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FINANCIAL ACCOUNTING STANDARDS NO. 87: RECURSION FORMULAS AND OTHER RELATED MATTERS

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INTRODUCTION

Statement of Financial Accounting Standards No. 87, *Employers' Accounting for Pensions*, was released in December 1985, culminating an effort that began in December of 1974 and took many turns, some of them startling. The final version contains 77 numbered paragraphs covering what amounts to an accounting/actuarial standard with significant income statement and balance sheet implications. It is a difficult document for both professions, since it freely mixes and adapts from each. There are no formulas in the 132 pages of Statement 87; the fundamentals are difficult to ferret out, despite 40 pages of examples. It is the purpose of this paper to present the essentials of Statement 87 in terms familiar to actuaries—recursion formulas and other symbolic representations—in order to provide insight into the underlying basis of this standard as well as formulas useful for checking purposes. Symbols will be defined as they occur in the text.

Statement 87 chooses the projected unit credit funding method as the sole approach to developing accrued liabilities and current service costs, or normal costs (*NC*), for use in determining pension expense (*PE*) and balance sheet liabilities. The *PE* set forth in the Statement, however, is quite different from that familiar to actuaries and based on the Employee Retirement Income Security Act of 1974 and Internal Revenue Service (IRS) regulations. Two different interest rates are used in calculating *PE*: (1) The discount rate (*DR*) is used to calculate the present values of liabilities and normal costs. (2) An expected long-term rate (*ELTR*) is used to calculate the expected return on plan assets. (Apparently, the authors of Statement 87 believe that $DR \leq ELTR$ more often than not.)

Two different asset valuation bases are used. Parts of the calculation of *PE* are a function of the market value of plan assets (*PA*), but the calculation of expected investment income applies the *ELTR* to a market-related value (*MRV*), a smoothed function of market value, with smoothing not to exceed five years. *MRV* is permitted to equal *PA*.

Net actuarial gains in excess of a corridor are amortized over a period of years. ("Corridor" is a misnomer; it is simply a cutoff point.)

For balance sheet purposes, as in current practice, there may be a prepaid pension cost (*PPC*) if prior funding contributions exceed prior accounting

pension expense. If pension expense has exceeded funding contributions, an analogous accrued pension cost (*APC*) occurs. In what follows, if *PPC* is negative, it becomes *APC*. A minimum balance sheet liability is introduced by the Statement, applicable to plans with unfunded liabilities, under the unit credit funding method. An "additional liability" and a somewhat corresponding intangible asset are also introduced; the latter with net worth implications.

Most of these can be represented by recursion formulas, which is a great convenience, since this serves as both a checking device and a considerable aid to comprehension. Some of the remaining items are simply rules which can be rationalized, but this is a bit dangerous since other equally appropriate rules could be designed and rationalized. In what follows, there is no attempt to provide any basis other than a formula representation, with the advantages that a formula has over a verbal description.

Many of these items will be disclosed in financial statements. It remains to be seen how helpful this information will be to readers of these statements.

ELEMENTS OF PENSION EXPENSE

Projected Benefit Obligation (PBO)

The *PBO* is the projected unit credit accrued liability and can be generalized as follows:

$$PBO_0 + i \cdot PBO_0 + {}_0NC_1 - {}_0B_1 - {}_0(LG)_1 + (AMENDT)_1 - (\Delta ASSUMTNS)_1 = PBO_1 \ldots \quad (1)$$

In formula 1, i is the *DR* over the year zero to one, and each term is assumed to include interest correctly over the interval (0,1), to be valued at time one. (*NC*) refers to the projected unit credit current service cost, or normal cost, for convenience. The last four terms refer to benefits paid (*B*), the accrued liability gain or loss (*LG*), amendments changing the benefits of the plan, and any changes in actuarial assumptions.

If the plan has salary-related benefits, a salary scale must be used. All actuarial assumptions are to be explicitly accurate.

Plan Assets (PA)

Similarly, the development of the market value of plan assets over the year can be represented as:

$$PA_0 + {}_0C_1 - {}_0B_1 + {}_0(EI)_1 + {}_0(IG)_1 = PA_1 \ldots \quad (2)$$

In formula 2, conventions about interest are the same as in formula 1. (*C*) represents the contributions during the period; (*EI*) is the expected in-

terest measured by the *ELTR* as applied to *PA* or *MRV* and (*IG*) is the excess interest gain. (Where *MRV* is adopted, replace *PA* appropriately in formula 2.)

These interest rates are described in Statement 87. The *DR* is intended to be a current rate at which pension benefits could be settled. Possibilities for determining this rate include: a current annuity buyout rate, current Pension Benefit Guaranty Corporation rates, a long-term-bond rate reflecting high-quality securities with a term similar to the period over which benefits will be paid. These alternative bases encompass a fairly wide range of acceptable rates at any point in time. The *ELTR* appears to be a proxy for the assumed valuation interest rate. We freely admit that other interpretations are possible and these brief comments are meant only to illustrate some alternatives in determining these interest rates.

Unrecognized Prior Service Cost (UPSC)

UPSC is zero at the date of adoption of Statement 87 and from that point forward refers to the increase in unrecognized liability included in the *PBO* as a result of any amendments changing the benefits of the plan.

UPSC is amortized by special rules which change the usual method of amortization; see paragraphs 25, 26 of Statement 87. For our purposes, *UPSC* is amortized over average future service. The development of *UPSC* over the year becomes:

$$UPSC_o - {}_0(AMORT\ UPSC)_1 + (AMENDT)_1 = UPSC_1 \ldots \quad (3)$$

Unrecognized Net Gain (UG)

UG is also zero at the date of adoption of Statement 87 and from that point forward can be represented as:

$$UG_0 + [(\Delta ASSMPTNS)_1 + {}_0(LG)_1 + {}_0(IG)_1] - {}_0(AMORT\ GAIN)_1 = UG_1 \ldots \quad (4)$$

The same comments about the amortization period of *UPSC* apply to *UG* as well. (See paragraphs 32, 33 of Statement 87 for some differences.) However, there is a complication, because *UG* is conditioned upon the operation of a trigger device, or "corridor." The recognized part of the process is defined as:

$$|G| - 10\% \cdot \text{Max}(PBO, MRV), \text{ zero if negative} \ldots \quad (5)$$

where *G* is the net actuarial gain in the year (*LG* + *IG*), increased by any unamortized actuarial gains from the prior year, and an additional amount,

when *MRV* is used at the time Statement 87 is adopted, equal to the appropriate amortization of the initial excess of *MRV* over *PA*. If (5) is positive, the excess is amortized over average future service.

Unrecognized Transition Obligation or Asset (UT)

Since *UPSC* and *UG* are zero at date of adoption of the Statement 87, there is an unrecognized transition obligation, or asset, with special amortization rules. The formula representation is best considered in two steps: the first is the definition of *UT* at the date the statement is adopted, and the second is the recursion formula.

$$UT_0 = PBO_0 - PA_0 + PPC_0 \ldots \quad (6)$$

$$UT_0 - {}_0(AMORT\ TRANS)_1 = UT_1 \ldots \quad (7)$$

UT_0 can be negative (an "asset") with the amortization period average future service or, if greater, 15 years. See paragraph 77 of Statement 87. (When *PPC* is defined, notice that formula 6 falls out at the date of adoption since $UPSC_0 = UG_0 = 0$. See formula 10.)

Pension Expense (PE)

PE is defined elaborately as:

$$\begin{aligned} {}_0PE_1 = {}& {}_0NC_1 + i \cdot PBO_0 - [{}_0(EI)_1 + {}_0(IG)_1] \\ & + {}_0(AMORT\ UPSC)_1 \\ & - {}_0(AMORT\ GAIN)_1 + {}_0(IG)_1 + {}_0(AMORT\ TRANS)_1 \ldots \end{aligned} \quad (8)$$

Formula 8 is unusual in that (*IG*) could be canceled but, presumably, the authors of Statement 87 preferred to show the third and sixth terms intact. As a result, although *PE* can be determined at the beginning of the plan year, the components of *PE* are not all known until year end, when *IG* can be calculated. *PE* can also be defined by recasting the preceding results as:

$$\begin{aligned} {}_0PE_1 = {}& -(PBO_0 - PBO_1) + (PA_0 - PA_1) \\ & + (UPSC_0 - UPSC_1) \\ & - (UG_0 - UG_1) + (UT_0 - UT_1) + {}_0C_1 \ldots \end{aligned} \quad (9)$$

Formula 9 provides some insight into how formula 8 was arrived at and also provides a check of the accuracy of the various calculations, since formula 8 must equal formula 9.

BALANCE SHEET ITEMS

These items include *PPC*, an Additional Liability, an Intangible Asset (*IA*) as well as a possible charge to equity. *PPC* is defined as follows:

$$PPC_1 = -PBO_1 + PA_1 + UPSC_1 - UG_1 + UT_1 \ldots \quad (10)$$

Formula 10 can be restated as:

$$PPC_1 = [-PBO_0 + PA_0 + UPSC_0 - UG_0 + UT_0] + {}_0C_1 - {}_0PE_1 \ldots \quad (11)$$

Notice that the term in the bracket, in formula 11, is *PPC* at time zero so that:

$$PPC_1 = PPC_0 + {}_0C_1 - {}_0PE_1 \text{ or}$$

$$PPC_1 - PPC_0 = {}_0C_1 - {}_0PE_1 \ldots \quad (12)$$

Formulas 10, 11, and 12 are useful in the same sense as formulas 8 and 9: they provide insight into the process and serve as checks that the calculations are correct. Similar formulas apply to *APC*, since *APC* is *PPC* negative. Note that formulas 9 and 12 say the same thing.

The Additional Liability, Intangible Asset (*IA*), and the possible charge to equity are related. To investigate these relationships, we need two terms, the Accumulated Benefit Obligation (*ABO*), which is the *PBO* without salary scale or, simply, the unit credit accrued liability based on the accrued pension benefit at the valuation date, and the Minimum Liability (*ML*), which is defined by:

$$ML = ABO - PA, \text{ zero if negative} \ldots \quad (13)$$

The Additional Liability is the sum of the Minimum Liability (*ML*) and the *PPC*, but only if $ML > 0$ and the Additional Liability itself is positive.

The *IA* is the lesser of the Additional Liability and a second amount equal to the *UT* plus *UPSC*. Therefore,

$$(IA) = \text{Lesser of } (ML + PPC, UT + UPSC) \ldots \quad (14)$$

$$= \text{Lesser of } (A, B) \ldots \quad (15)$$

The charge to equity, before tax recognition, is $A - B$, zero if negative, so that it exists only if $B < A$.

THE TIMING OF THE RESULTS

The beginning-of-year projected results are augmented by asset information developed at the measurement date, which must be within the three

month period preceding the end of the fiscal year. Formulas 1 and 2 would be redetermined at the end of the year, based on the appropriate *DR* and *ELTR*, and *G* would be found. Most of the disclosure items will have been determined or have obvious definitions. See paragraph 54 of Statement 87.

VOLATILITY OF THE PENSION EXPENSE AND BALANCE SHEET LIABILITIES

Pension expense and balance sheet liabilities likely will be volatile since the *DR* will change from year to year largely reflecting changes in yields on long-term bonds or direct placements with insurance companies.

Referring to formula 13, it is evident that *ML* will vary with changes in the *DR*. Clearly, *PA* might not vary in the same way. Thus, underfunded or barely funded plans on an *ABO* basis may see large swings in the *ML* unless *PA* correspondingly moves with the *DR*.

Pension expense, as defined in equation 8, also varies with *DR*. The expense elements *NC* and *iPBO* vary directly with the *DR*. Amortization Gain also varies with *DR* once the cumulative effect is larger than the 10 percent corridor for amortization.

The volatility of *PE* can be dampened by an investment policy in which asset gains counterbalance changes in the *PBO*'s *DR* changes. This might occur if an immunized bond portfolio is established for the entire *PBO*.

These comments are meant to be illustrative only of the relations necessary to dampen volatility and are not meant to suggest investment policy. We believe that companies should be wary of changing investment policy in response to accounting concerns in any way that may adversely affect the anticipated long-term return on plan assets.

A COMMENT ON AMORTIZATION

The amortization period under Statement 87 is generally tied to the average future working lifetime of those expected to receive benefits. Changes in actuarial assumptions can change the length of amortization periods calculated in the future. In particular, a change to (or from) select and ultimate turnover from (or to) a static turnover table can have a significant effect on the amortization period.

Although accounting, business, and government have long used compound interest amortization factors—as defined in a course in the mathematics of finance—this approach was not used.

SUMMARY AND CONCLUSIONS

It is hoped that these recursion formulas will provide insight into Statement 87, serve as an aid in checking calculations, and assist in reconciling results from one year to the next.

A comprehensive analysis of pension expense and balance sheet volatility, using simulations, would be of interest. A satisfactory frequency distribution for the discount rate would be needed. The use of different interest assumptions for discounting liabilities and in calculating expected interest on plan assets, the "corridor," and the method of amortization will affect the asset and liability gain/loss in ways yet to be fathomed.

A major purpose of Statement 87 was to introduce comparability into financial statements. It is questionable whether this objective has been achieved. In a few years, an evaluation of the range of interest assumptions used and their effects on income statements and balance sheets will be possible.

Whether adopting one funding method; separating pension expense from pension contribution; amortizing rapidly; ignoring IRS regulations as to the range of tax-deductible contributions; and using termination-type calculations instead of going-concern calculations result in better accounting—and more understanding of the process—remains to be seen and will be tested over the next several years.

GLOSSARY

ABO	= Accumulated Benefit Obligation
AMENDT	= Change in *PBO* from plan amendments
AMORT	= Amortization of *UPSC* or *G* or *UT*
APC	= Accrued Pension Cost ($PPC < 0$)
$\Delta ASSUMPTNS$	= Change in *PBO* from assumption changes
B	= Benefit payments made
C	= Contributions
EI	= Expected interest
ELTR	= Expected Long-Term Rate
G	= Gain or loss
i	= Discount Rate (*DR*)
IA	= Intangible asset
IG	= Excess interest gain (loss)
LG	= Liability gain (loss)
ML	= Minimum (balance sheet) liability
MRV	= Market related value of plan assets
NC	= Current Service Cost or Normal Cost
PA	= Market value of plan assets
PBO	= Pension Benefit Obligation
PE	= Pension Expense
PPC	= Prepaid pension cost
UG	= Unrecognized net gain (loss)
UPSC	= Unrecognized prior service cost
UT	= Unrecognized transition obligation (asset)

DISCUSSION OF PRECEDING PAPER

ELIAS S. W. SHIU:

This pedagogical note is motivated by the term "recursion formulas" in the title of the paper. The recursion formulas considered here are first-order linear difference equations. Analogous to the case of first-order linear differential equations which are solved by the method of integrating factors, first-order linear difference equations are solved by the method of summation factors.

First-order linear difference equations can be written in the form

$$x_{n+1} = a_n x_n + b_n \qquad n = 0, 1, 2 \ldots \tag{1}$$

where x_0, $\{a_n\}$ and $\{b_n\}$ are given. To solve for x_n, we divide both sides of (1) by

$$s_{n+1} = a_0 a_1 \ldots a_n.$$

Then (1) becomes

$$\frac{x_{n+1}}{s_{n+1}} = \frac{x_n}{s_n} + \frac{b_n}{s_{n+1}}$$

or

$$\Delta\left(\frac{x_n}{s_n}\right) = \frac{b_n}{s_{n+1}}. \tag{2}$$

Summing (2) we obtain

$$x_m = s_m \left(x_0 + \sum_{j=0}^{m-1} \frac{b_j}{s_{j+1}}\right). \tag{3}$$

Formula (3) can be very elegantly implemented in *APL* as follows:

$$S \times XO + +\backslash B \div S \leftarrow \times\backslash A$$

This beautiful *APL* algorithm, giving the values of $x_1, x_2, \ldots$ without looping, was pointed out ten years ago by John Gaboury in a letter to the Editor of *APL Quote Quad*.

In the new *Actuarial Mathematics* textbook [1], many concepts are expressed in terms of recursion formulas, which, as suggested in this paper,

is a great convenience. Frequently, such formulas are derived by the method of conditioning, that is, by applying the formula $E(X) = E(E(X|I))$.

REFERENCE

1. BOWERS, N.L., JR., GERBER, H.U., HICKMAN, J.C., JONES, D.A., AND NESBITT, C.J. *Actuarial Mathematics*. Itasca, Illinois: Society of Actuaries, 1986.