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EFFECT OF DEFERRED RETIREMENT ON PENSION PLAN LIABILITIES AND COST

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ABSTRACT

In the United States and in Canada, certain legislative developments have been concerned with providing retirement benefit credits to employees who defer retirement. The purpose of this paper is to provide a practical guide for the actuary for estimating the effect that deferred retirement has on the cost of a pension plan.

I. INTRODUCTION

In the United States, the Age Discrimination in Employment Act (ADEA) addressed the issue of providing additional retirement benefit credits for employees who continue to work past age 65. Under that law, as currently (1982) interpreted by the Equal Employment Opportunity Commission and the Department of Labor, credits for post-age-65 service need not be granted in defined benefit pension plans. Nevertheless, many plans do grant credit for post-65 service by recognizing additional credited service, salary earned after age 65, or an actuarial increase to reflect the deferred commencement of benefits.

In Canada, legislation effective April 1, 1982, in the Province of Quebec effectively prohibited mandatory retirement. Although an employee can draw a portion of his benefit from normal retirement date in order to compensate for any reduction in earnings, in most cases employees taking advantage of the law will defer their whole pension to actual retirement date or age 71 if earlier (when the federal tax authorities require that the pension commence). The Quebec law specifically considers the effect of postponed retirement and requires that the pension be "revalorized." It is still too early to tell what "revalorization" formula will be adopted, and which ones will be acceptable to the Quebec Pension Board.

Several common types of pension plan formulas are studied in the paper. The cost effect of alternative ways of recognizing post-65 service is derived. Alternative sets of actuarial assumptions are used in order to ana-

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lyze the effect that changes in assumptions have on the costs developed. Tables are included to illustrate the basic results. Examples are included to show how the basic tables are used in estimating the cost effect of deferred retirement for specific situations.

II. PLAN INFORMATION, ASSUMPTIONS, AND INDICES STUDIED

A. *Benefit Formulas*

1. *Final-pay formula.*—The formula used here is 1 percent times final-five-year average pay times years of service.
2. *Career-pay formula.*—The benefit accrued each year is 1 percent of that year's pay.
3. *Non-pay-related benefit formula.*—The results illustrate the formula one dollar for each year of service.
4. *Final-pay step-rate integrated formula.*—The formula here was chosen to represent a final-pay integrated plan that is not quite integrated to the maximum but that uses social security–covered compensation as the breakpoint. The formula is 1 percent times final-five-year average pay up to covered compensation plus 1½ percent times final pay in excess of covered compensation, all times years of service. The formula includes a test that provides that the accrued benefit will not decrease over time.
5. *Final-pay social security offset formula.*—The formula is 1 percent times final-five-year average pay less 1 percent times primary social security benefit, all times years of service. The social security benefit payable is developed assuming that age 65 occurs in 1981.

Results for the first three formulas can be generalized to other formulas of the same type. For example, the percentage changes in cost for 2 percent of final pay would be the same as for 1 percent of final pay. Assuming a constant salary scale, percentage results for a final-ten-year average pay plan would be the same as for a final-five-year average pay plan.

Results for the last two formulas are more difficult to generalize. Each formula may be thought of as being composed of integrated and nonintegrated portions. For each formula, the nonintegrated portion is 1 percent of final-five-year average pay times years of service and behaves identically with the nonintegrated final-pay formula. These results must be combined with the results for the integrated portion of the formula, with the variation from the nonintegrated results dependent on the levels of integration and salary. The nature of this relationship is described more fully in Section III, A.

B. *Methods of Crediting Benefits beyond Age 65*

1. No credit beyond age 65.
2. Recognition for salary increases only.
3. Credit for service only.
4. Both salary recognition and service credit.
5. Actuarial increase of the age 65 benefit.
6. Flat percentage increase of the age 65 benefit (the study uses 3 percent, and the increase is not compounded—e.g., the benefit at age 70 is 15 percent higher than the benefit at age 65).
7. Salary recognition and service credit together with an actuarial increase for the deferred commencement.

C. *Actuarial Assumptions*

UP-1984 mortality with ages set forward one year was used (conversion factors for female rates are given in Sec. III, E).

Interest rate and salary-scale combinations of 6 percent/4 percent, 8 percent/4 percent, and 8 percent/7 percent were used. These combinations will allow the reader to analyze the effect of changes in interest rate only or salary scale only.

Social security benefits are computed for age 65 in 1981, and increases in the Consumer Price Index and in the wage base are assumed at the same rate as stated in the salary-scale assumption. Covered compensation for deferral of retirement from age 65 in 1981 is the same as covered compensation for someone reaching age 65 in that year, assuming that increases in the 1981 wage base of $29,700 are at the stated salary scale. For these calculations each additional year's wage base was rounded to the nearest multiple of $300.

D. *Cost Measurement Indices*

The indices chosen for this paper to measure the cost effect of recognizing post-age-65 service in the pension computation are based on the assumption that age 65 is the plan's normal retirement age. Three measurements of the cost effect are developed.

1. ACTUARIAL PRESENT VALUE RATIO

The ratio of the actuarial present value of benefits under deferred retirement to the actuarial present value of benefits assuming retirement at 65, with all actuarial present values stated as of age 65.

Define BEN_x as the benefit payable if retirement occurs at age x. APV_x is the actuarial present value of benefits measured at age x. Let Δ be a symbol for ratio. Then the actuarial present value ratio is the following:

$$APV\Delta = \frac{APV_{65}(BEN_{65+n})}{APV_{65}(BEN_{65})} = \frac{BEN_{65+n}}{BEN_{65}} \frac{N^{(12)}_{65+n}}{N^{(12)}_{65}} .$$

It should be noted that the first factor reflects the change in the benefit amount, and the second factor, which is the reciprocal of an actuarially equivalent deferred retirement factor, reflects the delayed commencement of benefit payment.

2. ENTRY-AGE NORMAL COST RATIO

The ratio of the entry-age normal cost for the deferred retirement benefit to that for the normal age 65 benefit.

Similarly, we can develop the formulas for the two normal cost ratios. Under the entry-age normal actuarial cost method, the normal cost is equal to the actuarial present value of benefits at entry age divided by the actuarial present value of compensation at entry age. Letting e denote the entry age and using a superscript s to reflect the use of a salary scale, the entry-age normal cost factor ($EANC$) for an age 65 retirement is

$$EANC_{65} = \frac{BEN_{65}N^{(12)}_{65}}{D_e} \div \frac{{}^sN_e - {}^sN_{65}}{{}^sD_e} .$$

For a retirement at age $65 + n$, we have

$$EANC_{65+n} = \frac{BEN_{65+n}N^{(12)}_{65+n}}{D_e} \div \frac{{}^sN_n - {}^sN_{65+n}}{{}^sD_e} .$$

$EAN\Delta$, the ratio of these two normal cost factors, is then

$$EAN\Delta = \left(BEN_{65+n} \frac{N^{(12)}_{65+n}}{D_e} \div \frac{{}^sN_e - {}^sN_{65+n}}{{}^sD_e}\right) \Bigg/ \left(BEN_{65} \frac{N^{(12)}_{65}}{D_e} \div \frac{{}^sN_e - {}^sN_{65}}{{}^sD_e}\right)$$

$$= \frac{BEN_{65+n}}{BEN_{65}} \frac{N^{(12)}_{65+n}}{N^{(12)}_{65}} \frac{{}^sN_e - {}^sN_{65}}{{}^sN_e - {}^sN_{65+n}} .$$

This equation shows that the change in entry-age normal cost is the product of three factors. The first reflects the change in benefit amount, the second the delay in commencement of benefits, and the third the increase in compensation and the period over which the normal costs are spread. The first two factors of the equation are equal to $APV\Delta$. Therefore,

$$EAN\Delta = APV\Delta \frac{{}^{s}N_{e} - {}^{s}N_{65}}{{}^{s}N_{e} - {}^{s}N_{65+n}} .$$

Consequently, the change in entry-age normal cost can be viewed as being equal to the change in the actuarial present value ratio adjusted for the change in the funding period.

3. PROJECTED UNIT CREDIT NORMAL COST RATIO

The ratio of the projected unit credit normal cost for the deferred retirement benefit to that for the normal age 65 benefit. (Benefit allocations for the projected unit credit computations are based on a proration by service.)

Under the projected unit credit method, the normal cost ($PUCNC$) is equal to the cost of a pro rata share of the prospective benefit. Therefore,

$$PUCNC_{65} = \frac{BEN_{65}}{65 - e} \frac{N_{65}^{(12)}}{D_{z}} ,$$

where z is the attained age. Similarly, the normal cost under this method, assuming retirement at age $65 + n$, is

$$PUCNC_{65+n} = \frac{BEN_{65+n}}{65 + n - e} \frac{N_{65+n}^{(12)}}{D_{z}} .$$

$PUC\Delta$, the ratio of these two normal costs, is then determined as shown below:

$$PUC\Delta = \frac{BEN_{65+n}}{BEN_{65}} \frac{N_{65+n}^{(12)}}{N_{65}^{(12)}} \frac{65 - e}{65 + n - e} .$$

This is similar to the formula for $EAN\Delta$, but the adjustment term for the funding period reflects the difference in the funding method. Here again we can replace the first two factors by $APV\Delta$, which results in

$$PUC\Delta = APV\Delta \frac{65 - e}{65 + n - e},$$

showing that the change in the projected unit credit normal cost rate is equal to the change in the actuarial present value ratio adjusted for the change in the funding period.

III. STUDY RESULTS

A. *Actuarial Present Value Ratio—No Variation by Plan*

The *APV* ratio is a way of measuring the change in the actuarial present value of benefits at age 65. It will, in effect, measure the actuarial gain or loss produced by a deferred retirement when age 65 is the assumed retirement age.

When examining the effect of deferred retirement under the various approaches, it is convenient to first separate out those post-65 benefit adjustments for which the underlying plan is not a variable, that is, those adjustments producing the same change regardless of the plan the adjustments are imposed upon.

Besides producing results that do not vary by plan, these adjustments produce results that do not vary by salary either. Thus, the only relevant variables are assumed interest rate, age at retirement, and, in one instance, service at retirement.

By using ratios, Tables 1 and 2 demonstrate changes in the actuarial present value of benefits for these adjustments under respective 6 percent and 8 percent interest assumptions.

1. NO CREDIT (COL. 3)

This column demonstrates the extent of the actuarial gain produced by a postponed retirement when no adjustment is made to the benefit otherwise payable at age 65. The ratio is, of course, independent of plan, salary, and years of service. It merely represents the reduction in annuity cost on account of age at commencement, further reduced for assumed mortality and interest earned during the post-65 deferral period.

In this case the *APV* ratio is, in effect, the inverse of a deferred retirement factor. The figure is essentially similar in construction and interpretation to the inverse of an early retirement factor, but is applicable to the postponed retirement situation rather than early retirement.

2. SERVICE ONLY (COL. 4)

This column demonstrates the extent of actuarial gain produced when an employee's benefit is frozen at the age 65 benefit level but is prorated upward as a credit for the additional years of service after age 65. For example, an employee with fifteen years of service at age 65 and an earned benefit of \$100 per month at that age would receive (16 ÷ 15) × \$100 at age 66, (17 ÷ 15) × \$100 at age 67, and so forth. Other ages and benefits are calculated in a similar manner.

A service-only adjustment is not truly independent of the underlying plan in any plan in which benefit accrual is not directly proportional to service. For example, any front-loaded plan provides larger benefit accruals for early years of service than for later years. In this situation an adjustment of the type described in this section would overstate the benefit increase that a service-only credit based on the plan's benefit formula would provide. In like fashion, this approach would distort the benefit produced by added years of service under a back-loaded plan, in this case understating the benefit increase.

It can be seen that the values obtained in column 4 may be obtained directly from columns 2 and 3 by the simple equation

$$\text{Column 4} = \text{Column 3} \times \frac{\text{Service at age 65} + n}{\text{Service at age 65}} ,$$

where n is the number of years elapsed between age 65 and actual retirement.

3. ACTUARIAL INCREASE (COL. 5)

This column demonstrates that there is no actuarial gain or loss if the benefit earned at age 65 is adjusted upward to credit the retiring employee with assumed interest earnings during the period of postponement and is further increased to recognize the higher age and therefore shorter payment period at benefit commencement.

It is, of course, important to remember that the benefit has been adjusted for neither salary increases nor further service after age 65.

The relationship between columns 3 and 5 may be interesting. The ratio,

$$\frac{\text{Column 5}}{\text{Column 3}} = \frac{1}{\text{Column 3}} ,$$

may be considered the "actuarially equivalent late retirement factor." It is analogous to the actuarially equivalent early retirement factor applied to an age 65 benefit that commences prior to age 65.

4. THREE PERCENT ANNUAL INCREASE (COL. 6)

This column represents the actuarial gain resulting from a level annual percentage increase in benefits during the deferral period. Retiring employees are credited with an additional 3 percent of the benefit available at age 65 for each year retirement is postponed. There is no compounding of benefits in the cited example.

As with earlier examples, the change in the present value of benefits at age 65 if this adjustment is made may be simply calculated from the "no change" example (col. 3). It is given by

$$\text{Column 6} = \text{Column 3} \times [1.0 + (0.03 \times n)] .$$

5. OTHER ADJUSTMENTS

A basic pattern established above is that the extent of actuarial gain or loss due to a postponed retirement may be calculated as the ratio of the adjusted benefit divided by the normal benefit, all multiplied by the actuarial gain in the event there is no adjustment. Recognition of this pattern is useful in that it allows a simple estimation of the actuarial gain produced by more complicated forms of adjustment.

To produce the estimate, it is necessary first to determine the "no credit" or column 3 factors that are consistent with the plan's valuation assumptions. The second stage is to determine the approximate ratio of the benefit payable after adjustment to the benefit at 65. The product of these two factors is the estimated actuarial gain of the adjustment under study.

6. EFFECT OF INTEREST ASSUMPTION

A comparison of Tables 1 and 2 demonstrates that for a given benefit adjustment, a higher interest assumption will produce a greater actuarial gain. This is evidenced by the lower ratio of actuarial present value of benefits at age 65 for any particular adjustment.

The major factor in this increased actuarial gain is the additional investment income that the higher interest rate is presumed to generate.

TABLE 1

RATIO OF (*a*) ACTUARIAL PRESENT VALUE AT AGE 65 OF POSTPONED RETIREMENT BENEFIT TO (*b*) ACTUARIAL PRESENT VALUE AT AGE 65 OF NORMAL RETIREMENT BENEFIT ($=APV\Delta$)

ASSUMED 6 PERCENT INTEREST RATE

Age at Retirement (1)	Service at Age 65 (2)	No Credit (3)	Service Credit (4)	Actuarial Increase (5)	3% Annual Increase (6)
66	15	.8940	.9536	1.0000	.9208
66	25	.8940	.9298	1.0000	.9208
66	35	.8940	.9196	1.0000	.9208
67	15	.7966	.9028	1.0000	.8444
67	25	.7966	.8603	1.0000	.8444
67	35	.7966	.8421	1.0000	.8444
68	15	.7073	.8488	1.0000	.7710
68	25	.7073	.7922	1.0000	.7710
68	35	.7073	.7680	1.0000	.7710
69	15	.6257	.7925	1.0000	.7008
69	25	.6257	.7258	1.0000	.7008
69	35	.6257	.6972	1.0000	.7008
70	15	.5512	.7349	1.0000	.6339
70	25	.5512	.6614	1.0000	.6339
70	35	.5512	.6300	1.0000	.6339

TABLE 2

RATIO OF (*a*) ACTUARIAL PRESENT VALUE AT AGE 65 OF POSTPONED RETIREMENT BENEFIT TO (*b*) ACTUARIAL PRESENT VALUE AT AGE 65 OF NORMAL RETIREMENT BENEFIT ($=APV\Delta$)

ASSUMED 8 PERCENT INTEREST RATE

Age at Retirement (1)	Service at Age 65 (2)	No Credit (3)	Service Credit (4)	Actuarial Increase (5)	3% Annual Increase (6)
66	15	.8805	.9392	1.0000	.9070
66	25	.8805	.9158	1.0000	.9070
66	35	.8805	.9057	1.0000	.9070
67	15	.7728	.8758	1.0000	.8192
67	25	.7728	.8346	1.0000	.8192
67	35	.7728	.8170	1.0000	.8192
68	15	.6759	.8110	1.0000	.7367
68	25	.6759	.7570	1.0000	.7367
68	35	.6759	.7338	1.0000	.7367
69	15	.5889	.7459	1.0000	.6595
69	25	.5889	.6831	1.0000	.6595
69	35	.5889	.6562	1.0000	.6595
70	15	.5110	.6813	1.0000	.5876
70	25	.5110	.6132	1.0000	.5876
70	35	.5110	.5840	1.0000	.5876

B. *Actuarial Present Value Ratio—Variation by Plan*

There are several types of postretirement benefit adjustments where the impact of delayed retirement will vary according to the type of retirement benefit formula. Such adjustments are

1. Recognition for postretirement salary increases only.
2. Recognition for salary increases and additional service credit.
3. Salary recognition and service credit plus an actuarial adjustment.

Table 3 demonstrates, for three sets of assumptions, the relative change in actuarial present value of benefits when recognition for salary earned after age 65 is extended. Results vary by plan because of the differing role of salary recognition under each of the benefit formulas studied. This method did not seem an appropriate postretirement adjustment for career-pay programs; accordingly, only final-pay programs were analyzed. Since service credits are not involved, the results are independent of length of service at retirement. For final-pay step-rate and offset plans, results are shown for various age 65 salary levels.

Under the final-pay step-rate formula, results at $10,000 are equivalent to the nonintegrated final-pay plan because $10,000 is below the level of 1981 social security–covered compensation. This observation also applies to the other deferred retirement crediting methods examined in this section (Tables 4 and 5). For this reason, results for salaries between $10,000 and $20,000 should be obtained by extrapolation downward of the $20,000 and higher salary results.

It should be noted that it is possible, under certain circumstances, for the $APV\Delta$ factor to fall below the "no credit" factors (in Tables 1 and 2) even though salary beyond age 65 is being recognized in the computation of deferred retirement benefits. Recalling that

$$APV\Delta = \frac{BEN_{65+n}}{BEN_{65}} \frac{N_{65+n}^{(12)}}{N_{65}^{(12)}},$$

we see that the $APV\Delta$ factors reflecting salary recognition could only be less than the "no credit" factors (which are equal to $N_{65+n}^{(12)}/N_{65}^{(12)}$) if $BEN_{65+n} < BEN_{65}$, that is, if the computed accrued benefit actually decreased.

Any decrease in the computation of the accrued benefit would be due to the fact that the social security–covered compensation base had increased in such a manner as to overpower the increase in accrued benefit due *solely* to an increase in salary. This situation is more likely for salaries just in excess of the covered compensation base and for low salary scales

TABLE 3

RATIO OF (*a*) ACTUARIAL PRESENT VALUE AT AGE 65 OF POSTPONED RETIREMENT BENEFIT TO (*b*) ACTUARIAL PRESENT VALUE AT AGE 65 OF NORMAL RETIREMENT BENEFIT ($=APV\Delta$)
SALARY CREDIT ONLY

A. 6 PERCENT INTEREST, 4 PERCENT SALARY SCALE

AGE AT RETIREMENT	SERVICE AT AGE 65	FINAL PAY	STEP RATE				OFFSET				
			\$10,000	\$20,000	\$35,000	\$50,000	\$10,000*	\$10,000†	\$20,000	\$35,000	\$50,000
66	15	.9298	.9298	.9210	.9252	.9267	.8940	.7917	.9147	.9233	.9257
66	25	.9298	.9298	.9210	.9252	.9267	.8940	.7917	.9147	.9233	.9257
66	35	.9298	.9298	.9210	.9252	.9267	.8940	.7917	.9147	.9233	.9257
67	15	.8616	.8616	.8467	.8538	.8563	.7966	.6347	.8368	.8510	.8549
67	25	.8616	.8616	.8467	.8538	.8563	.7966	.6347	.8368	.8510	.8549
67	35	.8616	.8616	.8467	.8538	.8563	.7966	.6347	.8368	.8510	.8549
68	15	.7956	.7956	.7765	.7857	.7889	.7073	.4925	.7625	.7815	.7866
68	25	.7956	.7956	.7765	.7857	.7889	.7073	.4925	.7625	.7815	.7866
68	35	.7956	.7956	.7765	.7857	.7889	.7073	.4925	.7625	.7815	.7866
69	15	.7319	.7319	.7104	.7207	.7244	.6257	.3816	.6937	.7156	.7215
69	25	.7319	.7319	.7104	.7207	.7244	.6257	.3816	.6937	.7156	.7215
69	35	.7319	.7319	.7104	.7207	.7244	.6257	.3816	.6937	.7156	.7215
70	15	.6706	.6706	.6479	.6588	.6626	.5512	.3042	.6306	.6535	.6597
70	25	.6706	.6706	.6479	.6588	.6626	.5512	.3042	.6306	.6535	.6597
70	35	.6706	.6706	.6479	.6588	.6626	.5512	.3042	.6306	.6535	.6597

* Accrued benefit minimum applies.

† Results if accrued benefits were allowed to decrease; for use in interpolation.

TABLE 3—*Continued*

B. 8 PERCENT INTEREST, 4 PERCENT SALARY SCALE

AGE AT RETIREMENT	SERVICE AT AGE 65	FINAL PAY	STEP RATE				OFFSET				
			$10,000	$20,000	$35,000	$50,000	$10,000*	$10,000†	$20,000	$35,000	$50,000
66	15	.9158	.9158	.9071	.9113	.9127	.8805	.7798	.9009	.9094	.9117
66	25	.9158	.9158	.9071	.9113	.9127	.8805	.7798	.9009	.9094	.9117
66	35	.9158	.9158	.9071	.9113	.9127	.8805	.7798	.9009	.9094	.9117
67	15	.8359	.8359	.8214	.8283	.8308	.7728	.6158	.8118	.8256	.8293
67	25	.8359	.8359	.8214	.8283	.8308	.7728	.6158	.8118	.8256	.8293
67	35	.8359	.8359	.8214	.8283	.8308	.7728	.6158	.8118	.8256	.8293
68	15	.7603	.7603	.7420	.7507	.7538	.6759	.4706	.7286	.7467	.7516
68	25	.7603	.7603	.7420	.7507	.7538	.6759	.4706	.7286	.7467	.7516
68	35	.7603	.7603	.7420	.7507	.7538	.6759	.4706	.7286	.7467	.7516
69	15	.6889	.6889	.6686	.6783	.6817	.5889	.3591	.6528	.6735	.6791
69	25	.6889	.6889	.6686	.6783	.6817	.5889	.3591	.6528	.6735	.6791
69	35	.6889	.6889	.6686	.6783	.6817	.5889	.3591	.6528	.6735	.6791
70	15	.6217	.6217	.6006	.6107	.6143	.5110	.2820	.5846	.6058	.6116
70	25	.6217	.6217	.6006	.6107	.6143	.5110	.2820	.5846	.6058	.6116
70	35	.6217	.6217	.6006	.6107	.6143	.5110	.2820	.5846	.6058	.6116

* Accrued benefit minimum applies.

† Results if accrued benefits were allowed to decrease; for use in interpolation.

TABLE 3—*Continued*

C. 8 PERCENT INTEREST, 7 PERCENT SALARY SCALE

AGE AT RETIREMENT	SERVICE AT AGE 65	FINAL PAY	STEP RATE				OFFSET			
			$10,000	$20,000	$35,000	$50,000	$10,000	$20,000	$35,000	$50,000
66	15	.9422	.9422	.9393	.9407	.9412	1.0346	.9486	.9448	.9439
66	25	.9422	.9422	.9393	.9407	.9412	1.0346	.9486	.9448	.9439
66	35	.9422	.9422	.9393	.9407	.9412	1.0346	.9486	.9448	.9439
67	15	.8848	.8848	.8803	.8825	.8832	1.0858	.8987	.8906	.8884
67	25	.8848	.8848	.8803	.8825	.8832	1.0858	.8987	.8906	.8884
67	35	.8848	.8848	.8803	.8825	.8832	1.0858	.8987	.8906	.8884
68	15	.8280	.8280	.8229	.8253	.8262	1.1304	.8488	.8367	.8335
68	25	.8280	.8280	.8229	.8253	.8262	1.1304	.8488	.8367	.8335
68	35	.8280	.8280	.8229	.8253	.8262	1.1304	.8488	.8367	.8335
69	15	.7719	.7719	.7670	.7694	.7702	1.1652	.7990	.7832	.7790
69	25	.7719	.7719	.7670	.7694	.7702	1.1652	.7990	.7832	.7790
69	35	.7719	.7719	.7670	.7694	.7702	1.1652	.7990	.7832	.7790
70	15	.7167	.7167	.7126	.7145	.7152	1.2081	.7506	.7308	.7256
70	25	.7167	.7167	.7126	.7145	.7152	1.2081	.7506	.7308	.7256
70	35	.7167	.7167	.7126	.7145	.7152	1.2081	.7506	.7308	.7256

TABLE 4

RATIO OF (a) ACTUARIAL PRESENT VALUE AT AGE 65 OF POSTPONED RETIREMENT BENEFIT TO (b) ACTUARIAL PRESENT VALUE AT AGE 65 OF NORMAL RETIREMENT BENEFIT ($=APV\Delta$)
SALARY AND SERVICE CREDIT

A. 6 PERCENT INTEREST, 4 PERCENT SALARY SCALE

Age at Retirement	Service at Age 65	Final Pay	Career Pay	Step Rate				Offset				
				$10,000	$20,000	$35,000	$50,000	$10,000*	$10,000†	$20,000	$35,000	$50,000
66	15	.9918	.9744	.9918	.9824	.9869	.9885	.8940	.8445	.9757	.9849	.9874
66	25	.9670	.9512	.9670	.9578	.9622	.9637	.8940	.8233	.9513	.9602	.9627
66	35	.9563	.9419	.9563	.9473	.9516	.9532	.8940	.8143	.9408	.9497	.9521
67	15	.9765	.9428	.9765	.9596	.9677	.9705	.7966	.7194	.9484	.9645	.9688
67	25	.9305	.9006	.9305	.9144	.9221	.9249	.7966	.6855	.9038	.9191	.9233
67	35	.9108	.8837	.9108	.8951	.9026	.9053	.7966	.6710	.8846	.8996	.9037
68	15	.9548	.9059	.9548	.9318	.9428	.9467	.7073	.5910	.9150	.9378	.9439
68	25	.8911	.8486	.8911	.8697	.8799	.8836	.7073	.5516	.8540	.8752	.8810
68	35	.8638	.8256	.8638	.8431	.8530	.8565	.7073	.5347	.8279	.8484	.8540
69	15	.9271	.8647	.9271	.8998	.9129	.9175	.6257	.4833	.8786	.9064	.9139
69	25	.8491	.7958	.8491	.8241	.8360	.8402	.6257	.4426	.8047	.8301	.8370
69	35	.8156	.7681	.8156	.7916	.8031	.8071	.6257	.4252	.7729	.7973	.8040
70	15	.8942	.8197	.8942	.8639	.8784	.8835	.5512	.4056	.8408	.8713	.8796
70	25	.8048	.7423	.8048	.7775	.7906	.7952	.5512	.3650	.7567	.7842	.7917
70	35	.7664	.7111	.7664	.7405	.7529	.7573	.5512	.3476	.7207	.7468	.7540

* Accrued benefit minimum applies.

† Results if accrued benefits were allowed to decrease; for use in interpolation.

TABLE 4—*Continued*

B. 8 Percent Interest, 4 Percent Salary Scale

Age at Retirement	Service at Age 65	Final Pay	Career Pay	Step Rate				Offset				
				$10,000	$20,000	$35,000	$50,000	$10,000*	$10,000†	$20,000	$35,000	$50,000
66	15	.9768	.9597	.9768	.9676	.9720	.9736	.8805	.8317	.9610	.9700	.9725
66	25	.9524	.9369	.9524	.9434	.9477	.9492	.8805	.8109	.9369	.9458	.9482
66	35	.9419	.9277	.9419	.9330	.9373	.9388	.8805	.8020	.9266	.9354	.9378
67	15	.9473	.9146	.9473	.9309	.9387	.9415	.7728	.6979	.9201	.9356	.9399
67	25	.9027	.8737	.9027	.8871	.8946	.8972	.7728	.6650	.8768	.8916	.8957
67	35	.8836	.8573	.8836	.8683	.8756	.8782	.7728	.6510	.8582	.8727	.8767
68	15	.9123	.8657	.9123	.8904	.9009	.9046	.6759	.5647	.8743	.8960	.9020
68	25	.8515	.8110	.8515	.8310	.8408	.8443	.6759	.5270	.8160	.8363	.8418
68	35	.8254	.7889	.8254	.8056	.8151	.8184	.6759	.5109	.7911	.8107	.8161
69	15	.8726	.8138	.8726	.8469	.8592	.8635	.5889	.4549	.8269	.8530	.8602
69	25	.7991	.7490	.7991	.7756	.7868	.7908	.5889	.4166	.7573	.7812	.7877
69	35	.7676	.7229	.7676	.7450	.7558	.7596	.5889	.4001	.7275	.7504	.7567
70	15	.8289	.7599	.8289	.8009	.8143	.8190	.5110	.3760	.7794	.8077	.8154
70	25	.7460	.6882	.7460	.7208	.7329	.7371	.5110	.3384	.7015	.7269	.7339
70	35	.7105	.6593	.7105	.6865	.6980	.7020	.5110	.3222	.6681	.6923	.6989

* Accrued benefit minimum applies.

† Results if accrued benefits were allowed to decrease; for use in interpolation.

TABLE 4—*Continued*

C. 8 PERCENT INTEREST, 7 PERCENT SALARY SCALE

AGE AT RETIREMENT	SERVICE AT AGE 65	FINAL PAY	CAREER PAY	STEP RATE				OFFSET			
				$10,000	$20,000	$35,000	$50,000	$10,000	$20,000	$35,000	$50,000
66	15	1.0050	.9772	1.0050	1.0019	1.0034	1.0039	1.1036	1.0118	1.0078	1.0068
66	25	.9799	.9561	.9799	.9768	.9783	.9788	1.0760	.9865	.9826	.9816
66	35	.9691	.9485	.9691	.9661	.9675	.9680	1.0642	.9757	.9718	.9708
67	15	1.0028	9484	1.0028	.9977	1.0001	1.0010	1.2306	1.0185	1.0093	1.0069
67	25	.9556	.9101	.9556	.9508	.9531	.9539	1.1726	.9705	.9618	.9595
67	35	.9353	.8964	.9353	.9306	.9329	.9337	1.1478	.9500	.9415	.9392
68	15	.9936	.9145	.9936	.9875	.9904	.9914	1.3565	1.0186	1.0040	1.0002
68	25	.9273	.8624	.9273	.9216	.9244	.9253	1.2660	.9507	.9371	.9335
68	35	.8989	.8437	.8989	.8934	.8961	.8970	1.2273	.9216	.9084	.9049
69	15	.9777	.8760	.9777	.9716	.9745	.9756	1.4760	1.0121	.9920	.9868
69	25	.8954	.8133	.8954	.8898	.8925	.8934	1.3517	.9269	.9085	.9037
69	35	.8601	.7908	.8601	.8547	.8573	.8582	1.2984	.8903	8727	.8681
70	15	.9556	.8336	.9556	.9501	.9527	.9536	1.6108	1.0008	.9744	.9675
70	25	.8600	.7632	.8600	.8551	.8574	.8583	1.4497	.9007	.8770	.8707
70	35	.8190	.7380	.8190	.8144	.8166	.8174	1.3807	.8578	.8352	.8293

TABLE 5

RATIO OF (*a*) ACTUARIAL PRESENT VALUE AT AGE 65 OF POSTPONED RETIREMENT BENEFIT TO (*b*) ACTUARIAL PRESENT VALUE AT AGE 65 OF NORMAL RETIREMENT BENEFIT ($=APV\Delta$)
SALARY, SERVICE, AND ACTUARIAL INCREASE

A. 6 PERCENT INTEREST, 4 PERCENT SALARY SCALE

Age at Retirement	Service at Age 65	Final Pay	Career Pay	Step Rate				Offset				
				$10,000	$20,000	$35,000	$50,000	$10,000*	$10,000†	$20,000	$35,000	$50,000
66	15	1.1093	1.0899	1.1093	1.0989	1.1039	1.1056	1.0000	.9446	1.0913	1.1016	1.1044
66	25	1.0816	1.0640	1.0816	1.0714	1.0763	1.0780	1.0000	.9210	1.0640	1.0741	1.0768
66	35	1.0697	1.0536	1.0697	1.0596	1.0644	1.0662	1.0000	.9108	1.0524	1.0623	1.0650
67	15	1.2258	1.1835	1.2258	1.2046	1.2147	1.2183	1.0000	.9030	1.1905	1.2107	1.2162
67	25	1.1681	1.1306	1.1681	1.1479	1.1576	1.1610	1.0000	.8605	1.1345	1.1537	1.1590
67	35	1.1434	1.1093	1.1434	1.1236	1.1331	1.1364	1.0000	.8423	1.1105	1.1293	1.1344
68	15	1.3498	1.2808	1.3498	1.3174	1.3329	1.3384	1.0000	.8355	1.2936	1.3258	1.3345
68	25	1.2598	1.1998	1.2598	1.2295	1.2440	1.2492	1.0000	.7798	1.2074	1.2374	1.2456
68	35	1.2213	1.1673	1.2213	1.1919	1.2060	1.2109	1.0000	.7559	1.1704	1.1995	1.2074
69	15	1.4818	1.3820	1.4818	1.4382	1.4591	1.4664	1.0000	.7725	1.4043	1.4486	1.4607
69	25	1.3570	1.2719	1.3570	1.3171	1.3362	1.3429	1.0000	.7074	1.2861	1.3267	1.3377
69	35	1.3036	1.2276	1.3036	1.2652	1.2835	1.2900	1.0000	.6795	1.2354	1.2744	1.2850
70	15	1.6222	1.4871	1.6222	1.5673	1.5936	1.6029	1.0000	.7358	1.5254	1.5807	1.5958
70	25	1.4600	1.3467	1.4600	1.4106	1.4342	1.4426	1.0000	.6622	1.3728	1.4227	1.4362
70	35	1.3905	1.2901	1.3905	1.3434	1.3659	1.3739	1.0000	.6307	1.3074	1.3549	1.3679

* Accrued benefit minimum applies.

† Results if accrued benefits were allowed to decrease; for use in interpolation.

TABLE 5—*Continued*

B. 8 Percent Interest, 4 Percent Salary Scale

Age at Retirement	Service at Age 65	Final Pay	Career Pay	Step Rate				Offset				
				$10,000	$20,000	$35,000	$50,000	$10,000*	$10,000†	$20,000	$35,000	$50,000
66	15	1.1093	1.0899	1.1093	1.0989	1.1039	1.1056	1.0000	.9446	1.0913	1.1016	1.1044
66	25	1.0816	1.0640	1.0816	1.0714	1.0763	1.0780	1.0000	.9210	1.0640	1.0741	1.0768
66	35	1.0697	1.0536	1.0697	1.0596	1.0644	1.0662	1.0000	.9108	1.0524	1.0623	1.0650
67	15	1.2258	1.1835	1.2258	1.2046	1.2147	1.2183	1.0000	.9030	1.1905	1.2107	1.2162
67	25	1.1681	1.1306	1.1681	1.1479	1.1576	1.1610	1.0000	.8605	1.1345	1.1537	1.1590
67	35	1.1434	1.1093	1.1434	1.1236	1.1331	1.1364	1.0000	.8423	1.1105	1.1293	1.1344
68	15	1.3498	1.2808	1.3498	1.3174	1.3329	1.3384	1.0000	.8355	1.2936	1.3258	1.3345
68	25	1.2598	1.1998	1.2598	1.2295	1.2440	1.2492	1.0000	.7798	1.2074	1.2374	1.2456
68	35	1.2213	1.1673	1.2213	1.1919	1.2060	1.2109	1.0000	.7559	1.1704	1.1995	1.2074
69	15	1.4818	1.3820	1.4818	1.4382	1.4591	1.4664	1.0000	.7725	1.4043	1.4486	1.4607
69	25	1.3570	1.2719	1.3570	1.3171	1.3362	1.3429	1.0000	.7074	1.2861	1.3267	1.3377
69	35	1.3036	1.2276	1.3036	1.2652	1.2835	1.2900	1.0000	.6795	1.2354	1.2744	1.2850
70	15	1.6222	1.4871	1.6222	1.5673	1.5936	1.6029	1.0000	.7358	1.5254	1.5807	1.5958
70	25	1.4600	1.3467	1.4600	1.4106	1.4342	1.4426	1.0000	.6622	1.3728	1.4227	1.4362
70	35	1.3905	1.2901	1.3905	1.3434	1.3659	1.3739	1.0000	.6307	1.3074	1.3549	1.3679

* Accrued benefit minimum applies.

† Results if accrued benefits were allowed to decrease; for use in interpolation.

TABLE 5—*Continued*

C. 8 PERCENT INTEREST, 7 PERCENT SALARY SCALE

AGE AT RETIREMENT	SERVICE AT AGE 65	FINAL PAY	CAREER PAY	STEP RATE				OFFSET			
				$10,000	$20,000	$35,000	$50,000	$10,000	$20,000	$35,000	$50,000
66	15	1.1413	1.1098	1.1413	1.1378	1.1395	1.1401	1.2533	1.1491	1.1446	1.1434
66	25	1.1128	1.0859	1.1128	1.1093	1.1110	1.1116	1.2220	1.1203	1.1159	1.1148
66	35	1.1006	1.0772	1.1006	1.0971	1.0988	1.0994	1.2085	1.1080	1.1037	1.1025
67	15	1.2976	1.2272	1.2976	1.2910	1.2942	1.2953	1.5923	1.3179	1.3060	1.3029
67	25	1.2365	1.1777	1.2365	1.2303	1.2333	1.2343	1.5174	1.2559	1.2446	1.2416
67	35	1.2103	1.1599	1.2103	1.2042	1.2072	1.2082	1.4853	1.2293	1.2182	1.2153
68	15	1.4701	1.3530	1.4701	1.4610	1.4654	1.4669	2.0070	1.5071	1.4855	1.4798
68	25	1.3720	1.2759	1.3720	1.3636	1.3677	1.3691	1.8732	1.4066	1.3865	1.3812
68	35	1.3300	1.2483	1.3300	1.3219	1.3258	1.3272	1.8159	1.3636	1.3440	1.3389
69	15	1.6603	1.4875	1.6603	1.6499	1.6550	1.6567	2.5065	1.7187	1.6847	1.6757
69	25	1.5205	1.3810	1.5205	1.5110	1.5156	1.5172	2.2954	1.5740	1.5428	1.5346
69	35	1.4606	1.3428	1.4606	1.4515	1.4559	1.4574	2.2050	1.5120	1.4820	1.4741
70	15	1.8701	1.6313	1.8701	1.8594	1.8645	1.8663	3.1524	1.9586	1.9070	1.8934
70	25	1.6831	1.4935	1.6831	1.6734	1.6781	1.6797	2.8371	1.7627	1.7163	1.7041
70	35	1.6029	1.4442	1.6029	1.5937	1.5982	1.5997	2.7020	1.6788	1.6346	1.6229

(or when the salary scale is age-related and thus typically lower at higher attained ages than at younger ages). Its likelihood increases as the proportion of the total benefit provided via the "excess" portion of an integrated plan increases for a given salary level.

Accrued benefits do not decrease under the formulas and assumptions adopted in this paper; however, in the event that accrued benefits were to decrease, the "no credit" ratios should be used as minimum values for computation of changes in actuarial present values.

It should be noted from Table 3 that, as pay levels increase, the ratios of actuarial present value increase toward the level of the nonintegrated ratios. This would be true regardless of the particular step-rate formula in effect. To analyze the ratios for the offset plan, it is useful to review first the effect of postponed retirement on the social security benefit calculation. For each year of deferral, another year of salary is considered in computing average indexed monthly earnings, and the benefit is increased 3 percent to recognize its deferral. The result is an annual increase in benefits of between 5 and 6 percent, and this increase is not significantly different if 4 percent or 7 percent is used as the salary scale. Thus, with a 4 percent salary scale, at $10,000 annual salary the actual social security benefit rises more rapidly than the gross benefit under the offset formula, and the accrued benefit minimum applies. Thus the results at this pay level are equivalent to the "no credit" ratios of Tables 1 and 2. At higher pay levels, the leverage of the social security benefit declines, and the *APV* ratios trend toward the ratios of the nonintegrated plan.

Results are also shown for $10,000 salary where the accrued benefit is allowed to decline. These results are for use in interpolating for salaries between $10,000 and $20,000. In doing so, however, remember that the "no credit" ratios are a minimum.

Under a 7 percent salary assumption the leverage is reversed, with the gross benefit rising more rapidly than the social security benefit, producing large ratios at $10,000. Again, leverage declines as salaries increase, with the ratios trending toward the nonintegrated plan ratios.

Table 4 illustrates the granting of both salary recognition and service credits. A career-pay formula has been added. As described above, the step-rate table at $10,000 is equivalent to the final-pay column, and the accrued benefit minimum still applies to the $10,000 offset column at a 4 percent salary scale. The other columns are easily generated from Table 3 by multiplying the ratios in that table by the ratio of service at actual retirement to service at normal retirement. The career-pay ratios are uniformly lower than the final-pay ratios because the averaging period under the final-pay program shifts each year, upgrading the entire benefit. Under the career-pay plan, the salary recognition only affects the accrual for the years of service after age 65.

Table 5 illustrates salary recognition and service credit plus an actuarial increase in benefits and may be generated by dividing the ratios in Table 4 by the appropriate ratios given in Tables 1 and 2 representing no postponed retirement benefit increases. There is a minor technical problem in this point of view, since all benefits are being increased based on deferral from age 65. An alternative treatment would be actuarial adjustment of benefit accruals after age 65 according to the year in which they are earned. Such treatment would cause the factors in Table 5 to vary according to the relative size of the pre-65 and post-65 benefit accruals, and would require significantly more illustration than the approach shown here.

Note that Tables 5A and 5B are identical. This is because the actuarial increase factor is the inverse of the second term in the definition of $APV\Delta$ in Section II, D, 1 above. Thus the interest rate has no effect on $APV\Delta$ when there is an actuarial increase for deferred retirement.

In general, the tables in this section illustrate that results under integrated plans are highly dependent on the treatment of social security for persons retiring after age 65, especially at lower pay levels. Alternative treatments to the methods examined here would include freezing the social security benefit or covered compensation level at normal retirement age or projecting the social security benefit to actual retirement without recognizing the 3 percent benefit deferral adjustment.

C. *Ratios of Entry-Age Normal Cost*

As discussed earlier, the ratio of entry-age normal costs ($EAN\Delta$) can be expressed as follows:

$$EAN\Delta = APV\Delta \frac{{}^sN_e - {}^sN_{65}}{{}^sN_e - {}^sN_{65+n}} .$$

It should be noted that $APV\Delta$ is independent of attained age, and thus $EAN\Delta$ is also independent of attained age.

The second term in the foregoing formula for $EAN\Delta$ is independent of the type of benefit increases earned for post-65 employment; it depends only on the underlying actuarial assumptions. Thus, once the factor

$$\frac{{}^sN_e - {}^sN_{65}}{{}^sN_e - {}^sN_{65+n}}$$

is calculated for a given combination of entry age and retirement age, that factor can be applied to the value of $APV\Delta$ to derive $EAN\Delta$ for all forms of post-65 accruals.

Factors of $({}^sN_e - {}^sN_{65})/({}^sN_e - {}^sN_{65+n})$ have been developed on the basis of the actuarial assumptions used in this paper and are shown in Table 6.

All tabular results in this paper have been developed without a preretirement turnover assumption. Since in most practical applications there will be a turnover scale involved, it is useful to consider the effect that adding a turnover scale has on the results. There is, of course, no effect on the $APV\Delta$ factor as a result of adding or changing a turnover scale, since

$$APV\Delta = \frac{BEN_{65+n}}{BEN_{65}} \frac{N_{65+n}^{(12)}}{N_{65}^{(12)}},$$

and none of the above items are affected by the addition of a turnover assumption. Thus, since

$$EAN\Delta = APV\Delta \frac{{}^sN_e - {}^sN_{65}}{{}^sN_e - {}^sN_{65+n}},$$

TABLE 6

ENTRY-AGE NORMAL COST FACTOR

$({}^sN_e - {}^sN_{65})/({}^sN_e - {}^sN_{65+n})$

(Assumptions Include UP-1984 Mortality Table with Age Set Forward One Year for Males)

Age at Retirement	Service at Age 65	Interest Rate/Salary Scale		
		6%/4%	8%/4%	8%/7%
66	15	.9523	.9592	.9484
66	25	.9746	.9807	.9710
66	35	.9839	.9892	.9806
67	15	.9107	.9238	.9033
67	25	.9514	.9631	.9445
67	35	.9690	.9792	.9623
68	15	.8743	.8929	.8637
68	25	.9304	.9472	.9201
68	35	.9552	.9700	.9456
69	15	.8423	.8659	.8288
69	25	.9111	.9329	.8979
69	35	.9423	.9615	.9299
70	15	.8139	.8421	.7980
70	25	.8937	.9200	.8779
70	35	.9305	.9538	.9153

we can determine the effect of the turnover assumption by studying the effect on the second term,

$$\frac{{}^sN_e - {}^sN_{65}}{{}^sN_e - {}^sN_{65+n}} .$$

The table below sets out this ratio for retirement age 68 and various entry ages under the 8 percent interest/4 percent salary scale assumption.

Entry Age	No Turnover	Light Turnover	Medium Turnover	Heavy Turnover
30	.970	.973	.977	.984
40	.946	.948	.950	.954
50	.893	.893	.893	.893

Each of the turnover scales used was an aggregate scale with no turnover assumption past age 50. Thus, the ratio was constant for entry age 50.

At entry age 30, the change in normal cost (which would be measured by 1 minus the ratio shown) doubles when changing from heavy turnover to no turnover. However, the effect on overall plan cost is small. These results accord with those for other interest/salary scale combinations: turnover has relatively little impact on the factors. Therefore, the tables in this report appear appropriate for estimation of $EAN\Delta$ even where a turnover scale is present.

D. *Determination of Effect on Projected Unit Credit Normal Costs*

As seen earlier, in Section II, D, 3,

$$PUC\Delta = APV\Delta \frac{65 - e}{65 + n - e} ;$$

that is, the ratio of projected unit credit costs is a simple linear function of the ratio of the actuarial present value of benefits, regardless of assumptions, type of plan, or type of adjustment for deferred retirement past age 65. From this linear relationship, it is also clear that if, under the projected unit credit method, only credited service is used for normal cost proration purposes, and if no service credit is granted for benefit purposes after age 65, then $PUC\Delta = APV\Delta$. More typically, however, all service would be considered for proration purposes, so that the equality $PUC\Delta = APV\Delta$ would not hold. Table 7 provides the factors to be applied to the appropriate value of $APV\Delta$ to obtain $PUC\Delta$.

TABLE 7

PROJECTED UNIT CREDIT NORMAL COST FACTOR

$(65-e)/(65+n-e)$

Age at Retirement	Service at Age 65	Factor	Age at Retirement	Service at Age 65	Factor
66	15	.9375	69	15	.7895
66	25	.9615	69	25	.8621
66	35	.9722	69	35	.8974
67	15	.8824	70	15	.7500
67	25	.9259	70	25	.8333
67	35	.9459	70	35	.8750
68	15	.8333			
68	25	.8929			
68	35	.9211			

E. *Adjustments to Tabular Results for Female Mortality*

Thus far, tabular results have reflected male mortality only (based upon the UP-1984 Table with ages set forward one year). The effect of reflecting female mortality in computing the actuarial present value ratio ($APV\Delta$) and the entry-age normal cost ratio ($EAN\Delta$) must also be considered.

1. ACTUARIAL PRESENT VALUE RATIO

A simple multiplicative adjustment to $APV\Delta$ for males will provide $APV\Delta$ based upon female mortality. The adjustment is given by

$$\frac{{}^{f}N^{(12)}_{65+n}}{{}^{f}N^{(12)}_{65}} \div \frac{{}^{m}N^{(12)}_{65+n}}{{}^{m}N^{(12)}_{65}},$$

where the superscripts m and f simply stand for male and female, respectively. Using the UP-1984 Table set back four years for females, the adjustment factors shown in the table below are obtained.

Age	66	67	68	69	70
Adjustment factor	1.014	1.029	1.045	1.063	1.082

It should be noted that these adjustment factors apply under either a 6 percent interest assumption or an 8 percent interest assumption. When rounded to three decimals (as above), the factors are identical.

2. ENTRY-AGE NORMAL COST RATIO

As noted earlier,

$$EAN\Delta = APV\Delta \frac{{}^{s}N_{e} - {}^{s}N_{65}}{{}^{s}N_{e} - {}^{s}N_{65+n}} .$$

Since the effect of female mortality on $APV\Delta$ has already been analyzed, one need only study the effect that changing to female mortality has on the second term of this equation. As might be expected, the change from male mortality (UP-1984 set forward one year) to female mortality (UP-1984 set back four years) has only a minimal effect on this term. Thus, the table of factors found in Section III, C, "Entry-Age Normal Cost Factor" (Table 6) can be used for computing $EAN\Delta$ for females, once the value of the actuarial present value ration, $APV\Delta$, have been adjusted.

IV. EXAMPLES

The following examples illustrate how the tables in this paper might be used in a practical way. The first example illustrates the simpler approach of assuming 100 percent deferred retirements at a single retirement age. Example 2 illustrates the use of an assumed new distribution of retirement ages.

EXAMPLE 1

Plan type: Final pay, nonintegrated.
Post-65 adjustment: Salary credit only.
Demographics: Average entry age, 30; average attained age, 40.
Actuarial cost method: Aggregate.
Actuarial assumptions: 8 percent interest, 7 percent salary; UP-1984 mortality.
Old plan: All retirements at age 65.
New plan: All retirements at age 68.

OLD-PLAN VALUATION

1. Actuarial present value of benefits:	
Actives	\$10,000,000
Inactives	1,000,000
Total	\$11,000,000
2. Valuation assets	\$ 5,000,000
3. Actuarial present value of future contributions	\$ 6,000,000
4. Actuarial present value of future pay	\$40,000,000
5. Accrual rate	15.00%
6. Current compensation	\$ 1,000,000
7. Annual contribution (normal cost)	\$ 150,000

ANALYSIS OF PLAN CHANGES

1. *Actuarial Present Value of Benefits for Actives*

$$\text{New } APV \text{ actives} = \text{Old } APV \text{ actives} \times APV\Delta$$
$$= \$10{,}000{,}000 \times 0.8280 = \$8{,}280{,}000 .$$

Note that $APV\Delta$ comes from Table 3, C (Final-Pay column).

2. *Actuarial Present Value of Future Pay*

$$\text{New actuarial present value of future pay}$$
$$= \text{Old actuarial present value of future pay}$$
$$\times ({}^sN_e - {}^sN_{65+n})/({}^sN_e - {}^sN_{65})$$
$$= \$40{,}000{,}000 \times (1/0.9201) = \$43{,}473{,}535 .$$

The factor for adjusting the actuarial present value of future pay is obtained from Table 6. Note that it is the reciprocal of the figure in Table 6 and that the table is entered based on *attained* age.

The new-plan valuation can now be completed.

NEW-PLAN VALUATION

1. Actuarial present value of benefits:	
Actives	$ 8,280,000
Inactives	1,000,000
Total	$ 9,280,000
2. Valuation assets	$ 5,000,000
3. Actuarial present value of future contributions	$ 4,280,000
4. Actuarial present value of future pay	$43,473,535
5. Accrual rate	9.85%
6. Current compensation	$ 1,000,000
7. Annual contribution (normal cost)	$ 98,500

EXAMPLE 2

Plan type: Final pay, step rate.

Post-65 adjustment: Salary recognition and service credit.

Demographics: Average entry age, 30; average attained age, 40; average salary, $35,000.

Actuarial cost method: Projected unit credit; twenty-year funding of unfunded accrued liability.

Actuarial assumptions: 6 percent interest, 4 percent salary; UP-1984 mortality.

Old plan: One-third of actuarial present value of benefits from retirements at ages 60–64; two-thirds of actuarial present value of benefits from retirements at age 65.

New plan: Early retirements unaffected. Previous age 65 retirements now have retirement distribution as follows:

Age	65	66	67	68	69	70
Percentage	20	20	15	15	10	20

OLD-PLAN VALUATION

1. Accrued liability:
   Actives \$20,000,000
   Inactives 1,000,000
   Total \$21,000,000
2. Valuation assets \$10,000,000
3. Unfunded accrued liability \$11,000,000
4. Normal cost .... \$ 3,000,000
5. Annual contribution (normal cost + twenty-year amortization payment) .... \$ 3,904,745

ANALYSIS OF PLAN CHANGES

1. *Accrued Liability for Actives*

This item must be split into the part affected by the plan change and the part not affected. Thus

$$\begin{aligned}&\text{New accrued liability}\\ &\quad= (\text{Old accrued liability} \times \text{Percent unaffected})\\ &\qquad+ (\text{Old accrued liability} \times \text{Percent affected} \times \text{Change factors})\ .\end{aligned}$$

Note that the change factors come from Table 4A (\$35,000 Step-Rate column) and that the table is entered based on entry age. Thus

$$\begin{aligned}&\text{New accrued liability}\\ &\quad= (\$20{,}000{,}000 \times 1/3) + (\$20{,}000{,}000 \times 2/3)\\ &\qquad\times [(0.2 \times 1.0) + (0.2 \times 0.9516 \times 0.9722) + (0.15 \times 0.9026 \times 0.9459)\\ &\qquad\quad+ (0.15 \times 0.8530 \times 0.9211) + (0.10 \times 0.8031 \times 0.8974)\\ &\qquad\quad+ (0.2 \times 0.7529 \times 0.8750)]\\ &\quad= \$17{,}797{,}026\ .\end{aligned}$$

2. *Normal Cost*

Similarly, this item must be split into the part affected by the plan change and the part not affected. The part affected is multiplied by $PUC\Delta$, which is merely the product of $APV\Delta$ (Table 4A) and the spread factors from Table 7. Thus

$$
\begin{aligned}
&\text{New normal cost}\\
&= (\$3{,}000{,}000 \times 1/3) + (\$3{,}000{,}000 \times 2/3)\\
&\quad \times [(0.2 \times 1.0) + (0.2 \times 0.9516 \times 0.9722) + (0.15 \times 0.9026 \times 0.9459)\\
&\quad + (0.15 \times 0.8530 \times 0.9211) + (0.10 \times 0.8031 \times 0.8974)\\
&\quad + (0.2 \times 0.7529 \times 0.8750)]\\
&= \$2{,}669{,}554\ .
\end{aligned}
$$

The new-plan valuation can now be completed.

NEW-PLAN VALUATION

1. Accrued liability:	
Actives	$17,797,026
Inactives	1,000,000
Total	$18,797,026
2. Valuation assets	$10,000,000
3. Unfunded accrued liability	$ 8,797,026
4. Normal cost	$ 2,669,554
5. Annual contribution (normal cost + twenty-year amortization payment)	$ 3,393,106

V. SUMMARY AND CONCLUSIONS

The primary objective of this paper was to analyze the effect on pension cost of various approaches to adjusting benefit accruals after age 65. Also, it was intended that such analysis would be of particular use to actuaries in estimating pension cost changes. The examples provided in Section IV show how such changes can be computed from the study results. As indicated earlier, all of the common forms of defined benefit delivery were analyzed along with different interest rate and salary-scale assumptions and varying terms of service at age 65. In studying the effect on cost, three parameters were reviewed—$APV\Delta$, $EAN\Delta$, and $PUC\Delta$, all defined earlier in this paper. In addition to the tabular results presented, the more significant findings of this paper and some cautions in its use are summarized in the following paragraphs.

1. All estimates of the effect on pension cost of adjusting benefit accruals after age 65 can be computed or derived from $APV\Delta$.

2. $APV\Delta$ is shown for each age between 66 and 70 and for three different terms of service at age 65. If retirement rates after age 65 are assumed in the valuation, as opposed to an average retirement age, the average $APV\Delta$ must be determined on the basis of the tabular $APV\Delta$ at each age (66–70) by weighting such $APV\Delta$ by the proportion of persons assumed to continue working beyond age 65 and who will be retiring at each age (66–70).

3. $PUC\Delta$ is a simple linear function of $APV\Delta$.

4. $EAN\Delta$ is also a function of $APV\Delta$. Such relationship depends on the ratio of the temporary annuity from entry age to age 65 to the temporary annuity from entry age to retirement age after 65, that is,

$$\frac{{}^sN_e - {}^sN_{65}}{{}^sN_e - {}^sN_{65+n}} .$$

In general, this ratio will be only minimally affected by the level of preretirement turnover.

5. In estimating the effect on pension cost of adjusting benefit accruals after age 65, the change in the actuarial present value of benefits must be computed on the basis of the actuarial present value of benefits for retirement at age 65; that is, the total actuarial present value of retirement benefits must be split between benefits becoming payable prior to 65 and those becoming payable at 65. Only the latter actuarial present value should be adjusted by $APV\Delta$ in computing the cost impact.

6. Introduction of an adjustment to benefits becoming payable after age 65 will, in itself, most probably affect retirement patterns. Expected changes in retirement rates should be taken into consideration in estimating the proportion of actuarial present value of retirement benefits becoming payable before age 65 and at age 65 and later.

7. The tabular $APV\Delta$'s for "no adjustment," "service credit only," "actuarial increase," or "percentage increase" are independent of the type of plan.

8. For other adjustment types analyzed, that is, "salary recognition only," "salary recognition and service credit," and "salary, service, and actuarial increase," the value of $APV\Delta$ does vary by plan type.

9. $APV\Delta$ is presented in this paper reflecting male mortality only. $APV\Delta$ reflecting female mortality can be derived from the male $APV\Delta$ by using a multiplicative adjustment factor.