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Mis-Estimation Risk

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Actuaries often have to derive a mortality basis from the experience data of a portfolio. The most common application is for risk management, such as the annual valuation. However, it is also required for pricing block transfers, such as longevity swaps, reinsurance treaties and bulk annuities. In each case it is useful to know two things: (i) what uncertainty surrounds the mortality basis, and (ii) what financial impact that uncertainty has. Both of these questions come under the heading of mis-estimation risk, which is the subject of this article.

EXAMPLE SCENARIO

A U.K. pension scheme is considering a longevity swap. The scheme and insurer have agreed a basis for future mortality improvements, but both parties have to decide on a basis for current mortality rates. Furthermore, both parties want to understand the mis-estimation risk surrounding the basis, and thus the potential financial impact. The scheme has n=14,802 living pensioners and also has 2,265 records for past deaths observed over the period 2007–2012.

The two parties have slightly different rationales in wanting to understand the mis-estimation risk. The scheme wants to know the financial impact to judge if it is worth paying the insurer's premium to remove the risk. In contrast, the insurer wants to know if its pricing margin covers the risk of mis-estimation based on the scheme's recent experience. In particular, the insurer (or reinsurer) will have to hold regulatory capital for mis-estimation risk if the longevity swap is agreed.

A full assessment of a longevity swap will require other work, such as an assessment of the idiosyncratic risk through the simulation of the lifetimes of the individual lives. Such simulations presuppose that we know what the underlying risk factors are for each individual. However, we do not in fact know these risk factors precisely, as we can only estimate based on limited data. The mis-estimation assessment puts a financial value on this uncertainty.

MODELING CURRENT MORTALITY

There are many ways to analyze mortality, but one of the better approaches is to use survival models for individual lives. This involves a parametric model for the force of mortality, which makes the best use of all available information. The model fitted here is the time-varying version of the Makeham-Perks law:

$$\mu_{x,y} = \frac{e^{\epsilon} + e^{\alpha+\beta x+\delta(y-2000)}}{1 + e^{\alpha+\beta x+\delta(y-2000)}}$$

where $\mu_{x,y}$ is the force of mortality at age x and calendar time y. The offset of -2000 to the calendar time keeps the other parameters well scaled. Parameters α, β, δ, and ϵ are estimated by the method of maximum likelihood. At a very simple level we can allow for the fact that not all individuals are identical by giving each person their own personal value of α, α_i, defined as follows:

$$\begin{aligned}\alpha_i = \alpha_0 \\ &+ \alpha_{\text{Male}} z_{i,\text{Male}} \\ &+ \alpha_{\text{Mid-size pension}} z_{i,\text{Mid-size pension}} \\ &+ \alpha_{\text{Large pension}} z_{i,\text{Large pension}}\end{aligned}$$

where, for example, α_{Male} is the change in mortality from being male and $z_{i,\text{Male}}$ is an indicator variable taking the value 1 when life i is male and 0 otherwise. The other parameters and indicator variables are defined similarly. The model is fitted to the scheme's data and the resulting parameter estimates are shown in Table 1.

Table 1. Parameter estimates for minimally acceptable model for financial purposes.
Source: Richards (2014).

Parameter	Estimate	Standard error	Significance
Age (ß)	0.148	0.005	***
Gender.M (α_{Male})	0.479	0.060	***
Intercept (α_0)	-14.731	0.491	***
Makeham (ϵ)	-5.420	0.154	***
Mid-size pension (α Mid-size pension)	-0.180	0.078	*
Large pension (α Large pension)	-0.313	0.108	**
Time (δ)	-0.046	0.016	**

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CORRELATIONS AND CONCENTRATION OF RISK

The parameter estimates in Table 1 are shown with their standard errors. In a sense these standard errors are the beginning of understanding mis-estimation, as they tell us the degree of confidence we can have in each parameter estimate. For example, the estimate of the age parameter is 0.148 and an approximate 95 percent confidence interval for the true underlying value is (0.138, 0.158). At a superficial level, therefore, one might think that the standard errors are all we would need to assess mis-estimation. However, with all statistical models there are usually correlations between the parameters. Some of these correlations can be quite material, as shown in Table 2, and they must be taken into account when assessing mis-estimation risk.

Table 2. Percentage correlations between the estimates in Table 1. Source: Richards (2014).

Parameter	Age	Gender.M	Intercept	Makeham	Mid-size pension	Large pension	Time
Age (ß)	100%						
Gender.M (α_{Male})	23%	100%					
Intercept α_0	-94%	-26%	100%				
Makeham ϵ	72%	17%	-70%	100%			
Mid-size pension ($\alpha_{\text{mid-size pension}}$)	-7%	-17%	-70%	100%			
Large pension ($\alpha_{\text{large pension}}$)	-2%	-19%	2%	-2%	13%	100%	
Time δ	-2%	0%	-32%	-1%	-1%	0%	100%

Note that each parameter is perfectly correlated with itself, hence the leading diagonal is composed of 100 percent values. Also, the table is symmetric about the leading diagonal, so only the lower left values are shown.

The other aspect of mis-estimation risk is that it doesn't affect all lives equally, and that not all lives are of equal financial impact. For example, the large-pension cases account for the top 10 percent of lives, but they account for 39.8 percent of the total scheme pension. Table 1 shows that such cases have markedly lower mortality, but the standard error shows that there is relatively greater uncertainty over just how much lower. Furthermore, Table 2 shows that there is a correlation of -19 percent between the parameters for large-pension cases and males, so it is not sufficient to stress any one parameter in isolation.

QUANTIFYING THE RISK

If parameters are correlated to varying degrees, how can we perform a mis-estimation assessment? We cannot simply stress each parameter by a multiple of its standard error, as this ignores correlations. This is illustrated in Figure 1 (on page 11) for a simple Gompertz model with $\mu_x = e^{\alpha_0+\beta x}$. If we stress the value of α_0 downwards, the best estimate of β increases, as shown by the black line in Figure 1.

Our solution is to use the whole variance-covariance matrix to generate consistent alternative parameter groups. This not only allows for the uncertainty over the parameters themselves, but it also allows for their correlations. There is also the question of how to allow for the fact that individual liabilities are impacted to different extents. Our solution is to value the entire portfolio life-by-life with each alternative parameter set. We repeat this *m* times to generate a set, *S*, of alternative portfolio valuations. *S* describes the financial impact of parameter risk and parameter correlations, while allowing for all individual characteristics and concentrations of liability. The percentiles of *S* can be used to investigate the financial impact of mis-estimation risk, say by comparing the excess of a given percentile to the median.

RESULTS

For the pension scheme in question, we generated *m=10,000* sets of alternative parameter values with the covariance matrix. In each case we valued the in-force liabilities with each parameter set. The 99.5th percentile of *S* was 3.97 percent higher than the median (the median of *S* was very close to the mean). This compares loosely to a typical insurer pricing margin of around 4–5 percent. Of course, there are other sources of uncertainty to be considered and a fuller list is given in Richards (2014). The final price also has to include insurer expenses and the costs of capital.

It is also possible to express mis-estimation results as a percentage of a standard table using the equivalent-annuity calculation. For this portfolio the equivalent best-estimate percentages of S2PA were 88.5 percent for males and 87.2 percent for females. Using the appropriate percentiles of we can use the mis-estimation assessment to find a 95 percent confidence interval for these percentages. For males we get (78.7 percent, 99.5 percent) and for females we have (79.3 percent, 96.1 percent). The width of these intervals reflects the modest size of the scheme and the concentration of risk in a relatively small subset of lives. A larger portfolio would likely have a narrower confidence interval.

CONCLUSIONS

There are many potential risk factors which affect a demographic risk like mortality and the effect of these risk factors can be estimated using a parametric statistical model. The parameters in such a model have both uncertainty around their estimates and correlations with each other. Using the variance-covariance matrix for the estimated parameters, the mis-estimation risk for a portfolio can be straightforwardly assessed using the portfolio's own experience data. ■

REFERENCES

RICHARDS, S. J. (2014) Mis-estimation risk: measurement and impact, Longevitas working paper.

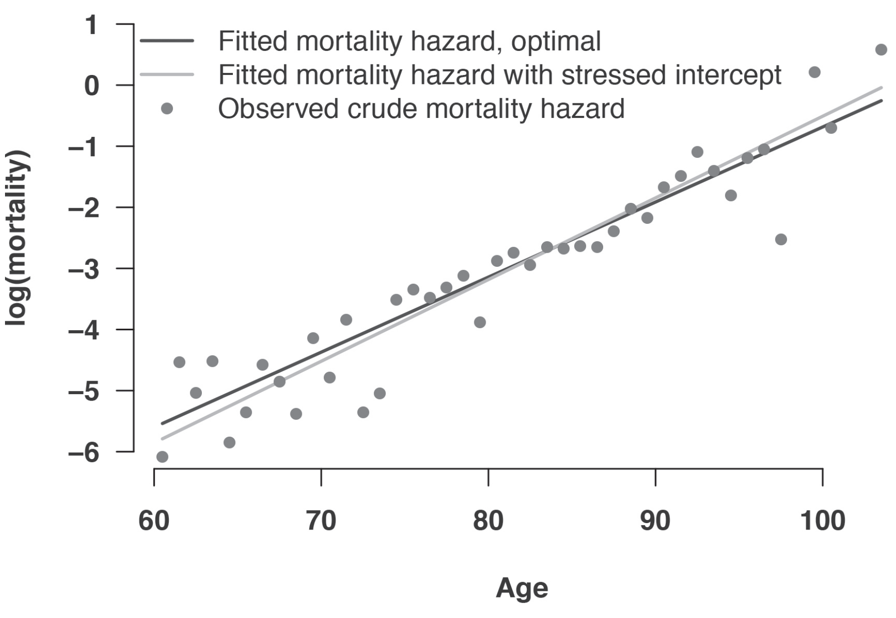

Figure 1. log(mortality) with best-estimate fit (black) and alternative fit with stressed intercept (grey).

Source: Richards (2014).