. Assuming a 10 percent declared interest rate and a 6 percent first-contract-year surrender charge, a death benefit equal to the account value would exist only for eight months (see below). The single premium *at issue* for such a benefit for a male aged 50 based on 1958 CSO, 5½ percent, curtate functions, age last birthday, would be 18.1 cents per $1,000 single premium, using the death benefits in column 4 of the accompanying table. If the actual policy reserve is available, it could be used in the calculation instead of the cash value.

Beginning of Month t (1)	Guaranteed Account Value (2)	Cash Value [0.94 × (2)] (3)	Death Benefit [$1,000 − (3)] (4)
1	$1,000	$ 940	$60
2	1,008	948	52
3	1,016	955	45
4	1,024	963	37
5	1,032	970	30
6	1,041	979	21
7	1,049	986	14
8	1,057	994	6
9	1,066	1,002	None

Since the extra reserve for the death benefit is quite small and the benefit disappears rapidly, it may be preferable to establish an approximate reserve. In the case of the standard policy, the procedure could be the following:

$$\text{Death benefit reserve} = A \times \frac{B}{12} \times \frac{C}{2} \times \frac{D}{2},$$

where

A = Premiums for single premium annuities received in the last twelve months prior to the valuation date;
B = Number of months death benefit exists;
C = First-year surrender charge; and
D = c_x, where x is the average issue age.

For example, for a company issuing \$10,000,000 in single premiums where the death benefit exists for eight months, the reserve for the extra death benefit would be

$$\text{Reserve} = \$10{,}000{,}000 \times \frac{8}{12} \times \frac{0.06}{2} \times \frac{0.00789}{2} = \$789$$

because

$$A = \$10{,}000{,}000\ , \quad B = 8\ , \quad C = 6 \text{ percent}\ ,$$

$$D = c_{50} = 0.00789 \text{ (1958 CSO, male, curtate, ALB, 5.5 percent)}\ .$$

Earlier, the reserve *at issue* per \$1,000 single premium was calculated as 18.1 cents, which is \$1,810 per \$10 million single premium. The \$789 is a reasonable estimate of the actual reserve because logically the reserve on a valuation date will be less than half the initial reserve. This occurs because the policies are usually issued rather uniformly over the twelve months preceding the valuation date and the benefit being reserved is a form of single premium decreasing term insurance.

The approximation of a small reserve is normally acceptable to state insurance and independent auditors as long as the error is nominal. The proposed formula calculates a reserve that should meet this criterion under the vast majority of circumstances.

B. *"Bail-out" or "Window" Option*

A growing number of annuities contain a provision that allows for surrender at the current value *without* penalty if the declared interest rate goes below some stated value. Does this mean that the company is actually guaranteeing at least the level of interest in the "bail-out" provision during the initial policy years where there is a surrender charge? If so, the guaranteed cash values to use in the CARM must be based on this guaranteed interest rate rather than the rate (or rates) stated in the policy. Or, does the bail-out provision mean that a company must establish a reserve at least equal to the current cash value plus any applicable surrender charge?

It is the author's opinion that a proper reserve for a single premium deferred annuity with this provision under the CARM must consider the effect of the bail-out provision. While it is not necessary to assume that the company will guarantee the bail-out interest rate, the reserve should at least be equal to the current cash values plus any applicable surrender charge.

Assuming that the bail-out interest rate is at least equal to the valuation rate, a reserve that assumes that future guaranteed interest rates will revert to the contract guarantee but ignores the waiving of the surrender charge may not be in compliance with the CARM, because it is not discounting the greatest potential cash surrender values. It does not seem proper to assume the best of both worlds—namely, a low future interest rate to avoid "deficiency reserves," and no waiving of the surrender charge.

When the bail-out is either temporary or only slightly above the valuation rate, a situation could occur in which the CARM reserve using the bail-out interest rate as the guaranteed rate would be lower than the current cash values plus any applicable surrender charge. Since the company has the option of paying at least the bail-out interest rate and can avoid waiving the surrender charge, it should be allowed to hold these reserves if they are lower. The choice of methods should be consistent within a block of business subject to the same current interest rate, since the company presumably must treat all policies in the block similarly.

A variation of the fixed-rate bail-out provision is the provision where the bail-out option is exercisable if the declared interest rate falls below a percentage of the prime rate or some other nonfixed interest rate. The CARM never contemplated this provision, nor is there a clear way to establish reserves for this provision under the CARM. It is not logical to ignore this provision when developing reserves if, as the author believes, a fixed-rate bail-out provision might require adding the surrender charge to the basic CARM reserve.

The most direct reserve for a policy with the non-fixed-rate bail-out provision would be the cash value plus the surrender charge (assuming that the effective future interest guarantees are at or lower than the valuation rate). While there may be some other logical reserve, the author does not see how any other reserve can be used at this time and still be consistent with the CARM. Of course, should the NAIC or individual states establish special reserve guidelines for this provision, they, rather than the proposed method, would govern.

C. *Money-back Guarantee*

Many single premium deferred annuities contain a clause that allows the policy to be surrendered at any time for the larger of the single premium and the cash value. The length of time during which the single premium is the amount that will be paid on surrender is a function of the interest rates that have been declared and the applicable surrender charge.

The proper reserve for a contract with the money-back guarantee provision is the larger of the single premium less any partial withdrawals and the CARM formula reserve held by the company. This comparison should be made for each individual policy.

Obviously, the higher the interest rates that have been declared by the company, the shorter the period during which the money-back guarantee affects the reserve. Larger surrender charges lengthen the period when the reserve will be the single premium rather than the CARM calculated reserve. For example, with current high interest rates, a 10 percent declared rate coupled with a 6 percent first-year surrender charge would require the use of the single premium as the reserve for eight months from the date of issue.

Another way to view the problem is to substitute, if appropriate, the single premium for the year-end cash value in the CARM formula. However, this approach will fail to recognize that the period prior to the end of the first contract year is normally the period when the single premium itself is the proper reserve. Even though by the end of the first contract year the single premium is no longer needed as the reserve, it must be used during the first contract year if it is greater than the CARM reserve.

D. *Free Partial Withdrawals*

It is not uncommon to find a policy that permits the withdrawal of up to 5 or 10 percent of the current contract value without imposition of any surrender charge, even during the policy years in which the surrender charge applies. This may be an attractive feature to an annuitant because it allows for some withdrawal of money (essentially a partial surrender) without any charge.

When a contract contains such a clause, it should be considered in the reserve formula. The guaranteed cash value to use in the CARM formula should include an allowance for the free partial withdrawal. The following example shows how this concept would operate.

Suppose that the policy permits a 10 percent free partial withdrawal per year and that the policy contains a 2 percent surrender charge for the current year followed by a 1 percent surrender charge in the year after. In the third year the surrender charge disappears. Further, for simplicity, assume that the guaranteed the valuation interest rates are both 5½ percent.

End of Year	Accumulated Value	Surrender Charge	Cash Value
1	$10,550	$211	$10,339
2	11,130	111	11,019
3	11,742	0	11,742

The guaranteed values under a $10,000 single premium policy are shown in the accompanying table. But at the end of year 1, a policyholder could actually receive 10 percent of $10,550 and 98 percent of 90 percent of $10,550, for a total of $10,360, because of the free partial withdrawal privileges. Therefore, it is necessary to determine CARM reserves using guaranteed cash values defined as follows:

$$CSV_t = [(1 - FPW)(1 - SC_t)(AC_t) + (FPW)(AC_t)](1 - FPW)^{n-1},$$

where

FPW = Free partial withdrawal percentage;
n = Years from valuation date to end of policy year t;
SC_t = Surrender charge for policy year t;
AC_t = Accumulated consideration at end of policy year t.

In the example, the proper table of cash surrender values for each policy year to use in establishing CARM reserves would be that shown below, rather than just the guaranteed cash values without considering any partial surrender.

Contract Year	Cash Values at End of Contract Year		
	1	2	3
1	$10,360	....	....
2	9,927	$11,030	....
3	9,511	10,568	$11,742

Obviously, because the cash surrender values including the free partial withdrawal privilege at the start of each policy year are slightly greater than those that do not assume any free partial withdrawals, this approach requires greater reserves than if the free partial withdrawals are ignored. Each year a new table of cash values must be generated and tested for each policy, based on its actual partial-withdrawal history.

Some policies treat partial withdrawals followed by complete withdrawals within a certain period of time as a single complete withdrawal, and charge the surrender charge against the partial withdrawal at the time of the complete withdrawal. This would reduce the effect of the free partial withdrawal privilege on the reserve, if the company is willing to calculate the exact CARM reserves for such a policy.

E. *No-Partial-Withdrawal Bonus*

At least one company credits an additional 10 percent of the original premium to the accumulated value at the end of the tenth policy year (and every five years thereafter), provided that no partial withdrawals have been made from the policy. It also has provision for partial credits if partial withdrawals do not exceed specified percentages of the original premium.

In the absence of any partial withdrawals, the proper valuation procedure under the CARM is to assume that 10 percent of the original premium will be credited to the cash value on policy anniversaries 10, 15, 20, and so on. Once partial withdrawals have been made, the valuation must include provision for whatever partial credit will be earned assuming no future partial withdrawals. Thus, the *total* cash values including the "no-partial-withdrawal bonuses" must be discounted in the CARM calculations. Since the cash value will increase at the bonus years, each of these cash values (after the increase) must be discounted, as well as the cash value at the point where the discounted value would otherwise be highest, in order to determine the CARM reserve.

The record keeping for this benefit is similar to that needed for disability policies with "good health" awards. It is definitely necessary to maintain a record of the partial withdrawals on a policy-by-policy basis, unless conservative reserves are to be held using percentages of the contract values calculated on the assumption that no partial withdrawals have been made.

F. *No Surrender Charge on Early Annuitization*

In the event that an annuitant elects to receive an early commencement of annuity benefits, some deferred annuity policies will waive any applicable surrender charge. The logical question raised by this provision is whether or not the cash value to be discounted in the CARM calculation can take credit for the surrender charge.

When the guaranteed annuity values on the contract do not exceed statutory maximums, the author believes that it would still be appropriate to base CARM reserves on the cash value after any applicable surrender charge. The reasoning behind this statement is that the frequency of conversion to annuity payout status is very low and that the annuity benefits offered normally make provision for profit margins that offset most of the loss that might occur from waiving a surrender charge.

Of course, the actuary must evaluate each set of circumstances to determine whether significant early annuitizations occur and whether the straightforward CARM makes adequate allowance for these conversions. If it does not, then it is incumbent upon the actuary to incorporate an adequate extra reserve to reflect the experience.

IV. OTHER RELATED AREAS

With the modern computer, companies should always credit interest on an exact daily basis if there is to be a proper matching of investment income and policyholder benefits. If a company were to credit 10 percent per year and figured the daily rate as $0.10/365 = 0.000273973$ rather than $(1.10)^{1/365} = 0.000261158$, it would pay an extra 0.52 percent interest per year. The additional amount credited would severely eat into the profit margins of many current products.

An important consideration for any company entering the annuity field is to carefully match its investment policy with its annuity objectives. A company cannot try to credit a high current yield if its new money is being invested in long-term bonds when short-term investments have a substantially greater yield.

In some instances policy loans may affect cash values. If so, they may also affect reserve calculations. Each set of policy provisions must be examined in light of CARM principles.

The current valuation law distinguishes between individual and group contracts. For the former type of contract, it permits a maximum valuation interest rate of 5½ percent for single premium deferred annuities, while for the latter, it allows 7½ percent or more. Where there are individual accounts held for each certificate holder of a group contract, it seems that the 7½ percent maximum valuation rate applies, rather than the higher rates generally used for other types of group annuity contracts.

The significant point is that a single premium deferred annuity sold as a group contract can result in a reduced reserve requirement as compared with the same contact sold on an individual basis. The 2 percent differential in the valuation rate between group and individual contracts will reduce surplus drain if interest guarantees or other benefits that bring the effective guaranteed interest rate above 5½ percent are offered.

The 1980 valuation amendments eliminate the difference between the valuation interest rates applicable to individual and group annuity contracts. This will remove the necessity to designate annuity contracts as group or individual solely for valuation purposes.

Variable rather than fixed deferred annuities are becoming increasingly popular. Among the reasons for variable annuities are the somewhat more liberal reserve requirements. For example, since future experience determines cash values, a variable annuity contract can credit a high and competitive return but not be burdened with having to discount future interest guarantees at 5½ percent for individual contracts. Thus, variable annuities do not involve "deficiency reserves."

Nevertheless, variable annuity reserves are not always straightforward. It would seem that back-end-loaded variable annuities must be reserved without taking credit for the possible surrender charge. This can be a severe handicap if a company does not have sufficient surplus to absorb the strain. For this reason, it is likely that a CARM for variable annuities will eventually be developed.

The popularity of annuities as vehicles for savings dollars makes them very important to the insurance industry. The CARM has gone a long way to answer actuaries' questions as to proper reserves for annuities. More questions remain, however, and it is hoped that this article will stimulate actuaries to resolve some of the pending issues.

DISCUSSION OF PRECEDING PAPER

HOWARD BLAKESLEE:

Mr. Jaffe's timely paper on Commissioners Annuity Reserve Method (CARM) reserves for "fixed single premium deferred annuities" handles some interesting questions, many of which we at New York Life have been pondering. Mr. Jaffe has given us a good track to run on in resolving these questions.

My intent in this discussion is to elaborate on some of the concepts covered by Mr. Jaffe and to mention the reserve impact of some additional policy provisions. I hope this will prove useful to those who must deal with the details of determining reserves that meet CARM requirements. Like Mr. Jaffe's paper, this discussion is applicable to any indeterminate premium plan, since CARM requires taking future premiums into account only if they are "required by the terms of such contract." This discussion will be organized as follows:

1. Definition of symbols, including concepts of a high guaranteed interest rate where the guarantee is renewed periodically, and two approaches to free partial withdrawal.
2. CARM formulas and relationships.
3. A discussion of CARM formulas as to how the job of coming up with CARM reserves might be simplified, and some possible pitfalls in simplification.
4. Other policy provisions and how they might be handled in computing reserves that meet CARM requirements.
5. Comments on contractually guaranteed rates.
6. Conclusion.

1. *Definition of Symbols*

Mr. Jaffe's symbols are used. In addition, the following symbols are needed.

Symbols for concepts discussed by Mr. Jaffe:

t = Current policy year.

$MV_{t-\Delta t}$ = CARM minimum reserve for a policy that has completed the $(1 - \Delta t)$th portion of policy year t (see below for definition of Δt).

$e_r^{\Delta r}$ = Effective interest rate (based on the cash surrender value) for the period from duration r to duration $r + \Delta r$. This rate equals $(CSV_{r+\Delta r}/CSV_r) - 1$. (Mr. Jaffe defines this rate and calculates a value of 8.42 percent (e_{10}^1) for a particular set of circumstances.)

g_r = Effective annual interest rate guaranteed contractually or otherwise during policy year r (note that ${}^c g$ overrides g_r during a portion of the current policy year; see below). For $r > t$ it is assumed that g_r either decreases or remains level as r increases.

i = Minimum basis valuation interest rate for individual annuities ($4\frac{1}{2}$ percent for flexible premium annuities and $5\frac{1}{2}$ percent for single premium annuities in states that have not yet adopted the 1980 NAIC amendments).

${}^a W_r$ = $(1 - FPW)(1 - SC_r) + FPW = 1 - SC_r(1 - FPW)$. Mr. Jaffe also includes the term $(1 - FPW)^{n-1}$, which I found confusing and which seems unnecessary in analyzing CARM. The prefixed superscript a stands for "approximate." See definition of ${}^e W_r$ for further explanation.

Symbols for other concepts:

${}^c g$ and Δc: Many companies now guarantee a high rate of interest for a relatively short period, this guarantee being renewed at the end of each period. In computing CARM reserves, it is necessary to take the current (effective annual) high guaranteed rate (denoted by ${}^c g$) into account. For the current policy year the remaining guaranteed interest rates are thus ${}^c g$ to the end of the current guarantee period, and g_t thereafter. The variable Δc will be used to denote the remaining portion of the current guarantee period as of the valuation date, expressed as a portion of a full year. Therefore, if the current guarantee period is three months and has two months to run, Δc is 2/12.

AAC_r and AF_r: Some companies make a deduction for administrative expenses, which I will call AAC_r (annual administrative charge, deducted at the end of policy year r). The symbol AF_r (accumulated fund after r years) will be used to denote the accumulated consideration (AC_r) less accumulated administrative expenses, and less any partial withdrawals occurring prior to the valuation date.

Δt: Since the actual valuation is done at some point during each policy year, the variable Δt will be used to denote the portion of the policy year from the point of valuation to the end of policy year t. Note that at the point of valuation, the rate ${}^c g$ applies for Δc of the policy year, and the rate g_t applies for $(\Delta t - \Delta c)$ of the policy year. This latter period will be negative if ${}^{c}g$ extends into policy year $t + 1$. In this case g_{t+1} would have to be adjusted to appropriately reflect ${}^{c}g$ for a portion of policy year $t + 1$.

${}^{e}W_r = (1 + g_r - FPW)(1 - SC_r) + FPW = 1 - SC_r(1 - FPW) + g_r(1 - SC_r) = {}^{a}W_r + g_r(1 - SC_r)$: This is the "exact" counterpart of ${}^{a}W_r$ and is applicable to the fund at the beginning of policy year r. Mr. Jaffe's expression is applicable to the fund at the end of each policy year, but as a practical matter FPW must be defined in terms of the fund at the beginning of each policy year. Thus, a 10 percent free partial withdrawal is assumed to mean that "during each policy year the policyowner may withdraw up to 10 percent of the fund at the beginning of that policy year without incurring a surrender charge."

$RFPW_t$: This is the remaining dollar amount of free partial withdrawal in the current policy year. Under the "exact" free partial withdrawal method, $RFPW_t$ would equal the greater of zero or the accumulated fund at the beginning of the current policy year (AF_{t-1}) times FPW, less any amounts actually withdrawn in the current policy year. This amount should be available on company records, since it is needed to administer the free partial withdrawal provision.

2. *CARM Relationships and Formulas*

$$AF_t = AF_{t-\Delta t}(1 + {}^{c}g)^{\Delta c}(1 + g_t)^{\Delta t - \Delta c} - AAC_t\,;$$

$$AF_{t+r} = AF_{t+r-1}(1 + g_{t+r}) - AAC_{t+r} \quad \text{for } r = 1, 2, \ldots .$$

$$CSV_{t-\Delta t} = (AF_{t-\Delta t} - RFPW_t)(1 - SC_t) + RFPW_t\,;$$

$$CSV_t = (AF_t - RFPW_t)(1 - SC_t) + RFPW_t\,;$$

CSV_{t+r} for $r = 1, 2, \ldots$ by exact method:

$$CSV_{t+r} = [AF_{t+r-1}(1 + g_{t+r}) - AAC_{t+r} - FPW(AF_{t+r-1})]$$
$$\times (1 - SC_{t+r})\, AF_{t+r-1} FPW$$
$$= {}^{e}W_{t+r}\, AF_{t+r-1} - AAC_{t+r}(1 - SC_{t+r})\,.$$

CSV_{t+r} by approximate method:[1]

$$CSV_{t+r} = {}^{a}W_{t+r}\, AF_{t+r}\,.$$

[1] Since reserves according to this method exceed the exact-method reserves and since this method is simpler, it seems appropriate to present it here.

$MV_{t-\Delta t}$ is the largest of the following:[2]

$$AF^{\psi}_{t-\Delta t}; \quad CSV^{\#}_{t-\Delta t}; \quad CSV^{\#}_{t} \div (1+i)^{\Delta t};$$

$$CSV_{t+r} \div (1+i)^{\Delta t+r} \quad \text{for } r = 1, 2, \ldots.$$

3. *Discussion of CARM Reserve Formula*

For any company with a computer, the easiest approach is to program the above calculations and tests and perform them for every policy. The resulting reserves could then be summarized and displayed in whatever manner was necessary. However, there are ways to reduce the complexity of the work and/or facilitate the checking process. One method is to use $AF_{t-\Delta t}$ as the reserve. There are two potential problems with this approach. First, if $AF_{t-\Delta t} > MV_{t-\Delta t}$, the surplus strain may be too high. Second, if $AF_{t-\Delta t} < MV_{t-\Delta t}$, then reserves will not meet minimum standards. Depending on the particular circumstances of each company's operations, the actuary may be able to ascertain that $AF_{t-\Delta t}$ produces an aggregate reserve that both exceeds minimum standards and produces an acceptable surplus strain. For example, this method will work for policies with short high interest guarantee periods and high surrender charges, since ignoring the surrender charge will offset the effect of the high interest rate.

A second way is to use the principle of CARM to find which cash surrender value will have the greatest present value. I would modify Mr. Jaffe's wording of the CARM principle by substituting the words "effective interest rate based on the cash surrender value" (i.e., $e_t^{\Delta t}$) for his words "combined effect of the guaranteed interest rate plus the reduction in the surrender charge" in the two places this phrase appears.

In order to use the principle of CARM, values of $e^{\Delta t}_{t-\Delta t}$ and e^{1}_{t+r} $(r = 0, 1, 2, \ldots)$ are needed:

$$e^{\Delta t}_{t-\Delta t} = (CSV_t/CSV_{t-\Delta t}) - 1, \quad e^{1}_{t+r} = (CSV_{t+r+1}/CSV_{t+r}) - 1.$$

Because of g, $e^{\Delta t}_{t-\Delta t}$ will normally exceed $(1+i)^{\Delta t} - 1$, and the problem will be to determine the values of e^{1}_{t+r} $(r = 0, 1, 2, \ldots)$.

[2] Throughout this paper, use term with superscript ψ only if there is a bail-out provision and *policy is in the "bail-out period"* (see below, under "Other Policy Provisions"). Superscript # means that for contracts with a money-back guarantee, premiums paid less partial withdrawal should be used.

In order to use the principle of CARM, values of $e_{t-\Delta t}^{\Delta t}$ and e_{t+r}^{1} $(r = 0, 1, 2, \ldots)$ are needed:

$$e_{t-\Delta t}^{\Delta t} = (CSV_t/CSV_{t-\Delta t}) - 1, \quad e_{t+r}^{1} = (CSV_{t+r+1}/CSV_{t+r}) - 1 .$$

Because of g, $e_{t-\Delta t}^{\Delta t}$ will normally exceed $(1 + i)^{\Delta t} - 1$, and the problem will be to determine the values of e_{t+r}^{1} $(r = 0, 1, 2, \ldots)$.

In the simplest case, where AAC is zero and the surrender charge is zero or is level, $e_{t+r}^{1} = g_{t+r}$. In most other cases the expression $(g_{t+r} + SC_{t+r} - SC_{t+r+1})$ is a good approximation to e_{t+r}^{1}, and its use can avoid the more complicated exact calculation of e_{t+r}^{1}. However, care must be taken in the following two situations:

1. $g_{t+r} + SC_{t+r} - SC_{t+r+1} \cong i$. In this case the error in approximating e_{t+r}^{1} is significant, since this error can be either positive or negative. Cases 5 and 6 in Table 1 of this discussion, for $g_{t+r} = 0.045$ (and thus for $e_{t+r}^{1} \cong 0.055$), show this problem. In case 5, since the errors are negative, e_{t+r}^{1} is actually less than 0.055, so that if the policy were a single premium retirement annuity, then i, which is 0.055 under the 1976 law, would exceed e_{t+r}^{1}, and the CARM minimum reserve would be the larger of $CSV_{t-\Delta t}^{*}$ and CSV_{t}^{*}. In case 6, however, e_{t+r}^{1} would exceed i until the end of the year in which the surrender charge is 2 percent, and the cash surrender value at that point would have the greatest present value.
2. *AAC is large relative to AF.* In this case the error in the approximation can exceed 1 percent (see cases 3 and 7 in Table 1). However, analysis is somewhat simplified by the fact that the error is always negative.

It can be seen that, using the principle of CARM, in many cases an actuary would be able to determine at issue the future year at the end of which the cash surrender value will produce the greatest present value. Even where this year has been determined, however, there will still be hurdles that have to be cleared in designing a valuation system. Since the starting point in any reserve calculation will be the current fund $(AF_{t-\Delta t})$, a valuation system capable of calculating CARM reserves must at least be able to do the following (assuming it has already been determined that the cash surrender value s years after the end of policy year t produces the greatest present value):

1. If $s > 0$, calculate $CSV_{t+s}/(1 + i)^{\Delta t+s}$.
2. If $s = 0$, calculate the greater of $CSV_{t-\Delta t}^{*}$ and $[CSV_t/(1 + i)^{\Delta t}]^{*}$, or $AF_{t-\Delta t}$ (if policy is in a "bail-out period").

Table 1 has been prepared using the exact method, but the values using the approximate method are very close (1 or 2 basis points) to those

TABLE 1

EXCESS OF EFFECTIVE INTEREST RATE OVER GUARANTEED RATE PLUS DECREASE IN SURRENDER CHARGE, TIMES 10,000

$$10{,}000(e'_t - g - SC_t + SC_{t+1})$$

t	100 × SC	Guaranteed Rates in All Years of:										
		0.040	0.045	0.050	0.055	0.060	0.070	0.080	0.090	0.110	0.130	0.150
	Case 1: $FPW = 0$; $AAC = 0$; AF_0—Does Not Affect Results											
1	10	16	16	17	17	18	19	20	21	23	26	28
2	9	14	15	15	16	16	18	19	20	22	24	26
3	8	13	14	14	15	15	16	17	18	21	23	25
4	7	12	12	13	13	14	15	16	17	19	22	24
5	6	11	11	12	12	13	14	15	16	18	20	22
6	5	9	10	11	11	12	13	14	15	17	19	21
7	4	8	9	9	10	10	11	12	14	16	18	20
8	3	7	8	8	9	9	10	11	12	14	16	19
9	2	6	7	7	8	8	9	10	11	13	15	17
10	1	5	6	6	7	7	8	9	10	12	14	16
	Case 2: $FPW = 0.10$; $AAC = 0$; AF_0—Does Not Affect Results											
1	10	3	4	4	5	6	7	8	9	11	13	16
2	9	2	3	3	4	5	6	7	8	10	12	14
3	8	1	2	2	3	3	5	6	7	9	11	13
4	7	0	1	1	2	2	4	5	6	8	10	12
5	6	−1	0	0	1	2	3	4	5	7	9	11
6	5	−2	−1	0	0	1	2	3	4	6	8	10
7	4	−2	−2	−1	−1	0	1	2	3	5	7	9
8	3	−3	−3	−2	−2	−1	0	1	2	4	6	8
9	2	−4	−4	−3	−3	−2	−1	0	1	3	5	7
10	1	−5	−5	−4	−4	−3	−2	−1	0	2	4	6

TABLE 1—*Continued*

t	100× SC	Guaranteed Rates in All Years of: 0.040	0.045	0.050	0.055	0.060	0.070	0.080	0.090	0.110	0.130	0.150
		Case 3: $FPW = 0$; AAC = Lesser of 1% or \$25; AF_0 = \$1,000										
1	10	−90	−90	−89	−89	−89	−89	−89	−89	−89	−89	−88
2	9	−91	−91	−91	−91	−91	−91	−91	−90	−90	−90	−90
3	8	−92	−92	−92	−92	−92	−92	−92	−92	−92	−91	−91
4	7	−93	−93	−93	−93	−93	−93	−93	−93	−93	−93	−93
5	6	−94	−94	−94	−94	−94	−94	−94	−94	−94	−94	−94
6	5	−96	−96	−96	−96	−96	−95	−95	−95	−95	−95	−95
7	4	−97	−97	−97	−97	−97	−97	−97	−97	−97	−96	−82
8	3	−98	−98	−98	−98	−98	−98	−98	−98	−98	−86	−71
9	2	−99	−99	−99	−99	−99	−99	−99	−99	−95	−77	−61
10	1	−100	−100	−100	−100	−100	−100	−100	−100	−86	−68	−52
		Case 4: $FPW = 0$; AAC = \$25; AF_0 = \$5,000										
1	10	−33	−32	−32	−31	−30	−29	−27	−25	−22	−19	−16
2	9	−33	−32	−31	−30	−29	−27	−25	−23	−19	−16	−12
3	8	−33	−31	−30	−29	−28	−26	−23	−21	−17	−13	−9
4	7	−32	−31	−29	−28	−27	−24	−22	−19	−14	−10	−6
5	6	−32	−30	−29	−27	−26	−23	−20	−18	−12	−8	−3
6	5	−32	−30	−28	−27	−25	−22	−19	−16	−11	−6	−1
7	4	−31	−29	−28	−26	−24	−21	−18	−15	−9	−4	0
8	3	−31	−29	−27	−25	−23	−20	−17	−14	−8	−3	2
9	2	−31	−29	−27	−25	−23	−19	−16	−13	−7	−2	3
10	1	−31	−28	−26	−24	−22	−19	−15	−12	−6	−1	3

TABLE 1—*Continued*

t	100 × SC	Guaranteed Rates in All Years of:										
		0.040	0.045	0.050	0.055	0.060	0.070	0.080	0.090	0.110	0.130	0.150
	Case 5: $FPW = 0$; $AAC = \$25$; $AF_0 = \$10{,}000$											
1	10	−9	−8	−7	−7	−6	−5	−3	−2	1	3	6
2	9	−9	−8	−8	−7	−6	−5	−3	−2	1	4	7
3	8	−10	−9	−8	−7	−6	−4	−3	−1	2	5	8
4	7	−10	−9	−8	−7	−6	−4	−3	−1	3	6	9
5	6	−10	−9	−8	−7	−6	−4	−2	−1	3	6	10
6	5	−11	−10	−9	−8	−6	−4	−2	0	3	7	10
7	4	−11	−10	−9	−8	−7	−4	−2	0	3	7	10
8	3	−12	−10	−9	−8	−7	−5	−3	0	3	7	10
9	2	−12	−11	−9	−8	−7	−5	−3	−1	3	7	10
10	1	−12	−11	−10	−9	−7	−5	−3	−1	3	7	10
	Case 6: $FPW = 0$; $AAC = \$25$; $AF_0 = \$25{,}000$											
1	10	6	6	7	8	8	9	11	12	14	17	19
2	9	5	6	6	7	7	9	10	11	14	16	19
3	8	4	5	5	6	7	8	9	11	13	16	18
4	7	3	4	5	5	6	7	9	10	13	15	18
5	6	2	3	4	4	5	7	8	9	12	15	17
6	5	1	2	3	4	4	6	7	9	11	14	17
7	4	1	1	2	3	4	5	7	8	11	13	16
8	3	0	1	1	2	3	4	6	7	10	13	15
9	2	−1	0	1	1	2	4	5	7	9	12	14
10	1	−2	−1	0	1	1	3	4	6	9	11	14

TABLE I—*Continued*

t	$100\times SC$	Guaranteed Rates in All Years of: 0.040	0.045	0.050	0.055	0.060	0.070	0.080	0.090	0.110	0.130	0.150
	Case 7: $FPW=0.10$; $AAC=$ Lesser of 1% or \$25; $AF_0=\$1{,}000$											
1	10	−102	−102	−102	−102	−102	−102	−101	−101	−101	−101	−101
2	9	−103	−103	−103	−103	−103	−103	−102	−102	−102	−102	−102
3	8	−104	−104	−104	−104	−104	−104	−104	−103	−103	−103	−103
4	7	−105	−105	−105	−105	−105	−105	−105	−104	−104	−104	−104
5	6	−106	−106	−106	−106	−106	−106	−106	−105	−105	−105	−105
6	5	−107	−107	−107	−107	−107	−107	−106	−106	−106	−106	−106
7	4	−108	−108	−108	−108	−107	−107	−107	−107	−107	−107	−93
8	3	−108	−108	−108	−108	−108	−108	−108	−108	−108	−97	−81
9	2	−109	−109	−109	−109	−109	−109	−109	−109	−105	−87	−71
10	1	−110	−110	−110	−110	−110	−110	−110	−110	−96	−78	−63
	Case 8: $FPW=0.10$; $AAC=\$25$; $AF_0=\$5{,}000$											
1	10	−46	−45	−44	−43	−42	−41	−39	−38	−35	−32	−29
2	9	−45	−44	−43	−42	−41	−39	−37	−35	−31	−28	−24
3	8	−44	−43	−42	−41	−40	−37	−35	−33	−29	−24	−20
4	7	−44	−42	−41	−40	−38	−36	−33	−31	−26	−21	−17
5	6	−43	−42	−40	−39	−37	−34	−31	−29	−24	−19	−14
6	5	−43	−41	−39	−38	−36	−33	−30	−27	−22	−17	−12
7	4	−42	−40	−38	−37	−35	−32	−29	−26	−20	−15	−10
8	3	−42	−40	−38	−36	−34	−31	−27	−24	−19	−14	−9
9	2	−41	−39	−37	−35	−33	−30	−26	−23	−17	−12	−8
10	1	−41	−39	−36	−34	−32	−29	−25	−22	−16	−11	−7

TABLE 1—*Continued*

t	100 × SC	Guaranteed Rates in All Years of:										
		0.040	0.045	0.050	0.055	0.060	0.070	0.080	0.090	0.110	0.130	0.150
	Case 9: FPW = 0.10; AAC = \$25; AF_0 = \$10,000											
1	10	−21	−20	−20	−19	−18	−17	−16	−14	−12	−9	−7
2	9	−21	−20	−20	−19	−18	−17	−15	−14	−11	−8	−5
3	8	−21	−20	−20	−19	−18	−16	−15	−13	−10	−7	−3
4	7	−21	−21	−20	−19	−18	−16	−14	−12	−9	−6	−2
5	6	−22	−21	−20	−19	−18	−16	−14	−12	−8	−5	−2
6	5	−22	−21	−20	−19	−17	−15	−13	−12	−8	−4	−1
7	4	−22	−21	−20	−19	−17	−15	−13	−11	−8	−4	−1
8	3	−22	−21	−20	−19	−17	−15	−13	−11	−7	−4	0
9	2	−22	−21	−20	−19	−17	−15	−13	−11	−7	−4	0
10	1	−23	−21	−20	−19	−17	−15	−13	−11	−7	−4	0
	Case 10: FPW = 0.10; AAC = \$25; AF_0 = \$25,000											
1	10	−6	−6	−5	−5	−4	−3	−2	0	2	4	7
2	9	−7	−6	−6	−5	−4	−3	−2	−1	2	4	7
3	8	−8	−7	−6	−6	−5	−4	−2	−1	2	4	7
4	7	−8	−8	−7	−6	−6	−4	−3	−1	1	4	6
5	6	−9	−8	−8	−7	−6	−5	−3	−2	1	3	6
6	5	−10	−9	−8	−7	−7	−5	−4	−2	0	3	6
7	4	−10	−9	−9	−8	−7	−6	−4	−3	0	3	5
8	3	−11	−10	−9	−8	−8	−6	−5	−3	−1	2	5
9	2	−11	−11	−10	−9	−8	−7	−5	−4	−1	2	4
10	1	−12	−11	−10	−10	−9	−7	−6	−4	−2	1	3

shown. Where an annual administrative charge is used, it has been calculated as the lesser of \$25 and 1 percent of the accumulated fund.

If *AAC* is zero, the calculations are obviously simplified, so that a company might choose to ignore any annual administrative charge. However, the reserve effect could be significant, as shown in Table 2, where the figures shown are the ratios of reserves including *AAC* to reserves excluding *AAC*; $\Delta c = \Delta t = 0$, and e^1 and *AAC* are constants.

For example, take a block of flexible premium annuities with an average fund of \$5,000, an effective rate of 5 percent, the maximum present value seven years in the future (as would be the case under the 1976 law if the surrender charge decreased by 1 percent a year to zero after seven years), and a \$25 annual administrative charge. Table 2 shows that ignoring *AAC* would result in holding approximately an additional 3.5 percent, or (1 − 0.966)/0.966, of reserve over CARM levels.

4. *Other Policy Provisions*

POLICY PROVISIONS WHERE SURRENDER CHARGES ARE WAIVED

Annuitization

I agree with Mr. Jaffe's conclusion that minimum reserves under CARM should not recognize the possibility of waiving any surrender charge on annuitization, but that the actuary must evaluate the adequacy of the reserves on the basis of the circumstances involved. Since an actuary may hold an extra reserve to allow for waiving the surrender charge on annuitization, and, further, since annuitization and bail-out can be handled similarly, formulas to adjust CARM values for them are shown below. For this purpose, assume a constant probability, p^a, of annuitizing.

Bail-out Provision

The bail-out provision presents another situation where money can be withdrawn without a surrender charge. As discussed in item 1 below, I do not agree with Mr. Jaffe that the minimum CARM reserve should be "the current cash values plus any applicable surrender charge" except in situation 2 below. Typically, the bail-out provision provides for a short (e.g., 60 days) "bail-out period" if the company declares a new high guaranteed interest rate that is less than the "bail-out rate" associated with the previous (expiring) high guaranteed rate. Under this type of bail-out provision, I see two potential reserve problems:

1. A future *high guaranteed interest rate could be so low as to trigger a bail-out period*. The reserve effect, if any, of this provision will depend on various factors peculiar to each company's circumstances and bail-out provision (e.g.,

TABLE 2

EFFECT OF OMITTING AAC FROM RESERVE CALCULATION

$$[AF_t(1 + e^1)^{r-t} - AAC(s_{\overline{r-t+1}|})] \div [AF_t(1 + e^1)^{r-t}]$$

e^1	AAC	$AF_t = \$1{,}000$			$AF_t = \$5{,}000$			$AF_t = \$10{,}000$		
		$r - t = 0$	$r - t = 3$	$r - t = 7$	$r - t = 0$	$r - t = 3$	$r - t = 7$	$r - t = 0$	$r - t = 3$	$r - t = 7$
.05.......	12	.988	.955	.919	.998	.991	.984	.999	.996	.992
	25	.975	.907	.830	.995	.981	.966	.998	.991	.983
.10.......	12	.988	.958	.930	.998	.992	.986	.999	.996	.993
	25	.975	.913	.853	.995	.983	.971	.998	.991	.985
.15.......	12	.988	.961	.938	.998	.992	.988	.999	.996	.994
	25	.975	.918	.871	.995	.984	.974	.998	.992	.987

the spread between the guaranteed rate and the bail-out rate). Therefore, as with annuitization, the decision of whether to hold an extra reserve should be a matter of actuarial judgment. Assuming a probability, p^b, that a bail-out period will be in effect at any given policy year-end, the mathematics involving p^b are as shown below. If the bail-out rate is guaranteed only for the current high interest guarantee period, then p^b would be used only with CSV_t (i.e., the cash surrender value at the end of the current policy year).

2. *A policy could actually be in a bail-out period at the time a valuation is performed.* In this case the CARM reserve cannot be less than $AF_{t-\Delta t}$, since the policyholder could surrender immediately and get this amount. This has been allowed for in the above formulas.

Formula Modifications to Allow for Annuitization and Future Bail-out (Let $p = p^a + p^b$)

$AF_{t-\Delta t}$ and $CSV^{\#}_{t-\Delta t}$ would not be affected. $CSV^{\#}_t$ is given by

$$\{pAF_t + (1 - p)[(AF_t - RFPW_t)(1 - SC_t) + RFPW_t]\}^{\#} .$$

This expression can be rearranged in a number of ways, one of which is

$$\{AF_t - AF_t(1 - p)SC_t(1 - RFPW_t/AF_t)\}^{\#} .$$

To find CSV_{t+r} by the approximate method, we use the same approach as for $CSV^{\#}_t$; rearranging, we obtain

$$AF_{t+r} - AF_{t+r}(1 - p)SC_{t+r}(1 - FPW) .$$

The expression obtained for CSV_{t+r} by the exact method is

$$AF_{t+r} - AF_{t+r-1}(1 - p)SC_{t+r}[(1 + g_t - FPW) - AAC_{t+r}/AF_{t+r-1}] ,$$

which is arrived at in the following way:

$$pAF_{t+r} + (1 - p)[{}^cW_{t+r}AF_{t+r-1} - AAC_{t+r}(1 - SC_{t+r})] .$$

The expression in brackets can be expanded (substituting for ${}^cW_{t+r}$) as follows:

$$[AF_{t+r} - AF_{t+r-1}SC_{t+r}(1 + g_t - FPW) + AAC_{t+r}SC_{t+r}] ;$$

multiplying this by $(1 - p)$ and canceling pAF_{t+r} yields the above expression for CSV_{t+r}.

Waiving of Surrender Charge Where Funds Are Used to Pay Premiums on Another Policy in Same Company

Here the treatment is different from annuitization or bail-out, since it is very likely that only a portion of the fund would be withdrawn to pay premiums on another company policy. If the actuary judged it necessary to add a specific extra reserve for such withdrawals, a means would probably have to be found to incorporate this policy provision into reserves by appropriately increasing either FPW or p (the latter only if the above approach was used for bail-out or annuitization).

POLICY PROVISION NOT EXPLICITLY COVERED BY ABOVE FORMULAS

AAC_r is charged only if AF_r is less than a certain amount (typically \$10,000).—This provision obviously complicates the calculation of CARM reserves, but must be taken into account if the relief from surplus strain afforded by the AAC is desired.

AAC is charged only if premiums are paid.—In this case AAC should be ignored, since the above formulas are for indeterminate premium plans where future premiums are not used in computing CARM reserves.

AAC is expressed as a percentage of the account balance.—This case could be handled simply by reducing g_t by AAC_t. However, most companies put a cap on AAC, such as \$25.

Surrender charges vary by issue-age grouping.—This complicates matters, but not too much if the number of age groups is small.

5. *Comments on Contractually Guaranteed Rates*

While CARM specifies that future guaranteed benefits shall be determined using "rates specified in such contracts for determining guaranteed benefits," it is obviously necessary to take any current high "guaranteed" interest rate into account in determining CARM reserves. This is true despite the fact that the actual high interest rates are not specified in the contract (although reference thereto probably would be). However, it does not follow that future high interest guarantees that may or may not occur should be taken into account in reserves. In fact, under CARM it is clear that they should not be used.

6. *Conclusion*

The foregoing shows many of the complexities that can arise in computing CARM and related reserves. While things can be greatly simplified if reserves are based on accumulated funds (ignoring any administrative charges and possibly ignoring even surrender charges), many companies may find the resulting surplus strains unacceptable. On the other hand, an actuary concerned with the overall adequacy of reserves for these deferred annuities (especially in light of the probability of future interest guarantees) might well consider this approach as a way to build in reasonable conservatism—especially if his or her company has a liberal bail-out provision and/or other provisions that significantly increase the likelihood of funds being withdrawn without a surrender charge.

I would like to thank Donna Berson for her help in preparing the tables presented herein and in producing other data that aided me in preparing this discussion. I would also like to thank Frank Alpert for the support and help he has given me in preparing this discussion.

GREGORY J. CARNEY:

I would like to congratulate Mr. Jaffe on a timely and well-written paper on the CARM and its application to fixed single premium deferred annuities (SPDA). It is important to note that the CARM was part of a model bill adopted by the National Association of Insurance Commissioners (NAIC) at its December 1976 meeting in Phoenix, Arizona. The model bill, known as "the 1976 amendments," was developed by the C-3 Technical Task Force chaired by John Montgomery, the chief actuary of the California Insurance Department. The 1976 amendments have since been adopted by the legislatures of all of the states.

In the interpretation of any law, one must consider the legislative intent. In the case of an NAIC model bill, one must certainly consider the discussions that occurred at the NAIC level during the deliberations on the model bill itself. With regard to the interpretation of the law as passed in a specific state, the legislative intent is critical to the interpretation of that law. And, of course, any changes made in the model bill by the state legislature, regardless of how small, can impact the interpretation of the specific state law. An example of changes can be seen in an analysis of the California and Texas laws. California added one sentence to the 1976 amendments. Texas modified one sentence. The results, however, are opposite. In Texas the 1976 amendments apply to all annuity contracts written since 1948. In California the 1976 amendments apply only to contracts issued after the operative date of the new law. My point is that, while Mr. Jaffe, I, and others may have our own views as to the proper interpretation of CARM, those views are only personal. The interpretation of the law is, of course, reserved to the state and ultimately to the courts.

In reviewing Mr. Jaffe's paper, I find only one area of disagreement. I disagree with his discussion of the "bail-out" or "window" option. Mr. Jaffe discusses two such options, a fixed window, and a variable window where the window is a percentage of some nonfixed interest rate. Mr. Jaffe states that CARM never contemplated this provision (variable window). This statement is not true. One company issued a product in 1974 that allowed for the removal of the surrender charge if the interest rate credited was ever below the passbook savings account rate at the five largest federally insured banks. This provision was probably the first variable window that was available, and it was considered during the deliberations regarding CARM. Another company utilized the same window provision but substituted prime for passbook. This contract was also available to those individuals responsible for the development of CARM. With regard to these contract provisions, their implication was discussed with members of the C-3 committee. It was the opinion of those individuals that the existence of a window did not require additional reserves until such time as the window was violated and the company could not collect its surrender charge.

One company tried to file a variable window product in a large eastern state in 1976. The state insisted that the company fix the rate, and, since the passbook rate was 5 percent at the time, the company used a fixed-window rate of 5 percent. Since the company had a minimum guaranteed rate of 3 percent, the company questioned the impact on reserves of the fixed window above their minimum guarantees. The state, which was a member of the C-3 committee, stated that extra reserves associated with the window provision would not be required until the window rate was violated.

I believe this conclusion can be reached by a careful analysis of section 4-a of the model Standard Valuation Law. This section states:

> 4-a. This section shall apply to all annuity and pure endowment contracts other than group annuity and pure endowment contracts purchased under a retirement plan or plan of deferred compensation, established or maintained by an employer (including a partnership or sole proprietorship) or by an employee organization, or by both, other than a plan providing individual retirement accounts or individual retirement annuities under Section 408 of the Internal Revenue Code, as now or hereafter amended.
>
> Reserves according to the commissioners annuity reserve method for benefits under annuity or pure endowment contracts, excluding any disability and accidental death benefits in such contracts, shall be the greatest of the respective excesses of the present values, at the date of valuation, of the future guaranteed benefits, including guaranteed nonforfeiture benefits, provided for by such contracts at the end of each respective contract year, over the present value, at the date of valuation, of any future valuation considerations derived from future gross considerations, required by the terms of such contract, that become payable prior to the end of such respective contract year. The future guaranteed benefits shall

be determined by using the mortality table, if any, and the interest rate, or rates, specified in such contracts for determining guaranteed benefits. The valuation considerations are the portions of the respective gross considerations applied under the terms of such contracts to determine nonforfeiture values.

Note that the term "future guaranteed benefits" is defined in terms of minimum guaranteed interest rates and load charges specified in the contract. With regard to load charges, the drafters of the model legislation were not trying to favor either front-end- or back-end-load products but were trying to produce consistent results among the two types of contracts. Therefore, surrender charges specified in the contract are interchangeable with front-end-load charges in the calculation of future guaranteed benefits. In other words, the valuation considerations could utilize the surrender charge during the years the surrender charge was applicable.

The law requires, then, for the calculation of minimum reserves, consideration of the minimum future guarantees as they relate to interest and load (surrender) charges and calculation of future guaranteed benefits under the conditions that exist "at the date of valuation." It is obvious that one must refer to the conditions at that date, since past excess interest credits or future guarantees may have been declared subsequent to the issue of the policy, and those must be considered for valuation purposes. If the company can impose the surrender charge at the valuation date, then the requirement of the law to utilize its minimum guarantees and load charges is satisfied by utilizing the surrender charge even if it is contingent. If the company had violated its window and could not impose the surrender charge on the date of valuation, then it could not assume the utilization of the surrender charge for the calculation of future guaranteed benefits.

Mr. Jaffe's concern, which I share, is that, since the magnitude of the window rates has increased, the probability of a company's violating the window has increased. Therefore, the minimum reserves under CARM may not be sufficient to cover the future liabilities. I believe that the actuary signing the annual statement must consider this under the "good and sufficient" provision and, on the basis of the probability of violating the window in any time frame, must establish appropriate additional reserves for that contingency. These reserves could be established, for example, by utilizing a wear-off of the surrender charge over the period where there is a high probability that the window will be violated.

Although Mr. Jaffe did not directly discuss calculation of future benefits in his paper, section 4-a quoted brings out an important point. The Standard Valuation Law allows the accumulation (calculation of minimum future guaranteed benefits) at an interest rate less than the discount rate used for present values (maximum valuation interest rate).

In another area of the paper, Mr. Jaffe makes an argument for an additional reserve for the death benefit prior to annuitization. While I believe that the theoretical justification is correct, as a practical matter I am not aware of any company specifically calculating the reserve. The reason that companies do not specifically calculate this reserve is that the CARM reserve in most instances exceeds the cash surrender value, and this excess also exceeds the additional death benefit reserve. Also, if a company wears off its surrender charge, its basic reserve exceeds CARM minimums and also will exceed the additional death benefit reserve. I believe Mr. Jaffe's point is especially well taken where the CARM reserve equals the cash surrender value or where the surrender charge is permanent. In other situations, the aggregate test will probably cover any additional reserve required.

In summary, I believe Mr. Jaffe did an excellent job in presenting the principles of CARM as they relate to fixed SPDA contracts. The annuity area is changing. New contracts and provisions are being developed. It is important for the actuary to understand fully the principles of CARM that Mr. Jaffe has discussed if a proper valuation of those benefits is to be achieved.

RICHARD R. MARKER AND STEVEN D. SOMMER:

Mr. Jaffe's paper is a timely addition to actuarial literature. It provides a clear explanation of how the CARM should be applied both to basic single premium deferred annuities (SPDAs) and to those with the additional policy benefits common in current products. We would like to supplement his discussion in three ways. First, we shall present a generalized recursive formula that simplifies the calculation of CARM reserves. Second, we shall discuss a problem not covered by Mr. Jaffe, that of off-anniversary cash values. Finally, we shall describe some of the considerations involved in calculating CARM reserves as of the valuation date, as opposed to the end of the policy year.

1. *The CARM Formula*

a) BASIC FORMULA

As Table 1 in Mr. Jaffe's paper demonstrates, a straightforward application of the CARM, even to a basic SPDA, is a lengthy procedure that involves discounting successive cash values back to the valuation date. We have developed a recursive formula that significantly reduces the number of necessary calculations. Even on the computer, using our formula instead of a straightforward application of the law results in a considerable savings in time. In its simplest version, applicable to SPDAs with no special policy benefits, the formula is as follows:

$$CARM_t = \max \begin{Bmatrix} CSV_t \\ {}^{\text{val}}v\,CARM_{t+1} \end{Bmatrix}, \tag{1}$$

where

$CARM_t$ = CARM reserve at the end of policy year t;
CSV_t = Cash surrender value at end of policy year t, based on the guarantees in effect as of the valuation date;
${}^{\text{val}}v$ = Discount factor at the valuation interest rate;

and "max" means to take the larger of the quantities in braces.

In using this formula, one would typically start by determining each successive guaranteed cash value, using the guarantees in effect as of the valuation date from the policy year-end immediately following the valuation date up through the maturity date (policy duration n). $CARM_n$ is then set equal to CSV_n, and one can determine the earlier reserves recursively, using the formula.

Inspection may show, of course, that it is not necessary to go all the way up to the maturity date, since it may be obvious that $CARM_t = CSV_t$ for all years past a certain point. More precisely, if we let SC_t denote the surrender charge for year t and ${}^{G}i_t$ be the guaranteed interest rate for year t, then we may begin our calculations by setting $CARM_t = CSV_t$ as long as

$$\frac{1 - SC_s}{1 - SC_{s+1}}\, {}^{G}v_{s+1} \geqslant {}^{\text{val}}v$$

for all years $s > t$. In other words, as long as the effective guaranteed interest (actual guaranteed interest plus the drop in the surrender charge) never exceeds the valuation rate in any year past the tth, then we may begin applying our formula in year t.

b) PRODUCT DESIGN

In Section II of his paper Mr. Jaffe describes how the principles of the CARM may be used to aid in product design. Our formula can also be used in this manner. The following analysis shows how one can determine, for example, how much the guaranteed interest rate can be increased, and how much the surrender charges can be decreased, without affecting the CARM reserve.

Four cases are possible for the change in reserve from one year to the next:

CASE A:	$CARM_t = CSV_t$,	$CARM_{t+1} = CSV_{t+1}$.
CASE B:	$CARM_t = CSV_t$,	$CARM_{t+1} > CSV_{t+1}$.
CASE C:	$CARM_t > CSV_t$,	$CARM_{t+1} = CSV_{t+1}$.
CASE D:	$CARM_t > CSV_t$,	$CARM_{t+1} > CSV_{t+1}$.

We shall now examine each of these cases separately for the SPDA.

Case A: $CSV_t > {}^{val}v\,CSV_{t+1}$.—Thus, ${}^{G}i$ could be increased, without affecting $CARM_t$, to

$$ {}^{G'}i = [(1 + {}^{val}i)(1 - SC_t)/(1 - SC_{t+1})] - 1 \ . $$

Alternatively, SC_t could be reduced using the same formula rearranged.

Case B: $CSV_t > {}^{val}v\,CARM_{t+1} > {}^{val}v\,CSV_{t+1}$.—In this case ${}^{G}i$ could be increased, without affecting $CARM_t$, to

$$ {}^{G'}i = [(1 + {}^{val}i)AC_t(1 - SC_t)/CARM_{t+1}] - 1 \ . $$

Case C: $CSV_t < {}^{val}v\,CSV_{t+1}$.—This is similar to Case A, except that a reduction of ${}^{G}i$ to ${}^{G'}i$ would produce a reduction in CARM$_t$.

Case D: $CSV_t < {}^{val}v\,CARM_{t+1}$.—This is similar to Case B, except that a reduction of ${}^{G}i$ to ${}^{G'}i$ would produce a reduction in $CARM_t$.

This example illustrates how the formula may be used. Surrender charge grade-offs or the guaranteed interest rate may be changed to the limits of the inequality, without affecting the reserves required in Cases A and B and without decreasing the required reserves in Cases C and D. These formulas may be expanded to include the other benefits described in the next section of our discussion.

c) GENERALIZED FORMULA

Our formula (1) can be expanded to cover the additional benefits found in currently issued products:

$$ CARM_t = \max\left\{ \begin{matrix} CSV_t \\ {}^{val}v\,CARM_{t+1} - NPREM_{t+1} + BEN_{t+1} \end{matrix} \right\} , \qquad (2) $$

where $NPREM_{t+1}$ is the nonforfeiture net premium for fixed annual deferred premium annuities but is zero for flexible premium annuities and SPDAs, and BEN_{t+1} consists of components representing various additional benefits, as outlined below.

1. *Death benefit prior to annuitization.* This benefit takes one of two forms. One form waives the surrender charge and pays the full account value as a death benefit. If we treat deaths as occurring at the end of the year, then

$$BEN_{t+1} = (AC_{t+1} - CARM_{t+1})c_{x+t} \nless 0 ,$$

   where AC_t is the account value at the end of the tth year (i.e., the accumulation of net premiums at the credited rate of interest) and c_{x+t-1} is the valuation cost of insurance for the tth year.

   The other form of this death benefit returns the premiums paid if they are greater than the cash value. For single premium deferred annuities for which this benefit does not exceed 12 months, BEN_1 may be approximated as

$$\tfrac{1}{2}(GPREM_1 - CARM_0)\frac{m}{12}c_x \nless 0 ,$$

   where $\tfrac{1}{2}(GPREM_1 - CARM_0)$ is the average death benefit, m is the number of months the benefit exists, and c_x is the valuation cost of insurance.
2. *Free partial withdrawals.* The formula for this benefit is

$$BEN_{t+1} = FPW\left(\max\begin{Bmatrix} {}^{val}v \\ {}^{G}v_{t+1}\end{Bmatrix}AC_{t+1} - {}^{val}vCARM_{t+1}\right) \nless 0 .$$

   When this benefit is offered, the formula for $CARM_t$ becomes

$$CARM_t = \max\begin{Bmatrix} CSV_t \\ {}^{val}vCARM_{t+1} + FPW\left(\max\begin{Bmatrix} {}^{val}v \\ {}^{G}v_{t+1}\end{Bmatrix}AC_{t+1} - {}^{val}vCARM_{t+1}\right)\end{Bmatrix} .$$

   [If the expression in parentheses is less than zero, replace it by zero.] Note that when free partial withdrawals are allowed, CSV_t should be calculated as

$$CSV_t = AC_t[1 - (1 - FPW)SC_t] .$$

   The above formula for $CARM_t$ can be justified by considering the largest effect that partial withdrawals during year $t + 1$ may have on the CARM reserve at the end of policy year t. The reserve for survivors of year $t + 1$ is ${}^{val}vCARM_{t+1}$.

The reserve for year-end free partial withdrawals is ${}^{val}vAC_{t+1}$, and the reserve for beginning-of-year free partial withdrawals is AC_t. Since AC_t may-be restated as ${}^{G}vAC_{t+1}$, the maximum present value of withdrawal benefits is

$$FPW\left(\max\left\{\begin{matrix} {}^{val}v \\ {}^{G}v \end{matrix} AC_{t+1}\right\}\right) .$$

The expression for the cost of the free partial withdrawal benefit is the maximum positive difference between the present value of withdrawal benefits and the survivor benefits.

The other special benefits discussed in the paper are handled in the calculation of CSV_t rather than included in the term BEN_t.

1. *"Bail-out" option.* Mr. Jaffe points out that in some cases guaranteeing the bail-out rate produces lower reserves because of the surrender charge, even though the actual guaranteed rate is less than the bail-out rate. To take the bail-out rate into account in our formula, the first step is to calculate each cash surrender value between the valuation date and the ultimate duration, using the following formula:

$$CSV_t = AC_{t-1}(1 + {}^{G}i_t)(1 - SC_t) .$$

   This formula should not be used when ${}^{G}i_t$ is less than the bail-out interest rate ${}^{b}i$. In that case, replace the factor $(1 + {}^{G}i_t)(1 - SC_t)$ by the lesser of $(1 + {}^{G}i_t)$ and $(1 + {}^{b}i)(1 - SC_t)$. In the first case SC_t should be set equal to zero in subsequent calculations, while in the second case ${}^{G}i_t$ should be set equal to ${}^{b}i$ in subsequent calculations.
2. *Money-back guarantee.* In applying the recursive formula, replace CSV_t by the sum of the gross premiums whenever this sum is larger than CSV_t.
3. *No surrender charge on early annuitization.* If the present value of the annuity exceeds the cash surrender value, it should be used as the cash surrender value when applying the recursive formula. Some states may accept Mr. Jaffe's suggestion of applying an election percentage to this benefit.
4. *No-partial-withdrawal bonus.* This benefit is handled by increasing the cash surrender value in the formula. To the extent that previous partial withdrawals would reduce the bonus, the cash value in the formula would be reduced.

2. *Off-Anniversary Cash Values*

The CARM states that the reserve is to be the greatest of the present values of the cash values available "at the end of each respective contract year." Normally one need consider only these policy year-end cash values. We suggest, however, that in some situations the actuary take into account the effect of off-anniversary cash values. Consider the following two examples:

1. Suppose an annual deferred premium annuity contract guarantees interest rates of 15 percent for the first policy year and 3 percent thereafter, with surrender charges of 7 percent in year 1, 4 percent in year 2, and then dropping 1 percent each year thereafter until the charges disappear in year 6. Assume for simplicity that there are no special benefits such as free partial withdrawals, and assume that the valuation interest rate is 7.5 percent. The table below shows the first three terminal accumulated considerations (AC_t), cash values (CSV_t), and CARM reserves ($CARM_t$) for a $1,000 single premium, using the initial guarantees:

End of Year t	AC_t	CSV_t	$CARM_t$
1	$1,150.00	$1,069.50	$1,069.50
2	1,184.50	1,137.12	1,137.12
3	1,220.04	1,183.43	1,183.43

If the policyholder surrenders at the end of year 1, he receives $1,069.50, and the CARM reserve is sufficient to cover this amount. If he waits one day after the end of year 1 to surrender, however, he receives $1,104.00, since in the second policy year the surrender charge drops from 7 to 4 percent. The CARM reserve is less than this cash value by $34.50.

2. Change example 1 by removing all surrender charges, and further assume that the initial 15 percent interest guarantee is for eighteen months rather than for twelve months.

End of Year t	AC_t	CSV_t	$CARM_t$
1	$1,150.00	$1,150.00	$1,164.28
2	1,251.60	1,251.60	1,251.60
3	1,289.15	1,289.15	1,289.15

The cash value at the end of eighteen months equals $1,150 × $(1.15)^{1/2}$, or $1,233.24. Discounting this value back to the end of policy year 1 (at 7.5 percent) produces a reserve of $1,189.44, as compared with the actual reserve at $1,164.28.

The actuary may want to consider interpreting the CARM as requiring that the reserve be the greatest of the present values of the future cash values available "at any future time," as opposed to "at the end of each respective contract year." At a minimum, he should be aware of the above-noted possibilities and make any reserve adjustments he feels are necessary. Guidance is provided by a gross premium valuation with suitably conservative assumptions. Considerable judgment must be used in choosing the "suitably conservative" assumptions, however, particularly the lapse assumption.

3. *Reserves at the Valuation Date*

While application of the CARM automatically guarantees that the terminal reserve for policy year t is never less than the corresponding cash value, it does not guarantee that the reserve at the valuation date always equals or exceeds the cash value available at that point. For example, suppose the cash value at the valuation date is \$1,000 and the interest rate guaranteed until the end of the policy year, six months after the valuation date, is 3 percent. If there are no surrender charges in the current year or in the future, and if the guaranteed rate remains at 3 percent, then the CARM reserve at the end of the policy year is $\$1{,}000 \times (1.03)^{1/2}$, or \$1,014.89. Discounting this value back to the valuation date using a valuation interest rate of 7.5 percent produces a reserve of \$978.85, which is less than the \$1,000 cash value.

The CARM does not require that the reserve at the valuation date exceed the cash value available at that point. Because of the requirements of Exhibit 8, Part G, of the Convention Blank, however, the total reserve held for each policy must never be less than the corresponding cash value. Then, in determining the CARM reserves at the valuation date, one should substitute the cash value for the reserve whenever it is greater. In the above example the valuation reserve would be \$1,000, not \$978.85.

We normally calculate valuation reserves by first calculating the CARM terminal reserve as of the next policy year-end and then discounting this reserve back to the valuation date using the valuation interest rate, but substituting the cash value at the valuation date if it is greater. We group the business by issue year and issue month, and then assume each month's issues to have been issued in the middle of the month. It would also be possible to do a seriatim valuation, using the actual issue dates. At the other extreme, one could group the business only by year of issue and then assume a July 1 issue date, but this method could produce inaccurate results if the issues were significantly skewed.

There is a problem with this approach, in that it ignores certain benefits that may be granted between the valuation date and the end of the policy year—namely, any death benefits and the money-back guarantee. This portion of the reserve for these benefits, if material, would have to be calculated separately.

DOUGLAS MENKES:

I would like to thank Mr. Jaffe for his timely article on the application of the CARM to single premium deferred annuities. Mr. Jaffe raises some very important issues that should be considered in the statutory valuation.

The bail-out provision presents special problems to the valuation actuary, since the relationship between the bail-out rate and the crediting rate, as well as the distribution of the underlying assets, will affect the behavior of policyholders. When this feature was originally introduced, many companies were using it as nothing more than window dressing. Assets were invested relatively long, and some bail-out rates were sufficiently less than the actual crediting rates (in some cases by at least 3 percent), so that the risk of not being able to credit the bail-out rate was minimal. As the bail-out rates began to approach the crediting rates and assets became shorter, the risk associated with this provision grew substantially. At least one state insurance department has unofficially required companies to maintain at least a 1 percent spread between the two. In situations with narrow spreads and very short assets, it is clear that the effect of the bail-out provision should be considered under CARM, since a drop in short-term rates would place an insurer in the unenviable position of either crediting higher rates than can be supported by the assets, or facing surrenders without surrender charges. Consider an insurer that invests in very short assets when short-term rates exceed long-term rates and sets its bail-out rate 2 percent lower than its crediting rate. The interest curve returns to normal sometime after issue, and the assets are invested longer at that point to lock in an equivalent or even higher return. Clearly, the bail-out should have been reflected when the policy was issued. If the policy has a declining surrender charge scale, it is quite possible that with the longer assets the surrender charge would be zero by the time the underlying assets could not support the bail-out rate (plus spread) under any reasonable scenario. Should this insurer be permitted to release any reserves that had been set up as a result of the bail-out?

Mr. Jaffe discusses the effect of no surrender charges on early annuitization. As he points out, if the frequency is low with single premium deferred annuities, then the effect of this provision on CARM reserves can be ignored. Recently, there has been a trend with flexible premium deferred annuities toward emphasizing early annuitization as a means of avoiding surrender charges. Annuitization can occur over twelve months in an extreme case. While current settlement option rates can be reduced for short payouts during periods of high surrender charges, there probably will be situations in which early annuitizations will result in losses when compared with surrenders. If high utilization is either expected as a result of experience or anticipated on the basis of marketing practices, the CARM reserves should reflect the effect of early annuitizations. One approach would be to hold the greater of the cash value plus the surrender charge less the present value of excess interest minus expenses during annuitization, and the CARM reserve ignoring the effect of early annuitization.

It appears that the 1980 valuation amendments will require the CARM for all individual deferred annuities and for group annuities used to fund IRAs and nonqualified programs. The valuation amendments permit valuation on either an issue-year basis or a change-in-fund basis. Under an issue-year valuation basis, the interest rate used to determine the minimum valuation standard for the entire duration of the annuity is the calendar-year valuation interest rate for the year of issue. Under a change-in-fund valuation basis the interest rate used to determine the minimum valuation standard applicable to each change in the fund held under the annuity is the calendar-year valuation interest rate for the year of the change in the fund. The advantage to an insurer of using a change-in-fund basis is that, all other things being equal, the maximum valuation interest rate is greater than when an issue-year basis is used. The term "fund held" is not defined in the NAIC model. As a result, many actuaries interpret a change-in-fund basis to be anything that is not an issue-year basis. For the purpose of this discussion, I will use the term *fund* to mean the accumulation value. This is the simplest change-in-fund basis—one that could lead to valuation rates which, as assets roll over, bear little or no relation to underlying yields. However, for this discussion, the type of change-in-fund basis is not as significant as the effect on the CARM of any change-in-fund basis.

Consider a fixed single premium deferred annuity contract with these provisions: there is no front-end load; surrender charges are 7 percent in year 1 decreasing 1 percent per year to zero in the eighth year; and guaranteed interest rates are 15 percent in year 1, 9 percent in years 2–3, and 4 percent thereafter.

For simplicity, assume there are no bail-out provisions, early annuitizations without surrender charge, or any other contractual feature that would require the actuary to consider additional reserves. Excess interest is declared in advance every six months on June 30 and December 31. For a \$10,000 policy issued July 1 of year Z, assume the following:

Six-Month Period	Declared Rate	Guarantee Rate	Accumulation Value, End of Period
$7/1/Z$–$12/31/Z$	15%	15%	\$10,724
$1/1/Z+1$–$6/30/Z+1$	16	15	11,550
$7/1/Z+1$–$12/31/Z+1$	16	9	12,440
$1/1/Z+2$–$6/30/Z+2$	16	9	....

Suppose the maximum valuation interest rate (for a change-in-fund basis) for new issues is 9 percent for calendar year Z and 11 percent for calendar year $Z + 1$. What is the CARM reserve as of $12/31/Z + 1$ if a change-in-fund basis of valuation is used?

At the end of year $Z + 1$, the fund consists of two pieces. The piece from year Z is \$10,724 and has a maximum valuation interest rate of 9 percent; the piece from year $Z + 1$ is \$1,716 and has a maximum valuation interest rate of 11 percent.

For the year Z piece, the largest present value of future guaranteed benefits is the present value of the accumulation value on the fourth policy anniversary:

$$(\$10{,}724)\,\frac{(1.16)^{1/2}(1.09)}{(1.09)^{3/2}}\,(0.96) = \$10{,}620\ .$$

(Clearly, after the fourth anniversary, the guaranteed interest rate drops to 4 percent, more than offsetting the decreasing surrender charge. On the third anniversary, the present value is 95/96 of the amount shown above.)

For the policy year $Z + 1$ piece, the largest present value of future guaranteed benefits is the present value of the accumulation value on the third policy anniversary:

$$(\$1{,}716)\left(\frac{1.16}{1.11}\right)^{1/2}(0.95) = \$1{,}667\ .$$

(The fourth-year present value is less because $1.09/1.11 < 0.95/0.96$.

We now have proposed that the CARM discounting process be applied to each fund separately and have shown that under certain circumstances the largest present value of guaranteed benefits need not originate from the same policy anniversary for all funds. The analysis thus far assumes that the surrender charge for the policy is allocated to each fund in proportion to fund values. Theoretically, this need not be the case. If the surrender charge is intended to recover unamortized acquisition expenses and market-value losses on cash-outs, then for the example shown above, virtually all of the surrender charge is allocable to the first fund, for two reasons:

1. The acquisition expenses were incurred in the first year.
2. The market value is less than book value for the first-year fund but is equal or nearly equal to book value for the second-year fund.

The interest on the unamortized acquisition cost could be allocated to the second year. For simplicity, if we recompute the reserve allocating all of the surrender charge to the first year (Z), we find that

$$Z \text{ piece} = \frac{(\$10{,}724)(1.16)^{1/2}(1.09) - \$584}{(1.09)^{3/2}} = \$10{,}549 \, ,$$

$$Z + 1 \text{ piece} = \frac{(\$1{,}716)(1.16)^{1/2} - \$0}{(1.11)^{1/2}} = \$1{,}754 \, .$$

The total reserve of \$12,303 exceeds the total reserve of \$12,287, which was derived assuming uniform allocation of surrender charges. In this particular case, the entire surrender charge is discounted at a lower interest rate, but this is more than offset by a lower total surrender charge. Under uniform allocation of surrender charges, the total surrender charge allowed as a reduction is 4 percent of most of the fund plus 5 percent of a small portion of the fund, valued one year earlier, or

$$0.04(\$10{,}724)(1.16)^{1/2}(1.09) + 0.05(\$1{,}716)(1.11)^{1/2} = \$594 \, .$$

If the surrender charge is allocated to the first-year fund, then the amount allowed as a reduction is

$$0.04(\$12{,}440)(1.16)^{1/2}(1.09) = \$584 \, .$$

What can we conclude? It is probably safe to say that the application of CARM to a change-in-fund basis was not a major consideration to the authors of the new valuation laws. I have demonstrated that in certain scenarios the allocation of surrender charges can result in procedures and results that differ from the traditional concept of applying CARM to one fund. The differences are small and probably not material for single premium deferred annuities. For flexible premium annuities, uniform allocation of the surrender charge may not produce satisfactory results under a nonlevel commission schedule. Each actuary using CARM will have to make his (her) own recommendations as to the application of CARM in the absence of specific guidelines. In situations where reasonable alternatives produce only minor differences, I would allocate the surrender charge to all funds proportionately and treat each fund as a separate policy.

ALFRED RAWES III:

By being the first to put his interpretation of the CARM in print, Mr. Jaffe has volunteered to serve as a focal point of the discussion on this topic. For this we owe him thanks, if not some hazardous duty pay as well. This discussion will consider some of the points made by Mr. Jaffe and some points he did not bring out.

Mr. Jaffe provides a clear explanation of how CARM applies to the basic annuity benefit. His Table 1 displays the present value of the cash surrender values to illustrate how CARM works. While the table is an aid to understanding the mechanics of the method, it suffers from two problems as a device to actually perform a valuation. The table is dependent on the history of past credited rates, as well as future guaranteed rates. Thus, if 9 percent were to be credited in year 2, instead of the guaranteed rate of 8 percent, then virtually the entire table would need to be recalculated. The table is also dependent upon there being no lapses.

Admittedly, Table 1 was not designed for valuation purposes, but it is easily transformed into one that is. Table 1 of this discussion is derived from Mr. Jaffe's Table 1 by dividing every number in each row by the projected accumulated values at the point in time given in the first column. The reserve factor from this table would be multiplied by the current accumulated value to get the actual reserve. The current accumulated value would reflect actual past lapse and interest history, so the reserve factor would not need to reflect these items.

Following the basic CARM, Mr. Jaffe turns to considering several ancillary benefits. The first is the death benefit inherent in the typical annuity. This death benefit has two parts. The first is the excess of premiums paid over the cash surrender value during the early months the policy is in force, and the second is the waiver of the surrender charge thereafter. On the basis of the parameters in his example, Mr. Jaffe calculates the reserve for the first part to be \$789 for each \$10,000,000 of premiums collected during the prior twelve months. Conservatism of the reserve system ought to allow the actuary to ignore this benefit and still be able to certify that reserves are adequate in the aggregate. The second part is mentioned in the article but is never really discussed. Before we examine this benefit, it is necessary to consider the philosophy underlying CARM.

Table 1 calculates the present value of future guaranteed cash surrender values by discounting with interest only. At duration t, the reserve per dollar of accumulated value is

$$\max_{j \geq t} \left\{ \prod_{n=t+1}^{j} [1 + r(n)] \right\} v^{j-t}(1 - SC_j) ,$$

TABLE I

PRESENT VALUE OF FUTURE GUARANTEED CASH SURRENDER VALUES, PER DOLLAR OF ACCUMULATED VALUE AND DISCOUNTED FOR INTEREST, AT END OF CONTRACT YEAR

CONTRACT YEAR OF VALUATION	CONTRACT YEAR										
	1	2	3	4	5	6	7	8	9	10*	11
0	.9299	.9625	.9961	1.0308	1.0665	1.0932	1.1008	1.1245	1.1405	1.1568	1.1293
1	.9000	.9316	.9641	.9977	1.0323	1.0581	1.0732	1.0884	1.1039	1.1196	1.0931
2		.9100	.9418	.9746	1.0084	1.0336	1.0483	1.0632	1.0784	1.0937	1.0678
3			.9200	.9520	.9851	1.0097	1.0241	1.0386	1.0534	1.0684	1.0431
4				.9300	.9623	.9863	1.0004	1.0146	1.0290	1.0436	1.0189
5					.9400	.9635	.9772	.9910	1.0052	1.0195	.9953
6						.9500	.9635	.9772	.9911	1.0052	.9814
7							.9500	.9635	.9772	.9911	.9676
8								.9500	.9635	.9772	.9541
9									.9500	.9635	.9407
10										.9500	.9275
11											.9500

* The present values in this column are largest.

where $r(n)$ is the guaranteed interest rate in year n, and v is based on the valuation interest rate. Assume this approach when determining the present value of future guaranteed death benefits. The typical policy form will state that the death benefit is the full accumulated value, without applying the surrender charge. This benefit will always be at least as large as the guaranteed cash surrender value. The reserve for this benefit is

$$\max_{j \geq t} \left\{ \prod_{n=t+1}^{j} [1 + r(n)] \right\} v^{j-t} ,$$

which is larger than the reserve given above.

CARM indicates that the greatest present value should be used in reserving. Hence, the reserve calculations will always be based on death benefits and never on cash surrender values. The reserve, in fact, is the greatest present value of future guaranteed accumulated values (before surrender charges). Because of the inclusion of surrender charges in the calculations published by the NAIC it is clear that the intention is to allow the insurance company to reflect those charges in its reserves. Yet the above demonstration shows that the company never is able to utilize the surrender charges in the reserves.

Suppose that the death benefit is viewed as being just the waiver of the surrender charge. Then the present value for the death benefit is

$$\max_{j \geq t} \left\{ \prod_{n=t+1}^{j} [1 + r(n)] \right\} v^{j-t} SC_j ,$$

which clearly will always be less than the present value based on the guaranteed cash surrender value. As a result of the greatest present value as the reserve, no reserve is set up for waiving the surrender charge at death. The net single premium at issue for this benefit is

$$\sum_{j=1}^{k} \prod_{n=1}^{j} [1 + r(n)] v^{j} {}_{j-1|}q_x SC_j ,$$

where k is the last policy year with a surrender charge and x is an average issue age. Using $x = 50$, 1958 CSO Mortality, and the specifics of the policy in the article, this premium is \$197,007 for \$10,000,000 of annuity premium. This is not an item that can be ignored.

One last way to resolve this is to add the reserve for the death benefit to the reserve for the cash value. But nowhere does CARM mention adding present values. CARM only allows using the maximum present value.

Having seen that interest-only present values have some inherent problems, now consider present values that also include probabilities of death and survivorship. The formula for the reserve considering only cash surrender values becomes

$$\max_{j \geq t} \left\{ \prod_{n=t+1}^{j} [1 + r(n)] \right\} v^{j-t} {}_{j-t}p_{x+t}(1 - SC_j) .$$

Table 2 of this discussion is the equivalent of Table 1, using this formula with $x = 50$ and 1958 CSO Mortality. The formula for the reserve considering only death benefits becomes

$$\max_{j \geq t} \left\{ \prod_{n=t+1}^{j} [1 + r(n)] \right\} v^{j-t} {}_{j-t-1|}q_{x+t} .$$

Table 3 displays these values.

Note that ${}_{j-t-1|}q_{x+t} = {}_{j-t-1}p_{x+t}q_{x+j-1}$ and ${}_{j-t}p_{x+t} = {}_{j-t-1}p_{x+t}p_{x+j-1}$. So the last two formulas may be combined to give the reserve considering both cash surrender values and death benefits:

$$\max_{j \geq t} \left(\left\{ \prod_{n=t+1}^{j} [1 + r(n)] \right\} v^{j-t} {}_{j-t-1}p_{x+t} \max \{q_{x+j-1}, p_{x+j-1}(1 - SC_j)\} \right) .$$

The term $p_{x+j-1}(1 - SC_j)$ will be greater than q_{x+j-1} whenever

$$p_{x+j-1} > \frac{1}{2 - SC_j} .$$

For a 10 percent surrender charge, $p_{x+j-1} > 0.526$, which is true for virtually every attained age. Smaller surrender charges merely produce lower limits for the range in which the inequality holds. The conclusion is that the greatest present value will come from consideration of the guaranteed cash surrender values, as opposed to the death benefits.

One more complication needs to be added. There is another guaranteed

TABLE 2

PRESENT VALUE OF FUTURE GUARANTEED CASH SURRENDER VALUES, PER DOLLAR OF ACCUMULATED VALUE AND DISCOUNTED FOR INTEREST AND SURVIVORSHIP, AT END OF CONTRACT YEAR

CONTRACT YEAR OF VALUATION	CONTRACT YEAR										
	1	2	3	4	5	6*	7	8	9	10	11
0	.9221	.9458	.9691	.9919	1.0141	1.0260	1.0258	1.0242	1.0211	1.0163	.9721
1	.9000	.9231	.9458	.9681	.9898	1.0013	1.0011	.9996	.9966	.9920	.9488
2		.9100	.9324	.9544	.9758	.9872	.9870	.9854	.9825	.9779	.9353
3			.9200	.9417	.9628	.9740	.9738	.9723	.9694	.9649	.9229
4				.9300	.9508	.9619	.9617	.9603	.9574	.9529	.9114
5					.9400	.9510	.9508	.9493	.9465	.9421	.9010
6						.9500	.9498	.9484	.9455	.9411	.9001
7							.9500	.9485	.9457	.9413	.9003
8								.9500	.9471	.9427	.9017
9									.9500	.9456	.9044
10										.9500	.9086
11											.9500

* The present values in this column are largest.

TABLE 3

PRESENT VALUE OF DEATH BENEFITS, PER DOLLAR OF ACCUMULATED VALUE AND DISCOUNTED FOR INTEREST AND MORTALITY, AT END OF CONTRACT YEAR

CONTRACT YEAR OF VALUATION	CONTRACT YEAR										
	1	2	3	4	5	6	7	21	22	23*	24
0	.0086	.0096	.0106	.0117	.0130	.0142	.0156	.0297	.0300	0.300	.0297
1		.0093	.0103	.0115	.0127	.0139	.0152	.0290	.0293	.0293	.0290
2			.0102	.0113	.0125	.0137	.0150	.0286	.0289	.0289	.0286
3				.0112	.0123	.0135	.0148	.0282	.0285	.0285	.0282
4					.0122	.0133	.0146	.0279	.0281	.0281	.0279
5						.0132	.0144	.0276	.0278	.0278	.0276
6							.0144	.0275	.0278	.0278	.0275
7								.0275	.0278	.0278	.0275
8								.0276	.0278	.0278	.0276
9								.0277	.0279	.0279	.0277
10								.0278	.0280	.0280	.0278
11								.0290	.0293	.0293	.0291

* The present values in this column are largest. Note: col. 23 equals col. 22 because $(1.03/1.05)(p_{71}/q_{71})q_{72} = 1.0002$.

benefit at all points in time. This benefit relates to annuitization. The guaranteed benefit at a specified time is found as follows:

1. Calculate the guaranteed accumulated value that can be applied to purchase an annuity.
2. Using the guaranteed purchase rates, convert this value to a monthly income.
3. Using the valuation basis specified in the policy form, calculate the present value of these monthly payments.

This would need to be done for each annuity option. If the value in step 3 is divided by the value in step 1, the result is a factor that is a function of the option elected (O), the date of valuation (t), the time in the future ($j \geq t$), and the assumed age (x). Denote this by $F(O, t, j, x)$. Then the present value of this benefit is

$$\max_{j \geq t} \left\{ \prod_{n=t+1}^{j} [1 + r(n)] \right\} v^{j-t} {}_{j-t}p_{x+t} F(O, t, j, x) .$$

When combined with the present value of guaranteed cash values, the reserve considering all guaranteed benefits is

$$\max_{j \geq t} \left\{ \prod_{n=t+1}^{j} [1 + r(n)] \right\} v^{j-t} {}_{j-t}p_{x+t} G(O, t, j, x) ,$$

where

$$G(O, t, j, x) = \max_{O} [F(O, t, j, x), (1 - SC_j)] .$$

A conservative set of guaranteed annuity purchase rates would easily produce $G(O, t, j, x) = 1 - SC_j$.

Mr. Jaffe's second consideration is the "bail-out" provision. I read the article to say that in applying the basic method to a future policy year in which the guaranteed rate is less than the bail-out rate, the guaranteed benefit at the end of that policy year should be the total accumulated value, not just the cash surrender value. Why is this necessary?

One concern is the excess lapses that a company might experience as a result of crediting less than the bail-out rate. Some lapses may well be attributed to a backlash type of reaction to the credited rate, but there should not be wholesale lapses. Competitively, a company must keep its rate in line with the industry. This would be especially true of a company with a bail-out provision. Thus, a credited rate below the bail-out rate is an indication of the general condition of economy. The policyholder has nowhere else to invest his money at rates better than he is getting from the annuity. If an annuity is a logical financial planning tool when interest rates are 15 percent, then it remains a logical tool when rates drop to 8 or 10 percent.

Another concern is the potential for forced liquidation of the assets underlying the annuity business if high lapses do occur. By matching guarantees and investments properly the company can insulate itself from such problems. A drop in the credited rate is caused by a drop in rates throughout the economy. This will cause the assets in the annuity line to increase in value. Any liquidations will realize capital gains that will offset the loss of surrender charges.

Many contracts currently being offered provide for a penalty-free termination only during the first ninety days after receiving notice of the credited rate being below the bail-out rate. Moreover, the penalty-free termination is available only after the first time this threshold is broken. So for these policies there is a very short exposure to any loss potential from lowering the credited rate.

To sum up, the arguments against the need to make provision in the reserve formula for the presence of a bail-out provision are stronger than the opposing arguments of theoretical completeness.

Mr. Jaffe considers policy provisions for free partial withdrawals. In the discussion he comments: "Each year a new table of cash values must be generated and tested for each policy, based on its actual partial-withdrawal history." While this has a theoretical appeal and while reserve systems for annuity products will need to operate at the policy level, this will be a practical nightmare. A conservative approach is to assume that the benefit guaranteed by the policy is the cash value given by

$$CSV_t = [(1 - FPW_t)(1 - SC_t) + FPW_t]AC_t ,$$

where FPW_t is the accumulated free partial withdrawal percentage available in year t assuming no prior partial withdrawals. This will be easier to apply and ignores actual withdrawal history.

I will now discuss two topics not addressed in Mr. Jaffe's article. First, how does CARM handle an interest guarantee that involves a published index? For example, a policy could have the following guarantee: (*a*) 14 percent for the first twelve months; (*b*) thereafter not less than 2 percent less than the average yield rate on one-year Treasury bills; but (*c*) never less than 4 percent. Is it appropriate to calculate the guaranteed cash values based on 14 percent followed by 4 percent in all renewal years? Does the answer differ depending on whether the reserves are on an issue-year basis or a change-in-fund basis? A negative response to the first question leads to another sequence of questions: For how many years must the index value be predicted? Will each state make its own forecast? Will a forecast be required for each index base?

Allowing the use of 14 percent followed by 4 percent has the advantage of being easy to understand and easy to apply. It also applies the letter of the law, since the minimum guaranteed rate is, in fact, 4 percent. Finally, it puts companies using an indexed product on an equal reserve footing with a company using a nonindexed product. No company can retain its market share by dropping its credited rate to the minimum contractual amount after the expiration of the initial guarantee. Why, then, should a company that tells the policyholder how the renewal credited rate is to be determined be governed by different valuation rules than a company that is silent on this point?

My second concern relates to the company's federal income tax. The typical tax problem with annuity products involves the treatment of interest credited in excess of the guaranteed rate. Because a nonlife company is not restricted in the amount of dividends it can deduct on its tax return, the problem can be solved by ceding the annuity business to a life insurance company that is taxed as a nonlife company. The difficulty over who has the right to declare the credited interest rate can be removed by creating a subsidiary to be the reinsurer. This avoids paying a reinsurer to do what the company can do for itself, and also allows the group to retain the entire risk. If the annuity reserves qualify as life reserves, then the treaty should be modified coinsurance with no section 820 election. If the reserves are not life reserves, then the treaty should be straight coinsurance. The distinction is necessary to maintain both companies' tax positions, so it becomes of great importance to know the status of the annuity reserves.

Life reserves need, among other requirements, to be calculated using a mortality table. The IRS appears to be allowing the reserve on any annuity product that contains permanent mortality guarantees to be considered a life reserve regardless of how it is calculated. The permanent guarantee takes the form of tables of settlement options involving life contingencies. Currently the reserves as set out by Mr. Jaffe are life reserves. The reserves described in this discussion are life reserves and so are independent of any IRS position. If the IRS changes its position at some point in the future, some companies could fail to qualify as life companies. The resulting potential for a Phase III tax after two years as a nonlife company argues strongly for the use of reserves clearly based on a mortality table.

A final concern on taxes is one of labels. Mr. Jaffe uses the term "deficiency reserves" in reference to annuity reserves. His use of quotes whenever the term appears indicates there is no true deficiency. The choice of a term that does not have negative tax connotations would have been more advisable.

ROBERT RICH:

My compliments on a very helpful paper on the application of the CARM, particularly with regard to some of the ancillary benefits that have been developing with regularity in the modern annuity products.

I do have a problem, though, and it is one that I have pondered at my own company. It is implicit in the paper, most particularly in Section D, where Mr. Jaffe believes it is possible to have a reserve that is less than the full accumulated value on the valuation date, despite having a guaranteed interest rate equal to the valuation rate and a surrender charge that becomes zero on some future date.

My feeling that this is not possible is based on my understanding that the CARM for a single premium annuity can be expressed as follows:

$$
{}_tV = \left[\begin{array}{l}
(AC_t)(1 - SC_t) \\
(AC_t)\left(\dfrac{1+g}{1+i}\right)(1 - SC_{t-1}) \\
(AC_t)\left(\dfrac{1+g}{1+i}\right)^2(1 - SC_{t+2}) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
(AC_t)\left(\dfrac{1+g}{1+i}\right)^{r-t}(1 - SC_r) \\
R
\end{array}\right] .
$$

where the brackets denote the greatest of the several terms, and i is the valuation interest rate, g is the guaranteed credited rate, r is the year of retirement, and R is the present value of the guaranteed retirement benefit. All other terms are those used in the paper.

If $i = g$ and the value of the retirement benefit is equal to the cash value at retirement, then this formula becomes

$$CV = \begin{bmatrix} (AC_t)(1 - SC_t) \\ (AC_t)(1 - SC_{t+1}) \\ \cdots\cdots\cdots\cdots \\ (AC_t)(1 - SC_r) \end{bmatrix}$$

The largest value in the brackets would occur when the surrender charge was the lowest. If the surrender charge was zero at any time, then the reserve would equal the full accumulated value.

It would seem that for a "no-load" annuity with a guaranteed interest rate equal to the valuation rate, and a surrender charge that ultimately disappears, the reserve will always be the accumulated value regardless of whether any of the ancillary benefits referred to in the paper are present.

(AUTHOR'S REVIEW OF DISCUSSION)

JAY M. JAFFE:

Interestingly, the six written discussions of my paper generally addressed two topics. The first topic was how to treat the "bail-out" or "window" provision. This is the policy provision that allows the annuitant to surrender the policy without penalty if the company's declared interest rate drops below a predetermined rate or "window." After reading the discussions, it is clear that there is more than one position one can use in reserving annuities with these provisions.

The purpose in writing the paper was to establish a framework for discussion about the CARM. As one discussant said, for this effort I "deserved hazardous duty pay." At least on the point of the bail-out option, the paper has served its purpose and caused serious discussion. The issue is not yet resolved, and this leads nicely into the second main topic of the discussants.

The reviewers consistently brought up the need for actuarial judgment to be applied in reserving annuities. I totally concur in this feeling. Each actuary must review particular policy provisions and the entire corporate picture before selecting an appropriate reserving methodology for deferred annuities. The methodology should consider both the written CARM and sound actuarial judgment.

My thanks to each of the discussants who took the time and effort to write a formal discussion. Their contributions add breadth to my paper and are appreciated by all actuaries who have been involved with the statutory reserving of deferred annuities.