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UNIVERSAL LIFE VALUATION AND NONFORFEITURE: A GENERALIZED MODEL

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ABSTRACT

In recent years there has been a proliferation of plans with the generic name of "universal life." The application of the Standard Valuation Law and Standard Nonforfeiture Law to these plans is still a matter of some controversy. Actuaries have long known that the mechanics of a side fund, with deductions for decreasing term insurance, parallel the mathematical structures of the reserve and cash value in traditional products. Applying this analysis to universal life, both the reserve and the cash surrender value have been expressed in terms of a side fund, or account value. We call this description the "Classical UL Model."

It is the authors' contention that this characterization of universal life is mathematically valid only under special conditions. Currently, most universal life products do not meet these conditions and thus cannot be viewed in this unbundled manner. We intend to set forth a generalized model (of which the Classical UL Model will be a special case) describing the application of current law (1976 NAIC model Standard Valuation Law and Standard Nonforfeiture Law) to products of the universal life type. To do this, we argue that the product must be conceptually "rebundled" so that its values can be judged in comparison with those associated with guaranteed future benefits.

I. INTRODUCTION

This creature called universal life has evolved from a combination of term insurance and a flexible premium annuity to a range of differing products with various design features. Common to any of these variations is the fact that future cash values cannot be determined completely at issue. Cash values (and other benefits) are produced by a formula that is specified in advance. Some of the various elements used in the formula, however, can be adjusted after issue by the insurer. Since the one common bond in all such products is the unknown cash value, throughout this paper we will refer to these products as "indeterminate cash value" products (ICVs).

We shall describe a general model of the relationship between ICVs and current (1976 NAIC) valuation and nonforfeiture laws. In this paper, we neither endorse nor criticize the philosophical bases of these laws. Our position is that the broader question of "ideal law" can be addressed adequately only in the context of all plans of insurance, not just ICVs. As urgent as that question is today, we have limited the scope of our investigation to viewing ICVs in relation to 1976 NAIC model laws. Our only goal is to demonstrate that direct interpretation and application of existing laws to ICVs result in consistency with conventional products, and yet allow for surprising flexibility in plan design.

2. RESERVES AND CASH VALUES FOR CONVENTIONAL PRODUCTS

Perhaps the best starting point in our description of reserve and nonforfeiture principles for ICVs is a review of those principles (1976 NAIC) for conventional products. We believe that the major concepts (for our purposes) of the 1976 Standard Nonforfeiture Law (SNFL) and Standard Valuation Law (SVL) can be summarized as follows:

2.1. Minimum cash values and minimum reserves are defined prospectively in terms of the present value of guaranteed future benefits under the contract.
2.2. Mathematically, both minimum cash values and reserves are defined prospectively as modified reserves.
2.3. For purposes of defining minimum values, adjusted premiums (modified net premiums for reserves) are defined to be a constant percentage of gross premiums.
2.4. Adjusted premiums (modified net premiums for reserves) are calculated on the basis of benefits guaranteed at issue.
2.5. Premium deficiency reserves (more properly, minimum reserves) are calculated on the minimum valuation standards of interest and mortality rather than on the contract valuation basis.
2.6. There is no required relationship between contract cash value and reserve bases with respect to either the rate of interest or the rates of mortality.

In discussion of the applicability of the 1976 model laws to ICVs, it is often stated that these laws did not anticipate contracts where unknown premium or benefit changes can occur. We believe that this opinion is only partially true. Both laws define a modified reserve equal to the present value of guaranteed future benefits, less the then present value of modified (adjusted) premiums. Both laws clearly define their respective modified premiums in terms of the guaranteed benefits at issue, and make no provision for modifying these premiums under any circumstance. This is the element of truth in the above-mentioned opinion.

However, these laws do anticipate post-issue changes in future benefits:

> Any cash surrender value available under the policy in the event of default in a premium payment due on any policy anniversary, whether or not required by section two, shall be an amount not less than the excess, if any, of the present value, *on such anniversary*, of the future guaranteed benefits which would have been provided for by the policy, *including any existing paid-up additions*, if there had been no default, over the sum of (a) the then present value of the adjusted premiums as defined in sections five, five-a and five-b corresponding to premiums which would have fallen due on and after such anniversary, and (b) the amount of any indebtedness to the company on the policy. [1976 SNFL, sec. 3 (emphasis added).]

> [R]eserves . . . shall be the excess, if any, of the present value, *at the dat of valuation*, of such future guaranteed benefits provided for by such policies, over the then present value of any future modified net premiums therefor. [1976 SVL, sec. 4 (emphasis added).].

To the authors, these sections clearly require that changes in future guaranteed benefits after issue be included in a prospective calculation of minimum reserves and cash values. These considerations lead us to an important conclusion:

2.7. The 1976 model laws require net single premium funding (increments to the reserve and the cash value) of any post-issue change in guaranteed future benefits.

This conclusion is important, but not very surprising. The 1980 model laws allow for recalculation of modified (adjusted) premiums after issue. This effectively provides for a more lenient annual pay funding of benefit changes after issue in the reserve and cash value. We infer from this that there has been a widespread awareness of the net single premium funding concept (2.7). There is also widespread acceptance of this rigid requirement for universal life reserves. However, there seems to be no recognition of this prospective principle for universal life cash values. We will return to this important topic later.

In what follows, we will apply this seven-point summary of the 1976 model laws to both fixed and flexible premium universal life contracts. We focus on the 1976 laws instead of the 1980 version for several reasons:

a) Most (perhaps all) existing universal life contracts are issued under the 1976 amendments.

b) The 1976 amendments require a more conservative approach to funding benefit changes (point 2.7).

c) Many companies will not be able to switch immediately to a contract based upon the 1980 amendments.

Our strategy will be to tackle the fixed premium ICV first. Since only the benefits vary after issue, this product slips easily into the framework of the current laws. With a thorough analysis of the fixed premium ICV behind us, we will be in a much better position to evaluate the difficult and subtle issues surrounding flexible premium ICVs.

3. THE CLASSICAL UL MODEL

It is appropriate to begin with a review of the methods currently in use to explain the mechanics of ICVs. We call this description the "Classical UL Model," although the label may seem a little unusual for something so new. The Classical Model is a mathematical description of universal life in which minimum reserves and cash values are expressed retrospectively in terms of the account value. This paradigm is described in the next few pages. A more detailed derivation can be found in Appendix A.

The heart of the boilerplate proof lies in showing that term with an annuity exhibits behavior identical with that of the amount at risk and reserve of a traditional policy. As an example, let us structure term plus a flexible premium deferred annuity to simulate a whole life policy with a death benefit of \$1 payable at the end of the year of death. The \$1 death benefit can come only from two sources: the account value of the annuity and the amount of term insurance purchased. Assuming that the charge for this term insurance is deducted from the account value at the end of the year, and a premium is deposited in the fund at the beginning of the year, the sources of the death benefit are visible in the equation below:

$$\$1 = [\text{Account value}] + [\text{Term benefit}] \ .$$

We contractually define the annuity account value so that

$$_{t+1}AV = ({}_tAV + P)(1 + i) - TC \ ,$$

where

${}_tAV$ = Account value of the deferred annuity, at the end of policy year t;
P = Net deposit to the account (premium less loads);
TC = Amount charged to the account for the term coverage (term charge).

Now let q equal the purchase rate for \$1 of term death benefit. Since $TC = q \times$ (Term benefit), our first equation can now be written as

$$\$1 = [({}_tAV + P)(1 + i) - TC] + [TC/q] \ .$$

Solving for TC, we find that

$$TC = [1 - ({}_tAV + P)(1 + i)] \frac{q}{1 - q} .$$

If we substitute this expression for TC in our recursive formula for the account value, we discover that

$${}_{t+1}AV = \frac{({}_tAV + P)(1 + i) - q}{1 - q} .$$

The latter equation analysis is a familiar formula for equating successive terminal reserves. From this we can conclude that, mathematically speaking, the account value is a reserve. Legally speaking, it is a reserve only for certain values of q and i (e.g., 1958 CSO mortality and 4½ percent interest). Further, to produce a Commissioners modification, a deduction may be taken from the account value equivalent to the Commissioners allowance normally taken in year 1. Since a certain amount of loading may be removed in the first year by policy design, the deduction takes the form of any excess of the Commissioners allowance over the loadings actually taken in the first year[1] amortized over the premium-paying period. If we assume level premiums, the CRVM reserve then becomes

$${}_tV^{\mathrm{CRVM}} = {}_tAV - \frac{\text{Commissioners allowance} - E^c}{\ddot{a}_x} \ddot{a}_{x+t} ,$$

where E^c represents the excess of first-year loads over renewal loads.

A similar approach[2] is taken in calculating cash values. The resulting formula (by the reasoning above) is

$${}_tCV = {}_tAV - \frac{E^1 - E^c}{\ddot{a}_x} \ddot{a}_{x+t} .$$

The conclusion commonly drawn from these results is that the account value (with adjustments for E^1 or Commissioners allowance is an ade-

[1] When we mention first-year loads, we refer to the excess of loads assessed in the first policy year over any level percentage renewal loads. The Classical UL Model requires that renewal loads be a level percentage of premiums.

[2] A derivation of both reserve and cash-value formulas can be found in Appendix A. In what follows, the familiar actuarial symbol $\ddot{a}$ represents in a general way the present value of any pattern of payments. It includes, but is not limited to, a level pattern of payments.

quate representation of reserves and cash values. Later in this paper we will demonstrate that, even if the level premium assumption is accepted, the approach outlined in this section yields proper results *only if certain conditions are satisfied*. These necessary conditions will be discussed in Section 5.

4. TEN PERCENT CASH VALUES?

We now digress somewhat to address a prevalent fallacy concerning cash values under the Classical UL Model. Defining the actual cash-value interest rate has been the source of much confusion. It is important to realize that *under the Classical UL Model the policy guarantees (interest and mortality) define the cash-value basis*. This is true regardless of whether more liberal interest or mortality was credited in the past. It is incorrect to assume that the *actual* interest rate credited represents the rate of interest used in calculating cash values.

Consider a universal life policy with 1958 CSO fixed mortality costs, an interest guarantee of 4 percent, and no mortality corridor.[3] Suppose that there are no front-end loads, no percentage-of-premium loads, and no surrender charges, and suppose that a policyholder pays an annual premium equal to P_x calculated using 1958 CSO mortality and 10 percent interest. If the company actually credits interest at 10 percent (the 10 percent is not guaranteed until credited) for the life of the policy, the plan of insurance will turn out to be whole life. The policyholder has obviously been given cash surrender values numerically equal to 10 percent whole life cash values (actually, they are equal to 1958 CSO, 10 percent net level reserves); however, it is *not* true that the cash-value interest rate was 10 percent.

The nonforfeiture laws attempt to define a cash surrender value commensurate with both the contractually *guaranteed* benefits forfeited and the outstanding contractual obligations on the part of the policyholder (future premiums, if any). The cash value in the above example is always based on 1958 CSO and 4 percent in relation to contractually guaranteed benefits forfeited. At issue, this universal life contract guarantees some form of term insurance benefits, or possibly term and an endowment for some fraction of the face amount. Unless the 10 percent interest rate is guaranteed *in advance*, our hypothetical universal life contract does not guarantee whole life benefits until the very instant of endowment, when the cash values on each interest basis are both equal to the face amount. The effect of any credited "excess interest" is both to increase the ac-

[3] The term *mortality corridor* refers to any minimum requirements on the term insurance amount.

count/cash value and to provide for additional guaranteed future benefits (through a longer term period, and/or an increased endowment value). As discussed in point 2.7, the 1976 SNFL requires that the resulting increment in the cash value be at least equal to the present value, at the nonforfeiture rates of interest and mortality, and the additional guaranteed future benefits.

In our example, the increment in the cash value is exactly *equal* to the present value of additional guaranteed benefits using 1958 CSO and 4 percent; it *exceeds* the present value of additional guaranteed benefits at 10 percent interest. Thus, far from being "10 percent cash values," these universal life cash values are prospectively *always* 1958 CSO 4 percent values. A similar statement can be made for reserves.

We are not venturing into new territory with this conclusion. A similarly fallacious "10 percent cash value" argument can be devised for all participating contracts. Dividends may be used to buy additional benefits or to reduce premiums. In either case, the *resultant* plan of insurance would produce cash values different from those required by law. As an example, it would be incorrect to require a "paid-up" cash value when in later durations whole life dividends exceed premiums.

5. WHY THE CLASSICAL APPROACH FAILS

The Classical view embodies constraints far more restrictive than any requirements for traditional plans. Assumptions inherent in the Classical Model unavoidably intertwine elements of the cost structure with corresponding elements of reserves and cash surrender values.

5.1. *Cash-Value Basis = Reserve Basis = Policy Guarantees*

As mentioned above, certain conditions *must* be satisfied in order that the Classical Model be valid. These are as follows:

5.1.1. The basis of account-value guarantees (both interest and mortality) cannot differ from either the reserve basis or the cash-value basis.

5.1.2. The method by which the account value accumulates must be identical to the way cash values and reserves accumulate. The amount of premium in each renewal year that is actually credited to the account value must be a constant percentage of gross premiums for that year. Under these conditions we say that the formula relating successive account values is *actuarial*.

5.1.3. There can be no benefit guarantees over and above those associated with the account value. (This requirement will become clear soon.)

These requirements must be satisfied to ensure that the account value is an adequate representation of the value (on both the nonforfeiture and valuation bases) of future guaranteed benefits. For example, consider one account-value premium deposit, or net deposit to the account value.[4] This premium will buy additional benefits based on the account-value guarantees. If the account-value guarantees match the reserve basis, and the accumulation formula matches the reserve method, then the present value of these benefits will be equal to the premium itself.

A parallel statement is true for nonforfeiture purposes. Therefore, the proper reserve or cash value (ignoring E^1 or Commissioners allowance) for benefits generated by this deposit is the deposit itself. This is the basic tenet of the Classical UL Model. Stated in other terms, *the account value represents the value of guaranteed future benefits only when the account value purchases benefits on a method and basis identical with that of the reserve and cash value.*

As a result of the above requirements, the classical approach will impose several limits on plan design:

a) We conclude as a direct result of the discussion above that insurers must guarantee to the policy holder, explicitly, parameters that simultaneously define a cash value and a statutory reserve. This consideration becomes paramount as we move to the 1980 CSO Mortality Table. The 1980 CSO Mortality Table may be conservative over the life of the policy, but insurers may not feel comfortable guaranteeing its level of mortality period by period. In addition, the slope of valuation table mortality rates may not be appropriate for the cost basis of the policy. Implicit expense loadings or profit margins may change the desired slope of the guaranteed mortality rates.

b) Traditionally, insurers are granted two forms of surrender charge (viewed from the reserve) when determining cash values: E^1 and the interest differential between the cash-value and reserve bases. The Classical UL approach eliminates the use of the interest differential. Thus, the Classical approach results in cash values greater than those required on an otherwise similar traditional policy.

c) Insurers must accumulate funds and make mortality deductions in an actuarially "perfect" way. Again, this is so that funds will accumulate to form benefits the present value of which equals the funds themselves.

d) Insurers cannot make short-term guarantees more liberal than the reserve basis. Such guarantees (for instance, guaranteeing the current interest rate for one year) would generate benefits having a greater value (on either the reserve or the nonforfeiture basis) than the account value itself.

e) Any renewal loadings must be a constant percentage of gross premiums. This precludes the use of fixed policy fees on a flexible premium form, for example.

Insurers have already deviated from the above requirements with policies currently on the market. Deductions are commonly not actuarially perfect,

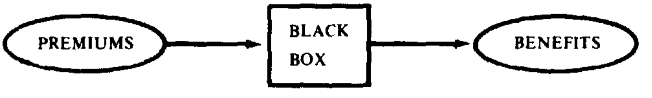
[4] If the net deposit to the account value is to represent a valuation premium, it must be a constant percentage of the corresponding gross premium.

because of monthly approximations or attempts to simplify the mechanics of the account value. Additionally, most policies carry liberal short-term guarantees without reflecting these short-term guarantees in the cash value.

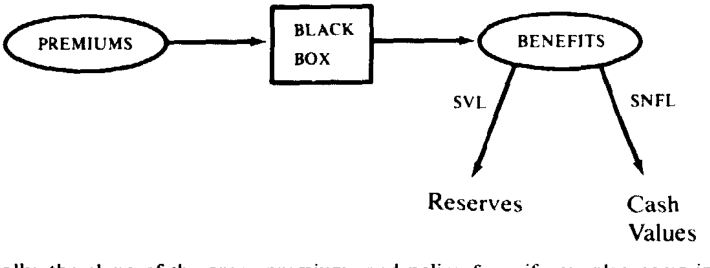
5.2. *Fallacy: Accounting Technique = Value of Benefits*

In the simple case where all of the requirements 5.1.1, 5.1.2, and 5.1.3 are met—reserve basis = cash-value basis = policy guarantees; the mechanics of the policy are actuarially perfect; there are no short-term guarantees more liberal than the reserve or nonforfeiture basis; and there are no benefit guarantees in addition to those benefits the account value will produce under policy guarantees—the account value *is* an accurate representation of the value of benefits for reserves and cash values.

When any deviation from these conditions occurs, however, the account value has merit only as an accounting technique for defining future benefits and may have no simple relation to the reserve or cash value. As insurers deviate from any of the conditions 5.1.1, 5.1.2, and 5.1.3, the Classical approach can no longer be considered the proper method for determining minimum cash values and reserves. The diagram below helps to illustrate the problem.



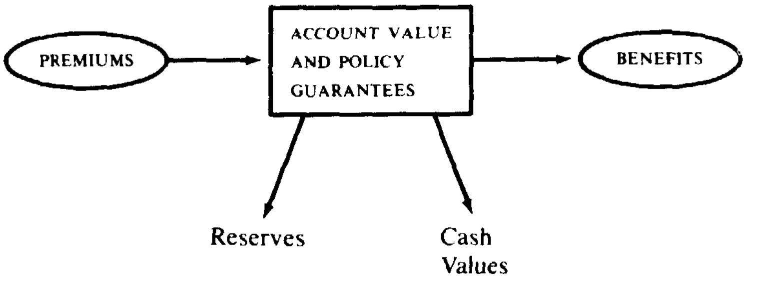
The "black box" represents the procedure that determines what amount and type of benefits a premium dollar buys. For many traditional policies, the transactions that take place within the black box are never disclosed. This poses no regulatory problem, however, since minimum reserves and cash values are defined only in terms of guaranteed policy benefits.[5] The following diagram illustrates the relationship of reserves and cash values to the previous diagram.



[5] Actually, the slope of the gross premiums and policy fees, if any, also come into play in the determination of minimum values.

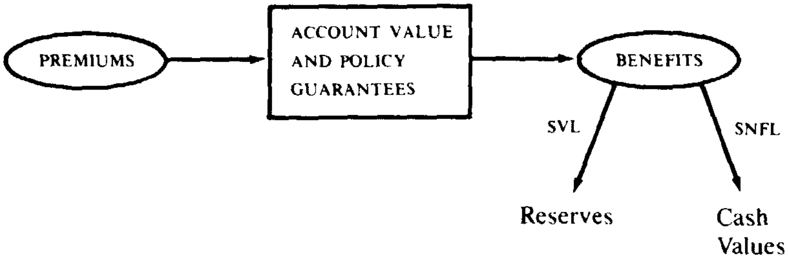
ICVs, however, substitute an explicit formula (or method) for determining the guaranteed policy benefits. This formula usually takes the form of an account value, mortality, interest, and expense guarantees, and a method of computing successive account values.

Consider the following picture illustrating the Classical UL Model.



The Classical UL Model looks at what takes place inside the "black box" and bases reserves and cash values on this information. This approach is mathematically coherent only as long as the three previously stated conditions are satisfied. If one or more of these conditions is not met, there is danger in looking into the black box to determine cash values and reserves.

As an example of what can go wrong, consider an ICV with an interest guarantee of 5 percent and a mortality guarantee of 90 percent of 1958 CSO mortality. For simplicity, we will ignore future premiums. If the reserve basis is 4½ percent and 1958 CSO (the legal minimum), then the value of future guaranteed benefits (for valuation purposes) is clearly not equal to the account value itself. The difference arises because guaranteed benefits are determined by accumulating the account value at 5 percent and levying mortality deductions at a rate less than 1958 CSO. The present value of these benefits, however, is determined using 4½ percent interest and 1958 CSO mortality, resulting in a larger number than the account value itself. In addition, deficiency reserves may be required (deficiency reserves will be discussed in Sec. 7). To consider the account value a proper representation of the value of future guaranteed benefits for reserve purposes is incorrect. In this case minimum reserves and cash values must be determined as the law requires, that is, in terms of benefits. The diagram at the top of the following page illustrates the only proper approach, given existing statutes. We will go into further detail as to how this affects actual calculations after introducing the required concepts. An example of the inequities that can occur if there are deviations from this approach is illustrated in Appendix B.



6. THE CONCEPT OF THE BENEFIT GENERATING ACCOUNT

We have mentioned several times that the function of the account value is to determine future benefits. Again we stress the fact that the account value may bear no simple relation to the reserve or the cash value required by law. For this reason (and to avoid confusion) we will refer to the account value as the *benefit generating account* (BGA). This term aptly describes the account value's primary purpose. The set of rules that determine the *guaranteed* benefits that the BGA will produce we will refer to as the *benefit generating function* (BGF). The BGF includes such items as the method of determining the term charge and amount at risk, the portion of the gross premium credited to the BGA, the method by which BGAs accumulate, and those policy guarantees defining future minimum BGAs. The portion of the gross premium actually credited to the BGA we will term the *BGF premium.*

If we restate the three necessary conditions for the Classical UL Model in terms of the BGA and the BGF, they become the following:

5.1.1. The reserve and cash-value bases must match the corresponding portions of the BGF.
5.1.2. The BGF must be actuarial.
5.1.3. All guaranteed benefits must be determined by the application of the BGF to the BGA.

One can easily find examples where the BGA does not equal the reserve. In fact, nearly every ICV being sold at this time falls into this category. The most prevalent reason is the presence of short-term guarantees. Many companies guarantee their current interest rate one year in advance. The following example demonstrates why this practice makes the BGA and the reserve different.

Consider a certain in-force ICV with a current death benefit of $1 and a BGA less than $1. For simplicity, assume there are no future premiums.

Further, assume that the BGF matches the reserve accumulation mechanics and basis in every respect except for a one-year current interest rate guarantee. After this one year is over, and provided that the current guarantee is not renewed, this product will satisfy all three conditions for the Classical UL Model. Therefore, the BGA will be equal to the reserve at that time. We can express the current reserve as the sum of the present value of the end-of-the-year BGA and death benefits paid. If i' is the current guaranteed interest rate, and i is the reserve rate, we find that

$$_tV = \frac{({}_{t+1}BGA)p + q}{1 + i} = \frac{1}{1 + i}\left\{\left[\frac{({}_tBGA)(1 + i') - q}{p}\right]p + q\right\},$$

or

$$_tV = {}_tBGA\,\frac{1 + i'}{1 + i}.$$

Many insurers recognize the above result and do hold this higher reserve.[6]

One can formulate many other realistic examples where the BGA does not equal the reserve or the minimum cash value. A few of these follow:

a) Differences between the mortality table used for reserves and the mortality rates guaranteed in the BGF will be likely as use of the 1980 CSO Mortality Table increases. An insurer may wish to reserve on the 1980 basis (with select mortality factors), yet for marketing reasons guarantee attained-age rates in the BGF.

b) The mortality rates guaranteed through the BGF may vary by risk classification. For instance, nonsmoker guaranteed rates may be lower than the corresponding rates for smokers. Both cash values, however, may be calculated using the same mortality table (perhaps 1958 CSO).

c) Guaranteed mortality rates for substandard risks or for guaranteed issue products may be greater than 1958 CSO, yet the cash value may still be calculated using the 1958 CSO Table.

d) As mentioned previously, any "current" rates (interest or mortality) guaranteed even one year in advance will impact both the minimum cash value and the reserve.

e) The insurer may wish to calculate case values using an interest rate different from that used in calculating reserves. For instance, the reserve might be based on 4½ percent interest, while the cash value is based on 5½ percent. This is not possible under the Classical UL Model.

[6] A similar result holds for cash values. However, the authors are unaware of any company that capitalizes BGF guarantees into its surrender values.

In addition to the above, there are theoretical reasons why the components of the BGF may be inappropriate as a reserve standard. The BGF is an accounting technique that generates future guaranteed benefits when applied to the BGA. All components of the BGF (formula, mortality, interest, and loads) together form the cost basis of the policy. In the general case where the BGF formula is not actuarial, comparisons between other components of the BGF and the reserve basis clearly can be misleading. Even in the simple case where the BGF formula is actuarial, we have at least three other components of the BGF that must be analyzed together: interest, mortality, and loadings. It would be incorrect to single out the guaranteed interest rate and assume this to be the overall rate of return guaranteed to the policyholder. As an example, an insurer may wish to guarantee a low interest rate yet offer aggressive mortality guarantees and very low loading charges. The reserve, however, might be calculated on a more traditional basis. Here, neither the interest nor the mortality guarantee would match that of the reserve basis. Only by analyzing the components together can it be determined whether the BGF is more liberal or conservative that the reserve basis. Once again, it would be inappropriate to consider the BGA an adequate representation of the reserve or the cash value.

It is unfortunate that the most common BGFs are so similar to the way in which reserves (and cash values) accumulate. This encourages searching for the reserve within the black box. Yet only in the special case where the three conditions for use of the Classical UL Model are satisfied can cash values and reserves be calculated without projecting future benefits.

7. FIXED PREMIUM INDETERMINATE CASH VALUE POLICIES

We have shown that, in order to be consistent with current law, future guaranteed benefits *must* be taken into account when determining minimum reserves or cash values. We will now begin our general model describing various relationships between reserves, cash values, and account values. Many ICVs require that a certain premium be paid. These fixed premium plans are a good starting point, since they resemble traditional products more closely than the flexible forms and fit directly into the framework of current law.

For purposes of this discussion, we construct a hypothetical product that has the following characteristics:

a) Premiums are fixed at issue and payable until age 95.

b) The death benefit is the greater of the account value (BGA) and \$1,000 (for simplicity, we assume no "corridor" exists).

c) The endowment amount (at age 95) is also the greater of the BGA and \$1,000.

The BGF is as follows:

Policy guarantees are $3\frac{1}{2}$ percent interest and 1958 CSO mortality. Successive annual BGAs accumulate by the formula below:

$$ {}_{t+1}BGA = ({}_{t}BGA + P)(1 + i) - {}_{t}TC \,, $$

where P = premium paid less loads, and ${}_{t}TC$ = term charge for year t.

$$ {}_{t}TC = (1{,}000 - {}_{t+1}BGA)q \,. $$

Loads (expense charges) = 50 percent of the first-year premium and are levied when the first-year premium is paid. There are no renewal loads.

Notice that there is a "secondary" benefit guarantee. We have guaranteed the benefits to be those of endowment at age 95 (E @ 95) for \$1,000 throughout, regardless of the performance of the BGA. In fact, E @ 95 benefits are guaranteed even if the BGA turns negative. We will refer to the benefits supported by the BGA as the primary guaranteed benefits, and the overriding E @ 95 guarantee as the secondary guaranteed benefits.

The formulas for equating successive BGAs and for determining the term charge follow the way in which curtate terminal reserves accumulate. In addition, renewal loads are a constant percentage (0 percent) of gross premiums paid. Therefore, the BGF is actuarial as described in requirement 5.1.2. For simplicity, we assume annual premium payments, with term charges and death claims payable at the end of the year.

We will calculate 1958 CSO 4 percent net level reserves and pay minimum 1958 CSO 5 percent cash surrender values. Both bases differ from the guaranteed BGF basis.

7.1. *Cash Values*

Using the hypothetical product described above, we will analyze three separate cases. Case 1 will assume that none of the three conditions 5.1.1, 5.1.2, and 5.1.3 for use of the Classical UL Model are satisfied. Restrictions will be added to this hypothetical policy one by one to create a total of three situations:

Case 1: None of conditions 5.1.1, 5.1.2, and 5.1.3 is satisfied.

Case 2: Conditions 5.1.1 and 5.1.2 are satisfied.

Case 3: All of conditions 5.1.1, 5.1.2, and 5.1.3 are satisfied.

For simplicity (so that we will not have to change it later on), we have constructed our example in such a way that the BGF is actuarial. Therefore condition 5.1.2 is satisfied. Nonetheless, we will approach the product under case 1, assuming that none of the conditions are satisfied.

We will determine cash values under cases 1, 2, and 3. With each added restriction, we can make simplifications in the cash-value calculation. We will show that the Classical UL Model, or Case 3, is only a special case of the generalized approach we are about to present.

CASE 1

Since we have not met the conditions for use of the Classical Model, we know that the BGA will not equal the minimum cash value. Minimum cash values must be calculated in terms of guaranteed future benefits.

First, we must determine what benefits are actually guaranteed at issue. We know that the death benefit is the greater of the BGA and $1,000. To determine the benefits guaranteed at issue, then, we must project the BGA to age 95 using policy guarantees, and compare this "guaranteed BGA" to the $1,000 guaranteed death and endowment benefits. Figure 7.1 represents the path of the BGA if interest and mortality are kept at guaranteed levels.[7]

Because the BGA guaranteed at issue never exceeds $1,000 (in fact, it turns negative at duration 44), the benefits guaranteed at issue are exactly those of E @ 95 (as a result of the secondary guarantee), that is, $1,000 on death or maturity.

Now that we have determined the benefits guaranteed at issue, we can calculate adjusted premiums, which, according to law, are determined on the basis of the benefits guaranteed at issue. In this case, the adjusted premiums (P^A) are those for E @ 95. The actual minimum cash value at any time (as discussed earlier) is the present value of the *then* guaranteed future benefits less the present value of the remaining adjusted premiums.

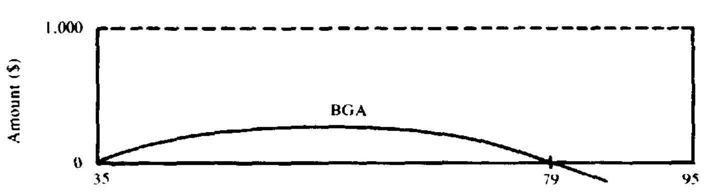


FIG. 7.1.—Comparison of benefit generating accounts (BGAs) by attained age. (Values on vertical scale are death benefit and endowment amounts.)

[7] Age 35, 12.88 annual premium.

The present value of the *then* guaranteed future benefits (all benefits guaranteed at the time the cash value is available) is composed of two parts: the present value of those benefits guaranteed at issue ($PVFB$) and the present value of any change in future guaranteed benefits since issue ($\Delta PVFB$). Using the symbols just introduced, and letting $PVADJP$ represent the present value of remaining adjusted premiums, the formula for the minimum cash value (CV) can be expressed as follows:

$$\text{Minimum } CV = PVFB + \Delta PVFB - PVADJP . \tag{7.1}$$

The adjusted premiums cannot be changed from those calculated at issue (see point 2.4). Notice that any changes in guaranteed benefits from those guaranteed at issue increase the cash value on a single premium basis. When $\Delta PVFB$ increases after issue, there is no corresponding change in $PVADJP$. In fact, for our hypothetical product we can rewrite formula (7.1) as

$$\text{Minimum } CV = (E @ 95)CV + \Delta PVFB , \tag{7.2}$$

since both $PVFB$ and $PVADJP$ are those of E @ 95.

The cash value changes from that of E @ 95 only when the guaranteed benefits change from E @ 95, that is, when $\Delta PVFB \neq 0$. When does this happen? Remember that the guaranteed BGA at issue turns negative at duration 44. However, there are additional benefit guarantees over those benefits generated by the BGA, since benefits are always guaranteed to be at least those of E @ 95. Hence, it is possible for the BGA to become somewhat larger than the level guaranteed at issue without increasing the original guaranteed benefits. This happens because there is a gap between those benefits generated by the BGA (viewed prospectively from issue) and the secondary benefit guarantee of E @ 95.

Consider a specific duration t. We can determine the amount of the BGA such that when it is projected forward using guaranteed rates, its benefits will exactly equal the secondary E @ 95 guarantee. We call this amount the "shadow fund" (SF). The shadow fund, together with future premiums, produces E @ 95 using the BGF. In the absence of any changes to the BGF after issue (short-term guarantees renewed after issue, for example) the shadow fund can be precalculated for all durations. We shall proceed on this basis. Let $PVFB^{BGF}$ and PVG^{BGF} be the present value of those benefits guaranteed at issue (E @ 95) and the present value of BGF premiums (net to the BGA), respectively, with present values based on the BGF.

We know that the shadow fund plus the present value of BGF premiums (on the BGF basis) must equal the present value of those benefits guaranteed at issue (evaluated on the BGF basis but defined by the more liberal secondary guarantee). Therefore,

$$SF + PVG^{BGF} = PVFB^{BGF} ,$$

or

$$SF = PVFB^{BGF} - PVG^{BGF} .$$

If the BGA is currently less than the shadow fund, the BGA projected using the BGF will be less than \$1,000 at age 95. If the BGA is greater than the shadow fund, the BGA will be greater than \$1,000 at age 95, and the guaranteed benefits will have increased (i.e., $\Delta PVFB > 0$). We can now give the shadow fund a formal definition.

> *Shadow fund:* The amount that the BGA must exceed at any time to generate guaranteed future benefits in excess of any secondary benefit guarantees.

For our hypothetical policy, values of the shadow fund and the minimum BGA guaranteed at issue are shown in Table 7.1 and illustrated in Figure 7.2. Looking forward from issue, the shadow fund represents the implicitly guaranteed BGA. The shadow fund differs from the BGA only when there exists a secondary benefit guarantee. In the case of our hypothetical product, we have guaranteed the benefits to be those of E @ 95, regardless of the performance of the BGA.

There are excellent economic reasons why an insurer may wish to offer an overriding guarantee such as this. The insurer may feel confident that he could guarantee an average rate of interest over the life of the contract at, for example, 5 percent. However, the insurer may not feel confident guaranteeing the 5 percent rate period by period. A period-by-period guarantee of only 3 percent may be more suitable, but for marketing reasons the insurer may not want to charge the redundant premium that will *ensure* that the BGA will support benefits with a 3 percent guarantee. The solution, then, is to guarantee year by year a lower rate such as 3 percent. (This 3 percent rate would be stated in the policy form as part of the BGF.) A higher guaranteed *average* rate over the life of the contract can be approximated with a secondary guarantee of benefits that the BGA will not support.

Products embodying the above concepts are not entirely new. Two examples of products with secondary guarantees not supported by the

TABLE 7.1

COMPARISON OF MINIMUM BENEFIT GENERATING ACCOUNT AND SHADOW FUND, BY DURATION

Duration	Min. BGA	Shadow Fund	Duration	Min. BGA	Shadow Fund
1.	$ 4.17	$ 56.76	31	$ 327.10	$ 536.72
2.	15.04	69.62	32	328.55	553.32
3.	26.17	82.82	33	327.81	569.65
4.	37.52	96.33	34	324.46	585.65
5.	49.08	110.14	35	318.04	601.29
6.	60.81	124.23	36	308.05	616.58
7.	72.71	138.61	37	293.93	631.53
8.	84.77	153.26	38	275.03	646.22
9.	96.97	168.18	39	250.58	660.80
10.	109.32	183.38	40	219.51	675.02
11.	121.77	198.85	41	180.39	689.17
12.	134.32	214.56	42	131.25	703.12
13.	146.93	230.50	43	69.42	716.79
14.	159.56	246.67	44	− 8.69	730.09
15.	172.18	263.03	45	− 107.75	742.96
16.	184.77	279.57	46	− 233.90	755.39
17.	197.24	296.28	47	− 395.28	767.40
18.	209.60	313.14	48	− 602.82	779.05
19.	221.80	330.13	49	− 871.43	790.44
20.	233.77	347.25	50	− 1,221.90	801.69
21.	245.48	364.47	51	− 1,683.81	812.93
22.	256.84	381.78	52	− 2,299.66	824.36
23.	267.78	398.13	53	− 3,131.72	836.23
24.	278.21	416.51	54	− 4,273.46	848.86
25.	288.04	433.90	55	− 5,869.11	862.74
26.	297.16	451.25	56	− 8,148.29	878.57
27.	305.45	468.56	57	− 11,489.76	897.44
28.	312.76	485.79	58	− 16,544.07	921.24
29.	318.94	502.91	59	− 24,481.61	953.30
30.	323.80	519.91	60	− 37,524.21	1,000.00

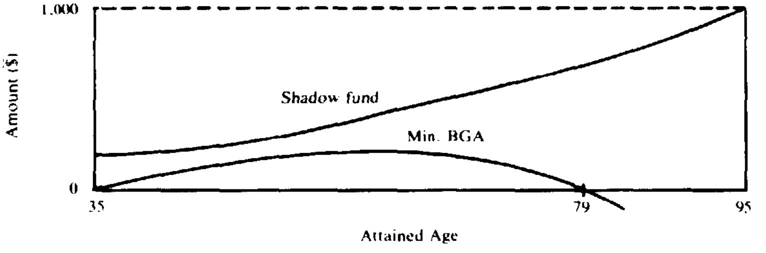


FIG. 7.2.—Comparison of minimum benefit generating account and shadow fund, by attained age.

formula for determining benefits (the BGF) are variable life and expanded dividend policies. The death benefit under a variable life contract is dependent on the value of the underlying assets. In no event, however, will the death benefit fall below the initial face amount. Conditions could arise where the benefit amount dependent on assets could be less than the overriding guarantee of the initial face amount. Expanded dividend policies (sometimes referred to as "economatic") carry with them an explicit BGF for determining the amount of benefits the dividends purchase. Many of these plans guarantee that the death benefit will not fall below a certain level for a stated number of years. This guarantee cannot be supported by formula (the BGF) on a guaranteed basis because dividends, of course, are not guaranteed.

Returning to our analysis, we know how to establish when the guaranteed future benefits will exceed those guaranteed at issue (i.e., when $\Delta PVFB > 0$). This occurs whenever the BGA exceeds the shadow fund. Remembering formula (7.1), we can express our minimum cash value as follows:

$$\text{If } BGA - SF \leq 0\,, \quad CV = PVFB - PVADJP\,.$$

$$\text{If } BGA - SF > 0\,, \quad CV = PVFB + \Delta PVFB - PVADJP\,.$$

Let us examine $\Delta PVFB$. In the general case, the actual extra future benefits can only be found by projecting the BGA into the future with policy guarantees (BGF) and observing the benefit pattern. The "extra" benefits may take the form of additional death benefits, a greater endowment amount, or both. Of course, this projection is necessary only if the BGA is greater than the shadow fund.

In practice the only way to evaluate these benefits is to project the current BGA using the BGF and then to discount the resulting death and endowment benefits on the cash-value basis.

Returning to our example, let us assume that interest is credited at a current rate of 10 percent, and mortality is charged at a current rate of 60 percent of 1958 CSO. We will determine the fifteenth-duration minimum cash value. The fifteenth-duration BGA is \$359.63, larger than the shadow fund. From this we know that $\Delta PVFB$ must be greater than zero. To determine $\Delta PVFB$, the fifteenth-duration BGA must be projected forward using the BGF. The benefits generated are shown in Table 7.2 and illustrated in Figure 7.3.

The extra guaranteed future benefits over those guaranteed at issue (E @ 95) are those death benefits greater than \$1,000 and the extra endow-

TABLE 7.2

COMPARISON OF DEATH BENEFIT AND BENEFIT GENERATING ACCOUNT BY DURATION

Duration	Death Benefit	BGA	Duration	Death Benefit	BGA
16	$1,000.00	$380.39	38	$1,038.51	$1,038.51
17	1,000.00	401.58	39	1,088.19	1,088.19
18	1,000.00	423.23	40	1,139.60	1,139.60
19	1,000.00	445.33	41	1,192.82	1,192.82
20	1,000.00	467.91	42	1,247.90	1,247.90
21	1,000.00	491.01	43	1,304.91	1,304.91
22	1,000.00	514.62	44	1,363.91	1,363.91
23	1,000.00	538.80	45	1,424.98	1,424.98
24	1,000.00	563.57	46	1,488.18	1,488.18
25	1,000.00	588.98	47	1,553.60	1,553.60
26	1,000.00	615.10	48	1,621.31	1,621.31
27	1,000.00	642.00	49	1,691.38	1,691.38
28	1,000.00	669.77	50	1,763.91	1,763.91
29	1,000.00	698.53	51	1,838.98	1,838.98
30	1,000.00	728.43	52	1,916.68	1,916.68
31	1,000.00	759.62	53	1,997.09	1,997.09
32	1,000.00	792.32	54	2,080.32	2,080.32
33	1,000.00	826.80	55	2,166.56	2,166.46
34	1,000.00	863.37	56	2,255.62	2,255.62
35	1,000.00	902.47	57	2,347.90	2,347.90
36	1,000.00	944.63	58	2,443.40	2,443.40
37	1,000.00	990.51	59	2,542.25	2,542.25
			60	2,644.56	2,644.56

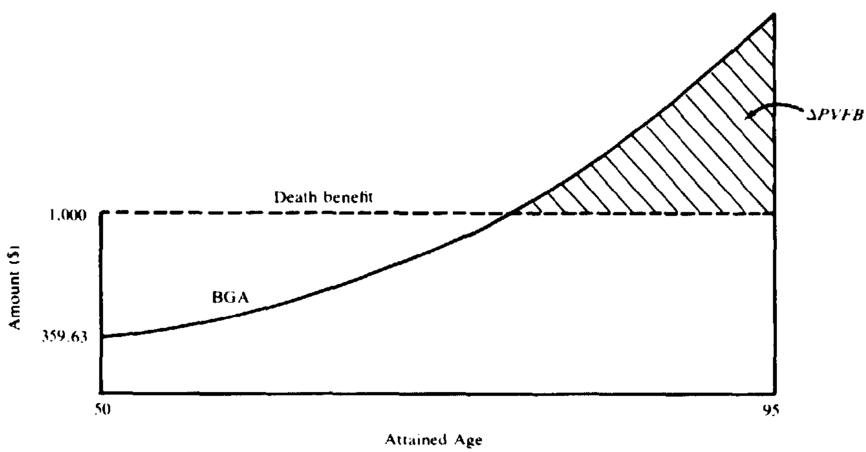


FIG. 7.3.—Comparison of death benefit and benefit generating account, by attained age, showing $\Delta PVFB$ for BGA greater than shadow fund.

ment amount (\$1,644.56). The present value of these benefits using 1958 CSO 5 percent is \$74.29. Using formula (7.1), $CV = PVFB + \Delta PVFB - PVADJP$, and realizing that $PVFB - PVADJP$ in this case is simply the minimum fifteenth-duration cash value for E @ 95, we now have our desired result:

$$_{15}CV = {}_{15}CV_{35:60} + \$74.29 .$$

In the general case, the required cash value must be found by projecting the current BGA forward using the BGF and actually observing the future guaranteed benefits. Obviously the effort involved in the projection of benefits is significant, but with further restrictions products may be arranged so that this projection is unnecessary. We will examine such a product when we look at case 2.

CASE 2

If we modify our example above so that the cash-value basis is 1958 CSO and 3½ percent, we satisfy conditions 5.1.1 and 5.1.2. Since the cash-value basis matches the BGF, we know that the BGA is an accurate representation of the value of benefits *produced by the BGA through the BGF*.[8]

A change of \$1 in the BGA (once the BGA exceeds the shadow fund)[9] results in new future guaranteed benefits, the present value of which on the nonforfeiture basis is exactly \$1. The \$1 is projected using the BGF and is discounted using the nonforfeiture basis. Since the mortality and interest elements of the BGF and the nonforfeiture basis are now identical, the projection and discounting are done on the same basis.

In order to define a unified approach to determining minimum cash values, we must translate *all* guaranteed benefits into BGA form. We previously demonstrated that any overriding benefit guarantees at issue can be translated into an implicitly guaranteed BGA, which we called the shadow fund. With the added restriction of conditions 5.1.1 and 5.1.2, by definition we can write

$$PVFB = PVFB^{BGF} \quad \text{and} \quad PVG = PVG^{BGF} .$$

[8] The last phrase is emphasized because the benefits produced by the BGA may not include all of the guaranteed benefits, since we still violate condition 5.1.3 with a secondary benefit guarantee.

[9] Notice that the above argument ignores any guarantees over and above those associated with the BGA. We know that until the BGA crosses the shadow fund, a \$1 increase in the BGA produces no new benefits whatsoever.

We can now express the shadow fund as follows:

$$SF = PVFB - PVG\ ,$$

where again PVG is the present value of BGF premiums.

We know that whenever the BGA is less than the shadow fund, the shadow fund is representative of guaranteed benefits in terms of the BGF. However, when the BGA exceeds the shadow fund, it is representative of the then guaranteed benefits in terms of the BGF.

It seems appropriate, then, to define a new entity, the "adjusted shadow fund" (ASF), to be the greater of the BGA and the shadow fund. At all times the adjusted shadow fund (with the BGF) defines future guaranteed benefits.

ASF differs from SF only when the BGA produces a change in guaranteed benefits (when the BGA exceeds SF). Since our nonforfeiture basis matches the BGF, the present value of the change in benefits equals the amount by which the BGA exceeds SF. From this it follows that, when conditions 5.1.1 and 5.1.2 are satisfied,

$$ASF = SF + \Delta PVFB\ .$$

Using our previous expression for SF,

$$ASF = PVFB + \Delta PVFB - PVG\ .$$

We now have the adjusted shadow fund expressed in a form similar to that of the minimum cash value.[10] We will combine this expression for ASF with formula (7.1). We now have the minimum cash value in terms of ASF:

$$\text{Minimum } CV = ASF - (PVADJP - PVG)\ .$$

The quantity in parentheses is commonly referred to as a surrender charge. Notice that the surrender charge can also be expressed in terms of SF and the E @ 95 cash value. Since

$$^{(E\ @\ 95)}CV = PVFB - PVADJP\ ,$$

[10] It is important to realize that the expression $PVFB$ in the formula for ASF is identical with $PVFB$ as it appears in the cash-value formula. This result holds true *only* when the cash-value basis matches the corresponding portions of the BGF. For our example, $PVFB$ in either case is the present value, using 1958 CSO and 3½ percent, of E @ 95 benefits, since these are the minimum benefits guaranteed at issue.

and

$$SF = PVFB - PVG \, ,$$

$$SF - {}^{E @ 95)}CV = PVADJP - PVG = \text{Surrender charge} \, .$$

Three important characteristics of this maximum surrender charge should now be apparent:

a) The legally maximum surrender charges are *fixed* at issue.
b) These maximum surrender charges are not directly related to first-year loads.
c) The surrender charge can be defined only in relation to the adjusted shadow fund, and *not* to the BGA.

CASE 3

As a final step in our cash-value analysis, we make one further simplifying assumption in our example, by removing the secondary benefit guarantee of E @ 95. We have now satisfied all three conditions for use of the Classical UL Model. We will apply our general model, however, and demonstrate equivalent results.

Our guaranteed benefits at issue now become term to age 79. Since these benefits are completely supported by the BGA, the shadow fund is equal to the BGA guaranteed at issue. Now, by definition, the adjusted shadow fund is equal to the actual BGA. Our minimum cash value formula becomes

$$\text{Minimum } CV = BGA - (PVADJP - PVG) \, .$$

The adjusted premiums are now those for term to age 79, those benefits guaranteed at issue. This adjusted premium can be expressed in terms of the net level premium.

$$P^{A} = P^{NL} + \frac{E^{1}}{\ddot{a}_{x}} \, ,$$

where E^{1} is the expense allowance for term to age 79 and all values are calculated on the cash-value basis (also the BGF basis). We can express the BGF premium similarly. Since

$$PVG - E^{c} = PVFB = P^{NL}\ddot{a}_{x} \, ,$$

we have

$$G = P^{NL} + \frac{E^c}{\ddot{a}_x},$$

where G is the BGF premium and E^c is the difference between the first-year BGF premium and renewal BGF premiums. Using the expression for the BGF premium and the adjusted premiums,

$$P^A - G = \frac{E^1 - E^c}{\ddot{a}_x}.$$

Now we have our surrender charge.

$$PVADJP - PVG = (P^A - G)\ddot{a}_{x+t} = \frac{E^1 - E^c}{\ddot{a}_x}\ddot{a}_{x+t}.$$

This can be recognized as the cash-value formula we introduced when discussing the Classical UL Model! This was the intended result, since we have satisfied the three conditions for applying the Classical UL Model:

a) The BGF matches the cash-value basis.
b) The BGF is actuarial.
c) There are no benefit guarantees over and above those benefits generated by the BGA through the BGF.

Thus, the Classical UL Model is a specific case of our more general cash-value approach.

7.2. *Reserves*

Everything we have discussed in terms of cash values applies to reserves, since minimum cash values are only a type of modified reserve. There is, however, one additional consideration that does not apply when discussing cash values. A test must be made for the possibility of a required deficiency reserve.

If we reexamine case 1 (3½ percent BGF, 4 percent reserve, 5 percent cash value) with reserves in mind, we notice that the net level valuation premium (we will choose to hold net level reserves) is \$13.91, while the gross premium is only \$12.88. The gross premium is deficient on our valuation basis, and deficiency reserves may be required. The Standard Valuation Law, however, requires deficiency reserves only if the premium is deficient on the most liberal valuation basis allowed by law. Since the net level valuation premium, at 1958 CSO and 4½ percent, of $12.878 is smaller than our gross premium, deficiency reserves are not required.

The above exercise illustrates the fact that deficiency reserve concepts apply to fixed premium ICVs just as they would to traditional plans.

7.3. *Summary of Formulas*

1. The minimum cash value required by law is the minimum cash value for benefits guaranteed at issue plus the present value (on the cash-value basis) of any additional benefits now guaranteed:

$$\text{Minimum } CV = PVFB + \Delta PVFB - PVADJP \, .$$

All present values in the above formula are calculated on the *cash-value basis*, which may differ from the BGF.

A similar formula can be written for reserves:

$$\text{Minimum reserve} = PVFB + \Delta PVFB - PV \text{ of valuation premiums} \, .$$

Here all present values are on the reserve basis.

2. Several simplifications can be made if (*a*) the BGF is actuarial and (*b*) the BGF matches the reserve and cash-value basis. Now,

$$\text{Minimum } CV = ASF - \text{Surrender charge} \, ,$$

where

$$\text{Surrender charge} = PVADJP - PVG \, ,$$

$$ASF = \text{Greater of } (SF, BGA) \, ,$$

$$SF = PVFB - PVG \, .$$

Similarly, for reserves,

$$\text{Minimum reserve} = ASF - \text{Surrender charge} \, ,$$

where

$$\text{Surrender charge} = PV \text{ valuation premiums} - PVG \, ,$$

and ASF and SF are as defined above.

3. We can use the Classical UL Model if (*a*) the BGF is actuarial, (*b*) the BGF matches the reserve and cash-value basis, and (*c*) there are no

other benefit guarantees over and above those benefits generated by the BGA. Now we find that

$$SF = \text{Minimum } BGA \text{ guaranteed at issue },$$

$$ASF = BGA ,$$

$$\text{Minimum } CV = BGA - \text{Surrender charge },$$

where

$$\text{Surrender charge} = PVADJP - PVG ,$$

$$= \frac{E^{t} - E^{c}}{\ddot{a}_x} \ddot{a}_{x+t} .$$

For reserves,

$$\text{Minimum reserve} = BGA - \text{Surrender charge },$$

where

$$\text{Surrender charge} = PV \text{ of valuation premiums } - PVG$$

$$= \frac{\text{Commissioners allowance } - E^{c}}{\ddot{a}_x} \ddot{a}_{x+t} .$$

4. Deficiency reserve considerations apply in the same way they apply for conventional policies.

8. FLEXIBLE PREMIUM INDETERMINATE CASH VALUE PLANS

In previous portions of this paper, we have demonstrated that fixed premium ICVs are explicitly handled by the SVL and the SNFL. These laws, however, do not contain specific provisions applying to plans of insurance where the pattern and amount of gross premiums cannot be determined at issue. We analyze several methods of applying the SVL and the SNFL to these plans, and discuss the ramifications of each method. Throughout this section we will work with cash values, realizing that parallel results hold for reserves.

8.1. *Limitations of the SNFL*

In Section 2 we highlighted seven characteristics of the 1976 model laws that were useful in our analysis. Consider points 2.3 and 2.4 of that summary. We state them again for reference.

2.3. For purposes of defining minimum values, adjusted premiums (modified premiums for reserves) are defined to be a constant percentage of gross premiums.
2.4. Adjusted premiums (modified premiums for reserves) are calculated on the basis of benefits guaranteed at issue.

Upon examination of these two requirements, we find that they are mutually exclusive for flexible premium plans. In order that adjusted premiums be calculated on the basis of benefits guaranteed at issue, an assumption must be made as to the future gross premium structure. When the policyholder deviates from the assumed premium structure, the adjusted premium is no longer a constant percentage of the gross premium actually paid. Thus, we have violated condition 2.3. Conversely, if the adjusted premiums are held to be a constant percentage of gross premiums, then the adjusted premiums cannot be calculated on the basis of benefits guaranteed at issue, thus violating 2.4.

Since the above results occur for flexible premium plans, regulators will have to rely on an interpretation of current law. We will discuss several logical interpretations.

It will be helpful in our subsequent analysis to expand the minimum cash value formula to four terms. Remembering that the adjusted premium can be expressed in terms of a net level premium, we can write

$$P^{A} = P^{NL} + \frac{E^{1}}{\ddot{a}_{x}}.$$

Utilizing this expression in formula (7.1), we find that

$$\text{Minimum } CV = PVFB + \Delta PVFB - PVNLP - PV\frac{E^{1}}{\ddot{a}_{x}}, \quad (8.1)$$

where $PVNLP$ is the present value of the remaining net level premiums, and $PV(E^{1}/\ddot{a}_{x})$ is the unamortized expense allowance.

Utilizing formula (8.1) for our analysis, we can discuss the determination of E^{1} separately from that of the other three terms. Sections 8.2 and 8.3 below will discuss cash values ignoring the term $PV(E^{1}/\ddot{a}_{x})$. This is done so that the effects of different methods of determining the other terms ($PVFB$, $\Delta PVFB$, and $PVNLP$) will not be clouded by variations in E^{1}. The determination of E^{1} will be discussed separately in Section 8.4.

8.2. *Considerations in Selecting a Cash-Value Method*

Two of the cash-value methods we will consider are dependent on a future gross premium assumption. A desirable characteristic of such methods is that they produce cash values as close as possible to the "correct" cash value. We will begin by defining this "correct" cash value.

In Section 8.1 we noted that the cash value prescribed by law cannot be determined unless one can see into the future and recognize in advance the gross premiums the policyholder will pay. The "correct" cash value is the one that would be calculated with direct application of the SNFL, assuming that one *could* accurately predict future gross premiums.

In fact, the correct cash values can be calculated only *after* the policy maturity date (assuming that the policyholder both survived and persisted). At that time the gross premium structure is known, and the adjusted premiums can be calculated. These adjusted premiums are calculated according to the guaranteed plan of insurance at issue. Note that this will differ from the resultant plan of insurance if there have been *any* deviations from original policy guarantees. To analyze this difference in plan, we must consider, once again, formula (7.1):

$$\text{Minimum } CV = PVFB + \Delta PVFB - PVADJP \,. \tag{7.1}$$

The resultant plan of insurance is composed of both *PVFB* and Δ*PVFB*. As an example, consider a certain ICV that has already matured on the policyholder's ninety-fifth birthday for $1,000, and provided $1,000 of death protection each year until maturity. Further, assume that policy guarantees were 4 percent interest and 1958 CSO mortality, but in fact interest was credited at 10 percent and mortality charges levied at only 60 percent of 1958 CSO.

To determine the guaranteed plan of insurance at issue, we would use the gross premiums actually paid and determine the guaranteed benefits by projecting ahead from issue using policy guarantees (4 percent, 1958 CSO). This plan of insurance might be thirty-year term. The resultant plan of insurance, however, was endowment at age 95.

Any benefits greater than those of thirty-year term enter the cash value on a single premium basis *as they become guaranteed.* We have previously seen the flaw in calculating cash values on the basis of the resultant plan of insurance (Sec. 4).

The "correct" cash value, then, can be calculated as follows:

a) Determine the guaranteed plan of insurance at issue, using policy guarantees together with gross premiums actually paid.

b) Calculate adjusted premiums utilizing the slope of gross premiums and the guaranteed benefits calculated in step *a*.

c) The cash value on any anniversary is the present value of the *then* guaranteed benefits less the present value of the future adjusted premiums that were calculated in step *b*.

Again, this method can be applied only after the policy has matured and the actual gross premium structure is known. At the time cash values must be determined, however, the future gross premium structure is not available information.

What are the implications of this for our cash-value methods? *Since we cannot accurately predict the future gross premium structure, we desire a method that is least sensitive to incorrect prediction*—in other words, a method that is reasonably insensitive to future premium assumptions.

Note that the Classical UL Model does, in fact, always produce the "correct" cash value. Under the Classical Model there is an identity between the BGF premium and a modified net premium.[11] Thus, the Classical UL Model is completely insensitive to future premium assumptions.

We stress again that the restrictions imposed by the Classical Model are severe. It is for this reason that we need a more general cash-value approach. Since most ICVs are designed under the Classical Model (with some variation), we require that any cash-value technique produce results identical with those produced by the Classical Model in those cases where all the prerequisites for the Classical Model apply.

8.3. *Determination of PVFB, ΔPVFB, and PVNLP*

We will discuss three general methods of calculating cash values. These can be characterized as follows:

Method 1: Comply with requirement 2.4, but not 2.3. Net premiums[12] are calculated at issue and may not be a constant percentage of the actual gross premiums throughout.

Method 2: Comply with requirement 2.3, but not 2.4. Net premiums are always a constant percentage of gross premiums and so must be recalculated whenever the gross premium deviates from that assumed.

[11] The BGF premium may not be identical with either the Commissioners Beta or the adjusted premium, since E^c may not be equal to the Commissioners allowance or E^1. However, adjustments to compensate for this are easily determined (see Appendix A).

[12] Net premiums, as discussed here, are premiums calculated on the cash-value basis with no consideration for E^1. In this way they represent net level premiums calculated using the *cash-value* assumptions of interest and mortality.

Method 3: Comply with a lenient interpretation of both 2.3 and 2.4. A net premium is calculated corresponding to each gross premium that is required to be paid under the contract. For many ICVs this would imply an assumption of no future premiums.

We will describe the three methods in detail. Although other methods are possible, the three methods chosen stand out as worthy of analysis because they are reasonable interpretations of current law. Also, each of the methods yields results identical with those of the Classical Model when the conditions for use of the model are satisfied; in other words, each of these methods encompasses the Classical Model as a special case.

Methods 1 and 2 will be considered in relation to the criteria described in Section 8.2. Since method 3 assumes no future premium flow, such an analysis is inappropriate.

METHOD 1: NET PREMIUMS CALCULATED AT ISSUE

In order to calculate net premiums, a certain future gross premium structure must be assumed. Once this is done, the guaranteed future benefits can be found, based on projections of the BGA (using policy guarantees) using the assumed gross premiums. The guaranteed future benefits, together with the pattern of gross premiums, determine the net premiums. To determine the *then* guaranteed future benefits at any point in the future, the actual BGA at that time is projected forward using policy guarantees and the remaining originally assumed gross premiums. The minimum cash value will be the present value of the resulting benefits less the present value of remaining net premiums, with both present values being calculated on the cash-value basis. The method is outlined below in algorithmic form.

At issue:

a) Assume a gross premium pattern.

b) Determine guaranteed future benefits using the BGF.

c) Calculate net premiums (on the cash-value basis) based on *a* and *b*.

To determine the minimum cash value at the current duration:

d) Determine current guaranteed future benefits by projecting the actual BGA forward using the originally assumed gross premiums and the BGF.

e) The cash value will be the present value of the benefits in *d* less the present value of remaining net premiums in *c* (both present values on the cash-value basis).

Note that if the gross premium pattern assumed exactly matches those gross premiums actually paid, we have calculated the correct cash value.

It is highly unlikely, however, that such prescience is widely available. In most cases, the flexible premium ICV policyholder will not keep to the assumed gross premium structure.

It is appropriate to analyze the sensitivity of the calculated cash value to a "wrong" gross premium assumption. Consider the following ICV:

Issue age: 35.
Face amount: $1,000.
Maturity date: Age 95.
BGF:

1. BGF is actuarial.
2. Policy guarantees: 1958 CSO mortality; interest rates shown in Table 8.1 and Figure 8.1.
3. No mortality corridor.

For simplicity we will assume level annual gross premiums. Table 8.1 shows the fifteenth- and fortieth-duration cash values at various assumed premium levels if the *actual gross premium paid* is $12.00 annually, and Figure 8.1 illustrates the comparison for the fifteenth duration. Three different relationships between the BGF interest guarantee and the cash-value interest rate (5½ percent) are illustrated. Note that it is not appropriate to make cash-value comparisons between the different BGF bases, since different plans of insurance are represented by different BGF guarantees.

At first glance, the results may seem surprising. When the cash-value basis is more liberal than the BGF, a larger assumed premium will yield a larger cash value. Conversely, when the cash-value basis is more conservative than the BGF, a larger assumed premium will yield a smaller cash value.[13] (See Appendix C.) Of course, when the cash-value basis is identical with the BGF, we have satisfied the constraints of the Classical Model and there is no sensitivity to the assumed premium.

We can summarize certain characteristics of cash-value method 1.

a) It is consistent with the Classical UL Model.
b) It allows flexibility between the cost structure (BGA, BGF) and the nonforfeiture structure.
c) Because net premiums are not always a constant percentage of gross premiums, method 1 diverges from current law.
d) There is a sensitivity to the premium assumption.

[13] An analysis of the forces involved in this phenomenon is deferred to Appendix C. The relationships of this paragraph will not always be true.

TABLE 8.1

COMPARISON OF FIFTEENTH- AND FORTIETH-DURATION CASH VALUES BY ASSUMED PREMIUM (METHOD 1)

Actual Gross Premium = $12.00
Actual Interest Credited = 10%
Actual Mortality Charged = 60% of 1958 CSO

Assumed Premium	BGF Interest = 4%		BGF Interest = 5½%		BGF Interest = 7%	
	$_{15}CV$	$_{40}CV$	$_{15}CV$	$_{40}CV$	$_{15}CV$	$_{40}CV$
$10.00........	$260.57	$4,460.85	$355.95	$5,006.65	$492.71	$5,628.86
10.50........	260.74	4,460.85	355.95	5,006.65	490.20	5,626.92
11.00........	261.09	4,461.34	355.95	5,006.65	487.88	5,625.07
11.50........	261.59	4,462.57	355.95	5,006.65	485.67	5,623.30
12.00........	262.26	4,464.03	355.95	5,006.65	483.56	5,621.57
12.50........	263.06	4,465.61	355.95	5,006.65	481.53	5,619.88
13.00........	264.04	4,467.56	355.95	5,006.65	479.57	5,618.22
13.50........	265.20	4,469.44	355.95	5,006.65	477.66	5,616.59
14.00........	266.12	4,470.84	355.95	5,006.65	475.79	5,614.98
14.50........	267.14	4,471.86	355.95	5,006.65	473.96	5,613.39
15.00........	268.10	4,472.84	355.95	5,006.65	472.15	5,611.82

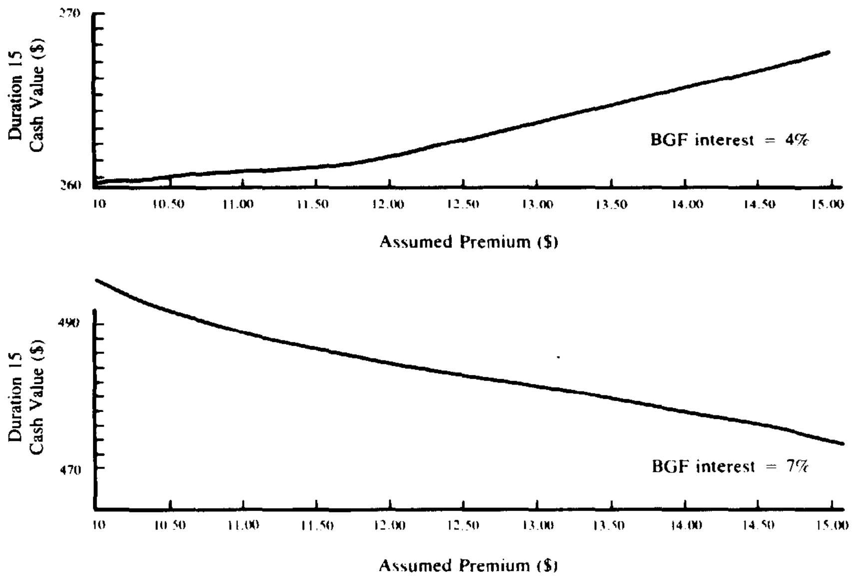


FIG. 8.1—Comparison of fifteenth-duration cash values by assumed premium (method 1).

We will discuss the basis for choosing an assumed premium after method 2 has been introduced, since the subject is relevant to both method 1 and method 2.

METHOD 2: NET PREMIUMS ALWAYS A CONSTANT PERCENTAGE OF GROSS PREMIUMS

For net premiums to be a constant percentage of gross premiums, all net premiums must be recalculated whenever a gross premium varies from the assumed premium. At any point in time we will calculate net premiums based on gross premiums paid to date together with an assumed future premium pattern. The benefits guaranteed at issue can be found by projecting the BGA from issue using this new premium structure and the BGF. The algorithm is as follows:

At issue:

a) Assume a gross premium pattern.
b) Determine guaranteed future benefits using the BGF.
c) Calculate net premiums (on the cash-value basis) based on *a* and *b*.
(At issue method 2 is identical with method 1.)

Annually:

d) Consider gross premiums actually paid to date together with future assumed premiums (from step *a* above) to be the gross premium structure.
e) Go back to the issue date and calculate guaranteed benefits using the BGF together with the gross premium structure of step *d*.
f) Calculate net premiums from issue based on the premium structure of step *d* and the guaranteed future benefits of step *e*.
g) The cash value is the present value of the currently guaranteed future benefits (found by projecting the current BGA through the BGF using future assumed premiums) less the present value of the future net premiums from step *f*.

At any point in time we have a set of net premiums that are a constant percentage of gross premiums paid in the past and gross premiums assumed in the future. Notice, as with method 1, that if the assumed gross premium pattern exactly matches those gross premiums already paid, we will calculate identically the "correct" cash value.

Once again we will examine the sensitivity of the cash value to the assumed premium. Table 8.2 shows and Figure 8.2 illustrates the calculated cash values for method 2 and compares them with those of method 1. Notice that, in general, method 2 demonstrates less sensitivity to the level of assumed future premiums. If the conditions for use of the Classical UL Model are satisfied, then method 2 exhibits no sensitivity to the assumed premium.

TABLE 8.2

Comparison of Fifteenth- and Fortieth-Duration Cash Values (Method 2)

Actual Gross Premium = $12.00
Actual Interest Credited = 10%
Actual Mortality Charged = 60% of 1958 CSO

Assumed Premium	BGF Interest = 4%		BGF Interest = 5½%		BGF Interest = 7%	
	$_{15}CV$	$_{40}CV$	$_{15}CV$	$_{40}CV$	$_{15}CV$	$_{40}CV$
$10.00........	$264.15	$4,463.51	$355.95	$5,006.65	$484.66	$5,624.73
10.50........	263.61	4,463.64	355.95	5,006.65	484.40	5,623.94
11.00........	263.13	4,463.77	355.95	5,006.65	484.13	5,623.15
11.50........	262.65	4,463.90	355.95	5,006.65	483.85	5,622.36
12.00........	262.26	4,464.03	355.95	5,006.65	483.56	5,621.57
12.50........	261.89	4,464.16	355.95	5,006.65	483.27	5,620.78
13.00........	261.57	4,464.29	355.95	5,006.65	483.00	5,619.99
13.50........	261.30	4,464.42	355.95	5,006.65	482.72	5,619.20
14.00........	261.07	4,464.55	355.95	5,006.65	482.44	5,618.42
14.50........	260.89	4,464.68	355.95	5,006.65	482.16	5,617.63
15.00........	260.77	4,464.81	355.95	5,006.65	481.87	5,616.85

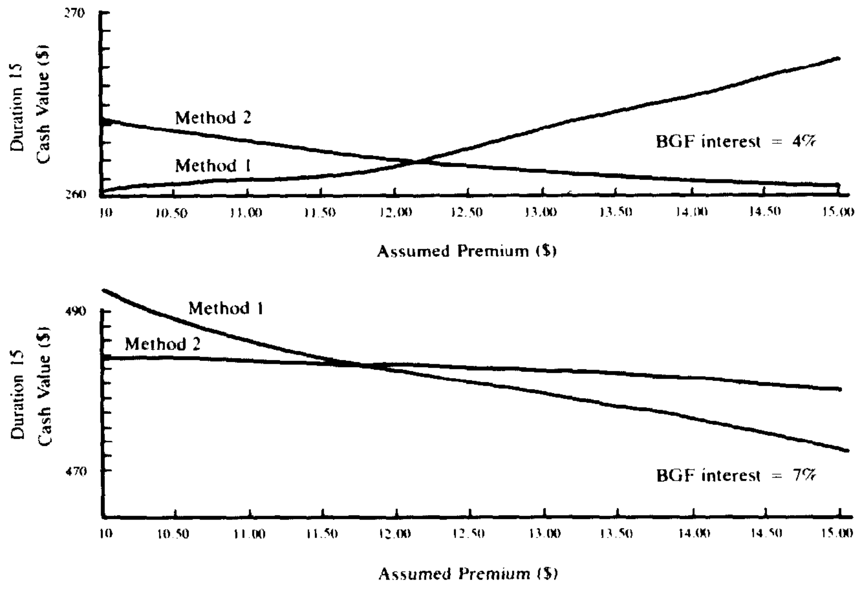


FIG. 8.2.—Comparison of fifteenth-duration cash values for methods 1 and 2, by assumed premium.

We can summarize method 2 as follows:

a) It is consistent with the Classical UL Model.
b) It allows flexibility between the cost structure and the nonforfeiture structure.
c) Net premiums are recalculated each time a gross premium differs from a corresponding assumed premium. This is contrary to current law.
d) There is less sensitivity to the assumed premium with method 2 than with method 1.
e) Method 2 is complicated to apply and more difficult to understand than method 1.

Since method 2 exhibits less sensitivity to the assumed premium, we find method 2 theoretically preferable to method 1. Practically speaking, however, we feel that its complexity completely outweighs this advantage.

Assumed Premium

Methods 1 and 2 both rely on an assumed premium structure. The assumed premium must be arbitrary in nature. Following are a few possibilities:

a) The assumed premium is that which will mature the contract for the face amount on a guaranteed basis (looking from issue).
b) The assumed premium is the premium structure "planned" by the policyholder. Many ICVs ask the policyholder to specify such a premium at the time of application.
c) The assumed premium is that which is required to be paid under the contract. In many cases this results in future assumed premiums of zero.

Strong arguments can be made in favor of *a* or *c*, but the planned premium has obvious disadvantages. Since the cash value would depend to some extent on the planned premium, two policyholders with identical policies and payment histories might develop differing minimum cash values. In addition, the administration of planned premium records and illustrations might be expensive. For these reasons we find the planned premium approach unacceptable. Approach *c* forms the basis of method 3.

METHOD 3: ASSUMED PREMIUM = MINIMUM PAYMENT REQUIRED UNDER CONTRACT

The Standard Nonforfeiture Law states that an adjusted premium must be calculated corresponding to each gross premium due under the contract:

Any cash surrender value available under the policy in the event of default in a premium payment due on any policy anniversary, whether or not required by section two, shall be an amount not less than the excess, if any, of the present value, on such anniversary, of the future guaranteed benefits which would have been provided for by the policy, including any existing paid-up additions, if there had been no default, over the sum of (a) the then present value of the adjusted premiums as defined in sections five, five-a and five-b *corresponding to premiums which would have fallen due on and after such anniversary*, and *(b)* the amount of any indebtedness to the company on the policy. [1976 SNFL, sec. 3 (emphasis added).]

Under most flexible premium ICVs there is no minimum premium due at any time after issue.[14] For these contracts, a lenient interpretation of the law would require that no net (adjusted) premiums be calculated.

Once such an assumption is made, matters become a little simpler. Formula (8.1) (ignoring E^1) becomes

$$\text{Minimum } CV = PVFB + \Delta PVFB \ . \tag{8.1a}$$

Since there are no net premiums corresponding to $PVFB$, it makes sense to combine $PVFB$ and $\Delta PVFB$ into one term: $PVFB'$ is the total of the guaranteed benefits at any time.

$$\text{Minimum } CV = PVFB' \ . \tag{8.1b}$$

At any particular duration the cash value would be simply the present value (on the cash-value basis) of the then future guaranteed benefits. These future guaranteed benefits would be found by projecting the actual BGA forward with the BGF assuming no future gross premiums. The cash value would be a net single premium *(on the cash-value basis)* for those benefits.

Figure 8.3 illustrates cash values under this method and compares them with the earlier methods. Note that the Classical UL Model is but a special case of method 3. We summarize the advantages of method 3 below:

a) It is consistent with the Classical UL Model.
b) It allows a flexibility between the cost structure and the nonforfeiture structure similar to that afforded traditional plans of insurance.
c) It relieves the burdens and inconsistencies of an assumed premium.
d) The method is relatively simple to understand.
e) It can be interpreted as being consistent with current law.

[14] It may be argued that the minimum premium due is that which will continue the contract in force. This would imply that adjusted premiums should be calculated corresponding to pure term premiums as the BGA runs dry. The effect of this assumption on cash values, however, is negligible. For this reason, and for the sake of simplicity, we assume no future gross premiums.

The above reasons seem to suggest method 3 as the most desirable for ICV cash values, from the standpoint of both current law and relative simplicity. It is also consistent with our fixed premium model (i.e., both fixed and flexible premium products can be handled within the same definition). There is, however, a possible shortcoming of method 3. Since there is no future premium assumption, any favorable guarantees associated with possible future premiums are not reflected in the minimum cash value. The most common "favorable guarantee" is a smaller loading in renewal years.

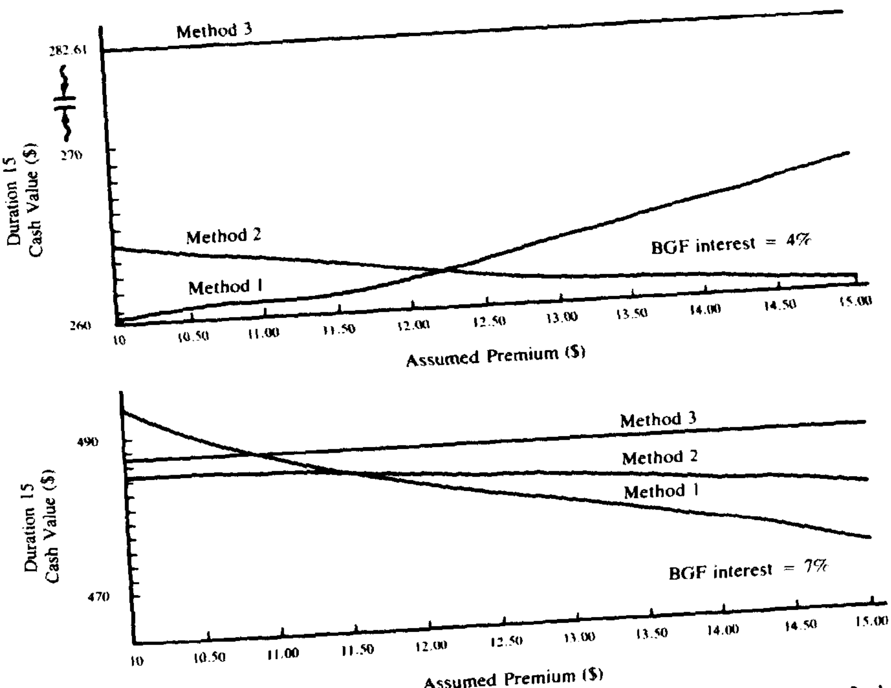


FIG. 8.3.—Comparison of fifteenth-duration cash values for methods 1, 2, and 3, by assumed premium.

Actual gross premium = \$12.00
Actual interest credited = 10%
Actual mortality charged = 60% of 1958 CSO

BGF interest = 4%:	BGF interest = 5½%:	BGF interest = 7%:
${}_{15}CV$ = \$ 282.61	${}_{15}CV$ = \$ 355.95	${}_{15}CV$ = \$ 487.07
${}_{40}CV$ = 4,460.85	${}_{40}CV$ = 5,006.65	${}_{40}CV$ = 5,640.83

TABLE 8.3

SUMMARY OF CHARACTERISTICS OF THREE DESCRIBED METHODS

Characteristic	Classical Model	Method 1	Method 2	Method 3
Consistent with Classical Model	Yes	Yes	Yes	Yes
Requires matching of cost and nonforfeiture structure	Yes	No	No	No
Sensitive to assumed premium	No	Yes	Yes	No
Consistent with current law	Yes	No	No	Yes

It might seem that consistency with current law requires that such factors affect the cash value. The authors, however, feel that the question becomes more of a philosophical one: To what extent should favorable policyholder options affect minimum cash values? We will not explore this question any further in this paper, although the reader should be aware of its existence.

Table 8.3 summarizes the characteristics of methods 1, 2, and 3.

8.4. *Expense Allowance*

The previous section dealt with the calculation of a net level reserve on the cash-value basis. Under the 1976 amendments, the minimum cash value is defined as a type of modified reserve. Formula (8.1) expressed the minimum cash value as a net level reserve less a deduction for any unamortized expense allowance.

$$\text{Minimum } CV = PVFB + \Delta PVFB - PVNLP - PV\frac{E^1}{\ddot{a}_x}. \tag{8.1}$$

We have discussed three methods of determining $PVFB$, $\Delta PVFB$, and $PVNLP$. All that remains, then, is a determination of E^1.

The quantity E^1 is dependent upon the guaranteed plan of insurance and the gross premium structure. Since we cannot pin down either the guaranteed plan of insurance or the future premium structure, we are faced with a problem quite similar to that of the previous section. Four possibilities follow:

E1. The expense allowance is the minimum of those prescribed for all the possible outcomes of the policy. Under the 1976 amendments the minimum expense allowance is $20 (for any premium-paying plan).

E2. The expense allowance is that appropriate for the plan of insurance and premium pattern defined by the insured's "planned" premium.

E3. There is no expense allowance for single premium plans. Since flexible premium ICVs typically require no premium after the initial payment, these plans could be considered single premium policies. Thus, no expense allowance is justified.

E4. The expense allowance is that for a level premium endowment at the latest maturity date.

Method E1 is that used by many insurers in conjunction with application of the Classical UL Model. It is clearly within the law, given the presumption that ICVs are *not* single premium plans.

Method E2 is undesirable for the reasons outlined when this method was discussed in conjunction with the assumed premium. The cash value would depend to some extent on the policyholder's declaration of a planned premium.

Although single premium plans are afforded no expense allowance under current law, the interest rate used in the calculation of cash values is often more liberal than that for premium-paying plans. This additional "surrender charge" may more than offset the loss of E^1. If method E3 is determined as appropriate for ICVs, it might appropriately be accompanied by a corresponding liberalization of the minimum cash value basis. This would be consistent with the 1976 amendments.

A strong case can be made for method E4. Since ICVs are largely marketed in lieu of more traditional *permanent* insurance, a comparable E^1 would seem in order. This method maintains the greatest parity with traditional forms of insurance.

Again, any method put into use (short of disallowing any E^1) requires some interpretation of current law. The authors favor method E4 for the reasons cited.

8.5. *Summary*

In general, flexible premium ICVs cannot comply directly with a strict interpretation of current law. Nevertheless, such plans that meet all the requirements of the Classical UL Model can be designed to comply with current law. The restrictions are severe (the authors are unaware of any plan that meets all the requirements) and place ICVs at a disadvantage when compared to more traditional forms of insurance.

An interpretation of current law is needed so that insurers have some direction to follow in designing flexible premium ICVs. The authors favor method 3 for the determination of $PVFB$ and $\Delta PVFB$, and method E4 for the determination of E^1. Method 3 has the advantage of being insensitive to the level of premiums (as discussed earlier), which can produce anomalous results in methods 1 and 2.

If we combine method E4 with method 3, the cash-value formula is as follows:

$$\text{Minimum } CV = PVFB' - \frac{E^1}{\ddot{a}_x}\,\ddot{a}_{x+t}\,.$$

This formula yields results identical with those of the Classical UL Model when the proper restrictions apply, and is conceptually simple. In addition, the burden of aligning the cost basis to match the nonforfeiture basis is removed, allowing greater flexibility and imagination in plan design.

9. CONCLUSIONS

Current law unambiguously defines minimum values in terms of guaranteed future benefits. The Classical UL paradigm, with its emphasis on unbundled retrospective surrender values, sidesteps the issues of prospective calculation. It is only by rebundling the product, by looking prospectively at guaranteed benefits, that we can calculate those values required by existing law.

We have demonstrated that minimum values for fixed premium ICVs are well defined by current (1976 NAIC) law.

We found that regulating the individual components of the BGF not only is unnecessary but is beyond the bounds of current law. A distinction must be made between the cost (BGA, BGF) basis of a contract, and its reserve and nonforfeiture bases. First-year loads need not be constrained to E^1, renewal loads need not be a level proportion of premiums, and there need be no requirement that the level or form of BGF guarantees matches that of a reserve or cash-value basis.

Of course, all of the above items may have an indirect bearing on the minimum cash value, but only through their effect on benefits.

Proposed limitations on these items, stemming from the application of an inappropriate model (Classical UL), are not supported either by statute or by actuarial theory.

In contrast with fixed premium plans, minimum values for flexible premium ICVs are not well defined under current law. An interpretation of these laws is needed. An assumption must be made with regard to the amount and timing of premiums.

The authors conclude that minimum reserves and cash values must be defined prospectively in terms of guaranteed benefits, and support either method 1 or method 3 together with method E4 for the determination of these values. Either method would relieve insurers of the restrictions inherent in the Classical approach.

The restrictions required for use of the Classical UL Model would place flexible premium ICVs on an unequal footing with traditional insurance forms. These restrictions do not apply to fixed premium ICVs.

10. ACKNOWLEDGMENTS

The authors wish to thank Bruce C. Dane for his extensive programming efforts in the technical development of this paper. Bruce's amazingly fast turnaround time reduced an almost impossible task to a manageable level. We would also like to express our appreciation for the initial encouragement and continued support we received from Claude Thau.

APPENDIX A

When the conditions for use of the Classical UL Model are met, we can derive our minimum reserve or cash value by making a simple adjustment to the account value. We start by reviewing several equations developed in Jordan's *Life Contingencies*.

In chapter 5, Jordan shows that successive net level reserves can be equated by the following formula:

$$({}_tV + P)(1 + i) = {}_{t+1}V + q_t(1 - {}_{t+1}V) \, . \tag{5.17}$$

From this it follows that the net level reserve can be expressed as a retrospective accumulation of premiums less mortality costs:

$${}_nV = P \cdot \ddot{s}_{\overline{n}|} - \sum_{t=0}^{n-1} (1 + i)^{n-t-1} \cdot K_t \, , \tag{5.18}$$

where $K_t = q_t(1 - {}_{t+1}V)$. In an analogous fashion, it is possible to express a modified reserve as an "interest-only" retrospective accumulation. We know that for any modified reserve,

$$\begin{aligned}
{}_tV^{\mathrm{Mod}} &= A_t - B\ddot{a}_t \\
&= vq_t + vp_tA_{t+1} - B - vp_tB\ddot{a}_{t+1} \\
&= vq_t - B + vp_t \, {}_{t+1}V^{\mathrm{Mod}}
\end{aligned}$$

Therefore,

$$({}_tV^{\mathrm{Mod}} + B)(1 + i) = {}_{t+1}V^{\mathrm{Mod}} + q_t(1 - {}_{t+1}V^{\mathrm{Mod}}) \, . \tag{A1}$$

Formula (A1) is the familiar equation (5.17) expressed for modified reserves. Following Jordan's development of (5.18), and remembering that ${}_0V^{\text{Mod}}$ is equal to $(\alpha - B)$, we obtain a valuable result:

$$ {}_nV^{\text{Mod}} = B\ddot{s}_{\overline{n}|} - (B - \alpha)(1 + i)^n - \sum_{t=0}^{n-1} (1 + i)^{n-t-1}K_t \,, \qquad \text{(A2)} $$

where now $K_t = q_t(1 - {}_{t+1}V^{\text{Mod}})$. Another familiar result from Jordan which we will find useful is the following:

$$ {}_nV^{\text{NL}} = {}_nV^{\text{Mod}} + \frac{B - \alpha}{\ddot{a}_0}\,\ddot{a}_n \,. \qquad \text{(A3)} $$

To relate all this to the Classical UL Model, we must make a few assumptions:

1. The formula for the account value is mathematically consistent with equation (A1); that is, the account value is a modified reserve. Note that this requirement rules out periodic, per-policy, contract charges.
2. We know in advance the amount and timing of premiums.
3. The reserve basis, the cash-value basis, and the contract guarantees are identical. This rules out prospective "current" interest and mortality guarantees.
4. Interest credited and mortality charged are at guaranteed levels.

Suppose our universal life contract has a level-percentage-of-premium load of $100r$ percent, and front-end charges of E^c. We can define a contract premium, P^c, which is equal to $(1 - r)GP$, where GP is the gross premium. Furthermore, we can project into the future to determine what the guaranteed plan of insurance is. In our example, the following equation will hold:

$$ A_0 = P^c\ddot{a}_0 - E^c \,. $$

Once our guaranteed benefits are determined, a net level premium can be calculated:

$$ P = \frac{A_0}{\ddot{a}_0} \,. $$

Given our first assumption, we can use equation (A2) to write:

$$ {}_nAV = P^c\ddot{s}_{\overline{n}|} - E^c(1 + i)^n - \sum_{t=0}^{n-1} (1 + i)^{n-t-1}K_t \,, \qquad \text{(A4)} $$

where $K_t = q_t(1 - {}_{t+1}AV)$. Also, from (A3),

$$ {}_nV^{NL} = {}_nAV + \frac{E^c}{\ddot{a}_0}\,\ddot{a}_n . \qquad \text{(A5)} $$

Now that we have both a plan of insurance and a net premium, we can develop retrospective expressions for minimum reserves and cash values. From (A2),

$$ {}_nCV^{Min} = P^A\ddot{s}_{\overline{n}|} - E^t(1 + i)^n - \sum_{t=0}^{n-1}(1 + i)^{n-t-1}K_t , \qquad \text{(A6)} $$

where $K_t = q_t(1 - {}_{t+1}CV^{Min})$. From (A3),

$$ {}_nV^{NL} = {}_nCV^{Min} + \frac{E^t}{\ddot{a}_0}\,\ddot{a}_n . \qquad \text{(A7)} $$

From (A2),

$$ {}_nV^{CRVM} = B^{CRVM}\ddot{s}_{\overline{n}|} - (B - \alpha)^{CRVM}(1 + i)^n - \sum_{t=0}^{n-1}(1 + i)^{n-t-1}K_t , \qquad \text{(A8)} $$

where $K_t = q_t(1 - {}_{t+1}V^{CRVM})$. From (A3),

$$ {}_nV^{NL} = {}_nV^{CRVM} + \frac{(B - \alpha)^{CRVM}}{\ddot{a}_0}\,\ddot{a}_n . \qquad \text{(A9)} $$

If we equate (A5) to (A7), we find one of our sought-after relationships:

$$ {}_nCV^{Min} = {}_nAV - \frac{E^t - E^c}{\ddot{a}_0}\,\ddot{a}_n , \qquad \text{(A10)} $$

and by equating (A5) to (A9), we find the other:

$$ {}_nV^{CRVM} = {}_nAV - \frac{(B - \alpha)^{CRVM} - E^c}{\ddot{a}_0}\,\ddot{a}_n . \qquad \text{(A11)} $$

One immediately wonders about the validity of these relationships when our four assumptions are relaxed. These issues are dealt with extensively in the main body of the paper. However, the last assumption merits some discussion here.

As long as the account value is positive on the guarantees throughout the first year, crediting higher interest or charging less for mortality will serve to increase projected future benefits. As long as our third assumption holds, the present value of these future benefits on the reserve and cash-value bases will exactly equal the amount of such increments to the account value. This means that, under 1976 NAIC model law (which requires NSP funding of changes in guaranteed benefits), equations (A10) and (A11) will remain value when we deviate from guarantees (assuming that the adjustment factors are calculated on guaranteed interest and mortality). However, a direct calculation from equations (A6) and (A8) could yield inappropriate values (P^A may be less that E^1).

Summary

We have shown that, under certain constraints, we can develop retrospective "interest-only" expressions for each of the account value, net level reserve, minimum cash value, and minimum reserve. Further, each of the latter three items can be obtained by a simple arithmetic adjustment to the account value.

APPENDIX B

As a concrete example of the limitations of the Classical UL Model, consider a fixed premium ICV (Plan A) with policy guarantees of 4 percent interest and 1958 CSO mortality. This particular plan will always credit 4 percent and charge mortality at 1958 CSO rates. There is no provision for any deviations from guarantees. The fixed premium is exactly equal to P^A for whole life calculated at 4 percent and 1958 CSO. The first-year load is equal to E^1, and there are no renewal loads.

With these specifications it is obvious that the guaranteed plan of insurance is whole life, and will remain so. The account values are equal to 4 percent, 1958 CSO cash values for whole life. We can represent this plan in "black box" notation.

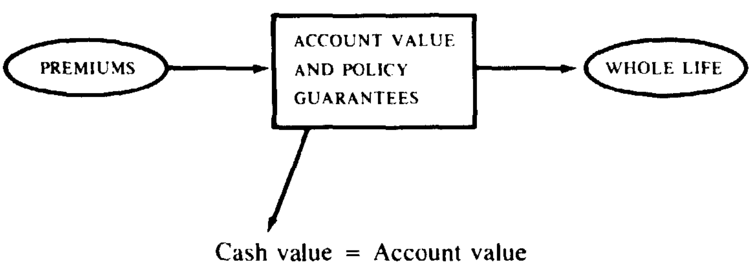
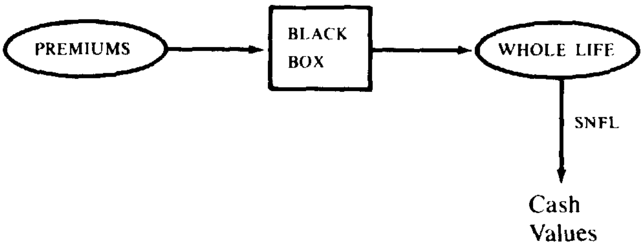


TABLE B1

COMPARISON OF REQUIRED CASH VALUES FOR PLANS A AND B

Duration (Years)	Plan A Classical Model	Plan B 5½%, 1958 CSO
5	$ 22	$ 13
10	95	72
15	175	141
20	262	219

Now, consider a traditional whole life contract (Plan B) with an identical gross premium. We can represent this plan as follows:



Obviously, the minimum cash values required for this plan are calculated using 5½ percent interest and 1958 CSO mortality (1976 SNFL). In contrast, the cash values required for the ICV under the Classical UL Model are 4 percent, 1958 CSO values. The required cash values for both Plan A and Plan B (issue age 35) are compared in Table B1. A major disadvantage of the Classical Model is now obvious. The cost basis must match the nonforfeiture basis. In this case we see identical premiums and benefits, yet different required[15] cash values.

APPENDIX C

The phenomenon illustrated by method 1 (Sec. 8.3) may seem unusual. A conceptual explanation follows.

Method 1 assumes a certain correspondence between BGF premiums and adjusted premiums. Method 1 would be insensitive to the assumed premium level if at each duration future BGF premiums purchased the same proportion of benefits as the corresponding net (adjusted) premiums.

[15] The cash values illustrated for the ICV are only "required" within the context of the Classical UL Model.

If this were true, then the value (on the cash-value basis) of the benefits generated by future assumed premiums would be identical with the present value (also on the cash-value basis) of future net premiums. In this way the assumed premiums have no effect on the resulting cash value. In general, however, no such parity exists between BGF premiums and the corresponding net premiums.

We know that BGF premiums can be considered to be net premiums on the BGF basis (this is the foundation of the Classical UL Model). With this fact in mind, we can analyze the relationships between BGF premiums and net premiums by comparing net premiums calculated on two different bases. We will use a simple whole life policy for our comparison.

Consider two sets of net level premiums calculated for a whole life policy. One set of net premiums (P^1) is calculated using 4 percent interest, while the other (P^2) is calculated at 5½ percent. Both assume 1958 CSO mortality.

It is obvious that the present value of each set of net premiums is sufficient to purchase whole life benefits on its own basis. However, if we single out a certain duration's net premium and determine the benefits it will support, we find that the results differ for each basis. Consider P^1 and P^2, both chosen from duration t. We can discount these premiums back to issue on the separate bases and determine the proportion of the total benefits that each premium will support.

$$\text{Percent whole life benefits} = \frac{P}{A_x} \, {}_tE_x \, .$$

Figure C1 illustrates the above percentages for each duration for both bases (issue age 35). Notice that early-duration P^2's are more "valuable" than early-duration P^1's, while for later durations the relationship is reversed.

Since the first duration P^2 is worth relatively more than the corresponding P^1, it follows that all the remaining P^2's are worth relatively less than all the remaining P^1's. This is logical considering that all P^1's will buy the same benefits on their own basis as all the P^2's on their own basis. For each duration, we can determine the relative value of all the remaining premiums by the following formula:

$$\text{Percent whole life benefits} = (P\ddot{a}_{x+t} \, {}_tE_x) \div A_x \, .$$

Figure C2 illustrates this percentage for both bases at each duration.

The remaining P^1's always support a higher proportion of benefits than the remaining P^2's. If we let the remaining premiums buy *attained-age* benefits, the relationship is similar. At each duration we will take the present value (at that duration) of remaining premiums and purchase whole life benefits.

$$\text{Amount of whole life benefits} = (P\ddot{a}_{x+t}) \div A_{x+t} .$$

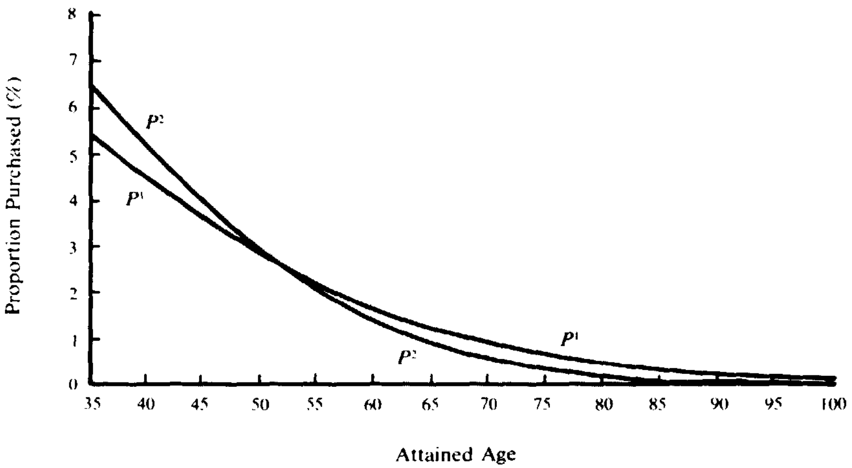


FIG. C1.—Proportion of whole life benefits purchased by the current net premium, by attained age

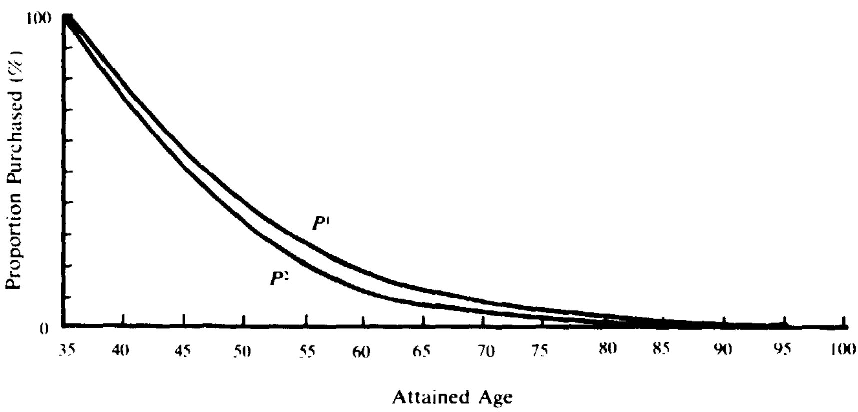


FIG. C2.—Proportion of age 35 benefits purchased by future net premiums, by attained age

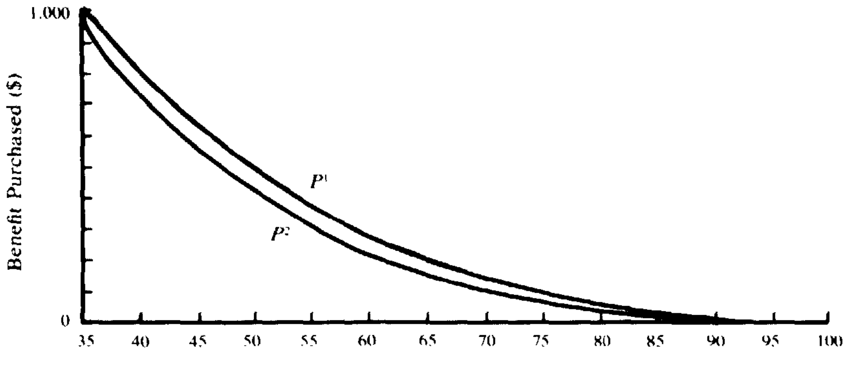


FIG. C3.—Benefits purchased by future premiums at attained age

These amounts on each basis are shown in Figure C3.

We can now tie this analysis to method 1 (Sec. 8.3). First, we express the minimum cash value formula (8.1) in different form.

$$\begin{aligned}\text{Minimum } CV & \\ &= PVFB + \Delta PVFB - PVADJP \\ &= \text{Present value of:} \\ &\quad [(\text{Benefits purchased by account value}) \\ &\quad + (\text{Benefits purchased by future assumed BGF premiums} \\ &\quad - \text{benefits purchased by future } P^A\text{'s})]\ . \end{aligned} \tag{8.1}$$

Let us analyze each of the terms. The first expression (benefits purchased by account value) is independent of the assumed future premium. The remaining terms (benefits purchased by future BGF premiums *less* benefits purchased by future P^A's), however, are dependent on the premium assumption.

We can think of this latter expression as the difference between the benefits purchased by remaining premiums on two different bases. The situation is quite similar in concept to our analysis above. In general, whenever the BGF basis is more liberal (e.g., with a higher interest rate) that the cash-value basis, the second term will be negative. This occurs because any remaining premiums calculated at a lower rate of interest will purchase more benefits than remaining premiums calculated at a higher interest rate. On the other hand, when the BGF basis is more conservative than the cash-value basis, this expressions will be positive.

In addition, the absolute value of this amount will grow with the size of the assumed premium, thus occasioning the sensitivity to the premium assumption.

This analysis requires that the form and incidence of benefits being purchased by the net premiums are substantially similar to those of whole life. For other forms of coverage, the relationships between net premiums will change.

The analysis of method 1 (Sec. 8.3) makes use of assumed premiums in the \$10–\$15 range. With these premiums, the benefits will fluctuate around those of E @ 95. At lower premium levels, the benefits purchased by the corresponding net premiums are shorter in duration, altering the relationships described above. Although considerably more complicated,[16] a similar analysis can be made for method 2 (Sec. 8.3).

APPENDIX D

The authors feel it worthwhile to comment briefly on the application of the generalized model to the 1980 Standard Valuation Law and Standard Nonforfeiture Law.

Reserves.—The 1980 Standard Valuation Law contains no new concepts significant to our analysis. Although the allowable rates of interest and mortality have changed, the mathematical model described in this paper applies equally well to the new valuation standard.

Nonforfeiture values.—There is one conceptual change in the new nonforfeiture law that may have an effect on ICVs. The passage is quoted below:

> In the case of policies which cause, on a basis guaranteed in the policy, unscheduled changes in benefits or premiums, or which provide an option for changes in benefits or premiums other than a change to a new policy, the adjusted premiums and present values shall initially be calculated on the assumption that future benefits and premiums do not change from those stipulated at the date of issue of the policy. At the time of any such change in the benefits or premiums, the future adjusted premiums, nonforfeiture net level premiums and present values shall be recalculated on the assumption that future benefits and premiums do not change from those stipulated by the policy immediately after the change. [1980 SNFL, sec. 5c.]

The above passage was originally written into the law to accommodate adjustable life policies. Briefly, it states that unscheduled policy changes allow for a recalculation of adjusted premiums, including an adjustment to E'.

[16] The relative value of earlier and later net premiums is shifted each year under method 2.

When applied to ICVs, this concept can produce some illogical results. For example, when future benefits are increased (through the extension of a liberal interest guarantee, for instance) the minimum cash value decreases. This happens because the present value of the increase in adjusted premiums is greater than the present value of the increase in benefits (due to an increased E^1). The authors feel that this effect is inappropriate for ICVs.

Nevertheless, the generalized model can be applied while incorporating this concept. As a example, formula (7.1) would be rewritten as

$$\text{Minimum } CV = PVFB + \Delta PVFB - PVADJP - \Sigma\Delta PVADJP \,.$$

The last term is the present value of all the changes made to the adjusted premium as a result of benefit enhancements. The authors will not present a detailed derivation here, but the reader should be aware that the main thrust of the paper remains intact: that minimum values are tied to benefits, and not to the internal accounting system of the policy.

GLOSSARY OF TERMS

Indeterminate cash value (ICV)—Any product where the cash value is determined by formula.

Classical UL Model—The demonstration whereby the account value and term amount are shown to have properties identical with those of a reserve and net amount at risk.

Benefit generating account (BGA)—Commonly called the "account value," the amount that determines future benefits.

Benefit generating function (BGF)—The formula that converts the BGA into guaranteed benefits (usually composed of mortality and interest guarantees as well as an accumulation formula).

BGF premium—The portion of the gross premium credited to the BGA.

PVFB—The present value of future guaranteed death and endowment benefits at issue, on either the valuation or the nonforfeiture basis.

ΔPVFB—Any changes in *PVFB* since issue.

PVADJP—The present value of future adjusted premiums on the nonforfeiture basis.

Secondary benefit guarantee—Any benefit guaranteed independent of the BGA.

Shadow fund (SF)—The amount which the BGA must exceed at any time to generate guaranteed future benefits in excess of any secondary benefit guarantees.

Adjusted shadow fund (ASF)—The greater of the BGA and the SF.

PVG—The present value of future BGF premiums.

$PVFB^{BGF}$ *and* PVG^{BGF}—*PVFB* and *PVG* as defined above, except that the present values are based on BGF guarantees.

DISCUSSION OF PRECEDING PAPER

ROBERT J. CALLAHAN:

The authors are to be commended for this excellent article setting forth a rationale for reconciling the statutory minimum cash values with the retrospective formula in the "universal life" plans. There is no unique definition of universal life. Most of the current forms fall under what the authors describe as the fixed premium indeterminate cash-value policies and the flexible premium indeterminate cash-value plans, although there has been some modification in benefit design as a result of the TEFRA (Tax Equity and Fiscal Responsibility Act) guidelines. The generation of values using a retrospective method appears to be the most common ingredient.

At this point every state has approved a form of universal life, in most if not all cases under laws similar to the 1976 NAIC (National Association of Insurance Commissioners) model. This product was in its infancy when the 1980 NAIC model was enacted. It was not specifically treated in the 1980 model, but before enactment, it was felt that the universal life plan could be accommodated under a new section pertaining to indeterminate premium plans and to "any plan of life insurance which is of such a nature that minimum values cannot be determined by the methods described" in preceding sections. From a conceptual standpoint it should be easier to demonstrate compliance with the 1980 version. However, the new section sets forth three conditions pertaining to (1) benefits as favorable, (2) benefits and pattern of premium not misleading, and (3) cash surrender and nonforfeiture benefits consistent with the principles of the Standard Nonforfeiture Law (SNFL) (see appendix to this discussion).

In demonstrating compliance with the 1976 laws, to generate guaranteed benefits most insurers use the same long-term guaranteed factors of interest and mortality as are permitted and used in calculating the minimum cash values and the amounts of paid-up or the periods of extended term insurance are equivalent to the cash value. The authors contend that guaranteed benefits can be generated by any formula, including a formula based on accumulation of premiums at interest less mortality costs, where such costs need not be related to the nonforfeiture basis. The application of this principle can lead to designs that may not meet the three conditions in the 1980 law.

Given a fixed amount of guaranteed benefits, actuarial textbooks demonstrate the equivalence of the prospective and retrospective method for reserves and cash values when the values are computed on a net premium basis using the same mortality and interest. There is no direct assessment of expenses, but there are expense allowances that affect the net premiums to be used in either discounting or accumulating.

In actual practice, inconsistencies can develop when minimum cash values and nonforfeiture benefits are defined prospectively but actual cash values are determined retrospectively with net premiums defined in terms of gross premiums. In a prospective calculation we start with the guaranteed benefit and solve for the net premiums. Prospectively, for a given amount of guaranteed benefit, the higher the interest rate used, the lower the cash values. In a retrospective calculation, we start with the net premium and solve for the guaranteed benefits. Retrospectively, for a given amount of net premium, the higher the interest, the higher the cash values. Similar observations could be made with respect to the loading (difference between gross and net premiums) and the mortality rates, with opposite effects as the loading and the mortality rates are varied.

In New York, insurance regulators had previous experience with a prospective minimum cash value law in which actual cash values were based on a retrospective formula. For individual deferred annuities, section 208(a) of the New York Insurance Law described minimum cash values prospectively with a maximum rate of interest and an initial expense allowance, generally 60 percent of the first year's nonforfeiture premium, but with no direct limitation on the magnitude of the loading. New York ran into many awkward situations, since the actual maturity value was determined from an accumulation of a portion of the gross premiums at interest. An insurer satisfying the law with a given maturity value may wish to increase the maturity value and each year's cash value by either decreasing the loadings in later years or increasing the interest rate. Based on the new higher maturity values, the values for earlier years, although higher than before, might not meet the prospectively calculated minimum cash values.

When individual deferred annuities became prominent, both the insurance industry and the NAIC developed a SNFL for annuities for the first time. Aware of the inconsistencies in the prospective method, in 1976, upon the recommendation of an industry advisory group, the NAIC adopted a retrospective formula based on maximum loadings and minimum rates of interest in spite of the appearance of rate regulation. In 1979 New York adopted a new section 208(c) with a retrospective formula.

From a solvency viewpoint, insurers were aware of the danger of long-term interest guarantees and resorted to periodic short-term higher guarantees. Even if the policies were labeled nonparticipating, they effectively became participating, a result beneficial to both insurer and policyholder. In New York the law requires any additional amounts to be declared on an equitable basis.

Based on the experience in the individual deferred annuity area, many felt that in the case of the determination of benefits under universal life using a retrospective method, the minimum cash value for such policies should be expressed retrospectively and that both the Standard Valuation Law (SVL) and the SNFL should be revised. Any revision would likely take a while since anything resembling rate regulation is far more controversial in the life insurance area than it was in the annuity area.

Meanwhile we are facing the implementation of the 1980 model SVL and SNFL with the 1980 CSO Table with low rates of mortality, dynamic interest bases with generally higher rates, and a new expense allowance formula. At the time of enactment, it was publicized that cash values and consequently gross premiums could be lowered. Such statements are based on a prospective method starting with a guaranteed benefit. The retrospective method starts with the gross premium and generates guaranteed benefits, and the low rates of mortality and high interest work to produce greater cash values.

Although some proponents claimed that the disclosure of the expenses would force the commission down, as the universal life product developed and insurers needed to pay normal commissions to market it, the initial expenses were covered up somewhat by replacing up-front loads with rear-end surrender charges based on amortizing the initial expense allowances. Many insurers could live with an initial expense allowance based on $20 per $1,000 of insurance plus 65 percent of a one-year term premium but cannot live with the new formula of $10 per $1,000 of insurance plus 125 percent of a one-year term premium. For flexible premium policies, this means determining a plan either based on that generated by assuming planned premiums and guaranteed cost factors or by assuming a level benefit level premium endowment plan for the maximum period during which premium may be paid. The authors prefer the endowment plan approach, which in turn has been endorsed by industry groups. This may prevent some manipulation on the part of agents and insureds but produces the same initial expense allowance regardless of the plan generated. This is contrary to the variance of initial allowance by plan in the present SNFL. The planned premium approach has been incorporated in guidelines by some states, such as New York and New Jersey.

The amortization period advocated by the authors and endorsed by industry groups is the maximum premium-paying period. In the interest of adequate disclosure, it may be necessary to require a shorter period, which could be adequately disclosed.

Some insurers provide for forfeiture of one of two years' excess interest in case of surrender. Some states may require legislation to permit such a penalty, presumably designed to prevent investment antiselection. New York law allows forfeiture of excess interest for the preceding twelve months. Accepting the premise that the cash-value interest rate for the present value of future benefits need not be the same as the guaranteed interest rates used to generate the guaranteed benefit, then by using a higher cash-value interest rate than the guaranteed benefit interest rate it is possible to generate another kind of surrender charge. This surrender charge is similar to that in the SNFL for individual deferred annuities that permits the calculation of cash value using an interest rate 1 percent higher than that used to generate the maturity value, that is, a surrender charge of about 1 percent for each year remaining to maturity. One wonders whether such a charge in universal life policies could be understood by the policyholder. (In actual practice, for annuities most insurers use an initial surrender charge as a percentage of the accumulation value, which grades down to zero at the end of five to ten years.)

To use a cash-value rate less than the guaranteed accumulation rate could result in both cash values and reserves, and possibly to premium deficiency reserves, in excess of the accumulation value. Insurers would want to avoid such results.

Therefore, as a practical matter, the cash-value interest rate should be set equal to the guaranteed interest rate. If the insurer wants to avoid additional reserves and possible surplus strain, the cash-value interest rate should be set no higher than the maximum reserve interest rate. While this may put practical restraint on the magnitude of the guaranteed interest rate, some restraint appears to be in the interest of insurer solvency. Competition and the results generated by the guaranteed factors may serve to put a floor on the amount of interest guarantee.

Although there is precedent for a secondary guarantee of benefits, one wonders whether this may be misleading or misunderstood in the universal life area, since for many years the crediting of excess interest may not result in either enhancement of the plan or an increased cash value or increased death benefits. Approval may depend on the manner of presentation, disclosure, and emphasis.

One of the major concerns is with the mortality levels under the 1980 CSO Table. Presumably, such rates contain adequate margins for standard issues, or they would not have been advocated by an industry advisory committee and adopted by the NAIC for valuation and nonforfeiture purposes. However, their use may force expenses to be up front and expressly identified as such. For substandard business, an insurer may use appropriate modifications of the 1980 CSO Table. Therefore, an insurer is able to use its own underwriting standards and accordingly may ensure the adequacy of the mortality rates.

As a practical matter, insurers may not want to guarantee mortality deductions more favorable than the 1980 table, not only because of the thin margins but also because discounting any guaranteed benefits might result in cash values and reserves greater than the fund. Current mortality deductions can be on a more favorable basis.

Regulators may find it difficult for standard issues to accept mortality deductions higher than those on a nonforfeiture basis. The use of the same mortality basis and interest rates for policy guarantees and for cash values, as well as no direct expense charges against the fund except for unamortized surrender charges, make the election of a nonforfeiture benefit unnecessary in the event of cessation of premiums. If higher guaranteed mortality rates are provided for flexible premium universal life policies in an active status, provision must be made for election of a nonforfeiture status. Some states would require disclosure of the amount and term for both statuses. It may be awkward to show a longer period for a policy in a nonforfeiture status.

While it is possible to justify use of higher extended term insurance rates in years in which no premiums are paid, it is awkward in practice. Perhaps it may be possible for flexible premium universal life policies to continue standard mortality deductions for a specified period of time, such as five years, during which no premiums are paid. Then, for policies less than a stated amount of accumulation value, or less than a stated amount per $1,000 of insurance, the insured would be notified that unless additional premiums are paid the policy will be placed in a nonforfeiture extended term insurance status and that mortality deductions will be on an appropriate extended term insurance table.

The authors do not mention rider benefits, such as term insurance on a spouse or children, guaranteed insurability options, payor benefit, accidental death benefits, and disability waiver of premiums. In the case of flexible premium policies, even if the total premium were increased to provide such benefits, direct payment of premium would be awkward, necessitating a priority in case of varying amounts and possible interruption of coverage in case of suspension of premium payments. It is more practical to provide for such benefits by deductions from the cash value. In the case of extended term insurance, it is customary to continue only the basic life insurance. Thus, even if guaranteed mortality and interest matched the nonforfeiture basis and no direct expenses were charged against the fund, where riders are present the similarity with extended term insurance breaks down.

On universal life policies on which no more premiums are paid, deductions for the rider benefits can be justified as being in the form of automatic withdrawal from the cash value to provide such benefits. Also, such a procedure is more favorable to the insured than the use of the automatic premium loan provision to pay direct premiums. New York has required that the deductions for basic life insurance on both the primary insured and a second life be on a tabular basis consistent with nonforfeiture mortality, but has allowed the deductions for other ancillary benefits on a basis comparable to charges for such benefits in traditional life policies.

The use of the 1980 CSO Table and the new expense allowance are not mandatory until January 1, 1989. In the meantime, most states are permitting a plan-by-plan election without requiring withdrawal of other plans previously approved. If an insurer wants to use the 1958 CSO for mortality guarantees under universal life policies, it would be wise to get its universal life policy into place before implementing the 1980 CSO for other plans. It is to be hoped that before January 1, 1989, with the assistance of industry advisory groups, the NAIC can either revise the law for universal life policies or can adopt by regulation the 1958 CSO Table for universal life policies. A revision in the law could embody direct recognition of the retrospective method with minimum factors and/or provide direct recognition of expense deductions from the cash value for expenses and for rider benefits.

We hope that this paper and discussion will lead to additional consideration and resolution of minimum cash values under the 1980 NAIC model that will be satisfactory to the insurers, the insureds, and the regulatory authorities.

The following section of the model law is quoted from the *NAIC Proceedings*, 1981, Vol. I:

APPENDIX

Section 8. Nonforfeiture Benefits for Indeterminate Premium Plans

In the case of any plan of life insurance which provides for future premium determination, the amounts of which are to be determined by the insurance company based on then estimates of future experience, or in the case of any plan of life insurance which is of such a nature that minimum values cannot be determined by the methods described in sections two, three, four, five, five-a, five-b, or five-c herein, then:

(a) the commissioner must be satisfied that the benefits provided under the plan are substantially as favorable to policyholders and insureds as the minimum benefits otherwise required by sections two, three, four, five, five-a, five-b, or five-c herein;

(b) the commissioner must be satisfied that the benefits and the pattern of premiums of that plan are not such as to mislead prospective policyholders or insureds;

(c) the cash surrender values and paid-up nonforfeiture benefits provided by such plan must not be less than the minimum values and benefits required for the plan computed by a method consistent with the principles of this Standard Nonforfeiture Law for Life Insurance, as determined by regulations promulgated by the commissioner:

Note: If desired the following provision may be added as subparagraph (d).

(d) notwithstanding any other provision in the laws of this state, any policy, contract or certificate providing life insurance under any such plan must be affirmatively approved by the commissioner before it can be marketed, issued, delivered or used in this state.

Drafting Comment: *If subparagraph (d) is enacted in a state where prior filing and approval of life insurance policy forms has not been previously required by statute, this subsection would mandate such action for plans requiring approval under section six. If subparagraph (d) is enacted in a state where approval is deemed under certain circumstances, such deemer provision would be overridden by the terms of this section six. In some states specific reference must be made to any statutory provision which is overridden.*

JEFF T. DUKES:

I enjoyed reading the authors' paper and thought it was well written. It should help to clarify the principles underlying the legal requirements for cash values and reserves, which have been obscured by decades of mechanical application of formulas to traditional fixed premium forms of insurance.

There was one area where I felt the paper was not as clearly written as elsewhere. The obscurity arises in defining the payment period for annuities. I assume (and I hope the authors will affirm or correct my assumptions) that for purposes of amortizing the initial expense allowance under their preferred E4 possibility (for flexible premium plans) the annuities run to the latest maturity date. For E2 they would run to the date the policy expires or matures under the "planned" premiums and policy guarantees in effect at issue. It is not clear what happens for possibility E1. Furthermore, I assume that the term *PVNLP* disappears if the policy goes beyond the period of coverage guaranteed at issue (so would *PVFB*, but that is not apparent or important in your algorithm).

I do not agree with two positions taken by the authors. The first has to do with recognizing current short-term guarantees in the cash surrender value. It does not make sense to me to pay out benefits that have not been earned. To my knowledge, upon surrender before the next policy anniversary, mutual companies do not pay more than the earned portion of any dividend declared for the current year. Why, therefore, should short-term guarantees of higher interest and/or lower mortality charges on a universal life plan be treated differently? On a practical basis, as the authors remarked, recognition of these short-term future guarantees can lead to cash surrender values greater than the current BGA value. In addition, such recognition renders it impossible, without capping the potential future guarantees at the time the policy is issued, to demonstrate compliance with the SNFL on back-end-loaded products where surrender charges are expressed as a percentage of the BGA.

The other point where I differ from the authors is in divorcing the expense allowance from the $PVFB + \Delta PVFB - PVNLP$ portion of the cash value. If method 1 or method 2 is adopted, why not calculate $PVFB + \Delta PVFB - PVADJP$ and dispense with splitting off the expense allowance? If method 3 is used, then I think a case can be made still for taking an expense allowance of the E1 variety and amortizing it over the period to maturity. A fourth method, applicable to many new plans, is to determine the guaranteed plan at issue based on an assumption that the minimum premium required by the company in the first year is paid in subsequent years as well. Adjusted premiums could be determined for this guaranteed plan at issue. In this case two insureds with identical issue dates, ages, premium payments, etc., could not have different minimum cash surrender values, because the minimum first-year premium is determined by the company for a given age, sex, and so on. I guess my problem can be stated thus: how can you emphasize that the guaranteed plan of insurance may be term to age 65 but then say it is all right to ignore that and take a whole life expense allowance calculated at a low interest rate because the plan is sold as whole life?

THOMAS G. KABELE:

Messrs. Chalke and Davlin have written an interesting paper on products of the "universal life" type. In Section 7 the authors point out that a universal life policy has both primary and secondary benefits. These are (1) the benefits supported by the benefit generating account (BGA) and (2) the overriding endowment at age 95 (E@95) benefits.

The same is true of a typical policy, where the BGA is the "cash value." The benefits generated by the cash-value account include loans, surrenders, extended term and reduced paid-up benefits, and annuity benefits. The E@95 benefits are a guaranteed renewable policy at guaranteed premiums with a guaranteed death benefit and a guaranteed maturity value.

The annual statement requires companies to hold the larger of the reserves for benefits of types 1 and 2. Thus, if cash values are greater than E@95 reserves, an additional reserve must be held in Exhibit 8G.

Illness Considerations

A flexible premium universal life plan may be even more "consumer-oriented" than companies realize. In many states the companies must tell the consumer how much extended term coverage he has, or at least whether his term coverage will expire during the next year. If the coverage period exceeds one year, and if he is "well," the policyholder can skip his premium. If he is "sick," and has little cash value, he can kick in an additional premium. If he is sick, and has excessive cash value, he can withdraw most of it and actually increase the net amount at risk (on some policies).

A typical whole life plan also allows skipped premiums through the automatic premium loan provision. The face amount of the coverage, however, decreases by the amount of the loan. In universal life the face amount does not necessarily go down.

"Stop-and-go" premiums have been discussed by Maurice LeVita.[1] LeVita, however, would require "evidence of insurability" for "inactive" accounts where the total of the premiums paid during the previous year were less than the whole life premium. There is no such requirement for universal life contracts.

Of course, the universal life company can recoup the adverse mortality costs by increasing its mortality charge, or even reducing excess interest margins.

Consumer Considerations

Chalke and Davlin point out that a policy that provides whole life benefits assuming 10 percent interest is not a whole life plan if the guaranteed cash value is only 4 percent. Such a plan is term insurance only for a period of years. On some contracts, it is not clear that the company must accept the "current year's mortality cost." In these cases the policy may expire when the person needs coverage the most.

On some forms it is not even clear whether the company must accept the "planned premiums." In these cases the consumer is really buying single premium term, and not E@95.

The 1958 CSO rates are artificially high at the advanced ages; in fact, the rates above age 92 were determined by a cubic. Under a typical E@95 policy the company guarantees to pay the face amount at maturity. Under universal life most of the face amount may be eaten up by "term charges."

Some universal life contracts are sold with the "cash-value addition" option for "excess interest." Thus excess interest is added only to cash values and not to death benefits. Therefore, on the death of the insured, the beneficiary forfeits the excess interest the company supposedly cred-

[1] "A Flexible or 'Stop and Go' Life Plan," *PCAPP*, XV (1965), 59.

ited to the policy. (There is a slight benefit to the cash-value additions, since the term insurance charges are reduced.)

In contrast to similar products that are called "participating," under universal life the companies are not subject to the typical 10 percent profit limitation imposed by certain states (2.5 percent limitation for Canadian companies). Therefore, the company may make more profits and give policyholders less.

Income Tax Considerations

Under current regulations, dividend and coupon accumulations are treated like "savings accounts." Thus the company must report interest added to the accumulation on a 1099 statement, and the policyholder must pay tax. Of course, the death benefit and cash value of these accounts are equal.

A dividend accumulation should be treated as either a "life reserve" (subject to the 101 exclusion at death) or an "annuity reserve" (subject to deferral), because the fund is subject to both permanent life insurance and permanent annuity purchase rates. On cessation of premium payments the company guarantees to apply the cash value of the dividend accumulation to buy extended term at guaranteed rates. On a settlement the company guarantees to buy an annuity at guaranteed rates.

With life contracts, the excess interest and premium additions do not increase the net amount at risk. Arguably these additions should be treated as additions to a savings account.

Recommended Design Changes

I would like to see several changes in product design. The companies should be required to accept the greater of the "planned premium" or the current year's mortality cost. The maximum mortality charges at advanced ages should be reduced, or else the company should be required to pay a significant maturity value at age 95. The cash-value addition option should be prohibited unless the policyholder and the beneficiary both sign a waiver. Flexibly priced products should be subject to the 10 percent profit limit, or at least a profit limitation should be disclosed to consumers. The companies should tighten the requirements on "stop-and-go" premiums and not permit cash withdrawals that do not reduce the face amount.

I am sure that many universal life contracts have solved the above problems, but on other contracts there may be uncertainties.

LARRY SILKES:

Life insurance, because it is a nontangible product, is extremely susceptible to being perceived as whatever people think it to be. The current conception of permanent life insurance is that it is really decreasing term plus a side fund.

When a conception has been around long enough, people start reacting to the concept accordingly. Not only is life insurance sold on the above theory; now there is the suggestion that the policy should be taxed as decreasing term plus a savings fund. I hope to demonstrate that there is an alternative view of life insurance, that of a term policy plus a fund that is forfeitable at death.

My problem starts with A, formulas (5.17) and (5.18) in Appendix A. Formula (5.18) states that the fund, cash value, or reserve accrues with interest, with the mortality charges being deducted with interest.

Formula (5.17) is derived from the following formula:

$$({}_tV + \pi)(1 + i) = q\,DB + p\,{}_{t+1}V\,. \tag{1}$$

This formula corresponds with general reasoning: the initial reserve increased with interest is equal to the death benefit for those who die plus a pure endowment of the year-end reserve for those who survive.

Some additional rearrangements are interesting.

$$({}_tV + \pi)\frac{1 + i}{p} - \frac{q\,DB}{p} = {}_{t+1}V\,. \tag{2}$$

Here we have the μ_x and κ_x formula for generating successive reserves. One more equivalence can be shown:

$$({}_tV + \pi)(1 + i) - \frac{q}{1 - q}\left[\frac{DB}{1 + i} - ({}_tV + \pi)\right](1 + i) = {}_{t+1}V\,; \tag{3}$$

$$({}_tV + \pi)(1 + i)\left(1 + \frac{q}{1 - q}\right) - \frac{q}{1 - q}DB = {}_{t+1}V\,; \tag{3a}$$

$$({}_tV + \pi)(1 + i)\frac{1}{p_x} - \frac{q}{p}DB = {}_{t+1}V\,. \tag{3b}$$

Formula (3) is the formula used by most companies to determine the next period's cash value in a typical universal life policy. Formula (2) is the most useful for analysis and understanding because all the death benefit terms are grouped together and all the cash-value items are grouped together.

If we examine the effect of increasing the initial reserve by adding Δ to the premium, and the DB function is not related to cash value (i.e., type 2 benefit), then the year-end cash value is increased with interest and survivorship.

$$({}_tV + \pi + \Delta)\frac{1+i}{p} - DB\frac{q}{p} = {}_{t+1}V + \Delta\frac{1+i}{p}.$$

Using the 1958 CSO Table, at age 65, we have $1/P = 1.033$, which means the fund increases an additional 3.3 percent because of survivorship. The total cash value does not increase at interest plus survivorship, but the marginal contribution increases at interest plus survivorship.

The concept of increasing for survivorship means that those who die forfeit their funds, which we then redistributed to the remaining survivors.

Survivorship does not increase the fund, but increases the individual survivor's shares.

The only way the cash value can increase faster than the interest rate is if some people forfeit their fund at death.

The above is a complete general formula for analyzing changes in reserve, fund, or cash value.

STEVEN D. SOMMER:

The authors of this excellent paper are to be congratulated for removing much of the confusion about reserve and cash-value calculations from indeterminate cash value (ICV) policies. The paper has certainly helped clarify my thinking about the differences between the BGA and the reserve and cash value.

I would like to supplement the paper, first by describing a deficiency reserve problem we have faced on fixed premium ICV policies, and second by presenting an analysis of the reserve calculations for flexible premium ICV policies that differs to a certain extent from that in the paper.

Fixed Premium ICV Policies

The authors briefly mention the possibility of deficiency reserve problems (Sec. 7.2) and properly state that the net premium used in the test may be calculated on the most liberal valuation basis allowed by law. If this net premium exceeds the gross premium, then deficiency reserves effectively must be held.

We have encountered another situation that may make deficiency reserves necessary. Consider the type of policy described by the authors at the beginning of Section 7. This policy can be thought of as a traditional E@95 policy with an additional feature: subject to certain rules, each gross premium paid is accumulated in a special account (the BGA); should that account value (possibly less a surrender charge) exceed the cash value as determined by the nonforfeiture law, the surrendering policyholder will receive that higher value.

For virtually all traditional forms of insurance, CRVM reserves equal or exceed minimum cash values. However, for these fixed premium ICV policies, it may happen, even under the minimum guarantees, that the BGA (less any surrender charge) exceeds the minimum cash value in some policy year. It may even exceed the CRVM reserve. In that case, is it necessary to grade the reserves up to these higher cash values, increasing the net premiums and possibly generating deficiency reserves?

I know of one state (California) that currently requires such an approach, and there may be others. To meet these requirements we have developed the following methodology:

1. Project the guaranteed BGA out to the maturity date.
2. Determine what effect, if any, this guaranteed BGA will have on the insurance amount. For example, for plans with a corridor death benefit, the BGA may become large enough to force the insurance amount above \$1,000 per unit.
3. Determine the CRVM reserves and minimum cash values for the policy by a straightforward application of the SVL and SNFL, using any adjusted insurance amounts produced in step 2.
4. Compare these CRVM reserves with the BGA less any surrender charge. If the latter exceeds the former at any policy duration, grade the reserves in such a way as to eliminate the excesses.
5. If any of the resulting net premiums exceed the corresponding gross premiums, apply the minimum reserve techniques defined in the SVL.

Whether or not this method generates any deficiency reserves, the resulting reserves will be adequate if the company credits the minimum interest rates and charges the maximum mortality rates. Because it is very likely that the company will be more liberal, the reserve will probably have to be adjusted upward.

Define

$_tV$ = CRVM reserve calculated in step 5 above;

$_tBGA$ = BGA calculated using the minimum guarantees, as in step 1 above; and

$_tBGA'$ = Actual BGA, calculated using the actual interest credits and mortality charges for the policy.

The actual reserve can then be calculated as follows:

1. If ${}_tV \leq {}_tBGA$, then the reserve equals ${}_tV + [{}_tBGA' - {}_tBGA]$, subject to a floor of ${}_tV$.
2. If ${}_tV > {}_tBGA$, then the reserve equals the greater of ${}_tV$ and ${}_tBGA'$.

The first formula simply states that the CRVM reserve should be increased by the excess, if any, of the actual over the minimum BGA. If the actual BGA happens to be less than the minimum, as it may for the disappearing-premium version of the policy, then the CRVM reserve is adequate.

The second formula states that, as long as ${}_tV > {}_tBGA$, the actual BGA can be increased up to the CRVM reserve with no reserve impact. Beyond that, however, the actual reserve must be set equal to the actual BGA.

Flexible Premium ICV Policies

It is possible to rework what the authors call the "boilerplate proof" of the Classical UL Model to remove the references to prospective and retrospective calculations. While the revised proof does not remove any of the policy design limitations that the authors describe, it does demonstrate that it is possible to apply the SVL and SNFL directly to some types of universal life policies, and avoid the contradictions described in Section 8.1.

The revised proof, which I have called the "arbitrary premium interpretation" of the law, is outlined below. Choose an arbitrary pattern of gross premiums. Calculate the corresponding CRVM reserves and the actual reserves that the company would hold for such a policy. Show that these actual reserves equal or exceed the CRVM reserves at each policy duration.

The SVL requirements are satisfied as long as the above demonstration works for every possible pattern of gross premiums. It is possible to show that such is the case, but only when the company holds the BGA as the actual reserve and when the universal life policy meets all the requirements listed by the authors.

Note that the SVL does not require us to calculate CRVM reserves; it only requires us to demonstrate that the reserves we are holding are equal to or exceed CRVM reserves. That fact can be demonstrated by the above proof, which does not force us to interpret or modify the SVL.

Admittedly, however, few plans contain the necessary policy design characteristics to make this or the boilerplate proof work, as pointed out by the authors. In such cases, I believe the SVL interpretation that does the least damage to the law is the paid-up interpretation (method 3 in the paper). The law defines the modified net premiums as a "uniform percentage of the respective contract premiums"; if there are no contract premiums, there can be no modified net premiums. The reserve, then, is just the present value of future guaranteed benefits as of the valuation date.

In Section 8.5 the authors state that they favor method 3 for determining the present value of future benefits, but they would reduce this reserve by the unamortized portion of an expense allowance. I would have trouble making this reduction. An expense allowance is an amount added to the present value of benefits (at issue) to determine the modified net premiums. If in method 3 there are no modified net premiums, I would not think an expense allowance deduction is justified.

(AUTHORS' REVIEW OF DISCUSSION)

SHANE A. CHALKE AND MICHAEL F. DAVLIN:

The authors wish to thank Messrs. Callahan, Dukes, Kabele, Silkes, and Sommer for their excellent discussions. We will address several of the points raised therein.

Mr. Callahan argues that "inconsistencies in the prospective method" necessitate a retrospective standard for ICV reserves and cash values. It is useful to examine carefully the apparent inconsistencies Mr. Callahan perceives in the prospective standard. Mr. Callahan observes that actuarial textbooks demonstrate the equivalence of prospective and retrospective values for a particular guaranteed benefit pattern and a given basis of calculation. He then states that basis of his concern: "Prospectively, for a given amount of guaranteed benefit, the higher the interest rate used, the lower the cash values. . . . Retrospectively, for a given amount of net premium, the higher the interest, the higher the cash values. Similar observations could be made with respect to the loading . . . and the mortality rates."

We assume that by the phrase "interest rate used," Mr. Callahan means the interest rate(s) guaranteed for the life of the contract. (Without this assumption, his comments would fall under the "10 percent cash value" fallacy discussed in Sec. 4 of our paper.) When a higher interest rate is used retrospectively, for a given amount of premium, it has the effect of both increasing death and endowment benefits as well as (paradoxically) *decreasing the cash value in relation to those benefits*. The resolution of the "inconsistency" lies in noting that as the guaranteed interest rate increases the plan of insurance changes. Using the new interest rate, and reflecting the change in benefits, the *prospective value will equal identically that calculated retrospectively;* however, each of these values will decrease in relation to the minimum prospective standard for the correct guaranteed benefits. There is no inconsistency between prospective and retrospective calculations. Similar arguments hold when varying expense and mortality rates are considered in isolation and in combination.

Mr. Callahan's position appears to be that the industry will eventually develop a retrospective standard for ICVs. In the interim period, he feels that *all* ICVs must comply with the subjective conditions specified in section 6 of the 1980 SNFL. The bulk of our paper served to demonstrate that for a large subset of ICVs, those with fixed premiums, the SNFL explicitly defines minimum reserves and surrender values. Therefore, the new catch-all section of the SNFL, by its own wording, does not apply to fixed premium ICVs. Of course, as Mr. Callahan correctly points out, not all designs that utilize a retrospective fund as a surrender value will comply with the SNFL. But does this mean that section 6 should be invoked in order to approve them? We think not. Instead, fixed premium ICVs should face the identical requirements and freedoms accorded participating whole life: a *prospective* minimum nonforfeiture standard (sections 2, 3, 4, 5, 5(a) 5(b), and 5(c) of the SNFL) and management control over both the BGA ("dividend fund") and BGF ("dividend formula" and "experience factors").

In Section 8 of our paper, we noted that it is more difficult to demonstrate compliance for a flexible premium ICV, but not impossible. The easiest route, other than limiting one's design to the Classical Model, is to choose a cost basis (BGA, BGF) that is "more expensive" than the policy nonforfeiture basis. Under this approach, a surrender value equal to the retrospective fund would comply with the SNFL.

The concerns that regulators have expressed about the levels of expense loadings and mortality costs cannot be justified by invoking the nonforfeiture statutes. To the contrary, the more expensive the cost basis (BGA, BGF), the less likely it is for a violation of the SNFL to occur. Although it may be counterintuitive, the cost basis will jeopardize nonforfeiture compliance only as it becomes inexpensive in relation to the nonforfeiture basis. We maintain that any regulatory limitations on the elements of the BGF, such as exists in the state of New York, not only "resemble" but are in fact rate regulations that are not legitimized by the nonforfeiture statutes.

Concerning Mr. Callahan's comments on nonforfeiture elections other than for cash, we note that the discrepancy he points out would not occur under a form that pays a surrender value equal to the minimum value calculated prospectively. In the case where the cash value equals the retrospective fund, the reduced paid-up option could be substituted for the extended term option. Also, the discrepancy would exist only where the requirement for mathematical equivalence or better exists.

Mr. Dukes is correct in his assumptions about our E2 and E4 proposals for the expense allowance. For method E1, the authors propose that the expense allowance be amortized over the maximum premium-paying period of the policy. The justification for this is largely simplicity. Realizing that the calculation and amortization of the expense allowance are an arbitrary procedure at best, the authors feel little attachment to any particular approach.

Mr. Dukes then brings up two points of contention. The first deals with the capitalization of short-term guarantees in the cash value. This is an interesting by-product of the nonforfeiture law and is worthy of greater attention than it has received in the past. Mr. Dukes finds it illogical that short-term current interest guarantees are reflected in the cash value prior to actual crediting of the interest. He states, "It does not make sense . . . to pay out benefits that have not been earned." The authors agree that this result makes little sense. However, this phenomenon is necessitated by a requirement of law. In writing this paper, we set out to discuss the application of current law to universal life, and not to comment on the appropriateness of the nonforfeiture law generally.

Although the phenomenon of which Mr. Dukes writes is unmistakably absurd, it is important to uncover precisely the defect in the nonforfeiture law that perpetrates the undesired effect. We will demonstrate by using a simple example.

Assume that a contract promises the payment of \$1.10 one year hence. Currently, such a contract might sell for \$1, reflecting a market rate of 10 percent. Now, suppose that we were to determine a cash value for this contract using the principles of our nonforfeiture laws. Such a cash value would be equal to the present value of future benefits. Using the market rate of 10 percent, the calculated cash value would be \$1, an amount equal to the purchase price. This answer has intuitive appeal and is "correct" when compared with the market-determined value of the contract. Everything fine so far. However, if we impose the further constraint that the maximum discount rate that can be used to calculate the present value of benefits is $5\frac{1}{2}$ percent, a curious result follows. The calculated cash value becomes \$1.04, an amount greater than the price actually paid for the contract. This is the result that rightly disturbs Mr. Dukes. Notice, however, that it is not the fact that guarantees are capitalized that produces the odd result, but the fact that the benefits were valued at a below-market rate of interest. With this observation in hand, we contend that the single premium funding concept of the nonforfeiture law is sound, but maximum discount rates below market can cause undesirable results.

The appropriateness of capitalizing guarantees in the cash value can be demonstrated from still another viewpoint. Consider two policies, identical in every respect except that one guarantees 12 percent interest over the next year and one does not. Which policy is worth more? Which policy would the company be willing to pay more to have lapsed? Simply ignoring the short-term guarantee denies economic reality.

Mr. Dukes's second point of contention involves the separation of the expense allowance from the premium-paying period. Justification is requested for divorcing the two items. Two comments are in order. The level of the expense allowance (in theory) should have some relationship to the expenses incurred in issuing the policy. Since most universal life first-year commissions are not based on any requirement that subsequent premiums be paid, an argument can be voiced for calculating an expense allowance independently of the guaranteed plan of insurance. It seems more appropriate to base the expense allowance on the "shell" of the policy, or what the policy *can* become. As for the period of amortization of the expense allowance, it is clear that the allowance should be amortized over a period no longer than the future profit flow. Universal life plans generate profit irrespective of the premium flow. It is not clear to us that the amortization must be within the premium payment period.

Mr. Kabele brings up several interesting topics. We will discuss those points relevant to valuation and nonforfeiture of universal life here, deferring comment on the remainder of Mr. Kabele's discussion to the end of our reply.

Mr. Kabele states that universal life policies have both primary and secondary benefits. This is generally not true. At the time of this writing, policies with secondary benefits are rare. Policies with secondary benefits are all of the fixed premium type, the secondary guarantee being a method of effecting a cumulative, rather than period-by-period, guarantee.

Although it is true that the reserve must be the greater of that for the primary and secondary benefits, the statement that "if cash values are greater than E@95 reserves, an additional reserve must be held in Exhibit 8G," is necessarily correct only in the case of the Classical Model. Reserve increases attributable to primary benefits exceeding secondary benefits should not be confused with the excess of cash values over reserves (those that appear in Exhibit 8G).

Mr. Sommer brings up an interesting point concerning fixed premium policies where the cash value exceeds the reserve. Traditionally, such amounts were held as reserves and appear in the annual statement as part of Exhibit 8, Section G. In contrast to setting up reserves for these "cash-value deficiencies" as they arise, Mr. Sommer suggests a method for the prefunding of such amounts.

Although the authors agree in principle with such an approach, reserving for cash values is certainly not a concept appearing in the Standard Valuation Law. But even though not required by law, the prefunding of cash-value deficiencies is sensible from both an actuarial and an economic point of view.

The most coherent way to effect such an approach is to borrow the methodology of the reserve technique for deferred annuities. The present value of future benefits would be the greater of that of the death and endowment benefits or the present value of the cash value at any duration. In this way the cash value is considered to be a benefit for reserve purposes.

We feel that the method proposed by Mr. Sommer is deficient in one respect: cash-value deficiencies are prefunded only at issue. Subsequent to issue, further cash-value deficiencies are not prefunded but are set up on a dollar-for-dollar basis as they arise. This funding method works well for the Classical UL Model but may not be appropriate where policy guarantees are more liberal than the reserve basis (Mr. Sommer mentions this case) or when the slope of guaranteed mortality rates is more shallow than the valuation table. In order that the method be consistent as well as logical, all cash-value deficiencies should be prefunded, not just those existing at issue. In other words, at every duration the present value of future benefits should take into consideration the scale of cash values guaranteed at that time.

In regard to flexible premium policies, Mr. Sommer outlines a method he terms the "arbitrary premium interpretation" of the law. The claim is made that this approach removes the references to prospective or retrospective calculations. In fact, the method described relies on a prospective proof by virtue of the fact that once an arbitrary pattern of premiums is chosen, a CRVM reserve is calculated according to the SVL, based on the benefits produced by that premium pattern.

Mr. Sommer rightly points out that this proof can be made only for a plan that conforms to the constraints of the Classical UL Model, and recommends method 3 for other plans. This is also the authors' favored interpretation. We have had a good deal of success with such demonstrations for quite some time.

Mr. Sommer concludes by questioning the validity of using an expense allowance while assuming no future premiums. We do not feel that there is a necessary tie between future premiums and the amortization of the expense allowance. The expense allowance is an artificial construct designed to produce reasonable levels of surrender values. Amortizing such an allowance over future profit flow rather than premium flow seems to be a reasonable alternative.

Mr. Kabele criticizes universal life for the policyholder's ability to make premium payments while in a poor state of health. The criticism that mortality rates in the future might be increased to reflect antiselection from unhealthy policyholders making withdrawals seems rather weak. One of the major advantages of universal life is its deterrence of antiselection, of both the financial and the health variety.

The "consumer considerations" mentioned by Mr. Kabele merit comment. First, the authors are unaware of any universal life policy that denies premium payments when the policyholder is in danger of lapsation. Second, Mr. Kabele alleges that most of a universal life policy's cash value may be "eaten up" by the artificially high 1958 CSO mortality rates. With universal life this is even less true than with traditional products. A typical basic whole life cash value accumulates using exactly 1958 CSO mortality rates. Universal life policies, however, typically have guaranteed rates less than 1958 CSO at the extreme upper ages. Although it is possible to design a policy where mortality rates at the upper ages are greater than 1958 CSO, Mr. Kabele's claim on this point is without merit for products currently on the market. Third, Mr. Kabele criticizes the fact that "excess interest" can increase the cash value but not the current face amount. He makes the point that the excess interest is forfeited upon death. The authors fail to be persuaded by this argument, since similar reasoning can be applied to any traditional plan of insurance which is not pure term. The purchaser of whole life certainly forfeits any prefunding of future death benefits in comparison to a term policyholder, given death in the first year of the policy. From this it can hardly be said that insurance plans with a prefunding element are detrimental to the consumer. The fourth consumer consideration referenced in the discussion is the lack of the "typical 10 percent profit limitation" imposed by two or three states on participating business written in a stock company. The authors are compelled to disagree with Mr. Kabele. Statutory profit limitations are rarely a consumer benefit in the long run. Profit levels are determined by the forces of competition and consumer demand elasticity. Any legal profit limitation can only have one of two effects:

1. If the statutory limitation is smaller than the natural profit level determined by market forces, then in the long run the regulated product will cease to be sold, since precious capital will gravitate toward larger potential profits.
2. If the statutory limitation is larger than the natural profit level determined by market forces, the limitation will have no effect.

From this it is evident that profit limitations are of limited consumer value. In addition, because of the intense competition in the universal life marketplace, profit levels are generally far below the 10 percent limitation referenced by Mr. Kabele. To make a statement such as, "Therefore, the company may make more profits and give policyholders less" is a denial of economic reality.

Mr. Kabele goes on to criticize premium payments and interest credits that do not increase the net amount at risk. The authors are confused by this concern with the net amount at risk. We wonder whether Mr. Kabele wishes to prohibit term conversions to whole life, since the premium payments increase while the net amount at risk drops.