Claims Reserving When There Are Negative Values in the Runoff Triangle: Bayesian analysis using the three-parameter log-normal distribution

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	1	2	3	4	5	6	7	8	9
1	33250.717	2097.059	78.897	21.117	-18.654	-0.121	-5.072	-1.292	-0.775
2	36717.578	2583.632	-34.240	19.080	10.120	-3.699	-2.492	1.259	
3	38155.786	2705.212	38.503	-0.247	6.442	-6.669	-9.525		
4	36180.233	2601.743	21.501	-8.662	-6.250	12.865			
5	35980.821	2892.427	52.478	10.982	-3.496				
6	37518.185	2901.650	-23.612	-39.496					
7	40213.152	3006.438	-14.591						
8	39105.807	3080.126							
9	41184.755								

					-11.84		-17.09	-0.033	-0.775
		1.0736	1.0004	1.0000	0.9999	1.0000	0.9999	0.9999	0.9999

Negative incremental values:

- result of salvage recoveries
- payments from third parties
- total or partial cancellation of outstanding claims
- initial over-estimation of the loss
- jury decision
- errors

Problem more with the data than with the methods

Correct the data before applying claims reserving methods

Z_{it} = incremental number (or amount) of claims in the *t-th* development year corresponding to year of origin (or accident year) *i*.

Do not need to assume $Z_{it} > 0$ for all $i = 1,\ldots,k$ and $t = 1,\ldots,s$.

We know the values Z_{it} for $i + t \le k + 1$

Presented in the form of a run-off triangle

$\{\lambda_j : j = 2,\ldots,n\}$ = typical chain-ladder development factors

$$\hat{\lambda}_j = \frac{\sum_{i=1}^{n-j+1} \sum_{t=1}^{j} Z_{it}}{\sum_{i=1}^{n-j+1} \sum_{t=1}^{j-1} Z_{it}}$$

Methods that can handle negative values

Chain Ladder

Over-dispersed Poisson GLM (Stochastic Chain Ladder)

$m_{ij} = E(Z_{ij}),\ V(Z_{ij}) = \phi m_{ij}$ scale parameter , $\phi > 0$

log-link function $log(m_{ij}) = \mu + \alpha_i + \beta_j$

$\hat{\mu}, \hat{\alpha}_i, \hat{\beta}_j$ obtained by ‘quasi-likelihood’

positivity constraints (no negative sums in columns)

Over-dispersed negative binomial GLM

mean $(\lambda_j - 1)W_{i,j-1}$

variance $\phi\lambda_j(\lambda_j - 1)W_{i,j-1}$, $W_{ij} = \sum_{k=1}^{j} Z_{ik}$ where

Normal approximation

$$Y_{it} = log(Z_{it} + \delta) = \mu + \alpha_i + \beta_t + \varepsilon_{ij} \qquad \varepsilon_{ij} \sim N(0,\sigma$$

$i=1,...,k,$ $t=1,...,k$ and $i+t \leq k+1$ so that

with $\mu_{it} = \mu + \alpha_i + \beta_t$

Z_{it} has a three parameter log-normal

$$f(Z_{it} \mid \mu, \alpha_i, \beta_t, \sigma^2, \delta) = \frac{1}{\sigma(z_{it} + \delta)\sqrt{2\pi}} exp[-\frac{1}{2\sigma^2}(log(z_{it} + \delta) - \mu - \alpha_i - \beta_t)^2]$$

“threshold” parameter $\delta > 0$ corrects the values so $(z_{it} + \delta) > 0$,

for $i,t=1,...,k,$ with $i+t \leq k+1$

Likelihood Function

$\underline{z} = \{ z_{it} ; i, t = 1, ..., k \ , i + t \leq k + 1$ be a T_U -dimension vector

$\underline{\theta} = (\mu, \alpha_2, ..., \alpha_k, \beta_2, ..., \beta_k)'$ is the vector of parameters

$$f(\underline{z} \mid \underline{\theta}, \sigma^2, \delta) = \frac{\sigma^{-T_U} (2\pi)^{-T/2}}{\prod (z_{it} + \delta)} exp[-\frac{1}{2\sigma^2} \sum_i \sum_t (log(z_{it} + \delta) - \mu - \alpha_i - \beta_t)^2]$$

Prior distribution

$$f(\underline{\theta}, \sigma^2, \delta) = f(\underline{\theta}) f(\sigma^2) f(\delta) = f(\mu) \times [\prod_{i=2}^{k} f(\alpha_i)][\prod_{t=2}^{k} f(\beta_t)] \times f(\sigma^2) \times f(\delta)$$

Posterior distribution

$$f(\underline{\theta}, \sigma^2, \delta \mid \underline{z}) \propto f(\underline{z} \mid \underline{\theta}, \sigma^2, \delta) f(\mu) \times [\prod_{i=2}^{k} f(\alpha_i)][\prod_{t=2}^{k} f(\beta_t)] \times f(\sigma^2) \times f(\delta)$$

PRIOR DISTRIBUTIONS

$$\mu \sim N(\mu_0, \sigma_0^{\,2}) \qquad \alpha_i \sim N(\mu_\alpha, \sigma_\alpha^{\,2}) \qquad \beta_t \sim N(\mu_\beta, \sigma_\beta^{\,2})$$

$$f(\sigma^2) = \frac{\lambda^v}{\Gamma(v)} (\sigma^2)^{-(v+1)} exp\{-\lambda / \sigma^2\}, \qquad \sigma^2 > 0,$$

Pareto prior for δ

$$f(\delta) = ac^a \delta^{-(a+1)}, \quad c > 0 \quad a > 0 \quad \delta \geq c,$$

where $c = -z_{(1)}$ and $z_{(1)} = min\{z_{it}; i,t = 1,\ldots,k\ , i+t \leq k+1\}$

ESTIMATING THE RESERVES

$$f(\underline{\theta},\sigma^2,\delta|\underline{z})\propto \frac{\sigma^{-T_U}}{\prod(z_{it}+\delta)} exp[-\frac{1}{2\sigma^2}\sum_i\sum_t(log(z_{it}+\delta)-\mu-\alpha_i-\beta_t)^2]\times f(\mu)\times[\prod_{i=2}^{k}f(\alpha_i)]$$

$$\times[\prod_{t=2}^{k}f(\beta_t)]\times f(\sigma^2)\times f(\delta)$$

In loss reserving: predict the observations in the lower triangle**,** z_{it}

$$f(z_{it}\mid\underline{z})=\int f(z_{it}\mid\underline{\theta},\sigma^2,\delta)f(\underline{\theta},\sigma^2,\delta\mid\underline{z})d\underline{\theta}d\sigma^2 d\delta$$

$$i=1,\ldots,k,\ \ t=1,\ldots,k,\ \ with\quad i+t>k+1$$

Bayes estimate of outstanding claims for year of business *i* =

$$\sum_{t>k-i+1}E(Z_{it}\mid\underline{z})$$

IMPLEMENTING MCMC IN BUGS

Stage 1:

$Y_{it} = log(Z_{it} + \delta) \sim N(\mu_{it}, \sigma^2)$

$\mu_{it} = \mu + \alpha_i + \beta_j$

$\alpha_1 = \beta_1 = 0$

$i = 1,\ldots,k, \quad j = 1,\ldots,k, \quad y \quad i + j \leq k + 1$

$k = 9$

Stage 2:

$\mu \sim N(0, \sigma^2_{\mu})$

$\alpha_i \sim N(0, \sigma^2_{\alpha_i})$

$\beta_t \sim N(0, \sigma^2_{\beta_t})$

$\sigma^2 \sim GI(\nu, \lambda)$

$\delta \sim Par(a, 3.95)$

Stage 3:

$\sigma^2_{\mu} \sim GI(0.1, 0.1)$

$\sigma^2_{\alpha_i} \sim GI(0.001, 0.001)$

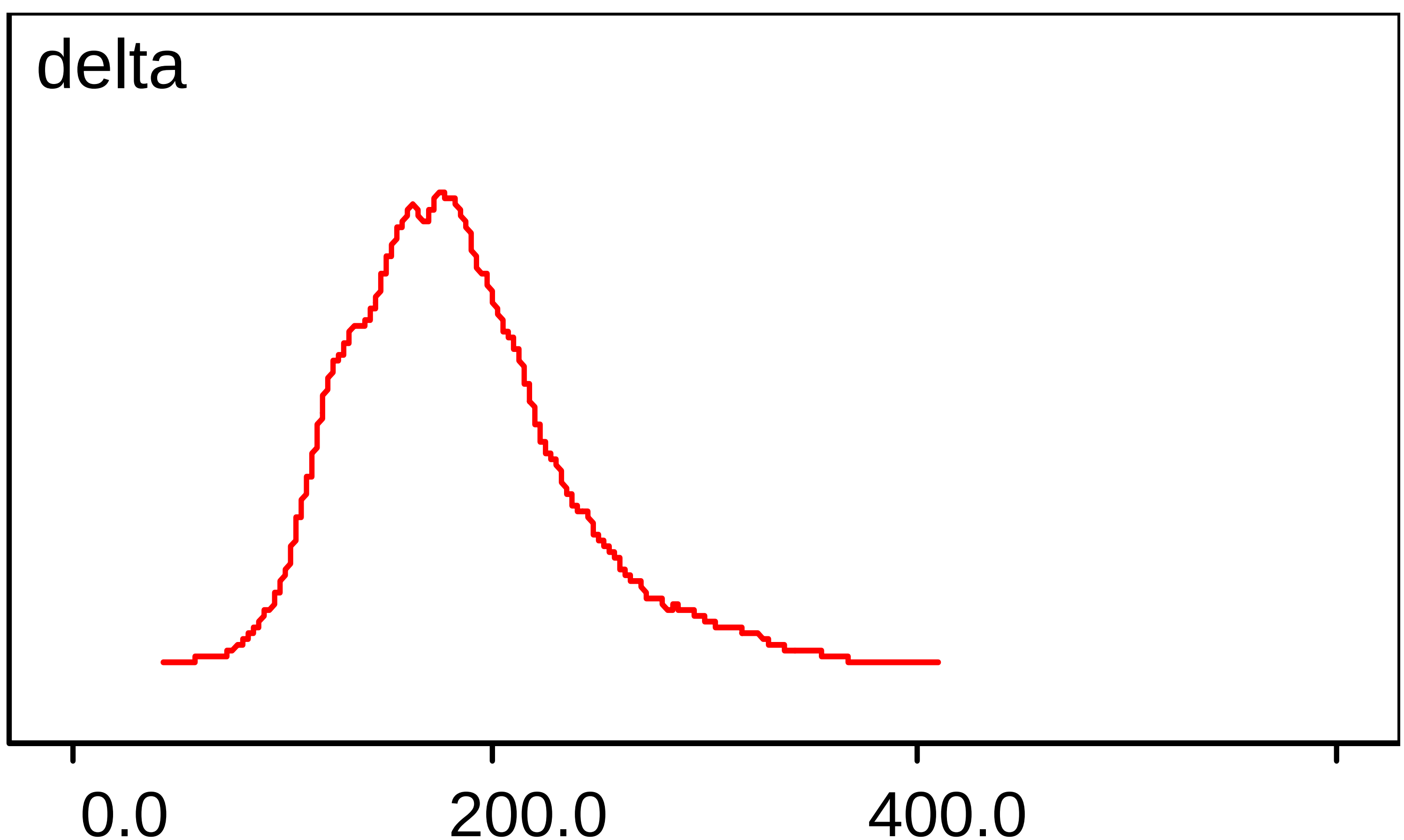
$\sigma^2_{\beta_j} \sim GI(0.001, 0.001)$

$\nu \sim G(2.5, 0.001)$

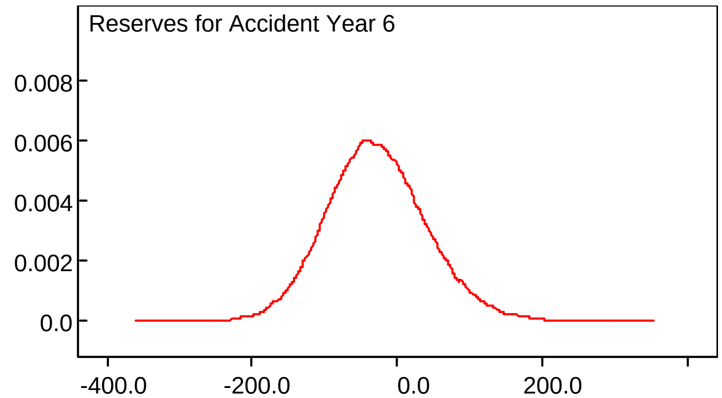
$\lambda \sim G(2, 0.1)$

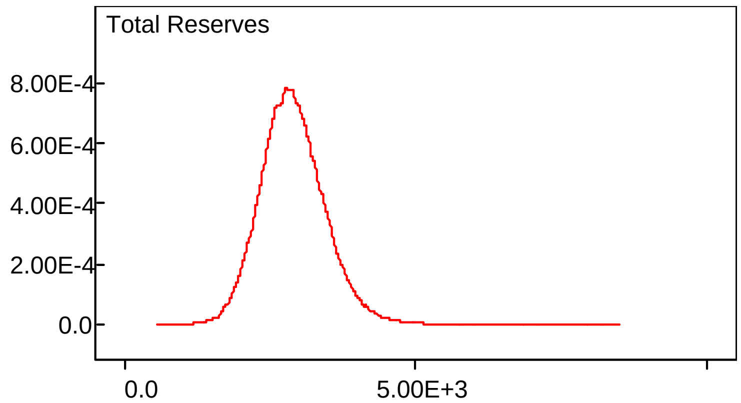
$a \sim G(0.001, 0.001)$

Parameter	Mean	Std. Dev.	Percentiles		
			2.50%	Median	97.50%
δ	182.0	50.0	102.5	176.6	302.3
μ	10.53	.0460	10.44	10.53	10.62
σ^2	.0167	.0091	.0068	.0145	.0397


delta
0.0
200.0
400.0

Year	Chain-Ladder	OD Poisson	Bayesian		Bayesian MCMC	
	Reserves	Reserves	Reserves	Std. Dev.	Reserves	Std. Dev.
2	-0.860	1.000	1.381	32.246	0.3587	32.22
3	-0.912	2.000	21.348	47.190	9.383	44.54
4	-6.601	3.000	5.810	57.877	1.187	52.91
5	-6.024	7.000	64.140	73.000	22.20	63.51
6	-8.715	-12.000	-54.890	81.640	-27.65	69.47
7	-8.817	22.000	77.690	107.550	7.945	79.73
8	9.513	48.000	244.080	148.150	49.17	94.72
9	3041.181	3085.000	3363.200	514.500	2835.0	468.8
TOTAL	3018.766	3182.000	3722.700	620.500	2897.0	545.3


Reserves for Accident Year 6
0.008
0.006
0.004
0.002
0.0
-400.0
-200.0
0.0
200.0


Total Reserves
8.00E-4
6.00E-4
4.00E-4
2.00E-4
0.0
0.0
5.00E+3